John W. Keating

John W. Keating, assistant professor, Department of Economics, Washington University in St. Louis, was a visiting scholar at the Federal Reserve Bank of St. Louis while this paper was written. Richard I. Jako provided research assistance.

Structural Approaches to Vector Autoregressions

THE VECTOR AUTOREGRESSION (VAR) model of Sims (1980) has become a popular tool in empirical macroeconomics and finance. The VAR is a reduced-form time series model of the economy that is estimated by ordinary least squares.[1] Initial interest in VARs arose because of the inability of economists to agree on the economy's true structure. VAR users thought that important dynamic characteristics of the economy could be revealed by these models without imposing structural restrictions from a particular economic theory.

Impulse response functions and variance decompositions, the hallmark of VAR analysis, illustrate the dynamic characteristics of empirical models. These dynamic indicators were initially obtained by a mechanical technique that some believed was unrelated to economic theory.[2] Cooley and LeRoy (1985), however, argued that this method, which is often described as atheoretical, actually implies a particular economic structure that is difficult to reconcile with economic theory.

This criticism led to the development of a "structural" VAR approach by Bernanke (1986), Blanchard and Watson (1986) and Sims (1986). This technique allows the researcher to use economic theory to transform the reduced-form VAR model into a system of structural equations. The parameters are estimated by imposing contemporaneous structural restrictions. The crucial difference between atheoretical and structural VARs is that the latter yield impulse responses and variance decompositions that can be given structural interpretations.

An alternative structural VAR method, developed by Shapiro and Watson (1988) and Blanchard and Quah (1989), utilizes long-run restrictions to identify the economic structure from the reduced form. Such models have long-run characteristics that are consistent with the theoretical restrictions used to identify parameters. Moreover, they often exhibit sensible short-run properties as well.

For these reasons, many economists believe that structural VARs may unlock economic information embedded in the reduced-form time series model. This paper serves as an introduction to this developing literature. The VAR model is shown to be a reduced-form for a linear simultaneous equations model. The contemporaneous and long-run approaches to identifying structural parameters are developed. Finally, estimates of contemporaneous and long-run structural VAR models using a common set of macroeconomic variables are presented. These models

[1]A VAR can be derived for a subset of the variables from a linear structural model. Furthermore, it is a linear approximation to any nonlinear structural model. The accuracy of the VAR approximation will depend on the features of the nonlinear structure.

[2]A Choleski decomposition of the covariance matrix for the VAR residuals.

are intended to provide a comparison between contemporaneous and long-run structural VAR modeling strategies. The implications of the empirical results are also discussed.

THE VAR REPRESENTATION OF A SIMULTANEOUS SYSTEM OF EQUATIONS

The standard, linear, simultaneous equations model is a useful starting point for understanding the structural VAR approach. A simultaneous equations system models the dynamic relationship between endogenous and exogenous variables. A vector representation of this system is

$$(1)\ Ax_t = C(L)x_{t-1} + Dz_t,$$

where x_t is a vector of endogenous variables and z_t is a vector of exogenous variables. The elements of the square matrix, A, are the structural parameters on the contemporaneous endogenous variables and C(L) is a kth degree matrix polynomial in the lag operator L, that is, $C(L) = C_0 + C_1L + C_2L^2 + \ldots + C_kL^k$, where all of the C matrices are square. The matrix D measures the contemporaneous response of endogenous variables to the exogenous variables.[3] In theory, some exogenous variables are observable while others are not. Observable exogenous variables typically do not appear in VARs because Sims (1980) argued forcefully against exogeneity. Hence, the vector z is assumed to consist of unobservable variables, which are interpreted as disturbances to the structural equations, and x_t and z_t are vectors with length equal to the number of structural equations in the model.[4]

A reduced-form for this system is

$$(2)\ x_t = A^{-1}C(L)\, x_{t-1} + A^{-1}Dz_t.$$

A particular structural specification for the "error term" z is required to obtain a VAR representation. Two alternative, commonly used and attractive assumptions are that shocks have either temporary or permanent effects. If shocks have temporary effects, z_t equals ε_t, a serially uncorrelated vector (vector white noise).[5] That is,

$$(3)\ z_t = \varepsilon_t.$$

Alternatively, z can be modeled as a unit root process, that is,

$$(4)\ z_t - z_{t-1} = \varepsilon_t.$$

Equation 4 implies that z equals the sum of all past and present realizations of ε. Hence, shocks to z are permanent. The assumptions in equations 3 and 4 are not as restrictive as they might appear. If these shock processes were specified as general autoregressions, the VARs would have additional lags. The procedures to identify structural parameters, however, would be unaffected.

Under the assumption that exogenous shocks have only temporary effects, equation 2 can be rewritten as,

$$(5)\ x_t = \beta(L)x_{t-1} + e_t,$$

where $\beta(L) = A^{-1}C(L)$ and $e_t = A^{-1}D\varepsilon_t$. The equation system in 5 is a VAR representation of the structural model because the last term in this expression is serially uncorrelated and each variable is a function of lagged values of all the variables.[6] The VAR coefficient matrix, $\beta(L)$, is a nonlinear function of the contemporaneous and the dynamic structural parameters.

If the shocks have permanent effects, the VAR model is obtained by applying the first difference operator ($\Delta = 1 - L$) to equation 2 and inserting equation 4 into the resulting expression, to obtain

$$(6)\ \Delta x_t = \beta(L)\Delta x_{t-1} + e_t,$$

with $\beta(L)$ and e_t previously defined.

This is a common VAR specification because many macroeconomic time series appear to have a unit root.[7] Because of the low power of tests

[3]This model can accommodate lags of z; this feature is omitted, however, to simplify the discussion.

[4]If observable exogenous variables exist, they are included as explanatory variables in the VAR.

[5]The individual elements in a vector white noise process, in theory, may be contemporaneously correlated. In structural VAR practice, they are typically assumed to be independent.

[6]The last term represents linear combinations of serially uncorrelated shocks, and these are serially uncorrelated as well. See any textbook covering the basics of time series analysis for a proof of this result.

[7]This model can also be written in levels form:

$$x_t = [A^{-1}C(L) + I]x_{t-1} - A^{-1}C(L)\, x_{t-2} + A^{-1}D\varepsilon_t.$$

Sims, Stock and Watson (1990) show that this reduced form is consistently estimated by OLS, but hypothesis tests may have non-standard distributions because the series have unit roots.

for unit roots, their existence is controversial. VARs can accommodate either side of the debate, however.[8]

The VAR is a general dynamic specification because each variable is a function of lagged values of all the variables. This generality, however, comes at a cost. Because each equation has many lags of each variable, the set of variables must not be too large. Otherwise, the model would exhaust the available data.[9] If all shocks have unit roots, equation 6 is estimated. If all shocks are stationary, equation 5 is used.[10] If some shocks have temporary effects while others have permanent effects, the empirical model must account for this.

Recently, Blanchard and Quah (1989) have estimated a VAR model where some variables were assumed to be stationary while others had unit roots. Alternatively, King, Plosser, Stock and Watson (1991) use a cointegrated model, where all the variables are difference stationary but some linear combinations of the variables are stationary. The stationary linear combinations are constructed by cointegration regressions prior to VAR estimation. They impose the cointegration constraints using the vector error-correction model of Engle and Granger (1987). Sometimes unit-root tests combined with theory suggest the coefficients for stationary linear combinations. Shapiro and Watson (1988), for example, present evidence that the nominal interest rate and inflation each have a unit root while the difference between these two variables is stationary. They impose the cointegration constraint by selecting this noisy proxy for the real rate of interest as a variable for the model.

Unrestricted versions of the VAR model (and the error-correction model) are estimated by ordinary least squares (OLS) because Zellner (1962) proved that OLS estimates of such a system are consistent and efficient if each equation has precisely the same set of explanatory variables. If the underlying structural model provides a set of over-identifying restrictions on the reduced form, however, OLS is no longer optimal.[11] The simultaneous equations system in a contemporaneous structural VAR, however, generally does not impose restrictions on the reduced form.

An alternative approach of Doan, Litterman and Sims (1984) estimates the VAR in levels with a Bayesian prior placed on the hypothesis that each time series has a unit root. The Bayesian VAR model permits more lags by imposing restrictions on the VAR coefficients, reducing the number of estimated parameters (called hyper-parameters). The reduction in parameters contributes to the Bayesian model's propensity to yield superior out-of-sample forecasts compared with unrestricted VARs with symmetric lag structure.

CONTEMPORANEOUS STRUCTURAL VAR MODELS

It is clear from equations 5 and 6 that, if the contemporaneous parameters in A and D were known, the dynamic structure represented by the parameters in C(L) could be calculated from the estimated VAR coefficients, that is, $C(L) = A\beta(L)$. Furthermore, the structural shocks, ε_t, could be derived from the estimated residuals, that is, $\varepsilon_t = D^{-1}Ae_t$. Because the coefficients in A and D are unknown, identification of structural parameters is achieved by imposing theoretical restrictions to reduce the number of unknown structural parameters to be less than or equal to the number of estimated parameters of the variance-covariance matrix of the VAR residuals. Specifically, the covariance matrix for the residuals, Σ_e, from either equation 5 or equation 6 is

$$(7)\ \Sigma_e = E[e_t e_t'] = A^{-1}DE[\varepsilon_t\varepsilon_t']D'A'^{-1} = A^{-1}D\Sigma_\varepsilon D'A'^{-1},$$

where E is the unconditional expectations operator, and Σ_ε is the covariance matrix for the shocks.

