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The Exchange Rate as an Instrument of Monetary Policy

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Abstract

Monetary policy research in small open economies has typically focused on "corner solutions": either the currency rate is fixed by the central bank, or it is left to be determined by market forces. We build an open-economy model with external habits to study the properties of a "new "class of monetary policy rules in which the monetary authority uses the exchange rate as the instrument. Different from a Taylor rule, the monetary authority announces the rate of expected currency appreciation by taking into account inflation and output fluctuations. We find that the exchange rate rule outperforms a standard Taylor rule in terms of welfare, regardless of the policy parameter values. The differences are driven by: (i) the behavior of the nominal exchange rate and interest rates under each rule, and (ii) deviations from UIP due to a time-varying risk premium.

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1 Introduction

Understanding the properties of alternative monetary policy rules and designing rules that maximize social welfare are important objectives both from a policy point of view and from an analytical perspective. The seminal paper by Taylor (1993) has generated an entire sub-field of monetary economics that studies the properties of alternative interest rate rules within empirical or theoretical frameworks. The majority of these papers focus on models of closed economies. The Taylor rule prescribes that the monetary authority adjusts interest rates in response to deviations in inflation from a prespecified implicit or explicit inflation target and to fluctuations in the output gap. The research based on Taylor's work has extended the policy rule by including additional variables, by implementing different estimation techniques to determine the reactions of central banks to key macroeconomic variables, and by building theoretical models to study the properties of these rules. Gali (2008) provides an insightful overview.

In small open economies, however, the exchange rate is an important element of the transmission of monetary policy (Svensson (2000)). The central banks in these economies generally prefer to keep the exchange rate under tight control. The economic literature incorporates exchange rate policies of various central banks in two ways. First, a large number of papers evaluate the costs and benefits of fixed exchange rates (including Friedman (1953) and Flood & Rose (1995)). A second approach to incorporating the exchange rate into discussions of monetary policy is by augmenting a closed-economy Taylor rule with the rate of currency depreciation. Under this approach the monetary policy instrument, the interest rate, reacts not only to inflation and the output gap but also to movements in the exchange rate. For example, De Paoli (2009) derives an optimal monetary policy rule within a DSGE model and shows that by putting some weight on real exchange rate fluctuations, a central bank can achieve improvements in social welfare.

In discussing how monetary authorities deal with exchange rates, monetary policy research focuses on "corner solutions": either the currency rate is fixed by the central bank or the government, or it is left to be determined by market forces. In this paper, we evaluate the properties of an alternative class of policy rules where the exchange rate is used as an instrument of monetary policy. The exchange rate is adjusted by the central bank in a manner similar to the use of the interest rate as an operating instrument. We build a model in which the rate of currency appreciation is determined as a reaction to the rate of inflation and the output gap. Our goal is to establish whether there are economic structures for which the use of this exchange rate-based rule delivers higher social welfare compared to policies based on interest rate rules, as well as rules in which the monetary authority commits to use its tools to keep the exchange rate fixed.

The motivation for writing the model comes from the Monetary Authority of Singapore (MAS). Unlike other central banks, MAS does not rely on the overnight interest rate or any monetary aggregate to implement its monetary policy. Since 1981, the operating instrument has been the exchange rate. Khor, Lee, Robinson & Supaat (2007), McCallum (2007), and MAS (2012) offer detailed descriptions of the policy regime in Singapore. Many authors view Williamson (1998) and Williamson (2001) as providing the analytical foundations of this system, since he proposed an intermediate exchange rate system in the form of an adjustable crawling peg with a band. The system is also referred to as BBC (basket, band, crawl) as the currency can be pegged against a basket of currencies in order to minimize misalignment with major trading partners. The crux of Williamson's argument is that the crawl or the level of the exchange rate must be adjusted to reflect differences in inflation and productivity trends between the domestic and foreign economies. In other words, the exchange rate must move over time towards its equilibrium value.

There can be little doubt that having the exchange rate aligned with the fundamentals is a valuable goal in the long run. In this paper, we argue that MAS goes beyond a simple adjustment towards equilibrium and uses the exchange rate to stabilize the business cycle. At business cycle frequencies, the central bank reacts to changes in inflation and the output gap. According to some casual metrics - macroeconomic stability, inflation, currency volatility, etc. - this policy has been quite successful over the past twenty years. And yet, it is not clear whether the success of the central bank is due to the novel policy rule or to some other factors like prudent fiscal policy. Therefore, an important question to ask is whether other countries would benefit from adopting a rule where the exchange rate is moved on a continuous basis as a reaction to the level of inflation and the output gap. To understand the benefits of this rule relative to a standard interest rate rule, we need the discipline of a model with optimizing agents, market clearing, and other features that are characteristic of recent advances in monetary policy analysis. Our paper aims to analyze the properties of exchange rate rules by building a DSGE model where the monetary authority adjusts the exchange rate in response to deviations of inflation from a target and fluctuations in the output gap.

Understanding the costs and benefits of an exchange rate policy rule within a fully specified model is not a trivial task. The immediate revelation is that if the model features an uncovered interest parity condition (UIP), then interest rate and exchange rate rules might generate similar, if not identical, outcomes. In our model, there are two reasons why the outcomes for the two rules differ. First, the actual implementation of the exchange rate rule is important. While the central bank technically can replicate any interest rate rule by moving the exchange rate today and announcing depreciation consistent with UIP, it is not the way that our rule operates. In our model, the exchange rate today is predetermined and the central bank announces the depreciation rate from time t to $t+1$. This implies for example, that the model may not feature the standard overshooting result as the currency rate both today and at $t+1$ are determined by the monetary authority.

The simulations of our model suggest that this feature does to generate differences between the two rules. These differences are amplified when UIP fails. Indeed, Alvarez, Atkeson & Kehoe (2007) argue forcefully that a key part of the impact of monetary policy on the economy goes through conditional variances of macroeconomic variables rather than conditional means. In terms of the UIP condition, their paper implies that the interest parity condition has a time-varying risk premium. Interest in time-varying risk premium has been growing in recent years and in the context of the interest parity condition, Verdelhan (2010) shows how consumption models with external habit formation can generate counter-cyclical risk premium that matches key stylized facts quite successfully. In our model, we adopt a similar approach by allowing external habit formation. To show the importance of the counter-cyclical risk premium, we report results for the first-order approximation, which wipes out the risk premium from UIP, and for the third-order approximation, which preserves time variation in the risk premium.[1]

[1] An alternative route for introducing risk premium in the UIP condition is by building in incomplete financial markets, as in Schmitt-Grohé & Uribe (2003), Turnovsky (1985), Benigno (2009) and De Paoli (2009). Under incomplete markets, deviations from UIP come from costs of adjusting holdings of foreign bonds. This requires the introduction of country's net foreign asset (NFA) position in the model dynamics. The cost of holding foreign bonds introduces a time-varying risk premium and deviations from UIP.

We start by writing down a relatively standard New-Keynesian small open economy model as in Gali & Monacelli (2005) that we extend to include external habit in consumption. We then analyze the performance of the model under two different policy rules: a standard Taylor rule in which the monetary authority sets interest rates, and an alternative monetary rule in which the monetary authority sets the depreciation rate of the nominal exchange rate. We show that if UIP holds (i.e. we use first-order approximation), these rules generate different responses to exogenous shocks. The Taylor rule implies overshooting of the exchange rate following a shock, generating a higher volatility of the exchange rate and other economic variables. We then introduce deviations from UIP. The goal is to analyze the performance of the two competing rules when the one-to-one relationship between exchange rates and interest rates breaks down. In this case, the differences between the two rules, in terms of the response of the economy to exogenous shocks, is amplified. The main reason is that the particular implementation of the rule has an effect on the volatility of the risk premium though a precautionary saving motive. The Taylor rule generates larger fluctuations of inflation and output gap, as the larger volatility of exchange rates increases the risk premium. The opposite is true for the exchange rate rule, as the exchange rates are less volatile and the monetary authority adjusts its path of appreciation to smooth economic fluctuations. In this regard, the exchange rate rule is different from a peg, in which the monetary authority fixes the exchange rate to a specified value. With the peg, the volatility of the exchange rate is zero, but the volatility of other economic variables is larger than with the exchange rate rule as the central bank is not allowed to respond to fluctuations of these variables.

To shed more light on the mechanism, we follow the methodology in Backus, Gavazzoni, Telmer & Zin (2010) and derive an analytical solution for the risk premium under the two monetary rules. We do it in the context of an endowment economy in which all variables are jointly log-normal. We find that the risk premium is different depending on the rule that the monetary authority follows. For the same parameter values, the monetary exchange rate rule implies a lower risk premium than the Taylor rule. To get a better understanding of these differences, we decompose the risk premium into the volatility of the exchange rate depreciation and the covariance between the stochastic discount factor and the exchange rate depreciation. The exchange rate rule generates both lower fluctuations of the exchange rate depreciation and a lower covariance, hence the lower the risk premium under this rule.

Finally, we find that the differences between the two rules are amplified when the economy is hit by a foreign shock rather than a domestic shock, which suggests that exchange rate rules may be more successful at smoothing economic fluctuations in small open economies that are exposed to foreign shocks, hence exchange rate risk. Indeed, we find that depending on the degree of openness of the economy exchange rate rules may generate lower volatility of CPI inflation compared to interest rate rules. We also report differences in the volatility of the main endogenous variables between the two regimes as well as the dynamics of these variables following a standard productivity shock.

The rest of the paper proceeds as follows. Section 2 lays out the details of the model. Section 3 presents the quantitative analysis. Section 4 analyzes the mechanism. Section 5 provides a discussion of our key findings, some ideas for future research and conclusions.

2 The Model

We introduce deviations from UIP by adding external habit in consumption to a standard small open economy model with sticky prices. Our model extends Gali & Monacelli (2005) by introducing a new policy rule based on using the exchange rate as a monetary policy instrument and adding external habit as in Campbell & Cochrane (1999), Jermann (1998), Verdelhan (2010) and De Paoli & Sondergaard (2009). Modeling assumptions are kept at a minimum to ensure that we can study the properties of the exchange rate instrument rule without introducing too many confounding factors.

2.1 Households

In each country, there is a representative household that maximizes life-time expected utility. The utility function of the household in country H is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t - hX_t, N_t) \tag{1}$$

where N_t is hours of labor, X_t is the level of habits defined below, and C_t is a composite consumption index defined by:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ denotes the consumption of domestic goods by the Home consumers, $C_{F,t}$ denotes the consumption of foreign goods by Home consumers, $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\alpha \in [0,1]$ is the degree of openness of the country (and the inverse of home bias). C_{Ht} and C_{Ft} are aggregates of intermediate products produced by H and F combined in the following way

$$C_{Ht} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_{Ft} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

with ε being the elasticity of substitution between varieties, which in turn are indexed by $i \in [0,1]$.

As in De Paoli & Sondergaard (2009) we assume that habits are external. We allow for flexibility in assessing the importance of habits by introducing in 1 parameter $h \in [0,1]$. When $h = 0$ the model collapses to the basic version of Gali-Monacelli, while $h = 1$ corresponds to the modeling assumptions in Campbell & Cochrane (1999) and Verdelhan (2010). The evolution of habits follows an AR(1) process with accumulation of habits based on last-period consumption:

$$X_t = \delta X_{t-1} + (1-\delta) C_{t-1}$$

Parameter $\delta \in [0,1]$ captures the degree of habit persistence. Again, this parameter allows us to consider various assumptions about habits with $\delta = 0$ corresponding to the assumptions in the earlier literature on habit-formation where habits are determined exclusively by the last-period consumption (e.g. Campbell (2003) and Jermann (1998)).

Consumers maximize 1 subject to the following budget constraint:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1 P_{F,t}(i)C_{F,t}(i)di + E_t\left\{\mathcal{M}_{t,t+1}B_{t+1}\right\} \leq B_t + W_tN_t$$

where $P_{H,t}(i)$ is the price of variety i produced at home, $P_{F,t}(i)$ is the price of variety i imported from Foreign (expressed in Home currency), $\mathcal{M}_{t,t+1}$ is the stochastic discount factor, B_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t, and W_t is the nominal wage.

The optimal allocation of expenditures within each variety gives the demand function for each product:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{Ht}}\right)^{-\varepsilon} C_{Ht};\ C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{Ft}}\right)^{-\varepsilon} C_{Ft}; \tag{3}$$

where $P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$ is the domestic price index and $P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}$ is the price index of imported goods (expressed in units of Home currency). From expression 3, $P_{Ht}C_{Ht} = \int_0^1 P_{H,t}(i)C_{H,t}(i)di$ and $P_{Ft}C_{Ft} = \int_0^1 P_{F,t}(i)C_{F,t}(i)di$.

The optimal allocation of expenditures between domestic and imported goods is:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t;\quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{4}$$

where $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI). From the previous equations, total consumption expenditures by the domestic households is $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. Therefore, we can re-write the budget constraint as

$$P_tC_t + E_t\left\{\mathcal{M}_{t,t+1}B_{t+1}\right\} \leq B_t + W_tN_t \tag{5}$$

The per period utility function takes the following form

$$U(C_t, X_t, N_t) \equiv \frac{(C_t - hX_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma}$$

The first order conditions for the household's problem are

$$(C_t - hX_t)^{\sigma}N_t^{\gamma} = \frac{W_t}{P_t} \tag{6}$$

$$\beta\left(\frac{C_{t+1}-hX_{t+1}}{C_t-hX_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)=\mathcal{M}_{t,t+1} \tag{7}$$

Taking expectations in both sides, we have the Euler Equation:

$$R_tE_t\left\{\beta\left(\frac{C_{t+1}-hX_{t+1}}{C_t-hX_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\}=1 \tag{8}$$

with $R_t=\left[\frac{1}{E_t\{\mathcal{M}_{t,t+1}\}}\right]$ the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$.

Below we elaborate on the need to use habit formation in this model but from the Euler equation it is already clear that marginal utility of consumption increases when consumption goes down relative to the acquired level of habit. This introduces a precautionary saving motive. As we discuss below, this modeling approach generates a time-varying coefficient of relative risk aversion. This leads to a counter-cyclical risk premium which plays an important role in driving a wedge between the interest rate differential and expected depreciation in the interest parity condition. Both Verdelhan (2010) and De Paoli & Sondergaard (2009) make this point quite forcefully.

2.2 Domestic inflation, CPI inflation, the RER, and the TOT

Before we proceed to the solution, we introduce some definitions, following Gali & Monacelli (2005).

2.2.1 Bilateral Terms of Trade (TOT)

The terms of trade, S_t, are defined as the price of Foreign good in terms of Home goods, so that

$$S_t\equiv\frac{P_{F,t}}{P_{H,t}}$$

We define the real exchange rate as the relative prices

$$Q_t=\frac{\mathcal{E}_tP_t^*}{P_t}$$

where $\mathcal{E}_t$ is the nominal exchange rate.

Inflation is defined the ratio of current and past CPI

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

We assume that the foreign economy is large relative to the home country, so the law of one price requires that the price index in the foreign country equals the price index of foreign goods in the come economy (when converted to the same currency).

$$P_{Ft} = \mathcal{E}_t P_t^*$$

2.3 International risk sharing

Under the assumption of complete international financial markets, the rest of the world must satisfy

$$\beta \left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \left(\frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}} \right) = \mathcal{M}_{t,t+1} \tag{9}$$

The exchange rate reflects the fact that the security bought by the foreign households is priced in terms of the small open economy. For simplicity, in this version we do not model habit formation in the foreign country.

Combining this expression with the one for the domestic households, we have

$$\left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \left(\frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}} \right) = \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right)$$

Using the expression for the real exchange rate

$$\left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \left(\frac{Q_t}{Q_{t+1}} \right) = \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \tag{10}$$

2.4 The Uncovered Interest Parity Condition

Under complete international financial markets, $R_t^* = \left(E_t \left\{ \mathcal{M}_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \right)^{(-1)}$ and since $R_t = \left(E_t \left\{ \mathcal{M}_{t,t+1} \right\} \right)^{(-1)}$ we obtain the following UIP condition

$$E_t \left\{ \mathcal{M}_{t,t+1} (R_t - R_t^* (\mathcal{E}_{t+1} / \mathcal{E}_t)) \right\} = 0$$

Log-linearization of this condition around perfect foresight steady state yields the standard uncovered interest parity condition:

$$r_t - r_t^* = E_t \Delta e_{t+1} \tag{11}$$

Alvarez et al. (2007) argue that assumptions leading to this simplified interest parity condition imply dynamics that are inconsistent with the data. Under assumptions of conditional log-normality of the stochastic discount factor, a time-varying risk premium emerges, as shown by Backus et al. (2010). Alternatively, a higher-order approximation of the Euler equation can also generate time-varying risk premium, as we show below.

