Richard G. Anderson is an assistant vice president at the Federal Reserve Bank of St. Louis. Barry E. Jones and Travis D. Nesmith are Ph.D. candidates at Washington University in St. Louis and visiting scholars at the Federal Reserve Bank of St. Louis. Mary C. Lohmann, Kelly M. Morris, and Cindy A. Gleit provided research assistance.

Building New Monetary Services Indexes: Concepts, Data and Methods

Richard G. Anderson, Barry E. Jones, and Travis D. Nesmith

This is the second of two articles that describe the monetary services index (MSI) project of the Federal Reserve Bank of St. Louis. The project's MSI database, which contains the monetary services index (MSI), its dual user cost index, and other related indexes and data, is available on the Bank's World Wide Web server.[1] To facilitate comparison with the monetary aggregates published by the Board of Governors of the Federal Reserve System, all of the indexes in the MSI database are provided for the same groupings of monetary assets as the Board's M1, M2, M3, and L aggregates.[2] Indexes are provided at monthly, quarterly, and annual frequencies. The St. Louis MSI database also contains all non-confidential data and computer programs used to construct the indexes.

Unlike the Board of Governor's monetary aggregates, the monetary services indexes and their dual user cost indexes are statistical index numbers, based on economic aggregation and statistical index number theory. The previous article in this *Review*, "Monetary Aggregation Theory and Statistical Index Numbers," surveys the literature on monetary aggregation theory and the use of statistical index number theory in monetary economics. Here, we discuss the construction of the monetary services index and related indexes.

In the first section, we define notation and introduce some key concepts that are used throughout the article. We emphasize the distinction between real and nominal monetary asset stocks and their user costs, and we review the concepts of the real monetary services index and its nominal dual user cost index. In the second section, we define each of the indexes in the monetary services indexes database, including the following: total expenditure on monetary assets; the nominal monetary services index; the real dual user cost index; the currency equivalent index; the simple sum index; and a set of indexes based on Theil's (1967) stochastic approach to index number theory. We emphasize that it is important to distinguish between real and nominal monetary index numbers: The aggregation theory underlying the monetary services indexes and related indexes is developed in terms of the real stocks of monetary assets, but actual monetary asset stock data are collected in *nominal* terms. We conclude that it is appropriate to construct a nominal monetary services index and thereafter to produce an approximation to the real monetary services index by deflating the nominal index.

In the third section, we describe the monetary asset stock data. We discuss the issue of weak separability, and we define the groupings of monetary assets for which we construct indexes. These groupings correspond to the assets contained in M1, M2, M3, and L, as well as the assets contained in M1A and MZM.[3] Because the aggregates are nested—each broader aggregate contains all the components of the previous, narrower aggregate—we refer to the groupings as *levels of aggregation*. M1A is the narrowest level of aggregation and L the broadest.

In the fourth section, we discuss the own rate of return data used in the

We offer special thanks to Professor William A. Barnett, Washington University in St. Louis, for his generous and invaluable guidance during this project. We also thank Professors W. Erwin Diewert, University of British Columbia, and Adrian R. Fleissig, St. Louis University, for advice and comments. For assistance with data, we thank the Savings Bond Operations Office of the U.S. Department of the Treasury and the Division of Monetary Affairs of the Board of Governors of the Federal Reserve System.

[1] The address is www.stls.frb.org/research.

[2] The Board of Governors' monetary aggregates are published weekly in the statistical release, *Money Stock, Liquid Assets, and Debt Measures*, and monthly in the *Federal Reserve Bulletin*, Table 1.21.

[3] Simple sum M1A (non-interest bearing M1) was produced as an official monetary aggregate from 1960 through April 1971 (Kavajecz, 1994). MZM was suggested by William Poole.

construction of the indexes, and we detail the sources of this data. Of special importance are the methods by which we construct own rates for particular monetary assets. Specific issues include the implicit rate of return on demand deposits, fixed and variable ceiling rates for rate-regulated monetary assets, and the market rate of return on savings bonds. When the sample period of the own-rate data is shorter than that of the associated asset-stock data, we construct proxies for the missing own-rate data. Finally, we review the own rate conversions and yield curve adjustments of particular rates that are necessary because not all own rates are reported on the same basis or for the same maturity.

In the fifth section, we detail the calculation of monetary asset user costs. Some published monetary asset stock data are, in fact, aggregates, or sub-indexes, of individual monetary assets with different user costs. It is necessary to obtain a single user cost index for these sub-indexes. Our solution to this problem, derived from unilateral index number theory, is described in this section. Finally, we discuss the concept of a benchmark asset and detail how we construct its rate of return.

The sixth section contains a discussion of some methodological difficulties associated with the project. The first is the introduction of new monetary assets; we implement Diewert's (1980) recommended solution to this problem. The second difficulty is created when published data for several monetary assets are combined into data for a single aggregate, or sub-index. We argue that it is inappropriate to treat this sub-index as a new monetary asset because doing so imputes economic relevance to the change in data reporting, when in fact there is none. We propose a solution to this problem that is based on the theory of splicing index numbers. The third issue is time aggregation. We use monthly data to construct the indexes. We implement Diewert's (1980) time aggregation methodology to produce quarterly and annual indexes from monthly data. Finally, we discuss seasonal adjustment.

NOTATION AND KEY CONCEPTS

In this section, we introduce notation and define some key concepts that will be used throughout the article. Readers are cautioned that this article's notation differs somewhat from that in "Monetary Aggregation Theory," because here we distinguish between real and nominal monetary assets.

Monetary Asset Stocks and User Costs

Assume that there are n monetary assets. Let m_{it}^{real} denote the optimal real stock of monetary asset i in period t, and let

$$m_t^{real} = (m_{1t}^{real}, \ldots, m_{nt}^{real})$$

be the vector of these optimal real stocks.[4] Similarly, let m_{it}^{nom} denote the optimal nominal stock of monetary asset i in period t, and let

$$m_t^{nom} = (m_{1t}^{nom}, \ldots, m_{nt}^{nom})$$

be the vector of these optimal nominal stocks. Real and nominal holdings of monetary assets are related by the identity

$$m_{it}^{real} = \left(m_{it}^{nom} / p_t^* \right),$$

where p_t^* is a true cost-of-living index.[5]

The *user cost* of an asset is the equivalent rental price of that asset. If an asset fully depreciates during the economic agent's decision period, it is said to be *non-durable* and the asset's user cost equals its market price. If an asset does not fully depreciate within the decision period, it is said to be *durable*, and the appropriate opportunity cost of the durable asset is its user cost. Monetary assets are assumed to be durable. Expressions for the user cost of monetary assets were derived for consumers in Barnett (1978), and for firms in Barnett (1987, 1990).

To define the user costs of monetary assets, we need the concept of a *benchmark asset*—a risk-free asset that can be used

[4] We assume that these stocks were chosen by an optimizing economic agent.

[5] The correct price index to be used depends on the context. We state the appropriate index for the consumer case. Barnett (1987) uses the same index for firms and financial intermediaries, although he notes that this is not strictly correct.

only for intertemporal transfer of wealth and provides no monetary services. Let r_{it} represent the nominal holding period yield on monetary asset *i* in period *t*, and let R_t be the nominal holding period yield on the benchmark asset, called the benchmark rate, in period *t*.[6] The *nominal user cost* of monetary asset *i* in period *t*, π_{it}^{nom}, is equal to the nominal value of interest income foregone by holding a unit of that asset for one period, $p_t^*(R_t - r_{it})$, discounted by $[1/(1+R_t)]$ to reflect the receipt of interest at the end of the period:

$$\pi_{it}^{nom} = p_t^* \left(R_t - r_{it}\right) / \left(1 + R_t\right) .$$

This form of the user cost for a monetary asset is valid for both consumers and firms. Note, however, that consumers and firms often face different market interest rates and prices; hence their user costs will differ.

The *real user cost* of monetary asset *i* in period *t*, π_{it}^{real}, is defined by

$$\pi_{it}^{real} = (R_t - r_{it})/(1+R_t),$$

and the nominal and real user costs are related by the identity

$$\pi_{it}^{real} = \left(\pi_{it}^{nom} / p_t^*\right).$$

The real stock of a durable asset multiplied by its nominal user cost is equal to the total expenditure on that asset. Thus, expenditure on monetary asset *i* in period *t* is given by the product $\pi_{it}^{nom} m_{it}^{real}$, and *total expenditure on monetary assets* in period *t* is given by

$$y_t = \sum_{i=1}^{n} \pi_{it}^{nom} m_{it}^{real} .$$

The Monetary Services Index and the Dual User Cost

Barnett (1980, 1987, 1990) derived the conditions under which monetary quantity and dual user cost aggregates will exist. These conditions are reviewed in our previous article in this *Review*, "Monetary Aggregation Theory." For consumers and firms, the *monetary quantity aggregate* is a measure of the flow of monetary services received by the holders of the monetary assets. Barnett (1980) first suggested the use of superlative *statistical index numbers* to track the flow of monetary services. Statistical index numbers, which contain no unknown parameters, are specification- and estimation-free functions of the prices and optimal quantities observed in two time periods.

Diewert (1976) showed that there exists a class of statistical index numbers, which he called *superlative*, that can provide second-order approximations to arbitrary economic aggregates in discrete time. Although there are many superlative index numbers, the Törnqvist-Theil index number is the only one known to retain its second-order tracking properties when some common aggregation theoretic assumptions are violated (Caves, Christensen, and Diewert, 1982; Anderson, Jones, and Nesmith, 1997).

Monetary quantity index numbers have been referred to by a variety of names in the past. We label our quantity indexes as *monetary services indexes* because of their close connection to the flow of monetary services in microeconomic demand models. The *real Törnqvist-Theil monetary services index* (chained Törnqvist-Theil quantity index formula), MSI_t^{real}, is defined by

$$MSI_t^{real} = MSI_{t-1}^{real} \prod_{i=1}^{n} \left(\frac{m_{it}^{real}}{m_{i,t-1}^{real}} \right)^{\overline{w}_{it}} ,$$

where $w_{it} = (\pi_{it}^{nom} m_{it}^{real} / y_t)$ and $w_{i,t-1} = (\pi_{i,t-1}^{nom} m_{i,t-1}^{real} / y_{t-1})$ are the expenditure shares of monetary asset *i* in periods *t* and *t*-1, respectively, and the average expenditure share of monetary asset *i* in period *t* is $\overline{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$.[7]

An index number that is dual to MSI_t^{real} can be used to measure the price of a unit of monetary services. A price index number is said to be *dual* to a quantity index number if their product is equal to the total expenditure on the component assets included in the indexes, a property

[6] All holding period yields are assumed to be reported on a common basis. This issue is discussed in the section of this article titled "Own Rate Data."

[7] Because of its connection with Divisia's (1925) continuous time index number, Barnett (1980) referred to this index as the Divisia Index.

called factor reversal. Dual to MSI_t^{real} is the *nominal dual user cost index*, Π_t^{nom}, which is defined using Fisher's (1922) weak factor reversal criterion by the formula

$$\Pi_t^{nom} = \Pi_{t-1}^{nom} \left(\frac{y_t / y_{t-1}}{MSI_t^{real} / MSI_{t-1}^{real}} \right).$$

Our real monetary services index, MSI_t^{real}, and its nominal dual user cost index, Π_t^{nom}, are constructed as chained superlative indexes. They therefore have the same statistical properties as other chained superlative quantity and price indexes—such as real gross domestic product (GDP), real personal consumption expenditures (PCE), and their price deflators—that are currently published by the U.S. Department of Commerce (Triplett, 1992).

INDEXES IN THE MSI DATABASE

In this section, we define the indexes in the MSI database. The formulas and definitions for these indexes are summarized in Table 1.

In this article, we distinguish carefully between nominal and real stocks of monetary assets. Monetary asset data collected by the Federal Reserve are necessarily in nominal terms, while monetary aggregation and statistical index number theory provide conditions for the aggregation of real stocks of monetary assets (Barnett, 1978, 1980, 1987, 1990; and our previous article in this *Review*, "Monetary Aggregation Theory."). Aggregation of nominal, rather than real, stocks of monetary assets requires some extension of the theory.