An OLS estimate of the VAR provides an estimate of Σ_e that can be used with equation 7 to obtain estimates of A, D and Σ_ε. The contemporaneous structural approach imposes restrictions on these three matrices. There are n^2 elements in A,

[8] Alternatively, the unit root could result from parameters in the dynamic structural model.

[9] The lag structure of a VAR can be shown to represent various sources of economic dynamics. Structural models with rational expectations predict restrictions on the VAR model. Dynamics in these models are often motivated by the costs of adjustment to desired or equilibrium positions. The lag structure of the VAR can also be motivated by dynamic processes for structural disturbances.

[10] VAR lag length is often selected by statistical criteria such as the modified likelihood ratio test of Sims (1980).

[11] A two-step structural VAR estimator will generally not be efficient if there are structural restrictions for C(L) since this implies restrictions on $\beta(L)$. For example, Sargent (1979) derives restrictions on VAR coefficients from a particular model of the term structure of interest rates under rational expectations. The full structural system is estimated by maximum likelihood.

n^2 elements in D, and $n(n+1)/2$ unique elements in Σ_ε, but only $n(n+1)/2$ unique elements in Σ_e. The maximum number of structural parameters is equal to the number of unique elements in Σ_e. Thus, a structural model will not be identified unless at least $2n^2$ restrictions are imposed on A, D and Σ_ε.

Often these restrictions are exclusion restrictions; of course, that need not be the case. Typically, Σ_ε is specified as a diagonal matrix because the primitive structural disturbances are assumed to originate from independent sources. The remaining parameters are identified by imposing additional restrictions on A and D. The main diagonal elements of A are set to unity because each structural equation is normalized on a particular endogenous variable. The main diagonal for D has this same specification since each equation has a structural shock. These normalizations provide 2n restrictions. Identification requires at least $3n(n-1)/2$ additional restrictions based on economic theory. Alternatively, the restrictions may be based on the contemporaneous information assumed available to particular economic agents following Sims (1986). Keating (1990) and West (1990) extend this approach by showing how rational expectations restrictions can be imposed in the contemporaneous structural VAR framework. Except for Bernanke (1986) and Blanchard (1989), existing models typically have not attempted to identify the structural parameters in D. Hence, D is usually taken to be the identity matrix, leaving at least $n(n-1)/2$ additional identifying restrictions to be imposed on A.

A two-step procedure is used to estimate structural VAR models. First, the reduced-form VAR, with enough lags of each variable to eliminate serial correlation from the residuals, is estimated with OLS. Next, a sufficient number of restrictions is imposed on A, D and Σ_ε to identify these parameters. This paper obtains the parameters in equation 7 with an algorithm for solving a nonlinear system of equations. Blanchard and Quah (1989) use this approach to estimate a structural VAR model.[12] Standard errors for the parameters, the impulse responses and the variance decompositions are calculated using the Monte Carlo approach of Runkle (1987), which simulates the VAR model to generate distributions for these results.[13]

The identification technique used in this paper is adequate for a model in which the number of parameters is equal to the number of unique elements in Σ_e. Alternative methods are needed to estimate a model with fewer parameters. Bernanke (1986) uses the method-of-moments approach of Hansen (1982) to estimate the parameters in equation 7 and obtain standard errors. Sims (1986) estimates the system of simultaneous equations for the residuals in equation 5 using maximum likelihood.[14] Blanchard and Watson (1986) also estimate the system of equations for residuals; however, they employ a sequential instrumental variables technique in which estimated structural shocks are used as instruments in all subsequent equations.[15]

The following four-equation contemporaneous structural VAR model is used to illustrate a particular set of such identifying restrictions. The residuals from a VAR consisting of the price level (p), output (y), the interest rate (r) and money (m) are used in the model. This model is used in the empirical work which follows. Equation 8 provides three restrictions by assuming that the price level is predetermined, except that producers can respond immediately to aggregate supply shocks. Equation 9 is a reduced-form IS equation that models output as a function of all the variables in the model. This approach was taken instead of explicitly modeling expected future inflation to calculate the real interest rate and explicitly modeling the term structure of interest rates to tie the short-term rate in the model with the long-term rate predicted by theory. The IS disturbance is also a factor in the output equation. The money supply function in equation 10 allows the Fed to adjust short-term interest rates to changes in the money stock. Two restrictions are obtained from assuming that the Fed does not immediately observe aggregate measures of output and price. The last equation is a short-run money demand function specifying nominal money holdings as

[12] Their model is identified by long-run restrictions.

[13] The actual residuals are randomly sampled, and the sampled residuals are used as shocks to the estimated VAR. After the artificial series are generated, they are used to perform the same structural VAR analysis. After 200 replications of the model, standard errors were calculated for the parameter estimates, the impulse responses and the variance decompositions.

[14] In contrast to the typical simultaneous equations model, this approach has no observable exogenous variables.

[15] This technique requires a structural model for which there are no estimated parameters in the first equation, the second equation has one parameter, the third has two parameters, etc. While the recursive model fits this description, this technique can estimate a much broader set of models.

a function of nominal GNP and the interest rate. This specification is motivated by a buffer stock theory where short-run money holdings rise in proportion to nominal income, yielding the final restriction for a just-identified model. Each equation includes a structural disturbance.

$$(8)\quad e_t^p = \varepsilon_t^{as}$$

$$(9)\quad e_t^y = A_1 e_t^p + A_2 e_t^r + A_3 e_t^m + \varepsilon_t^{is}$$

$$(10)\quad e_t^r = A_4 e_t^m + \varepsilon_t^{ms}$$

$$(11)\quad e_t^m = A_5(e_t^y + e_t^p) + A_6 e_t^r + \varepsilon_t^{md}$$

Standard VAR tools are employed after the structural parameters are estimated. Impulse response functions and variance decomposition functions conveniently summarize the dynamic response of the variables to the shocks, which is known as the moving average representation (MAR). The MAR for the VAR is obtained by applying simple algebra to a function of the lag operator. Take the VAR model for x:

$$x_t = \beta(L)x_{t-1} + e_t,$$

and subtract $\beta(L)x_{t-1}$ from both sides of this equation:

$$x_t - \beta(L)x_{t-1} = e_t.$$

Then factor terms in x_t using the lag operator,

$$[I - \beta(L)L]x_t = e_t.$$

Multiply both sides of this equation by the inverse of $[I-\beta(L)L]$:

$$x_t = [I - \beta(L)L]^{-1}e_t.$$

Insert the expression from equation 5 for e_t into this last equation:

$$(12)\quad x_t = [I - \beta(L)L]^{-1}A^{-1}D\varepsilon_t = \theta(L)\varepsilon_t,$$

where $\theta(L) = \sum_{i=0}^{\infty}\theta_i L^i$

and each θ_i is an n x n matrix of parameters from the structural model. Equation 12 implies that the response of x_{t+i} to ε_t is θ_i. Hence, the sequence of θ_i from i = 0,1,2,..., illustrates the dynamic response of the variables to the shocks. If the variables in x are stationary, then the impulse responses must approach zero as i becomes large.

Variance decompositions allocate each variable's forecast error variance to the individual shocks. These statistics measure the quantitative effect that the shocks have on the variables. If $E_{t-j}x_t$ is the expected value of x_t based on all information available at time t−j, the forecast error is:

$$x_t - E_{t-j}x_t = \sum_{i=0}^{j-1}\theta_i\,\varepsilon_{t-i},$$

since the information at time t−j includes all ε occurring at or before time t−j and the conditional expectation of future ε is zero because the shocks are serially uncorrelated. The forecast error variances for the individual series are the diagonal elements in the following matrix:

$$E(x_t - E_{t-j}x_t)(x_t - E_{t-j}x_t)' = \sum_{i=0}^{j-1}\theta_i\Sigma_\varepsilon\theta_i'.$$

If θ_{ivs} is the (v,s) element in θ_i and σ_s is the standard deviation for disturbance s (s = 1,...,n), the j-steps-ahead forecast variance of the v-th variable is easy to calculate:

$$E(x_{vt} - E_{t-j}x_{vt})^2 = \sum_{i=0}^{j-1}\sum_{s=1}^{n}\theta_{ivs}^2\sigma_s^2 \qquad v = 1, 2,..., n$$

The variance decomposition function (VDF) writes the j-steps-ahead percentage of forecast error variance for variable v atributable to the k-th shock:

$$(13)\quad VDF(v,k,j) = \frac{\sum_{i=0}^{j-1}\theta_{ivk}^2\sigma_k^2}{\sum_{i=0}^{j-1}\sum_{s=1}^{n}\theta_{ivs}^2\sigma_s^2} \times 100.$$

The same analysis can be used to derive the MAR for the VAR model in equation 6.

$$(14)\quad \Delta x_t = \theta(L)\varepsilon_t,$$

where $\theta(L) = [I-\beta(L)L]^{-1}A^{-1}D$. The response of x, rather than the change in x, is frequently of greater interest to economists. These impulse responses can be generated recursively by assuming that all the elements of ε at time zero and earlier are equal to zero.[16] For example,

[16]If the pre-sample ε is nonzero, its effects are lumped together with x_0 which represents the initial conditions.

$$x_1 = x_0 + \theta_0\varepsilon_1$$

and

$$x_2 = x_1 + \theta_0\varepsilon_2 + \theta_1\varepsilon_1.$$

Inserting the expression for x_1 into the x_2 equation yields:

$$x_2 = x_0 + \theta_0\varepsilon_2 + (\theta_0 + \theta_1)\varepsilon_1.$$

Repeating this operation for all x up to x_t, yields the following:

$$x_t = x_0 + \theta_0\varepsilon_t + (\theta_0 + \theta_1)\varepsilon_{t-1} + \dots + (\sum_{j=0}^{t-1} \theta_j)\varepsilon_1.$$

This result is equivalently written as

$$(15)\ x_t = x_0 + \Gamma(L)\varepsilon_t = x_0 + \sum_{i=0}^{t-1} \Gamma_i\varepsilon_{t-i},$$

where $\Gamma_i = \sum_{j=0}^{i} \theta_j$.