2.5 Firms

We now characterize the supply side of the economy. In each country there is a continuum of monopolistic competitive firms that use labor to produce a differentiated good (each firm is associated with a different variety). Labor is the only factor of production, and we assume it to be immobile across countries.

2.5.1 Technology

Each firm operates the linear technology

$$Y_t = A_t N_t \tag{12}$$

where $a_t \equiv log(A_t)$ follows the AR(1) process

$$a_t = \rho_a a_{t-1} + u_t \tag{13}$$

The real marginal cost MC_t is

$$W_t = MC_t P_{Ht} A_t \tag{14}$$

2.5.2 Price setting

Prices are set as in the Calvo model, in which a measure $1 - \theta$ of randomly selected firms set new prices every period. We need to define some auxiliary variables to express the pricing decision recursively:

$$\frac{\tilde{P}_{H,t}}{P_t} = \frac{\varepsilon}{\varepsilon - 1}\frac{H_t}{F_t} \tag{15}$$

$$F_t = \Lambda_t Y_t + \theta\beta E_t(F_{t+1}(\Pi_{t+1})^{\varepsilon-1})$$

$$H_t = \Lambda_t MC_t Y_t + \beta\theta E_t(H_{t+1}(\Pi_{t+1})^{\varepsilon})$$

$$\Lambda_t = (C_t - hX_t)^{-\sigma}$$

$$P_{Ht} = \left((1-\theta)\tilde{P}_{H,t}^{1-\varepsilon} + \theta P_{Ht-1}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{16}$$

This expression can be written in real terms as

$$\frac{P_{Ht}}{P_t} = \left((1-\theta)\left(\frac{\tilde{P}_{H,t}}{P_t}\right)^{1-\varepsilon} + \theta\left(\frac{P_{Ht-1}}{P_{t-1}}\right)^{1-\varepsilon}\Pi_t^{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}} \tag{17}$$

We take as a numeraire P_t and express every variable in real terms. Once we get an expression for Π_t we can obtain the price index using

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

2.6 The rest of the world

Because the foreign economy is exogenous to our small open economy, there is some flexibility in specifying the behavior of the foreign variables. We assume they follow AR(1) process:

$$log\left(Y_t^*\right) = \rho_y log\left(Y_{t-1}^*\right) + u_{y^*t} \tag{18}$$

We assume that $\Pi_t^* = 0$, there is habit in consumption as in the domestic economy, $Y_t^* = C_t^*$ and the foreign interest rate R_t^* is determined by the Euler equation as in the domestic economy.

2.7 Market clearing in the goods market

In the domestic economy, the goods market clearing condition is

$$Y_t = C_{H,t} + C^*_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{P_{F,t}}{P^*_t}\right)^{-\eta} C^*_t =$$

$$= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{S_t P_{H,t}}{P_t}\right)^{-\eta} C^*_t$$

Using the risk-sharing condition

$$C_t^{-\sigma} = Q_t C_t^{*-\sigma}$$

we have

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha Q_t^{\eta - \frac{1}{\sigma}} S_t^{-\eta}\right] = S_t^{\eta} Q_t{}^{-\eta} C_t \left[(1-\alpha) + \alpha Q_t^{\eta-\frac{1}{\sigma}} S_t^{-\eta}\right] \quad (19)$$

$$= C_t \left[(1-\alpha) S_t^{\eta} Q_t{}^{-\eta} + \alpha Q_t^{-\frac{1}{\sigma}}\right]$$

In the rest of the world, we have

$$Y^*_t = C^*_t \quad (20)$$

2.8 Monetary policy rules

The model is closed by specifying the monetary policy rule. First, we analyze the model under a standard Taylor rule, in which the monetary authority sets the nominal interest rate to smooth fluctuations in the output gap, CPI inflation and fluctuations in the nominal exchange rate. There is also interest rate smoothing.

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho} \left(\frac{Y_t}{\bar{Y}}\right)^{(1-\rho)\phi_y} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{(1-\rho)\phi_m} e^{u_t} \quad (21)$$

where $\rho \in (0,1)$ is the degree of interest rate smoothing.

Second, we consider a monetary policy rule in which the central bank uses the change in the nominal exchange rate as the instrument to stabilize output gap, CPI inflation, and movements in the nominal exchange rate. The policy is adjusted in response to anticipated deviations of these variables from their targets:

$$\frac{\mathcal{E}^*_{t+1}}{\mathcal{E}^*_t} = \frac{\bar{\mathcal{E}}_{t+1}}{\bar{\mathcal{E}}_t}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y^e}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_m^e} e^{v_t} \tag{22}$$

with $\overline{\mathcal{E}}_{t+1}/\bar{\mathcal{E}}_t$ the depreciation required to reach the long-run equilibrium nominal exchange rate.[2] We assume that there is some smoothing in the way the nominal exchange rate adjusts to its target level

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \left(\frac{\mathcal{E}^*_{t+1}}{\mathcal{E}^*_t}\right)^{(1-\rho_e)}\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\rho_e} \tag{23}$$

The exchange rate rule and its properties have been documented, for the case of Singapore, by Khor et al. (2007). The central bank announces a path of appreciation of the exchange rate that is given by Δe^*_t and this determines the evolution of the nominal exchange rate. Note that this rule corresponds to a managed float and it is between a completely fixed exchange rate regime in which $E_t\Delta e_{t+1} = 0$ and a flexible exchange rate regime, that would correspond to the central bank using as an instrument the nominal interest rate, letting the exchange rate fluctuations be driven by market forces.

Business cycle dynamics under the two rules

The two monetary rules have different implications for business cycle fluctuations. These differences are amplified at a third order approximation, due to the presence of an endogenous and time-varying risk premium that breaks down the UIP condition.

To understand how the two rules imply different business cycle dynamics, assume a shock that decreases both domestic inflation and output gap (i.e., a positive domestic supply shock or a negative domestic demand shock). First, we show the case in which UIP holds, which corresponds to a first order approximation of the model. To show this, we log-linearize the two rules.

From equation 21, we have

$$r_t = \rho r_{t-1} + (1-\rho)\phi_y\widetilde{y}_t + (1-\rho)\phi_m\widetilde{\pi}_t + u_t \tag{24}$$

[2] We follow the convention that an increase in the exchange rate implies depreciation of the domestic currency.

where the lower case variables represent log-deviations of the variable with respect to its steady state.

Similarly, if we put together equations 22 and 23, and we log-linearize the expression, we have

$$E_t \Delta e_{t+1} = -(1-\rho_e)\phi_y^e \widetilde{y}_t - (1-\rho_e)\phi_m^e \widetilde{\pi}_t + \rho_e \Delta e_t + v_t \tag{25}$$

Under a Taylor rule, the central bank reacts by decreasing the nominal interest rate (see equation 24). As a result, the domestic currency depreciates. When UIP holds, non-arbitrage determines a future appreciation of the domestic currency, which leads to an overshooting of the nominal exchange rate. Under an exchange rate rule, however, the overshooting does not happen. The reason is that, after the shock, the central bank reacts by announcing a slow depreciation of the currency (see equation 25). As forward-looking consumers expect that the currency will continue to depreciate, there is an excess supply of domestic bonds, which lowers the price of these bonds, hence increasing the nominal interest rate. When UIP holds, the increase in the nominal interest rate equals the expected future depreciation (see equation 11). Because there is no overshooting, this increase is lower than the decrease under the Taylor rule, and hence the exchange rate rule generates lower fluctuations of domestic variables. Therefore, the two rules imply differences in business cycle fluctuations due to the different response of the exchange rate depreciation and the nominal interest rate.

External habits introduce deviations of the UIP condition at higher order approximations. These deviations imply the existence of a risk premium, which is endogenous and time-varying at third order approximations. An endogenous risk premium amplifies the differences between the interest rate rule and the exchange rate rule on business cycle fluctuations. After a positive domestic productivity shock, there is an excess demand for foreign goods, and the domestic currency depreciates. Furthermore, the precautionary savings of the foreign consumers increase relative to that of the domestic investors and so the risk premium increase, both under the exchange rate rule and the Taylor rule. However, because the monetary authority is managing the exchange rate, the fluctuations of the risk premium are lower, and hence the nominal interest rate increases less than the decrease under the Taylor rule. This amplifies differences in economic variables under the two rules. After a positive foreign demand shock, the precautionary savings of the foreign investors decreases and the risk premium goes down. Again, the decrease is smaller under the exchange rate rule, which induces a lower decrease in the interest rate than the increase experienced under the Taylor rule. Hence, the presence of an endogenous risk premium amplifies business cycle fluctuations due to the effect of shocks in the precautionary savings of the consumers.

3 Quantitative exercise

We now calibrate the model and perform a quantitative analysis to compare the different performance of the two rules. We report and analyze impulse responses, second moments, and the evolution of the risk premium. The performance of different policy rules will depend critically on how they affect the dynamics of the risk premium. To capture a time-varying risk premium we solve for a third order approximation of the model (see Van Binsbergen, Fernandez-Villaverde, Koijen & Rubio-Ramirez (2012)). As in De Paoli & Sondergaard (2009), we use the log-linear version of the demand and supply conditions. In that way, our non-linear model isolates the role of a time-varying risk premium. We call this the hybrid model. In a robustness exercise, we present the simulation results after solving for a third order approximation of the full model. We use Dynare for our numerical exercises. In all of them do 10,000 simulations and use pruning for the third order approximation.

3.1 Calibration

The calibrated parameters are reported in Table 2. We follow closely the parametrization of De Paoli & Sondergaard (2009) and Gali & Monacelli (2005).

Parameter	Description	Value
h	Habit	0.85
δ	Degree habit	0.97
ε	Elast.subst.imports	1
η	Elast.subst.interm.	1
γ	Labor supply elast.	3/(1-h)
α	Openness	0.08
β	Discount Factor	0.99
θ	Price stickiness	0.75
σ	Intertemp. elast.	5

Table 2: Calibrated Parameters

We choose our parameters from De Paoli & Sondergaard (2009) and Gali & Monacelli (2005). The parameters of habit persistence are set to $h = 0.85$ and $\phi = 0.97$. The elasticity of substitution across traded goods, ε equals 1, and across intermediate goods is $\eta = 1$. As in De Paoli & Sondergaard (2009), we set the Frisch labor supply parameter, γ equal to $\frac{3}{1-h}$. The degree of openness, α, is set to 0.08 as in De Paoli & Sondergaard (2009) and Lubik & Schorfheide (2007). The discount factor is set at $\beta = 0.99$ which implies a steady-state interest rate of 4% in a quarterly model. We assume the degree of price stickiness to be $\theta = 0.75$, which is consistent with the average period of price adjustment of one year, and an intertemporal elasticity of substitution, to be $\sigma = 5$, which is within the range found by the empirical literature of $[2, 10]$.

As in Gali & Monacelli (2005), domestic productivity is assumed to have a standard deviation of 0.71% while the foreign productivity shock, has a standard deviation of 0.78%. The shocks are assumed to be positive correlated with $\rho(\sigma_a, \sigma_a^*) = 0.3$.

For simplicity, and to illustrate our mechanism, we perform the analysis with a simple rule in which the monetary authority reacts only to fluctuations of inflation, with a certain degree of smoothing in the instrument, as it is the case empirically. In particular, for the Taylor rule, we set $\phi_m = 1.5$, $\phi_y = 0$ and $\rho = 0.85$ as in Lubik & Schorfheide (2007), and $\phi_m^e = 0.28$, $\phi_y^e = 0$ and $\rho_e = 0.85$, Parrado (2004) estimated for the case of Singapore. The idea is that to assume that if the central bank of a country following an exchange rate rule were to use instead the nominal interest rate as its instrument, it would decide to follow a Taylor rule similar to other small open economies, as in Lubik & Schorfheide (2007). To make sure that the differences between the rules are not driven by an arbitrary choice of the exchange rate rule parameters, we also report moments for $\phi_m^e = 1.5$ and the extreme value $\phi_m^e = 3$.[3]. Later, we compare both rules under a wide range of alternative values of the policy parameters, to check whether there is any combinations of the parameters for which the two rules deliver similar business cycle dynamics. As a robustness check, we also extend the policy rules by allowing the monetary authority to react to the output gap in addition to inflation.

We then consider two cases. First, we do a first order Taylor expansion of the model to understand how the two rules differ when there are no deviations from UIP. Then, we do a third order approximation to introduce the effect of the time-varying risk premium that generates deviation from UIP.

We obtain impulse response functions and second moments that we compare for the two rules.

3.2 Impulse response functions

We compare the qualitative performance of the two rules by computing impulse response functions to a one deviation standard deviation domestic productivity shock and the foreign output shock. Figures 1 to 4 show the results.[4] In these exercises, we define the FX premium as the excess return on investing on domestic currency, that is, $fxp_t = r_t - (r_t^* - E_t \Delta e_{t+1})$.

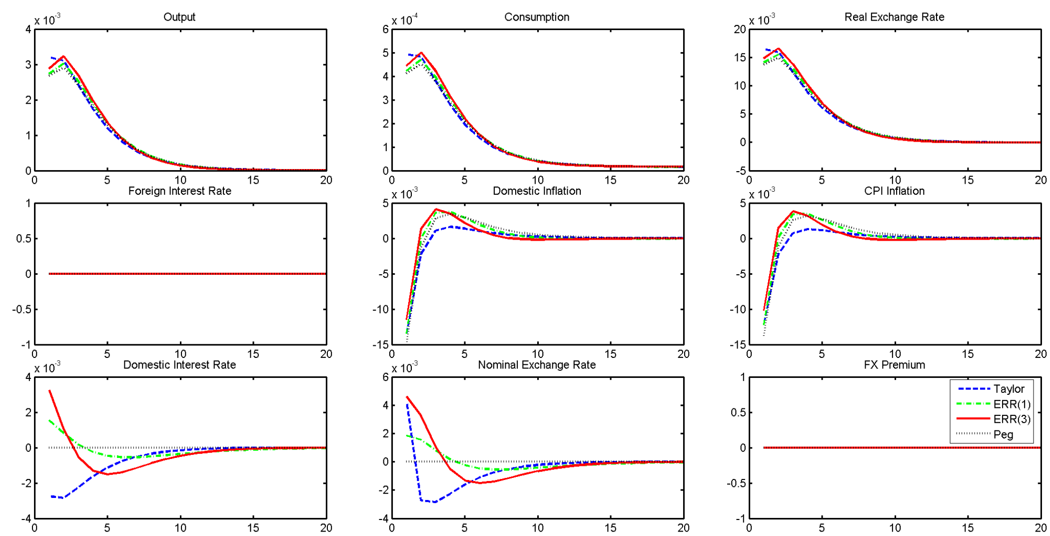
After a positive domestic productivity shock (figure 1), output and consumption go up. Because the potential output increases faster that the actual output (due to frictions in the economy), the output gap goes down. Domestic inflation decreases because the economy is now more productive.The reaction of the central bank to these fluctuations is different under the two rules. If the central bank follows a Taylor rule, it will decrease interest rates, which depreciates the currency (depreciation rate goes up). The initial depreciation is followed by an appreciation, since UIP holds and the initial increase in interest rate must be compensated by a future appreciation of the currency. There is overshooting. CPI inflation decreases because the initial decrease

[3]Several previous studies on optimal monetary policy tend to restrict $\phi_m \leq 3$ (see Schmitt-Grohé & Uribe (2007)). We do the same in our simulations.

[4]In the figures, ERR(1) and ERR(3) correspond to an exchange rate rule in which the coefficient of inflation is 1 and 3, respectively.

