A Comparison of the St. Louis Model and Two Variations: Predictive Performance and Policy Implications

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THE St. Louis Model was first published in the Federal Reserve Bank of St. Louis *Review* in April 1970.[1] This model, with modifications, has been used for years at the St. Louis Fed to provide alternate scenarios for the response of inflation, output and the unemployment rate under different monetary policy assumptions. In addition, it continues to be identified by those outside the St. Louis Federal Reserve as *the* model underlying the Bank's policy prescriptions.

This article has three basic themes. First, the structure of the St. Louis Model can be simplified and its predictive performance improved. Second, the St. Louis Model's specification of the demand slack variable in its Phillips Curve may bias the equation's estimate of inflation's response to demand slack and, therefore, could yield an overly optimistic assessment of the cost of reducing inflation in terms of the higher unemployment during the transition to a lower rate of inflation. Third, a monetarist reduced-form equation for inflation, in which inflation depends directly on current and past monetary growth, is not inconsistent with the existence of a Phillips Curve. This is demonstrated by comparing the predictive performance and policy implications of two variations of the St. Louis Model — one incorporating a Phillips Curve, the other a monetarist reduced-form for inflation. Both versions outperform the St. Louis Model's inflation predictions, and both yield nearly identical predictions and policy multipliers.

This article is organized as follows: The first section reviews the current version of the St. Louis Model. The second section introduces two alternative versions of the St. Louis-type model. The first version substitutes a simplified Phillips Curve for the St. Louis Model's price-change equation; the second version introduces a simple reduced-form equation for inflation in place of the Phillips Curve. The third section compares the predictive performance and policy implications of these three models. The final section summarizes our findings.

THE CURRENT VERSION OF THE ST. LOUIS MODEL

The St. Louis Model consists of five estimated equations and a number of identities. The key equations are the Andersen-Jordan or St. Louis nominal income reduced-form equation and the equation for the change in the price level. There are also equations for the unemployment rate, the long- and short-term interest rates, the anticipated change in the price level and the change in output. The only significant change since the model was introduced has been the substitution of a rate of change (or dot) version of the Andersen-Jordan nominal income

The research reported here was begun while Laurence H. Meyer was a Visiting Scholar at the Federal Reserve Bank of St. Louis. The results do not necessarily reflect the views of the St. Louis Federal Reserve Bank staff, nor should the models presented be viewed as new versions of the St. Louis Model.

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[1]Leonall C. Andersen and Keith M. Carlson, "A Monetarist Model for Economic Stabilization," this *Review* (April 1970), pp. 7-25.

reduced-form equation for the original first difference (or delta) version.[2]

The Andersen-Jordan Equation

The Andersen-Jordan equation is currently specified in rate-of-change or dot form; compound annual rates of change are used for nominal income (Y), the money supply (M1B is the definition of money currently used with the St. Louis Model, M), and the high-employment level of government expenditures (G). Dots over a variable indicate compound annual rates of change.

$$\text{(STL-1)}\quad \dot{Y}_t = a_0 + \sum_{i=0}^{4} a_{1i}\dot{M}_{t-i} + \sum_{i=0}^{4} a_{2i}\dot{G}_{t-i}$$

The parameter estimates (t-values in parentheses) for the equation estimated from I/1955 through IV/1980 are as follows:[3]

a_0 =	2.87	(3.26)			
a_{10} =	.46	(4.32)	a_{20} =	.061	(1.61)
a_{11} =	.45	(6.49)	a_{21} =	.048	(1.66)
a_{12} =	.24	(2.51)	a_{22} =	−.001	(−.034)
a_{13} =	.026	(.398)	a_{23} =	−.05	(−1.94)
a_{14} =	−.071	(−.12)	a_{24} =	−.06	(−1.78)
Σa_{1i} =	1.12	(7.44)	Σa_{2i} =	−.003	(−0.038)

R^2 = .44 SE = 3.6 DW = 2.04

The coefficients on the $\dot{M}$ variables approximately sum to unity while the coefficients on $\dot{G}$ sum approximately to zero. Thus, the estimated coefficients support the general conclusions associated with a monetarist viewpoint: Monetary change is the key variable explaining nominal income movements while fiscal variables have at best a minor and transitory effect.[4]

The Inflation Sector

The inflation sector of the St. Louis Model includes three equations: a price-change version of a Phillips Curve, an identity defining the anticipated change in the price level and an equation for the long-term interest rate. The weights in the distributed lag of inflation in the long-term interest rate equation are used to construct the anticipated-price-change variable; this variable, in turn, is included as an argument in the price-change equation. This structure is unnecessarily complicated. The predictive performance of the model with respect to inflation can be improved with a simpler and more conventional specification of the Phillips Curve in which the weights on the distributed lag on past inflation are estimated as part of the estimation of the Phillips Curve.[5]

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[2]The Andersen-Jordan equation was initially reported in Leonall C. Andersen and Jerry L. Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," this *Review* (November 1969), pp. 11-24. In that version, changes in the money supply had a strong and persistent influence on nominal income, while government expenditures had a weak initial impact that eroded to no effect at all after a single year, and tax changes had no effect at all. Benjamin Friedman noted subsequently that, when data were included through mid-1976, fiscal policy variables entered the reduced-form equation with larger, persistent effects. (Benjamin M. Friedman, "Even the St. Louis Model Now Believes In Fiscal Policy," *Journal of Money, Credit, and Banking* (May 1977), pp. 365-67.) In a reply, Carlson noted that the delta version of the St. Louis equation, when estimated with data through mid-1976, suffered from heteroscedasticity. (Keith M. Carlson, "Does the St. Louis Equation Now Believe in Fiscal Policy?" this *Review* (February 1978), pp. 13-19.) He therefore reestimated the equation in dot form, a standard approach to eliminating this problem. The dot version produced policy effects similar to the original delta version over the earlier time period: strong, persist monetary effects and weak, transitory fiscal effects.

[3]The equation is estimated using an Almon polynomial distributed lag (PDL) procedure with a fourth degree polynomial and with coefficients of the lag distributions restricted to zero at both ends of the lag distribution.