The response of x_{t+i} to ε_t is Γ_i. Since the differenced specification assumes that Δx is stationary, the θ_j matrix goes to zero as j gets large. This implies that Γ_i converges to the sum of coefficients in $\theta(L)$. Restrictions on this sum of coefficients are used to identify long-run structural VAR models. The variance decompositions for this model are identical to equation 13 except that θ is replaced by Γ.

In contrast to the atheoretical VAR models developed by Sims (1980), the structural approach yields impulse responses and variance decompositions that are derived using parameters from an explicit economic model.[17] Finding dynamic patterns consistent with the structural model used for identification would provide evidence in support of the theoretical model. Otherwise, the theory is invalid or the empirical model is somehow misspecified.

LONG-RUN STRUCTURAL VAR MODELS

Shapiro and Watson (1988) and Blanchard and Quah (1989) developed the alternative approach of imposing identifying restrictions on long-run multipliers for structural shocks. An advantage is that these models do not impose contemporaneous restrictions, but they allow the data to determine short-run dynamics based conditionally on a particular long-run model.[18]

If each shock has a permanent effect on at least one of the variables and if cointegration does not exist for the variables in x, the VAR in equation 6 can be estimated.[19] The impulse response function for x in equation 15 shows that the long-run effect of ε converges to the sum of coefficients in $\theta(L)$. It is obvious from the definition of $\theta(L)$ that replacing L by one yields the sum of coefficients. Hence, this sum is conveniently written as $\theta(1)$, and this matrix is used to parameterize long-run restrictions. The relationship between parameters of the structural MAR, contemporaneous structural parameters and VAR lag coefficients is given by

$$(16)\ \theta(L) = [I - \beta(L)L]^{-1}A^{-1}D.$$

The long-run multipliers are obtained by replacing L in equation 16 with unity.

Setting L equal to unity, solving equation 16 for $A^{-1}D$ and inserting the result into equation 7 yields

$$(17)\ [I-\beta(1)]^{-1}\Sigma_e\,[I-\beta(1)]^{-1\prime} = \theta(1)\Sigma_\varepsilon\theta(1)',$$

where the matrix $\beta(1)$ is the sum of VAR coefficients.

This equation can be used to identify the parameters in $\theta(1)$ and Σ_ε. A minimal set of restrictions on the long-run response of macroeconomic variables to structural disturbances is used to identify long-run structural VAR models. Estimates of the matrices on the left side of

[17]The relationship between structural and atheoretical VARs is addressed in the shaded insert at right.

[18]For example, agents may temporarily be away from long-run equilibrium positions or monetary policy may be non-neutral in the short run.

[19]Unit-root tests and cointegration tests support this assumption for the time series used in this paper. See Keating (1992) for this evidence.

The Relationship between Atheoretical and Structural VAR Approaches

Atheoretical VAR practitioners separate the residuals into orthogonal shocks by calculating a Choleski decomposition of the covariance matrix for the residuals. This decomposition is obtained by finding the unique lower triangular matrix λ that solves the following equation:

$$\Sigma_e = \lambda\lambda'.$$

This statistical decomposition depends on the sequence in which variables are ordered in x. The residuals' covariance matrix from a VAR ordered by output, the interest rate, money and the price level yields a Choleski decomposition that is algebraically equivalent to estimating the following four equations by ordinary least squares:

$$
\begin{aligned}
e^y_t &= \nu^y_t \\
e^r_t &= R_1 e^y_t + \nu^r_t \\
e^m_t &= R_2 e^y_t + R_3\, e^r_t + \nu^m_t \\
e^p_t &= R_4 e^y_t + R_5 e^r_t + R_6 e^m_t + \nu^p_t.
\end{aligned}
$$

Hence, each ν shock is uncorrelated with the other shocks by construction. This system implies that the first variable responds to its own exogenous shock, the second variable responds to the first variable plus an exogenous shock to the second variable, and so on. In practice, atheoretical VAR studies report results from various orderings. The total number of possible orderings of the system is n!, a number that increases rapidly with n.[1] Investigators sometimes note that certain properties of the model are insensitive to alternative orderings. Results sensitive to VAR orderings are difficult to interpret, especially if a recursive economic structure is implausible.

This atheoretical approach has been criticized by Cooley and LeRoy (1985). First, if the Choleski technique is in fact atheoretical, then the estimated shocks are not structural and will generally be linear combinations of the structural disturbances.[2] In this case, standard VAR analysis is difficult to interpret because the impulse responses and variance decompositions for the Choleski shocks will be complicated functions of the dynamic effects of all the structural disturbances. The second point attacks the claim that Choleski decompositions are atheoretical. The Choleski ordering can be interpreted as a recursive contemporaneous structural model. Unfortunately, most economic theories do not imply recursive contemporaneous systems. Such criticisms of the atheoretical approach inspired structural approaches to VAR modeling. If therory predicts a contemporaneous recursive economic structure, a particular Choleski factorization of the covariance matrix for the residuals is appropriate. But a researcher using the structural approach would not experiement with various orderings, unless these specifications were predicted by alternative theories.

[1]For example, 3! = 6 but 6! = 720.

[2]This result is easy to prove. The Choleski decomposition yields a system in which $e_t = R\nu_t$ but the true structural model is $e_t = A^{-1}D\varepsilon_t$, implying that the shocks from the Choleski decomposition are linear combinations of the structural distrubances; $\nu_t = R^{-1}A^{-1}D\varepsilon_t$.

equation 17 are obtained directly from the unconstrained VAR.[20] $\theta(1)$ has n^2 elements and Σ_ε has $n(n+1)/2$ unique elements. The $n(n+1)/2$ unique elements in the symmetric matrix on the left side of equation 17 is the number of parameters in a just-identified model.[21] Thus, at least n^2 identifying restrictions must be applied to $\theta(1)$ and Σ_ε. The elements of the main diagonal for $\theta(1)$ can each be set equal to one, analogously to the normalization used in the contemporaneous model. If each element of ε is assumed to be independent, then Σ_ε is diagonal. Hence, $n(n-1)/2$ additional restrictions are needed for $\theta(1)$ to identify the model.

Several alternative approaches for obtaining the structural parameters have been developed. Shapiro and Watson (1988) impose the long-run zero restrictions on $\theta(1)$ by estimating the simultaneous equations model with particular explanatory variables differenced one additional time. King, Plosser, Stock and Watson (1991) impose long-run restrictions using the vector error-correction model with some of the long-run features of the model chosen by cointegration regressions. Gali (1992) combines contemporaneous restrictions with long-run restrictions to identify a structural model. In the empirical section, we use the approach developed by Blanchard and Quah (1989).

Equations 18 through 21 present the long-run identifying restrictions used in the empirical example.[22] The time subscripts are omitted because the restrictions pertain to long-run behavior. Three restrictions come from equation 18, which specifies that aggregate supply shocks are the sole source of permanent movements in output.[23] Two more restrictions are obtained from the long-run IS or spending balance equation, 19, which specifies the interest rate as a function of output and the IS shock.[24] Note the coefficient S_1 should be negative. The final restriction comes from the money demand function, 20, which sets real money equal to an increasing function of output, a decreasing function of the interest rate, and a money demand shock. Equation 21 allows the supply of money to respond to all variables in the model and a money supply shock.[25]

$$(18)\ y = \varepsilon^{as}$$

$$(19)\ r = S_1 y + \varepsilon^{is}$$

$$(20)\ m-p = S_2 y + S_3 r + \varepsilon^{md}$$

$$(21)\ m = S_4 y + S_5 r + S_6(m-p) + \varepsilon^{ms}$$

EMPIRICAL EXAMPLES AND RESULTS

The examples from the previous two sections are estimated to illustrate the long-run and contemporaneous identification methods. Both models use real GNP to measure output, the GNP deflator for the price level, M1 as a measure of the stock of money, and the three month Treasury bill rate determined in the secondary market as the interest rate. The data are first-differenced. Statistical tests suggest that this transformation makes the data stationary.[26] The first step is to estimate the reduced-form VAR model. The estimated variance-covariance matrix from the reduced form is used to obtain the second-stage structural estimates. Four lags are used for the VAR model and the sample-period is from the first quarter of 1959 to the third quarter of 1991.