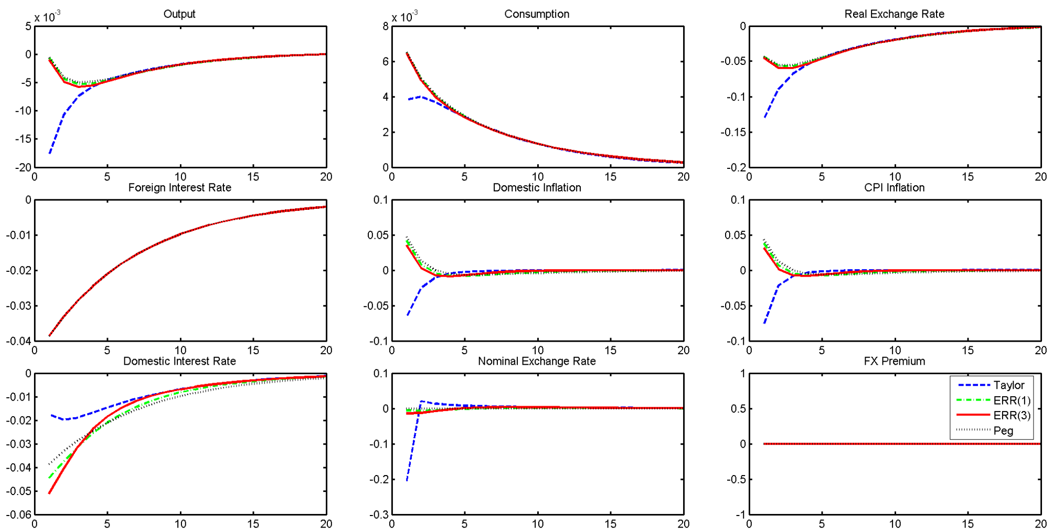
in domestic inflation dominates the depreciation of the currency. If, instead, the central bank follows an exchange rate rule, after the positive domestic TFP shock, the central bank reacts to the fall in inflation by announcing a depreciation of the exchange rate. In this case, the nominal interest rate increases, since, as households expect a depreciation of the currency there is an excess supply of domestic bonds. Therefore, the response of the nominal exchange rate and interest rate is different under the two rules, which translates into differences in business cycle dynamics. Quantitatively, however, the differences do not appear to be very large after a domestic shock.

Figure 1: Domestic productivity shock (order=1)



After a positive foreign shock, the business cycle dynamics become even more different under the two rules. The positive foreign shock induces an excess demand for domestic goods by foreigners, causing an appreciation of the domestic currency when exchange rates are flexible. The appreciation would cause a decrease in output and a decrease in domestic inflation. A central bank that follows a Taylor rule lowers the interest rate. Under an exchange rate rule, however, the exchange rate is determined by the central bank. As a result, the business cycle dynamics are different from the ones under a Taylor rule. After the positive foreign shock, domestic inflation increases and the central bank announces a path of expected appreciation of the currency. CPI inflation increases because the announced appreciation does not compensate the increase in domestic inflation. The foreign interest rate decreases substantially and hence, from the UIP condition, and since the appreciation is small, the domestic interest rate follows the foreign interest rate. Output and consumption go up under the exchange rate rule and decrease under the Taylor rule, since in the latter case there is a large appreciation of the domestic currency as a result of the decrease of the nominal interest rate.

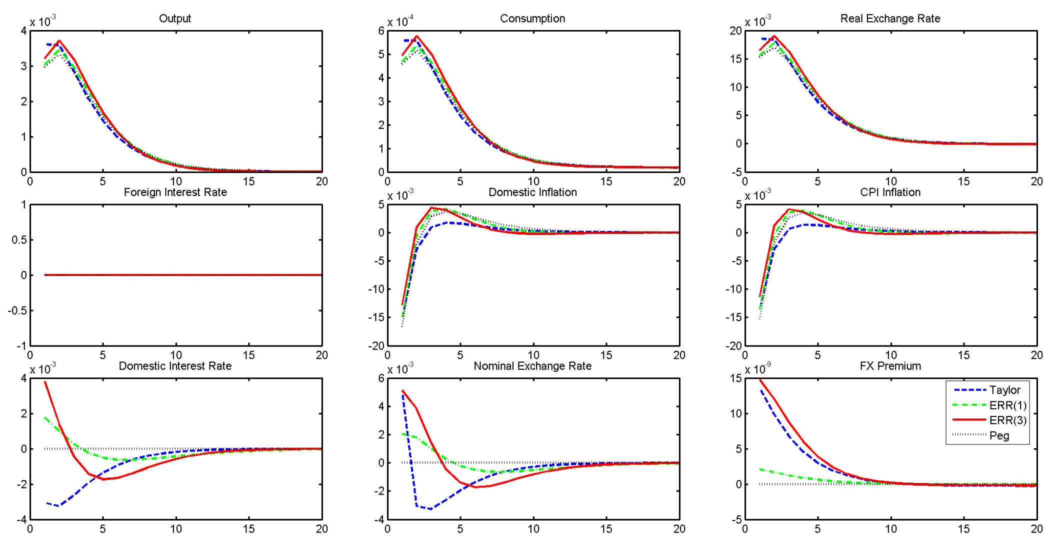
Figure 2: Foreign productivity shock (order=1)



Qualitative differences in business cycle dynamics between the two rules exist, especially when the small open economy experiences foreign shocks. The differences arise because with the exchange rate rule there is no overshooting, and this helps stabilize the variables that are affected by exchange rate fluctuation without increasing the volatility of domestic variables such as output and consumption. Domestic inflation however is more volatile under the exchange rate rule.

Differences are amplified at a third order approximation, owing to the existence of a time-varying risk premium. When there is a time-varying risk premium, a new channel is introduced (a precautionary saving channel) that may cancel or reinforce the no-overshooting effect. After a positive domestic productivity shock, there is an excess demand for foreign goods, and the domestic currency depreciates. Furthermore, the precautionary savings of the foreign consumers increase relative to that of the domestic investors and so the risk premium increases, both under the exchange rate rule and the Taylor rule. However, because the monetary authority is managing the exchange rate, the fluctuations of the risk premium are lower, and hence the nominal interest rate increases less than the decrease under the Taylor rule. This amplifies differences in economic variables under the two rules. After a positive foreign demand shock, the precautionary savings of the foreign investors decreases and the risk premium goes down. Again, the decrease is smaller under the exchange rate rule, which induces a lower decrease in the interest rate than the increase experienced under the Taylor rule. Hence, the presence of an endogenous risk premium amplifies business cycle fluctuations due to the effect of shocks in the precautionary savings of the consumers. Note that if the central bank reacts very intensively to fluctuations of inflation (i.e. higher ϕ_m^e) business cycle dynamics look, qualitatively, more similar to the Taylor rule. The reason is that, in this case, the central bank would like a larger depreciation of the domestic currency, which generates larger fluctuations in the nominal exchange rate and potentially overshooting.

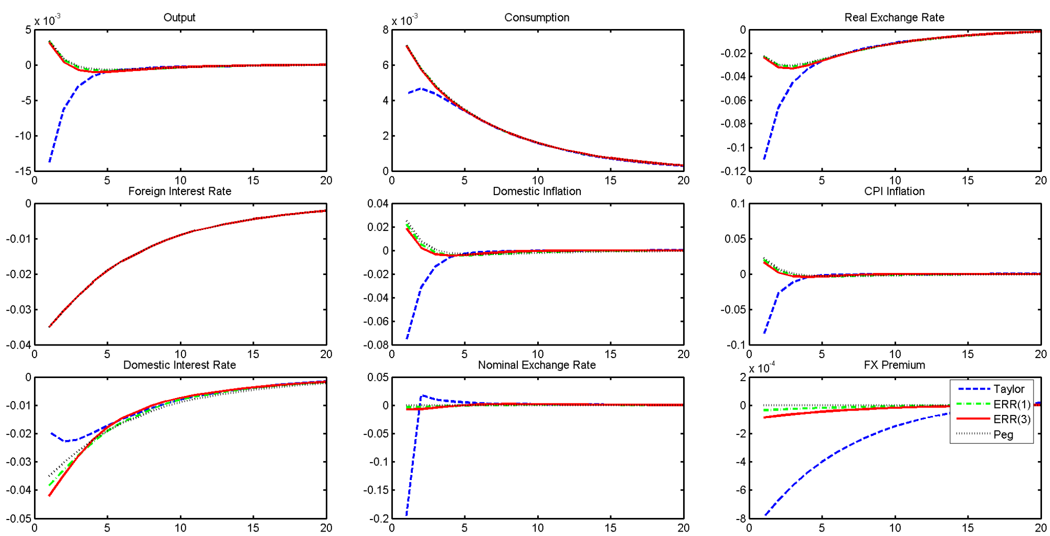
Figure 3: Domestic productivity shock (order=3)



After a positive foreign shock, foreigners experience a decrease in their risk aversion, and the risk premium for holding domestic currency decreases. In this case, the foreign interest rate decreases less than when UIP holds, since the precautionary savings motive by which foreigners save less would push the interest rate up. Different from the domestic productivity shock, in which the differences between the Taylor rule and exchange rate rules on business cycle dynamics depend on the value of the policy parameter, ϕ_m^e, after a foreign shock the exchange rate rule generates business cycle dynamics that are qualitatively very different from the interest rate rule, regardless of the intensity at which the central bank adjusts the exchange rate to react to inflation.

Our qualitative results show that the two monetary policy rules generate different business cycle dynamics. These differences are amplified when UIP does not hold due to a time-varying risk premium. The exchange rate rules seems to generate lower fluctuations than the Taylor rule, especially when the economy is hit by foreign shocks. This suggests that small open economies that are susceptible to shocks in the rest of the world may benefit from rules that use the exchange rate as their instrument to stabilize the economy.

Figure 4: Foreign productivity shock (order=3)



3.3 Moments

In this section, we report moments for the main economic variables under the two monetary policy rules. We do that for a first and third order approximation of the model, so that we can isolate the role of the risk premium in accounting for quantitative differences on business cycle dynamics.

Table 3 reports moments of key variables for a first order approximation of the model. In this case, differences on volatility of economic variables between the two rules are driven entirely by differences on how they are implemented. The exchange rate rule generates lower economic fluctuations in terms of output, and the real and nominal exchange rates, at the cost of increasing fluctuations in the nominal interest rate and consumption. As the impulse response functions showed, the central bank smooths fluctuations of the exchange rate with the objective of smoothing fluctuations of inflation. Inflation is much less volatile under the exchange rate rule, regardless of the value of the policy parameter. An exchange rate rule seems to be more successful at reducing economic fluctuations after shocks. And the impulse response analysis showed that this is especially the case when the small open economy experiences foreign shocks. Note that at a first order approximation, there is no risk premium, as the bottom panel of table 3 shows.

Table 3: Moments, order 1

	Taylor	ERR		Peg
		$\phi_\pi = 1$	$\phi_\pi = 3$	
$\sigma_{\Delta Y}$	1.89%	0.48%	0.51%	0.46%
$\sigma_{\Delta C}$	0.42%	0.69%	0.69%	0.70%
$\sigma_{\Delta Q}$	13.73%	4.53%	4.73%	4.46%
σ_R	4.72%	7.86%	8.11%	7.40%
σ_{R^*}	7.40%	7.40%	7.40%	7.40%
$\sigma_{\Delta E}$	21.02%	1.02%	2.13%	0.00%
σ_{π_H}	7.75%	4.29%	3.61%	4.85%
σ_π	8.61%	3.91%	3.22%	4.46%
σ_{fxp}	0.00%	0.00%	0.00%	0.00%
σ_{R-R^*}	2.89%	0.88%	1.68%	0.00%
$\sigma_{\Delta E^e}$	2.89%	0.88%	1.68%	0.00%
$\sigma_{R-R^*,\Delta E^e}$	2.89%	0.88%	1.68%	0.00%
$\sigma_{fxp,\Delta Y}$	0.00%	0.00%	0.00%	0.00%
$\sigma_{fxp,\Delta E}$	0.00%	0.00%	0.00%	0.00%
$\sigma_{fxp,\Delta E^e}$	0.00%	0.00%	0.00%	0.00%
$\hat{\beta}_{uip}$	0.00	0.00	0.00	0.00
$\sigma_{\hat{\beta}_{uip}}$	0.00	0.00	0.00	0.00

To account for the effect of the risk premium on business cycle dynamics, table 4 reports moments for a third order approximation of the model. In this case, the different performance between the two rules will be driven by the existence of a time-varying risk premium. At a third order approximation, the exchange rate rule is even more successful at reducing fluctuations of inflation. Note that, under the exchange rate rule, the risk premium becomes less volatile than under the Taylor rule. Therefore, the different effect that the two rules have on the risk premium is contributing to the different business cycle dynamics that arise quantitatively between the two rules. The volatility of the risk premium can be explained by the volatility of the interest rate differential, the volatility of the exchange rate depreciation and the covariance between the previous two variables. Table 4 how these three moments contribute to generating a lower volatility of the risk premium when the central bank uses the nominal exchange rate as its instrument. This result is even more evident under a third order approximation of the full model, or when the persistence of the shocks in high (see Appendix B).

A lower volatility of the risk premium implies that shocks that hit the economy will be stabilized more easily as consumers react to changes in expectations more smoothly, by experiencing lower increases or decreases in the risk premium. That is the case under an exchange rate rule.

Note that a fixed exchange rate regime (peg), generates larger fluctuations of inflation than the exchange rate rule. this result is even more evident when we take a full third order approximation of the model, as table 11 shows.

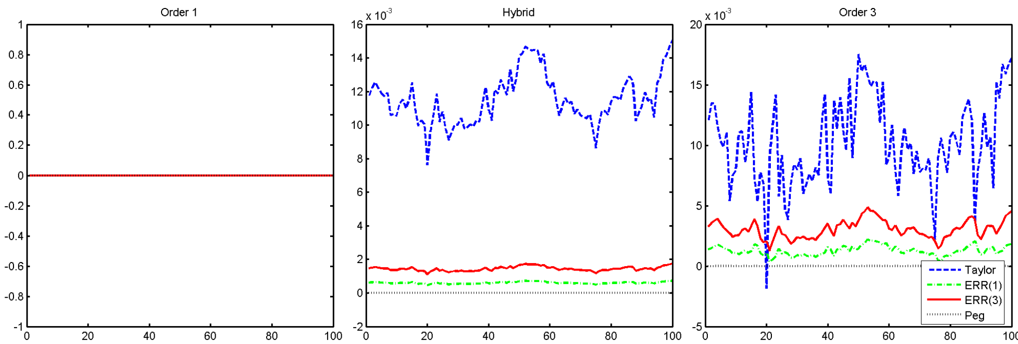
Table 4: Moments, order 3

	Taylor	ERR		Peg
		$\phi_\pi = 1$	$\phi_\pi = 3$	
$\sigma_{\Delta Y}$	1.62%	0.60%	0.61%	0.59%
$\sigma_{\Delta C}$	0.48%	0.76%	0.76%	0.76%
$\sigma_{\Delta Q}$	12.19%	2.58%	2.71%	2.53%
σ_R	5.66%	7.40%	7.55%	7.15%
σ_{R^*}	7.15%	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	20.26%	0.58%	1.22%	0.00%
σ_{π_H}	9.26%	2.45%	2.07%	2.75%
σ_π	9.94%	2.23%	1.85%	2.53%
σ_{fxp}	0.15%	0.01%	0.01%	0.00%
σ_{R-R^*}	1.98%	0.50%	0.98%	0.00%
$\sigma_{\Delta E^e}$	2.10%	0.50%	0.97%	0.00%
$\sigma_{R-R^*,\Delta E^e}$	2.04%	0.50%	0.98%	0.00%
$\sigma_{fxp,\Delta Y}$	0.00025%	0.00000%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E}$	-0.00293%	0.00001%	0.00005%	0.00000%
$\sigma_{fxp,\Delta E^e}$	-0.00245%	0.00001%	0.00005%	0.00000%
$\hat{\beta}_{uip}$	1.0839	0.9891	0.9851	0.0000
$\sigma_{\hat{\beta}_{uip}}$	0.1016	0.0060	0.0076	0.0000

Finally, we compute the simulated risk premium for our small open economy. We find that, consistent with our previous results, the risk premium is lower and less volatile under an exchange rate rule than under an interest rate rule (see figure 12).[5] These differences are key to understand the different business cycle dynamics generated by the two alternative rules, and they are driven by a lower volatility of the exchange rate and a higher covariance of the foreign exchange premium and the expected exchange rate depreciation.

[5]In figure 12, hybrid corresponds to a model in which the demand and supply conditions are linearized, but those equations that contain the risk premium are non-linear. Order 3 corresponds to the full third order approximation of the model and we discuss further about this case in Appendix B.

