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**International R&D Spillovers and Asset Prices**

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# International R&D Spillovers and Asset Prices

Federico Gavazzoni and Ana Maria Santacreu\*

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## Abstract

We provide new empirical evidence of a relationship between asset prices and trade-induced international R&D spillovers; in particular, we find that pairs of countries that share more research and development exhibit more highly correlated stock market returns and less volatile exchange rates. We develop an endogenous growth model of innovation and international technology diffusion that accounts for these empirical findings. A calibrated version of the model matches several important asset pricing and quantity moments, which helps explain some of the classical quantity–price puzzles highlighted in the literature on international macroeconomics.

Keywords: innovation, international diffusion, trade, asset prices, recursive preferences

JEL Classification: F3, F4, O3

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# 1 Introduction

Technological innovation is a fundamental source of sustained economic growth (Romer 1990). Asset prices reflect changes in the future growth prospects of the economy and thus capture variations in technological innovation (Kung and Schmid 2011). In an international setting, technology may diffuse across countries through trade in the products that incorporate that technology. Hence a country’s growth rate depends not only on its own innovation but also on the innovation efforts of its trading partners (Coe and Helpman 1995; Keller 1998). It follows that the dynamics of technological innovation both *within* and *across* countries may inform us about the comovements of international asset returns.

In this paper, we investigate the link between trade in varieties—our measure of how technological innovation diffuses across countries—and comovements in asset prices. Our first contribution is empirical. Using highly disaggregated bilateral trade data, we document the following empirical regularities. First, country pairs that share more research and development (R&D) by trading a higher number of varieties with each other exhibit *greater* correlation in stock market returns. Second, country pairs that share more R&D have *less* volatile exchange rates. We find that these patterns are robust to controlling for alternative measures of R&D and trade across each country pair, which suggests that international R&D spillovers play a distinct role in capturing how the international diffusion of technological innovation is related to asset prices.<sup>1</sup>

Our next contribution is theoretical. We build a two-country endogenous growth model of innovation and international technology diffusion through trade in varieties that rationalizes our empirical findings. Growth in each country is driven by the accumulation of technology through endogenous innovation. Our assumption is that the technology embodied in intermediate goods spreads across countries via international trade. As a result, the productivity level of a country depends on its own innovation *and* on the foreign innovations embodied in imported intermediate products. Preferences are recursive, so that consumers care about the timing of resolution of uncertainty and fear variation in the economy’s long-run future growth

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<sup>1</sup>The international trade literature has argued in favor of trade in varieties as a channel through which R&D diffuses across countries (Broda, Greenfield, and Weinstein 2006; Bøler, Moxnes, and Ulltveit-Moe 2012; Santacreu 2015). Other channels include the effect of multinationals in spreading the benefits of R&D across countries (Ramondo 2009; Guadalupe, Kuzmina, and Thomas 2010) and the effect of knowledge spillovers by way of international networks (Cai and Li 2012). The analysis of these additional channels is beyond the scope of this paper.

prospects. International financial markets are complete. Endogenous innovation, together with recursive preferences, makes the equilibrium growth path risky owing to its effect on the present discounted value of future profits of all firms in the economy.

Our endogenous growth mechanism works as follows. R&D drives a small and persistent component of equilibrium growth rates. International diffusion through trade in varieties renders this component common across countries. The intuition is that a technology shock in the home country affects the incentives to innovate not only in that country but also abroad, which in turn affects the prospects of global growth. Therefore, a short-run technology shock in the home country has a long-run effect on the dynamics of future growth rates in both the home and the foreign economy.

When preferences are recursive, variations in the economy's future prospects have a significant effect on asset prices: agents require a large risk premium for holding assets that are exposed to such variations. As technological innovation diffuses across countries through trade in varieties, it generates a sizable common component in asset returns that drives up the correlation in stock market returns while reducing the volatility of exchange rates. Our model can replicate these international asset pricing facts together with a realistic calibration of macroeconomic quantities. In particular, the model predicts—in line with the data—that realized output growth and realized consumption growth are only somewhat correlated across countries; the reason is that realized growth is driven mainly by exogenous technology shocks, which exhibit low levels of cross-country correlation.

We calibrate the model to match our empirical findings. Consistently with our predictions, we find that asset prices are largely driven by the economy's long-run future prospects, whereas international quantities are mostly driven by current technological levels. When growth is endogenous, stock market returns are highly correlated across countries—and even more so under increased international R&D spillovers. We further investigate our mechanism by decomposing stock market returns into two components. The first component, which we label return on *tangible* capital, captures the return on installed physical capital and is the standard measure of stock market return in real business cycle models that incorporate trade in intermediate goods. The second component is the return on *intangible* capital, which captures the effects of endogenous innovation and the international diffusion of technologies. We find that the return on tangible capital is only weakly correlated across countries, from which it follows that the strong cross-country correlation observed in overall stock market

returns mainly reflects the intangible component. Furthermore, the exchange rate in our model is as volatile as in the data, and its volatility increases when international R&D spillovers decrease.

The model presented here generates a novel set of testable implications. In particular, it predicts that both domestic innovation and foreign innovations embodied in imported intermediate goods are predictive of future domestic productivity growth. We test this theoretical prediction and provide empirical evidence in favor of our mechanism.

Our paper is related to several strands of the literature. First, the macroeconomic mechanism is related to the literature on endogenous growth through innovation. In our model, technological progress increases with the number of intermediate goods that incorporate technology. Kung and Schmid (2011) extend Romer (1990) to include recursive preferences, and they reproduce asset price dynamics that are consistent with the empirical literature. We develop our model along these lines and extend it to an international setting so that it accounts for our novel empirical findings on the relations among trade, R&D, and asset prices.<sup>2</sup>

The second related strand of literature is that on technology adoption and innovation through international trade in varieties (e.g., Broda, Greenfield, and Weinstein 2006; Santacreu 2015). Using highly disaggregated trade data, these papers find that adopting foreign innovations by trade in varieties affects the home country’s growth rate. However, these authors do not discuss the asset pricing implications of their mechanism, which is one of the principal contributions of our research.

Finally, this paper is related to the literature on asset pricing with long-run risk that began with the seminal one-country model of Bansal and Yaron (2004) and was then applied to the international setting by Bansal and Shaliastovich (2009), Colacito and Croce (2011), and Colacito and Croce (2013). Whereas these papers specify global long-run risk exogenously, our model shows how such risk—which is highly persistent within countries and highly correlated across countries—arises endogenously through innovation and the international diffusion of R&D. From a methodological standpoint, our paper is related to that of Croce, Nguyen, and Schmid (2013), who examine the role of fiscal policy in an international endogenous growth model with recursive preferences. Unlike that paper, however, here

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<sup>2</sup>Several papers have examined the link between technological growth and prices; examples include Pastor and Veronesi (2009), Garleanu, Panageas, and Yu (2012), and Garleanu, Kogan, and Panageas (2012). These papers assume that technology growth is *exogenous*. In contrast, and consistently with our empirical findings, we focus on the relation between international asset prices and *endogenous* growth through R&D.

the focus on the empirical links among trade, R&D, and asset prices in the context of an endogenous growth model with recursive preferences.

The rest of the paper is organized as follows. Section 2 reports our main empirical findings, and Section 3 describes our baseline model. Section 4 provides a description of the main mechanism at work. Section 5 presents the calibration and our quantitative results; in Section 6, we investigate the model’s predictions. Section 7 concludes.

## 2 Innovation, Trade, and Asset Prices: Empirical Evidence

A striking feature of international macroeconomics data is the low cross-country correlation of consumption growth relative to the correlation of stock market returns: the first two rows of Table 1 show that, at quarterly frequency, the former is 0.23 whereas the latter is nearly 3 times higher, about 0.71.<sup>3</sup> We also find that the annualized average volatility of the exchange rate depreciation for our sample of countries and time period of analysis is approximately 9.6%. Standard international macroeconomic models have had a hard time reconciling these empirical asset pricing and quantity moments.

We argue that the international spillovers of R&D resulting from trade in varieties are a significant driver of the dynamics of international asset prices. To the extent that the technology created by investing in R&D is embodied in a particular good, movements of goods across borders help diffuse those technologies (Coe and Helpman 1995, Keller 1998).

This section is devoted to investigating the empirical relevance of our proposed mechanism. We collect data on asset prices, international trade, and R&D before proceeding in two steps. First, we look at the correlation of broad measures of asset pricing moments with international trade and R&D. Then, following Liao and Santacreu (2015), we perform a regression analysis to investigate the drivers of this relationship. Our main data sources are: for international trade, the UN COMTRADE; for R&D, the World Development Indicators database of the World Bank; and for asset prices, Global Financial Data and Ken French’s website.<sup>4</sup>

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<sup>3</sup>The average is taken over a sample of 20 countries for the period 1993–2009. More specific details are given in Appendix A.

<sup>4</sup>[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

Table 1: Summary Statistics

Variable moment	Value
Correlation in consumption growth	0.23
Correlation in stock market return	0.71
Volatility of exchange rate depreciation	9.60

Note: Data is quarterly. Volatility is expressed in annualized percentage points.

Our main data set covers the period 1993–2009 period for a sample of 20 countries.<sup>5</sup> The choice of countries and time period was determined based on the availability of data for asset pricing, trade, and R&D. Details on these sources and on the construction of measures used in our analysis are given in Appendix A.

## 2.1 Correlation among Asset Pricing Moments, R&D, and International Trade

Here we analyze the correlations between broad measures of asset prices and measures of international trade and R&D. The goal is to determine whether the mechanism that we propose (i.e., R&D embodied in the trade in varieties) plays any role in asset prices. We consider two statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the exchange rate depreciation. We start with monthly observations and construct annual, non-overlapping cross-country correlation—for stock market returns—and volatility—for the exchange rate depreciation—per each ordered country pair.<sup>6</sup>

We then use data on R&D and international trade to construct several measures that capture our proposed mechanism. First we construct a measure of innovation for each country pair in our sample. We use R&D intensity—computed as R&D expenditures over gross domestic product (GDP)—and then compare this measure with our measures for asset prices

<sup>5</sup>The sample of countries includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

<sup>6</sup>We also consider 60-month overlapping measures with a 12-month overlap. The results are virtually unchanged.

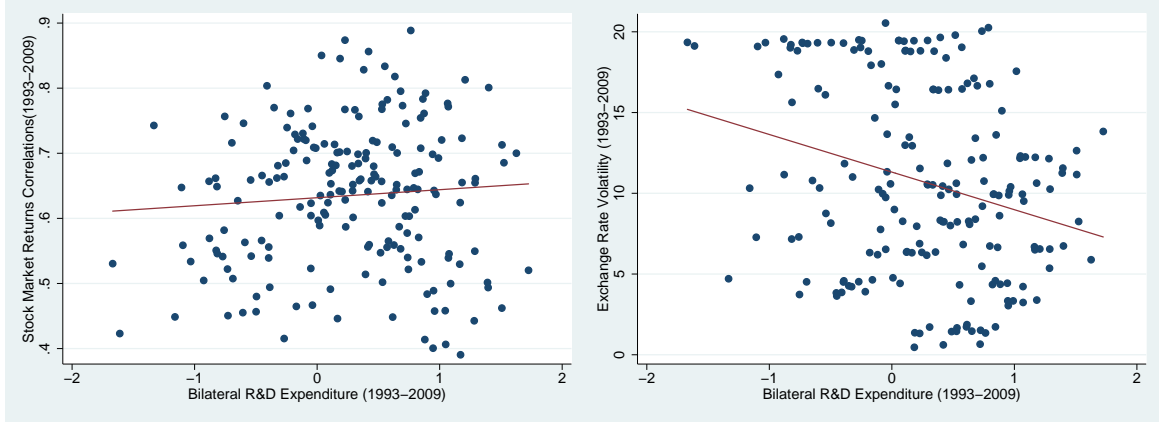


Figure 1: R&D intensity and asset prices

to determine whether there is any relation between them.<sup>7</sup> Figure 1 shows that country pairs with higher levels of R&D have more strongly correlated stock market returns and less volatile exchange rates. Although a clear pattern emerges, Figure 1 does not provide any insight regarding how R&D spreads across countries.

