
Controlling Money in an Open Economy: The German Case*

by MANFRED WILLMS

In recent years inflation has been a world-wide problem. To stem the tide of rising prices, stabilization authorities have called on all the economic tools available to them. During this period, there has been growing reliance on controlling growth of the money supplies of nations to prevent inflationary increases in total spending. However, it has been contended frequently that a country with a large foreign trade position could not effectively control its money stock in order to avoid "imported inflation." This is particularly important if a relatively large country, such as the United States, has persistent inflation and balance-of-payments deficits.

In the following article, Professor Manfred Willms presents a framework within which the various factors influencing growth of the money stock in open economies (possessing a relatively large and fluctuating stock of foreign reserves) can be analyzed. The ways that actions of the monetary authorities and the behavior of commercial banks and the nonbank public affect a nation's money stock are discussed, and the relative influence of these groups on the growth of money is estimated.

The article shows how changes in a nation's trade balance and/or net capital flows influence its stock of foreign reserves and growth of its monetary base. Next, there is a discussion of the conditions under which monetary authorities would be able to control the money stock.

Finally, the article presents empirical evidence which indicates that monetary authorities in Germany have maintained effective control in the short run over that nation's money stock, even though foreign reserves have fluctuated sharply and grown very rapidly on balance. The article also presents some special developments in Germany, such as an application of fiscal actions to control the money stock, which illustrate the interacting and opposing forces at work between monetary authorities, commercial banks and the public in a country with a large and volatile foreign sector. To help the general reader follow the main points in this article, the mathematical formulation of the money supply model and the statistical evidence supporting the conclusions are presented in footnotes and in appendices.

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UNDER THE PRESENT international monetary system of fixed exchange rates and free convertibility among major Western currencies, countries with a balance of payments deficit lose foreign reserves (gold and foreign currencies) and those with a surplus acquire foreign reserves. Such flows of foreign reserves may affect the growth of a nation's money stock. A controversial issue for each open economy (an economy with a large foreign sector) is whether its monetary authorities can offset the impact of an outflow or inflow of foreign reserves on the money stock, or whether the present system of fixed exchange rates constrains the domestic monetary policy of these economies. This question is especially relevant in some Western European countries which have accumulated very large U.S. dollar reserves over the last few years, and are also confronted with substantial swings in their dollar flows.

This article analyzes the controllability of the money supply in Germany, a country whose economy is both highly dependent on foreign trade, and well integrated into international financial markets. The article (1) examines two major hypotheses on the controllability of the money supply in an open economy; (2) describes the relationship between the balance of payments of a country and its foreign reserve position; (3) shows the relationship between the foreign reserve position of the central bank and the total amount of base money;¹ (4) develops a model of the money supply process for an open economy; and (5) presents some empirical estimates for the money supply process in Germany.

By arranging the article in this manner, the interrelationship between the balance of payments and the money supply is developed step by step. First, balance-of-payments influences on the stock of foreign reserves at the central bank are described; then the impact of foreign reserves on the creation of base money is discussed; and finally the influences of changes in base money and the money multiplier on the money stock are analyzed.

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¹"Base money" is defined as the net monetary liabilities of the central bank and the government held by commercial banks and the nonbank public. It is similar to the magnitude which Friedman, Schwartz, and Cagan call "high-powered money."

Two Views Regarding Money Control in an Open Economy

There are two alternative hypotheses concerning the controllability of the money supply in an open economy under fixed exchange rates. One hypothesis states that the money supply cannot be controlled in an open economy because any change in the interest rate differential between countries will lead to an inflow or outflow of foreign reserves, neutralizing the desired monetary impact on the domestic economy. According to this hypothesis, the interest rate elasticity of international capital flows is relatively high. The other hypothesis states that the interest rate elasticity of international capital flows is not so high that countries lose control over their money supply.

Hypothesis I

Those who suggest the first hypothesis argue that the monetary authorities, of a country such as Germany with continuous balance-of-payments surpluses, are unable to control the money supply and hence are unable to escape inflation without an adjustment in the exchange rate.² Economists of this group are in favor of more flexible exchange rates as a means of permitting greater national autonomy in the determination of the money supply and the price level. They blame fixed exchange rates for preventing national economies from adjusting to one another and from reconciling internal employment and price level objectives with external balance-of-payments objectives by using "sound" policy decisions.

To illustrate this viewpoint let us assume that world market prices for a country's major export goods are rising relative to prices in that country. Exports of this country will increase as foreign customers direct their demand to the relatively cheaper goods. The country realizes a trade surplus and receives foreign

²The view that the German Bundesbank is not in a position to conduct an independent monetary policy under the present international monetary system was recently expressed by Milton Friedman in an address at Frankfurt, West Germany. The impossibility of a successful fight against inflation in countries with a massive balance-of-payments surplus under a fixed exchange rate system is put forth by Friedrich A. Lutz and Egon Sohmen in *How Can a Country Escape Imported Inflation?* Appendix to the 1964/65 Annual Report of the German Council of Economic Experts; *Stable Money - Steady Growth* (Stuttgart: Kohlhammer 1965), pp. 157-167. See also the (1967/68) Annual Report of the German Council of Economic Experts; *Stability in Economic Growth* (Stuttgart: Kohlhammer 1967), p. 193; George W. McKenzie, "International Monetary Reform and the Crawling Peg," this *Review* (February 1969), pp. 15-23; Harry G. Johnson, "The Case for Flexible Exchange Rates, 1969," this *Review* (June 1969), pp. 12-24; Herbert Giersch, "Growth, Cycles, and Exchange Rates - The Experience of West Germany," *Wicksell Lectures 1970* (Stockholm: Almqvist & Wiksell, 1970), p. 26.

reserves which, unless offset by central bank actions, increase the stock of base money and hence exert an expansionary influence on the money stock. The outflow of goods and the expansion of the money stock increase the demand pressure in the country and lead to an increase in its price level. If the country introduces a restrictive monetary policy in the short run, domestic interest rates will rise relative to interest rates in other countries. This will attract international capital, which will increase the stock of base money and hinder attempts of the monetary authorities to slow the growth of the money stock and curb inflation.

A critical point in the argument of this group of economists is that capital flows between the advanced industrialized countries are highly responsive to changes in international interest rate differentials. Any restriction of the growth of a country's money stock which causes a deviation in its domestic interest rate from the international rate results in an increased inflow or outflow of foreign reserves until the previous interest rate differential is restored.

However, premiums and discounts in forward exchange markets indicate that interest rate differences do exist between countries, and that countries have some freedom to exercise independent monetary policy. This fact is not overlooked by the economists who question the feasibility of a fixed exchange rate system. Nevertheless, they consider the degree of independence of national economic policy to be rather small. Due to its impact on interest rates, monetary policy is considered to be of particularly limited effectiveness in a system of fixed exchange rates. Because the interest rate effect can be softened if fiscal policy is used, restrictive fiscal actions are considered to be more appropriate than monetary actions in reducing domestic demand in a country with balance-of-payments surpluses.³

To achieve this effect the fiscal actions must meet two necessary conditions: (1) they must restrain domestic demand; and (2) they must relieve any upward pressure on interest rates. These conditions are met if a tax increase is used to repay government debt held by the private sector or if government spending is reduced. However, as long as domestic prices are

not adjusted to prices in the world market, a growing volume of domestic goods is absorbed by other countries. Foreign demand merely replaces domestic demand without any relief in total demand pressure.

According to the above analysis, in a system of fixed exchange rates monetary and fiscal policies can have only a short-run effect in restricting domestic demand in countries with a balance-of-payments surplus. Sooner or later the domestic rate of inflation will reflect the price level trends in the world market. Only for a very limited period can an economy which is highly integrated in the world economy control its money stock and resist inflationary pressures from abroad.

Hypothesis II

A second group of economists have suggested the hypothesis that the monetary authorities can control the money stock in an open economy even under a system of fixed exchange rates.⁴ Economists of this group assume that the amount of foreign reserves attracted by a rise in domestic interest rates caused by monetary contraction is smaller than the reduction of base money by the monetary authorities. Thus, there will be a net restrictive effect on the growth of base money and the money stock. According to this view, short-term international capital flows will not react to the observed interest rate differential between two countries, but rather to the interest rate differential adjusted for the forward exchange rate. Movements of interest rates and the forward exchange rate have a tendency to offset each other, reducing the incentive for large movements of short-term international capital. Transaction costs and risks also restrict the mobility of international capital flows. For all these reasons, the interest elasticity of short-term international capital movements does not appear so high that a country would not be able to control its money supply.

The above discussion indicates that the main difference between the two viewpoints is that the first group assumes a rather high interest elasticity of international capital flows, while the second group considers this elasticity to be small. Which viewpoint is correct must be decided by empirical evidence. Before presenting evidence on this issue, the relationship

³For an analytical foundation of this statement see Robert A. Mundell, "Capital Mobility and Stabilization Policy Under Fixed and Flexible Exchange Rates," in *Canadian Journal of Economics and Political Science* (November 1963), pp. 509-517, reprinted in Robert A. Mundell, *International Economics*, (New York: Macmillan, 1968), pp. 250-262. See also Egon Sohmen, *Flexible Exchange Rates*, Revised Edition, (Chicago: Chicago University Press, 1969), p. 208 and p. 212.

⁴See J. Herbert Furth, "International Monetary Reform and the Crawling Peg - Comment," this *Review* (July 1969), pp. 21-25. The German Bundesbank and the German Government are obviously also very close to this view. See the official "Comments" by the German Federal Government on the (1964/65) Annual Report of the German Council of Economic Experts, p. 197.

between the balance of payments and the money stock of a nation will be explained.