The identities $m_{it}^{real} = (m_{it}^{nom} / p_t^*)$ and $\pi_{it}^{real} = (\pi_{it}^{nom} / p_t^*)$ can be used to demonstrate that total expenditure on monetary assets, y_t, may be represented in two equivalent ways. Total expenditure may be expressed as either: (1) the sum of the products of the real asset stocks and their nominal user costs, or as (2) the sum of the products of the nominal monetary asset stocks and their corresponding real user costs:

$$\begin{aligned} y_t &= \sum_{i=1}^{n} \pi_{it}^{nom} m_{it}^{real} \\ &= \sum_{i=1}^{n} p_t^* \pi_{it}^{real} \left(m_{it}^{nom} / p_t^* \right) \\ &= \sum_{i=1}^{n} \pi_{it}^{real} m_{it}^{nom} . \end{aligned}$$

This result implies that the expenditure shares do not depend on the price index, p_t^*, and hence can be calculated correctly using observed nominal asset stocks and real user costs. The expenditure shares may be interpreted as either expenditure on real assets based on nominal user costs,

$$\begin{aligned} w_{it} &= \left(\pi_{it}^{nom} m_{it}^{real} \right) / y_t \\ &= (R_s - r_{is}) m_{is}^{real} \Big/ \sum_{j=1}^{n} (R_s - r_{js}) m_{js}^{real} , \end{aligned}$$

or as expenditure on nominal assets based on real user costs,

$$\begin{aligned} w_{it} &= \left(\pi_{it}^{real} m_{it}^{nom} \right) / y_t \\ &= (R_s - r_{is}) m_{is}^{nom} \Big/ \sum_{j=1}^{n} (R_s - r_{js}) m_{js}^{nom} . \end{aligned}$$

These relationships are important because they permit us to measure the total quantity of real monetary services by first constructing a quantity index from the observable nominal monetary asset stocks and then deflating that quantity index.

Specifically, we define the *nominal Törnqvist-Theil monetary services index* (chained Törnqvist-Theil quantity index number formula), MSI_t^{nom}, by

$$MSI_t^{nom} = MSI_{t-1}^{nom} \prod_{i=1}^{n} \left(\frac{m_{it}^{nom}}{m_{i,t-1}^{nom}} \right)^{\overline{w}_{it}} ,$$

where $\overline{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$. Because the

Table 1

Definitions of Monetary Services Indexes, User Costs, and Related Indexes

Index	Formula
A. Monetary Services Indexes and User Costs	
Total expenditure on monetary assets	$y_t = \sum_{i=1}^{n} \pi_{it}^{real}\, m_{it}^{nom}$
Nominal Törnqvist-Theil monetary services index	$MSI_t^{nom} = MSI_{t-1}^{nom} \prod_{i=1}^{n} \left(\frac{m_{it}^{nom}}{m_{i,t-1}^{nom}} \right)^{\bar{w}_{it}}$, where $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$.
Real dual user cost index (This index is dual to the nominal Törnqvist-Theil monetary services index, that is, MSI_t^{nom} and Π_t^{real} satisfy the Fisher weak factor reversal criterion.)	$\Pi_t^{real} = \Pi_{t-1}^{real} \left(\frac{y_t / y_{t-1}}{MSI_t^{nom} / MSI_{t-1}^{nom}} \right)$
Currency equivalent index [a]	$CE_t = \left(\frac{1 + R_t}{R_t} \right) \sum_{i=1}^{n} \pi_{it}^{real}\, m_{it}^{nom}$
Simple sum index	$SS_t = \sum_{i=1}^{n} m_{it}^{nom}$
B. Divisia Second Moments and Related Indexes [b]	
Real Törnqvist-Theil user cost index (This index is not dual to the nominal Törnqvist-Theil monetary services index.)	$UC_t^{real} = UC_{t-1}^{real} \prod_{i=1}^{n} \left(\frac{\pi_{it}^{real}}{\pi_{i,t-1}^{real}} \right)^{\bar{w}_{it}}$
Törnqvist-Theil expenditure share index [c]	$S_t = S_{t-1} \prod_{i=1}^{n} \left(\frac{w_{it}}{w_{i,t-1}} \right)^{\bar{w}_{it}}$
Divisia quantity–growth rate variance	$K_t = \sum_{i=1}^{n} \bar{w}_{it} \left[\Delta \log\left(m_{it}^{nom}\right) - \Delta \log\left(MSI_t^{nom}\right) \right]^2$
Divisia user cost–growth rate variance	$J_t = \sum_{i=1}^{n} \bar{w}_{it} \left[\Delta \log\left(\pi_{it}^{real}\right) - \Delta \log\left(UC_t^{real}\right) \right]^2$
Divisia expenditure–share growth-rate variance	$\Psi_t = \sum_{i=1}^{n} \bar{w}_{it} \left[\Delta \log\left(w_{it}\right) - \Delta \log\left(S_t\right) \right]^2$

continued on next page

Table 1 *continued*	
Covariance of Divisia quantity growth rate and user-cost growth rate[c]	$\Gamma_t = (\Psi_t - K_t - J_t)/2$

[a] The standard formula is

$$CE_t = \sum_{i=1}^{n} m_{it}^{nom} \left(R_t - r_{it}\right)/R_t$$

(Rotemberg, Driscoll, and Poterba, 1995). We use the modified formula shown above for technical reasons that are discussed in the section of this paper titled "Unilateral Sub-Indexes."

[b] For discussions of these indexes, see Theil (1967); Barnett and Serletis (1990); Barnett, Jones, and Nesmith (1996); and Anderson, Jones, and Nesmith (1997).

[c] This covariance also may be written as

$$\Gamma_t = \sum_{i=1}^{n} \overline{w}_{it}\left[\Delta \log\left(\pi_{it}^{real}\right) - \Delta \log\left(UC_t^{real}\right)\right] \times \left[\Delta \log\left(m_{it}^{nom}\right) - \Delta \log\left(MSI_t^{nom}\right)\right]$$

(Theil, 1967).

individual expenditure shares may be interpreted as either nominal or real shares, this formula is simply the usual Törnqvist-Theil quantity index number formula applied to nominal, rather than real, stocks of monetary assets.

Similarly, define the *real dual user cost index*, Π_t^{real}, by

$$\Pi_t^{real} = \Pi_{t-1}^{real}\left(\frac{y_t/y_{t-1}}{MSI_t^{nom}/MSI_{t-1}^{nom}}\right).$$

Because the total expenditure on monetary assets can be defined in terms of nominal asset stocks and real user costs, this real dual user cost index will be dual to MSI_t^{nom}.

To simplify the discussion that follows, we define the log change operator as $\Delta \log(z_t) = \log(z_t) - \log(z_{t-1})$, where all logarithms are base *e*, or natural, logs. Then the real and nominal monetary services indexes, and their real and nominal dual user cost indexes, are related by

$$\Delta \log\left(MSI_t^{nom}/p_t^*\right) = \Delta \log(MSI_t^{real})$$

and

$$\Delta \log(\Pi_t^{real}) = \Delta \log\left(\Pi_t^{nom}/p_t^*\right),$$

respectively.[8] The real monetary services index may be constructed by aggregating over nominal asset stocks to produce the nominal monetary service index and then deflating this index; a similar relationship holds for the nominal and real dual user cost indexes.

The St. Louis MSI database includes the nominal monetary service index, MSI_t^{nom}, and its real dual user cost index, Π_t^{real}. Although the nominal monetary service index may be deflated to produce its real counterpart, we leave the choice of deflator to the user because choice of the appropriate deflator depends on the model being studied by the user. In consumer demand models, the appropriate price index is a measure of the true cost of

[8] In discrete time, these equalities are true up to a third-order error when the true cost of living index is measured by a superlative index number. If a non-superlative price index is used, the equality will be true only up to the tracking ability of the index.

living. In firm factor demand models, the appropriate price index is an index of factor input prices. There is a large set of published price indexes, any one of which may or may not be appropriate in a specific application. These indexes include the Consumer Price Index (CPI), the Producer Price Index (PPI), the GDP deflator, and the PCE deflator. It may also be appropriate to deflate the indexes by using a measure of the real wage rate. Caveat emptor.

In the remainder of this section, we discuss the additional indexes in the MSI database. Although the currency equivalent (CE) index (Rotemberg, Driscoll, and Poterba, 1995; Rotemberg, 1991) and the simple sum monetary aggregates (as published by the Federal Reserve Board) are both inferior to the Törnqvist-Theil monetary services index as measures of the flow of monetary services, they have interpretations as stock concepts. Our previous article in this *Review*, “Monetary Aggregation Theory,” provides a more complete discussion of these concepts.

Barnett (1991) proved that, under certain assumptions, the *currency equivalent index*

$$CE_t = \sum_{i=1}^{n} m_{it}^{nom} \left(R_t - r_{it}\right) \big/ R_t$$

measures the discounted present value of all current and future total expenditures on monetary assets.

Under the same assumptions, the *simple sum index*

$$SS_t = \sum_{i=1}^{n} m_{it}^{nom}$$

equals the sum of the discounted present value of the expected investment yields on current and future holdings of monetary assets, plus the CE index.

Theil (1967) noted that the Törnqvist-Theil price index number is not dual to the Törnqvist-Theil quantity index number—that is, the Törnqvist-Theil index formula is not *self-dual*. We can define the *real Törnqvist-Theil user cost index* (chained Törnqvist-Theil price index formula), UC_t^{real}, as

$$UC_t^{real} = UC_{t-1}^{real} \prod_{i=1}^{n} \left(\frac{\pi_{it}^{real}}{\pi_{i,t-1}^{real}} \right)^{\overline{w}_{it}} .$$

Theil’s (1967) result, applied to monetary indexes, shows that

$$\Delta \log(MSI_t^{nom}) + \Delta \log(UC_t^{real}) = \Delta \log(y_t) + \Delta \log(S_t),$$

where the *Törnqvist-Theil expenditure share index*, S_t, is defined by

$$S_t = S_{t-1} \prod_{i=1}^{n} \left(\frac{w_{it}}{w_{i,t-1}} \right)^{\overline{w}_{it}}$$

Theil (1967) also defined four indexes known as Divisia second moments: the Divisia quantity growth-rate variance, Divisia user-cost growth-rate variance, Divisia expenditure share growth-rate variance, and the covariance between the quantity and user-cost growth rates. Formulas for these indexes are shown in part B of Table 1.

Barnett and Serletis (1990) propose a dispersion dependency test, based on the Divisia second moments, for the failure of the principal assumptions of aggregation theory. The Divisia second moments may, for example, contain significant information during periods of regulatory change. The latter include the phased removal of Regulation Q ceilings on depository institutions’ offering rates between 1978 and 1986 and the introduction of new types of deposits, such as All Savers certificates in 1978 and money market deposit accounts in 1982.

The dispersion dependency tests are applied to U.S. monetary data in Barnett and Serletis (1990) and Barnett, Jones, and Nesmith (1996). These studies suggest that, for at least some time periods, movements in the monetary data are not consistent with the principal assumptions of aggregation theory. In this case, Barnett and Serletis (1990) suggest that including Divisia second moments in macroeconomic models might provide a correction for this aggregation error. For additional discussion, see our previous article in this *Review*, "Monetary Aggregation Theory."

ASSET STOCKS AND AGGREGATES

In this section, we describe our monetary asset stock data in detail, and we discuss the levels of aggregation of the indexes in the MSI database. Discussion of the own rate data is deferred until the following section.

The monetary aggregates published by the Federal Reserve Board—M1, M2, M3, and L—are constructed by summing over sets of monetary asset stocks at four nested levels of aggregation. In addition to these levels of aggregation, some economists have advocated two other levels of aggregation: M1A and MZM. M1A consists of the non-interest-bearing monetary assets in M1, and MZM includes the monetary assets in M2 which do not have a fixed maturity.[9] These levels of aggregation are summarized in Table 2.

Some economists have recently suggested that monetary indexes should contain, in addition, highly liquid capital-uncertain assets such as bond and equity mutual funds (Collins and Edwards, 1994; Orphanides, Reid, and Small, 1994). Although the theoretical procedures used in the construction of the St. Louis MSI database are valid only under the assumption of perfect certainty, extending the theory to include risk-neutral households and firms is straightforward (Barnett, 1994). Extending the theory to include risk-averse agents, however, is more difficult and requires the subtraction of a risk premium from the monetary-asset user costs (Barnett and Liu, 1994; Barnett, Liu, and Jensen, 1997). We leave as a topic for future research the measurement of monetary service indexes that include an allowance for risk aversion.