Next, we test for whether there is any relationship between the strength of international trade and our measures of asset prices. In particular, we construct a measure of overall bilateral trade as the value of trade between each pair of countries in our sample. We then decompose this measure into the so-called extensive and intensive margins of trade, which are (respectively) the number of different products that are traded between each pair of countries and how much of each product is traded. These measures are constructed in such a way that *overall* trade is equal to the product of the extensive and the intensive margins of trade. To facilitate comparisons, we normalize trade by GDP; we then compare these measures with those for asset prices to determine whether there is any correlation between them. Figures 2 and 3 show the results: countries that trade more with each other have more highly correlated stock market returns and less volatile exchange rate depreciations. To the extent that R&D is embodied in the products that are traded internationally, these results indicate that trade-induced international R&D spillovers may be an important factor in explaining the moments of asset pricing.

So far we have shown that both R&D and international trade are related to asset pricing

<sup>7</sup>As a robustness check, we also use a measure of R&D intensity computed as R&D expenditures over the stock of R&D. For this purpose, we use the “perpetual inventory” method to compute the stock of R&D while assuming an annual depreciation rate of 15%, as is standard in the literature (e.g., Coe and Helpman 1995, Nishioka and Ripoll 2012). The results are much the same when using other measures of R&D intensity.



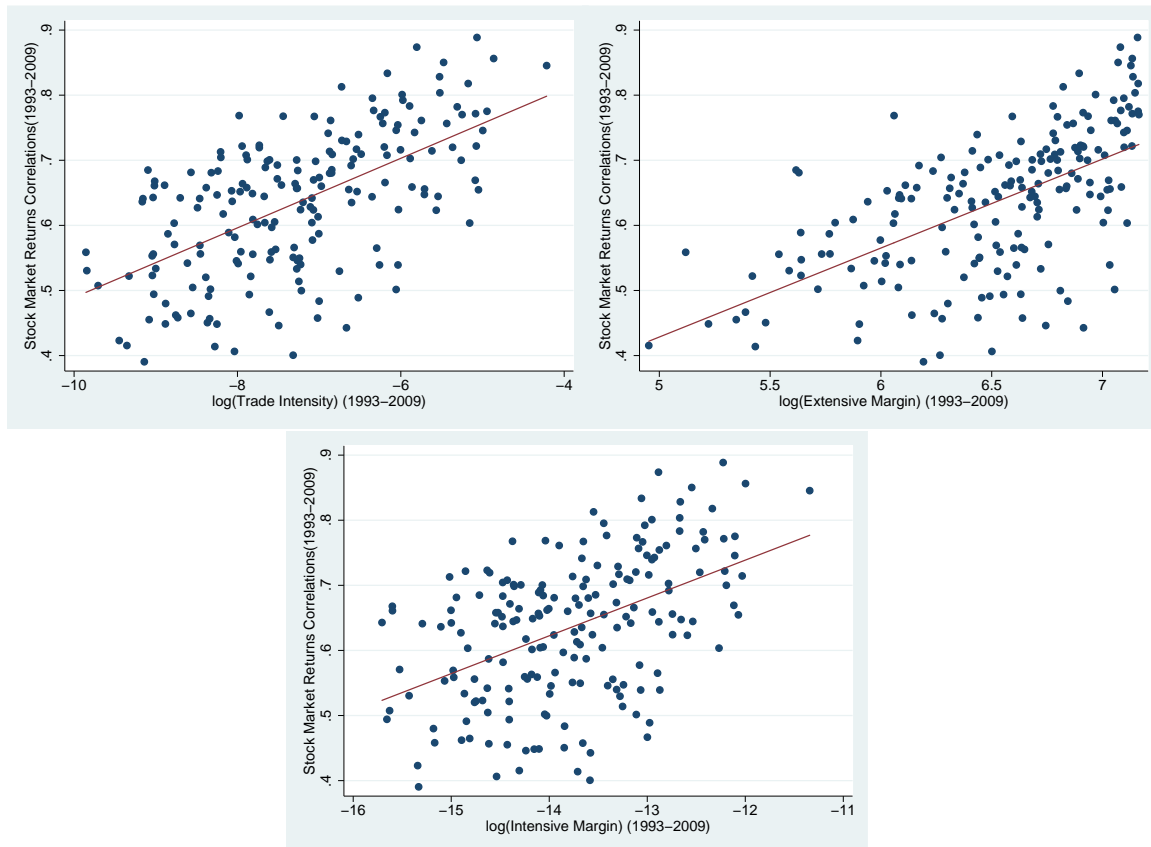


Figure 2: International trade and the correlation of stock market returns

moments. It would be ideal if we had *direct* measures of the extent to which R&D is embedded in international trade, but available trade data are not sufficiently disaggregated for this purpose. Hence our strategy is to construct an *indirect* measure that weights the bilateral trade of each country pair by the exporter's R&D. This measure places the most weight on imported intermediate products from relatively more innovative exporters.<sup>8</sup> Figures 4 and 5 reveal that pairs of countries whose trade is characterized by greater R&D content exhibit stock market returns that are more correlated and exchange rate depreciations that are less volatile.

Taken together, the graphs in Figure 1–5 suggest that R&D and international trade can inform our understanding of asset price drivers and dynamics.

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<sup>8</sup>The construction of this variable is detailed in Appendix A.

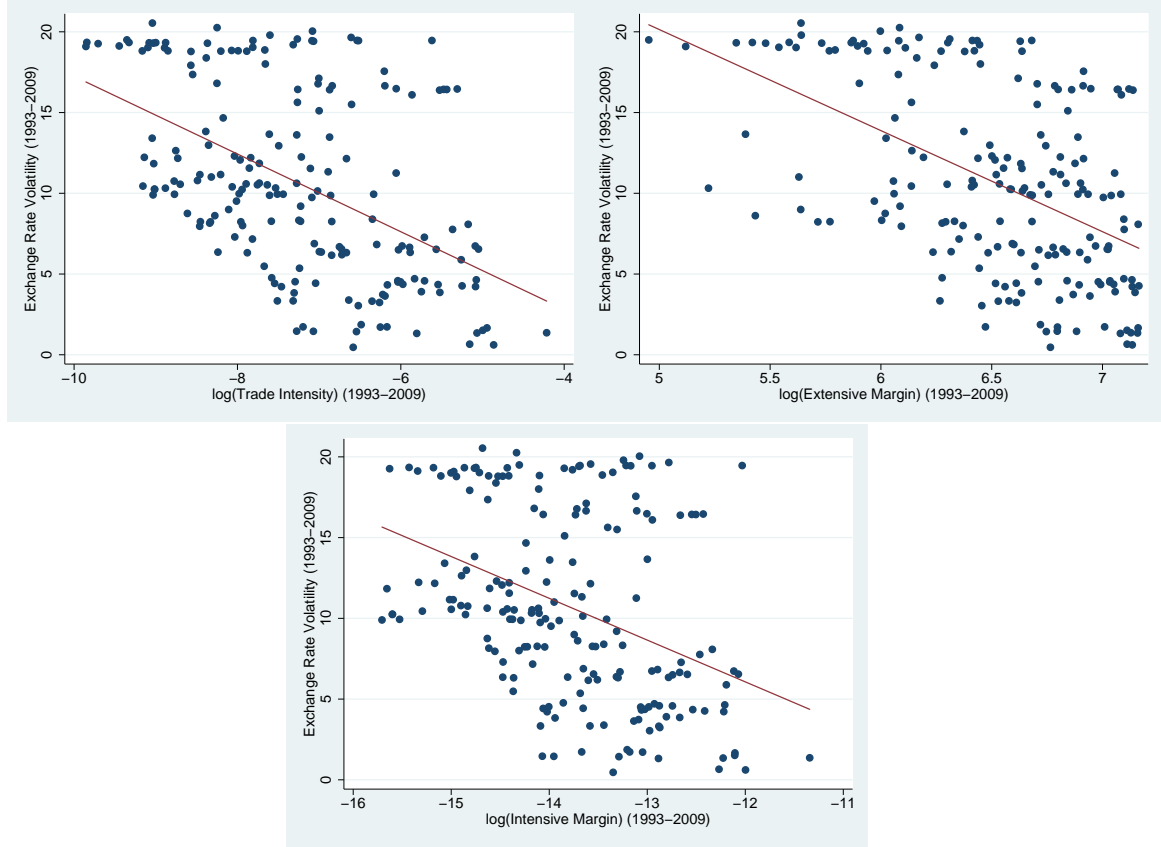


Figure 3: International trade and the volatility of exchange rates

## 2.2 Regression Analysis

We now undertake a formal regression analysis in order to address the economic and statistical significance of the results presented in Section 2.1. In particular, we regress the asset pricing moments of interest—namely, the correlation of stock market returns and the volatility of exchange rate depreciations—on our measures of R&D, international trade, and R&D embodied in trade.

In Table 2, for each country pair, we report the results from regressing those asset pricing moments on both the extensive margin (EM) and intensive margin (IM) of their respective international trade. We find that both coefficients are statistically significant, although the extensive margin accounts for most of the change in asset pricing moments. More specifically, Table 2 shows that if we hold the intensive margin constant, then a 1% increase in the extensive margin increases the correlation of asset returns by 0.094% (first column) and decreases the volatility of the exchange rate depreciation by 3.53% (second column); if instead we hold the extensive margin constant, then a 1% increase in the intensive margin increases



Figure 4: R&D embodied in trade and the correlation of stock market returns



Figure 5: R&D embodied in trade and the volatility of exchange rate depreciations

the correlation of stock market returns by 0.032% and decreases the volatility of the exchange rate depreciation by 1.384%. Thus the effect of the intensive margin of trade is weaker than that of the extensive margin. These findings are consistent with the mechanism that we propose. Indeed, if trade in varieties is the channel through which R&D spreads across countries, then variations in the extensive margin of trade (i.e., the number of varieties) and not in the intensive margin of trade (product quantities) should account for most of the changes in asset pricing moments.

Next we perform the regression analysis using our indirect measure of R&D embodied in trade; the results are summarized in Table 3. We find that trade involving greater R&D is associated with a higher cross-country correlation in stock market returns and a lower volatility of the exchange rate depreciation. It is worth emphasizing that only the R&D content of *extensive* trade margins has a statistically significant effect on volatility of exchange rate

Table 2: Regression of asset pricing moments on trade margins

	(1)	(2)
	Stock market correlation	Volatility of exchange rate depreciation
$\log(\text{EM})_{ij}$	0.094*** (0.006)	-3.523*** (0.160)
$\log(\text{IM})_{ij}$	0.032*** (0.003)	-1.384*** (0.085)
Constant	0.422*** (0.068)	13.443*** (1.818)
Observations	6,460	6,460
$R^2$	0.075	0.149

Note: standard errors are reported in parentheses.

\*\*\*  $p < 0.001$

depreciation. This additional finding lends empirical support to our model's main mechanism, whereby the international spillovers of R&D that result from trade in varieties have a significant impact effect on international asset pricing moments.