A Long-Run Structural Model

Table 1 reports the parameter estimates for the long-run model in equations 18 through 21. The first four parameters are standard deviations for the structural shocks, and each of these

[20] The Bayesian approach is not employed since a unit root is required to be certain that shocks have permanent effects.

[21] An over-identified long-run model will imply restrictions on the reduced-form coefficients.

[22] This is a simplified version of the model in Keating (1992).

[23] If interest rates affect capital accumulation, then IS shocks may permanently affect output and the restrictions in equation 19 may not be appropriate. If this is the only misspecification of the model, actual money supply and money demand shocks will be identified but the empirical aggregate supply and IS shocks will be mixtures of these real structural disturbances.

[24] Technically, the IS equation should use the real interest rate, an unobservable variable. However, if permanent movements in the nominal rate and the real rate are identical, this specification is legitimate. This would be true, for example, if the Fisher equation held and if inflation followed a stationary time series process.

[25] Thus, $\theta(1)$ is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -S_1 & 1 & 0 & 0 \\ -S_2 & -S_3 & 1 & 0 \\ -S_4 & -S_5 & -S_6 & 1 \end{bmatrix}^{-1}$$

[26] For empirical evidence, see the unit-root tests in Keating (1992).

Table 1
Estimates for the Long-Run Model

	Parameter	Standard error
σ_{as}	.0144*	.0041
σ_{is}	.0092*	.0028
σ_{md}	.0149*	.0031
σ_{ms}	.0172*	.0042
S_1	−.1171	.3068
S_2	.8722*	.4219
S_3	−2.276 *	.7083
S_4	−1.411 *	.5778
S_5	1.569	1.020
S_6	.9048*	.3842

NOTE: An asterisk (*) indicates significance at the 5 percent level.

estimates is significantly different from zero. The coefficient in the IS equation, S_1, is negative as hypothesized, but insignificantly different from zero. Each parameter in the money demand function is statistically significant and has the sign predicted by economic theory. The coefficient on real GNP, S_2, is not statistically different from one. Parameters for the money supply equation can be interpreted as a policy reaction function in which the Fed reduces money if output rises but increases money if interest rates rise. This last effect is not statistically significant. The increase in money in response to an increase in real money may reflect the fact that the Fed has typically smoothed interest rate fluctuations in the post-war period, so that a money demand shock that raises real money will be accommodated by a comparable increase in nominal money.

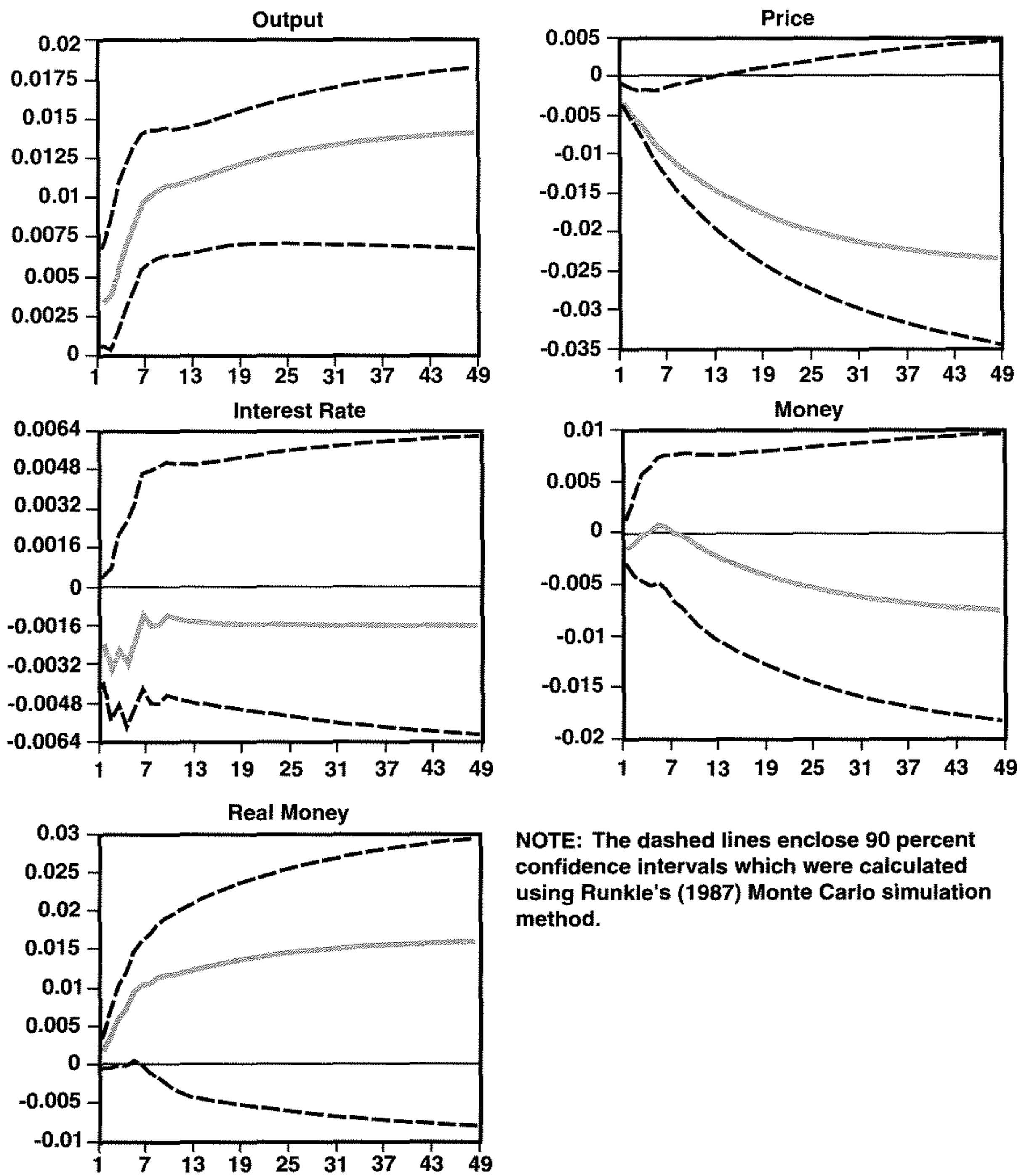
The impulse responses for the long-run model are shown in figures 1 through 4. Point estimates and 90 percent confidence intervals are graphed for the variables. If the long-run parameter estimates are consistent with the theoretical model, the asymptotic properties of the impulse responses must also be consistent with the theory. Economic restrictions are not imposed on the dynamic properties of the model. The empirical aggregate supply shock raises output and real money and lowers the interest rate, the price level and nominal money. The real spending shock raises output only temporarily because of the restriction that aggregate supply shocks are the only factor in long-run output movements. The interest rate and the price level rise, while the nominal and real measures of money decline after each variable initially rises by a small amount. The money demand shock has a strong positive effect on nominal and real money. The other effects are relatively weak, with prices falling, output temporarily falling and the interest rate temporarily rising. Money and the price level both rise in response to an increase in the money supply, which also causes a temporary decline in the interest rate and a temporary increase in output and real money. The impulse response functions provide evidence that the shocks affect each variable as theory predicts.

The variance decompositions in table 2 show the average amount of the variance of each variable attributable to each shock. Standard errors for these estimates are in parentheses. The output variance due to the supply shock one quarter in the future is only 17 percent. Eight quarters in the future, however, the estimate becomes nearly half of output's variance and, at 48 quarters, 90 percent of the variance of output is attributed to supply shocks. Variability in the price level is dominated by aggregate supply shocks, particularly in the short run. The other shocks have temporary output effects. This long-run feature is obtained because the model forces aggregate supply shocks to explain all permanent output movements. Short-run output movements are dominated by real spending shocks. This shock explains most of the interest rate variance and the variance of real money in the long run. The money supply shock has a gradual effect on output that peaks at 13 percent of the variance two years in the future. This shock accounts for a large portion of the short-run variance of the interest rate and the long-run movement in nominal money. The money demand shock has virtually no effect on output, interest rates or prices. Many theories predict that money demand shocks will not have much effect on these variables if the Fed uses the interest rate as its operating target. Money demand shocks have strong effects on nominal money and real money. In general, the results for this model are consistent with most economists' views about economic behavior, although some might be surprised by the relatively small effect on output of money supply disturbances.