The St. Louis MSI database contains monetary services indexes constructed over the same sets of assets (levels of aggregation) as the simple sum monetary aggregates M1A, M1, MZM, M2, M3, and L. We do not test for the weak separability of these levels of aggregation, although the correct level of aggregation of monetary assets should be determined by tests for weak separability.[10] Several previous studies have examined the weak separability of the assets included in M1, M2, M3, and L (Serletis, 1987; Swofford and Whitney, 1987, 1988). More recently, Swofford and Whitney (1994) and Spencer (1994) have noted that relaxation of the assumption of continuous complete portfolio adjustment, maintained in derivation of the monetary service index, significantly complicates separability testing. Testing the separability of the included assets is a topic for future research; researchers are encouraged to conduct their own tests using the disaggregated data provided in the database.

The asset stock data used to produce the indexes in the St. Louis MSI database are shown in Table 2. They include both seasonally adjusted and unadjusted data, except for the non-M3 components of L and Super NOW accounts at commercial banks and at thrift institutions, which are included only on an unadjusted basis. Most data were originally published by the Federal Reserve Board and have been later revised by Board staff. For discussion of Federal Reserve monetary aggregates and their components, see Anderson and Kavajecz (1994).

The data in Table 2 are reported at the most disaggregate level feasible.[11] Super NOW accounts have been separated from other checkable deposits from 1983.01–1985.12, the period in which separate data are available. Similarly, savings deposits and money market deposit accounts are separated during 1960.01–1991.08. In addition, the following asset categories are separated into thrift institution and commercial bank

[9] William Poole's MZM included institutional money market mutual funds. We exclude these funds because they do not follow the same accounting rules as retail money market funds, and are marketed only to larger investors.

[10] A weakly separable block could contain both monetary assets and consumption goods, but an aggregate formed over such a block would not usually be interpreted as a monetary service flow.

[11] The criterion for feasibility is that disaggregated data for the desired asset stock must be available and of good quality. In addition, reliable own-rate data for the category must exist.

Table 2

Monetary Asset Stock Data Used in the Monetary Services Index (by level of aggregation)

Monetary Asset	Period Asset is Included in the Index
M1A	
Currency	1960.01–present
Travelers checks	1960.01–present
Demand deposits	1960.01–present
M1 = M1A plus the following: [a]	
Other checkable deposits at commercial banks excluding Super NOW accounts	1974.01–1985.12
Other checkable deposits at thrift institutions excluding Super NOW accounts	1960.01–1985.12
Super NOW accounts at commercial banks	1983.01–1985.12
Super NOW accounts at thrift institutions	1983.01–1985.12
Other checkable deposits at commercial banks including Super NOW accounts	1986.01–present
Other checkable deposits at thrift institutions including Super NOW accounts	1986.01–present
MZM = M1 plus the following: [b]	
Money market deposit accounts at commercial banks	1982.12–1991.08
Money market deposit accounts at thrift institutions	1982.12–1991.08
Savings deposits at commercial banks excluding money market deposit accounts	1960.01–1991.08
Savings deposits at thrift institutions excluding money market deposit accounts	1960.01–1991.08
Savings deposits at commercial banks including money market deposit accounts	1991.09–present
Savings deposits at thrift institutions including money market deposit accounts	1991.09–present
Retail money market mutual funds	1973.02–present
M2 = MZM plus the following:	
Small-denomination time deposits at commercial banks	1960.01–present
Small-denomination time deposits at thrift institutions	1960.01–present
M3 = M2 plus the following:	
Total repurchase agreements [c]	1960.10–present
Total Eurodollar deposits [c]	1960.01–present
Total large-denomination time deposits (at commercial banks and thrift institutions)	1960.01–present
Institutional money market mutual funds	1974.01–present
L = M3 plus the following:	
U.S. savings bonds	1960.01–present
Short-term Treasury securities	1960.01–present
Bankers acceptances	1960.01–present
Commercial paper	1960.01–present

[a] We separate Super NOW accounts from other checkable deposits during the period 1982.12–1985.12. After 1985.12, Super NOW accounts are included in other checkable deposits. We do not seasonally adjust this category.

[b] Money market deposit accounts (MMDA), authorized by the Garn-St.Germain Act of 1982, were first issued in December 1982. Through August 1991, depository institutions reported to the Federal Reserve separate data on savings deposits and MMDA. In September 1991, depository institutions began reporting only the combined total of money market deposit accounts and savings deposits.

[c] This category includes both overnight (1-day) and term (more than 1-day) assets. Federal Reserve monetary aggregate data published through 1995 included overnight repurchase agreements and Eurodollar deposits in M2, and term repurchase agreements and Eurodollar deposits in M3. The M2 monetary aggregate was redefined in February 1996 to exclude repurchase agreements and Eurodollars; all these assets are now included in the non-M2 component of M3 (Whitesell and Collins, 1996).

categories: other checkable deposits, Super NOW accounts, small-denomination time deposits, savings deposits, and money market deposit accounts. The MSI database contains two exceptions to our practice of reporting data at its finest level of disaggregation. The first is that the sum of overnight and term Eurodollar deposits is included as total Eurodollar deposits, and the sum of overnight and term repurchase agreements is included as total repurchase agreements; the second is that large-denomination time deposits are not separated with respect to commercial banks and thrift institutions.[12]

OWN RATES OF MONETARY ASSETS

User costs of monetary assets are constructed from the assets' own rates of return. For most periods beginning October 1983, we base the user costs of deposits at commercial banks and thrift institutions on actual rates paid by depositories. These data have recently been revised and documented by Board staff. Although the Board of Governors has published deposit rate data for periods prior to October 1983, we choose to use the Regulation Q statutory rate ceilings due to (our) uncertainty whether the survey data are representative of rates generally paid.

In this section, we provide a detailed discussion of several aspects of the data, including our procedures for measuring the implicit rate of return on demand deposits and for estimating proxies for missing values, the market interest rates available on U.S. savings bonds, a set of own-rate conversions required prior to calculating user costs, and our yield curve adjustment of the own-rate data.[13] Table 3 lists the own-rate data used to calculate the indexes in the St. Louis MSI database.

The Implicit Rate of Return on Demand Deposits

To construct a user cost for demand deposits, we need to specify its own rate of return. Appropriate measures of this rate have been widely debated among economists because the Banking Acts of 1933 and 1935 forbade banks from paying explicit interest on demand deposits. Regardless, economists recognize that most financial institutions, during at least some recent time periods, have paid implicit interest on demand deposits in the form of free or reduced-cost bank services, or perhaps easier access to credit. Some economists have suggested that such non-price competition has allowed depositories to evade the prohibition of explicit interest on demand deposits. Startz (1979) discusses three competing hypotheses: the "traditional" hypothesis, which maintains that the prohibition on interest paid to demand deposits has been fully effective; the "competitive" hypothesis, which maintains that the prohibition of interest on demand deposits has been completely ineffective; and the modified competitive hypothesis, which maintains that the prohibition was partially effective.

Klein (1974) derived an expression for the fully competitive implicit rate of return on demand deposits. Assuming that banks earn no profit on demand deposits and that banks face perfectly competitive markets, the implicit rate of return for demand deposits is defined by

$$r_A - r_D = MC_D,$$

where r_D is the implicit interest rate on demand deposits, r_A is the interest rate on an alternative assets, and MC_D is the marginal cost of producing demand deposits. Under additional assumptions, Klein shows that this is equivalent to

$$r_D = (1-c)\, r_A,$$

where c is the ratio of reserves to deposits.

Startz (1979) advocates a modified competitive hypothesis. He argues, using functional cost analysis data, that the implicit demand deposit rate has been positive, well below the fully competitive Klein rate, and responsive to market interest rates.[14] Empirical evidence on the various hypotheses has been mixed (see

[12] We combine overnight and term repurchase agreements and Eurodollar deposits because we have no separate, reasonable own rates for their components. We combine large-denomination time deposits at commercial banks and thrift institutions for the same reason.

[13] Additional discussion can be found in Barnett and Spindt (1982), Farr and Johnson (1985), Thornton and Yue (1992), and Belongia (1995).

[14] Other implicit rates of return are discussed by Becker (1975) and Barro and Santomero (1972).

Table 3

Interest Rate Data Used in Calculation of Monetary Asset User Costs

A. Deposit Rates at Commercial Banks and Thrift Institutions [a]

	Sample Period
Super NOW accounts at commercial banks [b]	1983.10–1985.12
Super NOW accounts at thrift institutions [b]	1983.10–1985.12
Money market deposit accounts at commercial banks [b]	1983.10–1991.09
Money market deposit accounts at thrift institutions [b]	1983.10–1991.09
NOW accounts at commercial banks [c]	1986.01–present
NOW accounts at thrift institutions [c]	1986.01–present
Savings deposits and money market deposit accounts at commercial banks [c]	1986.04–present
Savings deposits and money market deposit accounts at thrift institutions [c]	1986.04–present
Small-denomination time deposits at commercial banks, 7- to 91-day maturity [c]	1983.10–present
Small-denomination time deposits at commercial banks, 92- to 182-day maturity [c]	1983.10–present
Small-denomination time deposits at commercial banks, 183-day to 1-year maturity [c]	1983.10–present
Small-denomination time deposits at commercial banks, 1- to 2.5-year maturity [c]	1983.10–present
Small-denomination time deposits at commercial banks, 2.5 years or longer maturity [c]	1983.10–present
Small-denomination time deposits at thrift institutions, 7- to 91-day maturity [c]	1983.10–present
Small-denomination time deposits at thrift institutions, 92- to 182-day maturity [c]	1983.10–present
Small-denomination time deposits at thrift institutions, 183-day to 1-year maturity [c]	1983.10–present
Small-denomination time deposits at thrift institutions, 1- to 2.5-year maturity c	1983.10–present
Small-denomination time deposit rate at thrift institutions, 2.5 years or longer maturity [c]	1983.10–present
Savings deposits excluding MMDA at commercial banks [c j]	1986.04–1991.09
Savings deposits excluding MMDA at thrift institutions [c j]	1986.04–1991.09

B. Rates and Yields on Money and Capital Market Instruments[d]

Overnight repurchase agreements [b j]	1972.02–present
Overnight Eurodollars [e]	1971.01–present
Overnight federal funds	1960.01–present
Commercial paper with 1 month to maturity	1971.04–present
Commercial paper with 3 months to maturity	1971.04–present
Commercial paper with 6 months to maturity	1960.01–present
Negotiable certificates of deposit with 1 month to maturity, secondary market rate	1965.12–present
Negotiable certificates of deposit with 3 months to maturity, secondary market rate	1964.06–present
Negotiable certificates of deposit with 6 months to maturity, secondary market rate	1964.06–present
Term Eurodollars, 1-month maturity	1971.01–present
Term Eurodollars, 3-month maturity [f]	1960.01–present
Term Eurodollars, 6-month maturity [g]	1963.05–present
Treasury bills with 1 month remaining to maturity, secondary market	1968.01–present
Treasury bills with 3 months remaining to maturity, secondary market	1960.01–present
Treasury bills with 6 months remaining to maturity, secondary market	1960.01–present
Constant-maturity yield on Treasury issues with 1 year to maturity	1960.01–present
Constant-maturity yield on Treasury issues with 2 years to maturity	1976.06–present
Constant-maturity yield on Treasury issues with 3 years to maturity	1960.01–present
Constant-maturity yield on Treasury issues with 5 years to maturity	1960.01–present
Treasury bills with 3 months to maturity, new issue, auction average	1960.01–1983.12

continued on next page

Table 3 *continued*

Treasury bills with 6 months to maturity, new issue, auction average	1960.01–1983.12
Treasury bills with 12 months to maturity, new issue, auction average	1960.01–1983.12
Money market mutual funds, average yield on all funds [b j]	1974.06–present
Moody's BAA bond rate	1960.01–present
Series E savings bonds, investment yield to maturity [h]	1960.01–1982.10
Bankers acceptances with 3 months to maturity [i]	1960.01–1972.12
Bankers acceptances with 3 months to maturity	1973.01–present
Bankers acceptances with 6 months to maturity	1976.01–present

Unless noted otherwise, all data were provided by the Board of Governors of the Federal Reserve System.

