# Understanding the Risk-Return Tradeoff in the Stock Market 

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#### Abstract

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## I. Introduction

Modern finance theories imply that risk-averse investors require a positive compensation for any extra risk they bear in the stock market, given everything else equal. This tenet is the cornerstone of rational expectations asset pricing models. Moreover, the static capital asset pricing model (CAPM) stipulates a positive relationship between stock market risk and return. Such a positive risk-return tradeoff, however, has been argued to be inconsistent with data in several studies. For example, Campbell (1987) reports a negative risk-return relation because the short-term interest rate is positively correlated with stock market variance, while it is negatively correlated with excess stock market returns. Similarly, Lettau and Ludvigson (2001b) find that the consumption-wealth ratio is negatively correlated with stock market variance and is positively correlated with excess stock market returns. ${ }^{1}$ In this paper, we show how the negative risk-return relation documented in these studies can be reconciled with the positive risk-return tradeoff derived from theories.

Our explanations are based on Merton's (1973) (continuous-time) intertemporal capital asset pricing model (ICAPM) that is summarized in equation (1):

$$
\begin{equation*}
E_{t} e_{M, t+1}=\left[\frac{J_{W W} W}{J_{W}}\right] E_{t} \sigma_{M, t+1}^{2}+\left[\frac{-J_{W F}}{J_{W}}\right] E_{t} \sigma_{M F, t+1} . \tag{1}
\end{equation*}
$$

The conditional excess stock market return $E_{t} e_{M, t+1}$, defined as the difference between the conditional stock market return $E_{t} r_{M, t+1}$ and the risk-free rate $r_{f, t+1}$, is a linear function of its conditional variance $E_{t} \sigma_{M, t+1}^{2}$ and its covariance with investment opportunities $E_{t} \sigma_{M F, t+1}$. We call the first term of equation (1) the risk component and the second term the hedge component.

[^1]$J(W(t+1), F(t+1), t+1)$ is an indirect utility function with subscripts denoting partial derivatives, where $W(t+1)$ is wealth and $F(t+1)$ is a vector of state variables that describe investment opportunities. [ $\frac{J_{W W} W}{J_{W}}$ ] is a measure of relative risk aversion and is positive if investors are risk-averse. For simplicity, we usually assume that investors have a power utility function and, therefore, relative risk aversion is constant. ${ }^{2}$

It is clear that the static CAPM holds only when the hedge component is zero or the investment opportunities are constant over time. The positive risk-return relation implied by the static CAPM, therefore, could be distorted in more general cases when the hedge component is economically important. In particular, if the hedge component is negatively correlated with the risk component, we may observe a negative risk-return relation even though the price of risk is positive. The relevant issue here is to disentangle the risk and hedge components of the conditional excess stock market return.

According to equation (1), macro variables forecast excess returns because they are correlated with either the risk component or the hedge component. Past stock market variance is a potentially good instrumental variable for the risk component because stock market variance is positively auto-correlated in the data. Following Merton (1980), we measure the quarterly realized stock market variance as the sum of the squared daily deviation of excess stock market

[^2]returns from the mean in each quarter. ${ }^{3}$ Interestingly, we find that past stock market variance is indeed significantly and positively correlated with future excess stock market returns in both insample and out-of-sample regressions. To our best knowledge, this is the first empirical evidence establishing the link between past stock market variance and future excess stock market returns. ${ }^{4}$

On the other hand, the consumption-wealth ratio is a good instrumental variable for the hedge component because as mentioned above, it implies a negative risk-return tradeoff. The short-term interest rate may also be used as an instrumental variable for the hedge component because it has strong forecasting power for excess returns, but not for variance. In our sample, the consumption-wealth ratio and the short-term interest rate subsume the information content of other variables that been found in the literature to forecast excess returns, such as the default premium and the term spread (Fama and French [1989]), and the dividend yield (Campbell, Lo, and MacKinlay [1997]). However, past stock market variance remains positive and statistically significant after we add these two variables in the excess return forecasting equation. Interestingly, the consumption-wealth ratio and past stock market variance jointly explain much more variations in future excess stock market returns than they do individually, in both in-sample and out-of-sample regressions. This result highlights the importance of both the risk and hedge components of the conditional excess stock market return.

[^3]To formally investigate whether stock market risk is positively priced, we estimate equation (1) of Merton's ICAPM directly. In particular, we assume that the hedge component is a linear function of the short-term interest rate and the consumption-wealth ratio, as we discussed above. As to the risk component, we first assume that conditional stock market variance is a linear function of the variables that have been found in the literature to forecast stock market variance. Because conditional stock market variance is a generated variable, we follow Pagan's (1984) advice and estimate equation (1) and a conditional variance equation jointly using twostage least square (2SLS) regressions. We find that the risk price is indeed positive and statistically significant, while the estimated risk and hedge components are negatively correlated. Therefore, the negative risk-return relation found in the early literature does not imply a negative risk price or a negative risk-return tradeoff.

Evidence suggests unstable relationships between stock market variance and some forecasting variables such as the dividend yield and, therefore, using these variables is likely to bias the estimation of conditional stock market variance in finite samples. ${ }^{5}$ To avoid this problem, we include only its own lags in the stock market variance equation; however, all the other forecasting variables remain in the instrumental set to improve the efficiency. Again, the risk price is found positive and statistically significant. As expected, the alternative specification shows substantial improvements over the specification above.

We obtain very similar point estimates of the risk price over different sample periods; however, there are some variations between monthly and quarterly data, possibly because the measurement error of the realized stock market variance is larger in monthly data than in
with the hedge component, Goyal and Santa-Clara emphasize the idiosyncratic risk. Second, while our strongest results come from the quarterly data, Goyal and Santa-Clara only look at the monthly data.
${ }^{5}$ To my best knowledge, Schwert (1989) first reports an unstable relationship between the dividend yield and stock market variance.
quarterly data, as mentioned in footnote 3 . Christensen and Prabhala (1998) show that volatility implied from S\&P 100 index options outperforms past volatility in forecasting future volatility of the S\&P 100 index. Guo and Whitelaw (2001) find that it is also a good predictor for aggregate stock market volatility. As expected, using the implied volatility as an instrumental variable for conditional stock market variance greatly improves the efficiency of our estimations-the risk price is very precisely identified and the point estimate is very close to what we obtain using quarterly data.

Existing theories do not provide many clues to the origins of the hedge component document in this paper. For example, Merton (1973) does not explain the nature of the state variables that generate the hedge demand. In fact, it is usually negligible in the frictionless intertemporal capital asset pricing models, in which utility is a function of consumption only. Because of the lack of theoretical foundation, many financial economists, i.e., Merton (1980) and Campbell (1987), argue that it might be empirically unimportant. One exception is the recent research by Guo (2000), who shows that it is possible to generate a substantial hedge demand for stocks in a limited stock market participation model and that such a hedge demand is important to understand the time-varying equity premium. Like equation (1) of Merton's ICAPM, the conditional excess stock market return also has two components in Guo (2000). That is, in addition to the risk premium in standard models, the stockholder also requires a liquidity premium because the stockholder cannot use stocks to hedge income risks due to limited stock market participation. The liquidity premium is not constant and, in fact, it is positively correlated with the dividend yield. Moreover, the dividend yield is negatively (positively) correlated with stock market variance when the dividend yield is relatively low (high). Stock market risk and return thus can be sometimes negatively correlated in his model, even though the risk price is
positive. As we discuss below, there is a close link between the consumption-wealth ratio and the dividend yield: in particular, the two are equivalent in Guo's model economy. Guo (2000), therefore, provides a consistent explanation to the empirical results documented in this paper.

Few empirical studies have analyzed the capital asset pricing model with hedge component in a time-series context. Scruggs (1998) is an important exception. Scruggs assumes that the hedge component is the covariance between stock and Treasury bond returns and estimates equation (1) using a multivariate EGARCH model. Scruggs also finds that the risk price is significantly positive after controlling for the hedge component. However, Scruggs, and Glabadanidis (2000) show that his findings are somewhat sensitive to the assumption of a constant correlation between stock and bond returns. Two reasons may explain the inconsistency in these two studies. First, there are no theoretical reasons why Treasury bond returns are a good instrumental variable for future investment opportunities. Second, as noted by French, Schwert, and Stambaugh (1987), the full information maximum likelihood (FIML) estimators such as EGARCH are sensitive to model misspecifications.

The remainder of the paper is organized as follows. Section II discusses the data used in the paper and section III shows that past stock market variance forecasts excess stock market returns. The estimation results of Merton's model are presented in section IV and some theoretical implications are discussed in section V. We conclude the paper in section VI.

## II. Data Description

As mentioned in footnote 3, most results reported in this paper are obtained using quarterly rather than monthly data. The consumption-wealth ratio (cay $)$, a crucial instrumental
variable used in this study, is available for the period 1952:Q4 to 2001:Q1 ${ }^{6}$. The stock market data are obtained from the Center for Research in Security Prices (CRSP), which are available until the year 2000. Our longest quarterly dataset thus spans from 1953:Q1 to 2000:Q4, given one lag used in the regressions.

For comparison, we report selected results obtained using monthly data as well, which span from 1959:1 to 2000:12. Christensen and Prabhala (1998) show that the monthly volatility implied from S\&P 100 index options outperforms past volatility in forecasting future S\&P 100 index volatility. Because it might improve the efficiency of our estimations, we also report selected results obtained using the implied volatility data, which span from 1983:10 to 1995:5. ${ }^{7}$

Given its important role in our empirical analysis, we first provide a brief description for the consumption-wealth ratio, although a thorough analysis can be found in Lettau and Ludvigson (2001a). In a representative agent model, if the intertemporal budget constraints hold, the consumption-wealth ratio can be written as a function of expected future asset returns and consumption growth. This is the theoretical background of its strong predictive ability for excess stock market returns documented by Lettau and Ludvigson (2001a). Interestingly, this argument is exactly the same one that Campbell and Shiller (1988) use for the dividend yield, which can be written as a function of expected future stock market returns and dividend growth. Indeed, the consumption-wealth ratio is equivalent to the dividend yield in the exchange economy studied by Lucas (1978) and Campbell and Cochrane (1999), among many others. Despite their strong theoretical link, the two variables are only moderately correlated in the data. Moreover, the consumption-wealth ratio has much stronger predictive power for excess returns than the

[^4]dividend yield does in the Post World War II data. One possible explanation is that the consumption-wealth ratio is a better measure of its theoretical counterpart than the dividend yield is.