[4]Although the Andersen-Jordan equation has been controversial since it was first introduced, attempts to develop more eclectic versions allowing for a permanent effect of fiscal variables on nominal income have generally been unsuccessful. For a survey of empirical evidence on the Andersen-Jordan equation, see Laurence H. Meyer and Robert H. Rasche, "Empirical Evidence on Stabilization Policies," in *Stabilization Policies: Lessons from the 1970s and Implications for the 1980s*," Proceedings of a Conference sponsored by the Center for the Study of American Business and the Federal Reserve Bank of St. Louis, 1980, pp. 41-102. There is, on the other hand, considerable evidence suggesting that simple reduced forms may yield unreliable estimates of policy multipliers. See, for example, Franco Modigliani and Albert Ando, "Impact of Fiscal Actions on Aggregate Income and the Monetarist Controversy," in Jerome L. Stein, ed., *Monetarism* (Amsterdam, North Holland; 1976), pp. 17-42; and Stephen M. Goldfeld and Alan S. Blinder, "Some Implications of Endogenous Stabilization Policy," *Brookings Papers on Economic Activity* (3:1972), pp. 585-640.

[5]Many of the criticisms of the St. Louis inflation sector discussed in this section were initially raised in comments by Nordhaus and Gordon at the time the St. Louis Model was presented at an NBER conference on price determination in 1970. See Otto Eckstein, ed., *The Econometrics of Price Determination*, Proceedings of a Conference sponsored by the Board of Governors of the Federal Reserve System and the Social Science Research Council, Federal Reserve System, June 1972. The St. Louis Model was described in the volume in Leonall C. Andersen and Keith M. Carlson, "An Econometric Analysis of the Relation of Monetary Variables to the Behavior of Prices and Unemployment," pp. 166-183. Comments on the St. Louis Model's modelling of inflation appear in William D. Nordhaus, "Recent Developments of Price Dynamics," pp. 16-49; and in Robert J. Gordon's discussion of the Andersen and Carlson paper, pp. 202-12.

The price-change equation — The price-change equation in the St. Louis Model is:

$$\text{(STL-2)} \quad \Delta P^*_t = b_0 + \sum_{i=0}^{5} b_{1i}\, DSL_{t-i} + b_2\, \Delta P^A_t$$

where Δ is the first difference operator, DSL is the demand slack variable (defined below), ΔP^A is the anticipated change in the price level (also defined below), and ΔP^*_t, the change in the price level, is specified as

$$\text{(STL-2a)} \quad \Delta P^*_t = \Delta P_t \cdot X_{t-1},$$

where X is the level of real GNP. The explanation for this form of the price change variable will be given below.

The parameter estimates when the equation is estimated over the period I/1955-IV/1980 are as follows:

b_0 = .65 (.77)
b_{10} = .012 (.53)
b_{11} = .028 (3.02)
b_{12} = .036 (5.59)
b_{13} = .038 (3.73)
b_{14} = .033 (2.97)
b_{15} = .019 (2.60)
Σb_{1i} = .166 (5.93)
b_2 = 1.29 (25.50)

R^2 = .88 SE = 5.6 DW = .83

Although the price-change equation is, in essence, a Phillips Curve equation, it has several unusual features. First, it explains the first difference in the price level (the implicit GNP deflator), while Phillips Curves are typically specified in terms of the inflation rate or the rate of change in nominal wages.[6] Its delta form reflects the now-abandoned delta specification of the Andersen-Jordan equation; it made the price-change equation dimensionally compatible with the income-change equation, allowing the change in output to be solved for via a simple "identity." Since the Andersen-Jordan equation is now used in dot form, the retention of the delta form for the price-change equation is unnecessary. Moreover, the delta specification, due to the possibility of heteroscedasticity, could produce an upward bias on the coefficients in that equation, including the coefficients on both the demand slack variable and on the anticipated-price-change variable.[7] These impacts would produce an upward bias in the model's response of inflation to monetary change.

A second unusual feature is that it uses a different demand slack variable than that used in most empirical Phillips Curves. Generally, either the unemployment rate or the (percentage) GNP gap (potential or full-employment output minus actual output) is used as the measure of demand slack. The St. Louis demand slack variable (DSL), on the other hand, is defined as

$$\text{(STL-2b)} \quad DSL_t = \Delta Y_t - (POTRT_t - X_{t-1}),$$

where POTRT is the level of potential output as measured by the Rasche-Tatom series.[8] This specification of the demand slack variable may seriously bias upward the equation's estimate of the response of inflation to demand slack, inasmuch as it allows changes in nominal income associated with changes in the price level to "explain" changes in the price level.[9]

The sum of the coefficients on the demand slack variable determines the degree to which decelerations in monetary growth are initially reflected in declines in the rate of growth of output and hence increases in the unemployment rate. Meyer and Rasche report simulations of the St. Louis Model with different values of this parameter (its value based on a sample through I/1975 and its value based on a sample through IV/1979 where the sum is three times larger) and demonstrate the dramatic differences in the implied responses of output and the unemployment rate to monetary decelerations.[10]

[6]More precisely, the price change variable is the change in the price level times lagged real output. See equation STL-2a above.

[7]This possible source of bias in the St. Louis price-change equation was noted by Gordon in his comments on the St. Louis Model in the Eckstein volume. Gordon argued that the results of the price-change equation "are plagued by heteroscedasticity" (p. 209). In response to the presence of heteroscedasticity, the nominal income was changed from a delta to a dot specification, although the price-change equation, where heteroscedasticity may have been more of a problem, was left in first difference form.

[8]See Robert H. Rasche and John A. Tatom, "Energy Resources and Potential Output," this *Review* (June 1977), pp. 10-24, for a discussion of this series.

[9]The possibility that the St. Louis specification of the price-change equation yields an upward biased estimate of the response of inflation to demand slack was suggested by the remarkable behavior of the sum of the estimated coefficients on the demand slack variable as additional years of data were added to the sample period during the 1970s. After 1975, the estimated coefficient begins to rise as more data is included; by the end of 1979, the coefficient is almost three times its value for the sample period ending before 1975. This pattern is consistent with what would be expected if the specification yielded biased estimates for the reason suggested above. This bias would be expected to become more serious during a period where changes in nominal income were dominated by changes in the price level.

[10]See Laurence H. Meyer and Robert H. Rasche, "On the Costs and Benefits of Anti-Inflation Policies," this *Review* (February 1980), pp. 3-14.

