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# Macroeconomic Policy Effects in a Monetary Union

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# MACROECONOMIC POLICY EFFECTS IN A MONETARY UNION


## ABSTRACT

This paper develops a two-country model of a monetary union. In order to analyze fully the linkages between the countries, the model specifies structural equations for the goods, money and bond markets in each country. Interdependencies arise through trade, the asset markets, and a common currency. The model also includes a supply side for each economy based on an expectations augmented Phillips curve. Using this model it is possible to trace the shifts in aggregate demand and aggregate supply in both countries resulting from a change in fiscal and monetary policies. The results suggest that given asymmetries in current account balances, fiscal policies may cause friction among countries in a European monetary union.



**KEYWORDS:** monetary union, fiscal policy, monetary policy, single currency, policy coordination, spillovers, convergence, asymmetries, net debtor/creditor

**JEL CLASSIFICATION:** E63, F33, F42



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## I. INTRODUCTION

In February 1992, the leaders of the European Community (EC) met in Maastricht, Holland, to sign a treaty of economic and monetary union. European economic and monetary union (EMU), as set forth by the Delors Report and formalized by the Maastricht treaty, is to be characterized by: a single market typified by the "complete freedom of movement for persons, goods, services and capital" (Committee for the Study of Economic and Monetary Union, 1989, para. 17); macroeconomic policy coordination among member governments; a single currency; and, a common monetary policy carried out through a European Central Bank (ECB). This single currency will be issued and controlled by the central bank which is to be independent both from the national governments and the European Community government. The central bank will be responsible for the formulation and implementation of the monetary policy for the entire Community (or members of the monetary union if these are less than the total EC countries). As stated in the Maastricht Treaty, the primary objective of the central bank will be price stability. To emphasize the weight the central bank should place on achievement of this objective, the treaty states that only without detriment to this objective should the central bank "support the general economic policies in the Community" (Article 105.1). To further emphasize the importance of price stability and the independence of the central bank, the bank is prohibited from directly financing government deficits.

While the member countries of the EC have agreed upon the basic structure of the monetary union, there is less agreement as to the degree of economic union that is necessary for monetary union -- in particular, the coordination and control of fiscal policies. There are two views on the need for fiscal coordination in a monetary union. The first argues that there is no need to establish binding rules for fiscal policy, as the monetary union will of its own accord lead to fiscal policy convergence. Cohen (1989) develops a model which supports this conclusion. He argues that monetary policy coordination, if it is credible, will "trigger the appropriate fiscal correction needed to make it sustainable" (Cohen, 1989, p. 304). Fiscal policy coordination might be welfare enhancing, but he notes it is not a prerequisite for monetary union. Glick and Hutchinson (1992) argue that all that may be necessary for the well functioning of a monetary union is that the central bank be prohibited from financing government deficits.

The second view argues that binding rules with respect to the size of government budget deficits are needed to ensure the proper functioning of a monetary union. This view is based on the premise that there would be a lack of fiscal restraint among the members of the monetary union which would crowd out investment within the union due to an increase the interest rate.[1] The increase in the interest rate would also increase the cost of financing deficits for all the member governments. Thus, countries that wanted to control their debt to GDP ratios would have to tighten their fiscal policies.[2] This lack of fiscal restraint could also put pressure on the

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[1] Given the existence of perfect capital mobility and if assets are perfect substitutes, one can think of an interest rate prevailing for the entire EC.

[2] See DeGrauwe (1992) for more on this point.

monetary authority to ease policy. Those countries pursuing expansionary policies might pressure the monetary authority to monetize their deficits, while the other countries might also favor an expansionary monetary policy in order to lower the interest rate. Fiscal laxity by some members could thus create problems for the entire union either through higher inflation or slower growth, which in turn could increase pressure within the fiscally sound countries to break away from the monetary union.