Table 3: Regression of asset pricing moments on R&amp;D embodied in trade

	(1)	(2)
	Stock market correlation	Volatility of exchange rate depreciation
$\log(\text{EM}_{ij}^{\text{R\&D}})$	0.031*** (0.007)	-1.810*** (0.183)
$\log(\text{IM}_{ij}^{\text{R\&D}})$	0.016*** (0.004)	-0.192 (0.122)
Constant	0.747*** (0.013)	3.081*** (0.343)
Observations	6,422	6,422
$R^2$	0.029	0.059

Note: standard errors are reported in parentheses

\*\*\*  $p < 0.001$

Finally, Table 4 provides additional empirical evidence that R&D matters for international asset prices. In particular, we find that country pairs characterized by higher levels of R&D have more correlated stock market returns and less volatile exchange rate depreciation.

The values reported in Tables 8–17 (Appendix D) establish that these results are robust to incorporating time fixed effects as well as time and country fixed effects.

Table 4: Regression of asset pricing moment on R&amp;D intensity

	(1)	(2)
	Stock market correlations	Volatility of exchange rate depreciation
$\log\left(\frac{\text{R\&D}_i}{\text{GDP}_i}\right) + \log\left(\frac{\text{R\&D}_j}{\text{GDP}_j}\right)$	0.054*** (0.004)	-1.183*** (0.122)
Constant	0.577*** (0.003)	9.900*** (0.095)
Observations	6,384	6,384
$R^2$	0.024	0.014

Note: standard errors are reported in parentheses

\*\*\* $p < 0.001$

### 3 Model

In this section, we present a model of innovation and international diffusion of R&D via trade in varieties that captures our empirical findings. Each country has a representative household, with recursive preferences, that consumes a final good. A final good producer uses labor, capital, and a composite of intermediate goods—which we call materials—to produce a nontradable final good that is used for consumption, investment in capital, and investment in R&D. Materials are produced using traded intermediate goods (varieties), both domestic and foreign, which are produced by monopolistic competitive firms. The production of materials reflects a love-for-variety effect; thus if expenditures are held constant then a higher number of varieties increases the country’s productivity. New varieties are introduced in each country through an endogenous process of innovation, after which they spread exogenously across countries through a slow process of adoption. Endogenous innovation and exogenous adoption, together with recursive preferences, are the new features at the core of our mechanism. The model is closed with an international risk-sharing condition.

Next we describe the domestic economy  $d$ . The foreign economy  $f$  is defined analogously.

#### 3.1 Households

The domestic representative household has Epstein and Zin (1989) recursive preferences over consumption:

$$U_{d,t} = \{(1 - \beta)C_{d,t}^\theta + \beta(E_t[U_{d,t+1}^{1-\gamma}])^{\theta/(1-\gamma)}\}^{1/\theta}; \quad (1)$$

here  $U$  is utility,  $t$  denotes time,  $\beta$  is the subjective discount factor,  $C$  denotes consumption,  $E$  is the expectation operator,  $\gamma$  is the constant relative risk aversion (CRRA),  $\theta = \frac{1-\gamma}{1-1/\psi}$ , and  $\psi \equiv \frac{1}{1-\theta}$  is the intertemporal elasticity of substitution. We assume that  $\psi > 1/\gamma$ , so that the representative agent has a preference for early resolution of uncertainty and fears variations in the economy's long-run prospects.

The stochastic discount factor is given by

$$M_{d,t+1} = \beta \left( \frac{C_{d,t+1}}{C_{d,t}} \right)^{\theta-1} \left( \frac{U_{d,t+1}}{E_t[U_{d,t+1}^{1-\gamma}]^{1/(1-\gamma)}} \right)^{1-\gamma-\theta}, \quad (2)$$

where the last term captures the agent's concerns about the uncertainty in future growth. The household consumes, supplies labor to the final producers, and makes investment/saving decisions participating in complete international financial markets. The household budget constraint is accordingly

$$C_{d,t} + E_t[M_{d,t+1}\mathcal{A}_{d,t+1}] = W_{d,t}L_{d,t} + \mathcal{A}_{d,t},$$

where  $W_{d,t}$  is the wage rate,  $L_{d,t}$  denotes hours worked, and  $\mathcal{A}_{d,t}$  is the state-contingent value of the household's financial wealth. Since there is no disutility of labor, the household supplies its entire endowment, which is normalized to 1.

### 3.2 Final Good Producers

Domestic final producers are perfectly competitive. They use capital ( $K_{d,t}$ ), labor ( $L_{d,t}$ ), and a composite of domestic and foreign intermediate goods ( $G_{d,t}$ ), to produce a nontraded final good ( $Y_{d,t}$ ) according to the following Cobb–Douglas production function:

$$Y_{d,t} = \left( K_{d,t}^\alpha (\Omega_{d,t} L_{d,t})^{(1-\alpha)} \right)^{(1-\xi)} G_{d,t}^\xi. \quad (3)$$

The composite good  $G_{d,t}$  is defined as

$$G_{d,t} = \left[ \sum_{i=1}^{N_{d,t}^d} (X_{d,i,t}^d)^\nu + \sum_{i=1}^{N_{f,t}^d} (X_{f,i,t}^d)^\nu \right]^{1/\nu}; \quad (4)$$

here  $X_{d,i,t}^d$  is the amount of domestically produced intermediate good  $i$  that is used for final production in the domestic economy,  $X_{f,i,t}^d$  is the amount of foreign-produced intermediate good  $i$  that is used for final production in the domestic economy,  $N_{d,t}^d$  (resp.  $N_{f,t}^d$ ) is the mass of domestic (resp. foreign) intermediate goods that is used by domestic final producers, and  $1/(1 - \nu)$  is the elasticity of substitution across intermediate goods with  $\nu < 1$ . The parameter  $\alpha$  governs the physical capital share, and  $\xi$  is the share of materials in final production (to which we shall later refer as intangible capital). Throughout the paper, a variable's subscript refers to the origin country and its superscript refers to the destination country. Note that intermediate goods are aggregated according to a constant elasticity of substitution' production function (à la Ethier 1979)—which implies that, if expenditures are held constant, then a higher number of varieties increases the productivity of the final producers. In this setup, the larger is the elasticity of substitution between intermediate varieties, the greater is the effect of varieties on productivity.

The exogenous process  $\Omega_{d,t}$  is the source of exogenous uncertainty in our model. We assume that  $\Omega_{d,t} = e^{a_{d,t}}$ , where  $a_{d,t}$  follows the first-order autoregressive, or AR(1), process

$$a_{d,t} = \varphi a_{d,t-1} + \varepsilon_{d,t},$$

and  $\varepsilon_{d,t} \sim N(0, \sigma^2)$ . We allow for cross-country correlation in the exogenous technology shocks and put  $\rho = \text{corr}(\varepsilon_{d,t}, \varepsilon_{f,t})$ .<sup>9</sup>

Final producers choose capital, labor, investment, and intermediate goods so as to maximize shareholder value subject to the production technology (3). Formally, we have

$$\max_{\{I_{d,t}, L_{d,t}, K_{d,t+1}, X_{d,i,t}^d, X_{f,i,t}^d\}_{t \geq 0}} E_0 \left[ \sum_{t=0}^{\infty} M_{d,t} D_{d,t} \right], \quad (5)$$

where firm's dividends are given by

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t} L_{d,t} - \sum_{i=1}^{N_{d,t}^d} P_{d,i,t}^d X_{d,i,t}^d - \sum_{i=1}^{N_{f,t}^d} P_{f,i,t}^d X_{f,i,t}^d. \quad (6)$$

Here  $M_{d,t}$  is the stochastic discount factor,  $W_{d,t}$  is the wage rate,  $I_{d,t}$  is investment in physical

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<sup>9</sup>When solving the model, we augment the AR(1) process for exogenous technology with an error correction term in the spirit of Colacito and Croce (2013). This term ensures the stability of our solution method and has virtually no effect on our results.

capital,  $P_{d,i,t}^d$  is the price of a domestically produced intermediate good, and  $P_{f,i,t}^d$  is the price of a foreign-produced intermediate good that is used for domestic production. Both prices are expressed in units of the domestic producer's final good.

The law of motion for physical capital is given by

$$K_{d,t+1} = (1 - \delta)K_{d,t} + \Lambda \left( \frac{I_{d,t}}{K_{d,t}} \right) K_{d,t}, \quad (7)$$

where  $\delta \in (0, 1)$  is the depreciation rate and  $\Lambda \left( \frac{I_{d,t}}{K_{d,t}} \right)$  captures convex capital adjustment costs.<sup>10</sup>

### 3.3 Intermediate Good Producers

In each country, a set of monopolistically competitive firms produces a differentiated good using final output according to a constant return to scale production function (one unit of final output is used to produce one unit of the intermediate good). All intermediate producers produce with the same efficiency. Intermediate producers produce for both the domestic and foreign market. To sell the good abroad, each producer incurs an iceberg transport cost  $\tau$ . Every period, each domestic intermediate producer  $i$  solves the following static profit maximization problem:

$$\begin{aligned} \max_{P_{d,i,t}^d, P_{d,i,t}^{f*}} \Pi_{d,i,t} &\equiv \max_{P_{d,i,t}^d, P_{d,i,t}^{f*}} (\pi_{d,i,t}^d + \pi_{d,i,t}^f) \\ &= \max_{P_{d,i,t}^d} P_{d,i,t}^d X_{d,i,t}^d(P_{d,i,t}^d) - X_{d,i,t}^d(P_{d,i,t}^d) \\ &\quad + \max_{P_{d,i,t}^{f*}} \underbrace{(P_{d,i,t}^f Q_t)}_{P_{d,i,t}^{f*}} X_{d,i,t}^f(\underbrace{P_{d,i,t}^f Q_t}_{P_{d,i,t}^{f*}}) - X_{d,i,t}^f(\underbrace{P_{d,i,t}^f Q_t}_{P_{d,i,t}^{f*}}), \end{aligned} \quad (8)$$

where  $\pi_{d,i,t}^d$  (resp.  $\pi_{d,i,t}^f$ ) are the profits from selling the domestic product at home (resp. abroad) and where  $P_{d,i,t}^{f*} = P_{d,i,t}^f Q_t$  is the price (in units of the domestic good) of a domestically produced intermediate good that is being exported. We use  $Q_t$  to denote the real exchange rate, defined as the number of domestic final goods per unit of foreign final good.<sup>11</sup>

<sup>10</sup> As in Jermann (1998),  $\Lambda_{d,t} \equiv (I_{d,t}/K_{d,t}) = (\alpha_1/\zeta)(I_{d,t}/K_{d,t})^\zeta + \alpha_2$ . The parameters  $\alpha_1$  and  $\alpha_2$  are chosen so that there are no adjustments costs in the steady state, and  $1/(1 - \zeta)$  is the elasticity of the investment rate with respect to Tobin's  $Q$ .

<sup>11</sup> We express real prices of the intermediate goods in units of the importers' final good. In particular, when the domestic (foreign) intermediate good is used for the production of the foreign (domestic) final output,



## 3.4 Innovation and Adoption

### 3.4.1 Innovation

In each country, innovators invest resources (final output) to introduce new prototypes of a product. If an innovator is successful then it starts producing the new good as an intermediate producer. Each domestic innovator  $i$  chooses  $S_{d,i,t}$  units of final output to maximize the present discounted value of future profits that it expects to obtain from selling the good to both domestic and foreign producers.

