FEDERAL RESERVE BANK of ST. LOUIS

RESEARCH DIVISION

Working Paper Series

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Working Paper 2018-006A
https://doi.org/10.20955/wp.2018.006

April 2018

FEDERAL RESERVE BANK OF ST. LOUIS
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P.O. Box 442
St. Louis, MO 63166

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Designing UISAs for Developing Countries

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April 2, 2018

Abstract

The benefits of implementing Unemployment Insurance Savings Accounts (UISAs) are studied in the presence of the multiple sources of information frictions often existing in developing countries. A benchmark incomplete markets economy is calibrated to Mexico in the early 2000s. The unconstrained optimal allocation would imply very large welfare gains relative to the benchmark economy (similar to an increase in consumption of 23% in every period). More importantly, in presence of multiple sources of information frictions, about half of those potential gains can be accrued through the implementation of UISAs with replacement rates between 40-50%, contribution rates between 10-15%, an initial liquidity transfer of about 20 quarters of average income, and higher payroll taxes to finance those initial stocks.

JEL classification: D82, H55, I38, J65.

Keywords: Unemployment Insurance, Informality, Moral Hazard, UISA.

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1 Introduction

We evaluate the benefits of implementing Unemployment Insurance Savings Accounts (UISAs) in an economy with the multiple sources of information frictions often existing in developing countries. More precisely, in addition to the typical moral hazard problem, in which agents exert unobservable effort to find and keep jobs, we add adverse selection via unobservable heterogeneous workers, the existence of an informal labor market, and the possibility that agents can secretly save.

In this environment we analytically characterize the first best allocation under full information in order to identify the key sources of inefficiencies. The potential gains of intervention are large. With respect to a benchmark incomplete market economy calibrated to Mexico in the early 2000s, the optimal contract under full information produces welfare gains similar to an increase in consumption of 23% in every period.

Then, we optimally choose the parameters characterizing UISAs to maximize ex-ante welfare. In the economy with UISAs, individuals make contributions to their savings account at a particular contribution rate during formal employment, and they withdraw from those accounts at a given extraction rate when they are not formally employed; there is an initial transfer to the savings account made by the government, which is funded by the general tax paid by the workers in the formal labor market. The resources left at retirement are included in their pensions. The key advantage of this system is that it can be implemented despite large informational frictions. We show that more than 50% of the potential gains can be accrued through the implementation of a simple menu of UISAs with (a) replacement rates between 40-50%, (b) contribution rates between 10-15%, (c) an initial liquidity transfers in UISAs of about 20 quarters of average income, and (d) higher payroll taxes to finance those initial stocks.

The novelty of this paper builds upon combining an unobservable labor market together with moral hazard and adverse selection. This is important not only theoretically but also empirically since, for example, the informal sector in Latin American countries produces between 25 to 76 % of output (Schneider and Enste, 2000).[1] This clear difference in the importance of the informal sector in developing countries and developed countries calls for the inclusion of informality when studying unemployment insurance in developing countries. The existence of a shadow source of income limits the possibility of financing an unemployment system since government revenues depend on labor taxes, which are not levied in this unobservable market: Higher unemployment payments need higher taxes, which diminish the benefits of searching for a taxable job, generating a Laffer effect.[2] Also, because agents can claim payments while working informally, lower

[1] In particular, about 33% of the labor force in Latin America reports being employed in a business of their own–mostly subsistence self-employment (in contrast, 9% in the United States). Moreover, salaried employees constitute 80% of the work force in the US but only 55% in Latin America.

[2] "A Laffer curve is a hump-shaped curve showing tax revenue as a function of the tax rate. Revenue

contributions can be replaced by income from the hidden market, and this limits the capacity to provide incentives. In addition to these informational issues, we will analyze how the optimal UISA contract depends on whether agents can privately save or not. When agents are allowed to save, they have a tool for self-insurance against unemployment risk, and the design of UISAs has less impact on incentives to exert search effort. Nevertheless, the importance of hidden savings may be limited in developing countries because its rate of return is often quite low (for instance, due to high inflation).

We model moral hazard as the seminal works of Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). This literature focuses on the moral hazard problem for the design of an optimal unemployment insurance, where agents are assumed to exert costly unobservable effort to find a job. In order to circumvent this moral hazard problem, this literature proposes monotonically decreasing benefits throughout the unemployment spell, replacement rates during unemployment, and taxes during employment that depend on the entire employment history of the worker. The last feature suggests that individual-specific unemployment accounts may be desirable and may motivate the evaluation of introducing UISAs.

We combine this sort of moral hazard friction in this literature with: (i) unobservable heterogeneity among workers, (ii) informality, and (iii) hidden savings. Previous studies have combined moral hazard with at most one of these three frictions at a time and have shown that information frictions interact in a way that has very important consequences for policy prescriptions.

Workers' unobservable heterogeneity in the context of unemployment insurance has been considered by Fuller (2014). This is an important feature because in the data there is a small share of workers who spend a significant part of their life in unemployment, and the rest of the workers spend most of their life employed (Mustre-del Rio, 2015). When the utility cost of exerting effort to find a job varies across agents, an adverse selection problem arises. Fuller (2014) shows that with the inclusion of this sort of adverse selection, the optimal contract is not necessarily decreasing unemployment insurance payments (consumption) with the unemployment spell and, in fact, it could be increasing payments. Wang and Williamson (2002) also study heterogeneity of workers, in this case across different industries, and found that the optimal unemployment insurance benefit schedule is non-monotonic, although its welfare gain is small relative to the current US system. We model this heterogeneity similar to Fuller (2014) since individuals have different costs in exerting search effort.

Combining moral hazard with informality is the main contribution of Alvarez-Parra and Sánchez (2009). They explore the impact of a shadow economy on the design of

initially increases with the tax rate but then can decrease if taxpayers reduce market labour supply and investments, switch compensation into non-taxable forms, and engage in tax evasion." See Fullerton (2008).

unemployment insurance and showed how this extension of the model breaks the identity between consumption and unemployment insurance payments. The non-observability of the participation in this market results in a flattening of the profile of payments for some time to prevent individuals from joining the informal sector. After that time, optimality dictates that the unemployment insurance payments should drop to zero and so unemployed workers should rely on the informal sector to obtain income. Our modeling of informality is similar to Alvarez-Parra and Sánchez (2009). However, we assume that finding a job in the informal sector is not always possible and calibrate that risk to transition rates in Mexico.

The role of savings in the design of optimal contract with moral hazard was first pointed out by Rogerson (1985). The key insight there was that agents will always choose to save under the optimal contract, suggesting that the policy prescription should be modified if agents can secretly save. That optimal contract with hidden savings was then characterized by Werning (2002), Cole and Kocherlakota (2001) and Abraham and Pavoni (2008). These results show that optimal unemployment benefits are not necessarily decreasing with unemployment duration. We allow for private savings in the same fashion that they do, but our analysis is simplified because we do not solve for the constrained efficient allocation; i.e. we restrict our attention to UISAs. This allows us to consider the role of this friction in economies with other informational frictions and other quantitatively relevant features for savings as a life-cycle profile for income.[3]

The papers mentioned above theoretically characterize the optimal unemployment insurance scheme in the presence of moral hazard and one additional friction. Note that in all cases these modifications are enough to alter the main prediction of the original model. Here, we consider including all these frictions to capture the economic environment in a developing country. The cost of our strategy is that analytic characterizations are hard to obtain. Our strategy to overcome this difficulty is two-fold. First, we analytically characterize the full-information allocation. This will be our reference framework to evaluate alternative unemployment insurance schemes. Second, we rely on numerical optimization of the parameters describing UISAs to find the optimal policy. The optimal constrained efficient allocation would imply welfare gains somewhere in between the gains from the full-information allocation and the gains associated with the optimal UISAs scheme.

Even when an analytic characterization of the constraint efficient allocation is available, an approximation by a relatively simple system (e.g., UISAs) is always desirable

[3]In different framework, Shimer and Werning (2008) study an economy where agents with constant absolute risk aversion (CARA) preference or constant relative risk aversion (CRRA) preference have access to credit market. There is no moral hazard on search effort but agents pass on job offers with low wages if they are given generous unemployment insurance payments. They show that the policy with constant benefits during unemployment, constant taxes during employment, and free access to savings using a risk-less asset is optimal under CARA preference and nearly optimal in CRRA preference.

to help guiding policy. Thus, it is not surprising that a similar approach to ours was previously pursued. For instance, Feldstein and Altman (2007) evaluated the benefits of UISAs for the US. More related to our work is the contribution of Hopenhayn and Hatchondo (2011). They develop a simpler model than ours to examine the performance of alternative designs of UISAs taking into account private information through savings and effort. Since we will rely on the quantitative evaluation of alternative schemes, our model is quantitatively richer. Importantly, we allowed for informality, which is arguably the most common difference between developing and developed countries.[4] Finally, Setty (forthcoming) looks for the optimal parameters of a hybrid system that combines UISA with a traditional unemployment system and is used if the worker run out of savings in the account. The only informational friction in his setup is moral hazard but his model is quantitatively richer.

The rest of the paper is organized as follows. The next section introduces the benchmark environment. Section 3 characterizes the full-information model. Section 4 presents the UISA system of menus. Section 5 shows the calibration results, and Section 6 studies the quantitative implications of the systems. All the formal results are explained in the main text, but the complete proofs are delegated to the Appendix.

2 The Environment

In each period $t = 0, 1, 2, 3...$, a new ex-ante identical generation of N_t individuals is born. The population growth rate is constant at $x \geq 0$; i.e., $N_t = (1+x)^t$. Each of these households has the following lifetime profile.