Figure 5: Risk premium



An interesting point to note is that as the central bank reacts more intensively to fluctuations of inflation under an exchange rate rule (i.e., as ϕ_m^e is higher), the business cycle dynamics under the two rules become more similar. In this case, there is some overshooting of the nominal exchange rate under the exchange rate rule. The business cycle properties of the risk premium in this case looks closer to Taylor. Therefore, the way in which implementation of the rule in has an effect on overshooting is key to determine different business cycle dynamics under the two rules. We shed more light about this point in the next section.

3.4 Equivalence of the monetary rules

In our numerical exercise, we have studied differences on business cycle dynamics of two alternative monetary policy rules, assuming that the central bank reacts to the same economic variables but with a different intensity (as shown by differences in the policy parameters). That is, we have assumed that under both rules the central bank reacts to smooth only fluctuations of inflation, with a certain degree of smoothing in the respective instruments. We then showed that, regardless of the value of the policy parameters, the exchange rate rule outperforms the Taylor rule by smoothing economic fluctuations. There are two reasons for this: (i) the way in which the exchange rate rule is implemented, and (ii) the effect that the particular implementation of the rule has on the risk premium. An interesting question to study is whether there exists an interest rate rule that is able to mimic the business cycle dynamics of an exchange rate rule. In this case, the two rules would be implemented in the same way, but the instruments used for monetary policy would be different.

We start by assuming an exchange rate rule in which the central bank reacts only to fluctuations of inflation. Following our previous analysis, we log-linearize the monetary policy rules around the steady state. That is,

$$\Delta e_t = -\phi_m^e \pi_t \tag{26}$$

Our goal is to study whether there exists an interest rate rule that delivers identical business cycle fluctuations. We do that by using the main equilibrium conditions of our model.

Let's start by writing

$$i_t = \phi_m^e \pi_t + \Delta e_t + i_t \tag{27}$$

From the definition of the real exchange rate

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^* \tag{28}$$

In the case in which the UIP condition holds, we can just work with the log-linearized version of equation 10

$$\Delta q_t = \sigma \Delta \tilde{c}_t - \sigma \Delta \tilde{c}_t^* \tag{29}$$

with $\tilde{C}_t = C_t - hX_t$.

Combining 27 and 28, we have

$$i_t = \phi_m^e \pi_t + \Delta q_t + \pi_t - \pi_t^* + i_t \tag{30}$$

Plugging equation 29

$$i_t = \phi_m^e \pi_t + \sigma \Delta \tilde{c}_t - \sigma \Delta \tilde{c}_t^* + \pi_t - \pi_t^* + i_t \tag{31}$$

Finally, using the UIP condition $i_t = \Delta e_{t+1} + i_t^*$ and equation 29 one period ahead, we have

$$i_t = \phi_m^e \pi_t + \sigma \Delta \tilde{c}_t - \sigma \Delta \tilde{c}_t^* + \pi_t - \pi_t^* + \sigma E_t \Delta \tilde{c}_{t+1} - \sigma E_t \Delta \tilde{c}_{t+1}^* + E_t \pi_{t+1} - E_t \pi_{t+1}^* + i_t^* \tag{32}$$

Rearranging,

$$i_t = (1 + \phi_m^e)\pi_t + \sigma\Delta\tilde{c}_t + \sigma E_t\Delta\tilde{c}_{t+1} + E_t\pi_{t+1} + u_t^* \tag{33}$$

with $u_t^* = -E_t\pi_{t+1}^* + i_t^* - \sigma\Delta\tilde{c}_t^* - \pi_t^* - \sigma E_t\Delta\tilde{c}_{t+1}^*$.

Therefore an interest rate rule that mimics the business cycle dynamics of the exchange rate rule 26,is one in which the monetary authority changes the nominal interest rate reacting to changes in current and future inflation, current and future changes in consumption and foreign shocks (see equation 33). Note that this rule implies that the central bank can successfully react to foreign shocks to stabilize the economy.

We study the business cycle dynamics of the two equivalent rules. At a first order approximation, they should generate the same business cycle dynamics, since they are implemented in the same way. At a third order approximation, the equivalence is not as clear, since there is the presence of a risk premium that moves endogenously with changes in the shocks. To isolate the effect of the risk premium in driving differences between the interest rate rule and the exchange rate rule, we simulate our model under 26 and 33, but considering a third order approximation of the risk sharing condition.

We show these results by simulating the model from Section 3 under the interest rate rule in equation 33 and the exchange rate rule in equation 26. Differences at a third order approximation would be entirely driven by the existence of a time-varying risk premium. However, we find that these differences are very small, both for domestic and foreign shocks (see figures 6 and 7).

Figure 6: Domestic productivity shock for equivalent rules (order=3)

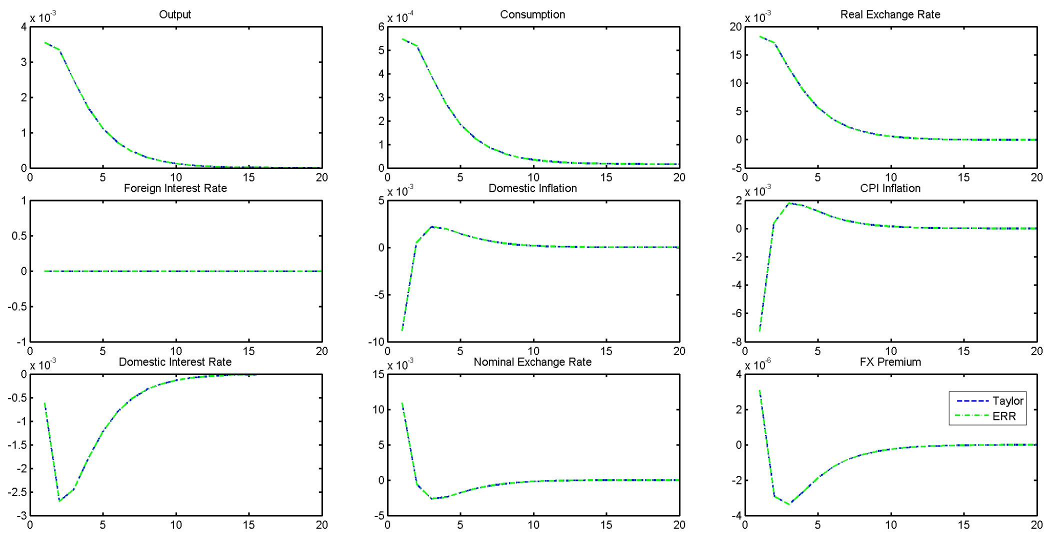
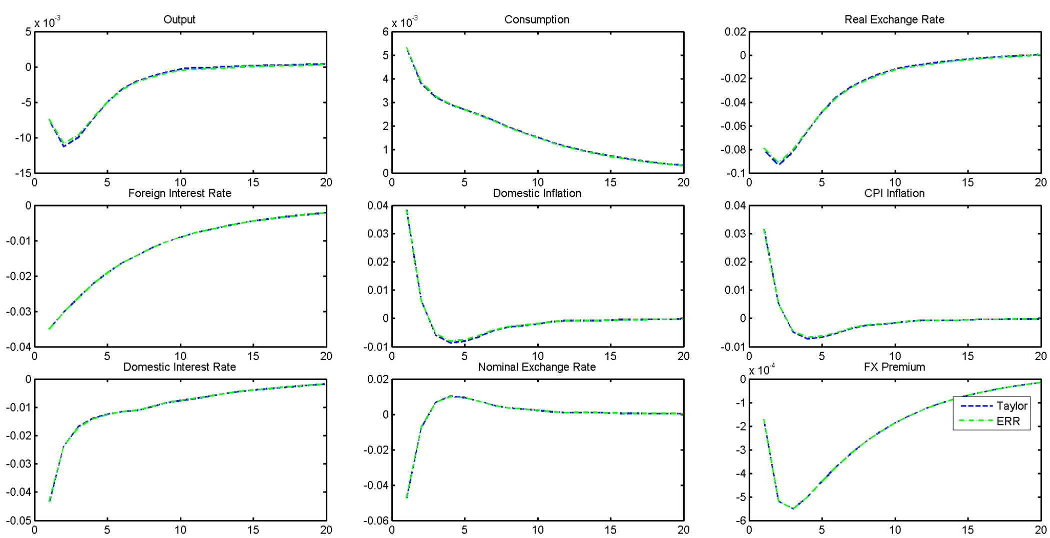


Figure 7: Foreign productivity shock for equivalent rules (order=3)



In Table 5, we report moments of key economic variables and the risk premium for a third order approximation of the model under our baseline calibration of the shocks, as well as for a larger persistence and volatility of the shocks. Consistent with the findings from our impulse response functions, the business cycle dynamics of the two rules are very similar, and only slight differences arise when the persistence or the volatility of the shocks are larger.

Table 5: Moments, order=3

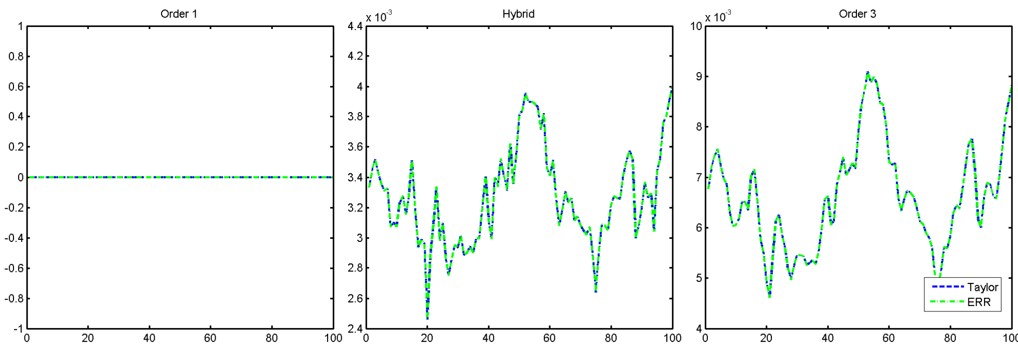
	Baseline		High persistence		High volatility	
	Taylor	ERR	Taylor	ERR	Taylor	ERR
$\sigma_{\Delta Y}$	0.83%	0.80%	1.03%	1.00%	3.91%	3.69%
$\sigma_{\Delta C}$	0.63%	0.63%	0.62%	0.62%	1.11%	1.13%
$\sigma_{\Delta Q}$	7.35%	7.20%	8.01%	7.82%	26.37%	25.17%
σ_{R}	6.66%	6.63%	1.84%	1.77%	12.09%	11.74%
σ_{R^*}	7.15%	7.15%	1.10%	1.10%	12.58%	12.58%
$\sigma_{\Delta E}$	4.40%	4.32%	4.80%	4.69%	15.76%	15.10%
σ_{π_H}	3.58%	3.51%	3.91%	3.81%	12.92%	12.26%
σ_{π}	2.95%	2.88%	3.22%	3.13%	10.62%	10.07%
σ_{fxp}	0.11%	0.11%	0.31%	0.31%	0.90%	0.90%
σ_{R-R^*}	1.64%	1.60%	1.56%	1.48%	6.93%	6.57%
$\sigma_{\Delta E^e}$	1.72%	1.68%	1.58%	1.52%	7.50%	7.16%
$\sigma_{R-R^*,\Delta E^e}$	1.68%	1.64%	1.55%	1.48%	7.19%	6.84%
$\sigma_{fxp,\Delta Y}$	-0.00005%	-0.00004%	-0.00027%	-0.00025%	-0.00518%	-0.00462%
$\sigma_{fxp,\Delta E}$	-0.00141%	-0.00141%	-0.00091%	-0.00125%	-0.04553%	-0.04527%
$\sigma_{fxp,\Delta E^e}$	-0.00139%	-0.00139%	-0.00079%	-0.00113%	-0.04480%	-0.04453%
$\hat{\beta}_{uip}$	1.0450	1.0477	0.9778	0.9938	1.0862	1.0955
$\sigma_{\hat{\beta}_{uip}}$	0.0246	0.0248	0.0292	0.0301	0.0200	0.0202

Our results in this section suggest that the main driving force in generating differences in business cycle dynamics between the two rules lies in the actual implementation of the rule. When we allow for the existence of a time-varying risk premium, the particular implementation of the rule interacts with the risk premium to generate larger differences. With the two equivalent rules considered in this section there is overshooting of the exchange rate and the effect on the time-varying risk premium is similar. This finding is corroborated in figure 8: The risk premium behaves identically under the two equivalent rules.

Our results imply that differences in economic fluctuations between the two rules arise at first order, and are amplified at third order, only when the rules are implemented differently, that is when the exchange rate rule involves a slow appreciation or depreciation of the currency, whereas the Taylor rule generates an overshooting of the exchange rate.

While the central bank technically can replicate any interest rate rule by moving the exchange rate today and announcing depreciation consistent with UIP, it is not the way that our rule operates.

Figure 8: Risk premium for equivalent rules



4 Risk Premium and UIP: Understanding the mechanism

In our model, quantitative differences between the performance of the Taylor rule and the exchange-rate-based rule are amplified substantially when we take into account dynamics of the uncovered interest parity condition. Here, we illustrate analytically the the mechanics of the two rules in generating fluctuations in the risk premium that drive the differences in business cycle dynamics. To ease the exposure of the analysis, we assume an endowment economy in which all variables are conditionally and jointly log-normal. In a production economy like ours, we will have additional dynamics, but the endowment economy allows us to show analytically how different the two rules perform differently in terms of the risk premium. We follow the methodology in Backus et al. (2010).

As above, the UIP condition can be written as:

$$E_t \left\{ \mathcal{M}_{t,t+1}(R_t - R_t^*(\mathcal{E}_{t+1}/\mathcal{E}_t)) \right\} = 0$$

Most models approximate the dynamics of this equation by using log-linearization, which yields the familiar expression implying that the expected rate of depreciation is equal to the interest rate differential between the home and the foreign economies:

$$r_t - r_t^* = E_t \Delta e_{t+1}$$

Under conditional log-normality, the UIP condition has a different form:

$$r_t - r_t^* = E_t \Delta e_{t+1} + \frac{1}{2} var_t(\Delta e_{t+1}) + cov_t(m_{t+1}, \Delta e_{t+1})$$

$$fxp_t = \frac{1}{2} var_t(\Delta e_{t+1}) + cov_t(m_{t+1}, \Delta e_{t+1}) = \frac{1}{2} var_t(m_{t+1}^*) - \frac{1}{2} var_t(m_{t+1}) \qquad (34)$$

The additional two terms in the UIP condition capture risk premium (possibly, time-varying), which we denote by RP_t.[6] Verdelhan (2010) and De Paoli & Sondergaard (2009) use this setup to explain the forward premium puzzle documented by Fama (1984). The introduction of second-order terms is not sufficient in itself to explain the empirical puzzle related to the interest parity condition. Therefore, Verdelhan (2010) and De Paoli & Sondergaard (2009) introduce habit formation in their models. In order to solve the UIP puzzle, these papers generate risk premium that co-varies negatively with the expected rate of depreciation. This negative correlation arises because models with habit formation generate time-varying coefficient of relative risk aversion. Under the assumptions of the model, the coefficient is given by:

$$CRRA_t = \sigma \frac{C_t}{C_t - hX_t}$$

To see the mechanics, let's consider a negative domestic TFP shock. Since consumption declines, the standard intertemporal substitution effect will make domestic consumers re-optimize by shifting future consumption to the present, which will cause a decrease in saving and hence an increase in interest rates. At the same time, the drop in domestic production generates excess demand for domestic goods and appreciation of the currency. As the economy is expected to return to steady state, this appreciation must be followed by expected depreciation. This is the standard UIP

[6] The second equality in the definition of RP_t comes from the international risk sharing assumption.

condition. In a model with habits, there is a second effect, however. As shown in the expression for $CRRA_t$, when consumption declines and gets closer to the level of habit, risk aversion increases. This generates a precautionary savings motif – consumers are not as willing to shift as much consumption from the future to today, which alleviates the pressure on the interest rate. If the precautionary savings channel is very strong, the economy may even get to a lower interest rate, i.e. expected currency depreciation will co-exist with lower interest rate, as documented in the empirical literature on the forward premium puzzle. We find that when the monetary authority follows an exchange rate rule, the precautionary savings motif is weaker, and the deviations from UIP are more muted. That is why the volatility of the risk premium is lower when the instrument of monetary policy is the exchange rate.