[a] From "Monthly Survey of Select Deposits" (FR2042) and its predecessors, Board of Governors of the Federal Reserve System. Current data are published in the Board's statistical release, *Money Stock, Liquid Assets, and Debt Measures* (H.6), and in the Federal Reserve Bulletin, Table 1.22. Earlier surveys are discussed by Mahoney (1987) and Farr and Johnson (1985).

[b] Regression-based proxy data for these rates during earlier periods are developed in Table 5.

[c] Regulation Q ceiling rates are used as proxy data for these rates during earlier periods; see Table 6.

[d] Originally published in the Board of Governor's statistical release, *Selected Interest Rates* (H.15). Constant-maturity yields on Treasury securities are calculated by the U.S. Department of the Treasury which statistically fits a smooth curve through the yields on all outstanding Treasury issues.

[e] Originally published in the Board of Governor's statistical release, *Selected Interest and Exchange Rates* (H.13).

[f] For dates from 1960.01 to 1970.12, these data are from Table I.a.1, page 148, of *OECD Financial Statistics* (1976). For dates beginning 1971.01, these data are from the Board of Governor's H.15 statistical release, *Selected Interest Rates*.

[g] For 1963.05 to 1970.12, from Table I.b.1, page 150, of *OECD Financial Statistics* (1976). Beginning 1971.01, these data are from the Board of Governor's H.15 statistical release, *Selected Interest Rates*.

[h] Provided by the Savings Bond Operations Office, Bureau of the Public Debt, U.S. Department of the Treasury. The data are also published, for dates through 1979, in Brennan and Schwartz (1979).

[i] These data are the 90–Day Prime Bankers Acceptance rate, Table 12.5, *Banking and Monetary Statistics*: 1941-1970 (1976).

[j] Unpublished data.

Rush, 1980; Carlson and Frew, 1980; Allen, 1983; and Rossiter and Lee, 1987).

In previous constructions of monetary index numbers, it has been assumed that the prohibition of interest on demand deposits is completely ineffective for business demand deposits and is fully effective for household demand deposits. In the calculation of the implicit rate of return on business demand deposits, the alternative asset was assumed to be commercial paper with one month remaining to maturity. In Farr and Johnson (1985) and Thornton and Yue (1992), the distinction between household and business demand deposits was based on the Federal Reserve's Demand Deposit Ownership Survey (Board of Governors, 1971-1991). Because that survey has been discontinued, we cannot base our indexes on the methods used in previous studies.

We apply the modified competitive hypothesis to all demand deposits. Startz (1979) has argued that the implicit rate of return on demand deposits is between 0.34 and 0.58 times the fully competitive Klein rate, using five-year Treasury notes as the alternative asset. Thus, the implicit rate of return on demand deposits is proxied as

$$r_D = (1-\tau)(r_A)(\alpha),$$

where r_A is the rate on 5-year Treasury notes, τ is (an estimate of) the maximum reserve requirement on demand deposits, and α is between 0.34 and 0.58. In this article, we set α equal to its maximum value of 0.58. This is equivalent to assuming that all demand deposits were issued by large banks, and that the deposit

Table 4

Statutory Maximum Reserve-Requirement Ratios for Transactions Deposits (percentage)

Reserve-Requirement Ratio	Applicable Dates
16.50 [a]	1960.01–1967.12
17.00 [a]	1968.01–1969.03
17.50 [b]	1969.04–1973.06
18.00 [b]	1973.07–1974.11
17.50 [b]	1974.12–1975.01
16.50 [c]	1975.02–1976.12
16.25 [c]	1977.01–1980.10
12.00 [d]	1980.11–1992.03
10.00	1992.04–present

[a] On net demand deposits at reserve city banks, from Table 10.4, *Banking and Monetary Statistics: 1941-1970* (1976).

[b] On net demand deposits over $5 million at reserve city banks, from *Annual Statistical Digest*.

[c] On net demand deposits over $400 million, from *Annual Statistical Digest*.

[d] On net transaction deposits after implementation of the Monetary Control Act, from *Annual Statistical Digest*. Because the act's reserve requirements were phased in, member banks faced a marginal ratio above 12 percent through January 1984, and nonmember institutions faced a ratio below 12 percent through 1987 (1991 in Hawaii). We use 12 percent because no data exist on average effective marginal reserve-requirement ratios during the phase-in period. For discussion, see Anderson and Rasche (1996).

holders regarded deposits at different-size banks as perfect substitutes. Our estimates of τ, the maximum reserve-requirement ratio on demand deposits, are shown in Table 4.

Regression-Based Proxies for Own-Rate Data

For some monetary assets, the asset stock data shown in Table 2 are available for dates before the earliest corresponding own-rate data shown in Table 3. Rather than discard these quantity data, we created proxies for the missing (unrecorded) own-rate data. For each such stock, we regressed the asset's available own-rate data for the later periods on one or more closely related rates and used the predicted values from the regression for earlier periods as proxies for the missing own-rate data. Our proxies, summarized in Table 5, are robust to reasonable alternative regression specifications.

Regulation Q Ceilings as Own-Rate Proxies

The regression method cannot be used for some commercial bank and thrift institution deposits prior to 1986. In these cases, we proxy the missing deposit own rates with the maximum rate that depositories were legally permitted to offer. These fixed and variable ceiling rates are summarized in parts A and B, respectively, of Table 6.

Negotiable order-of-withdrawal (NOW) accounts are checkable deposits currently included in the Federal Reserve's M1 monetary aggregate. Introduced in 1972 by a Massachusetts savings bank, NOW accounts spread rapidly: to all commercial banks and thrift institutions in Massachusetts and New Hampshire in January 1974, to the rest of the New England states in February 1976, to New York in November 1978, to New Jersey in December 1979, and nationwide in December 1980. Rates paid on NOW

Table 5

Regression-Based Proxy Data for Own Rates

Dependent Variable (Y)	Independent Variable (X)	Proxy Period	Estimated Regression $Y = a + bX$ (p-values in parentheses)		
			a	b	Estimation Sample Period
Super NOW account rate at commercial banks	One–month secondary market Treasury bill rate	1982.12–1983.09	2.88 (.000)	0.52 (.000)	1983.10–1985.12
Super NOW account rate at thrift institutions	One–month secondary market Treasury bill rate	1982.12–1983.09	3.67 (.000)	0.44 (.000)	1983.10–1985.12
Rate on money market deposit accounts at commercial banks	One–month secondary market Treasury bill rate	1982.12–1983.09	1.20 (.001)	0.78 (.000)	1983.10–1991.08
Rate on money market deposit accounts at thrift institutions	One–month secondary market Treasury bill rate	1982.12–1983.09	1.44 (.000)	0.80 (.000)	1983.10–1991.08
Overnight repurchase agreement rate	Overnight federal funds rate	1969.10–1972.01	0.25 (.001)	0.92 (.000)	1972.02–1983.12
Rate on money market mutual funds [a]	Overnight federal funds rate	1973.02–1974.05	0.67 (.002)	0.85 (.000)	1974.10–1983.12
One–month secondary market Treasury bill rate	Three–month secondary market Treasury bill rate	1960.01–1967.12	0.06 (.279)	0.95 (.000)	1968.01–1983.12

NOTE: All rates are adjusted to an annualized 1–month yield, on a bond-interest (365-day, coupon-equivalent) basis.

[a] Farr and Johnson (1985) and Thornton and Yue (1992) proxied this series using the large-denomination time deposit rate. Our proxy is statistically superior.

accounts became subject to legal ceilings beginning in January 1974. Ceiling rates also were in effect for savings deposits during most of our sample, through April 1986.

Small-denomination time deposits were subject to either fixed or variable ceiling rates during various periods of our sample; the latter were tied to Treasury market interest rates. We have constructed fixed ceiling-rate series for one-year maturity small-denomination time deposits at both commercial banks and thrift institutions. Because these rates, shown in Table 6, actually applied to deposits with a wide range of maturities, we caution the reader against overly precise interpretations. For large-denomination time deposits, we have assumed that depository institutions' offering rates, for dates since June 1964, have been approximately equal to secondary-market yields on negotiable certificates of deposit. The

Table 6

Regulation Q Maximum Deposit Offering Rates Used as Own Rate Proxy Data

A. Fixed Ceiling Rates

Asset Category	Rate	Effective Dates
NOW Accounts		
NOW accounts at commercial banks	5.00 [a]	1974.01–1980.12
	5.25 [a]	1981.01–1985.12
NOW accounts at thrift institutions	5.00 [a]	1974.01–1980.12
	5.25 [a]	1981.01–1985.12
Savings Deposits		
Savings deposits at commercial banks	3.00 [b]	1960.01–1961.12
	3.50 [b]	1962.01–1964.11
	4.00 [b]	1964.12–1970.01
	4.50 [a]	1970.02–1973.06
	5.00 [a]	1973.07–1979.06
	5.25 [a]	1979.07–1983.12
	5.50 [a]	1984.01–1986.03
Savings deposits at thrift institutions	4.75 [c]	1966.10–1969.12
	5.00 [c]	1970.01–1973.06
	5.25 [c]	1973.07–1979.06
	5.50 [c]	1979.07–1986.03
Time Deposits		
Time deposits at commercial banks, 1-year maturity	3.00 [d]	1960.01–1961.12
	3.50 [d]	1962.01–1963.07
	4.00 [d]	1963.08–1964.11
	4.50 [d]	1964.12–1965.11
	5.50 [e]	1965.12–1966.07
Small-denomination (less than $100,000) time deposits at commercial banks, 1-year maturity	5.50 [e]	1966.08–1966.09
	5.00 [f]	1966.10–1973.06
	5.50 [g]	1973.07–1979.12
	5.75 [g]	1980.01–1983.09
Small-denomination (less than $100,000) time deposits at thrift institutions, 1-year maturity	5.25 [h]	1966.10–1973.06
	5.75 [i]	1973.07–1979.12
	6.00 [i]	1980.01–1983.09

B. Variable Ceiling Rates

Variable Ceiling Rate	Linked Market Rate	Period Introduced
Money market time deposits (money market certificates) at commercial banks	Discount-basis auction-average rate on 6–month Treasury bills	1978.06

continued on next page

Table 6 *continued*

Variable Ceiling Rate	Linked Market Rate	Period Introduced
Money market time deposits (money market certificates) at thrift institutions	Discount-basis auction-average rate on 6–month Treasury bills	1978.06
12–month maturity All Savers certificates	Discount-basis auction-average rate on 1–year Treasury bills	1981.10
7– to 31–day maturity small-denomination time deposits at commercial banks	Discount-basis auction-average rate on 3–month Treasury bills	1982.09
7– to 31–day maturity small-denomination time deposits at thrift institutions	Discount-basis auction-average rate on 3–month Treasury bills	1982.09
91–day maturity small-denomination time deposits at commercial banks	Discount-basis auction-average rate on 3–month Treasury bills	1982.05
91–day maturity small-denomination time deposits at thrift institutions	Discount-basis auction-average rate on 3–month Treasury bills	1982.05

[a] Mahoney et.al. (1987) and *Annual Statistical Digest*, editions for 1970-79 and 1980 through 1986.

[b] On deposits held for less than 1 year, *Banking and Monetary Statistics: 1941- 1970*. After July 20, 1966, ceiling rates on savings deposits did not differ by the length of time the funds were held on deposit.

[c] Mahoney et.al. (1987) and *Annual Statistical Digest*, editions for1970-79 and for 1980 through 1986.

[d] On deposits payable in 6 months to 1 year through July 1966, *Banking and Monetary Statistics*: 1941-1970.

[e] On deposits payable in 6 months to 1 year between December 1965 and July 1966, and on single-maturity deposits with original maturity of 30 days to1 year between August 1966 and September 1966, *Banking and Monetary Statistics: 1941-1970*. Prior to August 1966, ceiling rates on time deposits did not differ by size.

[f] On deposits with original maturity of 30 days to 1 year, *Annual Statistical Digest*, edition for 1970-1979.

[g] On deposits with original maturity of 90 days to 1 year, *Annual Statistical Digest*: 1970-1979.