We measure the realized stock market variance by the sum of the squared deviation of the daily excess stock market return from its quarterly mean, or

$$
\begin{equation*}
\sigma_{M, t}^{2}=\sum_{k=1}^{\pi}\left(e_{M t, k}-\bar{e}_{M t}\right)^{2}, \tag{2}
\end{equation*}
$$

where $e_{M t, k}$ is the excess stock market return on the $k$ th day of quarter $t, \bar{e}_{M t}$ is its quarterly average, and $\tau t$ is the number of trading days. ${ }^{8}$ We use the daily stock market return data constructed by Schwert (1990a) before July 2, 1962, and use the daily value-weighted stock market return (VWRET) from CRSP thereafter. The daily risk-free rate is not directly available. Alternatively, we assume that the daily risk-free rate is constant within each month and equal to the monthly risk-free rate divided by the number of trading days. The monthly risk-free rate is also obtained from CRSP. It is well known that the October 19, 1987 stock market crash has a confounding effect on the stock market variance measured by equation (2). In particular, it is several times higher in the fourth quarter of 1987 than the second largest realized stock market variance in our sample. Following Campbell, Lettau, Malkiel, and Xu (2001), we replace the former with the latter in our empirical analysis. As we will show later, however, our results are not sensitive to such a modification in any qualitative ways.

[^5]The excess stock market return is the difference between the CRSP value-weighted stock market return and the risk-free rate. We also construct excess stock market return using the S\&P 500 index and find that it is almost perfectly correlated with the CRSP data.

Other variables used in the paper are as follows. The stochastically detrended risk-free rate $\left(\right.$ rrel $\left._{t}\right)$ is the difference between the risk-free rate and its last one-year average. The quarterly risk-free rate is approximated by the sum of the monthly risk-free rate and our source is CRSP. The dividend yield $\left(d p_{t}\right)$ is the sum of the dividends from most recent four quarters divided by the stock price using stocks listed in the S\&P 500 and our source is DRI. The default premium ( $d e f_{t}$ ) is the yield spread between the Baa-rated and Aaa-rated corporate bonds and our source is DRI. The term premium $\left(\right.$ term $\left._{t}\right)$ is the yield spread between the 10 -year Treasury bonds and the 3-month Treasury bills and our source is DRI. The commercial paper spread $\left(c p_{t}\right)$ is the yield spread between the 6-month commercial paper and the 3-month Treasury bills. The data of the 6month commercial paper is discontinued after August 1997, and we use the 3-month commercial paper data instead for the remaining periods. Our source is DRI.

Figure 1 plots the standardized stock market variance (thick solid line) along with excess returns and it reveals some interesting patterns. First, as noted by Schwert (1989), stock market variance is much higher during recessions than during expansions. In particular, it usually increases dramatically at the beginning of a recession and returns to a normal level when the recession is over. In contrast, excess stock market returns are more like white noise. Second, excess stock market returns and stock market variance usually move in opposite directions. Black (1976) explains such a contemporaneously negative relation as the leverage effect. However, French, Schwert, and Stambaugh (1987) argue that it is also consistent with a volatility feedback effect. That is, because stock market volatility is positively autocorrelated, a positive
shock today implies higher future volatility and thus higher expected return; as a result, the stock price has to fall today. Third, high stock market variance is likely to be followed by high excess returns in the next quarter or so. This observation is confirmed by our formal regression analysis reported in the next section.

Table 1 presents summary statistics for excess stock market returns, variance, and some variables that have predictive power for excess stock market returns. Consistent with Figure 1, stock market return and variance are negatively correlated with a correlation coefficient of -. 38 . Stock market variance is also negatively correlated with the consumption-wealth ratio and the dividend yield, while it is essentially not correlated with the stochastically detrended risk-free rate. As mentioned above, the consumption-wealth ratio is only mildly correlated with the dividend yield with a correlation coefficient of .36 , despite their theoretically close link.

Variables that forecast excess stock market returns are usually persistent. As shown in Table 1, the autocorrelation coefficient is .83 for the consumption-wealth ratio, .71 for the stochastically detrended risk-free rate, and .98 for the dividend yield. In contrast, it is only .43 for stock market variance.

## III. Past Stock Market Variance and Future Excess Stock Market Returns

In this section, we show that past stock market variance has significant forecasting power for future excess stock market returns. Our evidence includes in-sample, out-of-sample, and long-horizon regressions.

## i. Quarterly Forecasting Regressions

Table 2 reports the results of ordinary least square (OLS) regressions of the one-quarterahead excess stock market return on past stock market variance and other forecasting variables.

For all the results reported in this paper, Newey and West's (1987) method is used to correct the standard errors for heteroscedasticity and serial correlation in the residuals.

In Panel A, we report the regression results of the longest sample spanning from 1953:Q1 to 2000:Q4. Consistent with the theories, past stock market variance is positive and statistically significant in the univariate regression with a moderate adjusted $R^{2}$ of 3 percent, as shown in row 1 . For comparison, in rows 2 and 3 we replicate the well-known results that the consumption-wealth ratio and the stochastically detrended risk-free rate are strong predictors of excess stock market returns. Rows 4 to 6 show that the information content of past stock market variance is not subsumed by these two variables and vice versa. On the contrary, the predictive power is greatly enhanced when past stock market variance and the consumption-wealth ratio are both included as forecasting variables. They jointly explain 17 percent of variations in the quarterly excess stock market return in row 4 , compared with 3 percent and 8 percent in the univariate regressions in rows 1 and 2, respectively. The point estimates and the $t$-values of both variables are also much higher in row 4 than their counterparts in rows 1 and 2 . This result is explained by the fact that, while past stock market variance and the consumption-wealth ratio are negatively correlated, they both have positive coefficients in the forecasting equation. In contrast, a combination of past stock market variance and the stochastically detrended risk-free rate does not improve the fit over the univariate regressions. Overall, these three variables jointly explain 20 percent of variations in the quarterly excess stock market return, as reported in row 6 .

The dividend yield, the term spread, the default premium, and the commercial paper spread have been found to have some predictive power for excess stock market returns and/or stock market variance in the early literature. To check whether they provide additional information, we run a regression of excess stock market returns on all the forecasting variables.

Row 7 shows that only past stock market variance, the consumption-wealth ratio, and the stochastically detrended risk-free rate are statistically significant.

As discussed above, we adjust downward the realized stock market variance for the fourth quarter of 1987 because the October 19, 1987 stock market crash has a confounding effect on it. To check if our results are sensitive to such a modification, we do the following two experiments. First, we run the regressions again using unmodified stock market variance. We find that the past stock market variance is still marginally significant in the univariate regression, although the associated $R^{2}$ is small. Moreover, The multivariate regression results are very similar to that reported in panel A. ${ }^{9}$ Second, we run the regressions using subsamples 1953:Q11987:Q4 and 1988:Q2-2000:Q4 and report the results in Panels B and C of Table 2, respectively. The stock market variance of 1987:Q4 is not included in either subsample because of one lag between the dependent and independent variables. The results in panel B are essentially the same as those in panel A , which is not a surprise given large overlapping periods in the two samples. In panel C , none of the forecasting variables are statistically significant in the univariate regressions because of the small sample size. Interestingly, past stock market variance and the consumption-wealth ratio become significant or marginally significant when we include both variables in the regressions, as shown in rows 18,20 , and 21 . Moreover, their coefficients are very similar to the counterparts reported in panels A and B. To summarize, our results are not sensitive to the downward adjustment made for the stock market variance of 1987:Q4, and the forecasting equation is very stable over different sample periods.

Lastly, there is some evidence that the stock market variance constructed using daily data might be higher than that perceived by market participants for the fourth quarter of 1987. For

[^6]example, Schwert (1990b) argues that the October 19, 1987 stock market crash is unusual in many ways, and Seyhun (1990) shows that insiders did not react to it as much as outsiders. We also provide an informal test to this hypothesis, which is based on the stable relationship among excess stock market returns, past stock market variance, and the consumption-wealth ratio, as reported in Table 2. We run a regression of excess stock market returns on the unmodified stock market variance, a dummy variable for the fourth quarter of 1987, and the consumption-wealth ratio. All forecasting variables turn out to be statistically significant; the point estimate is 8.13 for past stock market variance and is -6.53 for the dummy variable. These results roughly suggest that the perceived stock market variance is only about 20 percent of the measured one, which is close to the adjustment that we make. ${ }^{10}$

## ii. Out-of-Sample Forecasting

In the preceding subsection, we find that past stock market variance has significant predictive power for excess stock market returns, especially when it is combined with the consumption-wealth ratio. However, the forecasting tests presented in Table 2 are subject to some criticisms. First, Stambaugh (1999) shows that the OLS estimate is biased in finite samples when the return innovation is correlated with the innovation in the forecasting variables. Second, Bossaerts and Hillion (1999) find that some variables have poor out-of-sample predictive ability, although they have good in-sample fit.

Following Lettau and Ludvigson (2001a), we address these problems using out-of-sample tests. Three statistics are used to gauge the out-of-sample performance of past stock market variance. The first is the standard mean-squared-error ratio. The second is the encompassing test ENC-NEW developed by Clark and McCracken (1999). It tests the null hypothesis that the
benchmark model encompasses all the relevant information for the next period's excess stock market return against the alternative hypothesis that past stock market variance provides additional information. The third is the equal forecast accuracy test MSE-F developed by McCraken (1999). The null hypothesis is that the benchmark model has a mean-squared forecasting error less than or equal to that of the model augmented by past stock market return; the alternative is that the augmented model has a smaller mean-squared error. Clark and McCracken (1999) provide asymptotic critical values for ENC-NEW and MSE-F tests. To address the finite sample problem, we also report their empirical distributions generated by the bootstrapping method.

We use the observations of the period 1953:Q1 to 1968:Q4 for initial in-sample estimation. The out-of-sample tests for various benchmarks are reported in Table 3. The Asy. CV column shows the 95 percent critical value from the asymptotic distribution, and the BS. CV column shows the 95 percent critical value from the empirical distribution.

In row 1, the benchmark model is that conditional excess stock market return is constant. We find that adding past stock market variance to the benchmark model lowers the meansquared error. Therefore, past stock market variance does improve the out-of-sample forecast for excess stock market return. Moreover, the formal tests show that such an improvement is statistically significant. Both ENC-NEW and MSE-F statistics are much higher than the corresponding 95 percent critical values of both asymptotic and empirical distributions. In row 2, the benchmark model is that excess stock market return follows an AR(1) process. Again, we find that the addition of past stock market variance significantly improves its out-of-sample performance.

[^7]In row 3, we test whether past stock market variance offers additional information beyond the consumption-wealth ratio, which Lettau and Ludvigson (2001a) find to have strong out-of-sample predictive power. Our results are striking. The mean-squared error of the unrestricted model is only 88 percent of that of the benchmark model, and the ENC-NEW and MSE-F statistics are highly significant. Past stock market variance thus greatly improves the out-of-sample performance of the consumption-wealth ratio. This is not a surprise. As we report in Table 2, past stock market variance greatly improves the in-sample forecasting power of the consumption-wealth ratio, and the forecasting equation is very stable over time.