The long-term interest rate equation — The equation for the long-term interest rate (R^L) is:

$$(STL\text{-}3)\ R^L_t = c_0 + c_1 \dot{M}_t + c_2 Z_t + \sum_{i=0}^{16} c_{3i} \dot{X}_{t-i} + \sum_{i=0}^{16} c_{4i} (\dot{P}_{t-i}/(U_{t-i}/UF_{t-i})),$$

where $\dot{X}$ is the rate of change in real GNP, Z is a dummy variable, allowing for a shift in the constant term over the period, U is the unemployment rate, and UF is a measure of the rate of unemployment at "full employment." The parameter estimates for the long-term interest rate equation are as follows:

c_0	=	.82	(1.42)
c_1	=	.02	(.65)
c_2	=	.82	(2.77)
Σc_{3i}	=	.29	(1.69)
Σc_{4i}	=	1.04	(14.23)

$R^2 = .89 \quad SE = .78 \quad DW = .17$

The measure of expected inflation in the above equation is a distributed lag on past inflation adjusted for the level of demand pressure as proxied by the ratio of the unemployment rate to the full-employment rate of unemployment (UF) where the latter is measured by series developed at the Council of Economic Advisors. This equation not only provides predictions of the long-term interest rate, it also provides the weights, the c_{4i} coefficients, used in the anticipated-price-change equation.

There are a number of questionable features of this long-term interest rate equation, particularly related to its role in providing the weights for an expected price-change variable. First, the weighted sum of current and past inflation rates can be viewed as a measure of the expected inflation rate only if we assume that a one percentage point increase in the expected inflation rate increases the long-term interest rate by one percentage point. We cannot, however, separate out the weights that convert current and past inflation rates into the expected rate of inflation and the coefficient that translates an increase in the expected inflation rate into an increase in the long-term interest rate. Recent work on the implications of specific tax structures for interest rate behavior in inflationary periods indicates that the simple Fisherian view that a percentage point increase in the expected inflation rate raises the long-term interest rate by a percentage point is no longer so obvious.[11]

Second, the expected-price-change variable that is derived from a long-term bond equation is likely to relate to a much longer horizon (the average term to maturity of long-term bonds) than is relevant to the formation of price expectations in the context of the Phillips Curve (the current period or at most an average of price change expected over the average length of contracts, implicit and explicit). This difference in horizon may affect the number of relevant lags and the weighting applied to past inflation.

There is one additional question about the specification of the long-term interest rate equation. One can derive a somewhat similar equation by beginning with a money demand equation in which the demand for money depends on the long-term interest rate and current and past rates of inflation and by solving that equation for the long-term interest rate as a function of the level of real money balances, the level of real output, and current and past rates of inflation. However, the long-term rate would depend on the *level* of real money balances rather than the rate of change in nominal money balances and on the *level* of real income rather than the rate of change in real income.

Finally, the Durbin-Watson statistic is very low, suggesting serious serial correlation of the residuals. Reestimating the equation using the Cochrane-Orcutt technique to correct for first-order serial correlation yields quite different parameter estimates for the money and output variables and an unimpressive equation in terms of the significance of key parameter values.

The anticipated-price-change equation — The equation for anticipated price change (ΔP^A) is an identity, given by

$$(STL\text{-}4)\ \Delta P^A_t \equiv Y_{t-1} \left(\left[\sum_{i=1}^{17} c_{4i} (\dot{P}_{t-i}/(U_{t-i}/UF_{t-i})) .01 + 1 \right]^{.25} - 1 \right).$$

This seemingly complicated equation transforms the weighted distributed lag on current and past inflation into the first difference of the price variable used in the St. Louis Model. This price change variable is *not* the first difference of the implicit price deflator; it is, instead, the first difference in the implicit deflator multiplied by the lagged value of the level of real output. This particular form of the price change variable is necessary because of the way that output is determined in the model.

[11]See, for example, Martin Feldstein, "Inflation, Income Taxes and the Rate of Interest: A Theoretical Analysis," *American Economic Review* (December 1976), pp. 809-20; and John A. Tatom and James E. Turley, "Inflation and Taxes: Disincentives for Capital Formation," this *Review* (January 1978), pp. 2-8.

To determine the dynamics associated with the price-change equation and, in particular, whether the St. Louis price-change equation implies a long-run trade-off or a vertical Phillips Curve, one must solve for the implied sum of the coefficients on lagged price changes. The equation for the change in the price level can be expressed directly as a function of demand slack and a distributed lag on past price changes:

$$\Delta P^*_t = b_0 + \sum_{i=0}^{5} b_{1i}\, DSL_{t-i} + \sum_{i=1}^{17} b'_{2i}\, \Delta P_{t-i}.$$

The sum of the coefficients on past price changes can in turn be related to the parameter on the anticipated-price-change variable in the price-change equation, STL-2, and the coefficients on past inflation in the long-term interest rate equation.

$$\sum b'_{2i} = b_2 \cdot \{ \sum c_{4i} \;/\; \text{Mean } (U/UF) \}$$

The $\Sigma b'_{2i}$ term equals 1.13, based on the St. Louis Model's estimates for b_2 and the c_{4i} parameters. The fact that the sum of coefficients on past price changes exceeds unity results in a dynamic instability in long-run simulations with the Model and a more rapid response of inflation to monetary change than if this sum were constrained to unity. This feature of the price-change equation reinforces the influence of the upward bias in the coefficient on the demand slack variable.

The Unemployment Gap Equation

The unemployment rate (U) is determined by the following equation:

$$\text{(STL-5)}\quad UGAP_t = d_0\, GAP_t + d_1\, GAP_{t-1},$$

where UGAP = U − UF and GAP is the percentage gap between potential output and actual output (GAP = ((POTRT − X)/POTRT)•100). The unemployment rate is then calculated from the identity,

$$\text{(STL-6)}\quad U_t \equiv UF_t + UGAP_t.$$

The parameter estimates for STL-5, based on the sample period I/1955-IV/1980 are:

$d_0 = .024 \quad (.38)$

$d_1 = .45 \quad (7.2)$

$R^2 = .78 \quad SE = .55 \quad DW = .38$

The pattern of coefficients on the gap variables in this equation are different from what might have been expected. The coefficient on the GAP variable in the contemporaneous period is essentially zero, implying that a change in the level of output relative to potential output has no impact on the unemployment rate in the same quarter. In addition, the Durbin-Watson statistic is low, suggesting the possible omission of other important explanatory variables.