The law of motion of new prototypes is

$$N_{d,t+1}^d = \vartheta_{d,t} S_{d,t} + (1 - \phi) N_{d,t}^d; \quad (9)$$

here  $N_{d,t}^d$  is the mass of new technologies arriving in country  $d$  at time  $t$ ,  $\phi$  is the exogenous probability that a new variety becomes obsolete, and  $\vartheta_{d,t}$  is the productivity of innovation. Following Comin and Gertler (2006), we assume that this productivity takes the following functional form:

$$\vartheta_{d,t} = \frac{\chi N_{d,t}^d}{S_{d,t}^{1-\eta} (N_{d,t}^d)^\eta}. \quad (10)$$

The process  $S_{d,t} = \sum_{i=1}^{N_{d,t}^d} S_{d,i,t}$  describes the total R&D expenditure in the domestic country (in terms of the domestic final good). The parameter  $\eta$  denotes the elasticity of innovation w.r.t. R&D and  $\chi$  is a scaling parameter. In this specification,  $\vartheta_{d,t}$  is an externality; it is taken as given when the innovators choose their optimal investment in R&D.

### 3.4.2 International Adoption

We assume that international adoption is exogenous and that, in every period, only a fraction  $\vartheta_d^f$  of intermediate goods from country  $d$  can be used by the final good producer in country  $f$ . The parameter  $\vartheta_d^f$  governs the speed of adoption and is crucial in our mechanism. The law of motion of domestic intermediate goods that can be used by the foreign final producer evolves according to

$$N_{d,t+1}^f = \vartheta_d^f (1 - \phi) (N_{d,t}^d - N_{d,t}^f) + (1 - \phi) N_{d,t}^f, \quad (11)$$

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we have  $P_{d,t}^f \equiv (1/\nu)\tau Q_t^{-1}$  and  $P_{f,t}^d \equiv (1/\nu)\tau Q_t$ .

where  $N_{d,t}^f$  is the number of domestic goods imported by the foreign economy. It follows that the mass of domestic varieties not yet adopted by the foreign country is equal to  $N_{d,t}^d - N_{d,t}^f$ .

### 3.4.3 Value Functions

We assume that every innovation produced in a country can immediately be used by the final producer of that country. However, a new intermediate product can be sold abroad only with probability  $\vartheta_d^f$ , so that  $N_{dt}^f/N_{dt}^d < 1$  is the fraction of domestically produced intermediate goods that are used by final producers in the foreign country.

The value of a domestic innovation,  $V_{d,i,t}$ , is given by the present discounted value of the profits that innovator  $i$  expects to obtain from selling the good domestically and abroad. Let the value of the domestic innovations that are immediately sold to the domestic (foreign) final producers be  $V_{d,i,t}^d$  ( $V_{d,i,t}^f$ ), and let the value of the innovations that can potentially be adopted by country  $f$  be  $J_{d,i,t}^f$ . Then

$$V_{d,i,t} = V_{d,i,t}^d + J_{d,i,t}^f, \quad (12)$$

where

$$V_{d,i,t}^d = \pi_{d,i,t}^d + (1 - \phi)E_t[M_{d,t+1}V_{d,i,t+1}^d], \quad (13)$$

$$V_{d,i,t}^f = \pi_{d,i,t}^f + (1 - \phi)E_t[M_{d,t+1}V_{d,i,t+1}^f], \quad (14)$$

$$J_{d,i,t}^f = (1 - \phi)E_t \left[ M_{d,t+1} \left( \vartheta_d^f V_{d,i,t+1}^f + (1 - \vartheta_d^f) J_{d,i,t+1}^f \right) \right]. \quad (15)$$

So with probability  $\vartheta_d^f$  the firm can sell the product abroad at  $t + 1$ , and with probability  $1 - \vartheta_d^f$  the product remains unadopted.

Discounted future profits on patents are the payoff to innovators. Because the R&D sector is competitive, the free-entry condition for R&D investment in the symmetric equilibrium in which all firms are identical is

$$S_{d,t} = E_t [M_{d,t+1}V_{d,t+1}] (N_{d,t+1}^d - (1 - \phi)N_{d,t}^d) \quad (16)$$

or, equivalently,

$$\frac{1}{\vartheta_{d,t}^f} = E_t [M_{d,t+1}V_{d,t+1}]. \quad (17)$$

### 3.5 Resource Constraint

Final output is used for consumption, intermediate goods production and investment in R&D. The resource constraint is therefore

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N_{d,t}^d X_{d,t}^d + N_{d,t}^f X_{d,t}^f .$$

### 3.6 Equilibrium and Steady State

We define a symmetric equilibrium as a set of equations according to which all firms within a country behave symmetrically.

For each country  $i = (d, f)$ , a *general symmetric equilibrium* is defined as an exogenous stochastic sequence of technology shocks  $\{\Omega_{i,t}\}_{t=0}^\infty$ , an initial vector  $\{N_{d,0}^d, N_{d,0}^f, N_{f,0}^f, N_{f,0}^d, K_{d,0}, K_{f,0}\}$ , a set of parameters  $\{\beta, \theta, \gamma, \psi, \alpha, \xi, \varphi, \sigma, \rho, \delta, \theta, \nu, \chi, \phi, \eta, \tau, \vartheta_d^f\}$ , a sequence of aggregate prices  $\{W_{i,t}, V_{i,t}, Q_t, q_{it}\}_{t=0}^\infty$ , value functions  $\{V_{it}^i, J_{dt}^f, J_{ft}^d, V_{dt}^f, V_{ft}^d\}_{t=0}^\infty$ , a sequence of intermediate good prices  $\{P_{d,t}^d, P_{d,t}^f, P_{f,t}^f, P_{f,t}^d\}_{t=0}^\infty$ , a sequence of aggregate quantities  $\{Y_{i,t}, G_{i,t}, C_{i,t}, I_{i,t}, L_{i,t}, S_{i,t}\}_{t=0}^\infty$ , quantities of intermediate goods  $\{X_{d,t}^d, X_{d,t}^f, X_{f,t}^f, X_{f,t}^d\}_{t=0}^\infty$ , a sequence of profits  $\{\Pi_{d,t}, \Pi_{f,t}, \pi_{d,t}^d, \pi_{d,t}^f, \pi_{f,t}^f, \pi_{f,t}^d\}_{t=0}^\infty$  and laws of motion  $\{N_{d,t+1}^d, N_{d,t+1}^f, N_{f,t+1}^f, N_{f,t+1}^d, K_{i,t+1}\}_{t=0}^\infty$  such that the following conditions hold.

- The state variables satisfy their respective laws of motion.
- The endogenous variables solve the producers', innovators', and representative households' problems.
- The resource constraint is satisfied.
- Prices are such that all markets clear.

The equilibrium conditions are given in Appendix B.

### 3.7 Asset Prices

We assume that stocks are claims to all the production sectors: the final good sector, the intermediate good sector, and also the innovation sector. Hence we define the aggregate dividend as the net payout from the production sector,

$$\mathcal{D}_{d,t} = D_{d,t} + N_{d,t}^d \pi_{d,t}^d + N_{f,t}^d \pi_{f,t}^d - S_{d,t}. \quad (18)$$

Optimality implies the following asset pricing condition:

$$\mathcal{P}_{d,t} = E_t[M_{d,t+1}(\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})],$$

where  $\mathcal{P}_{d,t}$  is the domestic stock market price and  $\mathcal{D}_{d,t}$  is the aggregate market dividend.

Since financial markets are complete, it follows that the exchange rate depreciation is given by the ratio of foreign to domestic stochastic discount factors; thus,

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{f,t+1}}{M_{d,t+1}}. \quad (19)$$

Recall that preferences are recursive and so the risk-sharing mechanism is nonstandard because agents fear not only current shocks but also variation in future utility. Formally, let  $\Upsilon_t = Q_t \left( \frac{C_{d,t}}{C_{f,t}} \right)^{\theta-1}$ . Using the expression for the stochastic discount factor in (2) together with the no-arbitrage condition (19), we can express  $\Upsilon_t$  recursively as

$$\Upsilon_{t+1} = \Upsilon_t \frac{M_{f,t+1}}{M_{d,t+1}} \frac{e^{(\theta-1)\Delta c_{d,t+1}}}{e^{(\theta-1)\Delta c_{f,t+1}}} . \quad (20)$$

Notice that  $\Upsilon_t$  is constant in the CRRA case. Under Epstein–Zin recursive preferences, however,  $\Upsilon_t$  evolves as a function of the cross-country realizations of the agents' continuation utilities. Colacito and Croce (2013) provide a thorough analysis of this mechanism.

## 4 The Mechanism: Aggregate Productivity and the Stock Market

In this section we present the expression for aggregate productivity as asset prices that are central to our mechanism. Domestic aggregate productivity can be expressed as

$$Z_{d,t} \equiv \Omega_{d,t} \left( \tilde{A} \right)^{1/(1-\alpha)} \left[ N_{d,t}^d + (\tau Q_t)^{\nu/(\nu-1)} N_{f,t}^d \right], \quad (21)$$

where  $\tilde{A} \equiv (\xi\nu)^{\xi/(1-\xi)}$ . Taking logs, we have

$$\log Z_{d,t} = \log \Omega_{d,t} + \log \left\{ \left( \tilde{A} \right)^{1/(1-\alpha)} \left[ N_{d,t}^d + (\tau Q_t)^{\nu/(\nu-1)} N_{f,t}^d \right] \right\}.$$

Thus aggregate productivity has both an exogenous and an endogenous component, that is

$$\log(\text{TFP}_{d,t}) = \log(\text{TFP}_{d,t}^{\text{EXO}}) + \log(\text{TFP}_{d,t}^{\text{ENDO}}).$$

Here

$$\log(\text{TFP}_{d,t}^{\text{EXO}}) \equiv \log \Omega_{d,t}$$

and

$$\log(\text{TFP}_{d,t}^{\text{ENDO}}) \equiv \log \left\{ \left( \tilde{A} \right)^{1/(1-\alpha)} \left[ N_{d,t}^d + (\tau Q_t)^{\nu/(\nu-1)} N_{f,t}^d \right] \right\} .$$

The exogenous component of aggregate productivity is given by the stochastic process  $\Omega_{d,t}$ ; the endogenous component, which plays a crucial role in our mechanism, depends on the number  $N_{d,t}^d$  of varieties that have been produced domestically and the number  $N_{f,t}^d$  of varieties that have been produced in the foreign country and are already adopted at home. We refer to  $N_{d,t}^d$  as the *domestic* component of endogenous aggregate productivity and to  $(\tau Q_t)^{\nu/(\nu-1)} N_{f,t}^d$  as its foreign component. Given the process of innovation and adoption in Section 3.4, we can show that endogenous aggregate productivity is positively affected both by home-country R&D and by the foreign R&D embodied in imports. Thus foreign R&D diffuses across countries through trade in varieties, generating a positive comovement of aggregate productivity across countries. This process of innovation and international diffusion results in a positive productivity shock that has a persistent effect on the productivity of a country and its trading partner, which helps to explain the positive correlation of quantities.<sup>12</sup> To understand why this channel has a quantitatively relevant effect on asset prices, we must explicate the role of recursive preferences.