The Balance of Payments and the Central Bank's Foreign Reserve Position

The simplest relationship between the balance of payments and the money stock is given by the classical gold standard mechanism. Whenever a country's exports of goods and services exceeded its imports, its gold stock tended to increase. Since there was a close relationship between the gold stock and the money stock, an increase in gold led to an increase in the country's money stock. On the other hand, a country which imported more than it exported, lost gold, and its money stock was reduced. Under such a mechanism, the money stock would be primarily a function of the country's balance of payments.

Under the present international monetary system, the close relationship between a nation's gold stock and its domestic money stock is broken. A country's stock of base money can be altered quite independently from changes in the stock of foreign reserves at the central bank.

Changes in foreign reserves at the central bank are the result of the total balance-of-payments situation. They are the joint reflection of conditions in domestic and foreign markets for goods and financial assets as well as of the domestic economic policy actions of the monetary authorities. To show this interrelationship and the impact of the balance-of-payments situation on the holdings of foreign reserves at the central bank, three broad categories of international transactions are distinguished. The first category consists of the current account items, primarily exports and imports of goods and services. The second category consists of two capital account items: changes in foreign assets and changes in foreign liabilities. Changes in these stocks reflect direct investments in real assets, portfolio investments in long-term financial assets, and investments in short-term financial assets. The third category of international transactions consists of monetary transactions involving changes in foreign currencies, official gold holdings, gold tranche positions and special drawing rights. These transfers can be considered the balancing items because they represent the means by which a surplus or deficit in the current account or the capital account is financed.

Changes in the stock of foreign reserves at the central bank occur within the following balance-of-payments constraint:

$$(Ex - Im) - \Delta(FA - FL) = \Delta FR$$

where: Ex = Flow of exports of goods and services
 Im = Flow of imports of goods and services
 FA = Stock of real and financial foreign assets
 FL = Stock of real and financial foreign liabilities
 FR = Stock of foreign reserves at the central bank
 Δ = Change in stock variables in one period

The first term in parentheses represents the current account, and the second term, the capital account.

The preceding equation shows that a surplus in the current account does not necessarily lead to an increase in the stock of foreign reserves at the central bank.⁵ Changes in foreign reserves also depend on the capital account. Foreign reserves at the central bank can increase with a balance or even a deficit in the current account. These situations require a surplus in the capital account, and in the latter case this surplus has to be greater than the deficit of the current account.

Factors Affecting the Different Balance-of-Payments Items

The items of the current account are primarily dependent upon domestic prices relative to prices in the world market and on domestic income as well as income in the rest of the world. For the purpose of this paper, the current account is treated as an exogenous variable. Therefore, the functional relationship is not discussed in detail.

The stocks of foreign assets and liabilities desired by private economic units are a function of domestic and foreign interest rates as well as uncovered and covered international interest rate differentials.⁶ The desired stock of foreign assets is postulated to be negatively related to domestic interest rates and positively related to interest rates in foreign markets as

⁵A surplus (deficit) in the current account is a situation where exports (imports) are greater than imports (exports). A surplus (deficit) in the capital account implies that changes in foreign liabilities (foreign assets) are greater than changes in foreign assets (foreign liabilities).

⁶The uncovered interest rate differential is defined as the foreign interest rate minus the domestic interest rate. The covered interest rate differential is the short-term international interest rate differential adjusted for forward premiums and discounts. For example, the covered interest rate differential between the United States and Germany (i_{dife}^{US}) is equal to the U.S. money market rate (i^{US}) minus the German money market rate (i^{GE}) minus a forward discount or plus a forward premium on the U.S. dollar:

$$i_{dife}^{US} = i^{US} - i^{GE} + \delta$$

The forward discount or premium on the dollar (δ) is expressed as a per cent per annum of the spot rate of exchange:

$$\delta = \frac{p_t - p_0}{p_0 \cdot t}$$

p_0 = spot rate of exchange
 p_t = forward rate of exchange
 t = time period expressed as fraction of a year

well as to international interest rate differentials. The desired stock of foreign liabilities is dependent on the same variables, however, with the opposite signs.

In addition, the desired stocks of foreign assets and foreign liabilities are influenced by speculation in connection with expected changes in the pegged foreign exchange rate. An expected revaluation reduces the desired stock of foreign assets and increases the desired stock of foreign liabilities.

Besides the independent variables mentioned above, the desired stocks of foreign assets and liabilities have been influenced by actions of the monetary authorities. Until September 1969, one of the most important instruments used by the German central bank for neutralizing the inflow of short-term foreign capital was to vary the forward exchange rate for U.S. dollars and to alter the maturity of its forward contracts with commercial banks.⁷ The Bundesbank fixed the forward dollar rate according to the spot exchange rate, the interest rate differentials between the domestic and foreign money markets, and the forward exchange rate in the free forward market for U.S. dollars. The fixed forward rate was only applied to forward transactions with commercial banks. Its main purpose was to increase commercial banks' holdings of short-term foreign assets.

While the forward exchange policy was undertaken to stimulate short-term capital exports of commercial banks, other measures were introduced to discourage capital imports. One of these measures was a higher legal reserve ratio for deposits of nonresidents than for residents. New deposits of nonresidents sometimes have been charged with a required reserve ratio of 100 per cent. Furthermore, interest payments on deposits of nonresidents were forbidden during certain periods.⁸

According to the above discussion, the desired stocks of foreign assets and foreign liabilities are a function of the domestic interest rates, the foreign interest rates, the uncovered interest rate differential, the covered interest rate differential in the free and

Table I
Functions Determining Desired Stocks of Foreign Assets (FA*) and Foreign Liabilities (FL*)

Dependent Variables	Independent Variables	Functional Relationships	
Foreign Assets (FA*)	domestic interest rate (i)	negative	
	foreign interest rate (ir)	positive	
	uncovered interest rate differential (iair)	positive	
	free covered interest rate differential (iairc)	positive	
	controlled covered interest rate differential (iairc)	positive	
	free forward rate (δ)	positive	
	controlled forward rate (δ̄)	positive	
	speculation variable (ρ)	negative	
	Foreign Liabilities (FL*)	domestic interest rate (i)	positive
		foreign interest rate (ir)	negative
uncovered interest rate differential (iair)		negative	
free covered interest rate differential (iairc)		negative	
controlled covered interest rate differential (iairc)		negative	
free forward rate (δ)		negative	
speculation variable (ρ)		positive	
required reserve ratio on foreign liabilities (rr)		negative	

Changes in the stock of foreign assets and foreign liabilities are assumed to follow an adjustment process of the type:

$$FA_t - FA_{t-1} = \lambda_1 (FA_t^* - FA_{t-1})$$

$$FL_t - FL_{t-1} = \lambda_2 (FL_t^* - FL_{t-1}); 0 < \lambda_1, \lambda_2 < 1$$

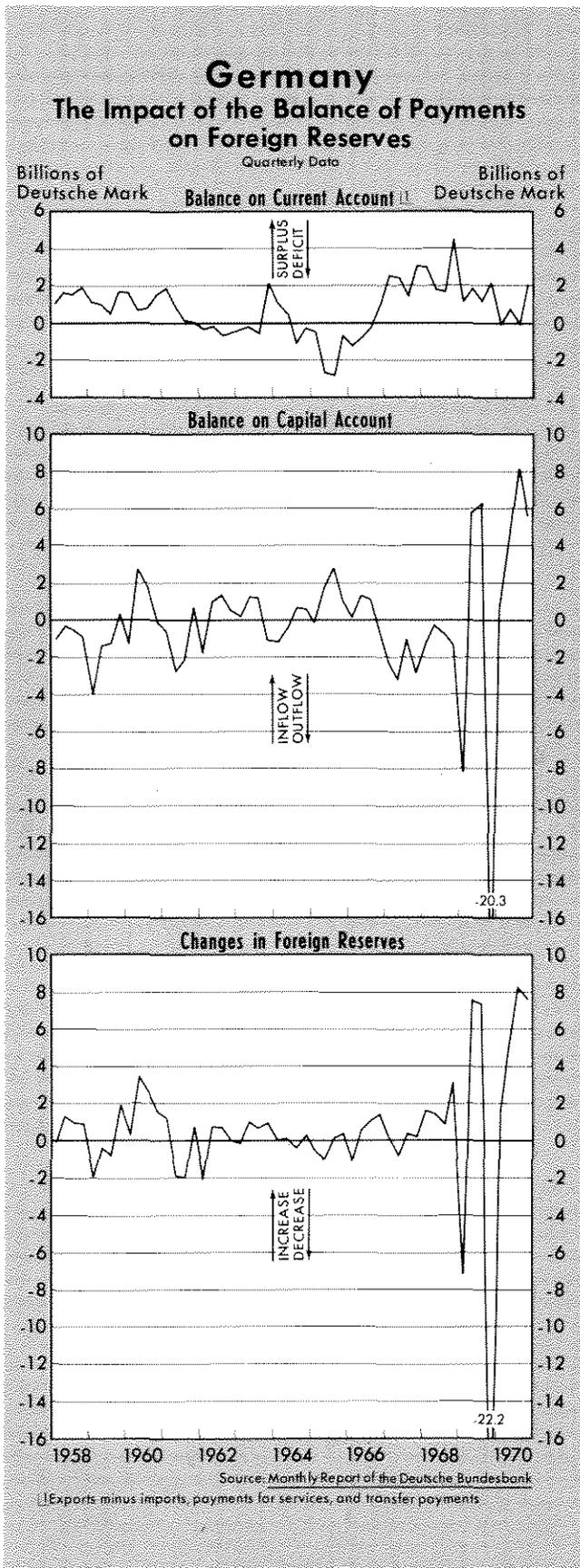
controlled forward market, the free and controlled forward rate, and a variable indicating speculative expectations with respect to an upward variation in the exchange rate.