The Short-Term Interest Rate Equation

We ignore the remaining equation in the St. Louis Model, the equation for the short-term interest rate (4- to 6-month commercial paper rate). This variable does not appear elsewhere in the model and we are not interested in the model's predictions for interest rates.

The Output Identity

The change in output is determined in the St. Louis Model via an "identity." Using first differences, ΔY can be expressed as

$$\Delta Y_t = P_{t-1}\, \Delta X_t + X_{t-1}\, \Delta P_t + \Delta X_t\, \Delta P_t.$$

The price change variable in the St. Louis Model is thus not ΔP, but rather $X_{-1}\Delta P$, the dollar change in total spending due to price changes (ignoring the interaction term, $\Delta X\, \Delta P$). The "change-in-output" variable in the St. Louis Model is then determined by an approximation to the actual identity since the interaction term is excluded. Thus the change in output in the St. Louis Model, $P_{-1}\, \Delta X$, is defined by

$$\text{(STL-7)}\quad P_{t-1}\, \Delta X_t \equiv \Delta Y_t - X_{t-1}\, \Delta P_t.$$

REFRAINS ON A ST. LOUIS MODEL THEME: PHILLIPS CURVE AND MONETARIST REDUCED-FORM APPROACHES TO THE INFLATION RATE

In this section, we present two variants of the St. Louis Model. The two versions differ from each other only in the equation used to explain the inflation rate. The first version includes a fairly conventional Phillips Curve and the second utilizes a monetarist reduced form instead. Thus, the inflation sector of the St. Louis Model is collapsed to a single equation in each of these two alternative models. Each of the revised versions includes the Andersen-Jordan equation and an unemployment equation. To avoid the appearance that either of these variants are intended to or actually have superceded the St. Louis Model previously presented, the two versions are labeled UCITYPC and UCITYRF, designating that they were developed in University City (alias UCITY), a suburb immediately west of the city of St. Louis and adjacent to Washington University. This is intended to remind the reader that these versions are close to the St. Louis Model, but not identical to it. Of course, the PC and RF refer to the differentiating feature of the two versions, the Phillips Curve (PC) or the monetarist reduced-form (RF) equation used to explain the rate of inflation.

First, we present the equations that the two versions have in common: the reduced-form equation for nominal income, the equation for the unemployment gap, and the identity that converts predicted values for nominal income and price level into predictions for output. Then, we will detail the two alternative specifications of the inflation equation.

The Identity Relating Nominal Income, Output and the Price Level

The relation between nominal income (Y), output (X) and the price level (P) can be expressed by the identity,

$$Y = P\,X.$$

We wish to avoid the use of an approximation to solve for output, as the current version of the St. Louis Model does. The model will yield solutions for the rate of change in both nominal income and the price level. However, the equation,

$$\dot{Y} = \dot{X} + \dot{P},$$

is only an approximation when the dot variables are measured by compound annual rates of change, an approximation that becomes poorer as the size of the rates of change increases. To solve this problem, the rates of change are defined as changes in the logs of Y, X, and P (delta log specification). Taking logs and then first differences of the equation, Y = P X, yields the identity,

$$\text{(UCITY-7)} \quad \Delta\ln Y = \Delta\ln X + \Delta\ln P.$$

The Andersen-Jordan and inflation equations both will be specified in terms of delta logs. The identity above will then be used to determine the change in the log of output.

The Andersen-Jordan Equation

There is, of course, little difference between the dot and delta log specifications of an equation. The delta log specification is given by,

$$\text{(UCITY-1)} \quad \Delta\ln Y_t = \alpha_0 + \sum_{i=0}^{4} \alpha_{1i}\,\Delta\ln M_{t-i} + \sum_{i=0}^{4} \alpha_{2i}\,\Delta\ln G_{t-i}.$$

The parameter estimates for the sample period I/1955-IV/1980 are:[12]

α_0 =	2.69	(3.26)			
α_{10} =	.45	(4.34)	α_{20} =	.062	(1.59)
α_{11} =	.44	(6.59)	α_{21} =	.054	(1.81)
α_{12} =	.24	(2.6)	α_{22} =	.004	(.116)
α_{13} =	.032	(.50)	α_{23} =	−.052	(−2.02)
α_{14} =	−.066	(−.59)	α_{24} =	−.069	(−2.01)
$\Sigma\alpha_{1i}$ =	1.10	(7.19)	$\Sigma\alpha_{2i}$ =	−.001	(−0.01)

R^2 = .45 SE = 3.33 DW = 2.04

The Unemployment-Gap Equation

The specification of the unemployment-gap equation is unchanged from the St. Louis Model (STL-5). The only modification is that it is estimated with a correction for second order autocorrelation. The estimates for this equation are:

d_0 = .26 (14.3)
d_1 = .17 (9.5)
ρ_1 = 1.17
ρ_2 = −.38
R^2 = .74 SE = .19 DW = 2.04

where ρ_1 and ρ_2 are the values of the rho coefficients on the first and second lagged values of the residual. Note the dramatic decline in the standard error of this equation, compared with the one in the St. Louis Model.[13]

The two revised versions include equation (STL-5), as reestimated above, and the identity (STL-6).

The Level of Output

Because the level of output is used in the GAP variable in the Phillips Curve, we must also include

[12]The change in log variables are all multiplied by 400 prior to estimation so that they approximate annual rates of change.

[13]The parameter estimates of the revised equation are quite close to those presented in John A. Tatom, "Economic Growth and Unemployment: A Reappraisal of the Conventional View," this *Review* (October 1978), pp. 16-22. Tatom corrects the levels equation for first-order serial correlation and also presents a first difference equation, also with a correction for first-order serial correlation.

an identity to determine the level of output from the predicted values of the change in the log of output and last period's level of output.

$$\text{(UCITY-8)}\ X_t = \exp(\ln X_{t-1} + \Delta\ln X_t)$$

The Phillips Curve

The Phillips Curve equation uses a delta log specification for the rate of change in the price level, measures the demand slack in the economy with the GNP gap variable, and proxies expected inflation with a distributed lag on past rates of inflation. The latter distributed lag can also be interpreted as capturing an element of inertia due, for example, to the existence of implicit or explicit contracts.