Unlike financial variables, macro variables are usually available with a one-month lag. To control for this informational lag in real time, we consider a benchmark model that uses as the forecasting variable a consumption-wealth ratio with a two-period lag. As shown in row 4, we again find that the augmented model outperforms the benchmark model by a very large margin, and the results are very similar to those in row 3 .

Row 5 shows that past stock market variance also provides additional information about future excess stock market returns beyond the stochastically detrended risk-free rate, although the rejection of the null hypotheses is not as strong as the results reported in rows 3 and 4 . This is consistent with the in-sample regression results reported in Table 2.

Lastly, we provide some assessments of the joint out-of-sample predictive power of these three variables. The benchmark model is constant excess stock market return, and we augment it by incorporating past stock market variance, the consumption-wealth ratio, and the stochastically detrended risk-free rate in the unrestricted model. For robustness, in rows 7 and 8 we use both one-period-lagged and two-period-lagged consumption-wealth ratio, respectively. As expected,
the forecasting variables greatly improve the out-of-sample predictive power of the benchmark model. The ratio of the mean-squared error is only 82 percent in row 7 and 88 percent in row 8 . Moreover, the ENC-NEW and MSE-F tests reject the null hypothesis at very high significance levels in both cases. Based on both in-sample and out-of-sample forecasting results, we conclude that a large portion of excess stock market returns is predictable.

## iii. Long-Horizon Forecasts

Table 4 reports the long-horizon predictability of excess stock market returns. In the univariate regression reported in row 1, past stock market variance has rather moderate predictive power for up to a one-year horizon. Again, when paired with the consumption-wealth ratio in row 4, its predictive power increases dramatically, and it remains significant up to a sixyear horizon. However, a comparison of the adjusted $R^{2}$ (in square bracket) in rows 2 and 4 suggest that the additional information provided by past stock market variance decreases quickly as forecasting horizons increase. The reason is as follows. Stock market variance is not very persistent; its predictive power for future stock market variance, and, therefore, future excess stock market returns, decreases quickly as horizons increase.

Consistent with the one-quarter-ahead regression reported in Table 2, we find no extra informational gain by including both past stock market variance and the stochastically detrended risk-free rate in the long-horizon forecasting regressions. Measured by the adjusted $R^{2}$, the joint predictive power reported in row 5 is approximately equal to the sum of their individual predictive power reported in rows 1 and 3 .

The early literature finds that the dividend yield is a strong predictor for excess stock market return, especially over long horizons. In row 6, we include the dividend yield in the regressions, along with past stock market variance, the consumption-wealth ratio, and the
stochastically detrended risk-free rate. While the dividend yield is insignificant at all horizons that we analyze, the predictive power of the other three variables is essentially unaffected.

Hodrick (1992) develops an alternative approach to analyze the long-horizon forecast. Rather than estimating them directly, he first estimates a VAR process for excess stock market returns and the forecasting variables, and then imputes long-horizon statistics from the estimated VAR process. One advantage of his approach is to avoid the small-sample biases that might occur when the horizon is large relative to the sample size. To address the effect of past stock market variance on the long-horizon excess return, two sets of VAR systems are estimated: with and without past stock market variance as a forecasting variable. Past stock market variance is included in the VAR system reported in panel B and is excluded from the VAR system reported in panel A. The imputed $R^{2}$ in panel B is very close to its counterparts reported in Table 4 over short horizons, although $R^{2}$ is somewhat upwardly biased in the single-equation regression over long horizons. Comparing panels A and B , past stock market variance greatly improves the forecasting ability over the short horizons; however, the imputed $R^{2}$ is actually smaller in panel B than that in panel A over long horizons. This suggests that there might be some nonlinearity in the stock market price.

## IV. Estimation of Merton's ICAPM

The objective of this paper is to investigate whether there is a positive risk-return tradeoff and we find some support for it in the preceding section. That is, past stock market variance is positively correlated with future excess returns and stock market variance. However, as we will show below, the consumption-wealth ratio is negatively correlated with stock market variance, although it has been found positively correlated with future excess returns. Evidence for a
positive risk-return relation, therefore, is sensitive to the instrumental variables used. In this section, we show that these seemingly conflicting results can be explained in the context of Merton's ICAPM, in which the hedge component also has an important effect on the asset price.

## i. Model Specifications

Two implications of Merton's ICAPM can help us to disentangle the risk and hedge components. First, any variables that forecast future stock market variance should also forecast future excess stock market returns. Second, if a variable captures variations in the risk component, its correlation with future stock market return and variance should have the same sign. Otherwise, it captures variations of the hedge component. Based on these two implications, we conclude that, while past stock market variance is a good instrumental variable for the risk component, the consumption-wealth ratio is a good instrumental variable for the hedge component. This hypothesis is plausible because the predictive power of past stock market variance is greatly enhanced if we also include the consumption-wealth ratio in the forecasting equation and vice versa. Also, the stochastically detrended risk-free rate may also track the hedge component because it only forecasts excess stock market returns.

To formalize the argument, we estimate Merton's ICAPM directly by using instrumental variables to approximate the risk and hedge components. In particular, we assume that conditional excess stock market return is a linear function of conditional stock market variance, and some instrumental variables for the hedge component, namely, the stochastically detrended risk-free rate and the consumption-wealth ratio. Moreover, we assume that relative risk aversion, or the coefficient of conditional stock market variance, $\gamma$, is constant. Thus, equation (1) can be rewritten as

$$
\begin{equation*}
e_{M, t+1}=\omega_{1}+\gamma E_{t} \sigma_{M, t+1}^{2}+\beta_{1} y_{t}+\eta_{t+1} \tag{3}
\end{equation*}
$$

where $\omega_{1}$ is a constant, $\beta_{1}$ is a vector of parameters, $y_{t}$ is a vector of instrumental variables for the hedge component and $\eta_{t+1}$ is the error term.

Conditional stock market variance is not directly observable. Following Whitelaw (1994), among many others, we assume that conditional stock market variance is a linear function of the state variables $x_{t}$ that have some predictive power for future stock market variance:

$$
\begin{equation*}
\sigma_{M, t+1}^{2}=\omega_{2}+\beta_{2} x_{t}+\varepsilon_{t+1} \tag{4}
\end{equation*}
$$

Whitelaw (1994) finds that stock market variance is forecasted by its own lags, the default premium, the dividend yield, the commercial paper spread and the yield on 1-year Treasury bonds. Lettau and Ludvigson (2001b) show that the consumption-wealth ratio is also a strong predictor of stock market variance. Table 6 presents the regression results of stock market variance on these variables using various sample periods. Panel A uses the longest sample, spanning from 1953:Q1 to 2000:Q4. Consistent with early results, the one-period-lagged stock market variance and the consumption-wealth ratio are strong predictors for future stock market variance in the univariate regressions; the coefficient is positive for the former and negative for the latter. In the multivariate regression, they both remain significant and have the same signs as their univariate counterparts. Moreover, except for the stochastically detrended risk-free rate, all other variables are also statistically significant. ${ }^{11}$

Panels B and C report the regression results using the subsamples of 1953:Q1-1987:Q3 and 1962:Q4-2000:Q4, respectively. While past stock market variance shows consistent patterns in the two subsamples, there are noticeable variations in the other forecasting variables. For example, the dividend yield has opposite signs in the two subsamples. Also, the predictive power of the consumption-wealth ratio is much weaker in the early period than in the late period. To
further investigate this, we calculate the correlation coefficients between the forecasting variables and one-quarter-ahead stock market variance using a ten-year rolling window. We find that correlation between the dividend yield and stock market variance does change signs over time. Although the consumption-wealth ratio and future stock market variance are negatively correlated during most periods, the relation is rather weak in the early sample periods. Only past stock market variance has a reliable relationship with future stock market variance. Our findings are nothing new to the literature. Schwert (1989) also reports an unstable relationship between stock market variance and the dividend yield using long historical data. Therefore, while variables such as the dividend yield have no predictive power for future stock market variance asymptotically, they may be strong predictors in finite samples.

To be robust, we adopt two specifications for stock market variance. First, we include all the forecasting variables in the conditional market variance equation, as in the early literature. Second, we include only past stock market variance in the stock market variance equation; however, all forecasting variables remain in the instrumental set. While the two specifications are asymptotically equivalent, the latter is less vulnerable to finite-sample bias.

One identification strategy is to estimate equation (4) first and then use the estimated stock market variance to estimate equation (3) using OLS. Pagan (1984) shows that this procedure generates a biased variance estimate for $\gamma$. However, the problem is easily overcome by jointly estimating the equations (3) and (4) using two-stage least square (2SLS) regressions, with a constant, $y_{t}, x_{t}$, and some other variables in instrumental sets:

$$
\begin{align*}
& e_{M, t+1}=\omega_{1}+\gamma \sigma_{M, t+1}^{2}+\beta_{1} y_{t}+\eta_{t+1}+\gamma \varepsilon_{t+1} \\
& \sigma_{M, t+1}^{2}=\omega_{2}+\beta_{2} x_{t}+\varepsilon_{t+1} \tag{5}
\end{align*}
$$

[^8]To control for heteroscedasticity and serial correlation in the residuals, we use general methods of moments (GMM) instead.

## ii. Empirical Results in Quarterly Data

As discussed above, we consider two specifications of conditional stock market variance in estimating equation (5). First, we include all the forecasting variables in the stock market variance equation, and the GMM estimation results are reported in Table 7. Panel A uses the longest sample spanning from 1953:Q1 to 2000:Q4. Row 1 shows that all the variables are statistically significant in the excess stock market return equation. The point estimate of relative risk aversion, $\gamma$, is 7.64 , indicating that stock market risk is indeed positively priced.

Interestingly, it is less than 10 and, therefore, falls into the plausible ranges stipulated by Mehra and Prescott (1985). Moreover, the coefficients of the consumption-wealth ratio and the stochastically detrended risk-free rate have the same sign as their OLS counterparts reported in Table 2. The stock market variance equation reported in row 2 is also similar to those reported in panel A of Table 6. The overidentifying restrictions are not rejected at the 5 percent significance level. We also estimate equation (5) using the subsamples 1953:Q1-1987:Q3 and 1962:Q42000:Q4, and report the estimation results in panels B and C of Table 7, respectively. The results are similar to those reported in panel A. Most strikingly, the point estimate of relative risk aversion, $\gamma$, is 7.46 in panel B and 8.23 in panel C , which are very close to the point estimate of 7.64 reported in panel A. These results suggest that relative risk aversion might be very stable over time. However, there is a noticeable difference between the full sample and the subsamples. The overidentifying restrictions are not rejected at much higher significance levels in the
subsamples than in the full sample. This difference possibly reflects the unstable relationships between some forecasting variables and stock market variance, as documented in Table 6.