Fiscal policy coordination, according to the Maastricht Treaty, will take place through the establishment of "broad guidelines" to which the member countries are to adhere. The Commission will report to the Council on the economies of each member state and the consistency of each member's policies with the established guidelines. If the Commission believes that a country's policies are "not consistent with the broad guidelines ... or that they risk jeopardizing the proper functioning of economic and monetary union," it may recommend that the Council suggest policy reforms to the errant country (Article 103). The emphasis of the Treaty, however, is not on coordinating policies, but restricting the ability of countries to pursue expansionary fiscal policies. The Treaty set reference points for the size of government deficits and the level of debt as percents of GDP. The Commission will monitor the budget deficits and government debt of the member countries and inform the Council of any country whose fiscal deficit rises above 3 percent of its GDP or whose debt is greater than 60 percent of its GDP. The Council may recommend corrective actions to be taken by the government(s) not meeting one of these criteria and can ultimately assess fines if these recommendations are ignored. Although this "excessive deficit" provision seems to place firm restrictions on fiscal policies, the treaty does allow the Commission to take into account the other information "including the medium-term economic and budgetary position" of the country in question before making its recommendation to the Council. Furthermore if the increase in the deficit or debt above the reference level is viewed as a temporary aberration it may be tolerated.

As noted in Buiter and Kletzer (1991), the asymmetry of constraints which is placed on fiscal policies, "can only be rationalized through a belief that absent these constraints there would be a bias towards government deficits that are too large rather than too small." Furthermore, while fiscal policy restrictions may be necessary to ensure the proper functioning of a monetary union, restricting the ability of a country to react to shocks, particularly idiosyncratic shocks may also place strain on a monetary union.

Before addressing the need for and usefulness of fiscal policy restrictions in a monetary union, it is practical to develop a model of a monetary union which allows one to understand the intra and cross-country effects of policies.

There has been much written concerning the process of establishing a monetary union, particularly with respect to the role of a central bank and the implementation of a common currency. While the process of creation of a monetary union will be a temporary one, the resultant union is expected to be permanent. Yet, there has been little written concerning the macroeconomies of the EC countries operating in a monetary union. Because of this, our understanding of how economies are linked in a monetary union is limited. Such an understanding is vital if the EC follows through on the final two stages of monetary union. This is particularly important given that the European economic and monetary union as set forth in the Maastricht treaty not only removes monetary policy from the control of each member country, through the creation of the independent supranational central bank, but also restricts the ability of a country to pursue an expansionary fiscal policy.

Modelling a monetary union, such as that expected to occur in the European Community, presents a problem because it does not fit into the mold of either a standard closed economy model, or an open economy fixed exchange rate model. The European Community system can not be modelled as a closed economy typical of federal systems, such as the United States or Canada, due to the difference in relative importance between the central government and regional authorities.[3] In modelling fiscal policy effects in a typical federal system one does not worry about the policy interactions and spillover effects among states, and between states and the central government. This is due to the dominant role of the central government as fiscal policy maker. Although states or regions in a federal system set taxation and revenue policies which have an impact upon the national economy, fiscal policy is still dominated by the actions of the federal government.

In the European Community, the members of the federation (the national governments) will continue to be the primary fiscal policy makers. Thus, the interactions of the regional players are of prime importance in modelling the monetary union. So, instead of ignoring the effects of fiscal policy decisions by the state governments (as in a closed economy model of the U.S.), the model of a monetary union presented here ignores the effects of fiscal policy decisions made by the Community government.

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[3] There are those, however who seem to indicate that such a model is applicable. Both Cohen (1989) and Portes (1990) in discussing the issue of fiscal policy coordination note that one does not worry about this between states in the U.S. and thus it may not be a problem for the EC.

This distinction can be justified by looking at the relative sizes of fiscal expenditures by U.S. states, versus the "states" of the European Community. The expenditures of the U.S. federal government constitute around 20-25 percent of U.S. GDP, while the expenditures of even the most populous states (California and New York) are less than 2 percent of GDP, and the total expenditures of all 50 states are only about 13 percent of GDP. In contrast, the expenditures of the European Community government are at present only slightly more than 1 percent of the GDP of the EC and are not expected to exceed 3 percent of GDP (Lamfalussy, 1989, pp. 107 and 111). Furthermore, the European Community government has no means for active fiscal policy.