For asset prices, the main mechanism works as follows. Risky growth through endogenous innovation and recursive preferences determine the optimal level of R&D and hence the level of current and future expected growth. Innovations then spread across countries via a process of technology adoption that we measure in terms of trade in varieties. Risky growth has a first-order impact on the stock market and governs its international correlation structure. To see this, recall from equation (18) that aggregate dividends in each country are given by the present discounted value of the future profits of all firms operating in that country. In Appendix C, we show that stock market returns can be decomposed into three components

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<sup>12</sup>See Liao and Santacreu (2015) for a cross-sectional analysis of the mechanism.

as follows:

1. price of installed capital,  $q_{d,t}^k K_{d,t+1}$ ;
2. value of domestic technologies adopted by both domestic and foreign final producers,

$$N_{d,t}^d(V_{d,t}^d - \pi_{d,t}^d) + N_{d,t}^f(V_{d,t}^f - \pi_{d,t}^f),$$

where  $V_{d,t}^d = J_{d,t}^d$  and where  $V_{d,t}^f = \pi_{d,t}^f + (1 - \phi)E_t [M_{d,t+1}V_{d,t+1}^f]$  is the present discounted value of the future profits of firms that are established in the market and are also selling abroad;

3. value of existing technologies that have not (yet) been adopted,

$$(1 - \phi)(N_{d,t}^d - N_{d,t}^f)E_t [M_{d,t+1}J_{d,t+1}^f],$$

where

$$J_{d,t}^f = (1 - \phi)E_t [M_{d,t+1}(\vartheta_d^f V_{d,t+1}^f + (1 - \vartheta_d^f)J_{d,t+1}^f)].$$

We refer to the first element as the price of the *tangible* component of the stock market and to the remaining two elements as the price of its *intangible* component. Accordingly, we define  $r_{d,t}^s$  as the log return of the overall stock market,  $r_{d,t}^{\text{tan}}$  as the log return of its tangible component, and  $r_{d,t}^{\text{int}}$  as the log return of its intangible component.

To understand how our mechanism delivers a milder effect on the cross-country correlation of quantities than on that of asset price returns, it is useful to analyze the effect of a positive domestic productivity shock. After an increase in domestic aggregate productivity, final producers demand more intermediate goods, both domestic and foreign. The higher demand of intermediate goods increases the value of an innovation and more resources are allocated into R&D. Through the process of international adoption, both quantities and asset prices become correlated across countries. Recursive preferences and complete markets imply that this effect is milder on quantities than on asset prices. Because the domestic economy is now more innovative, expected future productivity increases. In turn, the long-run growth prospects of the economy improve. With recursive preferences, agents experience a substantial drop in their marginal utility because they fear variations in future utility that are associated with news about future growth. Complete financial markets then imply that

domestic agents are willing to shift a substantial amount of resources to the foreign economy. The final result is a cross-country correlation in consumption that is lower than what we would have obtained in the absence of our recursive risk sharing mechanism. Asset prices, on the other hand, become even more highly correlated, since asset prices in both countries reflect the present discounted value of the prospects of growth and these prospects are shared across countries through international adoption.

## 5 Quantitative Implications

In this section we present the quantitative implications of our model and explore its ability to replicate key international moments for macroeconomic quantities, stock market returns, and exchange rates. Our baseline model is calibrated at a quarterly frequency.

### 5.1 Calibration

We need to specify a total of sixteen parameters, whose values are reported in Table 5. We start by discussing those that are relatively more standard. Values for the preference parameters are set in the spirit of the long-run risk literature (e.g., Bansal and Yaron 2004 and Colacito and Croce 2013). In particular, we set the coefficient  $\gamma$  of relative risk aversion equal to 10 and the coefficient  $\theta$  equal to  $1/3$ , which implies intertemporal elasticity of substitution (IES) of  $\psi = 1.5$ . Under this calibration of the preference parameters, agents in the economy fear shocks to expected future growth. The subjective discount factor is chosen to pin down the mean of the risk-free rate; thus,  $\beta = 0.984^{1/4}$ .

The parameters relating to the final good production technology are obtained from Kung and Schmid (2011). The capital share  $\alpha$  is set to 0.35 to match the average capital share, and the share of intangible capital  $\xi$  is set to 0.5 as in Comin and Gertler (2006). The depreciation rate of physical capital is set to 0.02 and  $\zeta$ , which pins down the elasticity of the investment rate with respect to Tobin's  $Q$ , is set to  $1/3$ . The parameter  $\nu$  is set to 0.5, which is also consistent with the literature; this parameter is related to the elasticity of substitution across intermediate goods and pins down the intermediate good markup in our model.

We set the autocorrelation of the exogenous technology shock to 0.95 and the volatility parameter  $\sigma$  at 1.08% in order to obtain a realistic volatility for consumption and output growth. Finally, we fit an AR(1) process to the TFP of each of the 20 countries in our

Table 5: Parameters for the baseline quarterly calibration.

Description	Parameter	Value
<i>Preferences</i>		
Risk aversion	$\gamma$	10
IES	$\psi = 1/(1 - \theta)$	1.5
Subjective discount factor	$\beta^4$	0.984
<i>Final Production</i>		
Capital share	$\alpha$	0.35
Share of materials	$\xi$	0.5
Autocorrelation of $\Omega = e^a$	$\varphi$	0.95
Volatility of exogenous shock $\varepsilon$	$\sigma$	1.08%
Cross-correlation of exogenous shock	$\rho$	0.35
Depreciation of capital stock	$\delta$	0.02
Investment adjustment cost	$\zeta$	0.33
Inverse markup	$\nu$	0.5
<i>Innovation and International Adoption</i>		
Scale	$\chi$	0.4240
Innovation obsolescence rate	$\phi$	0.0375
Elasticity of innovation w.r.t. R&D	$\eta$	0.60
Shipping cost	$\tau$	1.5
International adoption	$\vartheta_d^f$	0.01

sample and compute the cross-country correlation of the error term; this procedure yields a cross-country correlation in the exogenous TFP shocks of 0.35.

We now discuss the nonstandard parameters that govern the process of innovation and technology adoption. The parameter  $\chi$  is a pure scaling parameter; we set its value such that the steady-state growth rate of consumption has an annualized mean of 1.9%. Together with  $\nu$ ,  $\chi$  then gives us a value of  $\tilde{A}$  that is consistent with the balanced growth restriction. The parameter  $\eta$  governs the elasticity of new varieties with respect to R&D and is set to 0.6, a number within the range of estimates given by Griliches (1990). Finally, we set  $\phi = 0.0375$ , which corresponds to a 15% annualized depreciation rate of the R&D stock (as is standard in the literature).

The last parameters that we calibrate are those associated with international trade costs:  $\tau$  and  $\vartheta_d^f$ . We set the iceberg transport cost parameter  $\tau$  to 1.5 in accordance with international trade flows (cf. Santacreu 2015). Finally, we calibrate  $\vartheta_d^f$ , which governs the strength of



the international adoption process and is key to our model's mechanism. For this calibration we use the law of motion of newly adopted varieties in equation (11) as follows:

$$\vartheta_d^f = \frac{\phi N_{d,t}^f}{(1 - \phi) (N_{d,t}^d - N_{d,t}^f)}.$$

We then use annual data on R&D together with disaggregated bilateral trade data to measure the right-hand side of this equation. Specifically:  $N_{d,t}^f$  is measured using the number of varieties that are exported from country  $d$  to country  $f$ , (i.e., the extensive margin of trade); and  $N_{d,t}^d$  is measured as the stock of R&D, computed via the perpetual inventory method using R&D expenditures and an annual depreciation rate of 15% (thus,  $\phi = 0.0375$ ). We obtain a value for each pair of countries and each time period. Averaging over time reveals that  $\vartheta_d^f$  ranges between 0.048 and 0.084, which corresponds to a quarterly average value of  $\vartheta_d^f = 0.01$ . This is the value that we use in our baseline calibration.

Given these parameters, we use perturbation methods to solve our system of equations. We compute a third order approximation of our policy functions using the Dynare++ package. All variables included in our code are expressed in log-units.<sup>13</sup>

## 5.2 Results

Table 6 reports simulated moments of four different calibrations: Baseline, CRRA, Fast Adoption, and EXO. For the CRRA calibration, we impose  $\psi = 1/\gamma$  and leave all other parameters unaltered. For the calibration with Fast Adoption we increase  $\vartheta_d^f$  to 0.02. The EXO calibration corresponds to a model in which innovation is exogenous.<sup>14</sup> All results shown are averages of 1,000 simulations of 100 quarters each.

In terms of the moments of macroeconomic quantities, our baseline model with recursive preferences can generate reasonable means and standard deviations for both output and consumption growth. The mean growth is 1.90% for both variables, and their respective standard deviations are 1.42% and 1.21%. Note that our model endogenously generates an extremely high autocorrelation in the conditional mean of future consumption growth and total TFP growth, which suggests the existence of a slow-moving component that governs

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<sup>13</sup>For additional details concerning the solution and the approximation of recursive economies with multiple agents see, Colacito and Croce (2012, 2013) and Rabitsch, Stepanchuk, and Tsyrennikov (2015).

<sup>14</sup>In this version of the model, new prototypes arrive exogenously according to a Poisson process and so steady-state growth rate of consumption remains the same; hence growth is exogenous.

future growth prospects (cf. Bansal and Yaron 2004). Table 6 also reports moments of the properties of innovation and R&D expenditure. The cross-country correlation of R&D intensity is about 0.34 in our calibration and approximately 0.4 in the data. Similar values are obtained for the cross-country correlation of the growth rate in the number of varieties.

The model generates a cross-country correlation of consumption growth of 0.17. This relatively low correlation is in line with the empirical estimates of Section 2 and with previous studies. Observe that this value is lower than the calibrated cross-country correlation of exogenous TFP shocks—a consequence of the risk-sharing mechanism implied by recursive preferences. As argued by Colacito and Croce (2013), agents experience a sharp drop in their marginal utility when a positive long-term shock hits the domestic economy; the result is a substantial reallocation of resources towards the foreign country and, in the end, a lower cross-country correlation in consumption growth. The novel feature of our model is that long-run shocks are the endogenous outcome of the innovation mechanism.

With regard to asset pricing moments, our model generates a volatility of the exchange rate depreciation of 9.62% and a cross-country correlation in stock market returns ( $r^s$ ) as high as 0.49. This high correlation manifests the mechanism at work in our model. Indeed, when focusing on the various components of stock market returns, we note that the cross-country correlation in the returns on tangible capital is low (0.28). At the same time, the return on intangible capital is highly correlated across countries (0.78), which strongly suggests that the international adoption of foreign innovation is a significant driver of comovements in international asset prices. Finally, we note that our model can generate a low and persistent risk-free rate, as observed in the data.<sup>15</sup>

**Why recursive preferences? The CRRA case.** Recursive preferences are crucial to our mechanism because they allow for realistic dynamics of asset prices without compromising the performance of the model with respect to macroeconomic quantities. The second data column of Table 6 reports the results obtained in the standard CRRA case. The dynamics of macroeconomic quantities within each country are only marginally changed, but the model with CRRA preferences has several serious drawbacks. In particular, it cannot account for the sizable wedge—observed in the data—between cross-country correlations in consumption

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<sup>15</sup>In the current baseline calibration, the level and volatility of stock market returns are less than what we observe in the data. A similar issue arises in Kung and Schmid (2011) and in Colacito, Croce, Ho, and Howard (2012). This problem can be considerably alleviated by introducing—in the spirit of Boldrin, Christiano, and Fisher (2001)—leverage into the model.

Table 6: Simulated moments for macroeconomic quantities and asset prices. Results shown are averages of 1000 simulations of 100 quarters. The subscript  $d$  (domestic) and  $f$  (foreign) are suppressed when there is no ambiguity. ‘Baseline’ refers to our baseline calibration in Table 5. ‘CRRA’ refers to the constant relative risk aversion case and is obtained by setting  $\psi = 1/\gamma$ . ‘EXO’ refers to the model with exogenous growth. ‘Fast Adoption’ refers to a calibration with  $\vartheta_d^f = 0.02$ .