[h] On single-maturity deposits and all multiple-maturity deposits payable in 90 days to 1 year, *Annual Statistical Digest*, edition for 1970-1979. Thrifts were not subject to deposit rate ceilings prior to October 1966.

[i] On deposits of with original maturity of 90 days to 1 year, *Annual Statistical Digest*, editions for 1970-1979 and 1980 through 1983.

latter are not available in the Federal Reserve's databases for dates prior to June 1964, however, and we have used as a proxy the ceiling rate permitted by the Federal Reserve's Regulation Q on time deposits payable from six months to one year. The Regulation Q ceiling was generally binding during this period (*Federal Reserve Bulletin*, 1963a,b, 1964a,b).

With the introduction of money market certificates in June 1978, some small-denomination time deposits were subject to variable ceiling rates that were tied to market interest rates (part B, Table 6).[15] We constructed these variable ceiling rates for small-denomination time deposits from information contained in various issues of the Federal Reserve Board's *Annual Statistical Digest* covering 1970-82.12 [16]

Market Interest Rate on Savings Bonds

Investment yields to maturity for series-E savings bonds are available for January 1960–October 1982. Starting in November 1982, the Treasury Department issued bonds that paid a variable, market-based interest rate. This market rate is constructed according to the following procedure: The monthly five-year Treasury securities yield is averaged over

[15] Ceilings on small-denomination time deposits with original maturity of three and one-half years or longer were removed in May 1982, and on all other small-denomination time deposits on October 1, 1983.

[16] The linkages between variable ceiling rates and auction-average Treasury rates are discussed in Mahoney (1987) and *Annual Statistical Digest*, editions for 1970-1979 and 1980, 1981, and 1982.

Table 7

Own Rate Adjustments

A. To convert an annualized 1–month holding period yield on a bank interest (360-day) basis to an annualized 1–month holding period yield on a bond interest (365-day) basis:

Adjustment Formula	Own Rates Adjusted
$r^{adj} = r \times \left(\frac{365}{360}\right)$	Eurodollar deposits: 1- month, 3-month, 6-month Certificates of deposit: 1-month, 3-month, 6-month

B. To convert an annual effective yield on a bond interest basis to an annualized 1-month holding period yield on a bond interest basis:

Adjustment Formula	Own Rates Adjusted
$r^{adj} = \left[\left(1+\frac{\left(r/100\right)}{365}\right)^{30} - 1\right] \times \left(\frac{365}{30}\right) \times 100$	NOW accounts: thrift institutions, commercial banks Super NOW accounts: thrift institutions, commercial banks Small-denomination time deposits at commercial banks and thrift institutions: 7-day to 91-day 92-day to 182-day 183-day to 1-year 1-year to 2.5-year 2.5 year and longer MMDAs at commercial banks and thrift institutions Savings deposits at commercial banks and thrift institutions

C. To convert an annual effective yield on a bank interest basis to an annualized 1-month holding period yield on a bond interest basis:

Adjustment Formula	Own Rates Adjusted
$r^{adj} = \left[\left(1+\frac{\left(r/100\right)}{360}\right)^{30} - 1\right] \times \left(\frac{365}{30}\right) \times 100$	Overnight repurchase agreements Overnight Eurodollars Overnight federal funds

D. To convert an n-month bank discount basis rate to an annualized 1-month holding period yield on a bond interest basis:

Adjustment Formula	Own Rates Adjusted
$r^{adj} = \left[\frac{365\left(r/100\right)}{360 - 30\,n\left(r/100\right)}\right] \times 100$	Secondary market Treasury bill rate: 1-month, 3-month, 6-month Commercial paper rate: 1-month, 3-month, 6-month Bankers acceptance rate: 3-month, 6-month

six months, with six-month blocks beginning either on May 1 or November 1. The market-based savings bond rate for the next six months is equal to 85 percent of the average.[17]

Own-Rate Conversion

The application of aggregation theory and index-number methods to monetary data requires that all the own rates of return for the component assets be

[17] This methodology was supplied to us by the Savings Bond Operations Office of the U.S. Department of the Treasury.

measured on the same basis. This is generally not true in published data because different sources have different reporting conventions, and because own rates are reported for a variety of different maturities. For monetary aggregation, the choice of a common measurement basis is arbitrary; that is, the information content of the index numbers is unaffected by the choice. We have chosen to convert all available rate data to an annualized monthly yield, calculated on a bond (or, coupon) equivalent basis, primarily for consistency with past monetary aggregation research. In this subsection, we describe general procedures for adjusting various own rates to this common basis. Our adjustments of own-rate data are summarized in Table 7 by type of adjustment. In each case, r is the unadjusted own rate of the asset, r^{adj} is the adjusted own rate, and n is the maturity in months.

The simplest adjustment is to convert annualized one-month yields, quoted on a 360-day bank interest basis, to annualized one-month yields quoted on a 365-day bond coupon-equivalent basis. In this case, we simply multiply the unadjusted own rate by 365/360.

The second type of adjustment is to convert an annual effective yield, quoted in percentage points on a bond interest basis, to an annualized one-month holding-period yield on a bond interest basis. In this procedure, we convert the annual effective yield to a daily rate, compound that daily rate to a monthly rate, and then, assuming that all months contain 30 days, annualize the rate.

The third type of adjustment is to convert an annual effective yield on a bank interest basis to an annualized one-month holding-period yield on a bond interest basis, a procedure similar to the second one. We convert the bank-interest-basis annual effective yield to a daily rate, compound that daily rate to a monthly rate, and then, assuming a 30-day month, annualize to a bond interest basis.

In the fourth type of adjustment, we convert a rate quoted on a bank discount basis, for a monetary asset with a maturity of n months, to an annualized one-month holding-period yield. This conversion, which is discussed in detail by Farr and Johnson (1985), is valid only for rates with maturity of less than six months, and it assumes that each month has 30 days.

Yield Curve Adjustment

Own rates for monetary assets that have different maturities may have different term premiums, and hence are not directly comparable. Therefore, in addition to making the above adjustments, we need to remove a liquidity, or term, premium from each own rate. We *yield curve adjust* monetary assets' own rates by using the yield curve for U.S. Treasury securities. These adjustments of the own-rate data are summarized in Table 8.

We adjust the own rates by subtracting, from each own rate, an estimate of the liquidity premium obtained from the yield curve for Treasury securities. (Because these securities have no default risk, the slope of the Treasury yield curve provides a relatively "pure" estimate of the term premium.) The following discussion of yield curve adjustment assumes that all own rates (including Treasury bill rates) have been converted to an annualized one-month holding-period yield, on a bond interest basis.

Let r_n be an own rate for a monetary asset with a maturity of n months, let r_n^T be the own rate on Treasury securities that mature in n months, and let r_1^T be the one-month secondary-market Treasury bill rate. The own rate, r_n, is yield curve adjusted by subtracting the estimated liquidity premium $(r_n^T - r_1^T)$ from the own rate, such that the yield curve adjusted own rate, r_n^{YCA}, is equal to $r_n - (r_n^T - r_1^T)$. For a Treasury security that matures in n months (n = 1, 3, 6), r_n^T is the n-month secondary-market Treasury bill rate, adjusted from a bank discount basis to an annualized one-month holding-period yield on a bond interest basis. If maturity is in n years (n = 1, 2, 3), r_n^T is the corresponding constant-maturity Treasury security. Other values of r_n^T may be interpolated from the Treasury's constant-maturity yield curve.

Table 8

Yield Curve Adjustments for Own Rates on Five Groups of Monetary Assets (by Treasury security used for adjustment)

A. Three-Month Secondary-Market Treasury Bill Rate

Eurodollar deposits, 3-month maturity

Commercial paper, 3-month maturity

Bankers acceptances, 3-month maturity

Negotiable certificates of deposit, 3-month maturity

Small-denomination time deposits at commercial banks and thrift institutions, 7-day to 91-day maturity

Small-denomination time deposits at commercial banks and thrift institutions, variable ceiling rates on 91-day maturity

B. Six-Month Secondary-Market Treasury Bill Rate

Eurodollar deposits, 6-month maturity

Commercial paper, 6-month maturity

Bankers acceptances, 6-month maturity

Certificates of deposit, secondary-market, 6-month maturity

Small-denomination time deposits at commercial banks and thrift institutions, 92- to 182-day maturity

Money market time deposits at commercial banks and thrift institutions, variable ceiling rates on 6-month maturity

C. One-Year Constant-Maturity Treasury Security Rate

Time deposits at commercial banks, 1-year maturity

Small-denomination time deposits at commercial banks and thrift institutions, 183-day to 1-year maturity

Small-denomination time deposits at banks and thrift institutions, fixed ceiling rate on 1-year maturity

All Savers certificates, variable ceiling rate on 12-month maturity

D. Two-Year Constant-Maturity Treasury Security Rate

Small-denomination time deposits at commercial banks and thrift institutions, 1- to 2.5-year maturity

E. Three-Year Constant-Maturity Treasury Security Rate

Small-denomination time deposits at commercial banks and thrift institutions, 2.5-year and longer maturity

NOTE: All rates are adjusted to an annualized one–month yield on a bond interest (365-day, coupon equivalent) basis.

If a single monetary asset stock contains components with a range of maturities, we yield-curve adjust the own rate using the yield on a Treasury security with a maturity that falls within that range.

USER COSTS OF MONETARY ASSETS

In this section, we discuss in detail how we construct the user costs for monetary assets from the previous section's adjusted own-rate data. We address the problem that reported monetary asset stock data, such as the Federal Reserve Board's data on small-denomination time deposits, do not distinguish between monetary assets with different terms to maturity, and we construct the user costs of such assets as unilateral user cost sub-indexes.

Monetary Assets With Different Maturities

The definition of the real user cost of a monetary asset assumes that, in each period, each asset has a single applicable own rate and, hence, a unique user cost. Published Federal Reserve Board deposit data for commercial banks and thrift institutions do not distinguish adequately among monetary assets with different maturity. Only total dollar amounts, summed across all maturities, are reported for the following categories: small-denomination time deposits at commercial banks and thrift institutions; large-denomination time deposits; total Eurodollar deposits; total repurchase agreements; bankers acceptances; short-term Treasury securities; and commercial paper.[18] This bundling of assets with different maturities into monetary asset sub-indexes causes difficulty. Prior to measuring the sub-index's user cost, one should remove a liquidity (or term) premium from each component monetary asset's user cost. Because the own rates of the unobserved subcomponents may differ even after the yield curve adjustment, several user costs may apply to the sub-index.[19] A method must be found to combine the various user costs into a single user cost sub-index that corresponds to the reported asset stock.

A similar problem, in which a single price index is constructed from multiple individual prices without the use of quantity data, has been examined by Diewert (1995).[20] Price indexes constructed solely from price data, without quantity data, are called *unilateral price indexes*. Diewert (1995) advocates the use of a particular unilateral price index formula, called the *Jevons unilateral price index*. We construct such unilateral user cost indexes, based on the Jevons formula, for the following monetary asset categories: small-denomination time deposits at commercial banks and thrift institutions; large-denomination time deposits; total Eurodollar deposits; bankers' acceptances; and commercial paper.

User Costs by Component

In Table 9, we summarize the own rates used in the construction of each user cost. The own rates are (1) adjusted to a common basis, (2) yield curve adjusted, and (3) proxied, where appropriate (see Tables 5, 8, and 9). The own-rate series refer either to the own rate data shown in Table 4, or to the data discussed in the previous section of this article.

The construction of real user costs also requires the rate of return for a benchmark asset. The benchmark asset is a theoretical construct: It provides no monetary services, has no default risk, and is used by economic agents only to transfer wealth between periods. A theoretical lower bound for the benchmark asset can be identified; because monetary services are valued by households and firms, the user costs of monetary assets must be positive. Thus, the benchmark asset's rate of return must exceed the own rates on all assets that furnish monetary services.

A theoretical way of constructing the benchmark rate is to set it equal to the maximum rate of return over a large class of assets, both financial and non-financial. This method is inappropriate, however, because (unadjusted) rates of return on

[18] Data are not available on the outstanding quantities of monetary assets by remaining time to maturity. Data published by the Board of Governors regarding the outstanding stocks of small-denomination time deposits by original-issue maturity are not appropriate for calculating index numbers (see Table 1.22, Federal Reserve Bulletin, February 1997).