The symbols λ_1 and λ_2 in Table I stand for adjustment coefficients indicating the proportion of the gap between the actual and desired stock which is eliminated in one period. The closer λ is to 1, the faster is the adjustment. The adjustment process implies that for given values of the variables determining the desired stock, the rate of change of the stock variables decreases as the actual stock approaches the desired stock. The same variables determining the desired stocks of foreign assets and foreign liabilities influence the stock of foreign reserves at the central bank through their impact on the capital account.

In Germany, fluctuations in the stock of foreign reserves at the Bundesbank are dominated by international capital movements. This is shown in the following chart, where the data for the current account, the capital account, and changes in foreign reserve holdings of the Bundesbank are plotted for the period

⁷For a detailed discussion of the forward exchange policy of the Bundesbank, see Ekhard Brehmer, "Official Forward Exchange Operations: The German Experience," in *International Monetary Fund Staff Papers* (November 1964), pp. 389-413.

⁸For a further discussion of instruments to influence short-term capital flows, see Rodney M. Mills, "The Regulation of Short-Term Capital Movements: Western European Techniques in the 1960's," *Staff Economic Studies No. 46*, Board of Governors of the Federal Reserve System, Washington, 1968. See also Otmar Emminger, "Practical Aspects of the Problems of Balance of Payments Adjustment," in *Journal of Political Economy*, Supplement (August 1967), pp. 512-522.



from 1958 to 1970. During the periods 1958 through 1962 and 1969 through 1970, changes in the stock of foreign reserves at the central bank followed very closely the movements of the capital account. During the period 1963 through 1968, the impact of the heavy fluctuations in the current account on the foreign reserve position of the Bundesbank, to a large extent, was neutralized by offsetting movements in the capital account.

The movements in the capital account, to some extent, are the result of monetary policy actions. The Bundesbank used forward exchange policy very effectively in periods of increasing balance of payments surpluses. In 1960-61, when German interest rates were higher than U.S. rates, the Bundesbank fixed a premium for forward dollars, even though forward dollars were traded with a small discount in the free forward market. In 1968-69, interest rates in the United States exceeded those in Germany, and the Bundesbank offered the commercial banks a discount on forward U.S. dollars which was lower than the discount in the free forward market.

Sources and Uses of Base Money

In the preceding section, the impact of the balance of payments on changes in the stock of foreign reserves was described. Now the relationship between foreign reserves and base money will be examined.⁹

In Germany, base money consists of the net monetary liabilities of the Bundesbank and the Federal Government issued to the private sector. Consolidating the monetary accounts of the Bundesbank and the Government, a balance sheet can be constructed for deriving the sources and uses of base money.

A consolidated balance sheet for base money is illustrated in Table II on the following page. The left side of the balance sheet shows the different sources of base money. The right side shows the uses.

Sources of Base Money

The different terms of the source base reflect the impact of the foreign sector, the behavior of the central bank, the behavior of the government, the behavior of the commercial banks or a combination of these influences.

As indicated above, the stock of foreign reserves at the central bank is equal to the accumulated sum of

⁹Base money is defined in footnote 1.

Table II

Monetary Base in Germany
December 31, 1970
(Billions of marks)

Sources of the Base		Uses of the Base	
Foreign reserves (FR)	50.6	Currency held by the nonbank public (C)	36.9
Gold	14.3	Commercial bank reserves (R)	29.8
Foreign currency	28.4		
Others*	7.9		
Discount borrowing (DB)	18.7		
Government securities (GS ²)	1.2		
Government advances (GA)	2.0		
Government deposits (GD)	-6.7		
Special anticyclical deposits**	-5.4		
Others	-1.3		
Coin (CN)	3.0		
Others (U)	-2.1		
Source base (B^s)	66.7	Base	66.7
Reserve adjustment (B ^r)	-1.6	Reserve adjustment	-1.6
Monetary base (B)	65.1	Monetary base	65.1

*Mainly nonbank foreign assets, plus the reserve position at the IMF adjusted for special drawing rights.

**Special anticyclical deposits consist of (1) balances acquired by the Federal Government and the State Governments (2.9 billion marks), and (2) accumulated revenues from an anticyclical surtax on income (2.5 billion marks). Both measures were introduced in 1970 in order to reduce the growth of the money supply.

Note: In equation form, the source base is defined as:

$$B^s = FR + DB + GS^2 + GA - GD + CN - U$$

net balance-of-payments surpluses of preceding periods. In the short run, foreign reserves are to a large extent uncontrolled by the Bundesbank. The Bundesbank can only change its foreign reserve balances by influencing foreign assets and foreign liabilities of commercial banks through variations in the covered interest rate differential or the legal reserve ratio for deposits of nonresidents.

Commercial banks' borrowings through the discount window are controlled by the Bundesbank through changes in the discount rate and the discount quota. The discount quota is a limit for discount borrowings. It is fixed by the Bundesbank for each commercial bank according to its asset and liability structure. This limit for discount borrowings has been so high in the past that, with the exception of some short periods, it has had very little restrictive effect.

Open market policy works in a way similar to discount policy. Instead of determining the quantity of government securities it wants to sell or buy, the Bundesbank establishes the interest rate at which it sells or buys short-term government securities. With respect to the quantity, the Bundesbank is dependent

on the behavior of the commercial banks to which open market operations are restricted.¹⁰

Government advances and government deposits can be considered as exogenous variables.¹¹ Both are under the direct control of the Government. Particularly, Government deposits are an important instrument to influence the growth of base money. They consist of three items: regular deposits, special anticyclical deposits of the Federal Government and the State Governments, and deposits related to an anticyclical surtax on income. The last two measures were made available to the Government by the Stabilization Law of 1967.

The most important instrument of monetary policy with respect to its quantitative impact on the money supply has been required reserve policy.¹² Changes in required reserves do not influence the stock of the source base. However, they affect the money stock through the ability of commercial banks to create money. An increase in the

average required reserve ratio increases required reserves and, for a given stock of source base, exerts a contractionary influence on the money supply. The quantitative impact of changes in reserve requirements is reflected in the monetary base. The monetary base is equal to the source base adjusted for changes in reserve requirements.¹³

Table II shows the sources of the monetary base at the end of 1970, and the following chart illustrates the growth of the monetary base and its domi-

¹⁰For a theoretical analysis of the impacts of the two different types of open-market policy, see Manfred Willms, "Bankenverhalten und Offenmarktpolitik" (Commercial Bank Behavior and Open-Market Policy), in *Jahrbuecher fuer Nationaloekonomie und Statistik* (June 1970), pp. 159-172.

¹¹The German Constitution limits government advances to a total of 6 billion marks.

¹²See Juergen Siebke and Manfred Willms, "Das Geldangebot in der Bundesrepublik Deutschland. Eine Empirische Analyse fuer die Periode von 1958 bis 1968" (The Money Supply in the Federal Republic of Germany: An Empirical Analysis for the Period 1958 to 1968), in *Zeitschrift fuer die Gesamte Staatswissenschaft* (January 1970), pp. 55-74.

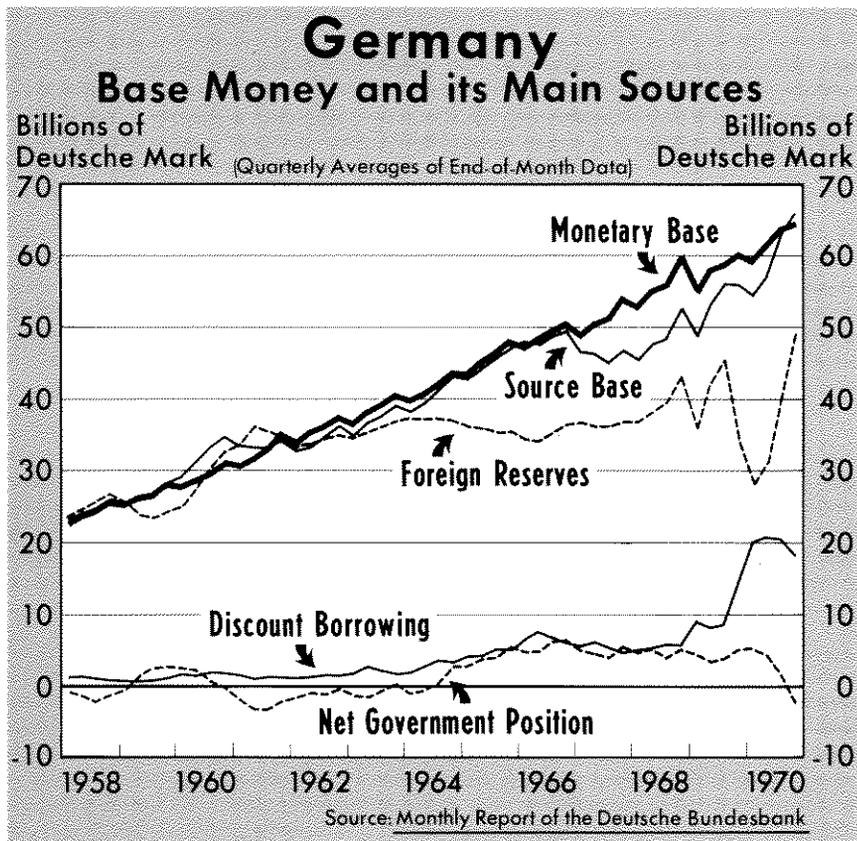
¹³ $B = B^s + B^r$

B = Monetary base

B^s = Source base

B^r = Adjustment component for changes in required reserves

The adjustment component B^r is the sum of monthly



the Bundesbank. Government deposits exceeded the stock of government securities held by the central bank, and the Bundesbank became a net debtor to the government.