The Phillips Curve also includes the differential in the rate of increase in the producers' price of energy relative to the rate of increase in the implicit price deflator for GNP. This variable, labeled ENERGY, is intended to capture a major source of "supply shocks" that dramatically have affected the inflation rate over a couple of periods in the data sample, in particular, during the latter part of 1973, throughout 1974 and, more recently, in 1979 and early 1980. This variable is lagged two periods, reflecting some experimentation with other simple lag patterns.

The Phillips Curve also includes a dummy variable to capture the influence of the price controls during the period from III/1971 through 1975. The variable, labeled CONTROLS, allows for a negative impact during the first part of the period and an offsetting positive influence associated with "catch-up" effects during the period after which controls were relaxed and then removed. The sum of the values the dummy variable takes on over this period is constrained so that the net price control effect on inflation is zero. Specifically, CONTROLS is 0 up to II/1971, 1 from III/1971-IV/1972, .2222 in I/1973, −.7778 from II/1973-I/1975 and 0 thereafter.[14]

The estimated Phillips Curve equation is

$$\text{(UCITY-2)}\ \Delta\ln P_t = \beta_0 + \beta_1\,GAP_{t-1} + \beta_2\,CONTROLS_t + \beta_3\,ENERGY_{t-2} + \sum_{i=1}^{20}\beta_{4i}\,\Delta\ln P_{t-i}.$$

[14]This specification of the controls variable was borrowed from John A. Tatom at the Federal Reserve Bank of St. Louis.

The distributed lag on inflation is estimated using a third degree polynomial with no end-point restrictions. We have employed the lagged GAP in this equation, preserving the simple recursive structure of the St. Louis Model. The empirical estimates with the contemporaneous GAP were almost indistinguishable from this equation.

The parameter estimates for the period I/1955-IV/1980 are:

β_0	.85	(3.14)			
β_1	−.22	(−4.3)			
β_2	−1.085	(−2.65)			
β_3	.044	(3.82)			
β_{41}	.19	(4.32)	β_{411}	.030	(2.63)
β_{42}	.14	(5.19)	β_{412}	.037	(3.24)
β_{43}	.097	(6.39)	β_{413}	.044	(3.54)
β_{44}	.066	(5.72)	β_{414}	.045	(3.66)
β_{45}	.043	(3.24)	β_{415}	.051	(3.77)
β_{46}	.028	(1.83)	β_{416}	.050	(3.9)
β_{47}	.020	(1.23)	β_{417}	.043	(3.8)
β_{48}	.017	(1.1)	β_{418}	.030	(2.43)
β_{49}	.019	(1.32)	β_{419}	.010	(.51)
β_{410}	.023	(1.87)	β_{420}	−.019	(−.56)

$\Sigma\beta_{4i}$ = .969 (14.04)

R^2 = .827 SE = 1.166 DW = 2.10

Note that the GAP variable is highly significant, that the sum of coefficients on the past inflation rates is not significantly different from unity, and that both the controls dummy and the energy differential variables are significant.

The Inflation Reduced-Form Equation

The inflation reduced-form equation explains the inflation rate in terms of current and lagged values of monetary growth and the energy inflation differential and controls dummy variables discussed above.[15] This equation is also specified in delta log form:

[15]Monetarist reduced-form equations for inflation have been employed for some time at the Federal Reserve Bank of St. Louis. The equation was initially reported in 1976 in Denis S. Karnosky, "The Link Between Money and Prices: 1971-76," this *Review* (June 1976), pp. 17-23. Some refinements have been presented in Keith M. Carlson, "The Lag from Money to Prices," this *Review* (October 1980), pp. 3-10, and in John A. Tatom, "Energy Prices and Short-Run Economic Performance," this *Review* (January 1981), pp. 3-17. A simple annual version of a monetarist reduced-form inflation equation was presented in Jerome L. Stein, "Inflation, Employment and Stagflation," *Journal of Monetary Economics* (April 1978), pp. 193-228.

$$(\text{UCITY-2}')\quad \Delta \ln P_t = \sum_{i=0}^{19} \gamma_{1i}\, \Delta \ln M_{t-i} + \gamma_2\, \text{CONTROLS}_t + \gamma_3\, \text{ENERGY}_{t-2}.$$

This distributed lag in monetary change is estimated using a third degree polynomial with no end-point restrictions. The parameter estimates for the sample period I/1955-IV/1980 are:

γ_{10}	.039	(1.34)	γ_{110}	.057	(5.28)
γ_{11}	.047	(2.45)	γ_{111}	.055	(4.81)
γ_{12}	.054	(4.02)	γ_{112}	.052	(4.30)
γ_{13}	.058	(5.19)	γ_{113}	.050	(3.89)
γ_{14}	.061	(5.38)	γ_{114}	.046	(3.66)
γ_{15}	.063	(5.27)	γ_{115}	.044	(3.59)
γ_{16}	.064	(5.25)	γ_{116}	.042	(3.12)
γ_{17}	.063	(5.36)	γ_{117}	.041	(2.22)
γ_{18}	.062	(5.50)	γ_{118}	.041	(1.11)
γ_{19}	.060	(5.52)	γ_{119}	.042	(.35)

$\Sigma\gamma_{1i}$ = 1.04 (33.5)

γ_2 −1.94 (−4.72)

γ_3 .045 (4.12)

R^2 = .822 SE = 1.173 DW = 1.62

The parameter estimates on the controls dummy and the energy inflation variable are both significant, and the coefficients on the monetary change variable sum to unity. The two inflation equations, UCITY-2 and UCITY-2′, perform quite similarly with respect to in-sample error, with a very slight edge to the Phillips Curve.

Summary of Differences of UCITY Models and the St. Louis Model

A summary of the St. Louis and UCITY models is given in table 1. The differences between the St. Louis Model and the UCITY models can be summarized as follows:

1. The nominal income and inflation equations are both specified symmetrically in delta log form in the UCITY models, allowing the change in the log of output to be solved for via an identity. In the St. Louis Model, the nominal income equation is in a rate-of-change specification, the price equation in first difference, and the change in output is solved for via an approximation.