[19] Total repurchase agreements have a unique user cost in each period because we use the only available rate, that on overnight agreements, for all maturities. Small-denomination time deposits also have several applicable own rates due to the existence of both variable and fixed ceiling-rate time deposits from 1978.06 to 1983.9.

[20] The Advisory Commission to Study The Consumer Price Index (1996) discusses a similar problem in the construction of lower-level price indexes, in which multiple prices are combined into a single price index.

Table 9

Own Rates Used to Construct the Monthly User Costs of Monetary Assets, including proxy data

Asset Stock	Sample Period	Own Rates
Assets included in M1A		
Currency	1960.01–present	zero
Travelers checks	1960.01–present	zero
Demand deposits	1960.01–present	Startz (1979) rate.[a]
Additional Assets included in M1		
Other checkable deposits at commercial banks, excluding Super NOW accounts	1974.01–1985.12	Fixed ceiling rate on NOW accounts at commercial banks.
Other checkable deposits at thrift institutions, excluding Super NOW accounts	1960.01–1973.12 1974.01–1985.12	Startz (1979) rate.[a] Fixed ceiling rate on NOW accounts at thrift institutions.
Super NOW accounts at commercial banks	1982.12–1985.12	Super NOW accounts at commercial banks.
Super NOW accounts at thrift institutions	1982.12–1985.12	Super NOW accounts at thrift institutions.
Other checkable deposits at commercial banks including Super NOW accounts	1985.12–present	NOW accounts at commercial banks.
Other checkable deposits at thrift institutions including Super NOW accounts	1985.12–present	NOW accounts at thrift institutions.
Additional Assets included in MZM		
Retail money funds	1973.02–present	Money market mutual funds.
Money market deposit accounts (MMDA) at commercial banks	1982.12–1991.09	MMDA at commercial banks.
Money market deposit accounts (MMDA) at thrift institutions	1982.12–1991.09	MMDA at thrift institutions.
Savings deposits, excluding MMDA, at commercial banks banks.	1960.01–1986.03 1986.04–1991.09	Fixed ceiling rate on savings deposits at commercial Other savings at commercial banks.
Savings deposits, excluding MMDA, at thrift institutions	1960.01–1966.09 1966.10–1986.03 1986.04–1991.09	Fixed ceiling rate on savings deposits at commercial banks.[b] Fixed ceiling rate on savings deposits at thrift institutions. Other saving deposits at thrift institutions.
Savings deposits including MMDA at commercial banks	1991.09–present	Savings deposits and MMDA at commercial banks
Savings deposits including MMDA at thrift institutions	1991.09–present	Savings deposits and MMDA at thrift institutions.

continued on next page

Table 9 *continued*

Additional Assets included in M2		
Small-denomination (less than $100,000) time deposits at commercial banks	1960.01–1966.07	Fixed ceiling rate on 1-year maturity other time deposits at commercial banks.
	1966.08–1978.05	Fixed ceiling rate on 1-year maturity small-denomination time deposits at commercial banks.
	1978.06–1983.09	Fixed ceiling rate on 1-year maturity small-denomination time deposits at commercial banks, and the variable ceiling rates on the following: money market time deposits at commercial banks, 12-month All Savers certificates, 7 to 31-day and 91-day small-denomination time deposits at commercial banks.
	1983.10–present	Small-denomination time deposits at commercial banks, with 7 to 91-day, 92 to 182-day, 183-day to 1-year, 1 to 2.5-year, and 2.5-year and longer maturity.
Small-denomination time deposits at thrift institutions	1960.01–1966.07	Fixed ceiling rate on 1-year time deposits at commercial banks
	1966.08–1966.09	Fixed ceiling rate on 1-year small-denomination time deposits at commercial banks plus 25 basis points [b]
	1960.10–1978.05	Fixed ceiling rate on 1-year small-denomination time deposits at thrift institutions
	1978.06–1983.09	Fixed ceiling rate on 1-year small-denomination time deposits at thrift institutions, and the variable ceiling rates on the following: money market time deposits at thrift institutions, 12-month All Savers certificates, 7- to 31-day and 91-day small-denomination time deposits at thrift institutions.
	1983.10–present	Small-denomination time deposits at thrift institutions, with 7- to 91-day, 92- to182-day, 183-day to 1-year, 1- to 2.5-year, and 2.5-year and longer maturity.
Additional Assets included in M3		
Large-denomination ($100,000 or more) time deposits	1960.01–1964.05	Ceiling rate on time deposits payable in 6 months to 1 year.
	1964.06–1965.11	Negotiable certificate of deposit, secondary market, 3- and 6-month maturity.
	1965.12–present	Negotiable certificate of deposit, secondary market, 1-, 3- and 6-month maturity.
Total repurchase agreements	1969.10–present	Overnight repurchase agreements.
Total Eurodollars	1960.01–1963.04	Term Eurodollar deposits, 3-month maturity.
	1963.05–1970.12	Term Eurodollar deposits, 3- and 6-month maturity.
	1971.01–present	Overnight Eurodollar deposits, and term Eurodollar deposits with 1-, 3-, and 6-month maturity.
Institutional money funds	1974.01–present	Average rate on money market mutual funds.
Additional Assets included in L		
Short-term Treasury securities	1960.01–present	Treasury bill, secondary market, 1-month maturity.
Bankers acceptances	1960.01–1975.12	Bankers acceptances, 3-month maturity.
	1976.01–present	Bankers acceptances, 3- and 6-month maturity.

continued on next page

Table 9 *continued*

Commercial paper	1960.01–1971.03 1971.04–present	Commercial paper, 6-month maturity. Commercial paper, 1-, 3-, and 6-month maturity.
U.S. Savings Bonds	1960.01–1982.10 1982.11–present	Series E savings bonds, investment yield to maturity. Savings bonds, market interest rate on new issues.

NOTE: Rates are proxied (if necessary), converted to a common basis, and yield curve adjusted, as summarized in Tables 5– 8. Where multiple rates are listed for the same period, the user cost is the Jevons user cost sub-index of the user costs calculated from each rate.

[a] The Startz rate is calculated as follows: (0.58) ((1.0 – maximum statutory reserve-requirement ratio on demand deposits) x (5–year Treasury constant maturity yield). The value 0.58 reflects our assumption that the public regards demand deposits issued by different commercial banks as perfect substitutes. Other checkable deposits at thrift institutions include some non-interest-bearing demand deposits. We assume that their implicit rate of return was equal that on demand deposits at commercial banks.

[b] We use the ceiling rate on savings deposits at commercial banks because thrift insitutions were not regulated prior to October 1996. On time deposits, thrift institutions offered about 25 basis points more than commercial banks (Farr and Johnson, 1985; Mahoney, 1987).

debt and equity contain risk premia. In empirical work, the traditional approach has been to identify the benchmark rate during each time period, t, as the "envelope" of the own rates of return on monetary assets and the rate on Moody's seasoned BAA bonds, $r_{BAA,t}$:

$$R_t = max\{r_{it}\ (i=1,2,...,n),\ r_{BAA,t}\}$$

(Barnett and Spindt, 1982; Farr and Johnson, 1985; and, Thornton and Yue, 1992). We adopt this practice, with a minor modification, and define the *benchmark rate* as

$$R_t^* = max\{r_{it}\ (i=1,2,...,n),\ r_{BAA,t}\} + c,$$

where c is a small constant. Although we typically set the value of the constant at one basis point or less, its inclusion guarantees that the benchmark rate is strictly greater than the rate on any monetary asset, and it allows us, in a previous section, to define Divisia second moments of our indexes. The indexes are robust experimentally to a large range of values for the constant.

Unilateral Index Number Theory

In this section, we provide the reader with a discussion of unilateral index number theory in the context of monetary aggregation, and we define and discuss our use of the Jevons unilateral price index formula.

Bilateral index numbers, such as the Törnqvist-Theil index number, are functions, in each period, of both observed prices and quantities. *Unilateral index numbers* are functions, in each period, of either the observed prices or the observed quantities, but not both. Unilateral indexes may be useful, therefore, when some of the price or quantity data required for a bilateral index have not been recorded. As previously noted, the Federal Reserve Board's monetary asset stock data often do not separate monetary assets with different maturities. The reported asset data—total dollar amounts summed across all maturities—are unilateral quantity indexes. We refer to these aggregates as *monetary quantity sub-indexes*, and the unreported individual assets with differing maturities as *sub-components*.

In an ideal world, we would treat each sub-component of each monetary quantity sub-index as a separate asset with its own user cost. In practice, the data collection process forces us to treat each monetary quantity sub-index as if it were a single asset. If the user costs of the sub-components are observed, we can construct a unilateral *user cost sub-index*, which can be viewed as the "user cost" of the monetary quantity sub-index.

Let m_s^i be the value of a monetary quantity sub-index in period s, let $r_s^i = (r_{1s}^i,...,r_{Ms}^i)$ be a vector of M own rates that apply to the sub-components of the monetary quantity sub-index. Then, $\pi_{js}^i = (R_s - r_{js}^i)/(1 + R_s)$ where j=1,...,M, are the real user costs that apply to the sub-components of the monetary quantity sub-index, m_s^i. (We remind the reader that the own rates in the vector $r_s^i = (r_{1s}^i,...,r_{Ms}^i)$ must all be converted to a common basis, and yield curve adjusted.) Diewert (1995) defines the Jevons and the Dutot unilateral price indexes.[21] For monetary aggregation, the *Jevons user cost sub-index* is defined by

$$\pi_{it}^{J} = \pi_{i,t-1}^{J}\left(\frac{\prod_{j=1}^{M}(\pi_{jt}^{i})^{(1/M)}}{\prod_{j=1}^{M}(\pi_{j,t-1}^{i})^{(1/M)}}\right),$$

and the *Dutot user cost sub-index* is defined by

$$\pi_{it}^{D} = \pi_{i,t-1}^{D}\left(\frac{1}{M}\sum_{j=1}^{M}\pi_{jt}^{i} \Big/ \frac{1}{M}\sum_{j=1}^{M}\pi_{j,t-1}^{i}\right).$$

Diewert (1995) defined a set of axioms that a reasonable unilateral price index should satisfy, and he showed that both the Jevons and the Dutot price indexes satisfy these axioms.[22] It can also be shown that a third unilateral price index, which we call the *Leontief price index*, satisfies weak versions of these same axioms.[23] In the present context, the *Leontief user cost sub-index*, π_{it}^L, is defined by

$$\pi_{it}^{L} = \pi_{i,t-1}^{L}\left(\frac{\min\{\pi_{1t}^{i},...,\pi_{Mt}^{i}\}}{\min\{\pi_{1,t-1}^{i},...,\pi_{M,t-1}^{i}\}}\right).$$

Because the Jevons, Dutot, and Leontief user cost sub-indexes all satisfy Diewert's axioms, we can compare the economic justifications of the indexes.

Diewert (1976) showed that the members of a class of bilateral statistical index numbers, called *superlative*, have strong economic justification as approximations of aggregator functions. Unfortunately, unilateral price indexes have only weak economic justifications; Diewert (1995) gives the stringent conditions under which the Jevons and Dutot user cost sub-indexes will be correct (exact).[24] The Jevons user cost sub-index will be correct only if two conditions hold: (1) the elasticities of substitution between the sub-components are unity, once the liquidity premium has been extracted; and (2) the expenditure shares on each sub-component are constant. The Dutot user cost sub-index will be correct if two conditions hold: (1) the elasticities substitution between the sub-components are zero, once the liquidity premium has been extracted; and (2) in each period the quantities of the sub-components are equal.

The Leontief user cost sub-index will be correct if the sub-components are, in fact, perfect substitutes after the liquidity premium has been removed. This assumption underlies the user cost sub-indexes that have been constructed, for certain subsets of assets, by Barnett and Spindt (1982), Farr and Johnson (1986), and Thornton and Yue (1992). In these articles, Barnett's (1978) user cost formula was applied, in each time period, to the maximum of the yield curve–adjusted own rates for the assets included in each subset, a procedure that is equivalent to the Leontief user cost sub-index. These sub-indexes were subsequently aggregated with other assets and user costs, using superlative index number formulas such as the Törnqvist-Theil or Fisher Ideal index formula.