A monetary union within the European Community also does not fit into the model of a fixed exchange rate system. First, the use of a single currency permanently fixes nominal exchange rates between the member countries. Revaluation or devaluation of the exchange rate is not possible. Most important is not the use of a single currency, but rather the common monetary policy that distinguishes the model of the European monetary union from that of an open economy fixed exchange rate model. Unlike the standard Mundell-Fleming fixed exchange rate model where monetary policy has no effects, in the model of a monetary union, the monetary policy actions of the central bank affect all countries in the union.

Open-economy models have been developed to analyze issues of interdependence and policy coordination between countries, and these were used as a starting point for developing a model of a monetary union. Cohen and Wyplosz (1989) develop a macroeconomic model of a monetary union in which aggregate demand in each country is given by one variable which is directly controlled by the fiscal policy maker, and inflation is a choice variable of the central bank. Cohen (1989), Bean (1985) and Pachecco (1985) use slightly more complex reduced form models to model coordination problems across countries. All of these use reduced-form models which are useful for examining issues where the policy effects across countries are well-known, but they are not adequate to address issues of policy effects within a monetary union. In the model developed here one can clearly distinguish the direct, spillover and feedback effects on aggregate demand and aggregate supply due to a change in fiscal or monetary policy.

The model developed in this paper is most similar to those of Oudiz and Sachs (1984), Sachs and Wyplosz (1984), Kole (1988), and Kenen (1989, 1990), all of which start from the explicit equations for the components of aggregate demand. Nevertheless, all of these models also have their limitations for modelling a monetary union. With the exception of Oudiz and Sachs, all of the models ignore the supply side of the economy. With the exception of Kole, they ignore interest-income terms in private absorption. This is a fairly typical restriction in open-economy model, as it simplifies the process of solving for aggregate demand and equilibrium output and prices. Inclusion of these terms allows last period's interest rate to have an effect on this period's consumption. Furthermore, in an open-economy model it introduces the importance of the net-debtor or net-creditor status of a country in the determination of policy effects.

The model developed here can be used to resolve some of the issues relating to fiscal policy coordination/restrictions in a monetary union. This is done by indicating the nature of the spillover effects of fiscal policy in a monetary union, and addressing the issue of crowding out both internally and in other countries within the monetary union. The model developed is useful in determining monetary policy effects in a monetary union.

The second section develops a two country model of a monetary union. The third section explains the linkages between the countries as captured in the aggregate supply and demand equations for each country. The fourth section gives the solution for equilibrium output and inflation in each country, and derives comparative statics which indicate how the policies of one country affect both countries within the monetary union. This section also examines how the existence of a monetary union changes the results of the standard two country open economy model. The final section presents the conclusions and indicates areas for further research.

## II. THE MODEL

There are two countries, indexed by 1 and 2, which are of similar size. Each country maintains independent control over its fiscal policy, but the monetary policy for the two countries is controlled by an independent central bank.

The countries produce goods which are imperfect substitutes. Goods are traded freely between the two countries. There are no transportation costs, but preferences for goods may vary across the countries. The government and the private sector in each country demand both domestic and foreign goods.

Each government issues one period bonds which are bought by the residents of each country and the central bank. The bonds are perfect substitutes, and capital is perfectly mobile. Thus, the nominal interest rate prevailing in each country at all times is the world interest rate (i.e.

$\iota_t=\iota_{1t}=\iota_{2t}$). There are no private issues of bonds, nor is there any capital accumulation.

There is a common currency, the ecu, which is issued by the central bank. Money creation is controlled solely by the central bank, through bond purchases.[4] At the end of each period, the governments repay their bonds plus interest. Thus, money is not held across periods. Since the real money supply is equal to the real value of bonds held by the central bank, the government makes its interest payments not in money but in goods. The central bank however, neither purchases goods, nor does it turn its profits over to the national governments. Therefore, one can assume that the goods payment of interest is immediately consumed by the central bank.[5]

All variables are measured in real terms. The deflator used to convert a country's nominal variables to real variables is its consumer price index. (See Table I for a listing of variables and parameters.) All stock variables are measured at the start of the period, which is indexed by t. Spending and portfolio decisions are made at the start of period t. Thus, $\iota_t$ is the t to t+1 interest rate, and $\pi_t$ is the t to t+1 inflation rate.

