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# Credit Search and Credit Cycles\*

Feng Dong<sup>†</sup>      Pengfei Wang<sup>‡</sup>      Yi Wen<sup>§</sup>

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## Abstract

The supply and demand of credit are not always well aligned and matched, as is reflected in the countercyclical excess reserve-to-deposit ratio and interest spread between the lending rate and the deposit rate. We develop a search-based theory of credit allocations to explain the cyclical fluctuations in both bank reserves and the interest spread. We show that search frictions in the credit market can not only naturally explain the countercyclical bank reserves and interest spread, but also generate endogenous business cycles driven primarily by the cyclical utilization rate of credit resources, as long conjectured by the Austrian school of the business cycle. In particular, we show that credit search can lead to endogenous local increasing returns to scale and variable capital utilization in a model with constant returns to scale production technology and matching functions, thus providing a micro-foundation for the indeterminacy literature of Benhabib and Farmer (1994) and Wen (1998).

*Keywords:* Search Frictions, Credit Utilization, Credit Rationing, Self-fulfilling Prophecy, Business Cycles.

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# 1 Introduction

The role of financial intermediation and credit supply in driving and amplifying the business cycle has long been analyzed in the history of economic thoughts at least since the Austrian school. The Austrian theory of the business cycle emphasizes bank issuance of credit as the main cause of economic fluctuations. It asserts that the banking sector's excessive credit supply and low interest policy (with loanable funds rate below the natural rate) drive firms' investment boom, and its tight credit and interest rate policy (with loanable funds rate above the natural rate) generate economic slump.

The history of financial crisis seemed consistent with the Austrian theory. A notable feature of financial crisis in 2007, for example, is that before the financial crisis both the bank interest rate and the reserve-to-deposit ratio were excessively low in the economic boom period leading to the financial crisis. On the verge of the financial crisis in late 2007 and early 2008, however, there was excessive demand for credit on the firm side (as reflected by the interest rate hike) and tightened credit supply on the bank side (as reflected in the significantly increased excess reserve-to-deposit ratio). For example, according to a survey on Chief Financial Officers (CFO) in 2008 by Campello, Graham and Harvey (2008), about 60 percent of U.S. CFOs states that their firms are financially constrained. Among them, 86% say that they have to pass attractive investment opportunities due to the inability to raise external financing. On the other hand, banks were building up their cash positions at unprecedented speed. Bank excessive reserves skyrocketed from 1.93 billion to 1043.30 billion from the second quarter of 2008 to the second quarter of 2010, while the growth of bank loans plunged 37.17% during the same period. Corresponding to the tight credit supply was the high interest rate on bank loans. Thus, we observe in Figure 1 a countercyclical movement in the excess reserve-to-deposit ratio (dashed line, re-scaled) and in Figure 2 a countercyclical movement in interest rate spread between the loan rate and the 3-month treasury bill rate (dashed line).

However, correlation is not causation. It is not clear from the figures whether the observed credit cycle and interest movements are endogenous outcome (symptoms) of the business cycle or the cause of it. This paper tries to shed light on such critical issues.

Specifically, we provide a search-based financial intermediation theory to explain the observed countercyclical excess reserve-to-deposit ratio and the countercyclical interest rate spread in the data. Our main starting point is that in the real world there are always agents with savings and agents with investment projects, but the demand side of the credit market (e.g., firms)

and the supply side of the credit market (e.g., households and banks) need to overcome search frictions to channel funds from savers to firms. This is especially the case in developing countries where financial markets are highly underdeveloped such that underground credit-search market and shadow banking are pervasive. We show that such search frictions can indeed lead to countercyclical excess reserve-to-deposit ratio and countercyclical interest rate spread. More importantly, they can also lead to endogenous business cycles driven by self-fulfilling beliefs on the tightness of credit conditions in the credit market. Such coordinated beliefs produce economic fluctuations through affecting the effective utilization rate of the aggregate credit resources. Moreover, our calibration analysis reveals that an endogenous multiplier-accelerator propagation mechanism that is rooted in credit search is not only theoretically appealing but also empirically plausible. The model captures many insights and predictions similar to those described by the Australian theory.

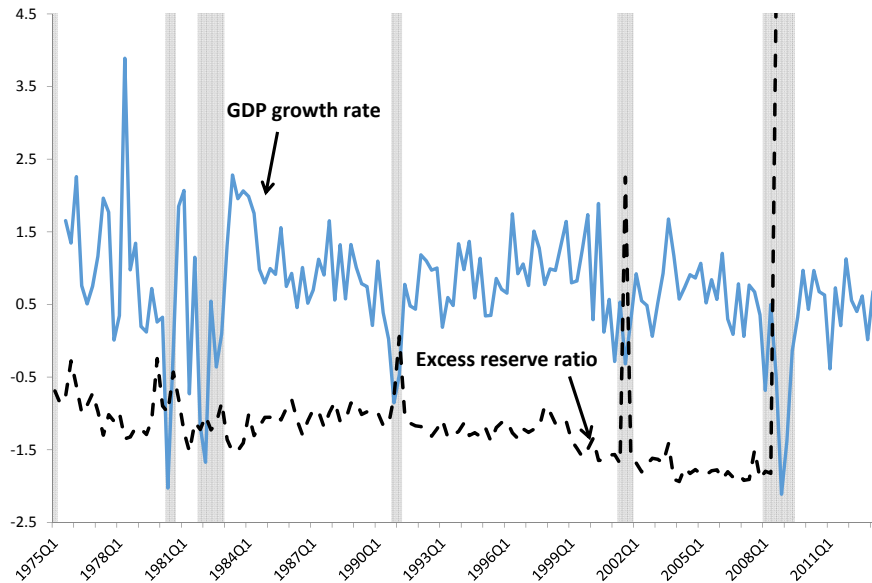


Figure 1: **GDP Growth Rate and Excess Reserve Ratio.** Data Source: Federal Reserve Economic Data (FRED).

Our model also sheds light on the issue of credit rationing. Credit rationing is not only of theoretical interest, but also plays a non-trivial role in real-world firm financing. As documented in Becchetti *et al* (2009) based on Italian firm data, around 20.24% of firms are subject to credit rationing in Italy. However, the literature on credit rationing is extremely thin despite the seminal work of Stiglitz and Weiss (1983). So, in addition to shedding light on

the cyclical behavior of bank reserves and interest spread and credit-led business cycles, our search-theoretical approach also provides a short cut to quantitatively study the business-cycle property of credit rationing.

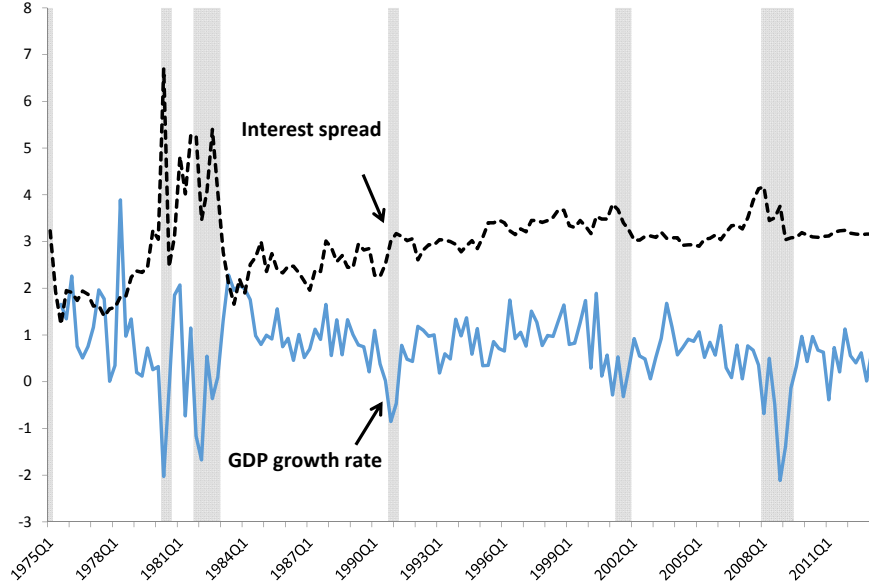


Figure 2: **GDP Growth Rate and Interest Spread (Loan Rate Minus Three-Month Treasury Bill). Data Source: FRED.**

To highlight the relevance of credit-market search frictions to the business cycle, our framework is by design kept extremely simple with off-the-shelf search-and-matching frictions. But we show that the results can be very powerful despite the simplicity of the model. Specifically, the benchmark model features three types of agents: a representative household with a continuum of *ex ante* identical members (depositors), a representative financial intermediary (bank) with a continuum of *ex ante* identical loan officers, and a continuum of firms. The banking sector accepts deposits from the household and then lends credit to firms through search and matching. We assume there are search frictions both between the household and the banking sector and between the banking system and firms. We will show which search frictions are more critical to generate self-fulfilling credit cycles.