During the first N periods, agents can participate in labor markets and work (i.e., working periods). When an individual reaches age $N+1$, she retires from the labor market. Once retired, individuals survive to the next period with probability ρ.

Workers have heterogenous abilities to find high productivity jobs. This is represented by the disutility of search effort, where $\theta \in \{\underline{\theta}, \overline{\theta}\}$. The fraction of workers with θ ability is given by $g(\theta) > 0$.

In each date t and given θ, the worker's date-t utility depends on consumption c_t and the corresponding effort level e_t, according to the utility function $u(c_t) - \theta e_t$. Lifetime expected discounted utility for an individual with preference shock θ is represented by

$$E\left\{\sum_{t=1}^{\infty} \beta^t [u(c_t) - \theta e_t]\right\},$$

where $\beta \in (0,1)$ is the discount factor and E is the expectation operator.

[4] Also, while our work emphasizes unemployment insurance dynamics in a developing country (Mexico), their work turns to a more developed labor market (Estonia).

2.1 Labor Market Decisions

We define the labor market decisions faced by an individual during her working periods. An individual of working age $n = 1, ..., N$ can either work in the formal sector, work in the informal sector, or be unemployed. The employment decisions in those three different states are the following.

First, consider a worker who enters her working period n with an offer in the formal sector. The worker's wage offered, denoted ω_n, is her productivity in the formal sector. As the offer is accepted, a worker who exerts effort e keeps this job in the formal sector next period with probability $q_f(e)$.

Second, consider a worker who enters her working period n with an offer in the informal sector. The worker's wage will be equal to the productivity in the informal sector, $\varpi_n < \omega_n$. If she accepts this offer, she must decide how much effort e to exert to receive an offer in the formal sector next period, with probability $q_i(e)$. Importantly, informal job offers are not only low-productivity jobs but also unobservable to third parties.

Finally, consider a worker who enters her working period n as unemployed; i.e., she does not receive either a formal or an informal job offer. The worker decides how much effort e to exert to receive an offer in the formal sector next period, with probability $q_u(e)$. Any rejected offer will make the worker unemployed immediately.

We assume that a worker can receive offers in both sectors at the same time. Therefore an active worker's opportunity status is denoted by $\{f, i, b, u\}$, which denote formal, informal, both offers, and unemployed states, respectively.

The technology to find a job in the formal sector satisfies $q_f(e) > q_u(e) > q_i(e)$ for all e. So effort is most productive in finding a formal job if the worker is already in this sector. On the other hand, working in the informal sector makes effort less productive than being unemployed. So workers choose a costly effort level e that translates into receiving a formal sector offer with probabilities increasing in e. This is a key feature when effort is unobservable because it implies that if individuals receive insurance against not having a formal job, they will have incentives to reduce effort; i.e., moral hazard.

The probabilities for a worker of age n to find a job in the informal sector are all exogenous but conditional upon the current employment status. The conditional probability of having an offer in the informal sector next period if the worker has worked informally during the current period is p_i; that is, $1-p_i$ is the separation rate in the informal sector. The conditional probability of having an offer in the informal sector next period if the worker has been unemployed in the current period is p_u. Finally, p_f is the conditional probability of having an offer in the informal sector next period if the worker has worked in the formal sector during the current period. We assume that $p_i > p_u > p_f$ for all n; i.e., informality is persistent as informal offers tomorrow are most likely if working informally today.

So there are two technological differences between the formal and the informal sector. First, workers are more productive (and so wages are higher) in the formal sector than in the informal sector. Second, job finding probabilities are different, because being employed in a particular sector is persistent.

2.2 Financial Markets

Agents have access to financial markets and undertake consumption-savings decisions. That is, an agent must allocate her resources (which will include financial income, as detailed below) between consumption and savings. While they are active, they have two assets to save (but not to borrow) that differ in the account observability. Individuals can save at the gross interest rate R in observable saving accounts. Alternatively, there is an asset with a lower return $r \leq R$, but the amount saved in that asset is unobservable. Once retired (i.e., age $n \geq N+1$), this is the only decision they must make and we suppose that only the high return asset is available. Finally, agents are endowed with $m_0 \geq 0$ when they are born.

The role of savings in this setting is key. Once a worker knows her type, the only source of risk is unemployment risk. So workers save for precautionary purposes. Also, workers face a life-cycle wage profile that is not flat, so savings is key to smooth consumption across age. Finally, they also save for retirement.

2.3 The Retired Worker's Problem

Retired workers always face the same problem at age $n \geq N+1$. Suppose that this individual survives and reaches period n with m asset holdings, and the person receives a transfer $h \geq 0$ from the government as a retirement payment. Denote $H(m)$ as her expected lifetime utility and so, as retired agents survive with probability ρ, H must solve

$$H(m) = \max_{m' \geq 0} \left[u(mR + h - m') + \beta\rho \, H(m') \right], \tag{1}$$

where m' denotes next period asset holdings.[5] Notice that retired agents do not exert any effort; i.e., $e_t = 0$ for all $t \geq N+1$.

3 Full Information

We first study the key features of an economy in which there is no informational friction, i.e., both effort and informal job opportunities are observable, and so full-insurance is

[5] In our setting with CRRA, as the budget constraint is homogeneous of degree one if $h = 0$, then H has a closed-form solution in this case; i.e., $H(w) = A \, (m)^{1-\sigma}$ for some constant $A > 0$.

attainable. To characterize the optimal allocation we consider the problem of a planner who allocates consumption and effort for the entire life cycle.

3.1 The Ex-Ante Representative Worker's Problem

We can divide the planner problem in two stages. First, the efficient allocation of consumption for the retired workers can be replicated without lost of generality, assuming $h = 0$ in (2.3) and letting them sign mortality contingent claims. Since death is the only source of uncertainty in this life period, this makes financial markets dynamically complete for retired workers, so they allocate resources efficiently. Consequently, the planner problem reduces only to allocate assets when beginning their retirement.

Second, the planner will allocate consumption and effort in the productive life of the worker while it has access to credit markets at the gross interest rate $R = \beta^{-1} = (1 + x)$.[6] Let $s_n \in \{f, i, b, u\}$ be the worker's job opportunity at age n. Let s^n denote a partial history up to age n with $\pi(s^n)$ as its corresponding (endogenous) probability. The planner insures the representative worker ex-ante (i.e., before her type θ has realized), and therefore the corresponding problem is the following.

$$\max_{((\tau_n, e_n)_{n=1}^{N}, m)} \sum_{\theta} \sum_{n=1}^{N} \sum_{s^n} \Big\{ \beta^{n-1} \pi(s^n) [u(w_{\theta,n}(s^n) + \tau_n(\theta, s^n)) - \theta e_n(\theta, s^n)] + \sum_{s^N} \beta^N \pi(s^N) H(m(\theta, s^N)) \Big\} g(\theta), \tag{2}$$

subject to

$$\sum_{\theta} \left\{ \sum_{n=1}^{N} \sum_{s^n} R^{-(n-1)} \pi(s^n) \tau_n(\theta, s^n) + \sum_{s^N} R^{-N} \pi(s^N) m(\theta, s^N) \right\} g(\theta) = m_0,$$

where $\tau_n(\theta, s^n)$ is a transfer.

In this stationary environment the fictitious planner can manipulate all the cross-sectional resources to maximize the ex-ante utility of the representative worker. The following result follows from first-order conditions in problem (2).

Lemma 3.1 *Under full information, full insurance is attained, so consumption and period $N + 1$ asset holdings are constant; i.e. $c_n(\theta, s^n) = c^*$ and $m(\theta, s^N) = m^*$ for all $n = 1, ..., N$, all s^n and all θ.*

So optimality dictates that workers are fully insured against both employment shocks and types shocks.

[6] This assumption is not innocuous as it is key in making the alternative schemes comparable and immune to differences in returns.

Let λ^* be the Lagrange multiplier corresponding to the constraint (3). It follows from necessary first order conditions that $\lambda^* = u'(c^*) = H'(m^*)$. This fact makes the planner problem (2) simplified as follows.

Lemma 3.2 *Given λ^*, problem (2) reduces to solve*

$$\max_{(\tau_n, w_n, e_n)_{n=1}^N} \sum_\theta \sum_{n=1}^N \sum_{s^n} R^{-(n-1)} \pi(s^n)[\lambda w_n(s^n) - \theta e_n(s^n)] g(\theta)$$
$$+\lambda^* R^{-N} m^* - \lambda^* \frac{1 - R^{-N+1}}{1 - R^{-1}} c^* + \frac{1 - R^{-N+1}}{1 - R^{-1}} u(c^*) + R^{-N} H(m^*),$$

i.e., it reduces to maximizing the expected discounted flow of income net the cost of effort, in utility units.

This problem becomes non-standard as the planner must undertake the discrete choice about whether the worker should make use of alternative job opportunities. The difficulty is not only when a low-productivity, informal job opportunity arrives. The opportunity of a formal job, either alone or jointly with an informal opportunity, does not make the optimal choice simpler either. To see this, notice that although both the worker's productivity (and so her wage) is high and the technology to find a high-productivity formal job next period dominates, the higher probability of becoming unemployed can make the option of a formal job undesirable. This will critically depend on the low value of being unemployed. All these facts are key to determine the optimal size of the informal sector. To tackle these issues, we solve the problem backwards. The characterization of the problem is presented in the Appendix.

It is important to remark that the optimal size of the informal sector which, probably confronting some of the conventional wisdom, is not necessarily 0. This follows because optimality in this setting dictates that some workers might indeed be allocated to work in the informal sector as the future costs of this low-productivity job are not "sufficiently high" to overcome the instantaneous/current payoff.