Using the point estimates reported in panel A of Table 7, we calculate the conditional excess stock market return and its components. Figure 2 plots the standardized conditional stock market return and Sharpe ratio. Interestingly, the two variables are almost perfectly correlated with one another and display strong business cycle patterns: they increase during recessions and decrease during expansions. Figure 3 plots the standardized hedge (thick solid line) and risk components. Although they both tend to increase during recessions and decrease during expansions, the risk component seems to lead the hedge component. As a result, they sometimes move in opposite directions. Figure 4 plots the coefficients of correlation between the risk and hedge components (thick solid line) and of that between the conditional stock market mean and variance using a ten-year rolling window. The risk and hedge components are almost always negatively correlated. In contrast, the conditional stock market mean and variance display an unstable relationship, although they are negatively correlated in the full sample. Therefore, the negative risk-return relation documented in the early literature reflects the negative relation between the risk and hedge components and does not imply a negative risk price.

In the second specification, we assume that conditional stock market variance is a linear function of only its own lags. However, we also include all other forecasting variables in the instrumental sets to improve the efficiency. We reestimate Equation (5) and report the results in Table 8, which are qualitatively similar to that reported in Table 7. Again, we find that relative risk aversion is positive and highly significant. Also, its point estimates are strikingly stable over different sample periods. As expected, there are strong indications that the specification in Table 8 fits the data much better than in Table 7. First, the $t$-values of relative risk aversion are much
larger in Table 8 than in Table 7. Second, the adjusted $R^{2}$ of the excess stock market return equation is higher in Table 8 than in Table 7. Third, evidence that the overidentifying restrictions are not rejected is much stronger in Table 8 than in Table 7, especially for the longest sample reported in panel A.

Using the point estimates reported in panel A of Table 8, we calculate the conditional excess stock market return and its components. Figure 5 plots the standardized conditional excess stock market return and Sharpe ratio. Figure 5 is almost identical to Figure 4, which is not a surprise because of the similar point estimates reported in Tables 7 and 8. Figure 6 plots the coefficients of correlation between the risk and hedge components (thick solid line) and of that between the stock market mean and variance. Figure 6 is somewhat different from Figure 4: while there is an unstable relationship between the risk and hedge components, stock market risk and return are always positively correlated.

## iii. Empirical Results in Monthly Data and Implied Volatility Data

For the sake of robustness, we also run the regressions using monthly data. However, we might not obtain a precise estimate of the risk price because of two potential problems associated with monthly data. First, the monthly consumption-wealth ratio is interpolated from quarterly data and, therefore, is vulnerable to measurement errors. Second, our measure of the realized stock market variance at monthly frequency might not be as reliable as at quarterly frequency because a smaller number of daily observations is utilized in monthly data. With these caveats in mind, we report selected results of the monthly data in Table 9. Panel A is the OLS regression of excess stock market return on selected forecasting variables. Interestingly, while one-periodlagged stock market variance is found insignificant in the univariate regression, the two-periodlagged one does have some predictive power for future excess stock market return in both
univariate and multivariate regressions. The consumption-wealth ratio and the stochastically detrended risk-free rate also appear significant in all the regressions. However, measured by adjusted $R^{2}$, the joint predictive power of past stock market variance and the consumptionwealth ratio is not higher than the sum of their individual predictive power. This is in contrast to what we find in quarterly data.

Equation (5) is also estimated using two different stock market variance specifications. The stock market variance is a linear function of all forecasting variables in panel B, while it includes only its own lags in panel C. Despite the potential data problems mentioned above, the results are qualitatively similar to what we obtain using quarterly data. Again, the price of risk is found to be significantly positive and the overidentifying restrictions are not rejected at conventional significance levels. However, there is a noticeable difference in the magnitude of the point estimate of the relative risk aversion coefficient-it is much smaller in monthly data than in quarterly data. For example, it is 5.90 in row 10 of Table 9 , which falls outside the two standard errors of its counterpart in quarterly data reported in row 1 of Table 8.

The discrepancy is possibly due to the fact that the quality of monthly data is not as good as quarterly data. One way to improve the data quality is to find efficient instrumental variables for conditional stock market variance. Christensen and Prabhala (1998) construct monthly implied volatility data using S\&P 100 index options. They find that implied volatility outperforms past stock market volatility in forecasting future volatility of the S\&P 100 index. Guo and Whitelaw (2001) also show that it is an efficient instrumental variable for aggregate stock market volatility because it subsumes the information content of past stock market
volatility and some other forecasting variables. ${ }^{12}$ Therefore, using Christensen and Prabhala's implied volatility can help us identify the price risk more precisely.

In Table 10, we report selected regression results using Christensen and Prabhala's implied volatility as an additional forecasting variable. Panel A is the OLS regression of excess stock market returns on forecasting variables. Row 1 shows that implied volatility has some predictive power in the univariate regression, although the adjusted $R^{2}$ is close to zero. ${ }^{13}$ The consumption-wealth ratio is also significant in the univariate regression. Row 3 is the multivariate regression that includes both implied volatility and the consumption-wealth ratio as forecasting variables. Similar to what we have found in quarterly data, the point estimates are larger and the standard errors are smaller in row 3 than their counterparts in the univariate regressions. Adjusted $R^{2}$ in row 3 is also larger than the sum of adjusted $R^{2}$ in rows 1 and 2 . In row 4, both variables remain significant when the stochastically detrended risk-free rate is also included as a forecasting variable.

We also report the GMM estimations of equation (5) using implied volatility data with three different specifications for the stock market variance equation. Conditional stock market variance is a linear function of all forecasting variables in panel B, a function of implied volatility only in panel C, and a linear function of implied volatility and the consumption-wealth ratio in panel D. In all specifications, the risk price is found to be positive and significant; the overidentifying restrictions are not rejected at very high significance levels. Strikingly, despite its much smaller sample size, the risk price is much more precisely identified in implied volatility data than in monthly data.

[^9]The point estimate of the risk price in implied volatility data is very close to what we have found in quarterly data. For example, it is 9.98 in row 7 of Table 10, compared with 12.61 in row 1 of Table 8 . Given that the two estimates are obtained using data with different frequencies and different sample periods, we conclude that constant relative risk aversion is a reasonable description of aggregate risk preference in the stock market. Although these estimates are much higher than those we have used in calibrating real business cycle models, they still fall into the high end of the plausible ranges considered by many financial economists, i.e., Mehra and Prescott (1985). Interestingly, Barsky, Kimball, Juster, and Shapiro (1997) report very similar estimate using survey data from the Health and Retirement Study.

## V. Some Theories

In the preceding sections, we note that, while the consumption-wealth ratio is positively correlated with future excess stock market returns, it is negatively correlated with future stock market variance. Although the early literature interprets this as evidence against a positive riskreturn tradeoff, we argue that it is also consistent with Merton's ICAPM, in which the hedge component is economically important and is negatively correlated with the risk component. Indeed, using the consumption-wealth ratio as an instrumental variable to control for the hedge component, we find that the risk price is positive in our formal estimation of Merton's ICAPM. Therefore, the negative risk-return relationship documented in the early literature is explained by the fact that the risk and hedge components are sometimes negatively correlated.

The existing theories, however, have little to say about the hedge component documented in this paper. For example, Merton (1973) does not give much intuition about the nature of the

[^10]time-varying investment opportunities, or about which state variables generate the hedge demand. In fact, it is usually negligible in a frictionless intertemporal capital asset pricing model in which the preference is a function of consumption only. Because of the lack of theoretical foundation, many financial economists, i.e., Merton (1980) and Campbell (1987), argue that the hedge component might be empirically unimportant.

Recent research by Guo (2000) is an important exception. Guo shows that it is possible to generate a substantial hedge demand for stocks in a limited stock market participation model and that such a demand is important to understand the time-varying equity premium. His model has two predictions that are relevant to our empirical findings. First, the dividend yield is positively correlated with future excess stock market returns because it is positively correlated with the hedge component. Second, there is an unstable relationship between the dividend yield and stock market variance: the dividend yield is negatively (positively) correlated with future stock market variance when the dividend yield is low (high). As a result, the hedge component tracked by the dividend yield might be negatively correlated with risk component when the dividend yield is low, as we observed in the data. Given that the consumption-wealth ratio is equivalent to the dividend yield in his model economy, our results, therefore, are well explained by the limited stock market participation model in Guo (2000). ${ }^{14}$

We can easily understand his results by looking at the log-linearized conditional excess return equation in Guo (2000):

[^11]\[

$$
\begin{equation*}
E_{t} e_{M, t+1}+E_{t} \frac{\sigma_{M, t+1}^{2}}{2} \approx \gamma E_{t} \sigma_{M C, t+1}+\left[r_{1, t+1}^{f}-\min \left\{r_{1, t+1}^{f}, r_{2, t+1}^{f}\right\}\right] \tag{6}
\end{equation*}
$$

\]

where the variance term $E_{t} \frac{\sigma_{M, t+1}^{2}}{2}$ on the left-hand side is a Jensen's Inequality adjustment, $E_{t} \sigma_{M C, t+1}$ is the conditional covariance between the stockholder's consumption growth and stock market returns, and $r_{1, t+1}^{f}\left(r_{2, t+1}^{f}\right)$ is the shadow risk-free rate of the stockholder (non-stockholder). Therefore, the conditional excess return also has two components in Guo (2000), as in Merton's ICAPM summarized by equation (1). If stock market return is on the efficient frontier, it is perfectly correlated with the pricing kernel or the stockholder's consumption growth rate. In other words, $E_{t} \sigma_{M C, t+1}$ in equation (6) is approximately proportional to the stock market variance $E_{t} \sigma_{M, t+1}^{2}$ in equation (1). We can think the second term of equation (6) $r_{1, t+1}^{f}-\min \left\{r_{1, t+1}^{f}, r_{2, t+1}^{f}\right\}$ as the hedge component in Merton's ICAPM. Because of borrowing constraints, $r_{1, t+1}^{f}$ and $r_{2, t+1}^{f}$ are not always equalized. As a result, $r_{1, t+1}^{f}-\min \left\{r_{1, t+1}^{f}, r_{2, t+1}^{f}\right\}$ is non-negative and is strictly positive when the stockholder's borrowing constraints are binding. Note that, it is this hedge term, $r_{1, t+1}^{f}-\min \left\{r_{1, t+1}^{f}, r_{2, t+1}^{f}\right\}$, that differentiates Guo (2000) from standard consumption-based capital asset pricing models (CAPM). For example, if the stockholder and the non-stockholder can freely borrow against one another, $r_{1, t+1}^{f}-\min \left\{r_{1, t+1}^{f}, r_{2, t+1}^{f}\right\}$ should always be zero and equation (6) is reduced to the standard consumption-based CAPM. Because the hedge component $r_{1, t+1}^{f}-\min \left\{r_{1, t+1}^{f}, r_{2, t+1}^{f}\right\}$ reflects the fact that the stockholder cannot trade stocks to hedge income risks due to limited stock market participation, it can also be interpreted as a liquidity premium. This liquidity premium is not constant-it is high when the stockholder needs to borrow, and it is low when he needs to save. As a result, it is positively correlated with the dividend yield. Stock
market variance also changes over time in Guo (2000). In particular, it is high when the stockholder finds it hard to smooth his consumption because (i) he cannot borrow anymore from the non-stockholder or (ii) he cannot lend anymore to the non-stockholder. The dividend yield, however, is relatively high in the first situation and relatively low in the second situation. Therefore, the dividend yield is positively correlated with stock market variance when the dividend yield is high and is negatively correlated with stock market variance when the dividend yield is low.