We now derive an analytical expression for the foreign exchange risk premium as a function of the parameters of an exchange rate monetary policy rule. We follow Backus et al. (2010), but depart from them by using a utility function with habits in consumption. Once we have an expression of the domestic nominal pricing kernel, we follow De Paoli & Sondergaard (2009) to derive the risk premium.

The key is to build a model that endogenously determines inflation. In the basic setup Backus et al. (2010) use two equations for two variables (i_t, π_t):

1. Nominal interest rate as a function of the log-linear pricing kernel (which depends on inflation)

2. Taylor rule determining nominal interest rate as a function of inflation.

We solve for the case in which the monetary authority reacts both to inflation and consumption growth, that is:

$$de_{t+1} = \phi_\pi \pi_{t+1} + \phi_c E_t(c_{t+1} - c_t) \tag{35}$$

and

$$i_t = -log(\beta) + \phi_\pi E_t \pi_{t+1} + \phi_c E_t(c_{t+1} - c_t) \tag{36}$$

After deriving a solution to inflation, one can express the nominal pricing kernel as a function of exogenous variables. With this, one can derive the foreign exchange risk premium (see Appendix C). We obtain the following analytical solution for the foreign excess return under the two rules:

1. ERR

$$fxp_t = \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma^2_{\epsilon^*} - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma^2_\epsilon}{(1-h)^2}\right] - \frac{b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma^2_\epsilon[(1-\phi-\rho_c)c_t+\phi x_t]$$

2. Taylor rule

$$fxp_t = \frac{1}{2}\sigma_{\epsilon^*}b^*_\epsilon - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right)^2 \sigma^2_\epsilon + (\psi_{\epsilon^*} + \psi_{c^*})^2\sigma^2_{\epsilon^*}\right] \quad (37)$$

$$-\sigma^2_\epsilon\frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right][(1-\phi-\rho_c)c_t+\phi x_t] \quad (38)$$

We observe that the risk premium has a different analytical expression, depending on the instrument of monetary policy. In particular, it depends on the volatility of the shock, the habit parameter, the persistence of the shock and the parameters of the monetary policy rules.

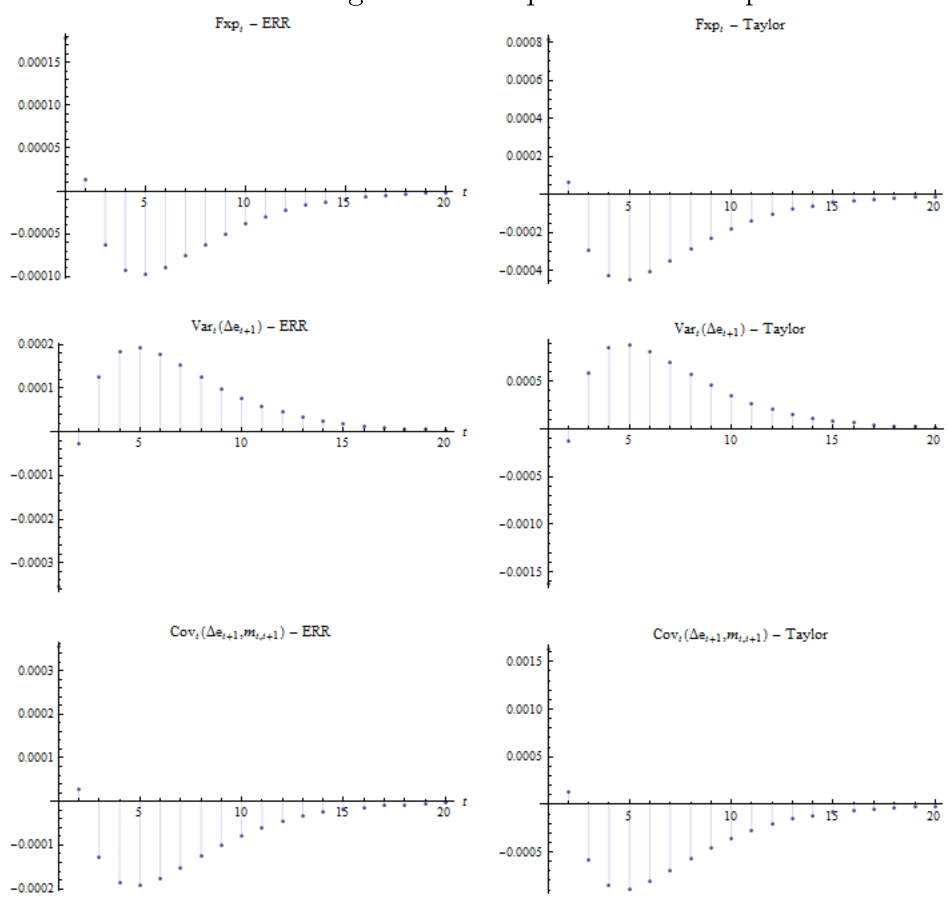
We perform a simple numerical analysis in which we plot the risk premium under the two rules, by setting values to the parameters in equations 37 and 38. In the top panel of figure 9, we plot the risk premium under the two rules. For simplicity, we assume a monetary rule in which the central bank reacts to fluctuations of inflation alone, that is we set $\phi_c = 0$ is equations 35 and 36. The risk premium is lower and less volatile under the exchange rate rule.

To get a better understanding of what drives differences in risk premium between the two rules, from equation 34, we decompose the risk premium into the conditional variance of the expected depreciation, $var_t(\Delta e_{t+1})$ rate and the covariance between the stochastic discount factor and the expected rate of depreciation of the currency, $cov_t(m_{t+1}, \Delta e_{t+1})$. In the middle and bottom panels of figure 9 we observe that both components are lower under the exchange rate rule, what explains the lower risk premium.

5 Conclusion

The use of the exchange rate as an instrument of monetary policy was pioneered by the Monetary Authority of Singapore and has supported the economic growth of the country by ensuring low inflation since its implementation in 1981. Certainly, credit

Figure 9: Risk premium decomposition



for this achievement also goes to other dimensions of economic policy. Nevertheless, this article has shown how a theoretical model based on optimizing behavior of households and producers is able to generate powerful conclusions about the desirability of the exchange rate as an instrument of monetary policy. More generally, we show that in a standard microfounded monetary model, relatively open economies can generate an improvement in social welfare by adopting an exchange rate rule. The improvement comes from a reduction in the volatility of key macroeconomic variables such as inflation and output, especially when households exhibit habit formation behavior.

The model reveals that there are two key sources of the reduction in volatility. First, in implementing the exchange rate rule, the central bank announces gradual depreciation rate, which avoids the standard overshooting result. For small open economies, where a large part of the price level is determined by prices of imported goods this policy already reduces the volatility of exchange rates and thus prices of imports. To identify the second channel, we follow recent advances in international monetary economics, which build into standard models counter-cyclical risk premia derived endogenously from habits in consumption. The time-varying risk premia drive a wedge between exchange rate movements and the interest rate differential thus further separating the implied dynamics of interest rate rules from the dynamics implied by adopting an exchange rate rule. We show that for open economies the time series properties of the risk premium differ considerably between the rules.

We have kept the model to a bare minimum in terms of its economic structure in order to identify the key factors behind the observed differences between the two rules. There are many directions in which the model can be extended to gain further insights in the desirability of exchange rate rules. First and foremost, it will be important to include a financial sector in the model possibly as in Bernanke, Gertler & Gilchrist (1999) or as recently built into a DSGE model by Christiano, Motto & Rostagno (2014). Second, our model does not distinguish between tradable and non-tradable goods. At the same time secular changes in relative prices due to convergence in income per capita, for example, might present interesting problems for the exchange rate rule. Finally, in the past few years, standard monetary policy rules have been put to test by the well-known zero lower bound on nominal interest rates. This minimum bound created a problem for economies that use the interest rate as an instrument of monetary policy, forcing them to switch to quantitative easing once interest rates reached zero. For an economy operating with an exchange rate rule, the challenge is slightly different. We can illustrate some of the issues with a simple example: Suppose the anchor currency is the US dollar and interest rates in the US are zero. Suppose that in the domestic economy, inflation increases and the central bank is forced to let the currency appreciate. If the currency is expected to appreciate slowly over time, the interest rate in the domestic economy must fall below the foreign rate (i.e. below zero) in order to preclude arbitrage opportunities. Because interest rates cannot go below zero (at least not for a sustained period of time), arbitrage becomes possible. But arbitrage also implies that capital inflows increase, which in turn generate an increase in liquidity and higher inflationary pressures. These pressures call for a stronger appreciation, thereby increasing the attractiveness of investing in the domestic currency and ultimately creating explosive dynamics. These three extensions are not only interesting from modeling point of view, but they are clearly relevant for the actual implementation of the exchange rate rule in small open economies. We leave this analysis for future research.

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Online Appendix

A Equations and steady state

A.1 Model equations

We describe the equilibrium in this model. The equilibrium determines the following variables:

$\left\{C_t, X_t, C_{Ht}, C_{Ft}, A_t, W_t, \frac{P_{Ht}}{P_t}, \frac{P_{Ft}}{P_t}, \Pi_t, N_t, R_t, S_t, Q_t, C_t^*, e_t, Y_t, N_t, MC_t, Y_t^*, \Pi_t^*, R_t^*\right\}$

The equations that determine the equilibrium are:

Households

$$R_t E_t \left\{ \beta \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} = 1$$

$$X_t = \delta X_{t-1} + (1-\delta) C_{t-1}$$

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t$$

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

$$(C_t - hX_t)^{\sigma} N_t^{\gamma} = \frac{W_t}{P_t}$$

Firms

$$Y_t = A_t N_t$$

$$a_t = \rho a_{t-1} + u_t$$

$$W_t = MC_t P_{Ht} A_t$$

Price setting

$$\frac{\tilde{P}_{H,t}}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{H_t}{F_t}$$

$$F_t = \Lambda_t Y_t + \theta\beta E_t(F_{t+1}(\Pi_{t+1})^{\varepsilon-1})$$

$$H_t = \Lambda_t MC_t Y_t + \beta\theta E_t(H_{t+1}(\Pi_{t+1})^{\varepsilon})$$

$$\Lambda_t = (C_t - hX_t)^{-\sigma}$$

$$\frac{P_{Ht}}{P_t} = \left((1-\theta)\left(\frac{\tilde{P}_{H,t}}{P_t}\right)^{1-\varepsilon} + \theta \left(\frac{P_{Ht-1}}{P_{t-1}}\right)^{1-\varepsilon} \Pi_t^{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}}$$

Market clearing condition

$$Y_t = C_t \left[(1-\alpha) S_t^{\eta} Q_t^{-\eta} + \alpha Q_t^{-\frac{1}{\sigma}}\right]$$

Monetary policy rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho} \left(\frac{Y_t}{\bar{Y}}\right)^{(1-\rho)\phi_y} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{(1-\rho)\phi_m}$$

$$\frac{e_{t+1}}{e_t} = \left(\frac{e^*_{t+1}}{e^*_t}\right)^{(1-\rho_e)} \left(\frac{e_t}{e_{t-1}}\right)^{\rho_e}$$

Terms of trade and real exchange rate

$$S_t = \frac{P_{Ft}}{P_{Ht}}$$

$$Q_t = \frac{P_{Ft}}{P_t}$$

$$P_{Ft} = \mathcal{E}_t P_t^*$$

Uncovered interest parity

$$E_t \left\{ \mathcal{M}_{t,t+1} \left(R_t - R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right\} = 0$$

International risk sharing condition

$$\beta \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = \mathcal{M}_{t,t+1}$$

Foreign economy

$$C_t^* = Y_t^*$$

$$\left(\frac{Y_t^*}{\bar{Y}} \right) = \left(\frac{Y_{t-1}^*}{\bar{Y}} \right)^{\rho_y} exp(u_t^y)$$

$$R_t^* E_t \left\{ \beta \left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \right\} = 1$$

A.2 The Steady State

We solve for the symmetric steady state. Here are the equations.

- $\tilde{C} = (1-h)C$
- $X = C$
- $\beta R = 1$
- $P_H = P_F = P$
- $C_H = (1-\alpha)C$

- $C_F = \alpha C$
- $Y = C$
- $\frac{W}{P} = \omega = \frac{\varepsilon - 1}{\varepsilon}$
- $Y = N = (1-h)^{\frac{-\sigma}{\sigma+\psi}} \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{\sigma+\psi}}$
- $Q = 1$
- $MC = \frac{\varepsilon-1}{\varepsilon}$
- $F = \frac{\Lambda Y}{1-\theta\beta}$
- $H = FMC$
- $\Lambda = C^{-\sigma}(1-h)^{-\sigma}$
- $\frac{\tilde{P}_H}{P_t} = 1$

B Robustness

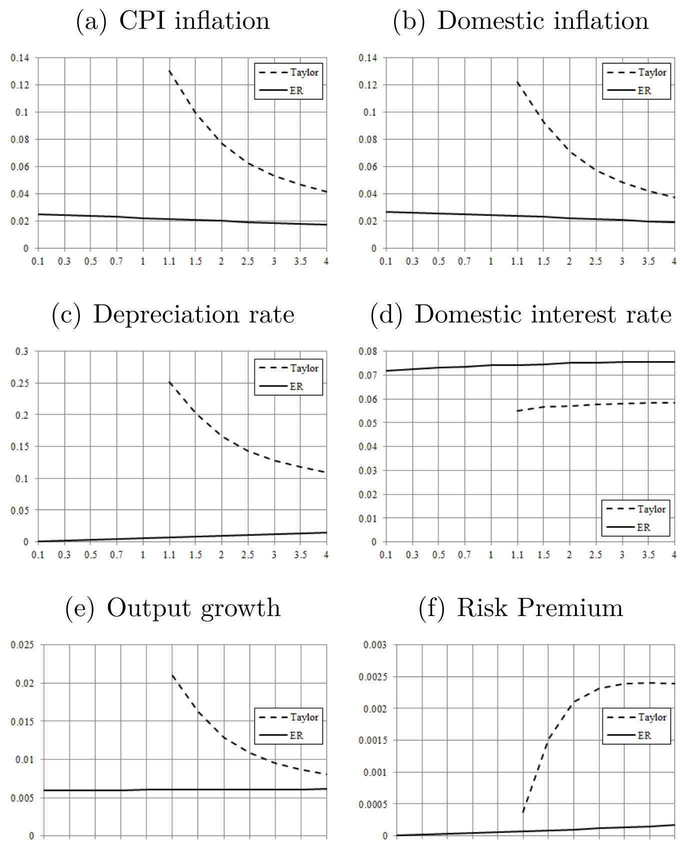
B.1 Comparing the two rules under a range of alternative policy parameters

In our numerical analysis, we have compared the performance of the two monetary policy rules for an arbitrary value of the policy parameters. Our findings suggest that, the exchange rate rule generates lower business cycle fluctuations than the interest rate rule. In this section, we compare the volatility of key economic variables using a wider range of policy parameters, to check whether exists any combination of parameters for which the two rules deliver identical business cycle fluctuations. We simulate the hybrid model by varying the parameter values of the two rules. In particular, we set $\phi_m(1.1, 3)$ and $\phi_m^e \in (0, 3)$. The range of the parameter values are chosen as follows: (i) To avoid indeterminacy in the Taylor rule , ϕ_m must be larger than 1. There is not a restriction in the lower bound of ϕ_m^e (ii) We set the upper limit following Schmitt-Grohé & Uribe (2007) who, when computing optimal policy, restrict the value of the reaction to inflation to be lower than 3. For each parameter value, we compute the volatility of inflation, output growth, the nominal interest rate, the exchange rate depreciation, domestic and CPI inflation, and the risk premium.

The results are shown in figure 10. For every combination of parameter values that we consider, we find that the exchange rate rule (solid line) outperforms the Taylor rule (dashed line), in terms of reducing the volatility of key economic variables. One exception is the nominal interest rate, which is always more volatile under the exchange rate rule. This finding is consistent with those in table 4 The figure also shows that, as the intensity at which the central bank reacts to inflation increase, the volatility of output growth, inflation and the exchange rate go down for both rules. However, the volatility of the risk premium increases.

Our results show that, when the central bank reacts to CPI inflation in its monetary policy rules, with a certain degree of smoothing of the instrument, the exchange rate rule generates lower fluctuations of economic variables than the Taylor rule.