4 A UISA Economy

This section describes a prototype UISA economy in which workers have personalized accounts to which they contribute in periods of formal employment and from which they draw funds when they are out of formality. The government administrates those accounts and has access to the high-return asset. Interest payments are credited and balances become available to the workers at retirement age. This system can provide correct incentives because individuals partially internalize the cost of unemployment.

The design of the system specifies rules for drawing from and contributing to the system and for the interest rate applied to balances. Our particular system under analysis consists of some relevant parameters: (i) an upper bound for savings in the worker's account, $\overline{s}$, above which the worker does not contribute to the system; (ii) a contribution rate ψ on the age-specific wage to the worker's savings account during employment in the formal sector if the total savings balance is smaller than the upper bound; (iii) an initial transfer to the savings account made by the government, s_0; (iv) a payment to those without a formal job, $b\omega_n$, which is received by individuals only if $s > 0$; and (v) a general tax paid in the formal market, $\widetilde{\tau}$.

Note that the government provides liquidity funds to the individuals when they just enter the job market. Workers are forced to deposit a fraction of their wages into a savings account, up to a limit. And later they withdraw from these while not working in the formal market, as long as they have funds available.

In addition to the parameters discussed above, there are two characteristics of the system that are taken as given. First, funds accumulated by the government on behalf of the workers are invested at the gross interest rate R. Second, balances on the account cannot reach negative amounts.

Workers may also have access to the low-return asset, so their balances m cannot be observed by any kind of governmental agency; i.e. hidden savings. Note that this means that the design of the UISA cannot be contingent on asset holdings, m. We will assume that agents do not save using the high return asset since those savings are observable and therefore the government could make the UISA payments contingent on them.

Period $n < N$

Here, we consider a worker with working age $n < N$. Suppose that the individual receives an offer in the informal sector. Her maximized lifetime utility, V_n^i, must solve

$$V_n^i(\theta, s, m) = \max \left\{ \underbrace{V_n^{i,a}(\theta, s, m)}_{\text{accept}}, \underbrace{V_n^u(\theta, s, m)}_{\text{reject}} \right\}.$$

The value of accepting the informal job offer, $V_n^{i,a}$, satisfies

$$\begin{aligned} V_n^{i,a}(\theta, s, m) \;=\; & \max_{e,m'} u\left(mr + \varpi_n + \eta_n - m'\right) - \theta e + \\ & \beta \left\{ (1 - q_i(e)) \left[p_i V_{n+1}^i(\theta, s', m') + (1 - p_i) V_{n+1}^u(\theta, s', m') \right] + \right. \\ & \left. q_i(e) \left[p_i V_{n+1}^b(\theta, s', m') + (1 - p_i) V_{n+1}^f(\theta, s', m') \right] \right\}, \end{aligned}$$

where $\eta_n = \min\{b\omega_n, s - \underline{s}\}$ is the UI payment and $s' = R(s - \eta)$ denotes next-period savings in the UI account.

Suppose that the worker receives an offer in the formal sector. Her maximized expected lifetime utility, V_n^f, must solve

$$V_n^f(\theta, s, m) = \max\left\{V_n^{f,a}(\theta, s, m), V_n^u(\theta, s, m)\right\}.$$

The value of accepting the formal job offer, $V_n^{f,a}$, must solve

$$\begin{aligned} V_n^{f,a}(\theta, s, m) &= \max_{e,m'} u\left(mr + \omega_n\left(1-\widetilde{\tau}\right) - \varphi_n - m'\right) - \theta e + \\ &\quad \beta\left\{(1-q_f(e))\left[p_f\ V_{n+1}^i(\theta, s', m') + (1-p_f)\ V_{n+1}^u(\theta, s', m')\right] + \right. \\ &\quad \left. q_f(e)\left[p_f V_{n+1}^b(\theta, s', m') + (1-p_f) V_{n+1}^f(\theta, s', m')\right]\right\}, \end{aligned}$$

where $\varphi_n = \min\{\psi\omega_n, \overline{s} - s\}$ is the payment to the UISA and $s' = R(s+\varphi)$ denotes next period savings in the UI account.

Suppose that the worker receives both offers, formal and informal. Her maximized expected lifetime utility, $V_n^b(\theta, s, m)$, must solve

$$V_n^b(\theta, s, m) = \max\left\{V_n^{f,a}(\theta, s, m), V_n^{i,a}(\theta, s, m), V_n^u(\theta, s, m)\right\}.$$

Finally, suppose that the worker receives no offer and is consequently unemployed. Her maximized expected lifetime utility is

$$\begin{aligned} V_n^u(\theta, s, m) &= \max_{e,m'} u\left(mr + \eta_n - m'\right) - \theta e + \\ &\quad \beta\left\{(1-q_u(e))\left[p_u\ V_{n+1}^i(\theta, s', m') + (1-p_u)\ V_{n+1}^u(\theta, s', m')\right] + \right. \\ &\quad \left. q_u(e)\left[p_u\ V_{n+1}^b(\theta, s', m') + (1-p_u)\ V_{n+1}^f(\theta, s', m')\right]\right\}, \end{aligned}$$

where $\eta_n = \min\{b\omega_n, s - \underline{s}\}$ and $s' = R(s-\eta)$.

Again, the value of rejecting all the job offers available and receiving none must necessarily coincide in equilibrium. The key difference is that in the first case the worker decides to be unemployed while in the second she is forced to be unemployed. Note that, in addition, the differences with the case of an individual working in the informal/formal sector are in the probabilities of finding a job.

The corresponding policy functions for effort levels and private savings are $\widetilde{e}_n^j(\theta, s, m)$ and $\widetilde{m}_n'^j(\theta, s, m)$, respectively, where $j \in \{f, i, b, u\}$.

Period $n = N$

The analysis at period N is similar but the continuation is simpler as the worker will be retired next period $N+1$, and therefore she does not exert any effort. Here we detail the case where a worker receives a formal offer. The other cases are similar.

Her maximized expected lifetime utility, V_N^f, must solve

$$V_N^f(\theta, s, m) = \max \left\{ V_N^{f,a}(\theta, s, m), V_N^u(\theta, s, m) \right\}.$$

The value of accepting the formal job offer, $V_N^{f,a}$, must solve

$$V_N^{f,a}(\theta, s, m) \;\; = \;\; \max_{m'} \left\{ u\left(mr + \omega_N \left(1 - \widetilde{\tau}\right) - \varphi_N - m'\right) + \beta H(m' + s') \right\},$$

where $\varphi_N = \min\{\psi\omega_N, \overline{s} - s\}$ is the payment to the UISA and $s' = R(s + \varphi)$ is the amount of savings in the UI account at retirement age $N + 1$.

Alternatively, the value of rejecting the formal offer and remaining unemployed must satisfy

$$V_n^u(\theta, s, m) \;\; = \;\; \max_{m'} \left\{ u\left(mr + \eta_N - m'\right) + \beta H(m' + s') \right\},$$

where $\eta_N = \min\{b\omega_N, s\}$ and $s' = R(s - \eta)$.

In the case where they only receive an informal offer and decide to accept it, the lifetime utility is

$$V_n^{i,a}(\theta, s, m) \;\; = \;\; \max_{m'} \left\{ u\left(mr + \varpi_n + \eta_n - m'\right) + \beta H(m' + s') \right\},$$

where $\eta_n = \min\{b\omega_n, s - \underline{s}\}$ is the UI payment and $s' = R(s - \eta)$ denotes next period savings in the UI account.

4.1 Optimal Scheme

Here we define the optimality concept that we follow to select UISA parameters. The corresponding set of policy parameters to provide both unemployment protection and liquidity provision is, $\Gamma = (\underline{s}, \overline{s}, \psi, b, s_0, \widetilde{\tau})$. Let $\Gamma(\theta)$ be the θ-contract in a separating scheme, while a pooling schemes simply reduces to $\Gamma(\theta_L) = \Gamma(\theta_H)$. Let $V_1^\kappa(\theta, m_0 \mid \Gamma)$ be the utility of the representative worker with preference shock θ, initial asset holdings m_0, and employment offer status κ. Let $T^{UISA}(\Gamma)$ and $G^{UISA}(\Gamma)$ denote the resources collected and spent by the scheme Γ, respectively. Also, let $\mu(\kappa)$ and $g(\theta)$ be the measure of agents with employment status κ and type θ, respectively.

Definition 4.1 *We define the optimal UISA scheme, Γ^*, as the one that solves*

$$\max_{\Gamma} \sum_{\theta} \sum_{\kappa} V_1^\kappa(\theta, m_0 \mid \Gamma(\theta)) \;\, \mu(\kappa) \;\;\, g(\theta),$$

subject to

$$\sum_{\theta} \left(T^{UISA}(\Gamma(\theta)) - G^{UISA}(\Gamma(\theta)) \right) = 0.$$

and the incentive compatibility constraint

$$\widetilde{V}_1^{\kappa}(\theta \mid \Gamma(\theta)) \geq \widetilde{V}_1^{\kappa}(\theta \mid \Gamma(\widetilde{\theta}))$$

for all κ.

Notice that, in this stationary environment, feasibility means that the fictitious planner can manipulate all the cross-sectional resources to maximize the ex-ante utility of the representative worker. In addition, to make the comparison with other systems, they must generate a balanced budget for the government.