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[4] Since the money supply is determined solely by the extent of bond purchases by the central bank, it is possible that the monetary authority might be constrained in its ability to increase the money supply. Given that this constraint is unlikely to be binding except in cases of hyperinflation, and given that the central bank's primary objective is price stability, it is assumed throughout that the constraint is nonbinding.

[5] This assumption is made to ensure that the central bank does not have control over goods or the allocation of seignorage in the model. An alternative would be to divide the seignorage in accordance with the provision of the Maastricht treaty, i.e. proportionate to each country's paid up share of capital in the central bank, which in turn is determined by the population and GDP of each country (Maastricht Treaty articles 32, par 5 and 29, par 1). This alternative was rejected as it would significantly complicate the model, without changing the nature of the results.

Country 1's Economy

The equations describing country 1's economy are listed in Table II. Equations (1)-(3) describe the supply side of the economy. Domestic price inflation, equation (1), is determined by the deviation of output from its natural level, and expected domestic price inflation. This gives a standard expectations augmented Phillips curve. The consumer price index is given by equation (2) as a weighted average of domestic and foreign prices. The weights are set in the initial period, as determined by the proportion of consumption consisting of domestic goods and imports, respectively.[6]

The demand side of the model, for country 1, is given by equations (4)-(13). Equations (4) and (5) are versions of the Fisher equation. They define the ex ante and ex post real interest rate, respectively. Equation (6) defines $\tilde{p}$ as the ratio of country 2's to country 1's consumer prices. This term is used to convert country 2's real variables, which appear in country 1's equations, into the same units as country 1's real variables. Thus, for example:

$$g_{2,t}\tilde{p}_t = \left(\frac{G_{2,t}}{p_{2,t}^c}\right)\left(\frac{p_{2,t}^c}{p_{1,t}^c}\right) = \frac{G_{2,t}}{p_{1,t}^c}$$

where $G_{2,t}$ is nominal spending by the government of country 2. Therefore, all of the real variables in country 1's demand and supply equations are in terms of country 1's consumer price index, and all of the real variables in country 2's demand and supply equations are in terms of country 2's consumer price index.

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[6] As is the practice with consumer price indexes, these weights are not adjusted each period, but may be updated after a number of years. For the purpose of this paper, there is no updating.

Equations (7)-(11) describe the goods market. Equation (7) is the national income identity. Real income (output) is equal to real private domestic absorption, $a_t$, real government spending, $g_t$, and real net exports, $nx_t$. Real private domestic absorption, equation (8), depends positively on real disposable income, $y_t^d$, and negatively on the ex ante real interest rate, $R_t^e$. Real disposable income, $y_t^d$, is given by equation (9), as real income less taxes, $t_t$, (where taxes are lump sum), plus real interest earnings and the repayment of bonds bought by the private sector last period (where these two terms are by definition the gross real interest earnings on private holdings of bonds, $r_{t-1}b^p_{t,1}$).[7] Real net exports are given by equation (10). They are positively related to country 2's private and government purchases of country 1's goods, and negatively related to country 1's private and government purchases of country 2's goods.[8] The government budget constraint is given by equation (11). Real government spending, $g_t$, is constrained by the real interest payments and repayments of last period's bonds (as noted

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[7] The term $r_{1,t-1}b^p_{1,t-1}$ is derived as follows:

$$b^p_{1,t-1}\left(\frac{p^c_{1,t-1}}{p^c_{1,t}}\right) + \imath_{t-1}b^p_{1,t-1}\left(\frac{p^c_{1,t-1}}{p^c_{1,t}}\right) = (1+\imath_{t-1})\,b^p_{1,t-1}\left(\frac{p^c_{1,t-1}}{p^c_{1,t}}\right)$$

$$= \left(\frac{i_{t-1}}{1+\pi_{1,t-1}}\right) b^p_{1,t-1}$$

$$= r_{1,t-1}\; b^p_{1,t-1}$$

$b^p_{1,t-1}$ denotes private bond holdings deflated by period t-1 consumer prices. Thus, to determine the real time t value of period t-1's bond purchases, it is necessary to multiply this term by the ratio of period t-1 to period t's consumer prices.