Similar to the standard Diamond-Mortensen-Pissarides (DMP) search-and-matching model of unemployment, our model features aggregate matching functions that determine the number of credit relationships between depositors and financial intermediaries, and between loan officers and firms. Such search frictions create un-utilized credit resources in equilibrium, analogous

to unemployed labor force in the DMP model. For example, when bank deposits are not matched with firms, they become idle (excess reserves) in the banking system, while firms that are unmatched with loans are considered as being denied for credit. This simple matching framework then provides a quantitative framework to analyze and explains the coexistence of excess reserves and credit rationing in the data. Since a booming economy encourages more costly search in the credit market, it increases the matching probability of credit resources. As a result, the reserve-to-deposit ratio is countercyclical over the business cycle. In addition, since the deposit rate facing the household sector is determined mainly by time preference (the natural rate) and the lending rate facing firms determined mainly by credit availability and firms' credit demand, the spread between the loan rate and the deposit rate may also be countercyclical under aggregate shocks. Thus, our framework provides a natural interpretation of the Australian school's concepts of the natural rate and loanable rate of interest.

In addition, we show that such an elastic supply of credit due to variable utilization rate of existing credit resources under search and matching can lead to *local* increasing returns to scale (IRS) in the aggregate production function even though the underlying production and matching technologies both exhibit constant returns to scale (CRS). This endogenous source of local IRS caused by procyclical credit utilization can lead to local indeterminacy and self-fulfilling credit cycles that feature a powerful multiplier-accelerator propagation mechanism.

In our model, an anticipated increase in credit supply from the banking sector (in the absence of any fundamental shocks) would entice firms to increase search efforts, resulting in more credit matches. So, more capital would be channeled from the financial sector to firms, provided that the cost of borrowing does not increase sufficiently to discourage entry. With more loans (capital) in hand, firms can increase production by hiring more labor, so households' labor supply would also increase, leading to higher aggregate income and household savings. If the increase in household savings is large enough, it would then increase bank deposits and raise bank's credit supply without too much upward pressure on the loanable funds rate, ratifying firms' initial optimistic expectations about credit conditions. But the process does not stop here. Because of higher deposit, competition among banks will reduce the loan rate, which induces more firm to enter the credit market to search for loans. More firm-entry in turn would further increase the matching probability, raising the effective capital stock used in firms' production even more. Consequently, the economy can enter a persistent boom period (after an initial shock) that features all the symptoms of a credit cycle described by the Austrian school; namely, a seemingly easy credit policy is associated with a higher lending volume,

higher aggregate production, and higher employment, which in turn generates higher household consumption, savings, and bank deposits with possibly further lowered interest rate. However, in the absence of true productivity (technological) growth, such an economic boom is not sustainable in the long run, because the finitely available credit resources in the economy will eventually be exhausted under concave production technologies. This means that the loanable funds rate will eventually rise high enough to clear the credit market and end the boom. Once a boom ends, a prolonged recession will then follow because the above multiplier-accelerator feedback mechanism then reverses itself.

Hence, our model produces genuine credit cycles described by the Austrian economists: An economic boom led by credit expansion will plant the seed for an economic downturn, and a downturn will plant the seed for the next boom.

Technically speaking, the persistence of an endogenous credit cycle lies in the local IRS, which originates from a subtle pecuniary externality (based on search and firm entry) instead of technological IRS based on production externality (as in Benhabib and Farmer, 1994). Under local IRS, a proportionate increase in household labor supply and savings would render firms' effective capital stock and aggregate production to increase more than proportionally. Also, as the matching probability of credit increases, the banking sector is able to pay a proportionately higher deposit rate relative to the loan rate to attract household deposits, leading to counter-cyclical interest spread. This will increase the rate of return to household savings even for those households who do not increase their saving rate, and decrease the cost of credit (interest payments) even for those firms that do not increase their borrowings, further reinforcing the positive feedback loop among saving, credit, and investment, as emphasized by the Austrian school. The IRS is local in nature because the utilization rate of credit resources in the aggregate economy cannot exceed 100 percent, at which point the highly elastic supply of credit ceases to exist and the model economy becomes identical to an standard real-business-cycle (RBC) model.

The endogenous local IRS in our model are appealing for several reasons. First, it is consistent with CRS production technologies. Second, aggregate demand shocks (such as preference shocks or government spending shocks) are now able to generate positive business cycle comovement among aggregate consumption, investment, and output. Demand shocks are widely believed to be important sources of business cycles, yet in standard RBC models they generally produce a negative comovement between consumption and investment. Third, the standard RBC model has been criticized for requiring large technology shocks to produce realistic

business cycles (see King and Rebelo (1999) for a survey of the literature). Thanks to the endogenous IRS in our model, small fundamental shocks (either demand or supply shocks, including news shocks) can generate large business cycle fluctuations with positive comovements, without assuming various types of adjustments costs and special utility functions. Fourth, our model can generate indeterminacy and self-fulfilling business cycles with hump-shaped output responses without productive externalities as in the model of Benhabib and Farmer (1994) and the variable capacity utilization model of Wen (1998).

Our paper is related to several strands of literature. First, the search friction is in line with approaches proposed by Den Haan, Ramey and Waston (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013). These researchers have explored the implication of credit search on macroeconomy, but does not study the possibility of indeterminacy. For simplicity and tractability, they have assumed linear utility. In contrast, we incorporate the credit search friction into an otherwise standard RBC model. This allows us to study a richer set of economic variables of interest. Our paper is also inspired by search frictions in goods market such as Bai *et al* (2012), and by search-theoretic models of asset trading such as Duffie, Gârleanu and Pederson (2005) and Lagos and Rocheteau (2009). Recently Cui and Radde (2014) incorporate such line of research into a dynamic general equilibrium model and show it can explain the interesting flight-to-liquidity phenomena observed in great recession.

Our model also provides a micro foundation for the Benhabib-Farmer (1994) model with IRS and the Wen (1998) model with variable capital utilization rate under IRS. We show that search frictions in the credit market can generate an economic structure isomorphic to the Benhabib-Farmer-Wen model with both increasing returns and elastic capacity utilization, yet without assuming IRS in the production technology of firms. Our paper is also closely related to several recent works on self-fulfilling business cycle due to credit market frictions, such as Gertler and Kiyotaki (2013), Miao and Wang (2012), Benhabib, Miao and Wang (2014), Azariadis, Kaas and Wen (2014), Pintus and Wen (2013), Liu and Wang (2104), and Benhabib, Dong and Wang (2014).

Finally, our model is in the same spirit of Acemoglu (1996), who shows that search friction in labor market generate increasing returns to human capital accumulation in a two-period model. In his model, the workers have to make human capital investments before they enter the labor market. An increased in the average human capital investment induce more physical investments from firms. So even some of workers who have not increased their humane capital will earn a higher return on their human capital if matched with firms. In other words, search



friction produce a positive pecuniary externality similar to the mechanism in our model. However, unlike Acemoglu (1996), we focus on search in credit market and explore its implication on indeterminacy and self-fulfilling expectation driven business cycles in an infinite-period model.

The rest of the paper is organized as follows. Section 2 and Section 3 lays out the baseline model, and examines its key properties respectively. Section 4 studies the model's business cycle implications under calibrated parameter values. Section 5 extends the baseline model, and Section 6 concludes the paper. The omitted proofs in the context are put in the appendix.

## 2 Model

### 2.1 Environment

Time is continuous. The economy is populated by three types of agents: a representative household composed of a continuum of depositors, a representative bank (financial intermediary) composed of a continuum of clerks or loan officers, and a continuum of firms. We assume perfect competition in all sectors. The household owns capital and firms, makes decision on labor supply and consumption, and deposits savings into the banking system, which channels capital (in the form of loans) to firms. To break the conventionally assumed accounting identity between aggregate household saving and firm investment (which was criticized by the Austrian school and Keynes), we assume search frictions among the three types of agents so that in equilibrium aggregate household saving does not automatically equal firm investment, but instead imposes an upper limit on firm investment for any given interest rate.

The time line of events in an interval of  $t$  and  $t + dt$  is as follows. First, loan officers search for depositors to collect bank reserves; or alternatively, depositors search for loan officers to deposit their savings (carried over from the last period) into the banking system. So there is a search-and-match friction between depositors and the banking system. Without loss of generality, we assume that the household pays for the search costs. After collecting deposits, the loan officers and firms engage in random search and match.<sup>1</sup> Again we assume that firms pay for the search costs. In order to enter the credit market, however, a firm needs to pay a fixed cost. If a firm gets matched successfully and obtains a loan, trading surplus is split between the firm and the bank and the credit relationship is then dissolved by the end of the period.<sup>2</sup> After obtaining a loan (capital), firms can start producing goods by hiring labor in

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<sup>1</sup>See Section 5.3 for an alternative setup with competitive search. All the qualitative results derived in this paper are preserved under competitive search.

<sup>2</sup>We address the issue of long-term credit relationships in a companion paper, but the basic results hold.

the spot market. The number of active firms engaging in production is then determined by the free entry condition: the expected surplus from a successful match equals the fixed search cost of entering the credit market. Finally, the household pools wage and profit incomes from the bank and firms and then makes decision on consumption and capital accumulation (next-period savings). The whole process is repeated analogously in the next time interval between  $t + dt$  and  $t + 2dt$ .

To facilitate the analysis, we assume that all depositors from the household are *ex ante* identical and assigned with the same amount of credit resources to be randomly matched with a continuum of *ex ante* identical loan officers. Any unmatched savings are kept as inventories and carried over to the next period by the household. Analogously, after collecting deposits from the household, all loan officers are assigned with the same amount of credit resources available and go out to be randomly matched with firms. Any unmatched loans are counted as excess reserves, and transferred lump sum back to the households in the end of each period.<sup>3</sup>

Specifically, denote  $S$  as the total savings of the household. Due to search frictions, only  $\tilde{S} < S$  units of savings are successfully matched and deposited into the banking system. After that, each loan officer is assigned with an equal fraction of the  $\tilde{S}$  units of deposits and goes out searching for potential borrows (firms).