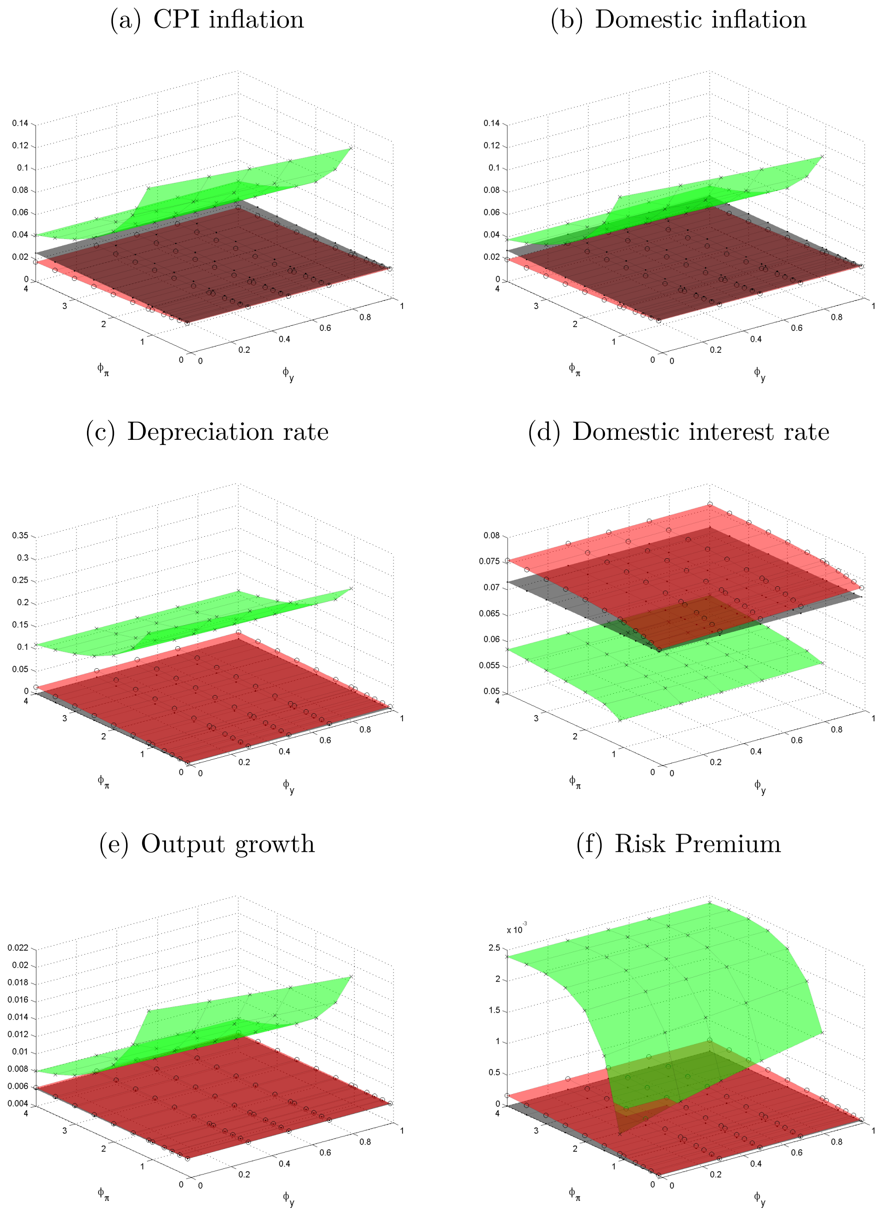
Figure 10: Comparing the two rules under alternative parameter values of ϕ_m



B.2 Augmented monetary rules

In this section, we compute the volatility of key economic variables and the risk premium when the central bank follows and augmented monetary policy rule in which it reacts to both inflation and deviations of output from its steady state, with a certain degree of smoothing of the instrument. We assume that the smoothing parameter for both rules is, as in our previous numerical analysis, $\rho = 0.85$. We then vary the parameters on inflation and deviations of output, and compute the corresponding volatility of key economic variables, for the Taylor rule, the exchange rate rule, and the case of the peg. Figure 11 displays the results. The exchange rate rule outperforms both the peg and the Taylor rule in reducing fluctuations of inflation and domestic inflation. As it was the case before, the exchange rate rule generates larger fluctuations in the nominal interest rate and lower risk premium volatility.

Figure 11: Augmented monetary rule (Standard deviations: (red (o) = ERR, green (x) = Taylor, black (.) = Peg))



B.3 Alternative values of the parameters

In this section, we simulate the hybrid model for alternative values of the degree of openness, the elasticity of substitution between intermediate goods, the labor supply elasticity and the smoothing parameter in the utility function, so that preferences

Table 6: Larger degree of openness($\alpha = 0.3$)

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	3.24%	0.59%	0.60%
$\sigma_{\Delta C}$	0.67%	0.79%	0.79%
$\sigma_{\Delta Q}$	5.27%	0.93%	0.95%
σ_R	7.05%	7.21%	7.23%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	18.64%	0.18%	0.38%
σ_{π_H}	12.79%	1.21%	1.07%
σ_π	14.38%	0.82%	0.68%
σ_{fxp}	0.18%	0.00%	0.00%
σ_{R-R^*}	0.70%	0.14%	0.26%
$\sigma_{\Delta E^e}$	0.73%	0.14%	0.26%
$\sigma_{R-R^*,\Delta E^e}$	0.71%	0.14%	0.26%
$\sigma_{fxp,\Delta Y}$	0.00047%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E}$	-0.00086%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E^e}$	-0.00037%	0.00000%	0.00000%
$\hat{\beta}_{uip}$	0.5323	0.9854	0.9804
$\sigma_{\hat{\beta}_{uip}}$	0.2654	0.0081	0.0105

become log-linear. The results are consistent with our previous findings. The most interesting result is that, as the degree of openness increases, the differences between the Taylor rule and the exchange rate rule become more striking. This suggests that as the economy is more exposed to foreign shocks, a monetary rule that uses the nominal exchange rate as its instrument will be more successful at reducing business cycle fluctuations.

In the case of log-utility, the differences between the exchange rate rule and the Taylor rule are less important.

Table 7: Higher elasticity of substitution ($\eta = 2$)

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	2.58%	0.59%	0.60%
$\sigma_{\Delta C}$	0.57%	0.77%	0.77%
$\sigma_{\Delta Q}$	9.16%	1.72%	1.79%
σ_R	6.41%	7.28%	7.34%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	19.34%	0.35%	0.75%
σ_{π_H}	11.37%	1.65%	1.40%
σ_π	11.90%	1.50%	1.26%
σ_{fxp}	0.17%	0.00%	0.01%
σ_{R-R^*}	1.28%	0.29%	0.56%
$\sigma_{\Delta E^e}$	1.38%	0.29%	0.55%
$\sigma_{R-R^*,\Delta E^e}$	1.33%	0.29%	0.55%
$\sigma_{fxp,\Delta Y}$	0.00039%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E}$	-0.00193%	0.00001%	0.00002%
$\sigma_{fxp,\Delta E^e}$	-0.00145%	0.00000%	0.00002%
$\hat{\beta}_{uip}$	1.0097	0.9870	0.9826
$\sigma_{\hat{\beta}_{uip}}$	0.1504	0.0071	0.0092

Table 8: Labor supply elasticity ($\psi = 3$)

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	2.69%	0.63%	0.65%
$\sigma_{\Delta C}$	0.30%	0.77%	0.77%
$\sigma_{\Delta Q}$	17.95%	2.12%	2.35%
σ_R	3.86%	7.52%	7.80%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	22.29%	0.62%	1.29%
σ_{π_H}	4.84%	1.90%	1.57%
σ_π	5.77%	1.72%	1.39%
σ_{fxp}	0.11%	0.00%	0.01%
σ_{R-R^*}	3.65%	0.59%	1.19%
$\sigma_{\Delta E^e}$	3.75%	0.59%	1.18%
$\sigma_{R-R^*,\Delta E^e}$	3.70%	0.59%	1.19%
$\sigma_{fxp,\Delta Y}$	0.00056%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E}$	-0.00417%	0.00002%	0.00007%
$\sigma_{fxp,\Delta E^e}$	-0.00375%	0.00002%	0.00007%
$\hat{\beta}_{uip}$	1.0939	0.9949	0.9924
$\sigma_{\hat{\beta}_{uip}}$	0.0600	0.0035	0.0044

Table 9: Log-utility ($\rho = 1$)

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	0.45%	0.64%	0.65%
$\sigma_{\Delta C}$	0.44%	0.68%	0.68%
$\sigma_{\Delta Q}$	3.23%	1.72%	1.79%
σ_R	0.93%	1.65%	1.76%
σ_{R^*}	1.50%	1.50%	1.50%
$\sigma_{\Delta E}$	4.49%	0.36%	0.76%
σ_{π_H}	1.88%	1.65%	1.40%
σ_π	1.99%	1.50%	1.26%
σ_{fxp}	0.00%	0.00%	0.00%
σ_{R-R^*}	0.78%	0.29%	0.57%
$\sigma_{\Delta E^e}$	0.79%	0.29%	0.57%
$\sigma_{R-R^*,\Delta E^e}$	0.79%	0.29%	0.57%
$\sigma_{fxp,\Delta Y}$	0.00000%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E}$	-0.00003%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E^e}$	-0.00003%	0.00000%	0.00000%
$\hat{\beta}_{uip}$	0.9758	0.9926	0.9881
$\sigma_{\hat{\beta}_{uip}}$	0.0562	0.0070	0.0089

B.4 The risk premium and the persistence of the shocks

In this section, we analyze the response of economic variables to domestic and foreign shocks under the two monetary policy rules, when the shocks are highly persistent. In particular, we set the persistence to 0.9977 as in De Paoli & Sondergaard (2009). They show that the effect of shocks on precautionary savings and hence the risk premium are amplified for high persistence shocks. The reason is that the precautionary saving motive and hence the volatility of the stochastic discount factor is larger the larger the persistence and volatility of the shocks.

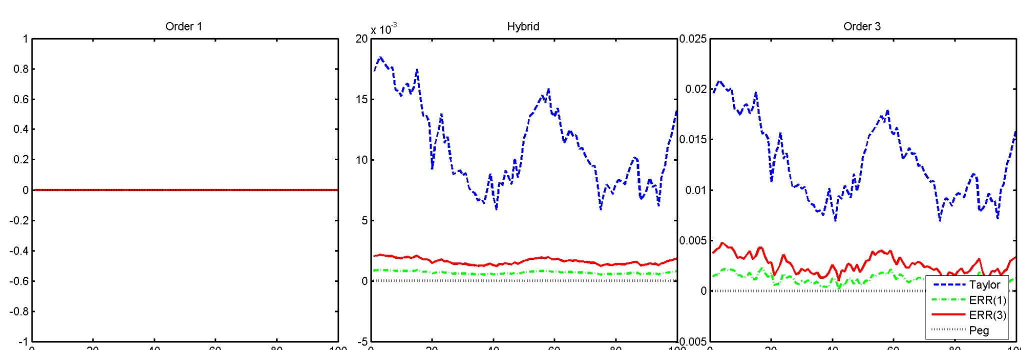
We can see that in the bottom of Table 10. The differences in the volatility of the risk premium between the two rules are larger. Furthermore, the coefficient that determines deviations from UIP, β_{UIP} is almost 1 for the exchange rate rule and 0.66 for the interest rate rule. This suggests that, under an exchange rate rule, deviations from UIP are weaker than under a Taylor rule.

Table 10: Moments, order=3, high persistence

	Taylor	ERR		Peg
		$\phi_\pi = 1$	$\phi_\pi = 3$	
$\sigma_{\Delta Y}$	0.65%	0.61%	0.62%	0.61%
$\sigma_{\Delta C}$	0.68%	0.74%	0.74%	0.74%
$\sigma_{\Delta Q}$	5.35%	3.03%	3.11%	3.01%
σ_R	3.38%	1.32%	1.71%	1.10%
σ_{R^*}	1.10%	1.10%	1.10%	1.10%
$\sigma_{\Delta E}$	6.80%	0.84%	1.58%	0.00%
σ_{π_H}	2.62%	2.75%	2.23%	3.28%
σ_π	2.74%	2.49%	1.98%	3.01%
σ_{fxp}	0.71%	0.02%	0.06%	0.00%
σ_{R-R^*}	2.59%	0.77%	1.34%	0.00%
$\sigma_{\Delta E^e}$	2.05%	0.76%	1.33%	0.00%
$\sigma_{R-R^*,\Delta E^e}$	2.28%	0.77%	1.34%	0.00%
$\sigma_{fxp,\Delta Y}$	-0.00014%	0.00000%	-0.00001%	0.00000%
$\sigma_{fxp,\Delta E}$	0.00858%	0.00004%	0.00013%	0.00000%
$\sigma_{fxp,\Delta E^e}$	0.00989%	0.00004%	0.00013%	0.00000%
$\hat{\beta}_{uip}$	0.6992	0.9962	0.9937	-0.0016
$\sigma_{\hat{\beta}_{uip}}$	0.0254	0.0045	0.0063	0.0000

Finally, note that the risk premium is relatively much more volatile with the Taylor rule when the persistence of the shock is high, relative to the volatility of the risk premium with the exchange rate rule.

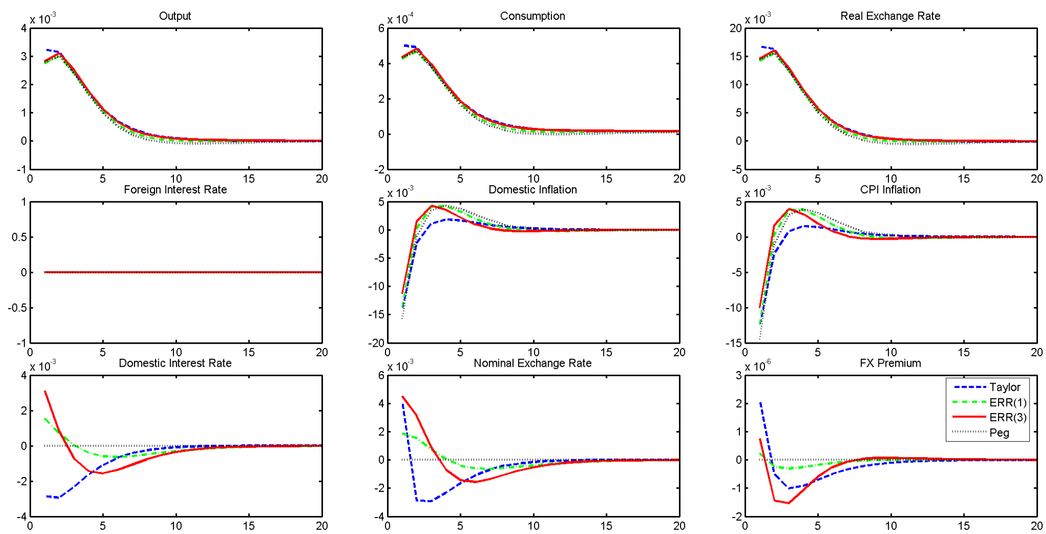
Figure 12: Risk premium



B.5 Third order approximation of the full model

In this section, we report impulse response functions and moments when we do a third order approximation of the full model. Recall that, so far, we have used a hybrid model, in which we linearize the supply and demand equations in order to isolate the role of the time-varying risk premium. The results are consistent with our previous findings. The two rules generate qualitatively different business cycle implications.

Figure 13: Domestic productivity shock (order=3)



Finally, as the results in Table 11 show, the exchange rate rule outperforms the Taylor rule in smoothing fluctuations of inflation.