5 Calibration

To evaluate the quantitative impact of policy reforms, we will proceed to calibrate and simulate the model and then analyze the predictions. We use Mexico as the benchmark economy because of the prevalence of the frictions included in our analysis.[7] To approximate the system of unemployment protection observed, we model an economy in which an individual who began working in the formal sector at age n and lost her job at age $n+1$ receives a severance payment as unemployment protection for one period, denoted by $b_n = b\omega_{n-1}$, where $b = 1$ is the replacement ratio in our calibration. The worker will have no right to receive severance payments in the case that he/she rejects a formal offer. The details of the system can be found in the Appendix.

As the goal is to evaluate the quantitative impact of policy reforms, we must select the key parameter of the model. This section briefly explains these choices, describes the parameters, and compares the model with the data (see the Appendix for more details).

The value of the parameters are set using three strategies. First, there is a group of parameters that can be obtained directly from data or taken from previous literature. Whenever possible, we follow that strategy. For the rest of the parameters, we choose specific targets and search for values to minimize the distance to the targets.

[7] Another advantage of using Mexico is data availability. Data are obtained from the *Instituto Nacional de Estadística y Geografía* (INEGI), Inter-American Development Bank and Bank of Mexico. See Appendix for more details.

5.1 Functional Forms

The utility function is the standard constant relative risk aversion (CRRA) form,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

with relative risk aversion parameter $\sigma > 0$. The functions describing the probability of getting formal job offers are

$$q_i(e) = 1 - \exp(\xi_I e), \quad q_u(e) = 1 - \exp(\xi_U e), \text{ and } q_f(e) = 1 - \exp(\xi_F e).$$

5.2 Calibration Results

As mentioned above, several parameters were obtained directly from data. The retirees survival probability, $\rho = 0.9875$, is set to match Mexican lifetime expectancy. The retirement payment, $d = 0.6$, is set to match the average amount of payment of retired people relative to the income at the peak of the life cycle (which is normalized to 1).

Other parameters are set at standard values in the literature. The coefficient of relative risk aversion is set at $\sigma = 2$. The discount factor is set at $\beta = 0.96^{1/4}$, and the returns on savings is set at $R = \frac{1}{\beta}$. Since there is no direct evidence on the initial stocks of assets in Mexico and since young workers in the United States have very little savings, we set m_0 such that the ratio of initial savings to yearly income is 10 % for Mexico.

The parameters that cannot be determined ex-ante were jointly calibrated to minimize the distance to specific targets. Table 1 presents the parameters resulting from the minimization routine.

Table 1: Calibrated Parameters

Parameter	Value
Effort efficiency for unemployment to employment probability, ξ_U	0.0022
Effort efficiency for informality to employment probability, ξ_I	0.0017
Effort efficiency for employment to employment probability, ξ_F	0.0517
Probability of informal offer $t+1$ given employment at t, p_f	0.6660
Probability of informal offer $t+1$ given unemployment at t, p_u	0.9374
Probability of informal offer $t+1$ given informality at t, p_i	0.9692
Utility cost of effort for low cost individuals, $\underline{\theta}$	0.0097
Utility cost of effort for high cost individuals, $\overline{\theta}$	0.0229
Initial share of individuals in informality, μ_i	0.5780
Initial share of individuals in employment, μ_f	0.1010
Share of low utility cost of effort individuals, $\underline{\theta}$	0.6058

Table 2 shows how the predictions of the model match the targeted moments. Since we force the fiscal deficit to be balanced, we find the labor tax τ to achieve that goal. Given the parameters, the resulting labor tax for our benchmark economy is $\tau = 0.235$, a bit lower than the actual number in Mexico (which is about 0.28-0.35).[8] As shown in the table, the model reproduces aggregate states quite well. In particular, it reproduces almost exactly the transition probabilities among the alternative labor market statuses (formal job, informal job, and unemployment).

Table 2: Calibration Targets and Fit

Statistic	Model	Data
Unemployment Rate	2.8	2.7
20-24 years	4.7	4.8
25-39 years	2.3	2.2
40+ years	2.7	1.0
Informality Rate	56.4	54.2
20-24 years	52.2	48.6
25-39 years	50.2	49.5
40+ years	67.3	64.1
Yearly transition prob. from formal to formal	84.8	86.0
Yearly transition prob. from formal to informal	13.3	12.9
Quarterly transition prob. from formal to unemployed	1.6	1.5
Quarterly transition prob. from informal to unemployed	3.0	3.1

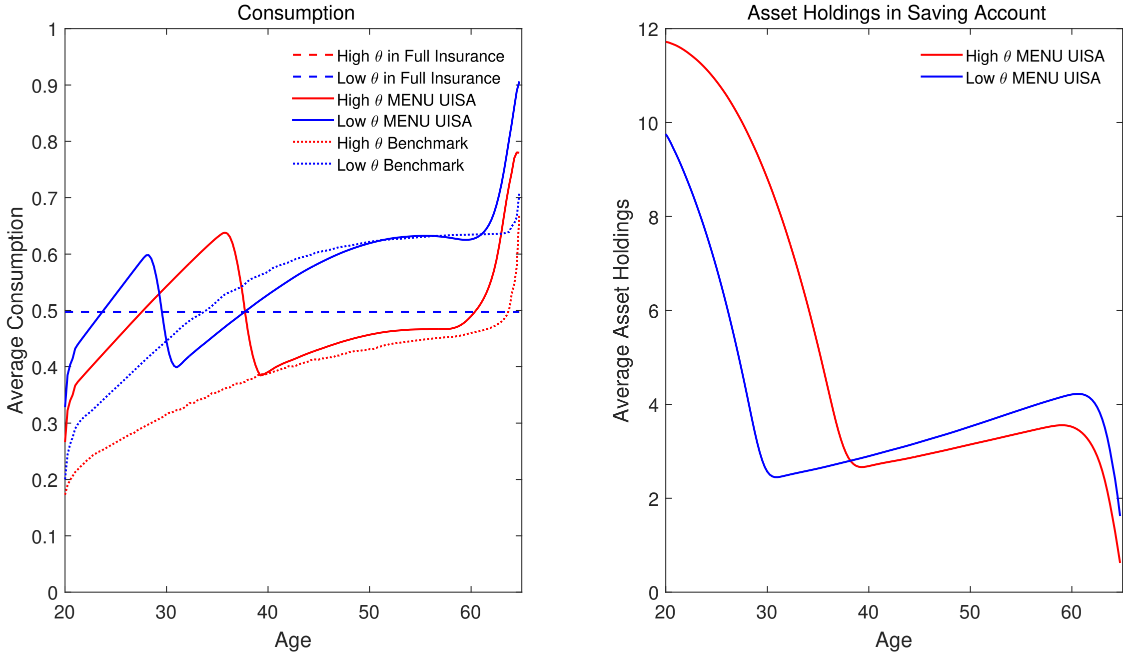
6 Quantitative evaluation

6.1 Full Information

This section describes the full information for the set of parameters determined in the previous section. From Lemma 3.1 we know that the full-information allocation is characterized by constant consumption along the life cycle. Since wages for the youth are low compared to adults, full-information allocation redistributes income from older to younger workers in order to smooth consumption. In addition, agents are fully insured against their type (their unobservable cost to find high-productivity formal jobs), that is, a full-information scheme provides cross-subsidies between types. Finally, an optimal scheme must try to alleviate the income volatility stemming from the labor market. The left panel of Figure 1 shows how the full-information average consumption over the life cycle differs from the benchmark. In the benchmark economy, individuals with low costs in exerting search effort have significantly higher consumption, and the consumption profile of all the individuals is quite steep.

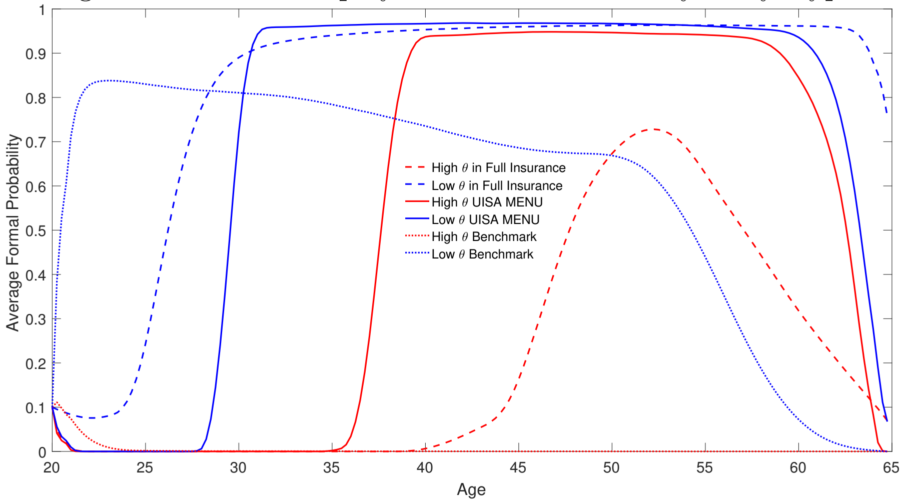
[8]See *Servicio de Administración Tributaria* (SAT) for information on income tax in Mexico.

Figure 1: Consumption over the Life-Cycle by Type



When consumption is fully insured, the full-information allocation of effort differs drastically from the one in the benchmark economy. Figure 2 displays the average formal job probability. When effort is observable, it is optimal to reduce it for young workers and increase it when they are older because the productivity of young workers is low and the present value of effort cost is high. However, in the benchmark economy, low-cost agents search for a formal job as soon as they enter the labor market in order to smooth consumption and accumulate assets for insurance. Recall that their consumption is quite low when they are young. Moreover, in the full-information allocation high-search cost agents must search for a formal job in their most productive years, between 45 and 60 years old.

Figure 2: Formal Employment over the Life-Cycle by Type



As a consequence of the reallocation of consumption and working time, the full-information economy produces large welfare gains of the order of 23.3% measured in consumption equivalent, as shown in Table 3.