2. The St. Louis Model employs a three-equation inflation structure. The UCITY models employ alternative single equations for inflation.

3. The St. Louis Phillips Curve uses an unusual demand slack variable, the change in nominal income minus the lagged real GNP gap; the UCITY Phillips Curve uses the GNP gap.

4. The weights on past inflation in the St. Louis Phillips Curve are derived from an equation for the long-term interest rate. In the UCITY model, the weights are estimated directly during estimation of the Phillips Curve.

5. One of the UCITY models substitutes a monetarist reduced-form equation for inflation for the Phillips Curve. The St. Louis inflation sector is built around a price-change version of a Phillips Curve.

6. The unemployment equation is estimated using a correction for second-order autocorrelation in the UCITY models. It is estimated using ordinary least squares in the St. Louis Model.

COMPARING THE THREE MODELS: PREDICTIVE PERFORMANCE AND POLICY IMPLICATIONS

This section compares the predictive performance and the policy implications of the three models. The results reported here bear directly on the three themes outlined at the beginning of the paper. First, in-sample and out-of-sample static simulations are used to compare the predictive performances of the St. Louis Model and the two UCITY models. Second, the responses of output, unemployment and inflation in the models to a deceleration in monetary growth are compared. Third, the two UCITY models are compared to determine whether any differences exist in their predictive performance or policy multipliers.

Predictive Performance of the Three Models

Because the two UCITY models include two significant variables not included in the St. Louis Model — the controls dummy and the energy inflation differential — it would not be surprising if they perform better than the St. Louis model. In order to determine the degree to which differences in predictive performance were due to the addition of these variables, two additional versions of each UCITY model were estimated: one without the controls dummy, the other without the controls dummy and the energy-inflation differential.

Table 1
Summary of St. Louis and UCITY Models

St. Louis Model	UCITY Models	
	Phillips Curve	Monetarist Reduced Form
(1) $\dot{Y}_t = a_0 + \sum_{i=0}^{4} a_{1i} \dot{M}_{t-i} + \sum_{i=0}^{4} a_{2i} \dot{G}_{t-i}$	$\Delta \ln Y_t = \alpha_0 + \sum_{i=0}^{4} \alpha_{1i} \Delta \ln M_{t-i} + \sum_{i=0}^{4} \alpha_{2i} \Delta \ln G_{t-i}$	Same as UCITYPC model
(2) $\Delta P^*_t = b_0 + \sum_{i=0}^{5} b_{1i} DSL_{t-i} + b_2 \Delta P^A_t$ (2a) $\Delta P^*_t = X_{t-1} \Delta P_t$ (2b) $DSL_t = \Delta Y_t - (POTRT_t - X_{t-1})$ (3) $R^L_t = c_0 + c_1 \dot{M}_t + c_2 Z_t + \sum_{i=0}^{16} c_{3i} \dot{X}_{t-1} + \sum_{i=0}^{16} c_{4i} (\dot{P}_{t-i}/(U_{t-i}/UF_{t-i}))$ (4) $\Delta P^A_t = Y_{t-1}\left([\sum_{i=1}^{17} c_{4i}(P_{t-i}/(U_{t-i}/UF_{t-i})).01 + 1]^{.25} - 1\right)$	$\Delta \ln P_t = \beta_0 + \beta_1 GAP_{t-1} + \beta_2 CONTROLS_t + \beta_3 ENERGY_{t-2} + \sum_{i=1}^{20} \beta_{4i} \Delta \ln P_{t-i}$	$\Delta \ln P_t = \sum_{i=0}^{19} \gamma_{1i} \Delta \ln M_{t-i} + \gamma_2 CONTROLS_t + \gamma_3 ENERGY_{t-2}$
(5) $UGAP_t = d_0 GAP_t + d_1 GAP_{t-1}$	Same as St. Louis Model but corrected for 2nd-order autocorrelation	Same as UCITYPC model
(6) $U_t = UF_t + UGAP_t$	Same as St. Louis Model	Same as St. Louis Model
(7) $P_{t-1} \Delta X_t = \Delta Y_t - X_{t-1} \Delta P_t$	$\Delta \ln Y = \Delta \ln X + \Delta \ln P$	Same as UCITYPC model
(8) $X_t = X_{t-1} + \frac{P_{t-1} \Delta X_t}{P_{t-1}}$	$X_t = \exp(\ln X_{t-1} + \Delta \ln X_t)$	Same as UCITY PC model

In-sample static simulation results — The in-sample root-mean-square errors (RMSEs) for inflation ($\dot{P}$), rate of change in nominal GNP ($\dot{Y}$), rate of change in real GNP ($\dot{X}$), level of real GNP (X), GNP Gap (GAP), and unemployment rate (U) for the various versions of the UCITY models and for the St. Louis Model are presented in table 2. Table 3 reports the percentage declines in RMSEs in the two UCITY models compared with the St. Louis Model. The two UCITY models uniformly predict more accurately than the St. Louis Model (the sole exception being the rate of change in nominal GNP for which the equations and hence predictions are virtually identical).

The improvement in the inflation forecast is quite large and, surprisingly, is accounted for to only a minor degree by the addition of the two new variables, although each does marginally improve the inflation predictions. The inflation RMSEs for the St. Louis Model, UCITYPC, and UCITYRF were 2.11, 1.12 and 1.14, respectively. This translates into a reduction in the RMSE for inflation of 47 percent and 46 percent in the UCITYPC and UCITYRF models, relative to the St. Louis Model. When *both* the controls dummy and energy inflation differential variables were excluded, the RMSEs in the UCITYPC model increased to 1.29 and UCITYRF to 1.42, still dramatically below the RMSE in the St. Louis Model.