	Baseline	CRRA	EXO	Fast adoption
<i>Macro Quantities</i>				
$E(\Delta c)$	1.900	1.900	1.900	1.900
$\text{Std}(\Delta c)$	1.213	1.106	1.325	1.211
$\text{ACF}_1 E_t(\Delta c_{t+1})$	0.899	0.901	0.903	0.892
$E(\Delta y)$	1.900	1.900	1.900	1.900
$\text{Std}(\Delta y)$	1.416	1.379	1.378	1.409
$E(\Delta z)$	1.900	1.949	1.899	1.901
$\text{Std}(\Delta z)$	2.185	2.129	2.126	2.174
$\text{ACF}_1 E_t(\Delta z_{t+1})$	0.900	0.900	0.899	0.899
$\text{Corr}(\Delta c_d, \Delta c_f)$	0.169	0.427	0.497	0.194
$\text{Corr}(\Delta y_d, \Delta y_f)$	0.322	0.388	0.435	0.332
$\text{Corr}(\Delta z_d, \Delta z_f)$	0.322	0.388	0.435	0.332
$\text{Corr}(\Delta s_d, \Delta s_f)$	0.338	0.320		0.318
$\text{Corr}(\Delta n_d, \Delta n_f)$	0.302	0.262		0.286
<i>Asset prices</i>				
$E(r^f)$	2.818	20.440	2.872	2.823
$\text{ACF}_1 (r^f)$	0.899	0.896	0.903	0.898
$\text{Std}(\Delta q)$	9.621	11.839	1.083	8.481
$\text{Corr}(r_d^s, r_f^s)$	0.491	0.376	0.371	0.520
$\text{Corr}(r_d^{\text{tan}}, r_f^{\text{tan}})$	0.277	0.350	0.461	0.271
$\text{Corr}(r_d^{\text{int}}, r_f^{\text{int}})$	0.782	0.405	0.288	0.828

growth and stock market returns, cross-country correlation in consumption growth is too high (0.43), and cross-country correlation in stock market returns is too low (0.38). With CRRA, agents do not fear variation about the economy’s future prospects, because their marginal rates of substitution reflect only current realization of consumption growth. The quantitative effect of this mechanism is evident from the cross-country correlation of the intangible part of stock market returns, which declines to 0.40 (from 0.72 in our Baseline calibration). We also note that the average of the risk-free rate is too high, a manifestation of the well-known risk-free rate puzzle.

**Why endogenous growth? The EXO case.** When growth is exogenous, the risk propagation mechanism at the core of our baseline calibration is muted. Put simply: short-run

risk, which comes from the exogenous TFP process, does not have long-run effects. In this case, the power of recursive preferences is limited. If there is little variation in the economy's future prospects, then agents will not attach a sizable price of risk to it, and so the dynamics of asset prices will resemble those in the standard CRRA case. Quantitatively, Table 6 shows that the cross-country correlation of consumption growth increases to 0.50, while the cross correlation of stock market returns drops to 0.37.

**The Role of International Adoption.** The last column of Table 6 shows the results we obtain after increasing the  $\vartheta_d^f$  parameter to 0.02. This value remains within the range of our empirical estimates yet implies foreign innovations are adopted more rapidly. According to our mechanism, faster adoption of foreign R&D has a significant effect on the dynamics of asset prices. We see in particular that, relative to our Baseline calibration, stock market returns are more correlated and exchange rates are less volatile in the Fast Adoption scenario. These results are consistent with our empirical findings that point to the effects of international R&D diffusion on asset prices.

## 6 Predictability Regressions

Our model implies that domestic R&D and foreign R&D embodied in imported intermediate products should predict domestic TFP growth at lower frequencies. We provide empirical evidence for this mechanism in our sample of 20 countries for the 1993–2009 period. Total factor productivity is computed as the Solow residual; thus, for each country  $i$ ,

$$\log(z_{i,t}) = \log(y_{i,t}) - \alpha \log(n_{i,t}) - (1 - \alpha) \log(k_{i,t}).$$

Here  $z_{i,t}$  denotes aggregate productivity,  $y_{i,t}$  real income,  $n_{i,t}$  total employment, and  $k_{i,t}$  real physical capital stock. Nominal GDP data (annual index in national currency) are collected from the International Financial Statistics (IFS) database of the International Monetary Fund. We take the gross fixed capital formation (GFCF) data from IFS and the employment index from IFS and the Organisation for Economic Co-operation and Development (OECD) database. For OECD countries, the GFCF data are given by the VOBARSA series (millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted); the employment data are from the OECD Labour Force Statistics (MEI, Main

Table 7: Band-pass filtered TFP forecast

	(1)	(2)	(3)	(4)	(5)	(6)
	t=0	t=1	t=2	t=3	t=4	t=5
$\log(\sum_{j \neq i} EM_j^i \frac{R\&D_j}{GDP_j})$	0.086*** (0.014)	0.146*** (0.028)	0.163*** (0.042)	0.214*** (0.059)	0.228** (0.076)	0.310** (0.095)
$\log(\frac{R\&D_i}{GDP_i})$	0.075*** (0.004)	0.152*** (0.008)	0.233*** (0.013)	0.326*** (0.018)	0.421*** (0.023)	0.515*** (0.028)
Constant	-0.217 (0.136)	-0.191 (0.273)	0.274 (0.418)	0.412 (0.577)	0.906 (0.745)	0.739 (0.929)
Observations	6099	6099	5757	5396	5035	4674
$R^2$	0.064	0.061	0.061	0.067	0.071	0.076

Note: standard errors in parentheses.

\*\*\* $p < 0.001$

Economic Indicators) data set (all persons, index OECD base year 2005 = 100, seasonally adjusted). For other countries, data are from the IFS database. The GFCF data are deflated by a GDP deflator (2005 = 100, also from the IFS database) to obtain real capital formation data. For countries and periods with regard to which quarterly data are not available, we interpolate the annual index while assuming a constant volume every quarter within a given year. As a robustness check, we exclude the periods when quarterly data are not available; this change does not affect our results.

Physical capital is constructed using the perpetual inventory method with a constant quarterly depreciation of 2.5% and while assuming that the initial capital stock is zero. We follow the literature in setting  $\alpha$ , the labor share of income in GDP, to 0.64 for all countries.<sup>16</sup>

We then determine the quarter-to-quarter growth rates of our TFP measure by calculating the log-difference of the series just computed. Because we are interested in capturing low-frequency movements of this variable, we apply a band-pass filter that removes frequencies higher than 32 quarters. The next step is to run the following regression to the filtered data for 1993–2009 using annual data for R&D and international trade:

$$\Delta(\text{TFP}_{i,t+p}) = a + b \log(\text{R\&D}_{i,t}) + c \log\left(\sum_j EM_{j,t}^i \times \text{R\&D}_{j,t}\right) + u_{i,t}$$

<sup>16</sup> As a robustness check, we also calculate aggregate productivity for emerging markets while setting  $\alpha = 0.5$ ; this change does not affect our results.

for  $p = 0, 1, 2, 3, 4, 5$  years. The first term on the right hand side corresponds to domestic R&D intensity, and the second term corresponds to the total foreign R&D intensity embodied in imported intermediate goods.

The results are reported in Table 7. There are two main findings. First, the coefficients for domestic R&D are all positive and statistically significant, and they increase as the time horizon increases; this result is consistent with Kung and Schmid (2011). Thus domestic R&D intensity predicts the medium-term component of TFP over horizons of 1–5 years. Second, we test for whether foreign R&D embodied in imported intermediate goods can also be used to predict the medium-term component of TFP. This is our paper’s novel mechanism—namely, the existence of a common component in the TFP of countries trading with each other that is (i) driven by their R&D and (ii) weighted by how *much* they trade with each other. Just as in the case of domestic R&D, we find that the coefficients for the foreign component of R&D are positive and statistically significant and that their values increase over longer horizons. So, in accordance with our model’s predictions, the data reveal that both domestic and foreign R&D can predict TFP over longer horizons. Furthermore, the  $R^2$  of the regression is between 0.064 and 0.076 and likewise increases with the horizon.

In the Appendix D we run two additional sets of predictability regressions in which we consider separately the effects of domestic R&D and of foreign R&D embodied in trade. In line with the results have just presented, both innovations have predictive power over the medium-term component of TFP. It is worth remarking that the  $R^2$  of the predictability regression on foreign R&D is 6 times larger than the corresponding value for domestic R&D.

## 7 Conclusion

In this paper we offer a quantitative analysis of a symmetric, two-country, endogenous growth model of innovation and international adoption of foreign innovations through trade in varieties. We have shown—both theoretically and by way of a calibration exercise—that recursive preferences, together with our endogenous growth channels, are key to accounting for the cross-country correlation of quantities is weaker than cross-country correlation of asset prices.

We also provide empirical evidence of our proposed mechanism. First we show that country pairs with a higher R&D content of international trade have more highly correlated

stock market returns and less volatile exchange rate depreciation. Second, we find that domestic R&D and also foreign R&D embodied in traded intermediate goods systematically drive a TFP predictable component at low frequencies and over long time horizons.

The model developed in this paper could be extended to address other international asset pricing puzzles. For instance, relaxing the symmetry assumption would allow one to analyze what role our mechanism might play in explaining deviations in the uncovered interest parity condition and the profitability of the currency carry trade. We leave these issues for future research.

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# Appendix

## A Trade Data, Asset Prices, and Comovements

In this note, we describe trade data and asset prices and construct the measures relevant to our analysis.

### Trade Data

The source of our trade data is UN COMTRADE. We collect product data at the 6-digit level of disaggregation. The data is annual and covers the 1985-2009 period. We focus on the trade that occurs between the importer  $i$  (identified with its IISCODE) and the exporter  $j$  (identified by its EISCODE), and collect data on the kind of product that is traded (the 6 digits identifying it) and the dollar value of the trade in each product (the per-product trade value).

**Preliminary stats:** calculate the fraction of world trade and world GDP that is accounted for by the countries in our sample. For the entire list, refer to the paper.

From this data, we construct the following measures:

#### *Step 1:*

- Trade Intensity  $(i, j)$ :  $TI_{i,j}$ , i.e. the sum of the trade value of all the products
- Extensive margin  $(i, j)$ :  $EM_{i,j}$ , i.e. the number (the“count”) of different kinds of good imported by country  $i$  from country  $j$
- Intensive Margin  $(i, j)$ :  $IM_{i,j} = TI_{i,j}/EM_{i,j}$ , i.e. “how much”, in dollars, country  $i$  is trading on average for each product imported from country  $j$

In order to compare these numbers across-country pairs, we normalize them taking into account each country’s GDP. In particular, we define the normalized measures as

$$\tilde{TI}_{i,j} = \frac{TI_{i,j}}{GDP_i + GDP_j}$$

$$\tilde{EM}_{i,j} = EM_{i,j}$$

$$\tilde{IM}_{i,j} = \frac{\tilde{TI}_{i,j}}{\tilde{EM}_{i,j}}$$

Note: this country pair is ordered:  $i$  is the importer and  $j$  is the exporter, i.e.,  $\tilde{T}I_{i,j}$  will usually be different from  $\tilde{T}I_{j,i}$

Aside: we want to make sure that the relationship  $\tilde{T}I_{i,j} = \tilde{E}M_{i,j}\tilde{I}M_{i,j}$  holds, so that, taking logs, we can easily run linear regressions.