Table 3: Welfare Gains

	Consumption Equivalence Gains	Relative to Full-Information Gains
Full Information	23.3%	100.0%
UISA MENU	13.0%	55.8%
UISA	11.0%	47.2%
UISA w/ hidden savings	6.2%	26.6%
Liquidity Transfer	4.8%	20.6%

The gains described above are derived from the fact that consumption is equalized across workers with different cost in exerting search efforts, across workers with and without a job, and over the life cycle, but jobs are also better allocated across sectors and the life cycle. Table 4 shows that under the full-information allocation there is a significant increase in output per worker (12.4% as compared with the benchmark economy). Note also that the size of the informal sector in the full-information economy is still quite significant; i.e., 51.45%. This occurs because a fraction of the working population has the cost of exerting the required effort to find a formal, high-productivity job, too high compared with the benefits in terms of the expected duration of a formal job. In this case, however, there is nothing informal about these jobs. It is a sector in which jobs are easier to find and have lower productivity. In the benchmark economy, this sector is even higher because individuals do not receive the total amount of resources they generate in formal employment, but only those after taxes. Thus, the need of taxing formal jobs for redistribution is one of the sources for this excessive level of informality.

Table 4: Employment States and Output per Worker

	Informality	Unemployment	Output per Worker	Output Gains from Benchmark
Benchmark	56.4	2.8	0.49	-
Full Information	51.5	2.4	0.55	12.4%
UISA MENU	49.6	3.2	0.58	17.8%
UISA	52.5	3.4	0.57	16.6%
UISA w/ hidden savings	54.3	2.6	0.52	5.1%
Liquidity Transfer	57.2	2.6	0.49	0.5%

Unemployment accounts with an initial liquidity transfer seem a proper system to deal with the issues described above. The initial transfer provides the liquidity needed to consume when the worker is young and acts as a way of redistribution since most of the costs are paid by those who spent more time formally employed. Also, savings in the account provide insurance against unemployment.

6.2 UISAs

Now, consider the case in which agents are offered a menu of contracts of unemployment insurance savings accounts. It will consist of two different contracts that will be chosen by agents as soon as they enter the labor force. At that point, they receive the initial transfer s_0 into their savings accounts. First, we consider the setting where savings are observable, which is equivalent to a setting in which agents cannot save privately, so they are forced to consume their net income. If an agent is formally employed, she must save a fraction of her salary given by the contribution rate ψ, up to the maximum of $\overline{s}$. She consumes the rest, net of taxes τ. While she is out of formality, she can work informally, consuming her salary and a fraction of her savings at a given extraction rate b as a percentage of formal salary, if she has enough funds above $\underline{s}$. Since the changes in welfare when $\overline{s}$ moves are really small and the parameter $\underline{s}$ and $\overline{s}$ turned out to be optimal at $\underline{s}$ very low and $\overline{s}$ very high, we set $\underline{s} = 0$ and $\overline{s} = 1.4$ for the rest of the paper.

This rigid structure of the system gives the designer great capacity to provide effort incentives. Although the presence of an informal labor market breaks the relationship between payments and consumption, this hidden sector is still risky for the worker and does not necessarily act as a way of insurance. So, when agents run out of savings in their accounts, they do not have assets for insurance purposes. This motivates incentive to search for formal jobs to accumulate in the savings account. When agents enter the formal labor market, they take advantage of the persistence in this sector and stay there longer.

Note that the system can also provide partial insurance against labor costs type by providing different initial liquidity transfers. These transfers would be financed mostly by workers with low search costs and therefore provide cross subsidies to the agents with higher search costs. Finally, the system also provides partial insurance for job-market risk since an agent that loses a formal job and has a positive amount in the savings account will subtract income from there.

The parameters of the optimal system are described in Table 5. The system is more generous in terms of UI payments for individuals with high-effort costs who have a more difficult time finding and keeping formal jobs. The optimal scheme provides a large liquidity transfers for both types. This allows agents to stay out of the costly formal market when they are young, which is optimal, and to consume by using assets from their accounts and (maybe) income from an informal job. Here, the extraction rate b and the initial transfer s_0 together determine the duration of this period out of formality. Since low-cost agents choose the contract with a smaller transfer and a large extraction rate (but with smaller taxes), then when the account begins to run out of assets and formal salary is high enough, they abandon informality and enter the formal market.

Table 5: Parameters

		τ	b	ψ	s_0
Benchmark		0.235	1		0.00
UISA MENU	Low θ	0.371	0.55	0.17	9.61
	High θ	0.540	0.38	0.12	11.6
UISA		0.468	0.48	0.1	11.4
UISA w/ hidden savings		0.231	0.45	0.035	1.88
Liquidity Transfer		0.214	-	-	0.74

The labor income tax is crucial to satisfy incentive compatibility restriction. Agents with higher search costs are offered a large liquidity transfer but with a higher income tax. This prevents agents with lower search costs from choosing the contract with larger liquidity.

Overall, a UISA system provides incentives that are in line with the ones that must be provided under full information. Having income in their savings accounts, it is optimal for young agents to increase their consumption relative to the benchmark, as seen in the left panel of Figure 1. They do so by depleting their stock of savings in the UISA, as seen in the right panel of Figure 1. Note that with the availability of resources in their accounts, they can avoid exerting effort when they are young and unproductive. When they begin to run out of resources in their UISAs and their productivity in the formal sector is high enough, they find jobs in the formal sector. As a consequence, they are formally employed during their most productive years. This creates an effort pattern similar to that which is optimal. Note in Figure 2 that the life-cycle profile of formal employment is much closer to the one in the full-information allocation than the profile in the benchmark economy. As a result, output per worker increases 18% relative to the benchmark, as shown in Table 4. Note that output increases even more than in the full-information allocation. This occurs because agents exert more effort than needed due to the lack of insurance when they are close to running out of assets. Note also that informality is less than in our benchmark but unemployment slightly increases.

The welfare gains, shown in Table 3, are quite high, given the simplicity of the system. UISAs provide gains equivalent to an increase in consumption of 13% in every period, which is equivalent to 56% of potential gains under full information.

6.3 Robustness

First, we analyze how much of the gains are due to using a menu instead of a unique contract, and we see if the main characteristics of the system are altered. We found that if only one contract was offered, the optimal policy would have the same characteristics as the menu: large liquidity transfer, positive contribution rate, extraction rate close to 50%, and a large income tax, as seen in Table 5. This UISA contract produces 47% of potential welfare gains, which is very similar to the one with a menu. This allows us to conclude that the bulk of the gains are not due to the possibility of offerring multiple contracts. As seen in Table 4, informality rate, unemployment rate, and output per worker all parallel that of the contract with a menu as well, indicating that the shape of the system's characteristics is almost unchanged.

Next, we allow agents to privately save without government monitoring. In order to keep things simple, we will analyze only a unique contract. We know from the previous section that the contract will have the essential characteristics and that the change in welfare is not significant. The presence of private savings vanishes the possibility of making the policy contingent on the assets; specifically, it breaks the temporal relationship between transfers and consumption since agents now can accumulate part of the transfers to consume later. Moreover, private assets will allow for self-insurance against unemployment risk, which takes away from the designer the most powerful tool to provide incentives.[9]

We assume that the technology to privately save for the agents is worse than that used in the savings accounts. This seems plausible since funds in the accounts are usually managed by professional agencies, while individuals save outside the financial system where resources cannot be monitored by the government.[10] In particular, we set the return on assets in the savings account equivalent to before (annual return of $(1/\beta)^4 = 4\%$) and set the return on private assets to an annual return of -10%, trying to simulate savings

[9] In Hopenhayn and Nicolini (1997) the optimality condition for the design of UI implies that the unemployed worker is willing to save. See Cole and Kocherlakota (2001).

[10] They will be, for instance, exposed to the risk of theft or inflation.

in cash in an inflationary economy. The fact that the return on the savings account is higher gives the agents a more-efficient tool for insurance than using their private account.[11] Also, they can use it to smooth consumption and keep savings for retirement. However, agents can only use this saving technology when formally employed, so they have more incentive than before to make an effort and get a formal job.

The optimal contract in this context is $\psi = 0.035, b = 0.45, s_0 = 1.88,$ and $\widetilde{\tau} = 0.23$, and the welfare gains that the system provides are 27% of potential gains under full information.

Finally, we analyze how much can be done using only initial liquidity. The results show that an important part of the gains can be accrued this way, but it increases the size of informality and does not achieve the increase in output per worker.

6.4 Discussion of main results and assumptions

In the benchmark economy, low-effort-cost agents work mostly in the formal sector, but that participation occurs predominantly during the earlier years of their life, when productivity low. This phenomenon occurs because at that time their marginal utility of consumption is high. In contrast, high-effort-cost agents work mostly in the informal sector—their participation in the formal sector is very small and it occurs only in the early years of their life.

The full information allocation is significantly different. Both, with low and high-effort-cost agents work more in the formal sector. This implies higher total output in the economy. In addition, their time in the formal sector occurs during the years in which their productivity is the highest. In fact, while most of the agents with low effort costs work in the formal sector between ages 25 and 65, high effort cost agents work formally only between the ages of 45 and 60—the rest of the time they work in the informal sector. This improvement in the allocation of time over the life cycle implies that, in addition to higher output, there is also higher output per worker in the full information allocation.

The UISA menu system approximates the full information allocation. However, low-effort-cost agents work slightly less but more concentrated during the years in which their productivity is the highest; high-effort-cost agents work significant more. This implies that both total output and output per worker are higher. The fact that high-effort-cost agents work harder is not desirable, otherwise it would be a feature of the full information allocation. This occurs because this system cannot differentiate as well as the full information allocation between agents with low and high cost of effort.