These results indicate that: (1) the inflation predic-

Table 2

Full In-Sample Static Stimulations, I/1955-IV/1980 (root-mean-square errors)

	St. Louis	UCITYPC	UCITYPC w/o ENERGY	UCITYPC w/o CONTROLS w/o ENERGY	UCITYRF	UCITYRF w/o ENERGY	UCITYRF w/o CONTROLS w/o ENERGY
$\dot{P}$	2.11	1.12	1.20	1.29	1.14	1.23	1.42
$\dot{Y}$	3.23	3.22	3.22	3.22	3.22	3.22	3.22
$\dot{X}$	3.24	2.98	3.06	3.08	3.07	3.13	3.14
X	8.80	8.09	8.29	8.62	8.32	8.54	8.58
GAP	.79	.72	.74	.75	.75	.76	.76
U	.55	.26	.27	.27	.26	.26	.27

tion in the St. Louis Model can be improved by substituting either a more traditional Phillips Curve or a monetarist reduced-form for the St. Louis Model's price-change equation; and (2) inflation predictions with the two versions of the UCITY model are very close, not surprising given the small differences in the standard errors in the two inflation equations.

The UCITY models also outperformed the St. Louis Model for the rate of change in output, the level of output, the GNP gap, and the unemployment rate, although the degree of improvement is smaller for the two output variables and GAP than for the inflation rate and the unemployment rate. For the rate of change in output, the RMSE in the St. Louis Model was 3.24, compared with 2.98 in the UCITYPC model and 3.07 in the UCITYRF model. This represents an improvement in the RMSEs of 5.5 percent to 8 percent in the two UCITY models, a much smaller improvement than might have been expected given the margin of improvement for inflation. As in the case of the inflation predictions, eliminating the controls dummy and inflation differential variables results in a small deterioration in the quality of the predictions from the UCITY models, but still leaves those predictions superior to those from the St. Louis Model. Interestingly, the improvement in the predictions for the unemployment rate was about as great as for the inflation rate, surprising in comparison with the much smaller improvement in predictions of the GAP, but less surprising in light of the particularly poor statistical quality of the St. Louis unemployment equation.

Out-of-sample static forecasts — The three models were re-estimated over the shorter period, I/1955-IV/1976, and static forecasts were made for the period I/1977-IV/1980. The results of the out-of-sample static forecasts were consistent with the in-sample results. The two UCITY models again outperformed the St. Louis Model for all variables (except nominal income, of course). The improvement for inflation was somewhat smaller (33 percent and 17 percent for UCITYPC and UCITYRF, respectively) while the improvement for the output and GAP variables was somewhat larger (9 percent to 15 percent in the UCITY models) than in the case of the in-sample results. Once again, the unemployment rate predictions for the St. Louis Model were poor compared with the UCITY results. The out-of-sample RMSEs for the various variables are reported in table 4, and the percent improvement in RMSEs in the UCITY models is given in table 5.

Table 3

Percent Reduction in In-Sample UCITY RMSEs (relative to St. Louis Model)

	UCITYPC	UCITYRF
$\dot{P}$	46.9%	46.0%
$\dot{X}$	8.0	5.2
X	8.1	5.5
GAP	8.9	5.1
U	49.1	49.1

The Response of Output, Unemployment and Inflation to Monetary Change in the Three Models

The UCITY models were developed, in part, to improve the predictions of inflation, output and

Table 4
Out-of-Sample Static Simulations: I/1977-IV/1980 (root mean square errors)

	St. Louis	UCITYPC	UCITYPC w/o CONTROLS	UCITYPC w/o CONTROLS w/o ENERGY	UCITYRF	UCITYRF w/o CONTROLS	UCITYRF w/o CONTROLS w/o ENERGY
$\dot{P}$	2.12	1.43	1.51	1.67	1.77	1.93	1.81
$\dot{Y}$	3.78	3.80	3.80	3.80	3.80	3.80	3.80
$\dot{X}$	3.92	3.34	3.58	3.47	3.58	3.84	3.55
X	14.36	12.12	13.00	12.59	13.04	14.04	12.92
GAP	.95	.81	.87	.84	.87	.93	.86
U	.59	.28	.29	.29	.30	.32	.30

Table 5
Percent Reduction in Out-of-Sample UCITY RMSEs (relative to St. Louis Model)

	UCITYPC	UCITYRF
$\dot{P}$	32.5%	16.5%
$\dot{X}$	14.8	8.7
X	15.6	9.2
GAP	14.7	8.4
U	52.5	49.2

unemployment from those available using the St. Louis Model presented here. Also of interest are the differences in the policy simulations obtained using the three models.

For the policy simulations, CEA projections for potential output and for high employment government expenditures were used for the period from I/1981-IV/1984. Two alternative monetary growth rates were used: as the "base" series, we used a constant rate of 5 percent per year, for the "policy" series we used 2 percent per year. We then computed the differences in the rates of change of nominal and real income, and differences in the level of real GNP, in the GNP gap, in the unemployment rate, and in inflation between the base and policy simulations. The results are reported in tables 6, 7 and 8 for the inflation rate, the rate of change in real output and the unemployment rate. The figures reported in each case are the values in the policy run minus the values in the base run.

The results confirmed our expectations about the direction of the differences, but the magnitude of the differences between the St. Louis and UCITY simulations were somewhat smaller than expected. Inflation declines more rapidly in the St. Louis Model and, as a consequence, the decline in the rate of growth of output and the increase in unemployment are smaller in the St. Louis Model.

For the inflation rate, all three models' projections are close during the first year, with inflation falling about 0.4 percentage points. By the end of the second year, inflation has fallen 1.8 percentage points in the St. Louis Model, compared with only 1.2 in the two UCITY models. By the end of the fourth year, inflation has fallen by 4.0 percentage points in the St. Louis Model compared with 2.8 and 2.9 percentage points in the UCITY models. Thus, while inflation has fallen more rapidly in the St. Louis Model, the decline by the end of the fourth year exceeds the equilibrium response, implying a tendency to overshoot.

In the St. Louis Model, the rate of increase in output declines for the first 12 quarters, the decline exceeding 2 percent per year for the first 6 quarters. In the two UCITY models, the rate of increase in output is lower throughout the 16-quarter simulation horizon, by 2 percent per year or more for eight quarters.

The unemployment results indicate that the monetary deceleration raises the unemployment rate for 16 consecutive quarters for each model, but that unemployment is about 0.6 percentage points higher in the two UCITY models at the end of the simulation horizon compared with the St. Louis Model.