Table 11: Moments, full order 3

	Taylor	ERR		Peg
		$\phi_\pi = 1$	$\phi_\pi = 3$	
$\sigma_{\Delta Y}$	12.22%	1.82%	1.14%	2.73%
$\sigma_{\Delta C}$	2.28%	0.61%	0.64%	0.59%
$\sigma_{\Delta Q}$	58.99%	11.94%	8.43%	16.67%
σ_R	27.88%	7.76%	7.98%	7.15%
σ_{R^*}	7.15%	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	25.27%	2.47%	3.57%	0.00%
σ_{π_H}	56.02%	11.46%	6.59%	18.12%
σ_π	51.34%	10.43%	5.90%	16.67%
σ_{fxp}	0.36%	0.04%	0.08%	0.00%
σ_{R-R^*}	22.21%	2.10%	2.78%	0.00%
$\sigma_{\Delta E^e}$	22.02%	2.12%	2.80%	0.00%
$\sigma_{R-R^*,\Delta E^e}$	22.11%	2.11%	2.79%	0.00%
$\sigma_{fxp,\Delta Y}$	-0.02272%	-0.00030%	-0.00010%	0.00000%
$\sigma_{fxp,\Delta E}$	0.04252%	-0.00036%	-0.00056%	0.00000%
$\sigma_{fxp,\Delta E^e}$	0.04175%	-0.00037%	-0.00057%	0.00000%
$\hat{\beta}_{uip}$	0.9893	1.0028	1.0022	0.0000
$\sigma_{\hat{\beta}_{uip}}$	0.0056	0.0061	0.0080	0.0000

C Deriving the risk premium under the two rules

We derive an analytical solution for the foreign exchange risk premium as a function of the parameters of the monetary rule. Following Backus et al. (2010), we show that the foreign exchange risk premium (therefore deviations from UIP) depend on the parameters of the monetary rule. We replicate their exercise using the ERR in addition to the Taylor rule. We follow Backus et al. (2010), but depart from them in that we use a utility function with external habits in consumption (instead of Epstein-Zin preferences). Once we have an expression of the domestic nominal pricing kernel, we follow and De Paoli & Sondergaard (2009) to derive the risk premium.

The key is to build a model that endogenously determines inflation. In the basic setup Backus et al. (2010) use two equations for two variables (i_t, π_t):

1. Nominal interest rate as a function of the log-linear pricing kernel (which depends on inflation)

2. Taylor rule determining nominal interest rate as a function of inflation.

After deriving a solution to inflation, one can express the nominal pricing kernel as a function of exogenous variables. With this, one can derive the foreign exchange risk premium.

Relative to the previous version, we start here first with the foreign economy.

Definitions

Backus et al. (2010) define the nominal interest differential $i_t - i_t^*$, the expected nominal depreciation $\mathbb{E}_t[de_{t+1}]$ and the exchange rate risk premium as fxp_t in terms of the domestic and foreign nominal pricing kernel, $M_{t,t+1}$ and $M^*_{t,t+1}$, respectively:

$$i_t - i_t^* = log\mathbb{E}_t[M^*_{t,t+1}] - log\mathbb{E}_t[M_{t,t+1}] \tag{39}$$

$$\mathbb{E}_t[de_{t+1}] = \mathbb{E}_t[logM^*_{t,t+1}] - \mathbb{E}_t[logM_{t,t+1}] \tag{40}$$

$$fxp_t = \frac{1}{2}\left[Var_t[logM^*_{t,t+1}] - Var_t[logM_{t,t+1}]\right] \tag{41}$$

Note that this follows from assuming *log-normality* of the nominal pricing kernel. Under external habit formation the nominal pricing kernel is

$$M_{t,t+1} = \beta\left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t}\right)^{-\rho}\Pi_{t+1}^{-1} \tag{42}$$

Defining surplus consumption $S_t = \frac{C_t - hX_t}{C_t}$ this can be written solely as a product[7]

$$M_{t,t+1} = \beta\left(\frac{S_{t+1}C_{t+1}}{S_tC_t}\right)^{-\rho}\Pi_{t+1}^{-1} \tag{43}$$

As in Backus et al. (2010), De Paoli & Sondergaard (2009) and Verdelhan (2010) consumption is assumed to follow an AR(1) process.

$$logC_{t+1} = \lambda logC_t + \epsilon_{c,t+1} \tag{44}$$

[7]Note that this is convenient, because it allows linearizing the pricing kernel by taking logs without using approximations.

Foreign consumption also follows an AR(1) process.

$$logC^*_{t+1} = \lambda^* logC^*_t + \epsilon_{c^*,t+1} \tag{45}$$

Solving for inflation

In this section we solve for inflation under (1) an exchange rate rule and (2) a standard Taylor rule.

Exchange rate rule

- First notice that equation (39) follows from taking expectations of the key Lucas (1984) equation relating the depreciation rate to the ratio of the nominal pricing kernels: $\frac{e_{t+1}}{e_t} = \frac{M^*_{t,t+1}}{M_{t,t+1}}$. It also holds here without the expectation terms. In log-linear form it writes:

$$de_{t+1} = logM^*_{t,t+1} - logM_{t,t+1} \tag{46}$$

- Now we need to find an expression for the log of the two pricing kernels. We assume that foreign inflation is zero (flexible prices in the rest of the world).

$$\begin{aligned} logM^*_{t,t+1} &= log\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\rho}\right] = log\beta - \rho[log(C^*_{t+1}) - log(C^*_t)] && (47)\\ &= log\beta + \rho(1-\rho^*_c)c^*_t - \rho\epsilon_{c^*,t+1} && (48)\end{aligned}$$

The domestic pricing kernel writes:

$$\begin{aligned} logM_{t,t+1} &= log\left[\beta\left(\frac{S_{t+1}C_{t+1}}{S_tC_t}\right)^{-\rho}\Pi^{-1}_{t+1}\right] && (49)\\ &= log\beta - \rho[c_{t+1} + s_{t+1} - c_t - s_t] - \pi_{t+1} && (50)\\ &= log\beta + \rho(1-\rho_c c_t - \rho[\epsilon_{c,t+1} + s_{t+1} - s_t] - \pi_{t+1} && (51)\end{aligned}$$

Note that we did not use any approximation until this point.

- It follows that (46) can be written as

$$\begin{aligned} de_{t+1} &= log\beta + \rho(1-\rho_c^*)c_t^* - \rho\epsilon_{c^*,t+1} \\ &- [log\beta + \rho(1-\rho_c)c_t - \rho[\epsilon_{c,t+1} + s_{t+1} - s_t] - \pi_{t+1}] \\ &= \rho[(1-\rho_c^*)c_t^* - (1-\rho_c)c_t] - \rho[\epsilon_{c^*,t+1} - \epsilon_{c,t+1}] + \rho[s_{t+1} - s_t] + \pi_{t+1} \end{aligned} \quad (52)$$

- We allow for an exchange rate rule that targets inflation and consumption growth, to make it as close as possible to the actual rule. When assuming an exchange rate rule there will be two equations in two unknowns (de_t, π_{t+1}), i.e.

$$de_{t+1} = \rho[(1-\rho_c^*)c_t^* - (1-\rho_c)c_t] - \rho[\epsilon_{c^*,t+1} - \epsilon_{c,t+1}] + \rho[s_{t+1} - s_t] + \pi_{t+1} \quad (53)$$

$$de_{t+1} = \phi_\pi \pi_{t+1} + \phi_c E_t(c_{t+1} - c_t) \quad (54)$$

The second equation is the exchange rate monetary policy rule.

- As in Backus et al. (2010), we use the **Method of undetermined coefficients**. We do not plug in the ERR rule into equation (53), but (1) guess a linear solution for inflation and (2) compare the exchange rate rule coefficients with the coefficients from equation (53).

We guess that the solution of inflation has the form:

$$\pi_t = \psi_c c_t + \psi_c^* c_t^* + \psi_s(s_t - s_{t-1}) + \psi_\epsilon \epsilon_t + \psi_{\epsilon^*}\epsilon_t^* \quad (55)$$

For convenience I write this solution at $t+1$:

$$\pi_{t+1} = \psi_c \rho_c c_t + \psi_c^* \rho_c^* c_t^* + \psi_s(s_{t+1} - s_t) + (\psi_c + \psi_\epsilon)\epsilon_{c,t+1} + (\psi_c^* + \psi_{\epsilon^*})\epsilon_{c^*,t+1} \quad (56)$$

- Collecting terms, the system of equation becomes:

$$\begin{aligned} de_{t+1} = [\rho(1-\rho_c^*) + \psi_c^*\rho_c^*]c_t^* + [\psi_c\rho_c - \rho(1-\rho_c)]c_t + (\psi_c^* + \psi_e^* - \rho)\epsilon_{c^*,t+1} \\ +(\psi_c + \rho + \psi_e)\epsilon_{c,t+1} + (\rho + \psi_s)(s_{t+1} - s_t) \end{aligned} \quad (57)$$

and rearranging,

$$de_{t+1} = [\phi_\pi \psi_c \rho_c - (1-\rho_c)\phi_c]c_t + \rho_c^* \phi_\pi \psi_c^* c_t^* + \phi_\pi \psi_s (s_{t+1} - s_t) \\ +[\phi_\pi(\psi_c + \psi_\epsilon) + \phi_c]\epsilon_{c,t+1} + [\phi_\pi(\psi_c^* + \psi_{\epsilon^*})]\epsilon_{c^*,t+1} \tag{58}$$

- Comparing coefficients gives solutions to ψ_i and therefore for domestic inflation. c

$$\begin{aligned}
\rho(1-\rho_c^*) + \psi_c^* \rho_c^* &= \rho_c^*[\phi_\pi \psi_c^*] \quad \Longrightarrow \; \psi_c^* = \frac{-\rho(1-\rho_c^*)}{\rho_c^*(1-\phi_\pi)} \\
\psi_c \rho_c - \rho(1-\rho_c) &= [\rho_c \phi_\pi \psi_c - (1-\rho_c)\phi_c] \quad \Longrightarrow \; \psi_c = \frac{(\rho - \phi_c)(1-\rho_c)}{\rho_c(1-\phi_\pi)} \\
\rho + \psi_s &= \phi_\pi \psi_s \quad \Longrightarrow \; \psi_s = \frac{-\rho}{1-\phi_\pi} \\
(\psi_c^* - \rho + \psi_e^*) &= [\phi_\pi(\psi_c^* + \psi_{\epsilon^*})] \quad \Longrightarrow \; \psi_e^* = \frac{\rho}{\rho_c^*(1-\phi_\pi)} \\
(\psi_c + \rho + \psi_\epsilon) &= [\phi_\pi(\psi_c + \psi_\epsilon) + \phi_c] \quad \Longrightarrow \; \psi_e = \frac{\phi_c - \rho}{\rho_c\,(1-\phi_\pi)}
\end{aligned}$$

- The solution therefore is:

$$\begin{aligned}
\psi_c^* &= \frac{-\rho(1-\rho_c^*)}{\rho_c^*(1-\phi_\pi)} \\
\psi_c &= \frac{(\rho - \phi_c)(1-\rho_c)}{\rho_c(1-\phi_\pi)} \\
\psi_{\epsilon^*} &= \frac{\rho}{\rho_c^*(1-\phi_\pi)} \\
\psi_s &= \frac{-\rho}{1-\phi_\pi} \\
\psi_\epsilon &= \frac{\phi_c - \rho}{\rho_c\,(1-\phi_\pi)}
\end{aligned}$$

Domestic pricing kernel dependent on ERR

We can now use the solution to inflation to derive a closed form solution of the domestic nominal pricing kernel using equation (51):

$$\begin{aligned} logM_{t,t+1} = log\beta + \rho(1-\rho_c)c_t - \rho[\epsilon_{c,t+1} + s_{t+1} - s_t] \\ -[\psi_c\rho_c c_t + \psi_c^*\rho_c^* c_t^* + \psi_s(s_{t+1} - s_t) + (\psi_c + \psi_\epsilon)\epsilon_{c,t+1} + (\psi_c^* + \psi_{\epsilon^*})\epsilon_{c^*,t+1}] \\ = log\beta + [\rho(1-\rho_c) - \psi_c\rho_c]c_t - (\psi_s + \rho)(s_{t+1} - s_t) - (\rho + \psi_c + \psi_\epsilon)\epsilon_{c,t+1} \\ -\psi_c^*\rho_c^* c_t^* - (\psi_c^* + \psi_{\epsilon^*})\epsilon_{c^*,t+1} \end{aligned}$$

Note that the domestic pricing kernel depends on the coefficients of the ERR, because it depends on the solution to inflation (ψ_i).

Derivation of the FXP

The following calculations now follow closely the Appendix A in De Paoli & Sondergaard (2009). Basically now, one would need to use the definition of the pricing kernel of section 3 and solve for

$$fxp_t \quad = \quad \frac{1}{2}\Big[Var_t[logM^*_{t,t+1}] - Var_t[logM_{t,t+1}]\Big] \tag{59}$$

- To save on notation, we introduce the following coefficients

$$\begin{aligned} b_c &= [\rho(1-\rho_c) - \psi_c\rho_c] && (60) \\ b_s &= -(\psi_s + \rho) && (61) \\ b_\epsilon &= -(\rho + \psi_c + \psi_\epsilon) && (62) \\ b_{c^*} &= -\psi_c^*\rho_c^* && (63) \\ b_{\epsilon^*} &= -(\psi_c^* + \psi_{\epsilon^*}) && (64) \end{aligned}$$

- and write the log-normal pricing kernel as

$$logM_{t,t+1} \quad = \quad log\beta + b_c c_t + b_s(s_{t+1} - s_t) + b_\epsilon \epsilon_{c,t+1} + b_c^* c_t^* + b_{\epsilon^*}\epsilon_{c^*,t+1}$$

- The conditional variance of the domestic nominal pricing kernel writes after dropping terms in t and constant terms:

$$\begin{aligned} Var_t[logM_{t,t+1}] &= Var_t[b_s s_{t+1} + b_\epsilon \epsilon_{t+1} + b_{\epsilon^*}\epsilon^*_{t+1}] \\ &= b_s^2 Var_t[s_{t+1}] + b_\epsilon^2 Var_t[\epsilon_{t+1}] + b_{\epsilon^*}^2 Var_t[\epsilon^*_{t+1}] + 2b_s b_\epsilon Cov_t[s_{t+1}, \epsilon_{t+1}] \quad (65) \\ &= b_s^2 Var_t[s_{t+1}] + b_\epsilon^2 \sigma_\epsilon^2 + b_{\epsilon^*}^2 \sigma_{\epsilon^*}^2 + 2b_s b_\epsilon Cov_t[s_{t+1}, \epsilon_{t+1}] \quad (66) \end{aligned}$$

note that $Cov_t[s_{t+1}, \epsilon^*_{t+1}] = 0$. The remaining terms are solved for in Appendix A in De Paoli & Sondergaard (2009) using a second order approximation to surplus consumption. Therefore the conditional variance of the domestic nominal pricing kernel is

$$Var_t[logM_{t,t+1}] = \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2 \sigma_\epsilon^2}{(1-h)^2} + b_{\epsilon^*}\sigma_{\epsilon^*}^2 + \frac{2b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma_\epsilon^2[(1-\phi-\rho_c)c_t + \phi x_t]$$

- Finally, the **foreign exchange risk premium** has the following expression:

$$\begin{aligned} fxp_t &= \\ &= \frac{1}{2}\left[Var_t[logM^*_{t,t+1}] - Var_t[logM_{t,t+1}]\right] \\ &= \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma_{\epsilon^*}^2 - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma_\epsilon^2}{(1-h)^2} - \frac{2b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma_\epsilon^2[(1-\phi-\rho_c)c_t + \phi x_t]\right] \\ &= \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma_{\epsilon^*}^2 - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma_\epsilon^2}{(1-h)^2}\right] - \frac{b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma_\epsilon^2[(1-\phi-\rho_c)c_t + \phi x_t] \end{aligned}$$

Taylor rule

- Notice the nominal interest rate can be written under lognormality as the following function of the nominal pricing kernel

$$\begin{aligned} i_t &= -log\mathbb{E}_t[M_{t,t+1}] \\ &= -\mathbb{E}_t[logM_{t,t+1}] - \frac{1}{2}Var_t[logM_{t,t+1}] \end{aligned}$$

- To solve for the nominal interest rate as a functin of inflation, consumption and surplus consumption, note that we can use the equation (51) for $log[M_{t,t+1}]$ and write the nominal interest rate as

$$\begin{aligned} i_t &= -log\beta - \rho(1-\rho_c)c_t + \rho\mathbb{E}_t[s_{t+1} - s_t] + \mathbb{E}_t[\pi_{t+1}] \\ &- \frac{1}{2}Var_t[-\rho[\epsilon_{c,t+1} + s_{t+1}] - \pi_{t+1}] \end{aligned}$$

- Assuming the same solution for inflation as above

$$\pi_t = \psi_c c_t + \psi_c^* c_t^* + \psi_s(s_t - s_{t-1}) + \psi_\epsilon \epsilon_t + \psi_{\epsilon^*}\epsilon_t^*$$