Diewert (1995) argued that, for calculating price indexes, the Jevons index formula is superior to the Dutot index formula because the Jevons index's conditions—unit elasticity and constant (or proportional) expenditures—are more plausible than the Dutot index's conditions: zero elasticity and constant (or proportional) quantities. The Advisory Commission to Study the Consumer Price Index (1996) also advocated the use of the Jevons index formula to calculate lower-

[21] These indexes also are defined in Diewert (1992).

[22] The growth rate of the Dutot price index is the ratio of the averages of prices in adjacent periods. An index based on the average of the ratios of prices in adjacent periods, the Carli (Diewert, 1992), does not satisfy the time reversal test, hence is not a reasonable unilateral price index.

[23] Specifically, the Leontief index satisfies the axiomatic tests if strict monotonicity is weakened to monotonicity.

[24] These unilateral index numbers are based on particular constant elasticity of substitution (CES) aggregator functions, which are not flexible functional forms. Hence the unilateral indexes are not superlative.

level price indexes for sub-components of the CPI. The Jevons index has been widely used as the benchmark for studying bias in lower-level price indexes in a number of countries; see Diewert (1995) for a review of these studies. The current consensus is that the Jevons index number formula should be used to calculate unilateral price indexes.

For measuring the user costs of our monetary service index, we concur with Diewert's preference for the Jevons index. We further argue that the Jevons index's unit elasticity condition is more reasonable than the Leontief index's assumption that monetary assets are perfect substitutes. The perfect substitutes condition would imply that, unless all user costs applying to the sub-index are equal, economic agents will hold only the least-expensive sub-component of each monetary sub-index—an implication that is usually rejected when it can be tested.[25] In our MSI database, we use the Jevons formula to create user cost sub-indexes for small-denomination time deposits at commercial banks and thrift institutions, large-denomination time deposits, total Eurodollar deposits, bankers acceptances, and commercial paper. The growth rate of the Jevons user cost sub-index, in two adjacent periods, is the ratio of the geometric means of the applicable user costs. We cardinalize the Jevons user cost sub-index by setting the initial value of the sub-index equal to the geometric mean of the user costs during the initial period.

After selection of the Jevons index, one more important difficulty remains: The number of own rates that are observed for the sub-components of a monetary quantity sub-index may change, from period to period, due to a number of factors, including changes in regulations and data-collection practices. In these cases, we calculate the growth rate of the Jevons user cost sub-index from the subset of user costs that are observed in the adjacent periods. This procedure is based on Diewert's (1980) new goods procedure, which is discussed in the section of this paper titled, "Introduction of New Monetary Assets." In a few cases, the set of observed sub-component user costs in adjacent periods changes completely. In such cases, we calculate, for both periods, the geometric means of the observed user costs and then calculate the Jevons index as the ratio of the current period's geometric mean divided by the geometric mean in the previous period.[26]

ADDITIONAL PROBLEMS

Several additional problems that arise in the construction of monetary services indexes are discussed in the following subsections: (1) the introduction of new monetary assets, (2) changes in the definitions of underlying monetary asset stock data, (3) the calculation of monetary service indexes and related indexes at different frequencies, and (4) seasonal adjustment of the indexes.

Introduction of New Monetary Assets

There have been many financial innovations during the time span of our monetary services indexes. New monetary assets have been created at various dates, and the indexes must be modified to include them.

The nominal Törnqvist-Theil monetary services index, MSI_t^{nom}, and its real dual price index, Π_t^{real}, are not well defined when new assets enter the indexes. The *real Fisher Ideal user cost index,*

$$P_t^F = P_{t-1}^F \sqrt{\frac{\sum_{j=1}^{n} \pi_{jt}^{real} m_{jt}^{nom}}{\sum_{j=1}^{n} \pi_{j,t-1}^{real} m_{jt}^{nom}} \cdot \frac{\sum_{j=1}^{n} \pi_{jt}^{real} m_{j,t-1}^{nom}}{\sum_{j=1}^{n} \pi_{j,t-1}^{real} m_{j,t-1}^{nom}}},$$

is well defined, and a corresponding quantity index may be obtained by Fisher's factoral reversal formula. We therefore switch to the Fisher Ideal index in periods when new monetary assets are introduced.[27]

[25] Prior to 1991:08, for example, economic agents held non-zero quantities of both MMDA and saving deposits assets, even though the user costs differed.

[26] Our calculations assume that each unit of a specific monetary asset stock has the same user cost, after necessary conversions and yield curve adjustment. Aggregation error will occur if deposit own rates vary with the size of the deposit. Some evidence on this practice is analyzed by Collins (1991).

[27] Farr and Johnson (1985) advocate the Fisher Ideal index because it is well-defined even when new assets are introduced. For all periods when data are available, the Törnqvist-Theil index is superior to the Fisher Ideal index because it is superlative in a stronger sense (Caves, Christensen, and Diewert, 1982).

To implement this approach, we need to develop an estimator for the new asset's user cost during the period prior to its introduction. Theoretically, the correct solution is to define a user cost, called the *reservation user cost*, that is sufficient to ensure that a zero quantity of the new asset would have been demanded at that user cost during the prior period if the asset had, in fact, existed. In practice, doing this correctly requires econometric estimation of the aggregator function (Diewert, 1980), whereas our primary motive for the use of statistical index numbers is to avoid such estimation.

Rather than estimate the reservation user cost, we use the following method, introduced by Diewert (1980) and used in Diewert and Smith (1994).[28] In the period when a new monetary asset is introduced, we calculate the Fisher Ideal real user cost index over all monetary assets except the new one, which we will call P_t^{**}. If monetary asset i is introduced in period t, P_t^{**} will be defined by

$$P_t^{**} = P_{t-1}^{**}\sqrt{\frac{\sum_{j\neq i}\pi_{jt}^{real}m_{jt}^{nom}}{\sum_{j\neq i}\pi_{j,t-1}^{real}m_{jt}^{nom}}\cdot\frac{\sum_{j\neq i}\pi_{jt}^{real}m_{j,t-1}^{nom}}{\sum_{j\neq i}\pi_{j,t-1}^{real}m_{j,t-1}^{nom}}}\,.$$

Diewert (1980) shows that this procedure will, in general, have lower bias than the other available alternatives, in the absence of strong information about the reservation user cost. The procedure is exactly correct in a special case: If the actual user cost of the new asset i in period t divided by the reservation user cost is equal to

$$\left(\sum_{j\neq i}\pi_{jt}^{real}m_{jt}^{nom}\Big/\sum_{j\neq i}\pi_{j,t-1}^{real}m_{jt}^{nom}\right),$$

then P^{**}_t will be exactly correct.

We form our real user cost indexes by switching to a Fisher Ideal index, calculated according to Diewert's recommended approach, during periods in which new monetary assets enter the indexes. The dual monetary services index is then defined implicitly by Fisher's weak factor reversal formula.

In Table 10, we list the periods in which new monetary assets are introduced.

Changes in the Definitions of Asset Stock Data

In the preceding section, we discussed the introduction of new monetary assets. A related problem is that, at times, the Federal Reserve has changed the definitions and the manner of reporting the components of its monetary aggregates. This happens twice in our series: (1) after 1985.12, Super NOW accounts are included in other checkable deposits (OCD), and (2) after 1991.08, money market deposit accounts (MMDA) and savings deposits are reported only on a combined basis, for thrift institutions and for commercial banks. In both of these cases, monetary assets that had been reported separately were combined into sub-indexes, and the sub-component data were no longer available.

These changes represent a redefinition of the asset stocks (and consequently the monetary services indexes), but they do not represent a meaningful change in the structure of the economy; in other words, these data-reporting changes are not economically relevant. The Federal Reserve Board's monetary aggregates are invariant to such changes because their aggregates are themselves sums of all the component data. Törnqvist-Theil monetary services indexes are not invariant to these changes because the change in reporting, from a group of assets to a single sub-index, represents a loss of information. In this section, we describe our approach to this problem.[29]

From 1983.01 through 1985.12, Super NOW accounts and savings deposits are included in our Törnqvist-Theil monetary services indexes as separate assets. Beginning in 1986.01, however, Super NOW accounts and savings deposits were reported only as a combined total. In response to this change, we define a second Törnqvist-Theil monetary services

[28] Diewert suggests this procedure in the general case; we state it here in the case of monetary aggregation.

[29] The CE and simple sum indexes are well-defined and do not require any modifications when asset stocks are redefined. The real user cost index that is dual to the Törnqvist-Theil monetary services index is calculated by factor reversal. The Törnqvist-Theil real user cost index and the Törnqvist-Theil expenditure share index are calculated according to the procedure described in this section.

index that begins in 1985.12 and contains the total of Super NOW accounts and other checkable deposits as a single asset. The value of the second index in its initial period, December 1985, is arbitrary, which permits us to scale the second index so that it equals the first index in 1985.12. This splices the two indexes so as to produce a single Törnqvist-Theil monetary services index over the entire period. We perform an analogous splice in 1991.08 when money market deposit accounts and savings deposits begin to be reported only on a combined basis.[30]

Data reporting changes that are not based on economic reasons, such as financial innovation or regulatory changes, represent a loss of information. We preserve as much information as possible during the periods when disaggregate data are available and avoid imputing economic relevancy to the data reporting change when it occurs. Our method draws on the literature of index number splicing (Hill and Fox, 1995).

Indexes at Different Frequencies

The disaggregated data in the MSI database are reported monthly. In some applications, monetary aggregates must be available at quarterly or annual frequency. In this section, we discuss a method developed by Diewert (1980) for constructing, from monthly data, indexes at quarterly and annual frequencies.

In the problem of constructing annual indexes from monthly indexes, the solution is to treat each asset, in each month, as a separate asset and then to aggregate over these assets. For example, demand deposits held in January and in February will be treated as different assets. Formally, let m_{it}^r be the nominal stock of monetary asset i in month r of year t. Similarly let

$$\pi_{it}^r = \left(R_t^r - r_{it}^r\right) / \left(1 + R_t^r\right)$$

be the real user cost associated with, m_{it}^r, where R_t^r is the rate of return on the benchmark asset in month r of year t, and r_t^r is the rate of return on the nominal stock of monetary asset i in the month r of year t. Then the log change of the annual Törnqvist-Theil nominal monetary services index, M_t^{annual} is defined by

$$\Delta \log(M_t^{annual}) = \sum_{i=1}^{n} \sum_{r=1}^{12} \overline{w}_{it}^{\,r} \Delta \log(m_{it}^{\,r}),$$

where

$$w_{it}^{\,r} = \left(\pi_{it}^{\,r} m_{it}^{\,r} \Big/ \sum_{j=1}^{n} \sum_{r=1}^{12} \pi_{jt}^{\,r} m_{jt}^{\,r} \right).$$

An analogous method can be used to define the quarterly indexes. In the MSI database, this method is used to produce both annual and quarterly indexes. Dual user cost indexes are obtained by Fisher's weak factor reversal criterion.[31]

Seasonal Adjustment

The issue of seasonal adjustment is a difficult one. Index number theoretic methods for dealing with seasonality, which are related to the issues discussed in the section of this paper dealing with

Table 10

Introduction of New Assets

New Asset	Introduction Date
Total repurchase agreements	1969.10
Retail money funds	1973.02
Other checkable deposits at commercial banks	1974.01
Institutional money funds	1974.01
Money Market deposit accounts at commercial banks	1982.12
Money Market deposit accounts at thrift institutions	1982.12
Super NOW accounts at commercial banks	1982.12
Super NOW accounts at thrift institutions	1982.12

[30] In 1985.12, the combined OCD asset stock is the sum of the asset stocks of its subcomponents, and the user cost for OCD is constructed from the weighted average of its subcomponents' own rates. A similar procedure is followed in August 1991 for savings deposits and MMDAs.

[31] At quarterly and annual frequencies, the splicing procedure described in the preceding subsection needs to be modified in a straightforward way.

indexes at different frequencies, can be found in Diewert (1980, 1983, 1996). Our approach is more traditional. We produce the indexes in the database by using both seasonally adjusted and unadjusted asset stock data, except for the non-M3 components of L, which are not seasonally adjusted in either set of indexes. Our seasonally adjusted data are produced with the Bureau of the Census X11 program, using default values for all options. We urge users of our unadjusted data to experiment with alternative seasonal adjustment methods.