Table 6
Dynamic Simulation Results: Inflation Rate (difference between the 2 percent and 5 percent simulations)

Date	St. Louis	UCITYPC	UCITYRF
1981 I	-.01	-.04	-.04
II	-.07	-.06	-.12
III	-.20	-.20	-.24
IV	-.43	-.38	-.40
1982 I	-.75	-.59	-.57
II	-1.11	-.80	-.77
III	-1.48	-1.02	-.98
IV	-1.84	-1.24	-1.20
1983 I	-2.17	-1.47	-1.42
II	-2.49	-1.69	-1.64
III	-2.79	-1.90	-1.86
IV	-3.06	-2.11	-2.08
1984 I	-3.32	-2.32	-2.28
II	-3.55	-2.52	-2.48
III	-3.67	-2.72	-2.66
IV	-3.95	-2.92	-2.82

Table 7
Dynamic Simulation Results: Rate of Change in Real GNP (difference between the 2 percent and 5 percent simulations)

Date	St. Louis	UCITYPC	UCITYRF
1981 I	-1.17	-1.57	-1.33
II	-2.51	-2.55	-2.57
III	-3.22	-3.08	-3.15
IV	-3.02	-3.03	-3.08
1982 I	-2.44	-2.63	-2.72
II	-2.05	-2.43	-2.53
III	-1.66	-2.21	-2.32
IV	-1.29	-1.98	-2.10
1983 I	-.97	-1.75	-1.88
II	-.65	-1.53	-1.65
III	-.34	-1.31	-1.44
IV	-.05	-1.12	-1.22
1984 I	.02	-.90	-1.02
II	.41	-.70	-.82
III	.63	-.51	-.64
IV	.83	-.31	-.48

Comparing the Predictive Performance and Policy Implications of the Phillips Curve and Monetarist Reduced-Form Inflation Equations

There is a considerable literature that views the Phillips Curve and the monetarist reduced form for inflation as mutually exclusive, alternative inflation equations.[16] Generally, these "competitive" alternative approaches are tested by investigating the consequences of adding monetary change to a Phillips Curve or introducing the unemployment rate into a monetarist reduced form.[17]

We do not view the Phillips Curve and monetarist reduced form as mutually exclusive, alternative models of the inflation process but rather as structural vs. reduced form approaches to explaining inflation. Because we view the two inflation equations as reasonable alternative specifications, we find no value in tests that add monetary change variables to the Phillips Curve or unemployment rates to the monetarist reduced form. Such experiments mix structural and reduced-form equations. We would not expect to be able to obtain significant coefficients on both monetary change and unemployment rates in an inflation equation, and, consequently, no such experiments were conducted. Instead, we compared the two inflation equations individually and as alternative components of a St. Louis-type model; we found that the two inflation equations were virtually indistinguishable in predictive performance and policy implications.

First, when the single equation performance of the Phillips Curve and monetarist reduced form

[16] See, for example, Keith M. Carlson, "Inflation, Unemployment, and Money: Comparing the Evidence from Two Simple Models," this *Review* (September 1978), pp. 2-6; and John A. Tatom, "Does the Stage of the Business Cycle Affect the Inflation Rate?" this *Review* (September 1978), pp. 7-15; and Stein, "Inflation, Employment, and Stagflation."

[17] Tests of this kind have been reported by Franco Modigliani and Lucas Papademos, "Targets for Monetary Policy in the Coming Years," *Brookings Papers on Economic Activity* (1:1975), pp. 141-63; George L. Perry, "Slowing the Wage-Price Spiral: The Macroeconomics View," *Brookings Papers on Economic Activity* (2:1978), pp. 259-91; Stein, "Inflation, Employment and Stagflation;" and John A. Tatom, "What Ever Happened to the Phillips Curve?" Federal Reserve Bank of St. Louis *Research Papers*, No. 81-008.

Table 8

Dynamic Simulation Results: Unemployment Rate (difference between the 2 percent and 5 percent simulations)

Date	St. Louis	UCITYPC	UCITYRF
1981 I	0	0.15	0.08
II	.15	0.40	0.30
III	.41	0.67	0.59
IV	.78	0.94	0.90
1982 I	.91	1.19	1.17
II	1.34	1.42	1.41
III	1.54	1.63	1.63
IV	1.70	1.81	1.82
1983 I	1.83	1.97	1.99
II	1.93	2.11	2.13
III	1.99	2.23	2.26
IV	2.03	2.33	2.36
1984 I	2.04	2.41	2.44
II	2.02	2.48	2.50
III	2.00	2.53	2.56
IV	1.94	2.56	2.60

were compared, the Phillips Curve and monetarist reduced forms yielded standard errors of 1.166 and 1.173, respectively. Thus the two equations fit the data almost equally well. Note the high level of significance of the key variables in both equations — the gap and the sum of the coefficients of past inflation in the Phillips Curve and on the sum of the coefficients of monetary change in the monetarist reduced form.

Second, the in-sample and out-of-sample static forecasts of the two UCITY models were compared, the only difference being that one includes a Phillips Curve while the other includes a monetarist reduced form. Looking at tables 2 and 4, we observe that the performance of the two models is very close, with a small but consistent edge to the Phillips Curve for virtually all variables in both in- and out-of-sample results.

Finally, policy simulations with the two UCITY models yielded remarkably similar results. Looking at tables 6, 7 and 8, we observe that the policy multipliers are nearly equal in both cases for inflation, output and unemployment. Rounded off to the first decimal point, they are almost identical, particularly after the first four quarters.

CONCLUSIONS

In this paper, we reviewed a current version of the St. Louis Model and presented two alternative versions, referred to as UCITY models, that retain the Andersen-Jordan nominal income reduced form but simplify the inflation sector and improve the estimation of the unemployment rate. In the UCITYPC version, we replaced the St. Louis Model's inflation sector with a more conventional Phillips Curve. In the UCITYRF version, we substituted a monetarist reduced form for inflation for the Phillips Curve.

We demonstrated that the UCITY versions yield improved predictive performance of the major economic variables of interest to policymakers when compared with the St. Louis Model. In addition, the St. Louis Model yields more rapid deceleration of inflation and a smaller temporary rise in unemployment in response to a deceleration monetary growth than in the UCITY models. Finally, the UCITY models yield very similar predictive performances and virtually identical policy multipliers, suggesting that the Phillips Curve and monetarist reduced form are both reasonable, alternative equations for explaining inflation and correspond to structural vs. reduced-form approaches to modeling inflation.