One period ahead inflation is

$$\pi_{t+1} = \psi_c c_{t+1} + \psi_c^* c_{t+1}^* + \psi_s(s_{t+1} - s_t) + \psi_\epsilon \epsilon_{t+1} + \psi_{\epsilon^*}\epsilon_{t+1}^*$$

and in expectation

$$E_t\pi_{t+1} = \psi_c\rho_c c_t + \psi_c^*\rho_c^* c_t^* + \psi_s(E_t s_{t+1} - s_t)$$

From here

$$var_t(\pi_{t+1}) = E_t(\psi_c\epsilon_{t+1} + \psi_c^*\epsilon_{t+1}^* + \psi_\epsilon\epsilon_{t+1} + \psi_{\epsilon^*}\epsilon_{t+1}^*)^2$$

- When we plug this solution into the expression for the nominal interest rate, use $c_{t+1} = \rho_c c_t + \epsilon_{t+1}$ and $c_{t+1}^* = \rho_c^* c_t^* + \epsilon_{t+1}^*$ and collect terms we find

$$\begin{aligned} i_t &= -log\beta - [\rho(1-\rho_c) - \psi_c\rho_c]c_t + (\psi_s + \rho)\mathbb{E}_t[s_{t+1} - s_t] + \psi_c^*\rho_c^* c_t^* \\ &- \frac{1}{2}Var_t[-(\rho + \psi_s)s_{t+1} - (\rho + \psi_\epsilon + \psi_c)\epsilon_{t+1} - (\psi_{\epsilon^*} + \psi_c^*)\epsilon_{t+1}^*] \end{aligned}$$

- We expand the conditional variance term $Var_t[...]$

$$\begin{aligned} i_t &= -log\beta - [\rho(1-\rho_c) - \psi_c\rho_c]c_t + (\psi_s + \rho)\mathbb{E}_t[s_{t+1} - s_t] + \psi_c^*\rho_c^* c_t^* \\ &- \frac{1}{2}\left[(\rho + \psi_s)^2 Var_t[s_{t+1}] + (\rho + \psi_\epsilon + \psi_c)^2\sigma_\epsilon^2(\psi_{\epsilon^*} + \psi_c^*)^2\sigma_{\epsilon^*}^2]\right] \\ &- (\rho + \psi_s)(\rho + \psi_\epsilon + \psi_c)Cov_t[s_{t+1}, \epsilon_{t+1}] \end{aligned}$$

Note that $Cov_t[\epsilon_{t+1}, \epsilon_{t+1}^*] = 0$ and $Cov_t[s_{t+1}, \epsilon_{t+1}^*] = 0$.

- As in the ERR case, red terms are solved for by using expressions from De Paoli & Sondergaard (2009):

$$\begin{aligned} Var_t[s_{t+1}] &= \left(\frac{h}{1-h}\right)^2 \left[\sigma_\epsilon^2 - (1-h)^{-1} 2\rho_c c_t \sigma_\epsilon^2 + 2\tilde{x}_t (1-h)^{-1}\sigma_\epsilon^2\right] \\ Cov_t[s_{t+1}, \epsilon_{t+1}] &= \frac{h}{1-h}\left[\sigma_\epsilon^2 - (1-h)^{-1}\rho_c c_t \sigma_\epsilon^2 + \tilde{x}_t(1-h)^{-1}\sigma_\epsilon^2\right] \end{aligned}$$

with $\tilde{x}_t = log(X_{t+1})$.

- Plugging these terms into the equation for the nominal interest rate gives the following

$$\begin{aligned} i_t &= -log\beta - [\rho(1-\rho_c) - \psi_c\rho_c]c_t + (\psi_s + \rho)\mathbb{E}_t[s_{t+1} - s_t] + \psi_c^*\rho_c^* c_t^* \\ &- \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right)^2 \sigma_\epsilon^2 + (\psi_{\epsilon^*} + \psi_{c^*})^2 \sigma_{\epsilon^*}^2\right] \\ &- \sigma_\epsilon^2 \frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right][(1-\phi-\rho_c)c_t + \phi x_t] \end{aligned}$$

- To solve for the coefficients of the guessed solution for inflation (ψ_i), we assume need another equation, here the Taylor rule. We assume a Taylor rule with the same targets as above:

$$i_t = -log(\beta) + \phi_\pi E_t \pi_{t+1} + \phi_c E_t(c_{t+1} - c_t)$$

- Plugging in the guessed solution for inflation into the Taylor rule gives:

$$i_t = -log(\beta) + (\phi_\pi\psi_c\rho_c - (1-\rho_c)\phi_c)c_t + (\phi_\pi\psi_c^*\rho_c^*)c_t^* + \phi_\pi\psi_x x_t + (\phi_\pi\psi_s + \phi_s)E_t(s_{t+1} - s_t) \quad (67)$$

- As before, equations (**??**) and (67) are used to solve for (ψ_i) by setting equal the coefficients of the variables:

$$-log\beta - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right)^2\sigma_\epsilon^2+(\psi_{\epsilon^*}+\psi_{c^*})^2\sigma_{\epsilon^*}^2\right] = -log(\beta)$$

c_t^* : $$\phi_\pi\psi_c^*\rho_c^* = \phi_\pi\psi_c^*+\phi_{c^*}$$

c_t : $$-[\rho(1-\rho_c)-\psi_c\rho_c] =$$

$$-\sigma_\epsilon^2\frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right][(1-\phi-\rho_c)]$$

s_t : $$(\phi_\pi\psi_s+\phi_s) = (\psi_s+\rho)$$

x_t : $$0 = \sigma_\epsilon^2\frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right][\phi]$$

This is a system of six equations and six unknowns.

- Note first that if $\phi_\pi \neq 0$ it must follow that

$$\begin{aligned}\psi_{c^*} &= \frac{-\phi_{c^*}}{\phi_\pi(1-\rho_c)}\\ \psi_c &= \frac{\rho(1-\rho_c)}{\rho_c}\\ \psi_s &= -\psi_{c^*}\\ \psi_{\epsilon^*} &= \frac{\phi_{c^*}}{\rho_c^*-\phi_\pi}\end{aligned}$$

$$\psi_\epsilon = -\frac{(\rho+\psi_s)h}{1-h}-\psi_c-\rho$$

- Therefore,

$$fxp_t = \frac{1}{2}\sigma_{\epsilon^*}b_\epsilon^* - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right)^2\sigma_\epsilon^2+(\psi_{\epsilon^*}+\psi_{c^*})^2\sigma_{\epsilon^*}^2\right]$$

$$-\sigma_\epsilon^2\frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right][(1-\phi-\rho_c)c_t+\phi x_t]$$

Comparing risk premiums for each rule

– ERR

$$fxp_t = = \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma^2_{\epsilon^*} - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma^2_\epsilon}{(1-h)^2} - \frac{2b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma^2_\epsilon[(1-\phi-\rho_c)c_t +\right.$$

$$= \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma^2_{\epsilon^*} - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma^2_\epsilon}{(1-h)^2}\right] - \frac{b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma^2_\epsilon[(1-\phi-\rho_c)c_t + \phi$$

– Taylor rule

$$fxp_t = \frac{1}{2}\sigma_{\epsilon^*}b^*_\epsilon - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right)^2\sigma^2_\epsilon + (\psi_{\epsilon^*} + \psi_{c^*})^2\sigma^2_{\epsilon^*}\right]$$

$$-\sigma^2_\epsilon\frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right][(1-\phi-\rho_c)c_t+\phi x_t]$$

We can decompose the risk premium into two components: the conditional variance of the exchange rate depreciation and the conditional covariance between the stochastic discount factor and the expected future depreciation of the currency.

The risk premium is also given by:

$$fxp_t = \frac{1}{2}var_t\left(\Delta e_{t+1}\right) + cov_t\left(\Delta e_{t+1}, log\left(M_{t,t+1}\right)\right)$$

C.1 Exchange rate rule

$$var_t\left(\Delta e_{t+1}\right) = \frac{(1-h)^2\rho^2\sigma^2_{\epsilon^*}\phi^2_\pi - \sigma^2_\epsilon((1-h)\phi_c - \rho\phi_\pi)^2}{(1-h)^2(1-\phi_\pi)^2} - \frac{2h\rho\sigma^2_\epsilon((1-h)\phi_c - \rho\phi_\pi)}{(1-h)^3(1-\phi_\pi)}\left[c_t(\rho_c + \phi - 1) - \phi x_t\right] \quad (68)$$

$$cov_t\left(\Delta e_{t+1}, m_{t,t+1}\right) = -\frac{(1-h)^2\rho^2\sigma^2_{\epsilon^*}\phi_\pi - \sigma^2_\epsilon((1-h)\phi_c - \rho\phi_\pi)^2}{(1-h)^2(1-\phi_\pi)^2} + \frac{2h\rho\sigma^2_\epsilon((1-h)\phi_c - \rho\phi_\pi)}{(1-h)^3(1-\phi_\pi)}\left[c_t(\rho_c + \phi - 1) - \phi x_t\right] \quad (69)$$

C.2 Taylor rule

$$
\begin{aligned}
var_t\left(\Delta e_{t+1}\right) &= \frac{\sigma_\epsilon^2\left(h^2\rho^4\sigma_\epsilon^2\sigma_{\epsilon^*}^2(\rho_c+\phi-1)^2+(1-h)^4\phi_c^2\right)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2} \\
&- \frac{2(1-h)^2\rho\sigma_\epsilon^2\phi_\pi\left(h\rho^2\sigma_{\epsilon^*}^2(\rho_c+\phi-1)+(1-h)\phi_c\right)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2} \\
&+ \frac{(1-h)^2\rho^2\phi_\pi^2\left((1-h)^2\sigma_{\epsilon^*}^2+\sigma_\epsilon^2\right)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(h-1)^2\phi_\pi\right)^2} \\
&+ \frac{2h\rho\sigma_\epsilon^2((1-h)\phi_c-\rho\phi_\pi)}{(1-h)\left((1-h)^2\phi_\pi-h\rho\sigma_\epsilon^2(\rho_c+\phi-1)\right)}\left[c_t(\rho_c+\phi-1)-\phi x_t\right]
\end{aligned} \tag{70}
$$

$$
\begin{aligned}
cov_t \quad & \left(\Delta e_{t+1}, m_{t,t+1}\right) \\
= \quad & -\frac{(1-h)^2((1-h)\phi_c-\rho\phi_\pi)\sigma_\epsilon^2((1-h)\phi_c-\rho\phi_\pi)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2} \\
- \quad & \frac{2h\rho\sigma_\epsilon^2((1-h)\phi_c-\rho\phi_\pi)\left((1-h)^2\phi_\pi-h\rho\sigma_\epsilon^2(\rho_c+\phi-1)\right)}{(1-h)\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2}\left[c_t(\rho_c+\phi-1)-\phi x_t\right]
\end{aligned} \tag{71}
$$

D Singapore economy

As we have mentioned, the desirability of the exchange rate as an instrument of monetary policy hinges on whether UIP holds or not. There is a substantial body of literature starting with Fama (1984) that shows that UIP fails for a large set of countries and time periods. Lewis (1995) reviews the early literature, whileChinn & Quayyum (2012) offers an overview of the studies since mid-1980s. Therefore building in a failure of the interest parity condition is not only theoretically interesting, but also it seems to be the empirically relevant approach to studying macroeconomic dynamics for open economies. As it turns out, however, there is an important twist in this rationalization. Empirical analysis shows that since late 1980s the interest parity condition between Singapore and the US holds (Khor et al. 2007). Incidentally, this is the period when MAS started using the new policy regime based on the exchange rate rule.

Singapore has attracted large capital inflows. Even though capital inflows tend to appreciate the currency, increase the domestic liquidity and inflation, thus generating a loss of competitiveness, the exchange rate has been quite stable sin 1981 (Figure

7).

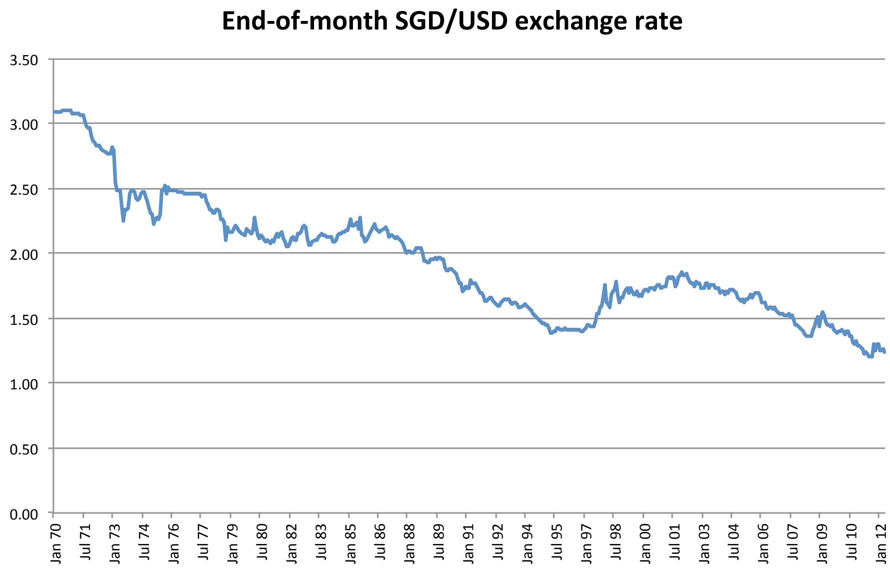


Figure 14: SGD/USD exchange rate

The monetary policy framework has been to use the exchange rate as the operating tool, instead of a benchmark interest rate. The main reason is that Singapore is very susceptible to imported inflation. If the ultimate goal is to stabilize inflation, the exchange rate might be a better tool. **?** studies the counter-cyclical role of Singapore monetary policy through the use of a monetary reaction function. A variant of a Taylor rule was estimated using changes in exchange rates instead of interest rates as the policy instrument. McCallum (2007), estimates a similar rule including deviations from the real exchange rate. To implement the policy, Singapore follows the BBC system proposed by Williamson. It is an intermediate exchange rate regime where there are: (i) A basket composed by the currencies of the main trading partners and competitors of Singapore (a trade-weighted index (TWI), in which neither the currencies nor the weights are made public). The MAS sets a goal parity of the SGD vis-a-vis the TWI; (ii) wide bands centered at the target parity for the TWI. The long-run equilibrium exchange rate (the target) is allowed to adjust gradually, without large steps like in the hard pegs. The bands are undisclosed; and (iii) a crawl, according to the underlying economic fundamentals, to avoid speculative attacks. The wide bands avoid that speculators make profits even when correctly anticipate the change in exchange rates. The soft margins that allow the bands to move also help to avoid speculative attacks. To control the exchange rate, the MAS undertakes intervention operations in foreign exchange rate movements.

To motivate further our investigation, we estimate a sequence of rolling regressions using monthly data for the 3-month Singapore-dollar interbank offer rate, the 3-month US-dollar interbank offer rate and the SGD/USD exchange rate. Figure 8 plots the coefficient on the interest rate differential, which under the null hypothesis must be equal to one.

Arbitrage implies the following condition

$$(1 + r_{USD}) = (1 + r_{SGD})\frac{e_t(SGD/USD)}{e_{t+1}(SGD/USD)}$$

where r_{USD} is the nominal interest rate in USD, r_{SGD} is the nominal interest rate in SGD, and e_t is the nominal exchange rate defined as the amount of SGD that are necessary to buy one unit of USD (an increase in e_t implies an appreciation of the USD). Taking logs, we obtain the regression equation that we use to test for the UIP condition:

$$\Delta e(SGD/USD) = \beta(log(1 + r_{SGD}) - log(1 + r_{USD})) + u_t$$

As it is common in these regressions, the coefficient is negative indicating bias in favor of US dollar denominated securities. Interestingly, towards late 1980s the coefficient becomes insignificantly different from one. The exception is the period of the Asian financial crisis from roughly the beginning of 1998 to the end of 2000. Does UIP holds because of the specific rule or for some other reason? The result is consistent with McCallum (1994), Anker (1999), and Backus et al. (2010) who argue that deviations from UIP are due to the behavior of monetary policy. Indeed, it is possible that by using the exchange rate as a policy instrument a central bank might not only reduce the volatility of CPI inflation, but may also reduce the risk premium associated with the fluctuations in the exchange rate and thus restore the one-to-one link between interest rates and exchange rate dynamics.