Contemporaneous Structural Model

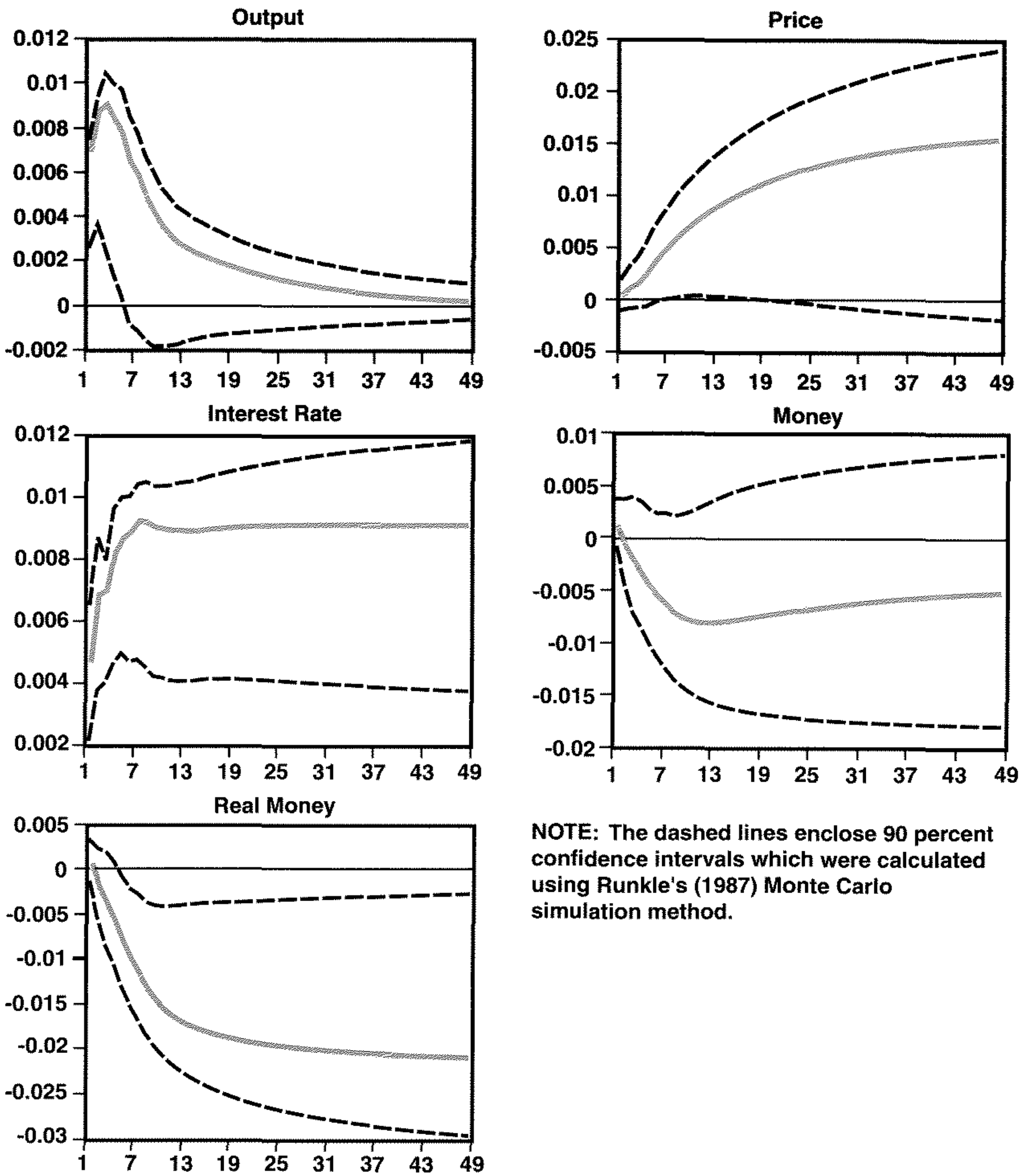
The parameter estimates for the contemporaneous model in equations 8 through 11 are reported in table 3. The coefficients in the reduced-form IS equation are all negative. The negative

Figure 1
Responses to an Aggregate Supply Shock in the Long-Run Model



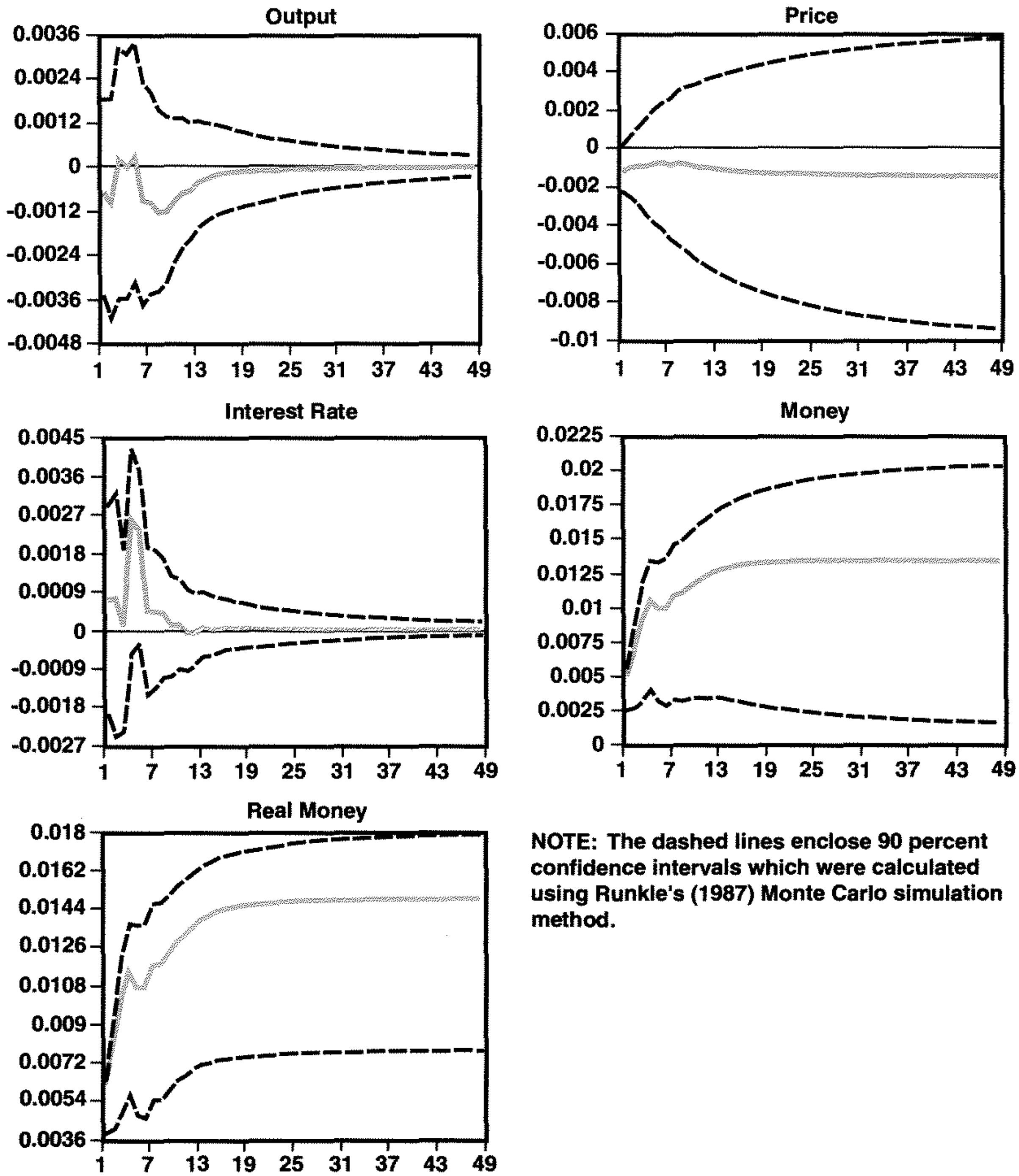
NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

Figure 2
Responses to a Real Spending Shock in the Long-Run Model



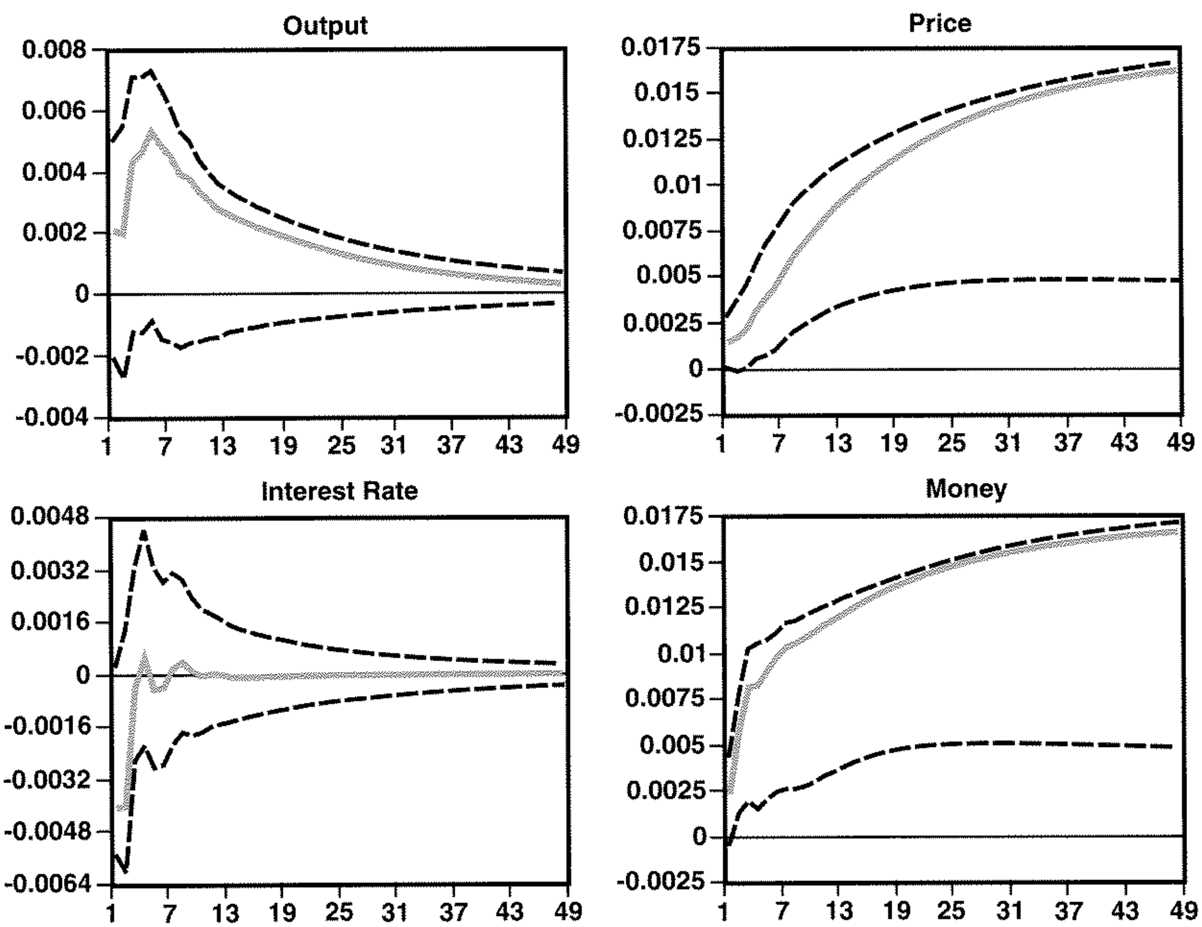
NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