Discount borrowings were relatively small until 1964. Since then their contribution to the monetary base has become more important, with a sharp increase in 1969-70. Over the last five years, borrowings through the discount window typically have moved inversely to foreign reserves. This indicates that the commercial banks reacted to any decline in base money due to a reduction in foreign reserves with an increase in discount borrowings.

The net government position consists of the stock of government securities at the Bundesbank plus government advances from the Bundesbank minus government deposits at the Bundesbank. Quantitatively, this variable is relatively unimportant in Germany compared with the United States or

England. However, in periods like 1961, 1965-66, and 1970, it was effectively used to offset at the margin

nant source components for the period 1958 through 1970. It is obvious that foreign reserves are the largest source component of the monetary base, contributing on the average 84 per cent to the monetary base (the corresponding figure for the U.S. economy is 24 per cent). Foreign reserves were at times greater than the monetary base. During these periods, the impact of foreign reserves was offset by an increase in government deposits at

England. However, in periods like 1961, 1965-66, and 1970, it was effectively used to offset at the margin

changes in the volume of required reserves due to changes in reserve requirement ratios:

$$B_t^r = \sum_{T=t_0}^t \frac{\Delta B_T^r}{T}$$

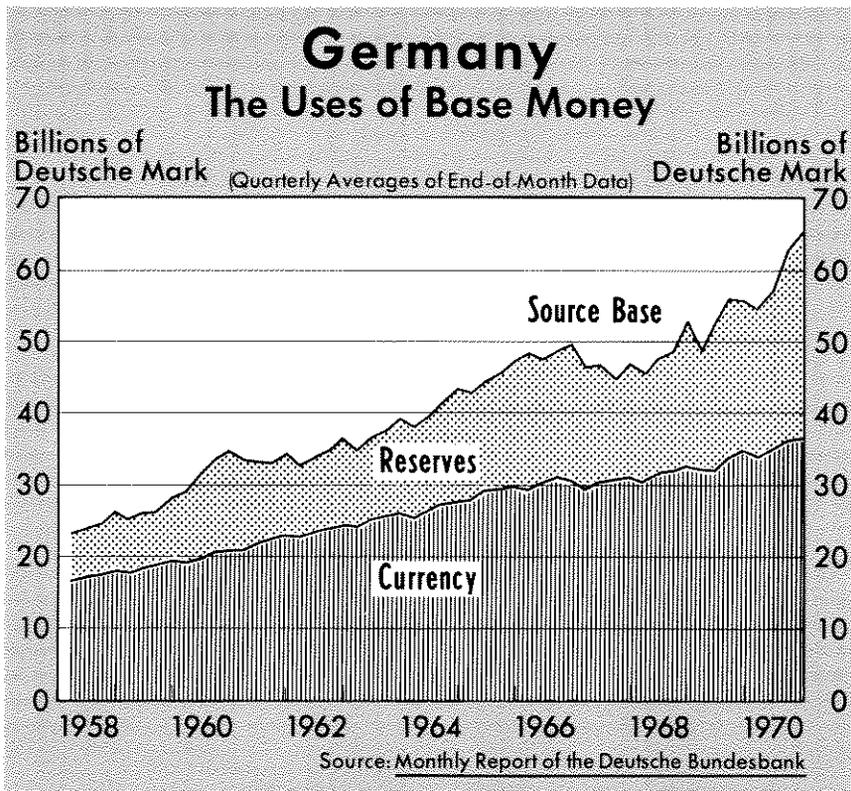
with $t_0 = \text{February 1949}$

ΔB_t^r is calculated as:

$$\Delta B_t^r = -(\bar{r}_t - \bar{r}_{t-1}) RV_{t-1}$$

where \bar{r} stands for the average required reserve ratio and RV for the deposits of commercial banks for which reserves are required.

For a detailed analysis see Leonall C. Andersen and Jerry L. Jordan, "The Monetary Base - Explanation and Analytical Use," this *Review* (August 1968) pp. 7-11.



the impact of an inflow and outflow of foreign reserves on the monetary base.

Uses of Base Money

Base money is used by the public as currency and by the commercial banks as reserves (that is, $B^s = C + R$). The distribution of base money between currency and reserves is shown in the preceding chart. The amount of notes and coins held by the public follows a relatively stable growth path. The major fluctuations in the source base are related to changes in the reserve position of commercial banks. Both the total amount of base money and its distribution between currency and reserves play an important role in the determination of the money supply. The relationship between these variables is derived in the model presented below.

A Model of the Money Supply Process

The above analysis indicated how the monetary base is influenced by the balance of payments and the actions of the monetary authorities. The analysis showed that some components of the monetary base, discount borrowings and net short-term foreign borrowings of commercial banks, are only indirectly controlled by the Bundesbank.¹⁴ Net short-term foreign borrowings of the nonbank public are completely uncontrolled.

In order to formulate a hypothesis on the controllability of the money supply process in an open economy, the monetary base has to be adjusted for these variables.¹⁵ This adjusted base series will be referred to as the "net monetary base."

The net monetary base (B^n) is defined as:

$$B^n = B - DB + (FA^B - FL^B) + (FA^P - FL^P)$$

where: B = Monetary base
 DB = Discount borrowings
 FA^B = Short-term foreign assets of commercial banks
 FL^B = Short-term foreign liabilities of commercial banks
 FA^P = Short-term foreign assets of the nonbank public
 FL^P = Short-term foreign liabilities of the nonbank public

¹⁴Within the German institutional framework, the term commercial banks includes private banks, savings banks and their central institutions the giro banks, state banks, credit cooperatives and mortgage banks.

¹⁵In this adjustment it is assumed that all foreign transactions culminate in changes in foreign reserves at the central bank.

The net monetary base can be considered as the exogenous part of the monetary base. The quantity of money supplied (currency and demand deposits in the hands of the public), and the quantity of domestic earning assets (bank credit) demanded by commercial banks, can be expressed, respectively, as the product of the net monetary base and a money multiplier, and the product of the net monetary base and a credit multiplier.¹⁶

$$M = m^n \cdot B^n$$

$$K^B = k^B \cdot B^n$$

B^n = Net monetary base
 M = Money stock
 m^n = Money multiplier related to net monetary base
 K^B = Bank credit
 k^B = Credit multiplier related to net monetary base

The determination of these multipliers (m^n and k^B) is explained in Appendix II. The multipliers reflect the behavior of the commercial banks and the public with respect to the supply of money and bank credit. They are assumed to depend on the market interest rate, the different rates on deposits, the short-term foreign rate, the covered interest rate differential in the free forward market, the covered interest rate differential in the controlled forward market, the free forward rate, the controlled forward rate, the discount rate, national income, nonhuman wealth, and the expected rate of inflation.

The policy variables exercise a direct and an indirect effect on the supply of money and bank credit. The direct effect consists of the impact of discount policy and forward market policy on the multipliers, and the impact of required reserve policy on the net monetary base. The indirect effects are on the multipliers, related to base-induced changes in the interest rates.

Interest rates affect the money multiplier and the credit multiplier through their impact on the behavior ratios which determine each multiplier. The desired ratio of discount borrowings to total deposits is assumed to depend on the market interest rate and the discount rate. The desired ratios of short-term foreign assets and short-term foreign liabilities to total deposits are a function of the market interest rate, the short-term foreign rate, the covered interest rate dif-

¹⁶This model of the money supply process is developed within the framework of the Brunner-Meltzer nonlinear money supply hypothesis. Karl Brunner and Allan H. Meltzer, "Some Further Investigations of Demand and Supply Functions for Money," *Journal of Finance* (May 1964), pp. 240-283; and "Liquidity Traps for Money, Bank Credit, and Interest Rates," *Journal of Political Economy* (January/February 1968), pp. 1-37. See also Albert E. Burger, *An Explanation of the Money Supply Process*, (Belmont, California: Wadsworth Publishing Company, forthcoming 1971).

ferentials, the forward rates, and the discount rate. The market interest rate and the different rates on demand deposits, time deposits, and savings deposits affect the multipliers through their impact on the currency ratio, the time deposit ratio, the savings deposit ratio, and the reserve ratio:

The demand for bank credit by the public is assumed to be a function of the market interest rate, the interest rates in other financial markets, the foreign money market rate, the expected rate of inflation, national income, and nonhuman wealth.

Equilibrium in the bank credit market occurs when the quantity of credit supplied by commercial banks is equal to the quantity of credit demanded by the public. The equilibrium in this market determines the market interest rate. Thus, the equilibrium interest rate can be derived by equating the supply function of credit and the demand function for credit. Appropriate substitution of the equilibrium value for the interest rate into the money supply equation leads to the determination of the equilibrium money stock. Thus, according to this approach, for a given net monetary base and other predetermined variables, the money stock is determined in the process of reaching equilibrium in the bank credit market.