We show that such a simple setup leads to a simple dynamic system that can generate (i) countercyclical excess reserve-to-deposit ratio, (ii) countercyclical interest rate spread between the loan rate and the deposit rate, and (iii) self-fulfilling business cycles with strong amplification and propagation mechanisms.

## 2.2 Deposit Search

We first consider search frictions between the household and the banking system. The matching function between household members and bank clerks is  $M^H(x_t H_t, B_t) = \gamma_H (x_t H_t)^{\varepsilon_H} B_t^{1-\varepsilon_H}$ , where  $x_t$  is the search effort chosen by household depositors. There is a unit measure of household depositors and bank clerks, *i.e.*,  $H_t = B_t = 1$ .<sup>4</sup> Thus, the probability of match is given by

$$e_t = \frac{M^H(x_t H_t, B_t)}{H_t} = \gamma_H x_t^{\varepsilon_H}. \quad (1)$$

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<sup>3</sup>In a follow-up project, we study the case with required reserves and interbank lending.

<sup>4</sup>Our results would be strengthened if we allow  $H_t$  and  $B_t$  to vary by costly entry as an additional margin of adjustment.

Denoting  $dt$  as the time interval, the budget constraint of the household can be written as [the original budget constraint is ]

$$C_t dt + S_{t+dt} = \left[ e_t (1 + R_t^d dt) - \phi^H x_t dt \right] S_t + (1 - e_t) S_t + W_t N_t dt + \Pi_t dt \quad (2)$$

subject to equation (1), where  $C_t$  denotes consumption,  $S_t$  total savings,  $e_t$  the fraction of aggregate savings successfully deposited into the banking system,  $R_t^d$  the deposit rate promised by the bank,  $x_t$  the search effort devoted by the household,  $W_t N_t$  the wage income and  $\Pi_t$  the profit income from banks and firms, which are to be specified later. Note the first term on the right-hand side (RHS) pertains to the cross rate of return to deposits per unit of savings,  $e_t (1 + R_t^d)$ , after subtracting the search cost per unit of savings in hand,  $\phi^H x_t$ . That is, we assume that the search cost for each depositor is proportional to her effort  $x_t$  and the stock of savings in hand.<sup>5</sup> The second term on the RHS,  $(1 - e_t) S_t$ , is the total idle (unmatched) credit resources, which is also the unmatched "inventory" stock of savings kept by the household.

The first-order-condition (FOC) of the effort choice is given by  $x_t = \left[ \left( \frac{\gamma_H \varepsilon_H}{\phi_H} \right) R_t^d \right]^{\frac{1}{1-\varepsilon_H}}$ . In turn, the aggregate utilization rate of household savings is  $e_t = \gamma_H \left[ \left( \frac{\gamma_H \varepsilon_H}{\phi_H} \right) R_t^d \right]^{\frac{\varepsilon_H}{1-\varepsilon_H}}$ . Because of the search costs, we can derive a pseudo "depreciation" function of the stock of savings as follows. Denoting  $\delta^0 \equiv \frac{\phi_H(1+\kappa)}{\gamma_H^{1+\kappa}}$  and  $\kappa \equiv \frac{1}{\varepsilon_H} - 1$ , we can define

$$\delta(e_t) \equiv \phi^H \left( \frac{e_t}{\gamma_H} \right)^{\frac{1}{\varepsilon_H}} \equiv \delta^0 \left( \frac{e_t^{1+\kappa}}{1+\kappa} \right), \quad (3)$$

as a "depreciation" function of the saving stock, which is convex in the utilization rate ( $e_t$ ) of savings, analogous to Wen's (1998) model. With this notation and taking the limit  $dt \rightarrow 0$ , the household budget constraint in equation (2) becomes

$$C_t + \dot{S}_t = W_t N_t + \left[ e_t R_t^d - \delta(e_t) \right] S_t + \Pi_t. \quad (4)$$

Then we can formulate the constrained optimization problem of the representative household in a continuous-time model as

$$\max \left\{ \int_0^{+\infty} e^{-\rho t} \left[ \log(C_t) - \psi \frac{N_t^{1+\xi}}{1+\xi} \right] \right\} \quad (5)$$

subject to equation (4), where  $\rho > 0$  is the discount factor,  $\psi > 0$  controls the utility weight on labor supply, and  $\xi > 0$  is the inverse Frisch elasticity of labor supply. The first order conditions

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<sup>5</sup>We choose this proportional cost function so as to compare with the fixed search cost on the firm side (to be specified below). This way we can show which form of the search costs leads to local IRS in our model.

of the household with respect to labor ( $N_t$ ), consumption ( $C_t$ ), saving ( $S_t$ ), and search effort ( $e_t$ ) are given by

$$\frac{\dot{C}_t}{C_t} = e_t R_t^d - \delta(e_t) - \rho, \quad (6)$$

$$\frac{W_t}{C_t} = \psi N_t^\xi, \quad (7)$$

$$R_t^d = \delta'(e_t) = \delta_0 e_t^\kappa \quad (8)$$

### 2.3 Loan Search

The loan market consists of a large number of credit lenders (loan officers) and borrowers (firms). More specifically, there are  $B_t$  number of loan officers and  $V_t$  number of firms. Note that the total deposits in the banking system are given by  $\tilde{S}_t = e_t S_t$ , which are divided equally among the loan officers (with measure  $B_t = 1$ ). Each firm needs to pay a fixed cost  $\phi_t$  to enter the credit market to search for lenders. If a firm is matched with a loan officer, it can produce  $y_t = A_t \tilde{S}_t^\alpha n_t^{1-\alpha}$  units of output, where  $n_t$  is the labor input of the matched firm. The search friction is captured by a matching technology  $M(B, V)$ , where  $V$  denotes the measure of firms entering the credit market after paying the fixed entry cost  $\phi$ . To make the results sharp and tractable, we also assume a Cobb-Douglas matching technology,  $M(B, V) = \gamma B^{1-\varepsilon} V^\varepsilon$ , with  $\varepsilon \in (0, 1)$ . Denoting  $\theta_t \equiv \frac{B_t}{V_t}$  as a measure of the credit market tightness, the probability that a firm can match with a credit supplier is

$$q_t \equiv \frac{M(B_t, V_t)}{V_t} = M(\theta_t, 1) = \gamma \theta_t^{1-\varepsilon}, \quad (9)$$

and the probability that a loan officer can match with a firm is given by

$$u_t \equiv \frac{M(B_t, V_t)}{B_t} = M\left(1, \frac{1}{\theta_t}\right) = \gamma \theta_t^{-\varepsilon}. \quad (10)$$

Note that  $u_t$  is also the utilization rate of total bank deposits. That is, the aggregate amount of capital lent out successfully to firms is  $u_t \tilde{S}_t$ . Notice that

$$V_t q_t = M(B_t, V_t) = B_t u_t. \quad (11)$$

Given real wage  $w_t$ , if a firm is successfully matched, its operating profit (the matching surplus) is given by  $\tilde{\Pi}_t = \max_{n_t \geq 0} \left\{ A_t \tilde{S}_t^\alpha n_t^{1-\alpha} - W_t n_t \right\}$ . Hence, based on the FOC of  $n_t$ , we obtain

$$n_t = \left[ A_t \left( \frac{1-\alpha}{W_t} \right) \right]^{\frac{1}{\alpha}} \tilde{S}_t, \quad (12)$$

$$\tilde{\Pi}_t = \alpha A_t \left[ A_t \left( \frac{1-\alpha}{W_t} \right) \right]^{\frac{1-\alpha}{\alpha}} \tilde{S}_t \equiv \pi_t \tilde{S}_t. \quad (13)$$

Notice from Equation (13) that a successfully matched firm's operating profit is proportional to the total bank deposits  $\tilde{S}_t$  (as we assume that total bank deposits are divided equally among the loan officers and the measure of loan officers is  $B_t = 1$ ).

For each successful match, the operating profit (surplus) is split between the firm and the loan officer by Nash bargaining, with the firm obtaining  $\eta \in [0, 1]$  fraction of the surplus. Denote  $R_t^l$  as the competitive interest rate on loans, then it must be true that the lending interest rate equals the expected rate of return to the loan:

$$R_t^l = (1 - \eta) \pi_t. \quad (14)$$

A firm's *ex ante* expected surplus before conducting credit search is  $q_t \eta \pi_t \tilde{S}_t$ . Hence, the free entry (zero profit) condition for the firms is given by

$$\phi_t = q_t \eta \pi_t \tilde{S}_t. \quad (15)$$

Then equations (9) and (15) together imply

$$q_t = \gamma \theta_t^{1-\varepsilon} = \frac{\phi_t}{\eta \pi_t \tilde{S}_t}. \quad (16)$$

This equation states that the probability of match for a firm,  $q_t$ , decreases with the volume of match surplus. The intuition is as follows. A higher match surplus will induce more firms to enter, hence reduces the probability of each firm's match with the given number of credit suppliers.