CONCLUSION

The St. Louis MSI database is an important resource for economists and policymakers studying the role of money in the economy. Monetary services are an important aspect of the economic behavior of households and firms, and the monetary service indexes provide new up-to-date measures of the flows of monetary services. The database also contains dual measures of the opportunity cost of monetary services and related stock and total expenditure variables.

The indexes in the MSI database are consistent with microeconomic aggregation theory and have the same statistical properties as commonly used macroeconomic indexes such as GDP and its deflator. In general, the monetary service index and its dual user cost index can be modeled in the same way as other macroeconomic quantity and price indexes and, in particular, models of money demand can be estimated by using the monetary service index.

In addition to our aggregate indexes, the MSI database contains disaggregate asset-stock and user-cost data that will allow researchers to study the demand for the disaggregated monetary assets in a way that is consistent with microeconomic models of decision making. The database is also comprehensive enough to allow researchers to experiment with alternative levels of aggregation, different measures of assets' own rates, and various seasonal adjustment techniques.

These data provide numerous opportunities for applied monetary research. Although monetary services indexes have been produced before by Barnett and Spindt (1982), Farr and Johnson (1985), and Thornton and Yue (1992), none of these studies furnished a broad enough set of indexes, the underlying data, or the computer programs necessary to build the indexes.

REFERENCES

Advisory Commission to Study the Consumer Price Index. *Toward a More Accurate Measure of the Cost of Living*, Final Report to the Senate Finance Committee (December 1996).

Allen, Stuart D. "A Note on the Implicit Interest Rate on Demand Deposits," *Journal of Macroeconomics* (Spring 1983), pp. 233-39.

Anderson, Richard G., and Kenneth A. Kavajecz. "A Historical Perspective on the Federal Reserve's Monetary Aggregates: Definition, Construction and Targeting," this *Review* (March/April 1994), pp. 1-31.

________and Robert H. Rasche. "Measuring the Adjusted Monetary Base in an Era of Financial Change," this *Review* (November/December 1996), pp. 3-37.

________, Barry E. Jones, and Travis D. Nesmith. "An Introduction to Monetary Aggregation Theory and Statistical Index Numbers," this *Review*, this issue.

Barnett, William A. "Reply [to Julio J. Rotemberg]," *Monetary Policy on the 75th Anniversary of the Federal Reserve System: Proceedings of the Fourteenth Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis*, Michael T. Belongia, ed., Kluwer Academic Publishers, 1991, pp. 232-43.

________. "Developments in Monetary Aggregation Theory," *Journal of Policy Modeling* (Summer 1990), pp. 205-57.

________. "The Microeconomic Theory of Monetary Aggregation," *New Approaches to Monetary Economics: Proceedings of the Second International Symposium in Economic Theory and Econometrics*, William A. Barnett and Kenneth J. Singleton, eds., Cambridge University Press, 1987, pp. 115-68.

________. "Economic Monetary Aggregates: An Application of Index Number and Aggregation Theory," *Journal of Econometrics* (Summer 1980), pp. 11-48.

________."The User Cost of Money," *Economic Letters* (vol.1, 1978), pp. 145-49.

________and Yi Liu. "Beyond the Risk Neutral Utility Function," paper presented at the University of Mississippi conference, "Divisia Monetary Aggregates: Theory and Practice," October 1994; conference volume, Macmillan, forthcoming.

________ and Apostolos Serletis. "A Dispersion-Dependency Diagnostic Test for Aggregation Error: With Applications to Monetary Economics and Income Distribution," *Journal of Econometrics* (January/February 1990), pp. 5-34.

________ and Paul A. Spindt. "Divisia Monetary Aggregates: Compilation, Data, and Historical Behavior," Staff Study 116, Board of Governors of the Federal Reserve System, May 1982.

________, Barry E. Jones, and Travis D. Nesmith. "Divisia Second Moments: An Application of Stochastic Index Number Theory," *International Review of Comparative Public Policy* (vol. 8, 1996), pp. 115-38.

________, Yi Liu, and Mark Jensen. "The CAPM Risk Adjustment for Exact Aggregation over Financial Assets," *Macroeconomic Dynamics* (May 1997), forthcoming.

Barro, Robert J., and Anthony J. Santomero. "Household Money Holdings and the Demand Deposit Rate," *Journal of Money, Credit, and Banking* (May 1972), pp. 397-413.

Becker, William E., Jr. "Determinants of the United States Currency-Demand Deposit Ratio," *The Journal of Finance* (March 1975), pp. 57-74.

Belongia, Michael T. "Weighted Monetary Aggregates: A Historical Survey," *Journal of International and Comparative Economics* (vol. 4, 1995), pp. 87-114.

Board of Governors of the Federal Reserve System. *Annual Statistical Digest,* editions for 1970-79, 1980-89, and annual editions for 1980 through 1995.

________. *Money Stock, Liquid Assets, and Debt Measures*, H.6 statistical release.

________. *Selected Interest and Exchange Rates*, H.13 statistical release (various issues).

________. *Selected Interest Rates*, H.15 statistical release (various issues).

________. *Federal Reserve Bulletin* (various issues).

________. "Demand Deposit Ownership Survey," *Federal Reserve Bulletin*, July issues, 1971–91.

________. *Banking and Monetary Statistics: 1941-1970*, September 1976.

________. "Survey of Time and Savings Deposits at Commercial Banks," *Federal Reserve Bulletin*, vols. 52 through 68.

________. "Flows Through Financial Intermediaries," *Federal Reserve Bulletin* (May 1964a), pp. 549-57.

________. "Bank Credit and Money in 1963," *Federal Reserve Bulletin* (February 1964b), pp. 141-47.

________. "Interest Rates on Time Deposits, Mid-February 1963," *Federal Reserve Bulletin* (June 1963b), pp. 766-72.

________. "Negotiable Time Certificates of Deposit," *Federal Reserve Bulletin* (April 1963b), pp. 458-68.

Brennan, Michael J., and Eduardo S. Schwartz. "Savings Bonds: Theory and Empirical Evidence," *Monograph Series in Finance and Economics*, Monograph 1979-4. Saloman Brothers Center for the Study of Financial Institutions, 1979.

Caves, Douglas W., Laurits R. Christensen, and W. Erwin Diewert. "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity," *Econometrica* (November 1982), pp. 1393-414.

Carlson, John A., and James R. Frew. "Money Demand Responsiveness to the Rate of Return on Money: A Methodological Critique," *Journal of Political Economy* (June 1980), pp. 598-607.

Collins, Sean. "Do Banks Price Discriminate Among Their Retail Customers? A Test Based on Tiered Deposit Rates," working paper, Division of Monetary Affairs, Board of Governors of the Federal Reserve System, September 1991.

________ and Cheryl L. Edwards. "An Alternative Monetary Aggregate: M2 Plus Household Holdings of Bond and Equity Mutual Funds," this *Review* (November/December 1994), pp. 7-29.

Diewert, W. E. "Seasonal Commodities, High Inflation and Index Number Theory," Working Paper DP 96-06, University of British Columbia, January 1996.

________"Axiomatic and Economic Approaches to Elementary Price Indexes," Working Paper No. 5104, National Bureau of Economic Research, May 1995.

________"The Treatment of Seasonality in a Cost-Of-Living-Index," *Price Level Measurement*, W.E. Diewert and C. Montmarquette, eds., Ottawa: Statistics Canada, 1983, pp. 1019-45.

________"Index Numbers," *The New Palgrave Dictionary of Money & Finance*, Vol. 2, Peter Newman, Murray Milgate, and John Eatwell, eds., Stockton Press, 1992, pp. 364-79.

________"Aggregation Problems in the Measurement of Capital," *The Measurement of Capital*, Dan Usher, ed., University of Chicago Press, 1980, pp. 433-538.

________"Exact and Superlative Index Numbers," *Journal of Econometrics* (May 1976), pp. 115-45.

________ and Ann Marie Smith. "Productivity Measurement for a Distribution Firm," *Journal of Productivity Analysis* (vol. 5, 1994), pp. 335-47.

Divisia, Francois. "L'Indice Monétaire et la Théorie de la Monnaie," *Revue d'Economie Politique* (1925), pp. 980-1008.

Farr, Helen T., and Deborah Johnson. "Revisions in the Monetary Services (Divisia) Indexes of the Monetary Aggregates," Staff Study 147, Board of Governors of the Federal Reserve System, December 1985.

Fisher, Irving. *The Making of Index Numbers: A Study of Their Varieties, Tests, and Reliability*, Houghton Mifflin Company, 1922.

Hill, Robert J., and Kevin J. Fox. "Splicing Index Numbers," working paper, School of Economics, The University of New South Wales, 1995; *Journal of Business and Economic Statistics*, forthcoming, Kavajecz, Kenneth A. "The Evolution of the Federal Reserve's Monetary Aggregates: A Timeline," this *Review* (March/April 1994), pp. 32-66.

Klein, Benjamin. "Competitive Interest Payments on Bank Deposits and the Long-Run Demand for Money," *American Economic Review* (December 1974), pp. 931-49.

Mahoney, Patrick I., et.al., "Responses to Deregulation: Retail Deposit Pricing from 1983 through 1985," Staff Study 151, Board of Governors of the Federal Reserve System, January 1987.

OECD Financial Statistics. Interest Rates 1960-1974, 1976.

Orphanides, Athanasios, Brian Reid, and David H. Small. "The Empirical Properties of a Monetary Aggregate That Adds Bond and Stock Funds to M2," this *Review* (November/December 1994), pp. 31-51.

Rossiter, Rosemary, and Tong Hun Lee. "Implicit Returns on Conventional Demand Deposits: An Empirical Comparison," *Journal of Macroeconomics* (Fall 1987), pp. 613-24.

Rotemberg, Julio J. "Commentary: Monetary Aggregates and Their Uses," *Monetary Policy on the 75th Anniversary of the Federal Reserve System: Proceedings of the Second International Symposium in Economic Theory and Econometrics*, Michael T. Belongia, ed., Kluwer Academic Publishers, 1991, pp. 223-31.

________, John C. Driscoll, and James M. Poterba. "Money, Output, and Prices: Evidence from a New Monetary Aggregate," *Journal of Business and Economic Statistics* (January 1995), pp. 67-83.

Rush, Mark. "Comment and Further Evidence on 'Implicit Interest on Demand Deposits'," *Journal of Monetary Economics* (July 1980), pp. 437-51.

Serletis, Apostolos. "Monetary Asset Separability Tests," *New Approaches to Monetary Economics: Proceedings of the Second International Symposium in Economic Theory and Econometrics*, William A. Barnett and Kenneth J. Singleton, eds., Cambridge University Press, 1987, pp. 169-82.

Spencer, Peter. "Portfolio Disequilibrium: Implications for the Divisia Approach to Monetary Aggregation," *The Manchester School of Economic and Social Studies* (June 1994), pp. 125-50.

Startz, Richard. "Implicit Interest on Demand Deposits," *Journal of Monetary Economics* (October 1979), pp. 515-34.

Swofford, James L., and Gerald A. Whitney. "A Revealed Preference Test for Weakly Separable Utility Maximization with Incomplete Adjustment," *Journal of Econometrics* (January/February 1994), pp. 235-49.

________and________. "A Comparison of Nonparametric Tests of Weak Separability for Annual and Quarterly Data on Consumption, Leisure, and Money," *Journal of Business and Economic Statistics* (April 1988), pp. 241-46.

________ and ________ "Nonparametric Tests of Utility Maximization and Weak Separability for Consumption, Leisure, and Money," *Review of Economics and Statistics* (August 1987), pp. 458-64.

Theil, Henri. *Economics and Information* Theory. Amsterdam: North Holland, 1967.

Thornton, Daniel L., and Piyu Yue. "An Extended Series of Divisia Monetary Aggregates," this *Review* (November/December 1992), pp. 35-52.

Triplett, Jack E. "Economic Theory and BEA's Alternative Quantity and Price Indexes," *Survey of Current Business* (April 1992), pp. 32-48.

Whitesell, William C., and Sean Collins. "A Minor Redefinition of M2," Finance and Economics Discussion Series, no. 96-7, Board of Governors of the Federal Reserve System, February 1996.