[8] Neither the private marginal propensity to consume, $\epsilon$, nor the government's marginal propensity to consume, $\epsilon_g$, is a function of relative prices. In an earlier version of this paper, a parameter was included in the net export equations of the two countries to capture the price substitution effect. The inclusion of this parameter only alters the subsequent analysis if there is a large divergence in prices between the two countries.

above, these two combined give by definition the gross real interest payments on bonds, $r_{t-1}b_{t-1}$),[9] less tax revenues, $t_t$, and new bond issues, $b_t$. In each period the government can choose, at most, two of the three contemporaneous variables: government spending, taxes, and/or bond issues. The creation of an independent central bank removes the ability of the government to finance a deficit through money creation.

Equations (12) and (13) describe the asset markets. Demand for real balances, equation (12), depends positively on real disposable income, via the transactions motive, and negatively on the nominal interest rate. Thus, an increase in the interest rate on government bonds will decrease the demand for money. As noted above, since the bonds issued by each country are perfect substitutes, there is only one nominal interest rate. The savings function is given by equation (13). All saving is through bond holdings, and real private bond demand is determined by the difference between real disposable income and real private domestic absorption.[10]

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[9] For the derivation of the term $r_{1,t-1}b_{1,t-1}$ see footnote 14.

[10] Equation (13) can also be used to derive the balance of payments equation. Substituting equations (8) and (9) into equation (13) yields:

$$\begin{aligned} b^p_{1,t} &= a_{1,t} + g_{1,t} + nx_{1,t} + r_{1,t-1}b^p_{1,t-1} - t_{1,t} - a_{1,t} \\ &= (g_{1,t} - t_{1,t}) + nx_{1,t} + r_{1,t-1}b^p_{1,t-1} \end{aligned}$$

Using equation (11) to substitute out for government spending net of taxes and rearranging, yields:

$$(b^p_{1,t} - b_{1,t}) - r_{1,t-1}\,(b^p_{1,t-1} - b_{1,t-1}) = nx_{1,t}$$

Now, using equation (28) to replace the bond variables with the individual components of bond demand results in the balance of payments equation:

(continued...)

Country 2's Economy

The parameter values in the model for country 2's economy are assumed to be the same as those for country 1.[11] Thus, the equations modelling country 2's economy, which are given in Table III, are basically the same as those for country 1. However, the variables for country 2's economy are deflated by its consumer price index. Furthermore, since the natural level of output $\bar{y}$ is measured in units of country 1's goods, division by $\tilde{p}$ is necessary in equation (18) to convert it into units of country 2's goods.

Market Equilibrium Conditions

The conditions for equilibrium in the bond, money and goods markets are presented in Table IV. The bonds issued by the government of country i (i=1,2) are held by three groups of agents: the public in country i, the public in country j, and the central bank. Equation (28) shows the equality between the supply of bonds issued by country 1, and the demand for these bonds, broken down by type of demander. Equation (29) presents the same information for country 2. Equation (30) is the world bond market equilibrium condition.

The actual holdings by the residents in the two countries of the bonds issued by each government is impossible to determine. This follows from the bonds being perfect substitutes and from the assumption of perfect capital

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[10](...continued)

$$b_{11,t}+b_{21,t}\tilde{p}_t-b_{11,t}-b_{12,t}-b_{1m,t} - r_{1,t-1}(b_{11,t-1}+b_{21,t-1}\tilde{p}_{t-1}-b_{11,t-1}-b_{12,t-1}-b_{1m,t-1}) = nx_{1,t}$$

$$b_{21,t}\tilde{p}_t-b_{12,t}-b_{1m,t} = nx_{1,t}+r_{1,t-1}(b_{21,t-1}\tilde{p}_{t-1}-b_{12,t-1}-b_{1m,t-1})$$

[11] This assumption is made both for simplicity and to keep the asymmetries between the two countries to a minimum.

mobility. Only by placing restrictions on the preferences of the individual bondholders (e.g. bondholders prefer to hold x% of their portfolio in their home country's bonds) can exact holdings be determined.[12] The central bank's holding of each bond is determined through the money market restrictions, as explained below.