The banking sector is perfectly competitive and thus makes zero profit. The bank needs to pay the depositors the deposit rate  $R_t^d$ , and earn the rate of return  $R_t^l$  (the lending rate) with probability  $u_t$ . Therefore the zero profit condition for the banking sector is given by

$$R_t^d = u_t R_t^l. \quad (17)$$

This equation captures the interest rate spread. Finally the aggregate net profit income distributed to the household is given by  $\Pi_t = (-R_t^d + u_t R_t^l) \tilde{S}_t + (-\phi + q_t \eta \pi_t) V_t = 0$ . Note that

although for simplicity we have assumed that the measures of depositors and loan officers are all unity ( $H_t = B_t = 1$ ), the number of firms  $V_t$  in the credit market is however a variable.

### 3 General Equilibrium Analysis

A general equilibrium is defined as a collection of prices  $\{W_t, R_t^d, R_t^l\}$  and quantities  $\{C_t, S_t, N_t, V_t, \pi_t, n_t, \tilde{S}_t, K_t, e_t, u_t, q_t\}$  such that (i) given prices and aggregate profit income  $\Pi_t$ , the allocation  $\{C_t, S_t, N_t\}$  solves household's utility maximization problem defined in (5); (ii) the surplus  $\pi_t$  and labor input  $n_t$  for a successfully matched firm are defined by (13) and (12); (iii) given the probability  $u_t$  of being matched with a bank loan officer, the equilibrium number of firms  $V_t$  is determined by the free entry condition (15); (iv) given the bank's probability of matching with a firm ( $u_t$ ), the bank earns zero expected profit as characterized by (17); (v) in the credit markets, the probability  $q_t$  and  $u_t$  are determined by (9) and (10), (vi) both labor markets and goods markets satisfy the standard market-clearing conditions.

#### 3.1 Aggregate Production Function

**Proposition 1** *In general equilibrium, the aggregate production function can be represented as*

$$Y_t = A_t (e_t u_t S_t)^\alpha N_t^{1-\alpha}. \quad (18)$$

**Proof:** See Appendix. ■

To complete the characterization of the aggregate system, we can show that the aggregate surplus from successful matches between loan officers and firms is then given by

$$\pi_t \equiv \alpha A_t \left[ A_t \left( \frac{1-\alpha}{W_t} \right) \right]^{\frac{1-\alpha}{\alpha}} = \alpha \left( \frac{Y_t}{K_t} \right), \quad (19)$$

which is equals to the marginal product of aggregate capital. The deposit rate is then given by  $R_t^d = u_t R_t^l = \alpha (1-\eta) \left( \frac{Y_t}{\tilde{S}_t} \right)$ . The last equality is obtained by combining  $K_t = V_t q_t \tilde{S}_t$  and Equation (11). Since  $B = 1$ , and  $u_t = \gamma \theta_t^{-\varepsilon}$ , the aggregate entry costs  $V\phi$  satisfy

$$V\phi = \left( \frac{B}{\theta} \right) \phi = \Delta(u) \equiv \Delta_0 \frac{u^{1+\lambda}}{1+\lambda}, \quad (20)$$

where  $\Delta(u)$  is convex with  $u$  with  $\Delta_0 \equiv \frac{\phi(1+\lambda)}{\gamma^{1+\lambda}}$  and  $\lambda \equiv \frac{1}{\varepsilon} - 1 > 0$ .

### 3.2 Local IRS and Local Indeterminacy

Combining equations (8), (17), (14) and (19) yields

$$e_t = \tilde{e} \left( \frac{Y_t}{S_t} \right)^{\varepsilon_H}, \quad (21)$$

where  $\tilde{e} \equiv \left( \frac{\alpha(1-\eta)(1-\sigma)}{\delta_0} \right)^{\frac{1}{1+\kappa}}$  and  $\kappa \equiv \frac{1}{\varepsilon_H} - 1$ . Additionally, the free entry condition (15) implies

$$V\phi = Vq\eta\pi\tilde{S} = \alpha\eta Y. \quad (22)$$

Combining equation (20) and (22) then yields

$$u_t = \tilde{u} Y_t^\varepsilon, \quad (23)$$

where  $\tilde{u} \equiv \left[ \frac{\alpha\eta(1+\lambda)}{\Delta_0} \right]^{\frac{1}{1+\lambda}}$ ,  $\Delta_0 \equiv \frac{\phi(1+\lambda)}{\gamma^{1+\lambda}}$ , and  $\lambda \equiv \frac{1}{\varepsilon} - 1$ .

**Proposition 2** *The aggregate production function in equation (18) exhibits local IRS (local increasing returns to scale) in household capital ( $S_t$ ) and labor supply ( $N_t$ ), because it can be rewritten as*

$$Y_t = \bar{Y} A_t^\tau S_t^{\alpha_s} N_t^{\alpha_n}, \quad (24)$$

where  $\bar{Y} \equiv \left[ (1-\eta)^{\varepsilon_H} \left( \frac{\eta}{\varepsilon} \right)^\varepsilon \left( \frac{\alpha}{\delta_0} \right)^{\varepsilon_H} \left( \frac{\alpha\eta}{\Delta_0} \right)^\varepsilon \right]^{\frac{1}{1-\alpha(\varepsilon+\varepsilon_H)}}$ ,  $\tau \equiv \frac{1}{1-\alpha(\varepsilon+\varepsilon_H)} > 1$ ,  $\alpha_s \equiv \alpha(1-\varepsilon_H)\tau > \alpha$ , and  $\alpha_n \equiv (1-\alpha)\tau > 1-\alpha$  with the degree of aggregate returns to scale given by

$$\alpha_s + \alpha_n > 1. \quad (25)$$

**Proof:** See Appendix. ■

As shown in condition (25), we obtain aggregate IRS in household capital  $S_t$  and labor supply  $N_t$  despite the lack of Benhabib-Farmer type production externalities. This is due to the endogenously time varying utilization rate of aggregate household savings and aggregate bank deposits, as suggested by Equations (21) and (23). However, the IRS are *local* because the capital utilization rates,  $e_t$  and  $u_t$ , are both bounded by the interval  $[0, 1]$ . Meanwhile, since  $\tau > 1$ , we also obtain the amplification effect on productivity shock.

It is obvious that our model based on credit search appears isomorphic to the IRS models of Benhabib and Farmer (1994) and Wen (1998). Hence, our model may also give rise to local indeterminacy and self-fulfilling business cycles with strong propagation mechanisms as in their models. To understand the intuition, consider a proportional increase in aggregate labor and

capital supply from the household. In a standard neoclassical model without credit search, such a proportional increase in labor and capital supply would only increase aggregate output one-for-one. However, in our model the increase of household savings leads to a higher credit supply in the banking system, which in turn would reduce the cost of borrowing and hence induce more firms to enter the credit market, which in turn increases the match probability of credit resources, raising the effective capital stock used in the production sector more than one-for-one, thus resulting in more than proportionate increase in aggregate output.

In addition to generating the IRS effects, the initial increase in household labor and capital supply can also become self-fulfilling. As the effective capital used in production increases, the returns to labor supply also increase for every household, reinforcing the initial increase in household labor supply. In addition, as the matching probability of bank increases, bank is able to pay a higher deposit rate. This will increase the returns to saving even for the households who do not increase their savings. Hence, the social increasing returns to scale originate from a subtle pecuniary externality that reinforces and multiplies itself in a positive feedback loop just like in the model of technological production externalities.

**Proposition 3** *The model is locally indeterminate if and only if either of the following conditions hold:*

1.  $\alpha \in (0, \frac{1}{2})$ ,  $\xi \in [0, \frac{\alpha}{1-2\alpha})$ ,  $\{\varepsilon, \varepsilon_H\} \in [0, 1]$  and  $\varepsilon + \varepsilon_H > \tilde{\varepsilon} \equiv (\frac{1}{\alpha}) \left( \frac{\alpha + \xi}{1 + \xi} \right)$ .
2.  $\alpha \in [\frac{1}{2}, 1)$ ,  $\varepsilon_H \in [0, 1]$  and  $0 \leq \varepsilon < \frac{1}{\alpha} - 1$ .

**Proof:** See Appendix. ■

Some comments are in order. First, when  $\alpha \in (0, \frac{1}{2})$ ,  $\xi \in [0, \frac{\alpha}{1-2\alpha})$ , which is line with our calibration in Table 1, then we know that  $(\frac{1}{\alpha}) \left( \frac{\alpha + \xi}{1 + \xi} \right) \in [1, 2)$  and thus  $\eta^* = \frac{\varepsilon}{\varepsilon + \varepsilon_H} < \varepsilon$ . The indeterminacy region for this scenario is illustrated in Figure 3. Second, note that the indeterminacy conditions have nothing to do with the bargaining power parameter  $\eta$ . Therefore, indeterminacy always exists under conditions specified in the above proposition regardless we adopt random search with Nash bargaining or competitive search (as shown in the Appendix). Third, our model provides a micro foundation to the models of Benhabib and Farmer (1994) and Wen (1998), which reply on exogenously assumed IRS in firms' production technologies. We show, instead, that such IRS technologies can be derived from credit search under CRS technologies and matching functions.