One feature of the optimal arrangements, both the full information allocation and the UISA menus, is that young agents work mostly in the informal sector and they con-

[11] Without lower return on hidden savings, the system reduces to a liquidity transfer since there is no incentive to force workers to save in the less liquid savings accounts. The results are available upon request.

centrate their years in the formal sector during the periods in which their productivity is the highest. This is a key distinction with the benchmark allocation and occurs because these optimal arrangements correctly separate the decision of generating resources from the allocation of consumption. Of course, in the optimal arrangement, it is also important that agents sustain high level of consumption when they are young. But that does not mean that they must generate resources at that age, when their productivity is low. In fact, these optimal arrangements dictate that working in the formal sector must be concentrated during the years with higher productivity. The consumption of young individuals in the economy with an UISA menu is financed with unemployment compensations withdrawn from the initial stock of resources in the UISAs.

This result, which is a common to all our optimal arrangements, could be potentially overturn if agents must work in the formal sector to accumulate general human capital. Note, however, that it is key for this argument that agents accumulate general human capital faster in formal jobs than in informal jobs. Actually, if individual human capital grows at the same rate in formal and informal jobs, we do not need to model human capital explicitly and the predictions of our model for the optimal arrangements would be still valid. The evidence for Mexico shows that in the early years of workers careers, general human capital accumulation on-the-job is indeed higher in the informal sector than in the formal sector (Cano-Urbina, 2016). Thus, we believe that a model that would incorporate human capital, and it is calibrated to this evidence, will also have this feature in the optimal arrangements.

7 Conclusion

We performed a quantitative study of UISAs in a life-cycle model with sequential search in the presence of multiple sources of information asymmetries. Information asymmetries arise from unobservable heterogeneity among workers in search costs, a hidden labor market, and hidden savings. The system under study is characterized by a set of parameters describing its generosity, and it is calibrated such that it is self-financed. Both the informational frictions under study and the quantitative evaluation of UISAs in this context are original contributions to the literature.

By characterizing the optimal contract under full information, we find that the potential gains of a system are large (about 23% in consumption equivalence). Also, we identify that an optimal system should contribute to smoothing consumption and to allocating labor across sectors and the life cycle.

We evaluate the performance of UISAs and test the robustness of our results. If savings are observable, UISAs are a powerful tool in providing incentives, and the optimal system is characterized by a large initial transfer, a positive contribution rate, and an extraction rate of around 50% of formal salary. The welfare gains of the UISA menu are large, approximately 56% of potential gains. If only a single contract is offered, the overall characterization of the optimal system remains similar to that of a UISA menu, with welfare gains decreasing slightly to approximately 47%. If agents can privately save, the design of UISAs has less impact on incentives, but the optimal system can still provide liquidity and a positive contribution rate, which gives 27% of potential gains.

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Appendices

A Characterization of Full Insurance Allocation

This section proves the lemmas in the main body of the paper.

A.1 Retired Worker

The full-insurance allocation for retired workers can be replicated as markets are complete. This is easy since the only source of uncertainty is mortality. Denoting m as the amount of mortality-contingent securities that pay 1 unit of the consumption good in case of survival, the retired worker's problem in this context is:

$$H(m) = \max_{m' \geq 0} \left[u(m - \mu m') + \beta\rho \; H(m') \right]. \tag{3}$$

The envelope condition implies that $H'(m^*) = u'(c_R^*)$ where c_R^* stands for the consumption of the retired worker. Hence, $c^* = c_R^*$, so $m^* = (1-\rho\beta)c^*$ and $H(m^*) = \frac{u'(c^*)^{1-\sigma}}{1-\rho\beta}$.

Suppose that in the last period N, one or two job opportunities are available. As a type-θ worker who needs not make any effort since she will be retired next period, the optimal choice is to make the individual work in the sector with higher productivity.

A.2 Period $N-1$

In the case where there is no job opportunity, the worker remains unemployed, and the optimal values and an effort level satisfy

$$\begin{aligned} \phi_{N-1}(\theta, u) &= [-\theta \widetilde{e}_{N-1}(\theta, i^r)] \\ &\quad + \beta\lambda^* \left(q_u(\widetilde{e}_{N-1}(\theta, i^r)) w_f + (1 - q_u(\widetilde{e}_{N-1}(\theta, i^r))) p_u w_i \right), \end{aligned}$$

where $\widetilde{e}_{N-1}(\theta, u))$ is uniquely determined by

$$\theta = \beta\lambda^* \; q_u'(\widetilde{e}_{N-1}(\theta, i^r)))(w_f - p_u w_i).$$

.

Suppose first that a formal job opportunity is drawn at period $N-1$. The optimal choice must satisfy:

$$\phi_{N-1}(\theta, f) = \max\{\phi_{N-1}(\theta, f^a), \phi_{N-1}(\theta, u)\},$$

and $\widetilde{e}_{N-1}(\theta, f))$ is a corresponding optimal level of effort. If the worker is allocated to work in the formal sector then

$$\begin{aligned}\phi_{N-1}(\theta, f^a) &= [\lambda^* w_{N-1,f} - \theta \widetilde{e}_{N-1}(\theta, f)] \\ &\quad + \beta\lambda^* \left(q_f(\widetilde{e}_{N-1}(\theta, f^a)) w_f + (1 - q_f(\widetilde{e}_{N-1}(\theta, f^a))) p_i w_i\right),\end{aligned}$$

where $\widetilde{e}_{N-1}(\theta, f^a)$ is uniquely determined by

$$\theta = \beta\lambda^* \, q_f'(\widetilde{e}_{N-1}(\theta, f^a))(w_f - p_i w_i).$$

Alternatively, if the worker is not making use of that job opportunity, she remains unemployed.

Similarly, if an informal job opportunity is drawn, the optimal choice must satisfy

$$\phi_{N-1}(\theta, i) = \max\{\phi_{N-1}(\theta, i^a), \phi_{N-1}(\theta, u)\},$$

and $\widetilde{e}_{N-1}(\theta, i))$ is a corresponding level of effort. Here if the worker is allocated to work in that sector

$$\begin{aligned}\phi_{N-1}(\theta, i^a) &= [\lambda^* w_i - \theta \widetilde{e}_{N-1}(\theta, i^a)] \\ &\quad + \beta\lambda^* \left(q_i(\widetilde{e}_{N-1}(\theta, i^a)) w_f + (1 - q_i(\widetilde{e}_{N-1}(\theta, i^a))) p_i w_i\right),\end{aligned}$$

as $\widetilde{e}_{N-1}(\theta, i^a)$ is uniquely determined by

$$\theta = \beta\lambda^* \, q_i'(\widetilde{e}_{N-1}(\theta, i^a))(w_f - p_i w_i).$$

Alternatively, if the worker is not making use of that job opportunity, she remains unemployed.

Finally, if both job opportunities are available, the optimal choice must satisfy $\phi_{N-1}(\theta, b) = \max\{\phi_{N-1}(\theta, f), \phi_{N-1}(\theta, i), \phi_{N-1}(\theta)\}$.

A.3 Backward Induction

Consider now any working period $1 \leq n < N - 1$, and notice that as we are solving this backwards, all the decisions regarding allocation of effort and jobs have already been made for any working period $t \geq n$. Let $\phi_{n+1}(\theta, s')$ be the next period net wealth for $s' \in \{f, i, b, u\}$.

If the worker receives a formal job opportunity the optimal choice then satisfies

$$\phi_n(\theta, f) = \max\{\phi_n(\theta, f^a), \phi_n(\theta, u)\},$$

so $\widetilde{e}_n(\theta, f))$ is a corresponding optimal level of effort.

If the planner allocates the worker to this high productivity job, then

$$\begin{aligned}\phi_n(\theta, f^a) &= [\lambda^* w_f - \theta \widetilde{e}_n(\theta, f^a)] + \beta(q_f(\widetilde{e}_n(\theta, f^a))\phi_{n+1}(\theta, f) \\ &\quad +[1 - q_f(\widetilde{e}_n(\theta, f))][p_f \phi_{n+1}(\theta, i) + (1 - p_f)\phi_{n+1}(\theta, u)]),\end{aligned}$$

where $\widetilde{e}_n(\theta, f^a)$ is uniquely determined by

$$\theta = \beta\lambda^* \, q_f'(\widetilde{e}_n(\theta, f^a)) \, (\phi_{n+1}(\theta, f) - (p_f \phi_{n+1}(\theta, i) + (1 - p_f)\phi_{n+1}(\theta, u))).$$

If the worker receives an informal job opportunity, the optimal choice then satisfies

$$\phi_n(\theta, i) = \max\{\phi_n(\theta, i^a), \phi_n(\theta, u)\},$$

where $\widetilde{e}_n(\theta, i))$ is corresponding optimal level of effort.