The interest rate elasticities of the money multiplier, the bank credit multiplier, and the demand for bank credit by the public play an important role in the determination of the money supply elasticities.

The ratio of these elasticities will be referred to as the "q-factor." Because of their importance, it is necessary to analyze these elasticities in more detail. The interest rate elasticity of the bank credit multiplier $[\epsilon(k^B, i)]$ is postulated to be positive, while the interest rate elasticity of the public's demand for bank credit $[\epsilon(k^B, i)]$ is assumed to be negative. Consequently, the denominator of the q-factor is positive. Therefore, the sign of the q-factor depends on the elasticity of the money multiplier with respect to the market interest rate $[\epsilon(m^n, i)]$.

The interest rate elasticity of the money multiplier is the sum of the following elasticities:

$$\begin{aligned} \epsilon(m^n, i) = & \epsilon(m^n, b) \cdot \epsilon(b, i) + \epsilon(m^n, f) \cdot \epsilon(f, i) \\ & + \epsilon(m^n, a) \cdot \epsilon(a, i) + \epsilon(m^n, r) \cdot \epsilon(r, i) \\ & + \epsilon(m^n, t) \cdot \epsilon(t, i) + \epsilon(m^n, s) \cdot \epsilon(s, i) \end{aligned}$$

The signs of the different elasticities are specified above each term. According to this equation, an increase in the market interest rate affects the money multiplier positively through its impact on the ratio for discount borrowings (b), the short-term foreign liability ratio (f), the short-term foreign asset ratio (a), and the total reserve ratio (r). A negative effect on the money multiplier emanates from the impact of an increase in the interest rate on the time deposit ratio (t) and the savings deposit ratio (s).

The interest rate elasticity of the money multiplier is the sum of positive and negative components. Therefore, its sign is theoretically undetermined. However, the first four of the six elasticity terms in the above equation are positive, while only the last two are negative. Empirical estimates for Germany show that the contribution of the borrowing ratio to the growth rate of the money stock was greater than the joint contribution of the time deposit ratio and the savings deposit ratio to the growth of the money stock.¹⁷ Therefore, it can be assumed that the interest elasticity of the money multiplier in Germany is positive. In that case, the value of the q-factor is also positive.

Determining the sign of the q-factor enables us to derive conclusions regarding the controllability of the money supply in an open economy. As long as the

Table III

Using logarithmic forms of the equations, elasticities of the money supply with respect to each of the predetermined variables can be derived. Some of the elasticities are shown below.

Elasticities of the Money Supply
With Respect to Some Exogenous Variables
of the Model

Elasticity of M with respect to the net monetary base $\epsilon(M, B^n) = 1 - q$

Elasticity of M with respect to the discount rate $\epsilon(M, i) = \epsilon(m^n, i) - q\epsilon(k^B, i)$

Elasticity of M with respect to the foreign money market rate $\epsilon(M, i_f) = \epsilon(m^n, i_f) + q[\epsilon(k^B, i_f) - \epsilon(k^P, i_f)]$

Note: $q = \frac{\epsilon(m^n, i)}{\epsilon(k^B, i) - \epsilon(k^P, i)}$

For instance, the notation $\epsilon(Y, X)$ is read: "the elasticity of Y with respect to X."

The elasticity is defined as the ratio of the relative percentage change in Y to the relative percentage change in X.

¹⁷Siebke and Willms, "Das Geldangebot in der Bundesrepublik Deutschland. Eine Empirische Analyse fuer die Periode von 1958 bis 1968" (The Money Supply in the Federal Republic of Germany: An Empirical Analysis for the Period 1958 to 1968), page 65 and page 69.

Table IV

GERMANY											
Regression Estimates of Reduced-form Equations for the Market Interest Rate and the Money Supply (Quarterly Data: I/1960 — II/1970)											
Regr. No.	Dependent Variable	Funct. Form	Constant	B ⁿ	Y	$\left(\frac{W}{P}\right)_{t-1}$	* Pt-1	iGE	R ²	D-W	S.E.
1	i	log	-.219	.427 (-4.643)	.603 (7.606)				.61	.321	.07
2	M	log	-.084	.228 (4.505)		.468 (4.528)	.132 (3.176)	.087 (4.192)	.99	.731	.02

Notes: i = Yield on long-term government securities as a proxy for the bank credit rate.
M = Currency and demand deposits in the hands of the public.
Bⁿ = Net monetary base.
Y = GNP, computed by Deutsches Institut fuer Wirtschaftsforschung, Berlin.
 $\left(\frac{W}{P}\right)_{t-1}$ = Sum of real financial assets held by the public at commercial banks lagged by one quarter.
* Pt-1 = Rate of change of consumer price index lagged by one quarter.
iGE = German money market rate.
R² = Coefficient of determination.
D-W = Durbin Watson statistics.
S.E. = Standard error of estimate.

value of the q-factor does not become equal to one, the elasticity of the money supply with respect to the net source base is still positive, and the central bank maintains control over the money supply. Only if the q-factor equals one does the response of the money supply to changes in the net monetary base become zero.

Those who argue that the monetary authorities are not able to control the money stock in an open economy implicitly assume that the q-factor is equal to one, and that such a value of the q-factor only results from the interest rate elasticity of short-term international capital movements. In other words, the interest rate elasticities of the money multiplier and the credit multiplier of commercial banks are dominated by the elasticities of short-term international capital transfers.

Others, who assert that the monetary authorities can control the money stock in an open economy, assume a value of the q-factor which is smaller than one. They may argue that any increase in the multiplier due to foreign borrowings of commercial banks can be neutralized by a reserve requirement of 100 per cent, and that foreign borrowings of private enterprises cannot be expanded indefinitely.

Some Empirical Observations of the Money Supply Process

The empirical section begins with some estimates of elasticities which are important for the explanation

of the money supply process in an open economy. The estimates are derived as reduced-form regression equations determining the market interest rate and the money supply. The estimates are presented in Table IV.

To conserve degrees of freedom given the limited number of sample observations, a much smaller number of independent variables appear in these reduced-form equations than in the theoretical analysis. Therefore, in the regressions the market interest rate is a function only of the net monetary base and national income, while the money supply is a function only of the net monetary base, real nonhuman wealth, the rate of change in the price level, and the domestic money market rate. In the estimates, the market interest rate has been approximated by the yield on long-term government securities, and nonhuman wealth by the sum of financial assets held by the public at commercial banks. To avoid spurious correlation between the money stock and the proxy variable for wealth, the latter has been lagged by one quarter. A similar lag was introduced for the rate of change in the price level. This variable is assumed to be a proxy for price expectations.

The regressions are performed in this manner in order to obtain estimates of the value of the q-factor, that is, the interest elasticity of the money multiplier divided by the interest elasticity of the credit multiplier minus the interest elasticity of the public's demand for bank credit. From the regression results the

Table V

GERMANY
Regression Estimates of Interest Rate Elasticities of Commercial Banks'
Short-term Foreign Assets, Short-term Foreign Liabilities,
and Discount Borrowings
 (Quarterly Data: I/1960 - II/1970)

Regr. No.	Dependent Variable	Funct. Form.	Constant	iGE	iEU	iUS	i	iEU difc	iUS difc	R ²	D-W	S.E.
3	a ^B	log	-3.463	-.678 (5.178)	.785 (6.378)					.50	.417	.20
4	a ^B	log	-3.210	-.519 (3.677)		.581 (4.628)				.34	.306	.23
5	a ^B	log	-.820		.821 (6.735)		-1.973 (5.544)			.52	.691	.19
6	a ^B	log	-.583			.732 (5.789)	-1.914 (4.907)			.45	.488	.21
7	a ^B	log	-3.145					.865 (2.646)		.13	.271	.26
8	f ^B	log	-3.438	.238 (1.967)		-.047 (.439)				.18	.185	.19
9	f ^B	log	-3.257						-.334 (2.252)	.10	.210	.19
10	b	log	-6.068	.676 (4.410)	.738 (5.121)					.75	1.495	.23
11	b	log	-5.903	.706 (5.381)		.733 (6.267)				.79	1.667	.21
12	b	log	-9.501		.587 (5.016)		2.490 (7.289)			.84	1.352	.18
13	b	log	-9.236			.561 (4.978)	2.454 (7.044)			.84	1.246	.18

Notes: a^B = Ratio of short-term foreign assets of commercial banks to total deposits.
 f^B = Ratio of short-term foreign liabilities of commercial banks to total deposits.
 b = Ratio of discount borrowings to total deposits.
 iGE = German money market rate.
 iEU = Eurodollar rate.
 iUS = U.S. money market rate.
 i = Yield on long-term government securities as a proxy for the bank loan rate.
 iEU difc = Eurodollar rate over German money market rate adjusted for forward premiums or discounts on the U.S. dollar.
 iUS difc = U.S. money market rate over German money market rate adjusted for forward premiums or discounts on the U.S. dollar.

value of the q-factor was estimated to be 0.77.¹⁸ Although this value of the q-factor is much greater than the corresponding values for the United States, it is smaller than its critical value of one.¹⁹ Thus, it can be

¹⁸The elasticities for the determination of the q-factor are calculated in the following way:

$$\epsilon(i, B^n) = -.427 \text{ (according to regression No. 1, Table IV)}$$

$$\epsilon(k^B, i) - \epsilon(k^P, i) = \frac{-1}{\epsilon(i, B^n)} = \frac{-1}{-.427} = 2.34$$

$$\epsilon(M, B^n) = .228 \text{ (according to regression No. 2, Table IV)}$$

$$\epsilon(M, B^n) = 1 | \epsilon(m^n, i) - \epsilon(i, B^n)$$

$$\epsilon(m^n, i) = \frac{\epsilon(M, B^n) - 1}{\epsilon(i, B^n)} = \frac{-.772}{-.427} = 1.81$$

$$q = \frac{\epsilon(m^n, i)}{\epsilon(k^B, i) - \epsilon(k^P, i)} = .77$$

¹⁹Karl Brunner and Allan H. Meltzer obtained a value of the q-factor for the United States of 0.48. Their regression estimate was 0.94 for the interest elasticity of the money

concluded that the Bundesbank had control over the money supply in the period I/1960-II/1970. However, this statement is only true for the average of the period in consideration, a period which included two revaluations of the German mark against the U.S. dollar. For some sub-periods of speculative inflows of foreign currencies related to an expected revaluation, the possibility cannot be excluded that the marginal value of the q-factor approached its critical value of one.