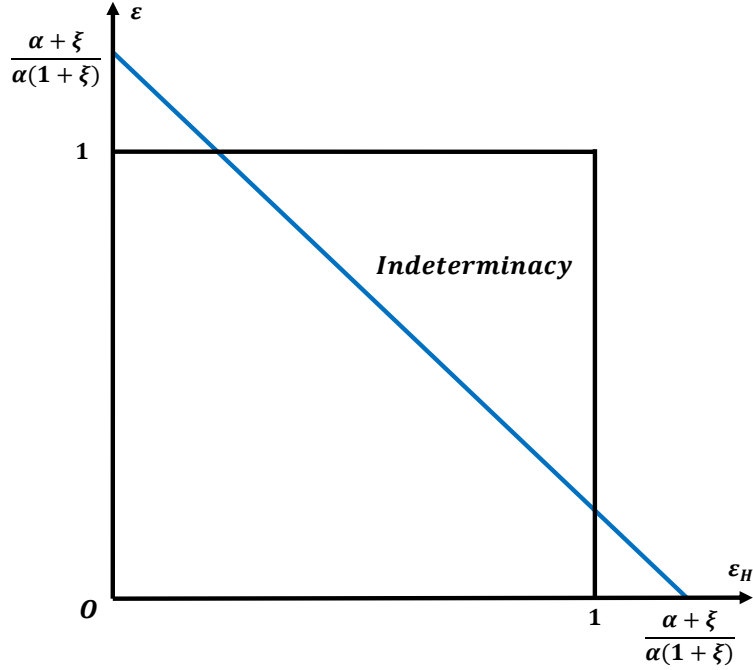


Figure 3: **Indeterminacy Region** (When  $\alpha \in (0, \frac{1}{2})$ ,  $\xi \in [0, \frac{\alpha}{1-2\alpha})$ )

## 4 Quantitative Exercises

### 4.1 Calibration

The time discounting factor is  $\rho = \frac{1}{\beta} - 1 = 0.01$ , where  $\beta = 0.99$  denotes the discount factor in discrete time models. We set the capital's share  $\alpha = 0.33$ , the coefficient of labor disutility  $\psi = 1.75$  and the inverse Frisch elasticity of labor supply  $\xi = 0.2$ . All of them are standard in the existing literature.

Now we have to calibrate the values of  $(\varepsilon_H, \phi, \eta, \gamma, \varepsilon)$ , which are specific to our model. First, as shown in the previous section,  $R^d = \frac{\rho(1+\kappa)}{\kappa}$  where  $\kappa \equiv \frac{1}{\varepsilon_H} - 1$ . We can obtain the average deposit rate  $R^d$  from Federal Reserve Economic Data (FRED), which implies  $\kappa = 0.23$ , and thus  $\varepsilon_H = \frac{1}{1+\kappa} = 0.82$ . Second, we have proved that  $\frac{S}{Y} = \frac{\alpha(1-\eta)}{R^d}$  in the steady state. Consequently, the bargaining power of firms,  $\eta$ , is obtained as  $\eta = 1 - \left(\frac{R^d}{\alpha}\right) \left(\frac{S}{Y}\right) = 0.187$ .

It remains to pin down  $(\phi, \gamma, \varepsilon)$ . First, we interpret  $\phi$  as the cost of intermediation to financing firm investment, which pertains to the size of the financial sector. Philippon (2012) argues that the share of financial industry is around 8% of GDP. Therefore, we set  $\frac{\phi}{Y} = 8\%$ . Note that  $\frac{\phi}{Y}$  is related to  $(\phi, \gamma, \varepsilon)$ . Second, we set  $u = 67\%$  according to data on the utilization

rate of capital in the manufacturing sector, and we also know that  $u$  is related to  $(\phi, \gamma, \varepsilon)$ . Finally, Becchetti *et al* (2009) documents that the proportion of Italian firms subject to credit rationing is around 20%. We assume the rate of credit rationing is 15% in the U.S. Then the proportion of matched firms is  $q = \gamma\theta^{1-\varepsilon} = 85\%$ , where  $\theta$  is also related to  $(\phi, \gamma, \varepsilon)$ . Therefore the three moments,  $(\frac{\phi}{Y}, u, q)$  jointly implies that  $\phi = 0.086$ ,  $\gamma = 0.797$  and  $\varepsilon = 0.729$ . Our calibration exercise shows that  $\varepsilon_H + \varepsilon > (\frac{1}{\alpha}) (\frac{\alpha + \xi}{1 + \xi})$ . Consequently indeterminacy due to credit search is empirically plausible in our model. The calibration is summarized in Table 1.

**Table 1. Calibration**

Parameter	Value	Description
$\rho$	0.01	Discount factor
$A$	1	Normalized aggregate productivity
$\alpha$	0.33	Capital income share
$\psi$	1.75	Coefficient of labor disutility
$\xi$	0.2	Inverse Frisch elasticity of labor supply
$\varepsilon_H$	0.82	Matching elasticity in 1st Stage Search
$\eta$	0.187	Firm's bargaining power
$\phi$	0.086	Vacancy cost to search for credit.
$\gamma$	0.797	Matching efficiency in 2nd stage search
$\varepsilon$	0.729	Matching elasticity in 2nd stage search

## 4.2 Impulse Responses

This subsection investigates the dynamic effect of TFP shocks and matching efficiency shocks ( $\gamma_t$ ) on aggregate output, the interest spread, the utilization rate of credit (the opposite of the reserve-to-deposit ratio), and credit rationing. All shocks have AR(1) persistence with the persistence coefficient of 0.9. We discretize our model to facilitate the analysis.

Figure 4 shows the model's impulse responses to a 1% TFP shock. Several striking features of the model are worth noticing. First, the responses of aggregate output are hump-shaped. Second, there exists a dynamic multiplier-acclerator effect or endogenous propagation mechanism such that not only the maximum response of output is postponed for several periods after the impact of the shock and it far exceeds 1%, but the impact of the shock is also long lasting with boom-bust cycles or an over-shooting and mean-reverting cyclical pattern. Third, both the reserve-to-deposit ratio (the negative of the top-right panel) and interest spread (lower-left panel) are countercyclical, consistent with the data.

Similarly, Figure 5 shows that a 1% credit matching efficiency shock can also generate the boom-bust cycles in aggregate output as well as countercyclical reserve-to-deposit ratio and

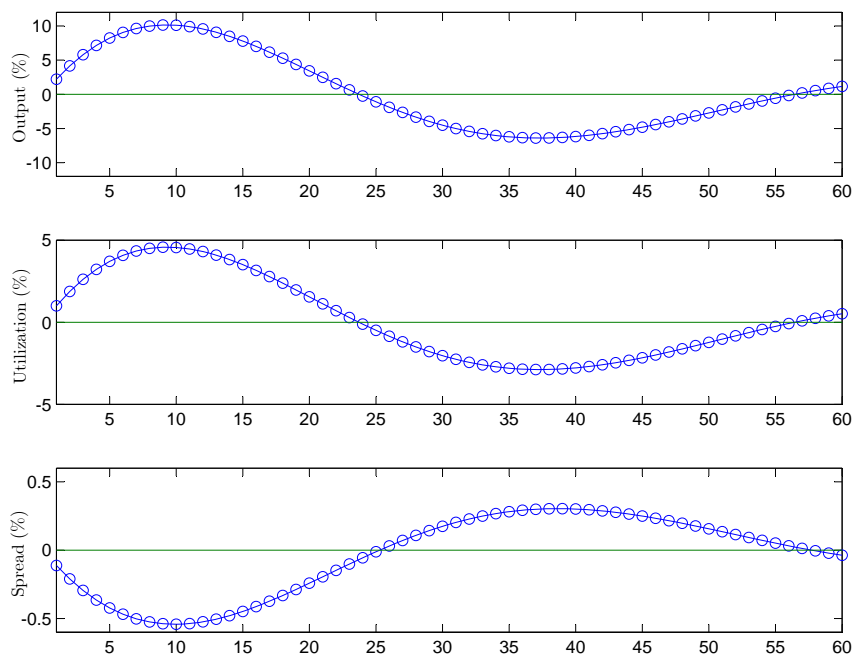


Figure 4: **Impulse Responses to TFP Shock (A)**

interest spread. Additionally, Figure 6 documents the impulse response driven by an *i.i.d.* sunspot shock to labor supply.<sup>6</sup> Despite the lack of any persistence in the sunspot shock, the responses of the economy to such a shock still exhibit high persistence with a boom-bust style similar to that under persistent fundamental shocks (except the lack of the initial hump).

To understand the intuition, consider an anticipated increase in credit supply from the banking sector. This optimistic expectation by firms would entice them to increase search efforts and compete for loans in the credit market, resulting in more banking capital being channeled into firms. Firms can thus increase production by hiring more labor, so households' labor supply also increases, leading to higher aggregate income and household savings. More household savings would then increase bank deposits and raise bank's credit supply without upward pressure on the loanable funds rate, fulfilling firms' initial optimistic expectations about cheap credit conditions.

However, an economic boom, once kick-started by a shock (whatever they are), is going to go through a "natural course" of continuous expansion before returning back to the steady

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<sup>6</sup>Alternatively, we can implement the impulse response by sunspot shock to consumption demand, which delivers a qualitatively similar result. See Wen (1998) for more details on how to introduce sunspot shocks in indeterminate DSGE models.

state. The positive feedback loop from firms' production to household income under local IRS means that any increase in firm production would lead to a more than proportionate increase in household savings and bank deposits in the initial periods of the boom, which induces more bank lending and firm-entry in the credit market to search for loans, especially if the shock is expected to persist despite with a damping magnitude.