## VI. Conclusion

In this paper, we find that stock market risk is indeed positively priced after we control for the hedge component advocated by Merton (1973). The negative risk-return relation documented in the early literature is thus explained by the negative correlation between the risk and hedge components. We conclude that the hedge component is an important pricing factor for stocks.

Our time-series evidence is consistent with the cross-section support for a multi-factor model. While Fama and French (1993) find that CAPM has no explanatory power for the return on portfolios sorted by size and book-to-market ratio, Lettau and Ludvigson (2001c) show that a conditional (consumption) CAPM that uses the consumption-wealth ratio as the scaling variable explains these cross-section returns very well. Lettau and Ludvigson explain that small-size or low book-to-market ratio stocks have higher (consumption) betas in bad times or when the consumption-wealth ratio is high than they have in good times or when the consumption-wealth ratio is low. These results are closely related to the credit channel of business cycle propagation in the macroeconomic literature. Firms of high book-to-market ratio, for example, have less
access to credit and, therefore, are more vulnerable to adverse liquidity shocks than firms of low book-to-market ratio, especially when business conditions bad. The "abnormal returns" on value stocks, therefore, are justified by the fact that liquidity is an important pricing factor and that value stocks are more sensitive to liquidity shocks than growth stocks when business conditions are bad.

We also want to stress the important role of limited stock market participation in explaining the observed asset price behavior. While some recent asset pricing models such as the habit formation model can explain many important features of the data, Lettau and Ludvigson (2001b) show that they cannot explain negative risk-return relation in the US stock market. In contrast, the risk-return tradeoff patterns documented in this paper are well explained by the limited stock market participation model developed by Guo (2000). In his model, the stockholder also requires a liquidity premium, in addition to the risk premium in standard models, because the stockholder cannot trade stocks to hedge income risks due to limited stock market participation. While this liquidity premium is positively correlated with the dividend yield, stock market risk is negatively correlated with the dividend yield when the dividend yield is relatively low. As a result, the stock market risk-return might be negatively correlated in his model, even though risk is positively priced. Moreover, Guo (2000) also shows that limited stock market participation also explains many puzzling phenomena in the finance literature, such as the equity premium puzzle, the risk-free rate puzzle and the excess volatility puzzle. Empirical studies by Mankiw and Zeldes (1991) and Vissing-Jorgensen (1998) also point out the promising role of limited stock market participation in resolving these puzzles. Admittedly, limited stock market participation is a relatively new idea in the literature. Given its great success in explaining asset price behaviors, it deserves more attention in future research.

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Table 1: Summary Statistics
We report summary statistics for the excess stock market return, $r_{t}-r_{f, t}$; the realized stock market variance, $\sigma_{M, t}^{2} ;$ the consumption-wealth ratio, cay ${ }_{t}$; the stochastically detrended risk-free rate, rrel $_{t}$; and the dividend yield, $d p_{t}$. All variables are measured at quarterly frequency and the sample spans from 1953:Q1 to 2000:Q4 with 192 observations total. See Section II for more details about the data.

$$
\begin{array}{lllll}
\hline \hline r_{t}-r_{f, t} & \sigma_{M, t}^{2} & c a y_{t} & \text { rrel }_{t} & d p_{t}
\end{array}
$$

Panel A: Correlation Matrix

|  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $r_{t}-r_{f, t}$ | 1 | -.38 | .30 | -.30 | -.10 |
| $\sigma_{M, t}^{2}$ |  | 1 | -.34 | -.02 | -.21 |
| cay $_{t}$ |  |  | 1 | -.16 | .36 |
| rel $_{t}$ |  |  |  | 1 | -.04 |
| $d p_{t}$ |  |  |  |  | 1 |

Panel B: Univariate Statistics

| Mean | .019 | .004 | .613 | .000 | -3.405 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Standard error | .081 | .004 | .012 | .003 | .352 |
| Autocorrelation | .06 | .43 | .83 | .71 | .98 |

Table 2: Forecasting Quarterly Excess Stock Market Return


In this table, we report the OLS regression results of the one-quarterly-ahead excess stock market return, $r_{t+1}-r_{f, t+1}$, on a variety of forecasting variables. Newey and West (1987) corrected standard errors are used to calculate the $t$-statistics, which are in parentheses. Coefficients that are significant at the 5 percent level are in bold. $\sigma_{M, t}^{2}$ is the realized stock market variance; $c a y_{t}$ is the consumption-wealth ratio; $r r e l_{t}$ is the stochastically detrended risk-free rate; $d p_{t}$ is the dividend yield; term ${ }_{t}$ is the yield spread between the 10 -year Treasury bonds and 3 -month Treasury bills; $d e f_{t}$ is the yield spread between the Baarated and Aaa-rated corporate bonds; and $c p_{t}$ is the yield spread between the 6 -month commercial paper and the 3 -month Treasury bills. Regression results obtained using three different sample periods are reported in panels A, B and C, respectively. All variables are measured at quarterly frequency. See Section II for more details about the data.

## Table 3: One-Quarter-Ahead Forecasts of Excess Stock Market Return: Nested Comparisons

In this table, we report the out-of-sample performance of past stock market variance, $\sigma_{M, t}^{2}$ in forecasting the one-quarter-ahead excess stock market return, $r_{t+1}-r_{f, t+1}$. Three statistics are reported: (i) the mean-squared-error ratio of unrestricted and restricted models, $\mathrm{MSE}_{\mathrm{u}} / \mathrm{MSEr}$, (ii) the encompassing test ENC-NEW developed by Clark and McCracken (1999), and (iii) the equal forecast accuracy test MSE-F developed by McCraken (1999). ENC-NEW tests the null hypothesis, that the benchmark model encompasses all the relevant information for the next quarter's excess stock market return, against the alternative hypothesis that past stock market variance contains additional information. MSE-F tests the null hypothesis, that the benchmark model has a mean-squared error that is less than or equal to the model augmented by past stock market variance, against the alternative hypothesis that the augmented model has smaller meansquared error. Observations for the period 1953:Q1 to 1968:Q4 are used to obtain the initial in-sample estimation, and the forecasting error is calculated for the remaining period from 1969:Q1 to 2000:Q4, recursively. For example, the forecast for the 1969:Q2 is based on the estimation using the sample 1953:Q1 to 1969:Q1 and so on. The Asy. CV column reports the asymptotic 95 percent critical values provided by Clark and McCracken (1999). The BS. CV column reports the empirical 95 percent critical values obtained using the bootstrapping method. To get the empirical distribution, we first estimate a VAR(1) process of the excess stock market return and the forecasting variables with the restrictions under the null hypothesis. The residuals are saved. We then generate the data using the estimated parameters. The initial values are set to their unconditional means. The shocks are drawn from the saved residuals with replacements. The ENC-NEW and MSE-F statistics are calculated using the simulated data and the whole process is repeated 10,000 times. The benchmark model is constant excess stock market return in row 1 ; first order autoregressive excess stock market return in row 2 ; a one-period-lagged consumption-wealth ratio, $c a y_{t}$, in row 3, a two-period-lagged consumption-wealth ratio, $c a y_{t-1}$, in row 4 ; and the stochastically detrended risk-free rate, rrel $_{t}$, in row 5 . The alternative hypothesis is that past stock market variance has additional information. In rows 6 and 7, the benchmark model is constant excess stock market return, the alternative hypothesis is that past stock market variance, the one-period-lagged (two-period-lagged in row 7) consumption-wealth ratio and the stochastically detrended risk-free rate have additional information. See Section II for more details about the data.

|  |  | ENC-NEW |  |  |  | MSE-F |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Row | Comparison | $\mathrm{MSE}_{\mathrm{u}} / \mathrm{MSEr}$ | Statistic | Asy. <br> CV | $\overline{\mathrm{BS}} .$ CV | Statistic | Asy. <br> CV | BS. <br> CV |
| 1 | $\mathrm{C}+\boldsymbol{\sigma}_{M, t}^{2}$ vs. C | . 973 | 6.30 | 2.09 | 2.06 | 3.59 | 1.52 | 1.38 |
| 2 | $\mathrm{C}+\boldsymbol{\sigma}_{M, t}^{2}+\mathrm{AR}$ vs. $\mathrm{C}+\mathrm{AR}$ | . 958 | 14.61 | 2.09 | 2.19 | 5.56 | 1.52 | 1.42 |
| 3 | $\mathrm{C}+\sigma_{M, t}^{2}+$ cay $_{t}$ vs. $\mathrm{C}+c a y_{t}$ | . 884 | 15.99 | 2.09 | 2.03 | 16.83 | 1.52 | 1.31 |
| 4 | $\mathrm{C}+\sigma_{M, t}^{2}+c a y_{t-1}$ vs. $\mathrm{C}+c a y_{t-1}$ | . 907 | 13.83 | 2.09 | 2.11 | 13.06 | 1.52 | 1.43 |
| 5 | $\mathrm{C}+\sigma_{M, t}^{2}+$ rrel $_{t}$ vs. $\mathrm{C}+$ rrel $_{t}$ | . 980 | 6.98 | 2.09 | 2.14 | 2.67 | 1.52 | 1.38 |
| 6 | $\mathrm{C}+\sigma_{M, t}^{2}+$ cay $_{t}+$ rrel $_{t}$ vs. C | . 819 | 35.81 | 3.56 | 3.92 | 27.98 | 1.61 | . 86 |
| 7 | $\mathrm{C}+\sigma_{M, t}^{2}+$ cay $_{t-1}+$ rrel $_{t}$ vs. C | . 876 | 28.05 | 3.56 | 3.89 | 17.66 | 1.61 | 1.04 |

## Table 4: Long-Horizon Regressions

In this table, we report the OLS regression results of the excess stock market return on a variety of forecasting variables over long horizons, as specified in the following equation:

$$
\sum_{i=1}^{H} e_{M, t+i}=\alpha+\beta x_{t}+\varepsilon_{t+H}
$$

where $H$ is the forecast horizon, $\alpha$ is the intercept, $\beta$ is a vector of slope coefficients, $x_{t}$ is a vector of forecasting variables, and $\varepsilon_{t+H}$ is the forecast error. Newey and West (1987) corrected standard errors are used to calculate the t-statistics, which are in parentheses. Parameters that are significant at the 5 percent levels are in bold. Adjusted $R^{2}$ is in brackets. Forecasting variables include the past stock market variance, $\sigma_{M, t}^{2}$, the consumptionwealth ratio, $c a y_{t}$, the stochastically detrended risk-free rate, $r r e l_{t}$, and the dividend yield, $d p_{t}$. The sample spans from 1953:Q1 to 2000:Q4. See Section II for more details about the data.