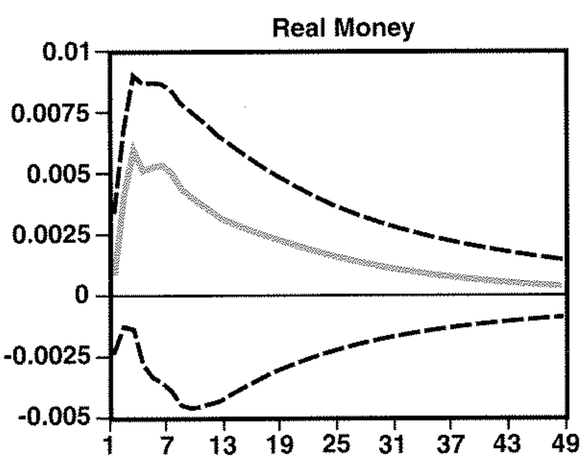
Figure 3
Responses to a Money Demand Shock in the Long-Run Model



NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

Figure 4
Responses to a Money Supply Shock in the Long-Run Model





NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

Table 2

Variance Decompositions for the Long-Run Model

Variable	Quarter(s) ahead	Aggregate supply shock	Real spending shock	Money demand shock	Money supply shock
Output					
	1	17 (24)	76 (24)	1 (8)	7 (16)
	2	16 (23)	78 (24)	1 (8)	5 (14)
	4	24 (25)	65 (24)	0 (6)	11 (14)
	8	45 (23)	42 (19)	0 (4)	13 (11)
	16	67 (15)	24 (13)	0 (2)	9 (7)
	32	84 (8)	11 (7)	0 (1)	5 (3)
	48	90 (5)	7 (5)	0 (1)	3 (2)
Interest Rate					
	1	13 (13)	49 (25)	1 (8)	37 (23)
	2	14 (13)	57 (23)	1 (7)	27 (19)
	4	13 (11)	71 (15)	3 (5)	13 (9)
	8	8 (10)	84 (12)	2 (3)	6 (6)
	16	5 (11)	91 (12)	1 (2)	3 (4)
	32	4 (13)	94 (13)	1 (1)	1 (2)
	48	4 (14)	95 (14)	0 (1)	1 (1)
Real Money					
	1	7 (11)	2 (10)	90 (17)	2 (11)
	2	12 (14)	3 (8)	73 (18)	12 (14)
	4	18 (17)	8 (12)	59 (19)	14 (14)
	8	26 (19)	24 (17)	41 (18)	9 (10)
	16	25 (19)	38 (20)	33 (18)	4 (5)
	32	26 (21)	44 (21)	29 (19)	2 (3)
	48	27 (22)	45 (21)	28 (19)	1 (1)
Money					
	1	6 (10)	5 (14)	73 (22)	15 (20)
	2	3 (9)	2 (10)	63 (22)	32 (21)
	4	1 (9)	3 (11)	59 (23)	38 (22)
	8	0 (10)	10 (15)	50 (21)	40 (21)
	16	1 (11)	14 (17)	46 (20)	39 (20)
	32	4 (13)	11 (17)	41 (19)	44 (20)
	48	6 (15)	9 (17)	39 (19)	46 (20)
Price					
	1	73 (27)	2 (12)	9 (13)	16 (23)
	2	78 (27)	4 (14)	5 (10)	12 (21)
	4	77 (28)	7 (16)	2 (8)	13 (21)
	8	69 (28)	15 (19)	1 (7)	16 (20)
	16	60 (27)	20 (21)	0 (7)	20 (19)
	32	55 (27)	22 (21)	0 (7)	23 (19)
	48	54 (27)	22 (22)	0 (7)	24 (19)

NOTE: Standard errors are in parentheses.

Table 3
Estimates for the Contemporaneous Model

	Parameter	Standard error
σ_{as}	.0038*	.0003
σ_{is}	.0086*	.0027
σ_{ms}	.0087	.0419
σ_{md}	.0087	.0324
A_1	−.1164	.3255
A_2	−.0469	.4122
A_3	−.3327	.6539
A_4	1.030	8.302
A_5	.5632	2.098
A_6	−.9397	5.737

NOTE: An asterisk (*) indicates significance at the 5 percent level.

coefficient on money would be unexpected in a structural IS equation; however, these estimates are reduced-form coefficients, not structural parameters. The coefficient on money in the interest rate equation is positive. This supports the view that the central bank attempts to stabilize money growth by raising interest rates. In the money demand equation, the coefficient on nominal spending is roughly one-half, and the interest rate coefficient is almost −1.0. Hence, the parameter estimates in this structural model are consistent with economic theory. Unfortunately, each of these structural parameters is statistically insignificant.

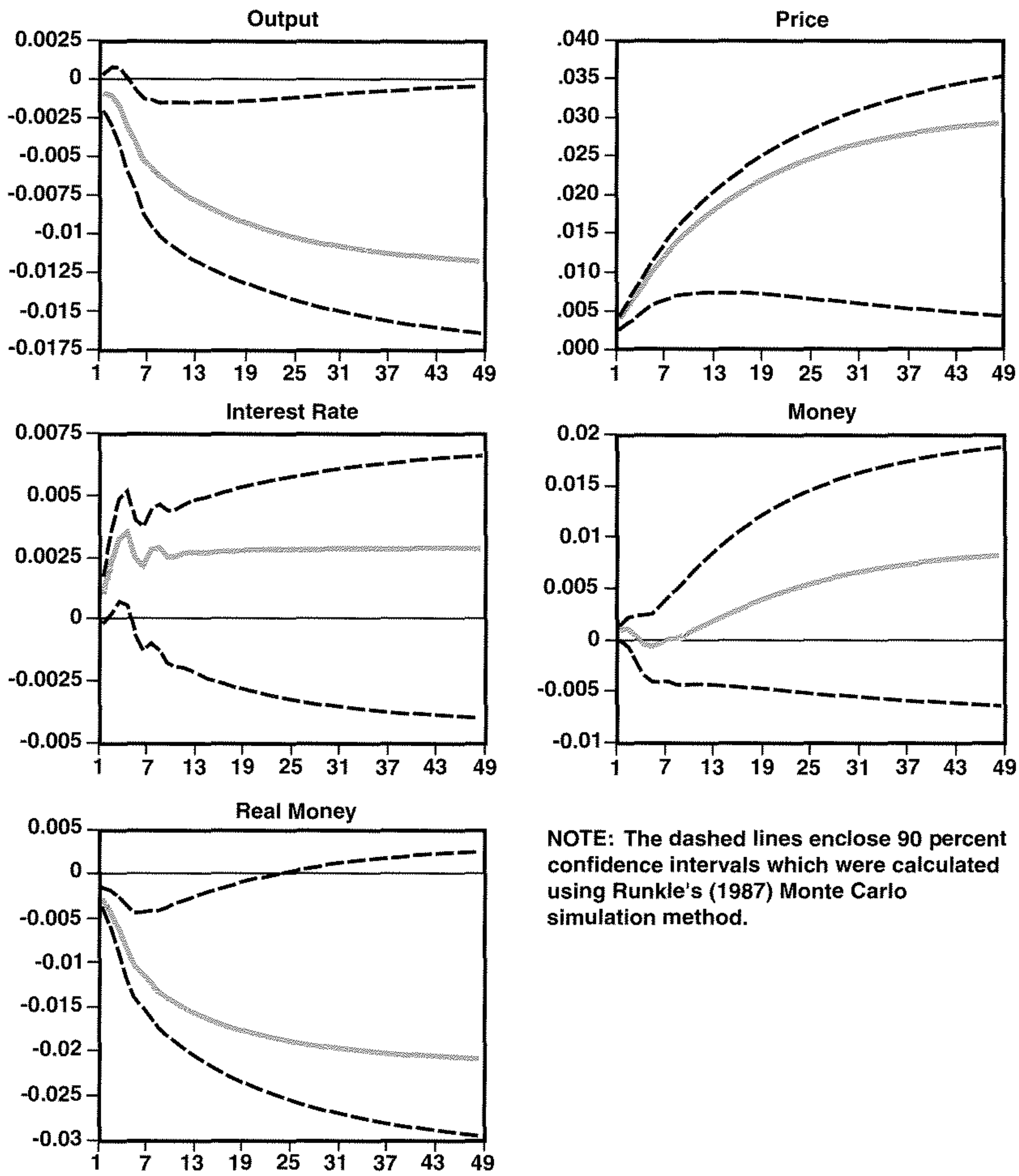
Figures 5 through 8 plot the impulse responses. In contrast to the long-run model, the aggregate supply equation is normalized on the price level. Hence, an aggregate supply shock raises the price level and reduces output. The aggregate supply shock has this expected effect on prices and output. Real money also decreases. The adverse supply shock has a weak positive effect on money and the interest rate. The IS shock raises prices, output and the interest rate. Real and nominal money initially increase, although both subsequently fall. The money supply equation is normalized on the interest rate so a reduction in the supply of money raise interest rates. This shock raises the interest rate and causes a decline in nominal money, real money and the price level. Surprisingly, output rises briefly before it begins to decline. The money demand shock causes the interest rate, nominal and real money to increase while output falls. The rising price level is inconsistent with theory, although this effect is not statistically significant. In contrast with the long-run model, there are a few unusual features in the impulse responses for the contemporaneous specification. Most of the dynamic patterns, however, are consistent with the structural model.