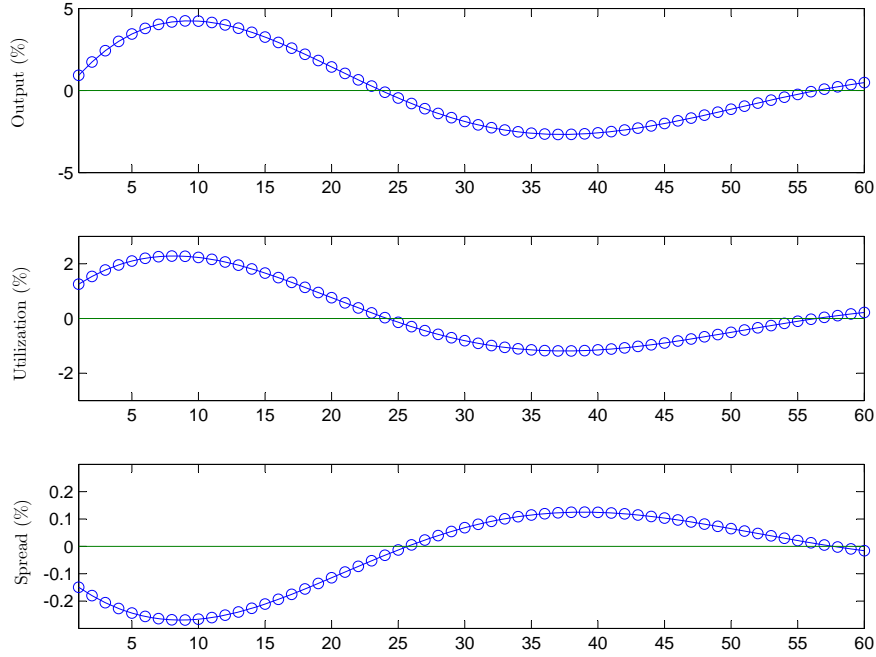


Figure 5: **Impulse Responses to Credit Shock, *i.e.*, Matching Efficiency Shock ( $\gamma$ )**

However, in the absence of permanent productivity (technological) growth, such an economic boom is not sustainable in the long run, because the IRS is only a local property. Once the utilization rate of aggregate credit resources becomes high enough before reaching 100%, the cost of borrowing will ultimately dominate the rate of return to capital (the marginal product of capital) under diminishing marginal product of capital in the production technology. Hence, the available credit resources in the economy will eventually be exhausted. This implies that after a peak in the boom period, in each subsequent round of the positive feedback mechanism between the banking sector and firms, the additional volume of loans unleashed from the banking sector becomes less and less, ultimately leading to rapid increases in the loanable funds rate. This would eventually choke off credit demand on the firm side because the rising costs of credit borrowing cannot be compensated by the falling (diminishing) marginal product

of capital as the boom continues. Hence, sooner or later the economy will stop growing and enter a contraction phase.

Once the contraction sets in, the above feedback (multiplier-accelerator) mechanism reverses itself and leads to a persistent period of recession. The recession features a shrinking credit market with both very little credit-search effort by firms and rapidly accumulating excess reserves in the banking sector. However, a prolonged recession cannot be an equilibrium state either, because the economy will eventually reach a point at which the marginal product of capital becomes so high (due to lack of effective investment) and the loanable funds rate becomes so low (due to lack of credit demand and an increasing excess reserve-to-deposit ratio), which will then cause another round of over-shooting (except with more dampened magnitude) as the economy converges back to the steady state from below.

Hence, our model produces genuine credit cycles broadly consistent with what the Austrian theory describes. Namely, an economic boom featuring a low interest spread (loanable funds rate below the natural rate) plants the seed for an economic recession, and a recession featuring a high interest spread (loanable funds rate above the natural rate) plants the seed for the next boom. The turning point of the business cycle appears to be determined by the relative magnitude of the loanable funds rate and the natural rate.

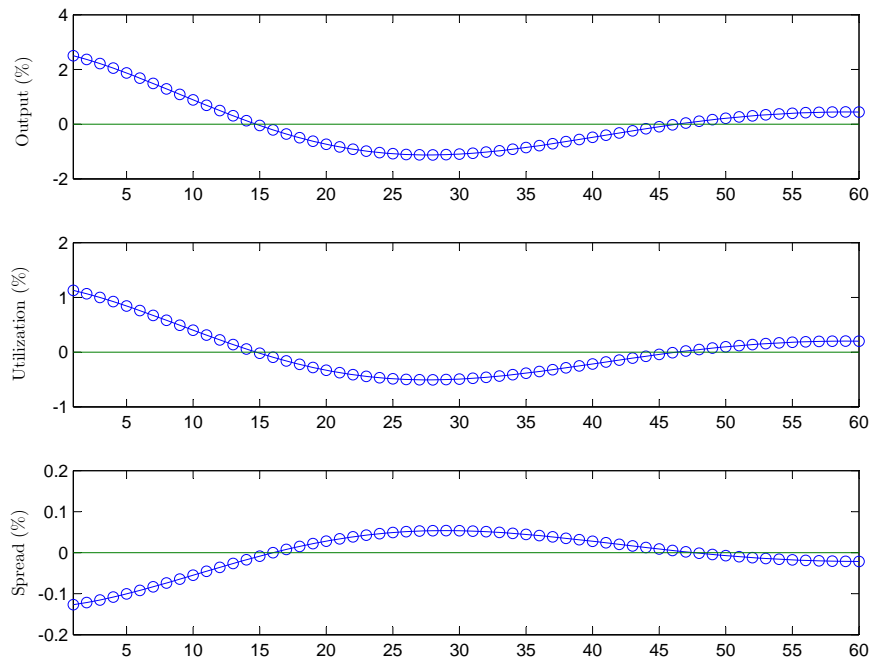


Figure 6: **Impulse Responses to Sunspot Shock**

## 5 Discussions

### 5.1 Eliminating Household Search Friction

Notice that if  $\varepsilon_H = 0$ , namely, if there is no household search, then  $e_t = 1$  and we obtain  $Y_t = A (u_t S_t)^\alpha N_t^{1-\alpha}$ , where  $u_t = \gamma \left( \frac{\alpha \eta Y_t}{\phi} \right)^\varepsilon$ . Hence, the aggregate production function becomes  $Y_t = \left[ \gamma \left( \frac{\alpha \eta}{\phi} \right)^\varepsilon \right]^{\alpha_s} A_t^\tau S_t^{\alpha_s} N_t^{\alpha_n}$ , where  $\alpha_s \equiv \alpha \tau$ ,  $\alpha_n \equiv (1 - \alpha) \tau$ , and  $\tau \equiv \frac{1}{1 - \alpha \varepsilon}$ . This production technology still exhibits local IRS because  $\alpha_s + \alpha_n = \frac{1}{1 - \alpha \varepsilon} > 1$ . So the model appears isomorphic to the Benhabib-Farmer model. However, because the IRS are local in our model whereas they are global in the Benhabib-Farmer model, indeterminacy is not possible in our model with  $\varepsilon_H = 0$  *although* we do have endogenous IRS, except locally. On the other hand, if we only allow household search but no firm search, i.e.,  $\varepsilon = 0$  and  $\varepsilon_H > 0$ , then the model becomes isomorphic to Wen's (1998) model without IRS, which is the Greenwood *et al* (1988) model. Hence, adding household search into the model is necessary to generate indeterminacy, analogous to Wen's (1998) finding that variable capacity utilization can significantly reduce the required degree of IRS in the Benhabib-Farmer model for indeterminacy. The well-known problem of the Benhabib-Farmer model is that it requires extremely large IRS to generate indeterminacy, which is empirically implausible. Wen's (1998) model can reduce the required IRS for indeterminacy down to empirically plausible range. Hence, our model provides a micro foundation for the indeterminacy literature pioneered by Benhabib and Farmer (1994) and Wen (1998) because we show that the Romer type IRS and Greenwood *et al* type capital utilization are not needed to generate essentially identical boom-bust business cycles obtained in Wen (1998).

### 5.2 Hosios Condition and Welfare

Since we have used random search to characterize frictions in the credit market, it is natural for us to check whether the Hosios (1990) condition holds in our environment. Given  $(A_t, S_t, N_t)$ , *i.e.*, if we control the technology and the supply of capital and labor, then the Hosios condition is obtained by solving the following constrained optimization problem of the social planner:

$$\max_{e_t, u_t} \{Y_t(e_t, u_t) - \delta(e_t) S_t - \Delta(u_t)\}, \quad (26)$$

where  $Y(e, u)$ ,  $\delta(e)$  and  $\Delta(u)$  are defined in equation (18), (3) and (20) respectively. Then we reach a modified Hosios condition as below.