The above estimates indicate the total impact of the market interest rate on the money multiplier. They do not provide information on the interest elasticities of the different ratios which determine the multiplier. In order to obtain information about the extent to which the interest elasticity of the money

multiplier and 1.96 for the interest elasticity of bank credit. See their "Liquidity Traps for Money, Bank Credit, and Interest Rates," in *Journal of Political Economy* (January/February 1968), Appendix III.

multiplier was influenced by the interest elasticities of international capital transfers and discount borrowings, some estimates have been made of the interest elasticities of the foreign asset ratio, the foreign borrowings ratio, and the domestic borrowings ratio for borrowings through the discount window. The results are summarized in Table V.

The regressions indicate that commercial banks do in fact respond to changes in domestic and foreign interest rates according to the hypotheses developed in the model of the money supply process. Particularly, their behavior with respect to short-term foreign assets and discount borrowings can be explained by foreign and domestic interest rates.

On the other hand, it is difficult to explain short-term foreign borrowings by reference only to interest rates or interest rate differentials. The best estimates of the short-term foreign borrowing ratio of commercial banks are reported in Table V. These estimates suggest that other variables are necessary for the explanations of the foreign borrowing ratio. For one thing, the foreign borrowing ratio is to a large extent influenced by expectations regarding the revaluation of the German mark. Due to the forward market operations of the Bundesbank, the same speculative impact on the foreign asset ratio was almost completely neutralized.

The estimates of Table V show that the interest elasticities of commercial banks with respect to foreign and domestic funds do not deviate substantially in absolute values.²⁰ This implies that commercial banks are indifferent with respect to foreign and domestic funds. It also implies that international capital movements, insofar as they are related to commercial banks, disturb the money supply process in the same way, and with a similar impact, as domestic sources of borrowing.

The previous analysis has been developed in order to formulate and test a hypothesis of the controllability of the money supply process in Germany. Particular attention was paid to the interest elasticities of endogenous variables related to the money supply process. A discussion of the technical details of the money supply process has been avoided. However, a few remarks with respect to the control process of the money supply seem to be appropriate.

Within the framework of the above model, the monetary authorities are assumed to be able to meas-

ure and control the net monetary base, while the money multiplier summarizes the endogenous non-policy controlled factors influencing the money supply process. If the monetary authorities, in the context of that model, want to control the growth of the money stock, they have to forecast the value of the money multiplier. Once the multiplier is predicted, the amount of base money which is needed to achieve the desired money stock is determined.²¹

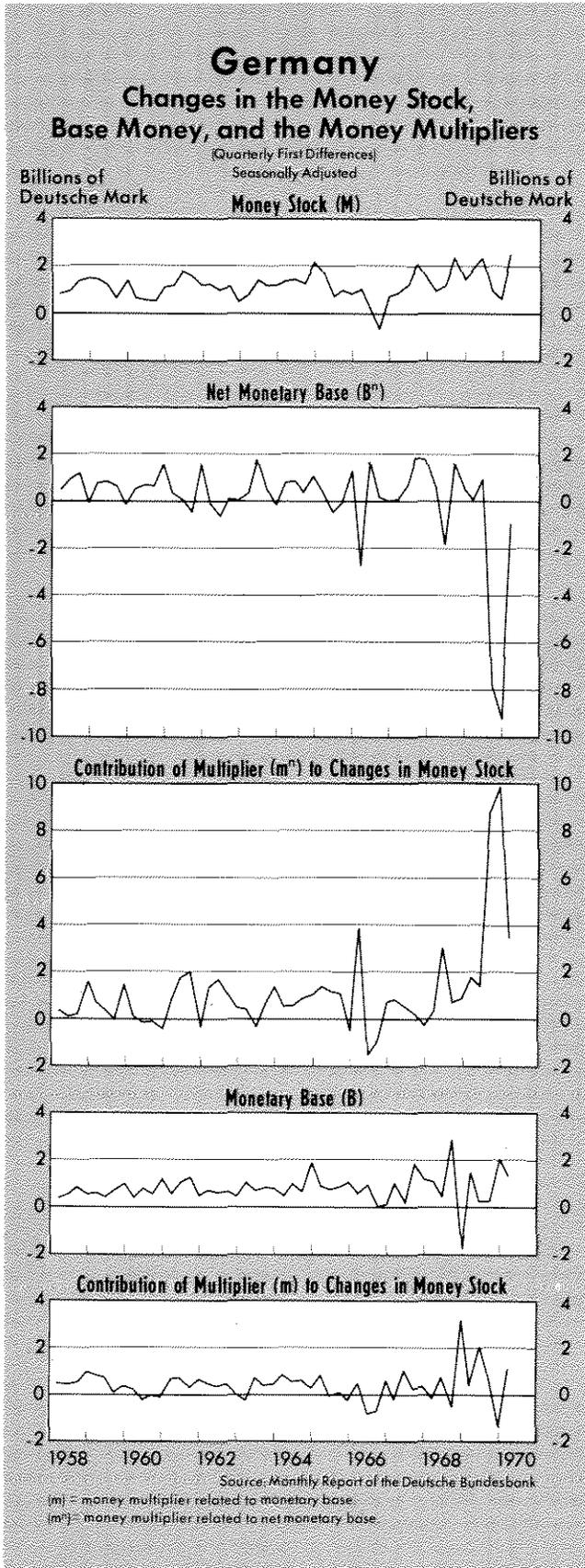
In order to obtain the desired money stock, it is important to make correct estimates of the money multiplier. As can be seen in the following chart, the money multiplier derived by the use of the above model is relatively unstable and its contribution to the money stock fluctuates considerably. Prediction of this multiplier would be difficult. Therefore, it is worthwhile to consider a slightly modified formulation of the multiplier-base concept in which the money multiplier is more predictable. The concept of the monetary base could be an alternative. The monetary base includes the discount borrowings and the net short-term foreign borrowings which the net monetary base does not include.

If the monetary base concept is used instead of the net monetary base concept, changes in the money multiplier reflect primarily the behavior of the public with respect to the allocation of their funds between currency and demand deposits and the allocation of their deposits between demand deposits, time deposits and savings deposits. Since these factors do not fluctuate significantly within short periods of time, the multiplier relating the monetary base and the money stock would be much more stable and predictable than the multiplier which relates the net monetary base to the money stock. The chart shows that the relationship between the monetary base and the money stock is much closer than the relationship between the net monetary base and the money stock. Regression estimates for the period I/1958 to II/1970 indicate that 80 per cent of the variance of quarterly changes in the money stock resulted from changes in the monetary base, while only 16 per cent of the variance of changes in the money stock were explained by changes in the net monetary base.

	R ²	D-W	S.E.
$\Delta M = .195 + 1.222\Delta B$ (12.189)	.80	2.490	.99
$\Delta M = 1.232 + .407\Delta B^n$ (2.841)	.16	2.393	2.03

²¹For a further analysis of a control process of the money supply along these lines, see Lionel Kalish, "A Study of Money Stock Control," *Journal of Finance* (September 1970), pp. 761-776.

²⁰That is, $|\epsilon(a^B, x)|$ is approximately equal to $|\epsilon(b, x)|$, where $x = i^{GE}, i^{EU}, i^{US}$, and i .



Of course, the observed close relationship between the monetary base and the money stock does not solve the control problem. In order to control the monetary base the monetary authorities have to offset movements in the uncontrolled components of the monetary base through changes in the controlled components. However, the monetary authorities gain information by predicting the multiplier which is related to the monetary base. The probability of a wrong forecast is much smaller for the base multiplier than for the net base multiplier.