The variance decompositions for the contemporaneous model are shown in table 4. Many features of this table are comparable to the long-run model's results. For example, the aggregate supply shock gradually explains most of output's variability, is the most important shock for the price level and is never an important source of interest rate movements. The IS shock is the most important source of short-run output movement, and it explains most of the long-run variance of the interest rate. Some results, however, are considerably different compared with the results from the long-run model. The money demand shock has its greatest effect on output in the long run. This shock explains a large amount of the short-run variance of the interest rate but virtually none of the long-run variance of real or nominal money balances. The money supply shock has essentially no effect on output, while accounting for a large amount of the variance in real money, even in the long run, and none of the variance of prices. These results are inconsistent with most macroeconomic theories.

CONCLUDING REMARKS

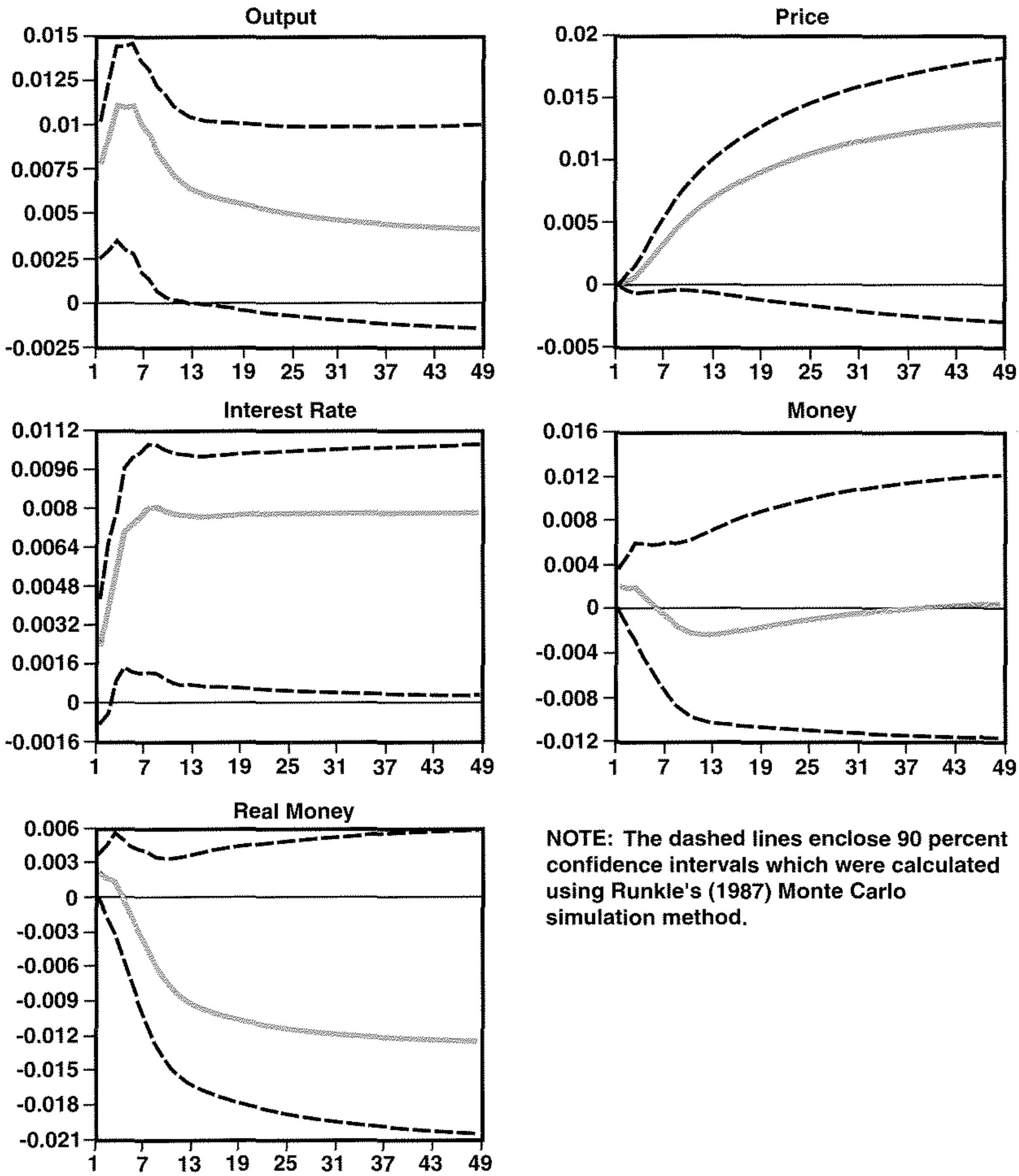
This paper outlines the basic theory behind structural VAR models and estimates two models using a standard set of macroeconomic data. The results for the two specifications are often similar. The long-run structural VAR model in this paper generally provides empirical results that are consistent with the structural model. Some of the variance decompositions and the impulse responses for the contemporaneous model were inconsistent with standard macroeconomic theory. These inconsistencies pertain to the effects of money supply and money demand disturbances. Another result is that structural parameters in the long-run model are more precisely estimated than parameters in the contemporaneous model. Wherever a significant discrepancy exists between the two models, the model with long-run restrictions yields sensible results, while the results from the contemporaneous model are inconsistent with standard economic theories.

Figure 5
Responses to an Aggregate Supply Shock in the Contemporaneous Model



NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

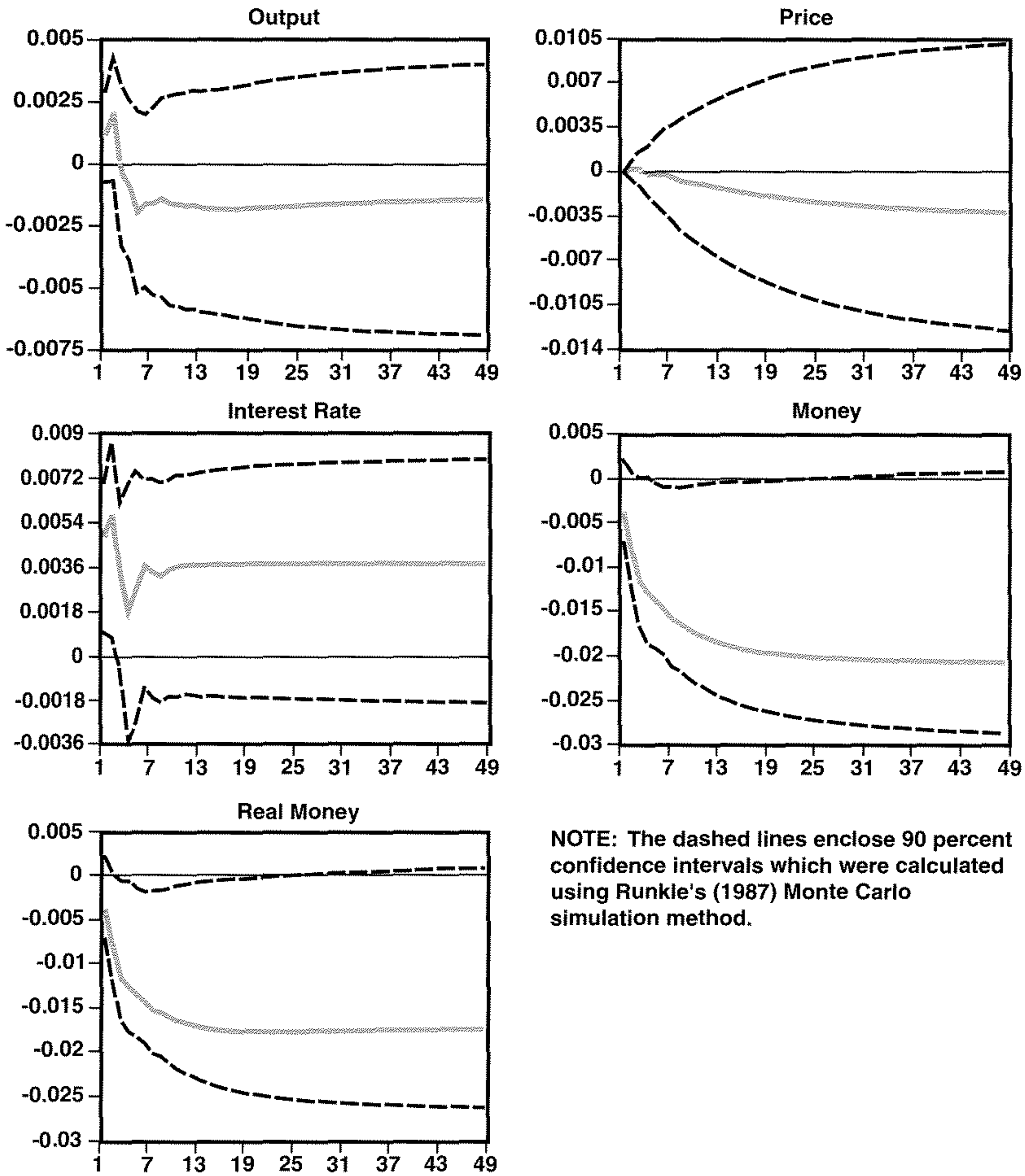
Figure 6
Responses to a Real Spending Shock in the Contemporaneous Model



NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

Figure 7

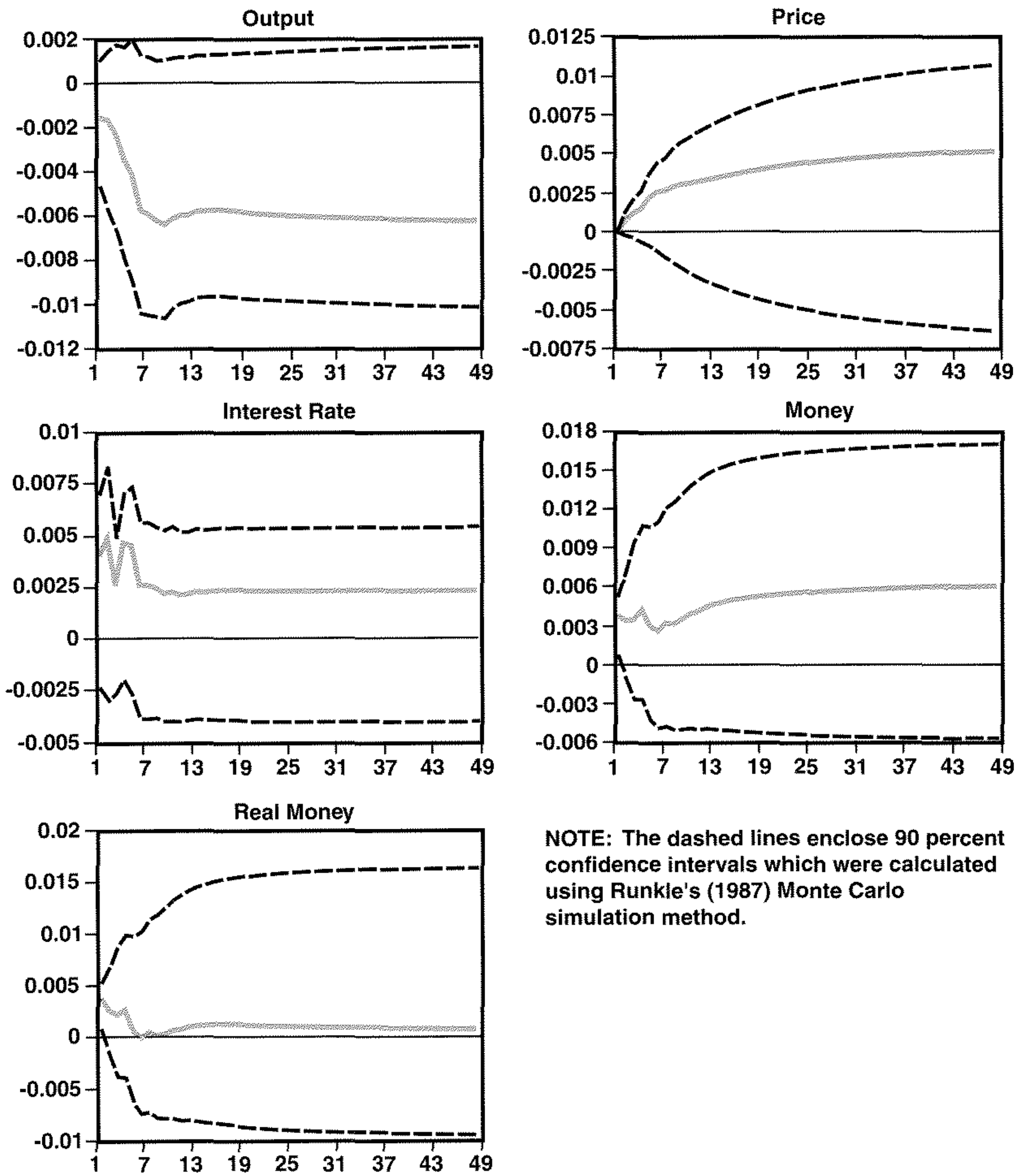
Responses to a Money Supply Shock in the Contemporaneous Model



NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

Figure 8

Responses to a Money Demand Shock in the Contemporaneous Model



NOTE: The dashed lines enclose 90 percent confidence intervals which were calculated using Runkle's (1987) Monte Carlo simulation method.

Table 4

Variance Decompositions for the Contemporaneous Model

Variable	Quarter(s) ahead	Aggregate supply shock	Real spending shock	Money demand shock	Money supply shock
Output					
	1	1 (2)	94 (16)	4 (14)	2 (4)
	2	1 (3)	93 (16)	3 (13)	3 (5)
	4	3 (5)	91 (15)	5 (13)	1 (3)
	8	12 (10)	73 (19)	13 (14)	2 (5)
	16	28 (16)	51 (20)	19 (14)	2 (7)
	32	47 (21)	31 (21)	20 (14)	2 (9)
	48	55 (23)	23 (22)	20 (14)	2 (10)
Interest Rate					
	1	2 (2)	12 (12)	37 (26)	49 (31)
	2	4 (4)	17 (13)	34 (24)	44 (30)
	4	10 (7)	38 (15)	27 (18)	25 (23)
	8	8 (6)	56 (18)	19 (16)	17 (20)
	16	8 (7)	63 (20)	12 (15)	16 (20)
	32	8 (10)	67 (22)	9 (14)	16 (20)
	48	8 (11)	68 (22)	8 (14)	15 (20)
Real Money					
	1	19 (7)	11 (14)	36 (21)	34 (22)
	2	18 (8)	5 (11)	16 (18)	60 (20)
	4	24 (11)	2 (8)	6 (16)	68 (19)
	8	34 (13)	4 (7)	2 (14)	61 (19)
	16	38 (16)	10 (10)	1 (15)	51 (21)
	32	42 (20)	14 (12)	0 (15)	44 (23)
	48	45 (21)	15 (12)	0 (16)	40 (24)
Money					
	1	2 (2)	14 (15)	44 (24)	41 (31)
	2	1 (2)	7 (12)	23 (22)	68 (27)
	4	0 (2)	3 (10)	13 (21)	83 (26)
	8	0 (3)	1 (9)	7 (20)	92 (25)
	16	1 (6)	1 (11)	6 (20)	92 (25)
	32	4 (10)	1 (11)	7 (20)	89 (25)
	48	7 (13)	0 (11)	7 (20)	86 (25)
Price					
	1	100 (0)	0 (0)	0 (0)	0 (0)
	2	98 (2)	0 (1)	2 (1)	0 (1)
	4	95 (4)	2 (2)	3 (3)	0 (2)
	8	89 (7)	6 (6)	4 (5)	0 (5)
	16	84 (10)	12 (8)	4 (6)	0 (7)
	32	82 (11)	14 (9)	3 (7)	1 (9)
	48	81 (12)	15 (10)	3 (7)	1 (9)

NOTE: Standard errors are in parentheses.

These comparisons between contemporaneous and long-run specifications may not generalize to all structural VAR applications, but they suggest that long-run structural VARs may yield theoretically predicted results more frequently than VARs identified with short-run restrictions. This result is not surprising. One reason is that economic theories may often have similar long-run properties but different short-run features. For example, while output movements are driven solely by aggregate supply shocks in a typical real business cycle model, supply shocks will account for permanent output movements in Keynesian models, but every shock may have some cyclical effect. In addition, long-run structural VAR models may provide superior results because they typically do not impose contemporaneous exclusion restrictions. Keating (1990) shows that contemporaneous "zero" restrictions may be inappropriate in an environment with forward-looking agents who have rational expectations. The intuition behind this result is that any observable contemporaneous variable may provide information about future events. One implication from that paper is that different short-run restrictions can be obtained from alternative assumptions about available information. Further research should investigate other examples of contemporaneous and long-run structural VAR models to determine whether the superior performance of this paper's long-run model is a special case or a more general result.

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