**Proposition 4 (Modified Hosios Condition)** Given  $(A_t, S_t, N_t)$ , The ratio of output in the decentralized economy to that in the social planner economy is given by

$$\frac{Y^{DE}}{Y^{SP}} = \left[ (1 - \eta)^{\varepsilon_H} \left( \frac{\eta}{\varepsilon} \right)^\varepsilon \right]^{\frac{\alpha}{1 - \alpha(\varepsilon + \varepsilon_H)}}, \quad (27)$$

and thus

$$\eta^* = \arg \max_{\eta \in [0, 1]} \left( \frac{Y^{DE}}{Y^{SP}} \right) = \frac{\varepsilon}{\varepsilon + \varepsilon_H}. \quad (28)$$

Several remarks are in order. First, when search frictions exist only between banks and firms, *i.e.*,  $\varepsilon_H = 0$ , then  $Y^{DE} = Y^{SP}$  if and only if  $\eta = \varepsilon$ , which shares a flavor of the classic Hosios condition. However, in contrast to the classic Hosios condition,  $\eta = \varepsilon$  does not maximize  $Y^{DE}$ . Second and more interestingly, we contribute to the literature by detecting a modified Hosios condition in the presence of dual search frictions, *i.e.*,  $\varepsilon_H > 0$  and  $\varepsilon > 0$ . Intuitively, the increase of firm's bargaining power  $\eta$  delivers two competing effects. On the one hand, the increase of  $\eta$  intensifies firm's search for credit by inducing more firm entry. This in turn increases  $u$ , the utilization rate of credit in the second link of the search-and-matching chain, and thus drives up output. On the other hand, a higher  $\eta$  dampens the profit share of the bank by lowering the loan rate, which translates into a lower deposit rate, and therefore discourages the search effort of household for the decision of deposit making. Therefore Equation (28) strikes a balance between these two competing effects. In particular,  $\eta^*$  increases with  $\varepsilon$  (the matching elasticity of firms searching for credit) and decreases with  $\varepsilon_H$  (the matching elasticity of household searching for financial intermediation). Notice that  $\eta^* = \varepsilon$  if and only if  $\varepsilon + \varepsilon_H = 1$ . As shown in the next subsection,  $\varepsilon + \varepsilon_H > 1$  when indeterminacy emerges, and thus  $\eta^* < \varepsilon$  in that scenario.

Finally, in deriving the Hosios conditions, we have so far followed the literature by holding the supply of labor and capital as fixed. This restriction is fine when it comes to the standard setup of macro labor economics *a la* DMP, which typically assumes inelastic labor supply and does not take into account capital accumulation, in addition to the assumption of risk neutral firms and workers. However, our paper has to address both of these issues since the household in our model is allowed to make decision on labor supply as well as capital accumulation. Moreover, the household is risk averse when it comes to consumption. As a result, neither the classic nor the modified Hosios condition can guarantee a constrained optimum in welfare. Instead, we have to take into account the effect of  $\eta$  on both consumption and leisure decisions of the household over the lifetime horizon. Note that the household welfare in the steady state

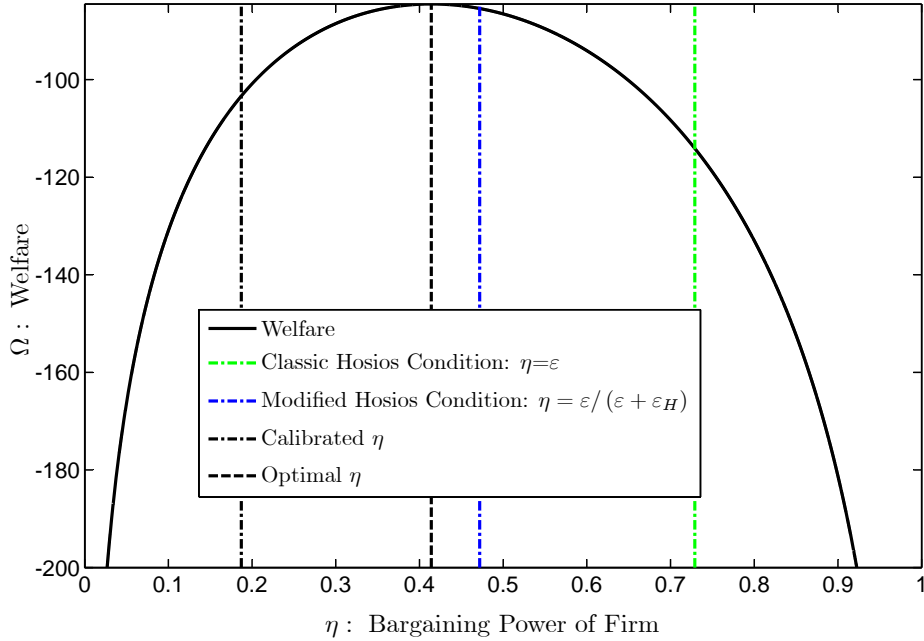


Figure 7: **Hosios Conditions and Welfare: A Numerical Example (See Table 1 for Parameterization)**

is

$$\Omega = \frac{1}{\rho} \left\{ \log \left[ \left( \frac{C}{Y} \right) Y \right] - \psi \frac{N^{1+\xi}}{1+\xi} \right\},$$

where  $\frac{C}{Y} = \left(1 - \frac{\alpha}{1+\kappa}\right) - \left(\frac{\kappa}{1+\kappa}\right) \alpha \eta$ ,  $N = \left(\frac{1-\alpha}{\psi} \frac{1}{C/Y}\right)^{\frac{1}{1+\xi}}$ , and  $Y = \left[A \left(\gamma \left(\frac{\alpha \eta}{\phi}\right)^{\varepsilon}\right)^{\alpha} \left(\frac{S}{Y}\right)^{\alpha} N^{1-\alpha}\right]^{\frac{1}{1-\alpha(1+\varepsilon)}}$  denote respectively the steady state of the ratio of consumption to output, labor supply and output, all of which are obtained from the simplified dynamical system in the proof of Proposition 3. Figure 7 indicates that in the presence of risk aversion, endogenous capital accumulation, and elastic labor supply, neither the classic Hosios condition (*i.e.*,  $\eta = \varepsilon$ ) nor the modified Hosios condition (*i.e.*,  $\eta = \frac{\varepsilon}{\varepsilon + \varepsilon_H}$ ) maximizes the household's welfare in steady state.

### 5.3 Competitive Search

For simplicity, we have adopted random search in the baseline model. Alternatively, we can use competitive search *a la* Moen (1997). More specifically, each loan officer can post her own terms of trade,  $R^l$ , in sub-market  $\theta$  such that

$$\max \left\{ u(\theta) R^l(\theta) \tilde{S} \right\} \quad (29)$$



subject to

$$q(\theta) \left[ \pi - R^l(\theta) \right] \tilde{S} = \phi \text{ for all } \theta, \quad (30)$$

where  $u(\theta) = \frac{M(B(\theta), V(\theta))}{B(\theta)}$ ,  $q(\theta) = \frac{M(B(\theta), V(\theta))}{V(\theta)}$ . If  $M(B, V) = \gamma B^{1-\varepsilon} V^\varepsilon$ , then we can easily check that, the equilibrium loan rate is determined by  $R^l = (1 - \varepsilon)\pi$ , which is qualitatively to the loan rate under Nash bargaining in equation (14). Additionally, we can check that the indeterminacy condition characterized in Proposition 3 still holds. Therefore the sunspot condition is unrelated to the bargaining protocol.

## 6 Conclusion

The critical role that credit supply and financial intermediation play in generating and amplifying the business cycle has long been noted by economists at least since the Austrian school, as manifested historically in the countercyclical excess reserve-to-deposit ratio, the countercyclical interest rate spread between the loan rate and the deposit rate, as well as the countercyclical proportion of firms subject to credit rationing. However, simple correlation between credit expansion/contraction and economic boom/bust does not tell us causality. This paper provides a framework to rationalize the Austrian theory and the observed credit cycles. Our framework is based on a simple idea: In an industrial economy with the division of labor and segregation of decision making between consumption demand and output supply and between household saving and firm investment, savers (lenders) with "idle" credit resources need to search and match with investors (borrowers) with projects to utilize the available saving/credit resources and make them productive. But search and matching are costly due to informational frictions and various transaction costs and commitment technologies. It also requires efforts and coordinations from borrowers and lenders. Hence, in equilibrium, credit resources in the economy are not always fully utilized, creating an important margin for elastic credit supply—excess reserves and an endogenous utilization rate of available credit resources. So economic booms and busts are closely associated with credit expansions and contractions and changes in interest spread. Meanwhile, the under-utilization of credit resources coexists with the prevalence of credit rationing to firms. Our highly stylized model nonetheless demonstrates the fundamental nature of credit-driven economic boom-bust cycles. Finally, we show that such a margin of elastic credit supply turns out critical not only for understanding the countercyclical excess reserve-to-deposit ratio and interest rate spread, but also for providing a micro-foundation for the powerful amplification and propagation mechanisms underling the endogenous business cy-

cle literature studied by Benhabib and Farmer (1994) and Wen (1998) based on the assumption of IRS in production technologies.