| Row | Regressors | Forecast Horizon H |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 8 | 12 | 16 | 24 |
| 1 | $\sigma_{M, t}^{2}$ | 4.60 | 6.61 | 6.96 | 8.99 | 11.25 | 6.31 | 3.61 | 18.97 |
|  |  | (2.82) | (2.02) | (2.13) | (2.66) | (1.81) | (.91) | (.53) | (1.81) |
|  |  | [.03] | [.03] | [.02] | [.03] | [.03] | [.00] | [-.00] | [.02] |
| 2 | $c a y_{t}$ | 1.93 | 3.29 | 4.51 | 5.41 | 8.14 | 10.63 | 11.18 | 16.23 |
|  |  | (3.21) | (3.17) | (3.05) | (2.96) | (3.74) | (4.21) | (4.10) | (3.94) |
|  |  | [.08] | [.11] | [.14] | [.15] | [.19] | [.25] | [.24] | [.27] |
| 3 | $\mathrm{rrel}_{t}$ | -7.86 | -12.95 | -17.79 | -19.66 | -7.01 | -5.68 | -8.33 | -16.46 |
|  |  | (-3.93) | (-3.82) | (-3.81) | (-3.11) | (-1.09) | (-.85) | (-1.03) | (-1.52) |
|  |  | [.07] | [.09] | [.11] | [.11] | [.00] | [.00] | [.00] | [.02] |
| 4 | $\sigma_{M, t}^{2}$ | 7.68 | 11.62 | 12.75 | 15.08 | 18.59 | 13.12 | 10.17 | 26.74 |
|  |  | (5.28) | (4.67) | (4.94) | (5.17) | (3.15) | (2.33) | (2.24) | (3.35) |
|  | cay ${ }_{\text {t }}$ | 2.69 | 4.43 | 5.56 | 6.50 | 9.35 | 11.24 | 11.61 | 17.20 |
|  |  | (5.64) | (5.29) | (4.64) | (4.31) | (4.47) | (4.28) | (4.24) | (4.32) |
|  |  | [.17] | [.21] | [.21] | [.23] | [.27] | [.28] | [.25] | [.33] |
| 5 | $\sigma_{M, t}^{2}$ | 4.38 | 6.21 | 6.14 | 7.85 | 10.89 | 6.02 | 3.17 | 18.27 |
|  |  | (2.91) | (2.01) | (1.85) | (2.19) | (1.75) | (.86) | (.47) | (1.74) |
|  | rrel $_{t}$ | -7.65 | -12.64 | -17.37 | -19.02 | -6.14 | -5.31 | -8.14 | -15.57 |
|  |  | (-3.64) | (-3.61) | (-3.62) | (-2.92) | (-.94) | (-.78) | (-1.03) | (-1.42) |
|  |  | [.10] | [.12] | [.13] | [.13] | [.03] | [.00] | [.00] | [.04] |
| 6 | $\sigma_{M, t}^{2}$ | 7.22 | 10.90 | 11.47 | 13.46 | 18.13 | 11.74 | 7.68 | 18.33 |
|  |  | (5.32) | (4.68) | (4.11) | (3.98) | (2.82) | (1.71) | (1.26) | (2.36) |
|  | $c a y_{t}$ | 2.40 | 3.89 | 4.81 | 5.66 | 9.09 | 11.21 | 11.42 | 15.44 |
|  |  | (5.40) | (5.25) | (4.78) | (4.47) | (4.30) | (3.69) | (3.42) | (3.38) |
|  | $\mathrm{rrel}_{t}$ | -5.82 | -9.76 | -13.89 | -15.15 | . 04 | 2.62 | -. 09 | -3.24 |
|  |  | (-4.51) | (-4.08) | (-4.45) | (-3.95) | (.01) | (.27) | (-.01) | (-.32) |
|  | $d p_{t}$ | . 00 | . 01 | . 02 | . 03 | . 06 | . 09 | . 10 | . 36 |
|  |  | (.19) | (.46) | (.38) | (-.42) | (.40) | (.49) | (.49) | (1.29) |
|  |  | [.20] | [.25] | [.28] | [.29] | [.27] | [.28] | [.25] | [.37] |

Table 5: VAR of Excess Stock Market Return and Imputed Long-Horizon $R^{2}$
In this table, we report imputed long-horizon $R^{2}$. Following Hodrick (1992), we first estimate a VAR(1) process of the excess stock market return, $r_{t+1}-r_{f, t+1}$ and the information variables, including the consumption-wealth ratio, $c a y_{t+1}$, the stochastically detrended risk-free rate, rrel $_{t+1}$, the dividend yield, $d p_{t+1}$, and past stock market variance, $\sigma_{M, t+1}^{2}$. Newey and West (1987) corrected standard errors are used to calculate the $t$-statistics, which are in parentheses. Parameters that are significant at the 5 percent levels are in bold. We then calculate the implied long-horizon $R^{2}$ using the estimated parameters and the covariance matrix of the VAR system. For comparison, the VAR (1) process is estimated with (panel B) and without (panel A) past stock market variance. Quarterly data is used, and the sample spans from 1953:Q1 to 2000:Q4. See Section II for more details about the data.

| Dependent Variables | Constant | $r_{t}-r_{f, t}$ | $\boldsymbol{\sigma}_{M, t}^{2}$ | $c a y_{t}$ | $\mathrm{rrel}_{t}$ | $d p_{t}$ | $\overline{R^{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel: Excluding $\sigma_{M, t+1}^{2}$ |  |  |  |  |  |  |  |
| $r_{t+1}-r_{f, t+1}$ | $\begin{aligned} & \hline-\mathbf{1 . 1 7 2} \\ & (-3.913) \end{aligned}$ | $\begin{gathered} -.095 \\ (-1.713) \end{gathered}$ |  | $\begin{gathered} \mathbf{1 . 9 1 1} \\ (4.136) \end{gathered}$ | $\begin{gathered} -7.352 \\ (-4.642) \end{gathered}$ | $\begin{gathered} \hline-.006 \\ (-.314) \end{gathered}$ | . 12 |
| $c a y_{t+1}$ | $\begin{gathered} . \mathbf{0 6 3} \\ (2.492) \end{gathered}$ | $\begin{gathered} -.053 \\ (-7.801) \end{gathered}$ |  | $\begin{gathered} .909 \\ (23.830) \end{gathered}$ | $\begin{gathered} -.272 \\ (-1.914) \end{gathered}$ | $\begin{gathered} .002 \\ (1.661) \end{gathered}$ | . 80 |
| $r r e l_{t+1}$ | $\begin{gathered} .010 \\ (1.059) \end{gathered}$ | $\begin{gathered} .003 \\ (1.874) \end{gathered}$ |  | $\begin{gathered} -.018 \\ (-1.296) \end{gathered}$ | $\begin{gathered} .7222 \\ (15.180) \end{gathered}$ | $\begin{gathered} -.000 \\ (-.773) \end{gathered}$ | . 51 |
| $d p_{t+1}$ | $\begin{gathered} \mathbf{8 2 4} \\ (3.518) \end{gathered}$ | $\begin{gathered} .020 \\ (.386) \end{gathered}$ |  | $\begin{aligned} & -\mathbf{1 . 2 8 9} \\ & (-3.559) \end{aligned}$ | $\begin{gathered} 7.569 \\ (4.844) \end{gathered}$ | $\begin{gathered} 1.013 \\ (57.851) \end{gathered}$ | . 96 |
| H | 2 | 3 | 4 | 8 | 12 | 16 | 24 |
| Implied $R^{2}$ | . 21 | . 25 | . 27 | . 28 | . 26 | . 25 | . 23 |
| Panel: Including $\sigma_{M, t+1}^{2}$ |  |  |  |  |  |  |  |
| $r_{t+1}-r_{f, t+1}$ | $\begin{gathered} -1.442 \\ (-4.733) \end{gathered}$ | $\begin{gathered} .026 \\ (.487) \end{gathered}$ | $\begin{gathered} 7.422 \\ (5.462) \end{gathered}$ | $\begin{gathered} \mathbf{2 . 3 6 1} \\ (5.307) \end{gathered}$ | $\begin{gathered} \mathbf{- 5 . 6 0 5} \\ (-4.025) \end{gathered}$ | $\begin{gathered} .004 \\ (.279) \end{gathered}$ | . 20 |
| $\sigma_{M, t+1}^{2}$ | $\begin{gathered} .040 \\ (2.609) \end{gathered}$ | $\begin{gathered} .003 \\ (1.133) \end{gathered}$ | $\begin{gathered} \mathbf{3 6 9} \\ (4.350) \end{gathered}$ | $\begin{gathered} -.069 \\ (-2.82) \end{gathered}$ | $\begin{gathered} .057 \\ (.795) \end{gathered}$ | $\begin{gathered} -.001 \\ (-1.575) \end{gathered}$ | . 25 |
| $c^{\text {cay }}{ }_{\text {t+1 }}$ | $\begin{gathered} .056 \\ (2.158) \end{gathered}$ | $\begin{gathered} -.049 \\ (-6.84) \end{gathered}$ | $\begin{gathered} .208 \\ (1.864) \end{gathered}$ | $\begin{gathered} . \mathbf{9 2 1} \\ (24.012) \end{gathered}$ | $\begin{gathered} -.224 \\ (-1.624) \end{gathered}$ | $\begin{gathered} .002 \\ (1.820) \end{gathered}$ | . 81 |
| rrel $_{t+1}$ | $\begin{gathered} .012 \\ (1.305) \end{gathered}$ | $\begin{gathered} .002 \\ (.969) \end{gathered}$ | $\begin{gathered} -.069 \\ (-1.454) \end{gathered}$ | $\begin{gathered} -.022 \\ (-1.561) \end{gathered}$ | $\begin{gathered} .706 \\ (14.542) \end{gathered}$ | $\begin{gathered} -.000 \\ (-1.029) \end{gathered}$ | . 51 |
| $d p_{t+1}$ | $\begin{gathered} \mathbf{1 . 0 8 0} \\ (4.323) \end{gathered}$ | $\begin{gathered} -.095 \\ (-1.888) \end{gathered}$ | $\begin{gathered} -.702 \\ (-6.930) \end{gathered}$ | $\begin{aligned} & -1.714 \\ & (-4.779) \end{aligned}$ | $\begin{gathered} 5.918 \\ (4.227) \end{gathered}$ | $\begin{gathered} \mathbf{1 . 0 0 3} \\ (81.532) \end{gathered}$ | . 97 |
| H | 2 | 3 | 4 | 8 | 12 | 16 | 24 |
| Implied $R^{2}$ | . 29 | . 30 | . 29 | . 24 | . 20 | . 18 | . 15 |

Table 6: Forecasting Quarterly Stock Market Variance
In this table, we report the OLS regression results of one-quarter-ahead realized stock market variance, $\sigma_{M, t+1}^{2}$, on a variety of forecasting variables. Newey-West (1987) corrected standard errors are used to calculate the $t$-statistics, which are in parentheses. Parameters that are significant at the 5 percent level are bold. The forecasting variables include past stock market variance, $\sigma_{M, t}^{2}$, the consumption-wealth ratio, $c a y_{t}$, the stochastically detrended riskfree rate, $r r e l_{t}$, the dividend yield, $d p_{t}$, the yield spread between the Baa-rated and Aaa-rated corporate bonds, $d p_{t}$, and the yield spread between the 6-month commercial paper and the 3month Treasury bills, $c p_{t}$. Quarterly data is used, and three different samples are reported in panels A, B, and C, respectively. See Section II for more details about the data.


