



# Commentary

Thomas J. Sargent

In several recent papers—including the paper from this conference, Bansal (2007)—Ravi Bansal and his coauthors have constructed an interpretation of some asset pricing puzzles that I think macroeconomists should pay attention to. Why? Because a representative agent’s consumption Euler equation that links a one-period real interest rate to the consumption growth rate is the “IS curve” that is central to the policy transmission mechanism in today’s New Keynesian models. A long list of empirical failures called puzzles come from applying the stochastic discount factor implied by that Euler equation. Until we succeed in getting a consumption-based asset pricing model that works well, the New Keynesian IS curve is built on sand.

In several exciting papers, Bansal and his coauthors propose a way to explain some of those asset pricing puzzles by (i) specifying the intertemporal structure of risks to put long-run risks into consumption and assets’ cash flows and (ii) altering preferences to make the representative consumer care more about those long-run risks.

## LONG-RUN RISK

Let  $c_t$  be the logarithm of aggregate consumption. A workhorse model that does a good job of fitting per capita U.S. consumption of nondurables and services makes  $c_t$  a random walk with constant drift and i.i.d. Gaussian innovations. Bansal

and his coauthors begin from the observation that it is difficult to distinguish that specification from an alternative one in which the drift in log consumption growth is itself a highly persistent covariance stationary process with low conditional volatility but high unconditional volatility. Thus, the drift itself is almost but not quite a random walk. The high unconditional volatility of the drift confronts the representative consumer with what Bansal and his coauthors call long-run risk because the conditional mean of consumption growth is not constant but wanders.

Bansal also posits that cash flows on particular portfolios differ in the extent to which they are subject to long-run risks that are more or less correlated with the long-run risk in aggregate consumption. For example, Bansal and coauthors as well as Hansen, Heaton, and Li (2006) have offered evidence that the cash flows from Fama and French’s high book-to-market portfolios have long-run components that are more highly correlated with long-run components of consumption than are the cash flows from low book-to-market portfolios. Can the need to compensate the representative consumer for that higher long-run correlation with consumption explain why those high book-to-market portfolios have higher returns?

## PREFERENCES

The answer is no if the representative consumer’s preferences are usual ones assumed by

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## Sargent

macroeconomists—for example, time-separable logarithmic preferences with discount factor  $\beta \in (0, 1)$ . Why? Those preferences lead to the usual stochastic discount factor whose logarithm is

$$s_{t+1,t} = \log \beta - \Delta c_{t+1},$$

which has the property that the representative consumer just doesn't care enough about those long-run risks to pump up the returns on those high book-to-market portfolios. Therefore, Bansal and his coauthors adopt a preference specification of Epstein and Zin (1989) that separates the intertemporal elasticity of substitution (IES) from the reciprocal of the coefficient of relative risk aversion, both of which are unity for the log preference specification mentioned above. With the IES held fixed at 1, those preferences imply a stochastic discount factor whose logarithm is

$$s_{t+1,t} = \log \beta - \Delta c_{t+1} - (\gamma - 1) \sum_{j=0}^{\infty} (E_{t+1} - E_t) \Delta c_{t+j+1} - \frac{1}{2} (\gamma - 1)^2 \text{var}_t \left( \sum_{j=0}^{\infty} \beta^j (E_{t+1} - E_t) \Delta c_{t+j+1} \right),$$

where  $\gamma$  is the coefficient of relative risk aversion and  $E_t$  denotes mathematical expectation conditioned on time- $t$  information. Setting  $\gamma > 1$  adds forward-looking terms to the stochastic discount factor that make the representative consumer care today about rates of log consumption growth far in the future. For a  $\gamma$  high enough, the representative consumer does have to be compensated for long-run cash flow risk correlated with long-run consumption growth risk in amounts that are big enough to explain how the market prices those Fama and French portfolios. Furthermore, Tallarini (2000), Bansal and Yaron (2004) and others have shown that with a high enough  $\gamma$  this kind of preference specification can provide a neat explanation for both the risk-free rate and the equity premium puzzles.