# Appendix

## A Proofs

**Proof of Proposition 1:** Since each match between the household and the banking sector utilizes  $\tilde{S}_t = e_t S_t$  units of household capital (savings), and each match between the banking sector and the production sector (firms) utilizes  $K_t = u_t \tilde{S}_t$  units of bank capital (deposits), given the total initial available credit resources  $S_t$  in the economy, the fraction of aggregate credit resources being utilized ultimately in production is hence given by

$$K_t = u_t e_t S_t. \quad (\text{A.1})$$

As each matched firm employs  $n_t$  units of labor, the labor market equilibrium then requires

$$N_t = V_t q_t n_t = V_t q_t \left[ A_t \left( \frac{1 - \alpha}{W_t} \right) \right]^{\frac{1}{\alpha}} \tilde{S}_t. \quad (\text{A.2})$$

Finally, it is easy to show that the total output produced by all firms is given by

$$Y_t = V_t q_t y_t = V_t q_t A_t \tilde{S}_t^\alpha n_t^{1-\alpha} = V_t q_t A_t \tilde{S}_t^\alpha \left( \frac{N_t}{V_t q_t} \right)^{1-\alpha} = A_t \left( V_t q_t \tilde{S}_t \right)^\alpha N_t^{1-\alpha} = A_t K_t^\alpha N_t^{1-\alpha}, \quad (\text{A.3})$$

where  $K_t$  is determined by equation (A.1). Since  $K_t = e_t u_t S_t$ , the aggregate production function in equation (A.3) can also be written equation (18). *QED*

**Proof of Proposition 2:** Substituting equation (21) and (23) into equation (18) yields equation (24). *QED*

**Proof of Proposition 3:** Equations (12) and (A.2) together imply

$$W_t = (1 - \alpha) \left( \frac{Y_t}{N_t} \right). \quad (\text{A.4})$$

Substituting Equation (A.4) into Equations (4), (6), and (7) yields

$$\dot{S}_t = (1 - \alpha\eta) Y_t - \delta(e_t) S_t - C_t, \quad (\text{A.5})$$

$$\frac{\dot{C}_t}{C_t} = (1 - \eta) \alpha \left( \frac{Y_t}{S_t} \right) - \delta(e_t) - \rho, \quad (\text{A.6})$$

$$\psi N_t^\xi = (1 - \alpha) \left( \frac{Y_t}{N_t} \right) \left( \frac{1}{C_t} \right). \quad (\text{A.7})$$

Consequently, we can reduced the dynamic system of  $\{C_t, S_t, N_t, W_t, R_t^d, R_t^l, \pi_t, K_t, e_t, u_t, q_t, \theta_t, Y_t, V_t\}$  to a simplified one with fewer variables in  $\{C_t, S_t, e_t, u_t, Y_t, N_t\}$  with Equations (18), (21), (23), (A.5), (A.6), and (A.7), where  $\delta(e)$  is defined in equation (3). The FOCs indicate  $\delta'(e_t) = (1 + \kappa) \left( \frac{\delta(e_t)}{e_t} \right) = R_t^d$ , thus,

$$\delta(e_t) = \frac{R_t^d e_t}{1 + \kappa} = \varepsilon_H \alpha (1 - \eta) \left( \frac{Y_t}{S_t} \right).$$

Log-linearizing the above simplified transition dynamics yields

$$\begin{aligned} \dot{c}_t &= (1 - \varepsilon_H) \left( \frac{Y}{S} \right) (1 + \hat{y}_t - \hat{s}_t) - \rho \\ \dot{s}_t &= [(1 - \alpha\eta) - \varepsilon_H \alpha (1 - \eta)] \left( \frac{Y}{S} \right) (1 + \hat{y}_t - \hat{s}_t) - \left( \frac{C/Y}{S/Y} \right) (1 + \hat{c}_t - \hat{s}_t) \\ \hat{y}_t &= \alpha (\hat{e}_t + \hat{u}_t + \hat{s}_t) + (1 - \alpha) \hat{n}_t \\ \hat{e}_t &= \varepsilon_H (-\hat{s}_t) \\ \hat{u}_t &= \varepsilon \hat{y}_t \\ (1 + \xi) \hat{n}_t &= (1 - \alpha) (\hat{y}_t - \hat{c}_t). \end{aligned}$$

Consequently, we obtain the simplified dynamic system on  $(s_t, c_t)$  as

$$\begin{bmatrix} \dot{s}_t \\ \dot{c}_t \end{bmatrix} = J \cdot \begin{bmatrix} \hat{s}_t \\ \hat{c}_t \end{bmatrix},$$

where

$$J \equiv \delta \cdot \begin{bmatrix} \left( \frac{1+\kappa}{\alpha} - 1 \right) \left( \frac{1}{1-\eta} \right) \lambda_s & \left( \frac{1+\kappa}{\alpha} - 1 \right) \left( \frac{1}{1-\eta} \right) (\lambda_c - 1) \\ \kappa (\lambda_s - 1) & \kappa \lambda_c \end{bmatrix}.$$

$$\kappa \equiv \frac{1}{\varepsilon_H} - 1, \alpha_s \equiv \frac{\alpha(1-\varepsilon_H)}{1-\alpha(\varepsilon+\varepsilon_H)}, \alpha_n \equiv \frac{1-\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}, \lambda_s \equiv \frac{\alpha_s(1+\xi)}{1+\xi-\alpha_n}, \text{ and } \lambda_c \equiv \frac{-\alpha_n}{1+\xi-\alpha_n}.$$

Note that the local dynamics around the steady state is then determined by the eigenvalues of  $J$ . If both eigenvalues of  $J$  are negative, then the model is indeterminate. As a result, the model can experience endogenous fluctuations driven by sunspots. The eigenvalues of  $J$ ,  $x_1$  and  $x_2$ , satisfy

$$\begin{aligned} x_1 + x_2 &= \text{Trace}(J) = \delta \left[ \left( \frac{1+\kappa}{\alpha} - 1 \right) \left( \frac{1}{1-\eta} \right) \lambda_s + \kappa \lambda_c \right], \\ x_1 x_2 &= \text{Det}(J) = \delta^2 \left( \frac{1+\kappa}{\alpha} - 1 \right) \left( \frac{\kappa}{1-\eta} \right) (\lambda_s - \lambda_c - 1). \end{aligned}$$

Therefore indeterminacy emerges if and only if  $\text{Trace}(J) < 0$  and  $\text{Det}(J) > 0$ . We can prove that  $\text{Trace}(J) < 0$  and  $\text{Det}(J) > 0$  hold if and only if the following four conditions hold, in

addition to the restriction that  $\varepsilon, \varepsilon_H \in [0, 1]$ :

$$\varepsilon + \varepsilon_H < \frac{1}{\alpha} \quad (\text{A.8})$$

$$\varepsilon + \varepsilon_H > \left(\frac{1}{\alpha}\right) \left(\frac{\alpha + \xi}{1 + \xi}\right) \quad (\text{A.9})$$

$$\varepsilon_H < 1 - \frac{(1 - \eta)(1 - \alpha)\kappa}{(1 + \kappa - \alpha)(1 + \xi)} \quad (\text{A.10})$$

$$\varepsilon < \frac{1}{\alpha} - 1. \quad (\text{A.11})$$

First, since  $\varepsilon_H \in [0, 1]$ , then comparing Conditions (A.8) and (A.11) suggests that the former is never binding. Secondly, note that  $\kappa \equiv \frac{1}{\varepsilon_H} - 1$ . Thus the Condition (A.10) can be rewritten as

$$\varepsilon_H < \left[ \frac{1 + \xi - (1 - \eta)(1 - \alpha)}{1 + \xi} \right] \left(\frac{1}{\alpha}\right).$$

Since  $\xi \geq 0$ , we have  $\left[ \frac{1 + \xi - (1 - \eta)(1 - \alpha)}{1 + \xi} \right] \left(\frac{1}{\alpha}\right) > [1 - (1 - \eta)(1 - \alpha)] \left(\frac{1}{\alpha}\right) > 1$ , and therefore we know that Condition (A.10) is not binding. Finally, if  $\alpha \in [\frac{1}{2}, 1)$ , then we know that  $\frac{1}{\alpha} - 1 \in (0, 1]$ , and we must have  $0 \leq \varepsilon < \frac{1}{\alpha} - 1$ . Besides, we know that  $\tilde{\varepsilon} \equiv \left(\frac{1}{\alpha}\right) \left(1 - \frac{1 - \alpha}{1 + \xi}\right) > 2$  when  $\alpha \in [\frac{1}{2}, 1)$ . Therefore Condition (A.9) always holds in this case. In contrast, when  $\alpha \in (0, \frac{1}{2})$ , we have  $\frac{1}{\alpha} - 1 > 1 > \varepsilon$ , and thus Condition (A.11) always holds. Meanwhile, since  $\varepsilon + \varepsilon_H \leq 2$ , to guarantee that Condition (A.9) can be satisfied, we must have  $\tilde{\varepsilon} \equiv \left(\frac{1}{\alpha}\right) \left(1 - \frac{1 - \alpha}{1 + \xi}\right) < 2$ , *i.e.*,  $\xi \in [0, \frac{\alpha}{1 - 2\alpha})$ . *QED*

**Proof of Proposition 4:** The FOCs are given by

$$\delta^0 e_t^\kappa = \frac{\alpha Y_t}{e_t S_t} \quad (\text{A.12})$$

$$\Delta^0 u_t^\lambda = \frac{\alpha Y_t}{u_t} \quad (\text{A.13})$$

Substituting Equations (A.12) and (A.13) into Equation (18) yields

$$Y^{SP} = \tilde{Y}^{SP} A_t^\tau S_t^{\alpha_s} N_t^{\alpha_n}, \quad (\text{A.14})$$

where  $\tilde{Y}^{SP} = \left[ \left(\frac{\alpha}{\delta^0}\right)^{\varepsilon_H} \left(\frac{\alpha\eta}{\Delta^0}\right)^\varepsilon \right]^{\frac{\alpha}{1 - \alpha(\varepsilon + \varepsilon_H)}}$ . Dividing Equation (24) by Equation (A.14) yields Equation (27). Then FOC of Equation (27) yields  $\eta^*$ . *QED*

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