Table 7: GMM Estimation of Equation (5) Using Quarterly Data
In this table, we report the GMM estimation of equation (5):

$$
\begin{aligned}
& e_{M, t+1}=\omega_{1}+\gamma \sigma_{M, t+1}^{2}+\beta_{1} y_{t}+\eta_{t+1}+\gamma \varepsilon_{t+1} \\
& \sigma_{M, t+1}^{2}=\omega_{2}+\beta_{2} x_{t}+\varepsilon_{t+1}
\end{aligned}
$$

Newey-West (1987) corrected standard errors are used to calculate the $t$-statistics, which are in parentheses. Parameters that are significant at the 5 percent level are in bold. We assume that the hedge component is a linear function of the consumption-wealth ratio, $c a y_{t}$, and the stochastically detrended risk-free rate, rrel $_{t}$. Conditional stock market variance is a linear function of past stock market variance, $\sigma_{M, t}^{2}$; the consumption-wealth ratio, $c a y_{t}$; the dividend yield, $d p_{t}$; the yield spread between the Baarated and Aaa-rated corporate bonds, $d p_{t}$; and the yield spread between the 6 -month commercial paper and the 3 -month Treasury bills $c p_{t}$. Instrumental variables include a constant and all the independent variables in both equations. There are 10 parameters to be estimated and 14 identifying relations. Therefore, the overidentifying restrictions have 4 degrees of freedom. Quarterly data is used, and three different samples are reported in panels A, B, and C, respectively. See Section II for more details about the data.

| Constant | $\sigma_{M, t+1}^{2}$ | $\sigma_{M, t}^{2}$ | $c a y_{t}$ | $r_{r e l}^{t}$ | $d p_{t}$ | $d e f_{t}$ | $c p_{t}$ | $\bar{R}^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |



Table 8: GMM Estimation of Equation (5) Using Quarterly Data with Alternative Stock Market Variance Specification
In this table, we report the GMM estimation of equation (5):

$$
\begin{aligned}
& e_{M, t+1}=\omega_{1}+\gamma \sigma_{M, t+1}^{2}+\beta_{1} y_{t}+\eta_{t+1}+\gamma \varepsilon_{t+1} \\
& \sigma_{M, t+1}^{2}=\omega_{2}+\beta_{2} x_{t}+\varepsilon_{t+1}
\end{aligned}
$$

Newey-West (1987) corrected standard errors are used to calculate the $t$-statistics, which are in parentheses. Parameters that are significant at the 5 percent level are in bold. We assume that the hedge component is a linear function of the consumption-wealth ratio, $c a y_{t}$, and the stochastically detrended risk-free rate, $r$ rel $l_{t}$. Conditional stock market variance is a linear function of only past stock market variance, $\sigma_{M, t}^{2}$ and $\sigma_{M, t-1}^{2}$. In addition to a constant and the independent variables in both equations, the dividend yield, $d p_{t}$; the yield spread between the Baa-rated and Aaa-rated corporate bonds; and the yield spread between the 6 month commercial paper and the 3 -month treasury bills, $c p_{t}$, are also included as the instrumental variables. There are 7 parameters to be estimated and 16 identifying relations. Therefore, the overidentifying restriction has 9 degrees of freedom. Quarterly data is used, and three different samples are reported in panels A, B, and C, respectively. See Section II for more details about the data.

|  | Constant | $\sigma_{M, t+1}^{2}$ | $\sigma_{M, t}^{2}$ | $\sigma_{M, t-1}^{2}$ | $c a y_{t}$ | $\mathrm{rrel}_{t}$ | $\bar{R}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A 1953:Q1-2000:Q4Excess Return Equation |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | $\begin{gathered} -1.177 \\ (-6.358) \end{gathered}$ | $\begin{aligned} & \mathbf{1 2 . 6 1 0} \\ & (5.026) \end{aligned}$ |  |  | $\begin{gathered} 1.877 \\ (6.430) \end{gathered}$ | $\begin{gathered} -4.735 \\ (-3.937) \end{gathered}$ | . 20 |
|  |  |  | Stock Mar | et Varian | Equation |  |  |
| 2 | $\begin{gathered} .002 \\ (7.486) \end{gathered}$ |  | $\begin{gathered} .428 \\ (9.835) \end{gathered}$ | $\begin{gathered} .123 \\ (3.078) \end{gathered}$ |  |  | . 21 |
| Overidentifying Restrictions $\chi^{2}(9)=10.0631, \mathrm{p}$-value .345 |  |  |  |  |  |  |  |
| Panel B 1953:Q1-1987:Q3 |  |  |  |  |  |  |  |
| 3 | Excess Return Equation |  |  |  |  |  |  |
|  | $\begin{aligned} & -1.357 \\ & (-6.757) \end{aligned}$ | $\begin{aligned} & 15.171 \\ & (6.603) \end{aligned}$ |  |  | $\begin{gathered} 2.168 \\ (6.636) \end{gathered}$ | $\begin{gathered} -6.691 \\ (-7.003) \end{gathered}$ | . 23 |
|  |  |  | Stock | Market V | ance |  |  |
| 4 | $\begin{gathered} .002 \\ (8.481) \end{gathered}$ |  | $\begin{gathered} .415 \\ (7.709) \end{gathered}$ | $\begin{gathered} .088 \\ (2.243) \end{gathered}$ |  |  | . 16 |
|  | Overidentifying Restrictions $\chi^{2}(9)=7.65166, \mathrm{p}$-value . 570 |  |  |  |  |  |  |
| Panel C 1962:Q4-2000:Q4 |  |  |  |  |  |  |  |
| 5 | Excess Return Equation |  |  |  |  |  |  |
|  | $\begin{aligned} & -1.215 \\ & (-6.542) \end{aligned}$ | $\begin{gathered} \mathbf{1 3 . 0 3} \\ (5.150) \end{gathered}$ |  |  | $\begin{gathered} 1.931 \\ (6.612) \end{gathered}$ | $\begin{gathered} -\mathbf{3 . 9 8 7} \\ (-3.106) \end{gathered}$ | . 18 |
| 6 |  |  | Stock Mar | et Varian | Equation |  |  |
|  | $\begin{gathered} .002 \\ (5.934) \end{gathered}$ |  | $\begin{gathered} .459 \\ (12.72) \end{gathered}$ | $\begin{gathered} .112 \\ (2.957) \end{gathered}$ |  |  | . 25 |
|  | Overidentifying Restrictions $\chi^{2}(9)=8.59075, \mathrm{p}$-value .476 |  |  |  |  |  |  |

Table 9: Selected Results from Monthly Data
In this table, we report selected results using monthly data. Newey-West (1987) corrected standard errors are used to calculate the $t$-statistics, which are in parentheses. Parameters that are significant at the 5 percent level are in bold. Panel A reports the OLS regression results of the excess stock market return, $r_{t+1}-r_{f, t+1}$, on a variety of forecasting variables, including past stock market variance, $\sigma_{M, t}^{2}$ and $\sigma_{M, t-1}^{2}$; the consumption-wealth ratio, cay ${ }_{t}$; and the stochastically detrended risk-free rate rrel $_{t}$. Panels B and C report the GMM estimation of equation (5) using different specifications. In both panels, we assume that the hedge component is a linear function of the consumption-wealth ratio, $c a y_{t}$, and the stochastically detrended risk-free rate, $r r e l_{t}$. In panel B, conditional stock market variance is a linear function of past stock market variance, $\sigma_{M, t}^{2}$ and $\sigma_{M, t-1}^{2}$; the consumption-wealth ratio, $c a y_{t}$; the dividend yield, $d p_{t}$; the yield spread between the Baa-rated and Aaa-rated corporate bonds, $d e f_{t}$; and the yield spread between the 6 -month commercial paper and the 3 -month Treasury bills, $c p_{t}$. In panel C, conditional stock market variance is a linear function of only the past stock market variance, $\sigma_{M, t}^{2}$ and $\sigma_{M, t-1}^{2}$. Monthly data is used, and the sample spans from 1959:1 to 2000:12. See Section II for more details about the data.


| 8 | $\begin{gathered} -.179 \\ (-4.387) \end{gathered}$ | $\begin{gathered} \hline \mathbf{4 . 5 8 3} \\ (2.169) \end{gathered}$ |  |  | $\begin{gathered} .472 \\ (4.458) \end{gathered}$ | $\begin{gathered} \hline-\mathbf{3 . 9 7 8} \\ (-2.553) \end{gathered}$ |  |  |  | . 04 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | $\begin{gathered} . \mathbf{0 0 3} \\ (2.390) \end{gathered}$ |  | $\underset{(6.837)}{.355}$ | $\xrightarrow[(3.215)]{. \mathbf{1 4 5}}$ | $\begin{gathered} -.005 \\ (-1.705) \end{gathered}$ |  | $\begin{gathered} -.001 \\ (-3.967) \end{gathered}$ | $\begin{gathered} .000 \\ (4.737) \end{gathered}$ | $\begin{gathered} .001 \\ (4.342) \end{gathered}$ | . 35 |
|  | Overidentifying Restrictions $\chi^{2}(5)=6.17113$, p -value . 290 |  |  |  |  |  |  |  |  |  |

Panel C: GMM Estimate of Merton's Model, variance equation includes only lagged variance

| Excess Return Equation |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | $\begin{gathered} -.129 \\ (-3.939) \end{gathered}$ | $\begin{gathered} \mathbf{5 . 8 9 6} \\ (3.039) \end{gathered}$ |  | $\begin{gathered} .339 \\ (3.975) \end{gathered}$ | $\begin{gathered} -3.971 \\ (-2.584) \end{gathered}$ | . 04 |
| 11 | $\begin{gathered} .000 \\ (7.818) \end{gathered}$ |  | $\begin{gathered} .428 \\ (9.722) \end{gathered}$ | Stock Market Varia $\begin{gathered} .177 \\ (4.001) \end{gathered}$ | Equatio | . 31 |
| Overidentifying Restrictions $\chi^{2}(9)=13.8609, \mathrm{p}$-value . 127 |  |  |  |  |  |  |