### **Step 2:**

For the measures above, we calculate their R&D *intensity*. In order to do so, we collect data on the percentage of each country's GDP that comes from expenditure in R&D. We obtain the R&D *intensity* of the trade intensity, of the extensive margin, and of the intensive margin as follows ( $k$  indexes the countries from which country  $i$  is importing):

$$\tilde{T}I_{i,j}^{R\&D} = \frac{TI_{i,j} \% R\&D_{GDP(j)}}{\sum_k TI_{i,k} \% R\&D_{GDP(k)}}$$

$$\tilde{E}M_{i,j}^{R\&D} = \frac{EM_{i,j} \% R\&D_{GDP(j)}}{\sum_k EM_{i,k} \% R\&D_{GDP(k)}}$$

$$\tilde{I}M_{i,j}^{R\&D} = \frac{IM_{i,j} \% R\&D_{GDP(j)}}{\sum_k IM_{i,k} \% R\&D_{GDP(k)}}$$

## **Asset Prices**

We consider two main statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the currency depreciation rate. We use monthly observations for the 1993/2009 period from the following sources: i) Ken French's website for stock market data, and ii) Global Financial Data exchange rates. From this data, we construct the following measures:

### **Stock Market**

- Cross-country stock market return correlations between country  $i$  and country  $j$ :  $corr(r_{i,t}^s, r_{j,t}^s)$ , for the entire sample. The return  $r^s$  is per quarter.

### **Exchange Rate**

- Quarterly log depreciation rate for currency  $i$  w.r.t. currency  $j$ :  $\Delta q_{i,t}^j = q_{i,t}^j - q_{i,t-1}^j$ , where  $q_{i,t}^j$  is the log exchange rate level at time  $t$  for country  $i$  (in units of currency  $i$  per one unit the  $j$  currency)
- Volatility of currency  $i$  depreciation rate w.r.t. currency  $j$ :  $vol(\Delta q_{i,t}^j)$  for the entire sample.

## B Model Equations

Here, we present the equations for the domestic econmy. The foreign economy is represented by similar equations.

### Preferences

$$U_{d,t} = \left\{ (1 - \beta) C_{d,t}^\theta + \beta \left( E_t \left( U_{d,t+1}^{1-\gamma} \right) \right)^{\frac{\theta}{1-\gamma}} \right\}^{\frac{1}{\theta}}$$

### Stochastic discount factor

$$M_{d,t+1} = \beta \left( \frac{C_{d,t+1}}{C_{d,t}} \right)^{\theta-1} \left( \frac{U_{d,t+1}}{E_t \left( U_{d,t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right)^{1-\gamma-\theta}$$

### Final producers

$$Y_{d,t} = (Z_{d,t} L_{d,t})^{1-\alpha} K_{d,t}^\alpha$$

### Labor

$$L_{d,t} = 1$$

### Aggregate productivity

$$Z_{d,t} \equiv \Omega_{d,t} \left( \tilde{A} \right)^{\frac{1}{1-\alpha}} \left[ N_{d,t}^d + (\tau Q_t)^{\frac{\nu}{\nu-1}} N_{f,t}^d \right]$$

$$\tilde{A} = (\xi \nu)^{\frac{\xi}{1-\xi}}$$

$$\Omega_{d,t} = e^{a_{d,t}}$$

$$a_{d,t} = \varphi a_{d,t-1} + \varepsilon_{d,t}$$

**First order condition of labor**

$$W_{d,t} = (1 - \alpha)(1 - \xi) \frac{Y_{d,t}}{L_{d,t}}$$

**First order condition of investment**

$$q_{d,t} = \frac{1}{\Lambda'_{d,t}}$$

$$1 = E_t \left[ M_{d,t+1} \left\{ \frac{1}{q_{d,t}} \left( \alpha(1 - \xi) \frac{Y_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}(1 - \delta) - \frac{I_{d,t+1}}{K_{d,t+1}} + q_{d,t+1} \Lambda_{d,t+1} \right) \right\} \right]$$

**Law of motion of capital**

$$K_{d,t+1} = (1 - \delta)K_{d,t} + \Lambda_{d,t}K_{d,t}$$

**Investment adjustment costs**

$$\Lambda_{d,t} \equiv \Lambda \left( \frac{I_{d,t}}{K_{d,t}} \right) = \frac{\alpha_1}{\zeta} \left( \frac{I_{d,t}}{K_{d,t}} \right)^\zeta + \alpha_2$$

$$\Lambda'_{d,t} = \alpha_1 \left( \frac{I_{d,t}}{K_{d,t}} \right)^{\zeta-1}$$

**Demand for domestic intermediate goods**

$$X_{d,t}^d = \left( \xi \nu Y_{d,t} G_{d,t}^{-\nu} \right)^{\frac{1}{1-\nu}}$$

**Demand for foreign intermediate goods (imports)**

$$X_{f,t}^d = \left( \xi \nu Y_{d,t} G_{d,t}^{-\nu} \right)^{\frac{1}{1-\nu}} (\tau Q_t)^{\frac{1}{\nu-1}} = X_{d,t}^d (\tau Q_t)^{\frac{1}{\nu-1}}$$

**Materials (intermediate goods)**

$$G_{d,t} = \xi \nu Y_{d,t} \left[ N_{d,t}^d + (\tau Q_t)^{\frac{\nu}{\nu-1}} N_{f,t}^d \right]^{\frac{1-\nu}{\nu}}$$

Profits of intermediate producers

$$\Pi_{d,t}N_{d,t}^d = \pi_{d,t}^d N_{d,t}^d + \pi_{d,t}^f N_{d,t}^f$$

Profits of domestic producers in the domestic market

$$\pi_{d,t}^d = \left(\frac{1}{\nu} - 1\right) X_{d,t}^d$$

Profits of domestic producers in the foreign market

$$\pi_{d,t}^f = \left(\frac{\tau}{\nu} - 1\right) X_{d,t}^f$$

Present Discounted Value (PDV) of a domestic producers selling in the domestic market

$$V_{d,t}^d = \pi_{d,t}^d + (1 - \phi)E_t[M_{d,t+1}V_{d,t+1}^d]$$

PDV of a domestic producers selling in the domestic market

$$V_{d,t}^f = \pi_{d,t}^f + (1 - \phi)E_t[M_{d,t+1}V_{d,t+1}^f]$$

PDV of a domestic producers not-yet selling in the domestic market

$$J_{d,t}^f = (1 - \phi)E_t \left[ M_{d,t+1} \left( \vartheta_d^f \pi_{dt+1}^f + (1 - \vartheta_d^f) J_{d,t+1}^f \right) \right]$$

Value of an innovation

$$V_{d,t} = V_{d,t}^d + J_{d,t}^f$$

Law of motion of new technologies

$$N_{d,t+1}^d = \vartheta_{d,t} S_{d,t} + (1 - \phi)N_{d,t}^d$$

$$\vartheta_{d,t} = \frac{\chi N_{d,t}^d}{S_{d,t}^{1-\eta} (N_{d,t}^d)^\eta}$$

## Free entry condition of innovation

$$S_{dt} = E_t [M_{d,t+1} V_{d,t+1}] (N_{d,t+1}^d - (1 - \phi) N_{d,t}^d)$$

## Law of motion of adopted technologies

$$N_{d,t+1}^f = \vartheta_d^f (1 - \phi) (N_{d,t}^d - N_{d,t}^f) + (1 - \phi) N_{d,t}^f$$

## Resource Constraint

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N_{d,t}^d X_{d,t}^d + N_{d,t}^f X_{d,t}^f$$

## International risk sharing

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{f,t+1}}{M_{d,t+1}}$$

# C Deriving the Stock Market

Dividends are generated by the final producers, the intermediate producers, and the innovators. The stock market is the present discounted value of the future dividends generated by all the firms in the economy. Optimality implies the following asset pricing condition:

$$\mathcal{P}_{d,t} = E_t [M_{d,t+1} (\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})] ,$$

where  $\mathcal{P}_{d,t}$  is the domestic stock market price, and  $\mathcal{D}_{d,t}$  is the aggregate market dividend. Substituting forward, we have

$$\mathcal{P}_{d,t} = E_t \sum_{i=0}^{\infty} M_{d,t+i+1} \mathcal{D}_{d,t+i+1}$$

Total dividends are

$$\mathcal{D}_{d,t} = D_{d,t} + N_{d,t}^d \pi_{d,t}^d + N_{f,t}^d \pi_{f,t}^d - S_{d,t} ,$$

with the dividends of the final producers,  $D_{d,t}$ , evolving according to

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t} L_{d,t} - N_{d,t}^d P_{d,t}^d X_{d,t}^d - N_{f,t}^d P_{f,t}^d X_{f,t}^d .$$

Consider the present discounted value of the dividends of the final producers,  $\mathcal{P}_{d,t}^{tan}$ . We have

$$\mathcal{P}_{d,t}^{tan} = E_t \left[ \sum_{i=0}^{\infty} M_{d,t+i+1} D_{d,t+i+1} \right]$$

or, in recursive form,  $\mathcal{P}_{d,t}^{tan} = E_t \left[ M_{d,t+1} \left( \mathcal{P}_{d,t+1}^{tan} + D_{d,t+1} \right) \right]$ .

**Result.**  $\mathcal{P}_{d,t}^{tan} = q_{d,t} K_{d,t+1}$ .

*Proof.* Consider the following expression:

$$E_t(M_{d,t+1} D_{d,t+1}) = E_t \left[ M_{d,t+1} (Y_{d,t+1} - I_{d,t+1} - W_{d,t+1} L_{d,t+1} - N_{d,t+1}^d P_{d,t+1}^d X_{d,t+1}^d - N_{f,t+1}^d P_{f,t+1}^d X_{f,t+1}^d) \right] .$$

From the FOC of labor, we have

$$W_{d,t+1} L_{d,t+1} = (1 - \alpha)(1 - \varepsilon) Y_{d,t+1} .$$

Use the first order conditions for intermediate producers to rewrite the expression

$$N_{d,t}^d P_{d,t}^d X_{d,t}^d + N_{f,t}^d P_{f,t}^d X_{f,t}^d$$

or, substituting for the prices of intermediate goods,

$$N_{d,t}^d \frac{1}{\nu} X_{d,t}^d + N_{f,t}^d \frac{\tau}{\nu} Q_t X_{f,t}^d .$$

Using

$$X_{f,t}^d = X_{d,t}^d (\tau Q_t)^{\frac{1}{\nu-1}} ,$$

we have

$$\left( N_{d,t}^d \frac{1}{\nu} + N_{f,t}^d \frac{\tau}{\nu} Q_t (\tau Q_t)^{\frac{1}{\nu-1}} \right) X_{d,t}^d = \left( N_{d,t}^d + N_{f,t}^d (\tau Q_t)^{\frac{\nu}{\nu-1}} \right) \frac{1}{\nu} X_{d,t}^d .$$

Similarly, using

$$X_{d,t}^d = \left( \varepsilon \nu Y_{d,t} G_{d,t}^{-\nu} \right)^{\frac{1}{1-\nu}}$$

and substituting  $G_{d,t} = \varepsilon \nu Y_{d,t} \left( N_{d,t}^d + N_{f,t}^d (\tau Q_t)^{\frac{\nu}{\nu-1}} \right)$ , we have

$$X_{d,t}^d = \varepsilon \nu Y_{d,t} \left( N_{d,t}^d + N_{f,t}^d (\tau Q_t)^{\frac{\nu}{\nu-1}} \right)^{-1} .$$