To analyze whether, and to what degree, the German monetary authorities have in the past been neutralizing undesired influences on the monetary base, a regression equation was estimated in which it was assumed that discount borrowings and net short-term foreign borrowings are the noncontrolled components of the monetary base. If the monetary authorities offset all or part of the movements of the monetary base components which are not subject to their direct control, the regression coefficient in the following equation should be negative and statistically significant. For a perfect offset, the regression coefficient should be equal to minus one.

$$\Delta(B-DB+FA-FL) = \alpha_0 + \alpha_1 \Delta(DB-FA+FL)$$

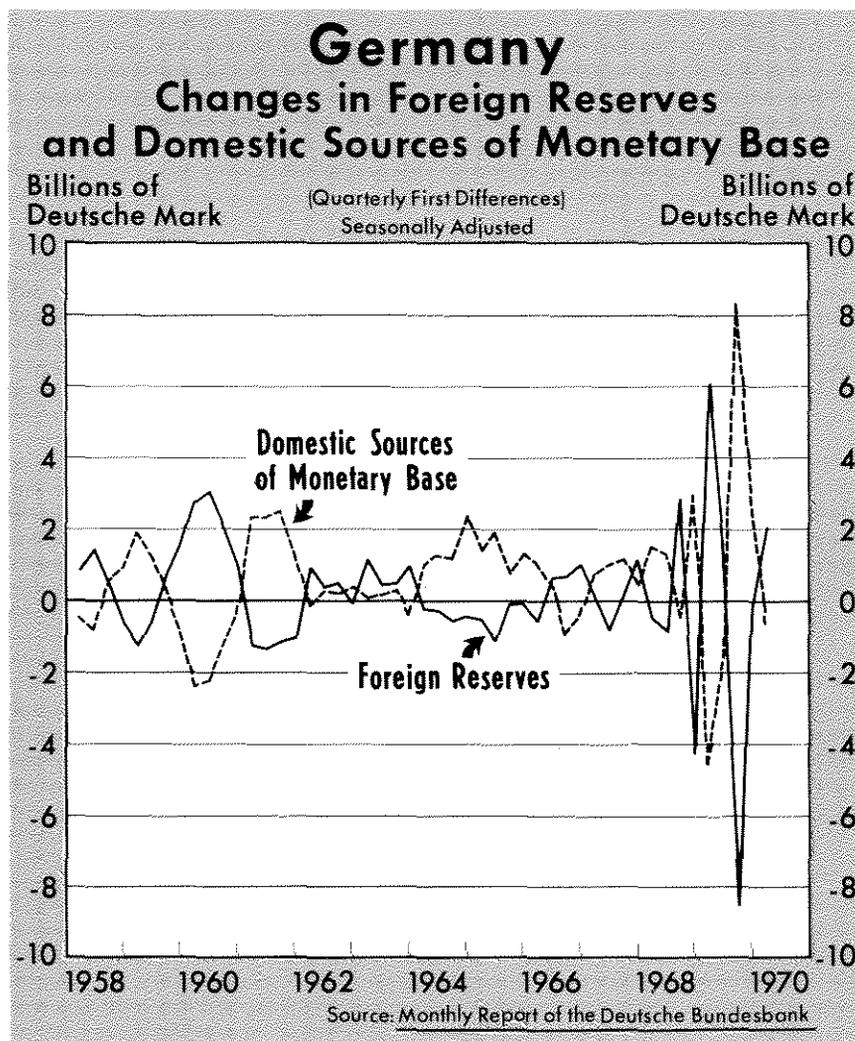
Using central differences of quarterly data for the period I/1958-II/1970 produced the following results:

	R ²	D-W	S.E.
$\Delta(B-DB+FA-FL) =$			
$.824 - .873\Delta(DB-FA+FL)$.82	1.578	.76
(-13.566)			

The estimates indicate that the monetary authorities responded to changes in the components of the monetary base which are not under their direct control. According to these results, the monetary authorities neutralized on the average 87 marks for each 100 mark change in discount borrowings and net foreign borrowings in the period under consideration.

With respect to the question of the controllability of the monetary base in an open economy, it is of interest to analyze the offsetting behavior of the monetary authorities with respect to changes in the total amount of foreign reserves. If a change in the stock of foreign reserves is considered to be the variable to which the monetary authorities adjust the domestic component of the monetary base, the regression coefficient for changes in foreign reserves should be negative and statistically significant.

$$\Delta(B-FR) = \beta_1 + \beta_2 \Delta FR$$



Using quarterly central differences of data for the period I/1958-II/1970, the estimates for this equation are:

$$\Delta(B - FR) = .795 - .863\Delta FR$$

	R^2	$D-W$	$S.E.$
	.79	1.693	.70

(-12.114)

According to the results obtained, the monetary authorities on average offset about 86 marks out of each 100 mark change in foreign reserves through opposite changes in the domestic component of the monetary base.

The offsetting behavior of the monetary authorities with respect to changes in the stock of foreign reserves also becomes obvious in the above chart. Quarterly first differences of seasonally adjusted data of foreign reserves and the domestic part of the monetary base are plotted for the period from 1958 to 1970. The chart shows that the variations of these variables are more or less mirror images of each other, indicat-

ing that the monetary authorities responded strongly to the inflow or outflow of foreign reserves.

Conclusions

In recent years the German economy has experienced a heavy inflow of foreign reserves, primarily U.S. dollars. In real terms the cost of this inflow is a loss in goods and services. In financial terms the primary influence of the flow of foreign reserves in the short-run is on the money supply process. One way to eliminate the continuing inflow of foreign reserves into Germany is to develop a more flexible exchange rate policy. The other alternative is to maintain the present fixed exchange rate system and have the monetary authorities neutralize the impact of changes in foreign reserves on the money supply process.

In the past the German monetary authorities have been relatively successful in neutralizing the impact of the noncontrolled or indirectly controlled components of the money supply process by changing the directly controlled components. The most important instruments for offsetting the impact of changes in the noncontrolled com-

ponents of the money supply process have been the required reserve policy and a change in Government deposits or special anticyclical deposits at the Bundesbank. According to the theoretical analysis of this paper, if the monetary authorities can control the net monetary base, very extreme conditions must occur before the money supply cannot be controlled. The empirical estimates of the interest elasticities of the endogenous variables of the money supply process indicate that the observed elasticities are sufficiently low that control of the money supply can be maintained in the short-run.

In the long-run the use of monetary and fiscal policies to offset domestic inflationary pressure arising from an inflow of foreign reserves means that Germany trades investment and consumption goods for foreign currency. Hence, in real terms a policy of controlling inflation in a fixed exchange rate system results in welfare losses for the German economy.

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APPENDIX I

Alphabetical List of Symbols

a^B	Ratio of short-term foreign assets of commercial banks to total deposits	i_d	Interest rate on deposits
a^P	Ratio of short-term foreign assets of the nonbank public to total deposits	i_g	Rate on government securities
B	Monetary base	i_r	Foreign money market rate
B^n	Net monetary base	i_{d1fc}	Covered interest rate differential
B^r	Adjustment of source base for changes in required reserves	K^B	Supply of bank credit to the public
B^s	Source base	k^B	Credit multiplier
b	Ratio of discount borrowings to total deposits	k^P	Demand function for bank credit by the public
C	Currency in the hands of the nonbank public	M	Money stock
CN	Coin	m	Money multiplier related to monetary base
c	Ratio of currency to demand deposits of the nonbank public	m^n	Money multiplier related to net monetary base
D	Demand deposits	p	Domestic price level
DB	Discount borrowings	R	Total reserves of commercial banks
Ex	Exports of goods and services	\bar{R}_d	Required reserves related to domestic deposits
e^B	Ratio of bank credit to total deposits	\bar{R}_f	Required reserves related to foreign deposits
FA	Short-term foreign assets	R^e	Excess reserves
FL	Short-term foreign liabilities	r	Ratio of total reserves to domestic and foreign deposits
FR	Foreign reserves at the central bank	\bar{r}	Average required reserve ratio on domestic and foreign deposits
f^B	Ratio of short-term foreign liabilities of commercial banks to total deposits	\bar{r}_h	Average required reserve ratio for domestic deposits
f^P	Ratio of short-term foreign liabilities of the nonbank public to total deposits	\bar{r}_f	Required reserve ratio on foreign deposits
GA	Advances of the central bank to the government	\bar{r}_d	Required reserve ratio on demand deposits
GD	Government deposits at the central bank	\bar{r}_t	Required reserve ratio on time deposits
GS^B	Government securities in the hands of commercial banks	\bar{r}_s	Required reserve ratio on savings deposits
GS^Z	Government securities in the hands of the central bank	\bar{r}	Ratio of reserve adjustment component to total deposits
g^B	Ratio of government securities in the hands of commercial banks to total deposits	S	Savings deposits
Im	Imports of goods and services	s	Ratio of savings deposits to demand deposits
i	Market interest rate	T	Time deposits
\bar{i}	Discount rate	t	Ratio of time deposits to demand deposits
		Y	National Income
		W	Nonhuman wealth
		δ	Forward rate
		π	Expected rate of return on real capital

APPENDIX II

Behavior Functions and Multipliers
Determining the Money Supply Model

In the model developed in this article the money supply is determined by the joint behavior of the monetary authorities, the commercial banks and the nonbank public.

The behavior of the monetary authorities is mainly reflected in the movement of the net monetary base:

$$(A1) \quad B^n = B - DB + (FA^B - FL^B) + (FA^P - FL^P)$$

The commercial banks influence the money supply process through their portfolio behavior, that is, through the adjustment of their assets and liabilities according to changes in policy variables and relative prices of financial assets. In the following analysis the consolidated balance sheet of all commercial banks consists of these items:

$$(A2) \quad GS^B + K^B + FA^B + R = D + T + S + DB + FL^B$$

The assets and the borrowings from the central bank and from abroad are related by coefficients to the total amount of deposits. The following functions and their derivatives specify the hypothesis regarding the behavior of commercial banks:

Supply function of commercial banks' funds to the government (= demand function of commercial banks for Government securities)

$$(A3) \quad G^B = g^B(i, \bar{i}, \bar{i}_g, \bar{i}_t, i \frac{\bar{r}}{\text{diff}}, \bar{r}) [D+T+S]$$

with $g_3^B > 0$ and $g_1^B, g_2^B, g_4^B, g_5^B, g_6^B < 0$

Equation (A3) indicates that commercial banks will increase their demand for Government securities if the central bank raises the rates for these assets. On the other hand, an increase in the market interest rate, the discount rate, the rate in foreign money markets, the covered interest rate differential in the controlled forward market, or the average required reserve ratio will induce a reduction in the demand for Government securities by commercial banks.