## Table 10: Selected Results from Implied Volatility Data

In this table, we report selected results using the implied volatility data constructed by Christensen and Prabhala (1998) as an instrumental variable for conditional stock market variance. Newey-West (1987) corrected standard errors are used to calculate the t-statistics, which are in parentheses. Parameters that are significant at the 5 percent level are in bold. Panel A reports the OLS regression results of the excess stock market return, $r_{t+1}-r_{f, t+1}$, on a variety of forecasting variables, including the implied volatility, $I V O L_{t}$; the consumption-wealth ratio, cay ${ }_{t}$; and the stochastically detrended risk-free rate, rrel $_{t}$. Panels B, C, and D report the GMM estimation of equation (5). We assume that the hedge component is a linear function of $c a y_{t}$ and rrel $_{t}$. In panel B , conditional stock market variance is a linear function of $I V O L_{t}$; cay ; the dividend yield, $d p_{t}$; the yield spread between the Baa-rated and Aaa-rated corporate bonds, $\operatorname{def}_{t}$; and the yield spread between the 6-month commercial paper and the 3-month Treasury bills, $c p_{t}$. In panel C, conditional stock market variance is a linear function of $I V O L_{t}$, and in panel D , it is a linear function of $I V O L_{t}$ and $c a y_{t}$. See Section II for more details about the data.

|  | Constant | $\sigma_{M, t+1}^{2}$ | $I V O L_{t}$ | $\sigma_{M, t}^{2}$ | $c a y_{t}$ | $\mathrm{rrel}_{t}$ | $d p_{t}$ | $d e f_{t}$ | $c p_{t}$ | $\overline{R^{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Forecasting Monthly Excess Returns |  |  |  |  |  |  |  |  |  |  |
| 1 | . 006 |  | 1.351 |  |  |  |  |  |  | . 00 |
|  | (2.361) |  | (2.512) |  |  |  |  |  |  |  |
| 2 | -. 164 |  |  |  | . 445 |  |  |  |  | . 01 |
|  | (-2.475) |  |  |  | (2.588) |  |  |  |  |  |
| 3 | -. 224 |  | 2.388 |  | . 587 |  |  |  |  | . 02 |
|  | (-2.880) |  | (4.236) |  | (2.949) |  |  |  |  |  |
| 4 | -. 260 |  | 2.662 |  | . 674 | -5.296 |  |  |  | . 03 |
|  | (-3.284) |  | (4.380) |  | (3.370) | (-1.438) |  |  |  |  |


| 5 | $\begin{gathered} -.323 \\ (-4.264) \end{gathered}$ | $\begin{gathered} 6.247 \\ (5.002) \end{gathered}$ |  |  | $\begin{gathered} .830 \\ (4.320) \end{gathered}$ | $\begin{gathered} -\mathbf{6 . 0 1 8} \\ (-2.683) \end{gathered}$ |  |  |  | . 04 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | $\begin{gathered} .001 \\ (1.831) \end{gathered}$ |  | $\begin{gathered} .327 \\ (13.075) \end{gathered}$ | $\begin{gathered} . \mathbf{1 9 6} \\ (2.599) \end{gathered}$ | $\begin{gathered} -.020 \\ (-3.318) \end{gathered}$ |  | $\begin{gathered} -.000 \\ (-1.198) \end{gathered}$ | $\begin{gathered} .000 \\ (.149) \end{gathered}$ | $\stackrel{.001}{(3.537)}$ | . 46 |
| Overidentifying Restrictions $\chi^{2}(5)=4.99534$, p-value . 416 |  |  |  |  |  |  |  |  |  |  |
| Panel C: GMM Estimate of Merton's Model, variance equation includes only IVOL |  |  |  |  |  |  |  |  |  |  |
| Excess Return Equation |  |  |  |  |  |  |  |  |  |  |
| 7 | $\begin{gathered} -.258 \\ (-3.820) \end{gathered}$ | $\begin{gathered} \mathbf{9 . 9 8 2} \\ (5.411) \end{gathered}$ |  |  | $\begin{gathered} .651 \\ (3.845) \end{gathered}$ | $\begin{gathered} -5.298 \\ (-2.199) \end{gathered}$ |  |  |  | . 02 |
| Stock Market Variance Equation |  |  |  |  |  |  |  |  |  |  |
| 8 | $\begin{gathered} .000 \\ (6.939) \end{gathered}$ |  | $\stackrel{.460}{(26.264)}$ |  |  |  |  |  |  | . 41 |
| Overidentifying Restrictions $\chi^{2}(8)=7.49026, \mathrm{p}$-value . 485 |  |  |  |  |  |  |  |  |  |  |

Panel D: GMM Estimate of Merton's Model, variance equation includes only IVOL and $c a y_{t}$

| Excess Return Equation |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | -. 370 | 10.741 |  | . 939 | -4.567 | . 03 |
|  | (-4.779) | (5.610) |  | (4.777) | (-1.803) |  |
| Stock Market Variance Equation |  |  |  |  |  |  |
| 10 | . 008 |  | . 412 | -. 370 |  | . 42 |
|  | (4.301) |  | (15.971) | (-4.779) |  |  |
|  | Overidentifying Restrictions $\chi^{2}(7)=5.23637$, p-value . 631 |  |  |  |  |  |



Figure 1: Standardized Stock Market Return and Variance (Thick Solid Line)


Figure 2: Standardized Conditional Excess Return and Sharpe Ratio (Thick Solid Line) Using Point Estimates from Panel A of Table 7


Figure 3: Standardized Hedge Component (Thick Solid Line) and Risk Component Using Point Estimates from Panel A of Table 7


Figure 4: 10-Year Rolling Correlations between Conditional Risk and Return, and Risk and Hedge Components (Thick Solid Line) Using Point Estimates from Panel A of Table 7


Figure 5: Standardized Conditional Excess Return and Sharpe Ratio (Thick Solid Line) Using Point Estimates from Panel A of Table 8


Figure 6: 10-Year Rolling Correlation between Conditional Risk and Return, and Risk and Hedge Components (Thick Solid Line) Using Point Estimates from Panel A of Table 8


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[^1]:    ${ }^{1}$ Researchers have drawn conflicting conclusions on the empirical relationship between stock market risk and return. While the two papers mentioned above find a negative risk-return relation, some other authors, i.e., French, Schwert, and Stambaugh (1987), argue for a positive one. See Scruggs (1998) for an overview of this literature.

[^2]:    ${ }^{2}$ Assuming a recursive utility function, Campbell (1993) derives a discrete-time version of equation 1 based on Lucas' (1978) consumption-based capital asset pricing model. In Campbell's equation, $\left[\frac{J_{W W} W}{J_{W}}\right]=\gamma$, $\left[\frac{-J_{W F}}{J_{W}}\right]=\gamma-1$, and $E_{t-1} \sigma_{M F, t}=\operatorname{cov}\left[r_{m, t}-E_{t-1} r_{m, t},\left(E_{t}-E_{t-1}\right) \sum_{j=1}^{\infty} \rho^{j} r_{m, t+j}\right]$, where $\gamma$ is the relative risk aversion coefficient and $\rho$ is a constant slightly less than 1 .

[^3]:    ${ }^{3}$ Most results reported in this paper are generated using quarterly data rather than monthly data for two reasons. First, Merton (1980) shows the stock market variance can be accurately measured if the number of daily observations used to construct it is sufficiently large. Therefore, the stock market variance is likely to be better estimated at quarterly frequency than at monthly frequency. Indeed, our measured stock market variance displays stronger predictive power for the excess stock market return in quarterly data than in monthly data. Second, the consumption-wealth ratio, an important instrumental variable used in this study, is reliably available only at quarterly frequency. However, we do find qualitatively similar results using monthly data.
    ${ }^{4}$ In an independent research, Goyal and Santa-Clara (2001) report that average stock market variance, defined as the average variance of all stocks, forecasts excess stock market returns. They also find that one-period lagged stock market variance has no predictive ability for excess returns in the monthly data, a result we confirm in this paper; however, we report that two-period lagged variance does have some forecasting power. Therefore, Goyal and SantaClara and our paper differ in two important dimensions. First, while we focus on systematic risk and its interaction

[^4]:    ${ }^{6}$ The consumption-wealth ratio data are downloaded from Martin Lettau's website http://www.newyorkfed.org /rmaghome/economist/lettau/data.html. The original series is constructed using quarterly data. Lettau also provides the interpolated monthly data, which span from 1959:1 to 2001:7.
    ${ }^{7}$ We thank Prabhala for providing the implied volatility data.

[^5]:    ${ }^{8}$ As a robustness check, we compare our measure of stock market variance defined by equation 2 with those constructed using different methods and data. First, we use the sum of the squared daily excess stock market return. Second, we use the daily index of the S\&P 500. Third, we use the square of the quarterly excess stock market return. The first two alternative variance measures are almost perfectly correlated with the measure used in the paper, with correlation coefficients of about 1 and .98 , respectively. The correlation coefficients between the third variance measure and the other two are above .5 . However, its information content is completely subsumed by the other measures in forecasting excess stock market returns, indicating the superiority of the realized stock market variance constructed from daily data.

[^6]:    ${ }^{9}$ To save the space, the regression results using unmodified variance are not reported.

[^7]:    ${ }^{10}$ Stock market variance is .2 for the fourth quarter of 1987 , while the second largest stock variance in our sample is

[^8]:    ${ }^{11}$ We do not include the yield on 1-year Treasury bonds in the multivariate regression because it is non-stationary,

[^9]:    ${ }^{12}$ We find that implied volatility does not subsume the information content of the consumption-wealth ratio, which is not considered by Guo and Whitelaw (2001).

[^10]:    ${ }^{13}$ Our results are somewhat sensitive to the number of lags used in Newey-West's (1987) correction for the standard

[^11]:    errors. The implied volatility becomes insignificant if we do not correct for the serial correlation in the residuals. However, other results reported in Table 10 are not sensitive to the choice of the lags.
    ${ }^{14}$ During the stock market boom in late 1990 s, stock market variance increased dramatically, while the dividend yield and the consumption wealth ratio fell to their historically low levels. This observation explains why the correlation between the dividend yield and stock market variance changes sign from positive in the early periods to negative in the late periods, as shown in Table 6. It also explains why the consumption-wealth ratio is more negatively correlated with stock market variance in the late periods than in the early periods. Moreover, we suspect that the dividend yield and the consumption-wealth ratio were strongly positively correlated with stock market variance in 1930s, when stock prices fell to its historical low levels.