As the worker is allocated to this low-productivity, informal job, the value is

$$\begin{aligned}\phi_n(\theta, i^a) &= [\lambda^* w_i - \theta \widetilde{e}_n(\theta, i^a)] + \beta(q_i(\widetilde{e}_n(\theta, i^a))\phi_{n+1}(\theta, f) \\ &\quad +[1 - q_i(\widetilde{e}_n(\theta, i^a))][p_i \phi_{n+1}(\theta, i) + (1 - p_i)\phi_{n+1}(\theta, u)]),\end{aligned}$$

where $\widetilde{e}_n(\theta, i^a)$ is uniquely determined by

$$\theta = \beta\lambda^* \, q_i'(\widetilde{e}_n(\theta, i^a)) \, (\phi_{n+1}(\theta, f) - (p_i \phi_{n+1}(\theta, i) + (1 - p_i)\phi_{n+1}(\theta, u))).$$

Alternatively, if the worker is not allocated to either the formal or informal job, the worker stays unemployed, so these values coincide as before and satisfy

$$\begin{aligned}\phi_n(\theta, u) &= -\theta \widetilde{e}_n(\theta, u) + \beta(q_u(\widetilde{e}_n(\theta, u))\phi_{n+1}(\theta, f) \\ &\quad +[1 - q_u(\widetilde{e}_n(\theta, u))][p_u \phi_{n+1}(\theta, i) + (1 - p_u)\phi_{n+1}(\theta, u)]),\end{aligned}$$

where $\widetilde{e}_n(\theta, i^a) = \widetilde{e}_n(\theta, u)$, and they are uniquely determined by

$$\theta = \beta\lambda^* \, q_i'(\widetilde{e}_n(\theta, u)) \, (\phi_{n+1}(\theta, f) - (p_u \phi_{n+1}(\theta, i) + (1 - p_u)\phi_{n+1}(\theta, u))).$$

Finally, if both job opportunities are available, the optimal choice must satisfy $\phi_n(\theta, b) = \max\{\phi_n(\theta, f), \phi_n(\theta, i), \phi_n(\theta)\}$.

B Setup for the Benchmark Economy Calibrated to Mexico

To provide an alternative for both positive and normative analysis, we will study a simple unemployment insurance scheme that captures most of the main features of the status quo in the economy under analysis. We assume that an individual who began working in the formal sector at age $n-1$ and lost her job at age n will receive an insurance payment during one period, where the payment during her period out of formality is a complete formal wage corresponding to her age; that is, $b = 1$ is a fraction of the formal salary that she will receive.

Rejected formal job offers are observable by the government and the worker has no right to file for unemployment insurance in the case that she rejects a formal offer. When the worker retires, the government provides d every period as retirement payment.

B.1 Period $n < N$

Now we describe the decision problem faced by a representative worker who is of working age $n < N$, who has received a preference shock θ, and who has m assets. In this scheme, we need to keep track of the history of unemployment payments, and we denote $z = (0, 1)$ as the right to receive payments, where $z = 1$ means that the agent will be paid the unemployment insurance and $z = 0$ that she has no right to receive. Therefore, if we index the fraction of the payments with z, then $b_1 = 1$ and $b_0 = 0$.

If the worker receives an offer in the informal sector, she must decide whether to accept the offer (a, accept) or not and remain unemployed (u, reject/unemployed). Her maximized lifetime utility, $V_n^i(\theta, m, z)$, solves

$$V_n^i(\theta, m, z) = \max\left\{V_n^{i,a}(\theta, m, z), V_n^u(\theta, m, z)\right\}.$$

Here, $V_n^{i,a}$ denotes the value of accepting the informal job offer and it satisfies

$$\begin{aligned} V_n^{i,a}(\theta, m, z) \quad = \quad & \max_{e,m'} u\left(mR - m' + \varpi_n + b_z\omega_n\right) - \theta e + \\ & \beta\left\{(1 - q_i(e))\left[p_{i,n}V_{n+1}^i(\theta, m', 0) + (1 - p_{i,n})V_{n+1}^u(\theta, m', 0)\right] + \right. \\ & \left. q_i(e)\left[p_{i,n}V_{n+1}^b(\theta, m', 0) + (1 - p_{i,n})V_{n+1}^f(\theta, m', 0)\right]\right\}, \end{aligned}$$

where the corresponding policy functions for effort levels and savings are given by $e_n^{i,a}(\theta, m, z)$ and $m_n^{i,a}(\theta, m, z)$, respectively.

The value of rejecting the informal job and remaining unemployed, $V_n^u(\theta, m, z)$, is detailed below.

If the worker receives an offer in the formal sector, her maximized lifetime utility,

$V_n^f(\theta, m)$, must solve

$$V_n^f(\theta, m, z) = \max\left\{V_n^{f,a}(\theta, m, z), V_n^u(\theta, m, z)\right\}.$$

Here $V_n^{f,a}$ is the value of accepting or rejecting the offer and must solve

$$\begin{aligned} V_n^{f,a}(\theta, m, z) &= \max_{e,m'}\{u\left(mR - m' + \omega_n(1-\tau)\right) - \theta e + \\ &\quad \beta\left\{(1 - q_f(e))\left(p_{f,n}V_{n+1}^i(\theta, m', 1) + (1 - p_{f,n})V_{n+1}^u(\theta, m', 1)\right) + \right. \\ &\quad \left. q_f(e)\left(p_{f,n}V_{n+1}^b(\theta, m', 0) + (1 - p_{f,n})V_{n+1}^f(\theta, m', 0)\right)\right\}, \end{aligned}$$

where corresponding policy functions for effort levels and savings are given by $e = e_n^{f,a}(\theta, m, z)$ and $m' = m_n^{f,a}(\theta, m, z)$, respectively.

If the worker receives offers in both the formal and informal sectors, her maximized lifetime utility, $V_n^b(\theta, m, z)$, must solve

$$V_n^b(\theta, m, z) = \max\left\{V_n^{f,a}(\theta, m, z), V_n^{i,a}(\theta, m, z), V_n^u(\theta, m, z)\right\}.$$

If the worker receives no offer, she is unemployed and her maximized lifetime utility, $V_n^u(\theta, m, z)$, satisfies

$$\begin{aligned} V_n^u(\theta, m, z) &= \max_{e,m'}\{u\left(mR - m' + b_z\omega_n\right) - \theta e + \\ &\quad \beta\left[(1 - q_u(e))\left(p_{u,n}V_{n+1}^i(\theta, m', 0) + (1 - p_{u,n})V_{n+1}^u(\theta, m', 0)\right) + \right. \\ &\quad \left. q_u(e)\left(p_{u,n}V_{n+1}^b(\theta, m', 0) + (1 - p_{u,n})V_{n+1}^f(\theta, m', 0)\right)\right]\Big\}, \end{aligned}$$

where the corresponding policy functions for effort levels and savings are given by $e_n^u(\theta, m, z)$ and $m_n^u(\theta, m, z)$, respectively.[12]

B.2 Period $n = N$

Finally, consider the decision problem faced by an agent at working age $n = N$. He does not exert any effort to find a job in the formal sector as she will be retired in the next period and only decides how much to consume and save (θ is immaterial).

If employed in the formal sector, the worker solves

$$V_N^f(m, z) = \max\left\{V_N^{f,a}(m, z), V_N^u(m, z)\right\},$$

[12]It is important to highlight that the value of rejecting all job offers available and receive none must necessarily coincide in equilibrium. The key difference is that in the first case the worker decides to be unemployed while in the second she is forced to be unemployed.

where

$$V_N^{f,a}(m,z) = \max_{m'} \left\{ u(\omega_N(1-\tau) + mR - m') + \beta H(m') \right\},$$

$$V_N^{u}(m,z) = \max_{m'} \left\{ u(mR - m' + b_z\omega_N) + \beta H(m') \right\}.$$

If employed in the informal sector, the worker solves

$$V_N^{i}(m,z) = \max \left\{ V_N^{i,a}(m,z), V_N^{u}(m,z) \right\},$$

where

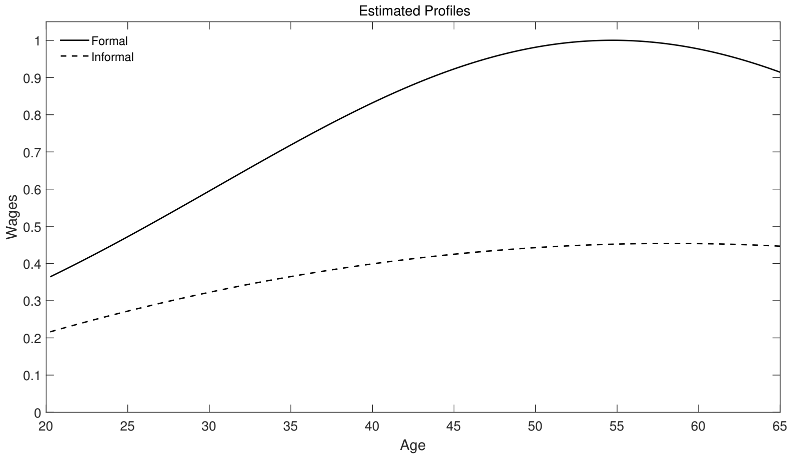
$$V_N^{i,a}(m,z) \quad = \quad \max_{m'} \left\{ u(\varpi_N + mR - m' + b_z\omega_N) + \beta H(m') \right\}.$$

C Data Used for Calibration

The unemployment rate and informality rate used for calibration were taken from the household survey *Encuesta Nacional de Ocupacioón y Empleo* (ENOE) and *Encuesta Nacional de Empleo Urbano* (ENEU) from *Instituto Nacional de Estadística y Geografía* (INEGI), and each is an average from 1998 to 2007. Yearly transition probability from formal to formal and informal is also taken from the INEGI. Quarterly transition probability from formal and informal to unemployed is taken from Inter-American Development Bank and is an average from 1987Q1 to 2010Q1.

The formal wages ω follow the life-cycle estimated profile for Mexico by Polachek (2008). Informal wages, ϖ, are calibrated such that the relative wage (before taxes) of informal workers to formal workers is 45.5%. This number was calculated as a ratio of the average nominal wage of workers without access to social security to the average nominal wage of workers with access to social security in 2005. Note also that wages have been inflation-adjusted to 2012 peso using data from the Bank of Mexico. Average wage differences in returns to age between formal and informal sectors is about 0.4% per quarter, as documented by Treviño (2007).