Plugging this expression into the total spending for intermediate producers, we have

$$\begin{aligned} \left( N_{d,t}^d \frac{1}{\nu} + N_{f,t}^d \frac{\tau}{\nu} Q_t (\tau Q_t)^{\frac{1}{\nu-1}} \right) X_{d,t}^d &= \left( N_{d,t}^d + N_{f,t}^d (\tau Q_t)^{\frac{\nu}{\nu-1}} \right) \frac{1}{\nu} X_{d,t}^d = \\ &= \left( N_{d,t}^d + N_{f,t}^d (\tau Q_t)^{\frac{\nu}{\nu-1}} \right) \frac{1}{\nu} \varepsilon \nu Y_{d,t} \left( N_{d,t}^d + N_{f,t}^d (\tau Q_t)^{\frac{\nu}{\nu-1}} \right)^{-1} = \varepsilon Y_{d,t} . \end{aligned}$$

Finally, consider the FOC for investment and rearrange it to obtain

$$q_{d,t} K_{d,t+1} = E_t [M_{d,t+1} (\alpha(1 - \varepsilon) Y_{d,t+1} - I_{d,t+1})] + E_t [M_{d,t+1} q_{d,t+1} ((1 - \delta) + \Lambda_{d,t+1}) K_{d,t+1}] .$$

From the law of motion of capital, we have

$$\left( \frac{(1 - \delta) + \Lambda_{d,t+1}}{K_{d,t+1}} \right) = \frac{K_{d,t+2}}{K_{d,t+1}}$$



and, substituting in the previous expression, we obtain

$$q_{d,t}K_{d,t+1} = E_t [M_{d,t+1} (\alpha(1 - \varepsilon)Y_{d,t+1} - I_{d,t+1})] + E_t [M_{d,t+1}q_{d,t+1}K_{d,t+2}] \quad .$$

Letting  $\hat{q}_t = q_t K_{t+1}$ , solving the expression above recursively, and imposing the standard transversality condition, we have

$$\hat{q}_t = E_t \left[ \sum_{i=0}^{\infty} M_{t+i+1} (\alpha(1 - \varepsilon) Y_{t+i+1} - I_{t+i+1}) \right] \quad .$$

Combining these results together, we have

$$\begin{aligned} \mathcal{P}_{d,t}^{tan} &= E_t \left[ M_{d,t+1} \left( \mathcal{P}_{d,t+1}^{tan} + D_{d,t+1} \right) \right] \\ &= E_t \left[ \sum_{i=0}^{\infty} M_{d,t+i+1} (-I_{d,t+i+1} + (\alpha)(1 - \varepsilon)Y_{d,t+i+1}) \right] = \hat{q}_{d,t} \end{aligned}$$

□

We now need to compute the present discounted values of the remaining terms in the expression for the market dividends. To compute the present discounted value of the profits of all the existing intermediate producers, we need take into account that there are two types of intermediate producers in the economy. First, there are firms that are selling both to the domestic and the foreign market. Second, there are firms that are not yet selling to the foreign market but have the possibility of doing that in the future.

The present discounted value of each firm that sells both to the domestic and foreign market today. These firms keep selling to both markets unless they do disappear with probability  $\phi$ . Let's call the present discounted value of the dividends of these firms as  $\pi_{d,t}$ , which are given in recursive form by

$$\pi_{d,t} = \pi_{d,t}^d + \pi_{d,t}^f + (1 - \phi)E_t [M_{d,t+1}\pi_{d,t+1}] = V_{d,t}^d + V_{d,t}^f$$

where the last equality uses the definition of the value function for one firm that is selling in the domestic market and the value function of one firm that is already selling in the foreign market. From the previous expression, the expected present discounted value of the future dividends for one firm that sells both in the domestic and in the foreign market, which is the component that we need to compute the stock market and we call  $V_{1t}$ , is

$$V_{1t} = (1 - \phi)E_t [M_{d,t+1}\pi_{d,t+1}] = (V_{d,t}^d - \pi_{d,t}^d) + (V_{d,t}^f - \pi_{d,t}^f)$$

Finally, the present discounted value of the dividends firms that only sell today in the domestic market but have a chance to sell tomorrow to the export market is given, in recursive form, by

$$\pi_{d,t}^d + (1 - \phi)E_t [M_{d,t+1} (\pi_{d,t+1}^d + J_{d,t+1}^f)] = V_{d,t}^d + (1 - \phi)E_t [M_{d,t+1} J_{d,t+1}^f]$$

From the previous expression, the expected present discounted value of the future dividends for one firm that sells only in the domestic market, which is the component that we need to compute the stock market and we call  $V_{2t}$ , is

$$V_{2t} = (1 - \phi)E_t [M_{d,t+1} (\pi_{d,t+1}^d + J_{d,t+1}^f)] = (V_{d,t}^d - \pi_{d,t}^d) + (1 - \phi)E_t [M_{d,t+1} J_{d,t+1}^f]$$

Since there are  $N_{d,t}^f$  firms selling both in the domestic and in the foreign market and  $(N_{d,t}^d - N_{d,t}^f)$  firms selling only in the foreign market, aggregating the previous expressions we can obtain the component of the stock market that is driven by the already established intermediate producers as

$$\begin{aligned}
& N_{d,t}^f V_{1t} + (N_{d,t}^d - N_{d,t}^f) V_{2t} = \\
& = N_{d,t}^f \left[ (V_{d,t}^d - \pi_{d,t}^d) + (V_{d,t}^f - \pi_{d,t}^f) \right] + (N_{d,t}^d - N_{d,t}^f) \left[ (V_{d,t}^d - \pi_{d,t}^d) + (1 - \phi) E_t \left[ M_{d,t+1} J_{d,t+1}^f \right] \right] \\
& = N_{d,t}^d (V_{d,t}^d - \pi_{d,t}^d) + N_{d,t}^f (V_{d,t}^f - \pi_{d,t}^f) + (N_{d,t}^d - N_{d,t}^f) \left( (1 - \phi) E_t \left[ M_{d,t+1} J_{d,t+1}^f \right] \right)
\end{aligned}$$

Therefore, there are three components to the stock market

1. Price of installed capital

$$q_{d,t} K_{d,t+1}$$

2. Value of intangible capital

$$N_{d,t}^d (V_{d,t}^d - \pi_{d,t}^d) + N_{d,t}^f (V_{d,t}^f - \pi_{d,t}^f)$$

3. Value of intangible capital that can potentially be sold abroad

$$(1 - \phi)(N_{d,t+1}^d - N_{d,t+1}^f) E_t \left[ M_{d,t+1} J_{d,t+1}^f \right]$$

## D Additional Empirical Results

Table 8: International trade and the correlation of stock market returns

	(1)	(2)	(3)
$\log(EM)_{ij}$	0.094*** (0.006)	0.085*** (0.005)	0.018 (0.009)
$\log(IM)_{ij}$	0.032*** (0.003)	0.038*** (0.003)	0.029*** (0.004)
Constant	0.422*** (0.068)	0.432*** (0.057)	0.841*** (0.102)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6460	6460	6460
$R^2$	0.075	0.361	0.442

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 9: R&D embodied trade and the correlation of stock market returns

	(1)	(2)	(3)
$\log(EM_{ij}^{R\&D})$	0.031*** (0.007)	0.021*** (0.006)	0.004 (0.009)
$\log(IM_{ij}^{R\&D})$	0.016*** (0.004)	0.019*** (0.004)	0.036*** (0.004)
Constant	0.747*** (0.013)	0.848*** (0.015)	0.812*** (0.044)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6422	6422	6422
$R^2$	0.029	0.310	0.441

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 10: R&amp;D intensity and the correlation of stock market returns

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i}) + \log(\frac{R\&D_j}{GDP_j})$	0.054*** (0.004)	0.025*** (0.004)	0.019 (0.014)
Constant	0.577*** (0.003)	0.695*** (0.012)	0.392*** (0.030)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6384	6384	6384
$R^2$	0.024	0.288	0.431

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ 

Table 11: Domestic and foreign R&amp;D intensity and the correlation of stock market returns

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i})$	0.051*** (0.008)	0.030*** (0.007)	0.055* (0.027)
$\log(\frac{R\&D_j}{GDP_j})$	0.061*** (0.010)	0.016 (0.009)	-0.019 (0.028)
Constant	0.577*** (0.005)	0.697*** (0.017)	0.401*** (0.047)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	3192	3192	3192
$R^2$	0.024	0.288	0.434

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 12: International trade and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(EM)_{ij}$	-3.523*** (0.160)	-3.476*** (0.148)	-3.731*** (0.233)
$\log(IM)_{ij}$	-1.384*** (0.085)	-1.354*** (0.079)	-1.217*** (0.088)
Constant	13.443*** (1.818)	15.295*** (1.697)	25.743*** (2.514)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6460	6460	6460
$R^2$	0.149	0.278	0.565

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ 

Table 13: R&amp;D embodied trade and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(EM_{ij}^{R\&D})$	-1.810*** (0.183)	-1.949*** (0.170)	-2.011*** (0.229)
$\log(IM_{ij}^{R\&D})$	-0.192 (0.122)	-0.134 (0.113)	-1.678*** (0.109)
Constant	3.081*** (0.343)	7.725*** (0.449)	-0.938 (1.091)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6422	6422	6422
$R^2$	0.059	0.198	0.561

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 14: R&amp;D intensity and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i}) + \log(\frac{R\&D_j}{GDP_j})$	-1.183*** (0.122)	-1.611*** (0.117)	0.782* (0.363)
Constant	9.900*** (0.095)	15.315*** (0.359)	27.581*** (0.765)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6384	6384	6384
$R^2$	0.014	0.163	0.515

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ 

Table 15: Domestic and foreign R&amp;D intensity and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i})$	-1.044*** (0.215)	-1.367*** (0.201)	1.082 (0.689)
$\log(\frac{R\&D_j}{GDP_j})$	-1.427*** (0.283)	-2.067*** (0.270)	0.444 (0.708)
Constant	9.916*** (0.135)	15.388*** (0.509)	26.640*** (1.187)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	3192	3192	3192
$R^2$	0.015	0.164	0.540

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 16: Band-pass filtered TFP forecast: Domestic R&amp;D intensity

	(1)	(2)	(3)	(4)	(5)	(6)
	t=0	t=1	t=2	t=3	t=4	t=5
$\log(\sum_{j \neq i} EM_{ij} \frac{R\&D_j}{GDP_j})$	0.140*** (0.014)	0.254*** (0.028)	0.340*** (0.043)	0.472*** (0.060)	0.580*** (0.077)	0.762*** (0.096)
Constant	-0.740*** (0.138)	-1.234*** (0.277)	-1.443*** (0.425)	-2.106*** (0.587)	-2.525*** (0.758)	-3.671*** (0.946)
Observations	6137	6137	5776	5415	5054	4693
$R^2$	0.016	0.013	0.011	0.011	0.011	0.013

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ 

Table 17: Band-pass filtered TFP forecast (foreign R&amp;D intensity embodied in trade

	(1)	(2)	(3)	(4)	(5)	(6)
	t=0	t=1	t=2	t=3	t=4	t=5
$\log(\frac{R\&D_i}{GDP_i})$	0.080*** (0.004)	0.159*** (0.008)	0.242*** (0.013)	0.339*** (0.017)	0.436*** (0.022)	0.536*** (0.028)
Constant	0.627*** (0.002)	1.252*** (0.005)	1.883*** (0.007)	2.516*** (0.010)	3.150*** (0.012)	3.784*** (0.015)
Observations	6099	6099	5757	5396	5035	4674
$R^2$	0.058	0.057	0.059	0.065	0.069	0.073

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$