## ASSESSMENT

It is possible to reinterpret the above stochastic discount factor with IES = 1 and atemporal

risk aversion  $\gamma > 1$  in terms of a representative consumer who has IES and risk aversion both equal to 1, but where now  $\gamma > 1$  measures his doubts about the stochastic specification of his model for consumption growth and cash flows. This reinterpretation is achieved by noting that the continuation value in Epstein and Zin's formulas equals the indirect utility function for a robust valuation problem in which a malevolent nature helps the decisionmaker construct valuations that are robust to misspecification by choosing a worst-case model from a set of models surrounding the decisionmaker's approximation model. Now  $\gamma$  acquires the interpretation of a penalty parameter on the relative entropy between the approximating and the distorted model. The additional terms in the log stochastic discount factor that appear when  $\gamma > 1$  encode the likelihood ratio of the worst case to the approximating model.<sup>1</sup> This reinterpretation is of special interest for the work of Bansal and his coauthors, who reason as follows:

1. Statistically, it is difficult to distinguish a stochastic specification with long run-risk from one without it.
2. Therefore, without attributing wacky ideas to their representative consumer, Bansal and coauthors can assume that the representative consumer assigns probability 1 to the long-run risk model and takes our original i.i.d. log consumption growth model off the table.
3. Besides, by using the rational expectations cross-equation restrictions associated with the consumption Euler equation, we can infer that the representative consumer has to believe the long-run risk specification to explain the asset pricing data.

There is more to say here. Although long-run risks are difficult to detect (assertion 1), Epstein-Zin preferences or concerns about robustness make it vital for the representative consumer to

<sup>1</sup> Barillas, Hansen, and Sargent (2007) show that by interpreting  $\gamma$  as measuring fear of model misspecification rather than risk-aversion, a moderate fear of model misspecification can do most of the job of the large risk-aversion parameters of Tallarini (2000) and Bansal and Yaron (2004) in explaining the equity premium.

care about them. The tenuous part of this argument is how the representative consumer can come to be sure about the presence of these long-run risks when they are so difficult to detect statistically. Assertion 3 differs from arguments in the least-squares learning literature that typically have an agent learn about a forcing process by way of a least-squares learning algorithm on that process itself, not by using prices and the rational expectations cross-equation restrictions to reverse engineer what that process must be.

The robustness interpretation can help with these learning issues. Hansen and Sargent (2007) address these in the context of a model with a representative consumer who responds to assertion 1 by leaving both the i.i.d. and long-run risk models for log consumption growth on the table, attaching a prior initialized at the equal ignorance value of 0.5 to the long-run risk model, then updating by Bayes' law. We show that a consumer who distrusts both submodels and the posterior over submodels that emerges from Bayes' law will behave in a way that supports much of what Bansal and his coauthors do. In particular, because the long-run risk model is worse for the representative consumer, his worst-case probabilities become slanted toward that model and possibly put almost all of the mass on that model. This is nice because it provides an alternative defense of Bansal's assumption that the representative consumer acts as if he puts probability 1 on the long-run risk model.

But the structure of Hansen and Sargent (2007) yields other interesting outcomes, too. Even when the robust investor slants his worst-case probability to put probability 1 on the long-run risk model, the gap between the ordinary Bayesian probability and this worst-case probability contributes a potential source of time-varying countercyclical risk premia.

## CONCLUSION

Bansal and Yaron's idea of stressing long-run risks that are difficult to detect but, with Epstein-Zin preferences or fear of model misspecification,

easy to care about is worth taking seriously. When many macroeconomists are now busy attaching loosely interpreted shocks or wedges to agents' first-order condition to make our dynamic stochastic general equilibrium (DSGE) models fit better, I welcome Bansal's new approach.

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