Figure 3: Estimated Life-Cycle Profile of Wages by Sectors



D Measurement

At any date t as population growth is constant, we normalize the population with working age $n \leq N$ as $(1+x)^{-(n-1)}$. As individuals retire they survive with probability ρ for $n = N + k$ for all $k \geq 1$; the size of the population of that age is $\rho^k (1+x)^{-(N+k-1)}$.

Let $F_n^j(\theta, m, s, es)$ denote the fraction of individuals in the economy $j \in \{FI, BE, UISA\}$ with preference shock θ, asset holdings m, savings s, age n, and employment status $es \in \{f, i, u\}$ (i.e., formal employee, informal employee, unemployed). Denote $c_n^j(\theta, m, es)$ as the corresponding consumption policy functions.

In order to carry out measurement, we assume that the law of large number holds so that $F_n^j(\theta, m, s, es)$ also denotes the date-1 probability that a worker reaches age n with with preference shock θ, asset holdings m, savings s, and employment status $es \in \{f, i, u\}$ in the economy $j \in \{FI, BE, UISA\}$.

D.1 Fiscal Accounting: Benchmark Economy

Now we compare the effect of a fiscal reform, taking into account the impact on the fiscal budget. Total taxes collected by the government and its corresponding expenditures in the economy BE are

$$\begin{aligned} T^{BE} &= \sum_{n=1}^{N} \sum_{\theta} \int_m (1+x)^{-(n-1)} F_n^{BE}(\theta, m, f) \; w_n \tau \; g(\theta) dm, \\ G^{BE} &= \sum_{n=1}^{N-1} \sum_{\theta} \int_m b \; (1+x)^{-n} F_n^{BE}(\theta, m, f)(1 - q_f(e_n^f(\theta, m, s))) \; g(\theta) dm \\ &\quad + \sum_{k \geq 1} \rho^k (1+x)^{-(N+k-1)} \; d, \end{aligned}$$

since $F_n^{BE}(\theta, m, s, f)(1 - q_f(e_n^f(\theta, m, s)))$ is the fraction of workers who at working age $n+1$ get fired from the formal sector and collect unemployment benefits b. That is, they were employed in the formal sector the previous period n and were exerting effort $e_n^f(\theta, m, s)$. Each of them gets fired with probability $(1 - q_f(e_n^f(\theta, m, s)))$.

So we define the fiscal surplus in the BE economy as

$$S^{BE} = T^{BE} - G^{BE}.$$

Notice that under our assumptions the implicit amount of net wealth left to the individuals at date 1 is $m^0 - S^{BE}$. We will always use $S^{BE} = 0$.

D.2 Fiscal Accounting: UISA Economy

Total taxes collected to finance transfers to workers entering the labor market are

$$T^{UISA} = \sum_{n=1}^{N} \sum_{\theta} \int_{m} (1+x)^{-(n-1)} F_n^{UISA}(\theta, m, s, f) w_n \, \widetilde{\tau} \, g(\theta) \, dm,$$

while the expenditures needed to finance those transfers are

$$G^{UISA} = (s_0 - m_0) + \sum_{k \geq 1} \rho^k (1+x)^{-(N+k-1)} \, d,$$

since s_0 is uncontingent with respect to θ.

Notice that this means the government can force the workers to deposit their initial wealth in the unemployment insurance savings account. To make both systems comparable, we restrict to UISA to

$$T^{UISA} - G^{UISA} = S^{BE}.$$

D.3 Labor Market

It is useful to first compute the levels of formal employment, informal employment, and unemployment for each economy $j \in \{FI, BE, UISA\}$. The level of employment in the formal sector is given by

$$F^j = \sum_{n=1}^{N} \sum_{\theta} \int_{m} (1+x)^{-(n-1)} F_n^j(\theta, m, s, f) \, g(\theta) \, dm.$$

The corresponding level of employment in the informal sector, our measure of informality in the economy, is given by

$$I^j = \sum_{n=1}^{N}\sum_{\theta}\int_{m}(1+x)^{-(n-1)}F_n^j(\theta,m,s,i)g(\theta)dm.$$

Finally, the unemployment level for each economy is defined as

$$U^j = \sum_{n=1}^{N}\sum_{\theta}\int_{m}(1+x)^{-(n-1)}F_n^j(\theta,m,s,u)g(\theta)dm.$$

These represent total numbers of workers for each employment status. We can translate the numbers into shares by simply writing.

$$S_f^j = \frac{F^j}{F^j+I^j},$$
$$S_i^j = \frac{I^j}{F^j+I^j},$$
$$U^j = \frac{U^j}{F^j+I^j+U^j},$$

where S_f^j, S_i^j, and U^j stand for the fraction of workers employed in the formal sector, the fraction of workers employed in the informal sector, and the rate of unemployment, respectively, for the economy j.

D.4 Welfare Comparisons

Let $\nu(j,j')$ be the percentage change in consumption needed to make an ex-ante representative worker indifferent between the allocations in the economies j and j', which deliver ex-ante expected utility V^j and $V^{j'}$. As the utility function of the representative worker is assumed to be homogeneous of degree $(1-\sigma)$ with respect to consumption for $\sigma > 0$ (i.e., CRRA preferences), $\nu(j,j')$ can be directly computed as follows

$$\nu(j,j') = \left[\frac{V^{j'} + \sum_\theta \int_m \sum_s \left(V_e^j(\theta,m,s) - \beta^N H^j(\theta,m,s)\right)\mu(s)dmg(\theta)\big)}{\sum_\theta \int_m \sum_s V_c^j(\theta,m,s)\mu(s)dmg(\theta)}\right]^{\frac{1}{(1-\sigma)}},$$

where

$$V_e^j(\theta,m,s) = \sum_{n=1}^{N}\beta^{n-1}\sum_{es\in\{f,i,u\}}F_n^j(\theta,m,s,es)\theta e_n^j(\theta,m,s,es)$$
$$V_c^j(\theta,m,s) = \sum_{n=1}^{N}\beta^{n-1}\sum_{es\in\{f,i,u\}}F_n^j(\theta,m,s,es)u(c_n^j(\theta,m,s,es)).$$

E Algorithm

- Step 0: Define a (negative) real number $Def(\theta_L; \mathbf{P}_L)$ that will be the deficit hold by agents with low search costs.
- Step 1: Find policy values and income taxes such that $\mathbf{P}_L^* = \arg\max_{\{\mathbf{P_L}\}} V(\theta_L; \mathbf{P}_L)$ s.t. $Def(\theta_L; \mathbf{P}_L)$.
- Step 2: Find income taxes from high θ agents that balance the budget for all possible combinations of P_H and find the utility. That is, find $V(\theta_H; \mathbf{P}_H)$ s.t. $\alpha_L Def(\theta_L; \mathbf{P}_L^*) = X - \alpha_H Def(\theta_H; \mathbf{P}_H)$ so that (BC) holds by construction.
- Step 3: Check if incentive compatibility holds for each combinations of parameters. Intuitively, (IC_L) may not hold since θ_L is subsidizing θ_H workers.
- Step 4: Among all possible combinations that satisfy incentive compatibility constraints, keep the one that maximizes ex-ante utility. That is, define $[\mathbf{P}_L^*, \mathbf{P}_H^*] = \arg\max_{\{\mathbf{P_H}\}} \alpha_L V(\theta_L; \mathbf{P}_L^*) + \alpha_H V(\theta_L; \mathbf{P}_H)$
- Step 5: Return to Step 1 for all possible combinations of deficits.

The optimal parameters will be the ones that maximize ex-ante utility for all possible deficits hold by agents with low search costs.

F Informality and Productivity

Here we provide a simple general equilibrium interpretation to construct appropriate TFP measures. We assume that there is a representative firm for each sector (formal and informal) that operates a constant returns to scale technology

$$y^h = \left[\sum_{n=1}^{N} \varphi_n^h l_n^h \right],$$

where h is the sector $(h = F, I)$, n indexes ages, l_n^i is the amount of labor hired by sector h of age n, and φ_n^h is the marginal productivity of the worker of age n in sector h.

Firms solve the problem

$$\max \left[\sum_{n=1}^{N} \varphi_n^h l_n^h - \sum_{n=1}^{N} l_n^h w_n^h \right],$$

where w_n^h is the wage paid to workers of age n in sector h per quarter.

In a competitive equilibrium with free entry, wage will be determined by marginal productivity at age n, so $w_n^i = \varphi_n^i$.

To construct TFP measures, we can re-write with the aggregate technology as follows

$$Y = \sum_{n=1}^{N} \left\{ \left[\varphi_n^F \pi_n^F + \varphi_n^I \left(1 - \pi_n^F\right) \right] l_n \right\}, \tag{4}$$

where $l_n = l_n^F + l_n^I$ is the total employment of age n, and $\pi_n^F = \frac{l_n^F}{l_n}$ is the fraction of people of age n employed in the formal sector. So the aggregate production function can be expressed as

$$Y = \widetilde{A} \left[\sum_{n=1}^{N} l_n \right], \tag{5}$$

where

$$\widetilde{A} = \sum_{n=1}^{N} \left[\varphi_n^F \pi_n^F + \varphi_n^I \left(1 - \pi_n^F\right) \right] \frac{l_n}{\sum_{n=1}^{N} l_n}$$

is the average productivity in the economy, weighted by sector and age dependent factors. Also, our measure of productivity is equivalent to output per worker:

$$\widetilde{A} = \frac{Y}{\sum\limits_{n=1}^{N} l_n}. \tag{6}$$