Supply function of bank credit to the public (= demand function of commercial banks for private domestic earning assets):

$$(A4) \quad K^B = e^B(i, \bar{i}, \bar{i}_g, \bar{i}_t, i \frac{\bar{r}}{\text{diff}}, \bar{r}) [D+T+S]$$

with $e_1^B > 0$ and $e_2^B, e_3^B, e_4^B, e_5^B, e_6^B < 0$

Supply function of commercial banks' funds to foreign markets (= demand function of commercial banks for foreign earning assets):

$$(A5) \quad FA^B = a^B(i, \bar{i}, \bar{i}_g, \bar{i}_t, i \frac{\bar{r}}{\text{diff}}, \bar{r}) [D+T+S]$$

with $a_4^B, a_5^B > 0$ and $a_1^B, a_2^B, a_3^B, a_6^B < 0$

Demand function of commercial banks for discount borrowing:

$$(A6) \quad DB = b(i, \bar{i}, \bar{i}_g, \bar{i}_t, \bar{r}) [D+T+S]$$

with $b_1, b_3, b_4, b_5 > 0$ and $b_2 < 0$

Demand function of commercial banks for foreign liabilities:

$$(A7) \quad FL^B = f^B(i, \bar{i}, \bar{i}_g, \bar{i}_t, i \frac{\bar{r}}{\text{diff}}, \bar{r}) [D+T+S]$$

with $f_1^B, f_2^B, f_3^B, f_6^B > 0$ and $f_4^B, f_5^B < 0$

Demand function of commercial banks for reserves:

$$(A8) \quad R = r(i, \bar{r}_h, \bar{r}_t) [D+T+S]$$

with $r_2, r_3 > 0$ and $r_1 < 0$

Total reserves (R) consist of reserves required against domestic deposits (\bar{R}_h), reserves required against foreign deposits (\bar{R}_t), and excess reserves (R^e), where

$$\begin{aligned} \bar{R}_h &= \bar{r}_d D + \bar{r}_t T + \bar{r}_s S = \bar{r}_h (D+T+S), \\ \bar{R}_t &= \bar{r}_t + FL^B, \\ R^e &= r^e (D+T+S) \end{aligned}$$

Since $R = \bar{R}_h + \bar{R}_t + R^e$,
and $FL^B = f^B (D+T+S)$,
then $R = \bar{r}_h (D+T+S) + \bar{r}_t [f^B (D+T+S)] + r^e (D+T+S)$;
simplifying
 $R = (\bar{r}_h + \bar{r}_t f^B + r^e) (D+T+S)$.

The desired total reserve ratio (r) is a weighted average of the required reserve ratios on demand deposits (\bar{r}_d), time deposits (\bar{r}_t), savings deposits (\bar{r}_s), foreign liabilities of commercial banks (\bar{r}_l), and the excess reserve ratio (r^e):

$$r = \bar{r}_h + \bar{r}_t f^B + r^e = \frac{R}{D+T+S}$$

The behavior of the public is described by three allocation parameters and the demand function for commercial bank credits.^o Equations (A9) through (A11) relate currency and two different types of deposits by allocation parameters to demand deposits of the public.

Demand function for currency by the public:

$$(A9) \quad C = c(i, i_d, Y, W) D$$

Demand function for time deposits by the public:

$$(A10) \quad T = t(i, i_d, Y, W) D$$

Demand function for savings deposits by the public:

$$(A11) \quad S = s(i, i_d, Y, W) D$$

The demand for bank credit by the public is assumed to be positively related to the interest rate in another financial market, to the expected rate of return on real capital, as well as to national income and nonhuman wealth, while it is negatively related to the market interest rate and the foreign interest rate.

Demand function for bank credit by the public:

$$(A12) \quad K^P = k^P(i, i_{fin}, i_t, \pi, Y, W)$$

with $k_2^P, k_4^P, k_5^P, k_6^P, k_7^P > 0$ and $k_1^P, k_3^P < 0$

Using the following definition of the net monetary base

$$(A13) \quad B^n = C + R + B^r - DB + (FA^B - FL^B) + (FA^P - FL^P)$$

$$= [c + (r + \bar{r} - b + a^B - f^B + a^P - f^P) (1 + t + s)] D$$

and the definition of the money stock

$$(A14) \quad M = C + D = (1 + c) D$$

the relation between B^n and M is given by

$$(A15) \quad M = \left[\frac{1 + c}{c + (r + \bar{r} - b + a^B - f^B + a^P - f^P) (1 + t + s)} \right] B^n$$

The term in the brackets is the money multiplier which is related to the net monetary base. The money multiplier related to the monetary base is described in (A16).

$$(A16) \quad M = \left[\frac{1 + c}{c + (r + \bar{r}) (1 + t + s)} \right] B$$

^oShort-term foreign assets and liabilities of the nonbank public in this model are related to total deposits of commercial banks by allocation coefficients similar to those in equations (A5) and (A7). The reasoning is that short-term international capital transactions of commercial banks and the nonbank public are considered to be close substitutes for each other. In addition, they are to a large extent dependent on the same arguments with the same signs of the derivatives of the behavior functions.

^{*}The symbol g^B ($j = 1, 2, \dots, 6$) represents the derivative $\frac{g^B}{j}$.

The independent variables with a bar are policy variables.

The explicit formulation of the credit multiplier is derived from the balance sheet of the commercial banks. According to this balance sheet, bank credit is defined as:

$$(A17) \quad K^B = D+T+S+FL^B+FL^P+DB-R-CS^B-FA \\ = (1+f^B+f^P+b-r-g^B-a) (1+t+s)D$$

The relation between bank credit and the net monetary base is given by:

$$(A18) \quad K^B = \left[\frac{1+f^B+f^P+b-r-g^B-a}{c+(r+\tilde{r}-b-f^B-f^P+a)} (1+t+s) \right] B^0$$

The term in brackets is the credit multiplier.

APPENDIX III

Calculation of the Elasticities

The equilibrium condition in the bank credit market is specified by equating the credit supply and the credit demand functions. In logarithmic form this can be written as:

$$(A19) \quad \log B^n + \log k^B = \log k^P$$

Total differentiation leads to:

$$(A20) \quad \frac{dB^n}{B^n} + \frac{1}{k^B} \left(\frac{\partial k^B}{\partial i} di + \frac{\partial k^B}{\partial i_d} di_d + \frac{\partial k^B}{\partial i_r} di_r + \frac{\partial k^B}{\partial i} di + \dots \right)^3 \\ = \frac{1}{k^P} \left(\frac{\partial k^P}{\partial i} di + \frac{\partial k^P}{\partial i_{fin}} di_{fin} + \frac{\partial k^P}{\partial i_r} di_r + \frac{\partial k^P}{\partial \pi} d\pi + \dots \right)$$

Equation (A20) can be written as:

$$(A21) \quad d \log B^n + \varepsilon(k^B, i) d \log i + \varepsilon(k^B, i_d) d \log i_d + \varepsilon(k^B, i_r) d \log i_r \\ + \varepsilon(k^B, \bar{i}) d \log \bar{i} + \dots \\ = \varepsilon(k^P, i) d \log i + \varepsilon(k^P, i_{fin}) d \log i_{fin} + \varepsilon(k^P, i_r) d \log i_r \\ + \varepsilon(k^P, \pi) d \log \pi + \dots$$

³The ellipses indicate that similar terms for i_{dife} , i_{dite} , δ , $\bar{\delta}$, Y , and W have to be added.

Solving for the bank credit rate gives:

$$(A22) \quad d \log i = \frac{1}{\varepsilon(k^B, i) - \varepsilon(k^P, i)} \left\{ -d \log B^n + [\varepsilon(k^P, i_r) - \varepsilon(k^B, i_r)] d \log i_r + [\varepsilon(k^P, Y) - \varepsilon(k^B, Y)] d \log Y \right. \\ \left. + [\varepsilon(k^P, W) - \varepsilon(k^B, W)] d \log W + \varepsilon(k^P, \pi) d \log \pi \right. \\ \left. + \varepsilon(k^P, i_{fin}) + \varepsilon(k^B, i_d) d \log i_d - \varepsilon(k^B, i_d) d \log i \right\}$$

Total differentiation of the logarithmic form of the money supply equation leads to:

$$(A23) \quad d \log M = d \log B^n + \varepsilon(m^n, i) d \log i + \varepsilon(m^n, i_d) d \log i_d \\ + \varepsilon(m^n, i_r) d \log i_r + \varepsilon(m^n, \bar{i}) d \log \bar{i} + \dots$$

Substitution of equation (A22) into (A23) results in:

$$(A24) \quad d \log M = (1-q) d \log B^n + [\varepsilon(m^n, i_d) - q\varepsilon(k^B, i_d)] d \log i_d \\ + [\varepsilon(m^n, \bar{i}) - q\varepsilon(k^B, \bar{i})] d \log \bar{i} + \\ q\varepsilon(k^P, \pi) d \log \pi + q\varepsilon(k^P, i_{fin}) d \log i_{fin} \\ + [\varepsilon(m^n, i_r) + q[\varepsilon(k^P, i_r) - \varepsilon(k^B, i_r)]] d \log i_r + \dots$$

The elasticities in Table V in the text are derived from the last equation.