Equation (31) gives the money market clearing condition, stating that the supply of real balances by the central bank is equal to the combined demand of the two countries. The supply of real balances is determined by the central bank's purchases of each government's bonds, as given by equation (32). The next two equations, (33) and (34), determine the central bank's holdings of each country's bonds. For simplicity, it is assumed that the central bank buys half of its bonds from country 1 and half from country 2.[13] These four equations indicate that while the initial distribution of real balances between the countries is determined by the central bank, the ultimate distribution is determined by demand conditions in each country.

Equation (35) gives the goods market clearing condition for either country. Real disposable income less domestic absorption and the government deficit must be equal to the real current account balance.

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[12] Such restrictions would change the magnitude of the interest rate effects due to an increase in bond issues by the governments. Furthermore, the magnitude of such effects could differ depending on which government issued the bonds if the residents of both countries prefer the bonds of one government over the bonds of the other. This in turn could change the fiscal policy effects derived in this model.

[13] Relaxing this assumption will not affect the model.

**Assumptions on the Parameters**

It is assumed that $\gamma > 1/2$, which indicates that the residents of each country prefer their own goods to foreign goods. The marginal propensity to consume domestic goods out of disposable income, c, is assumed to be greater than 1/2, and thus the marginal propensity to consume imported goods, $\epsilon$, must be less than 1/2. These two assumptions also correspond to the preference for home goods over foreign goods in each country. Furthermore, the government's marginal propensity to import, $\epsilon_g$, is constrained to be not greater than the private marginal propensity to import, $\epsilon$.[14]

## III. AGGREGATE SUPPLY AND AGGREGATE DEMAND

Using equations (1)-(3) and (14)-(16) one can derive the aggregate supply equations for country 1 and country 2:[15]

$$(36) \quad y_{1,t} = \bar{y} + \frac{\bar{y}}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\,[\gamma_3(\pi_{1,t-1} - \pi^e_{1,t-1}) - \gamma_2(\pi_{2,t-1} - \pi^e_{2,t-1})]$$

$$(37) \quad y_{2,t} = \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}/\tilde{p}_t}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\,[\gamma_1(\pi_{2t-1} - \pi^e_{2t-1}) - \gamma_4(\pi_{1t-1} - \pi^e_{1t-1})]$$

where:

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[14] This restriction is made since in practice a large portion of government spending goes towards the salaries of government workers, and governments generally do not have a weaker preference for domestically produced goods over foreign goods than do their citizens.

[15] See Appendix A for the derivation.

$$\gamma_1 = \frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1} + (1-\gamma) p_{2,t-1}}, \qquad \gamma_2 = \frac{(1-\gamma) p_{2,t-1}}{\gamma p_{1,t-1} + (1-\gamma) p_{2,t-1}}$$

$$\gamma_3 = \frac{\gamma p_{2,t-1}}{\gamma p_{2,t-1} + (1-\gamma) p_{1,t-1}}, \qquad \gamma_4 = \frac{(1-\gamma) p_{1,t-1}}{\gamma p_{2,t-1} + (1-\gamma) p_{1,t-1}}$$

Each country's aggregate supply is determined by the natural level of output, the period t-1 to t expected change in the consumer price index and the period t-1 to t actual change in the consumer price index in both countries. Prices in country 2 influence supply in country 1 (and vice versa) through their effect on consumer prices in each country.

Given the restrictions on the weights in the consumer price indices of the two countries, it is possible to determine the sign of the coefficients on the inflation terms in each aggregate supply equation. Since $\gamma>1/2$, it follows that:

$$\gamma_1\gamma_3 - \gamma_2\gamma_4 > 0$$

An increase in a country's own inflation rate, holding inflationary expectations constant, has a positive effect on its output, which implies that the short-run aggregate supply curve is upward sloping in inflation-output space. The increase in inflation will decrease short-run aggregate supply in the other country. The long-run aggregate supply curve for each country is vertical.

The process of solving for aggregate demand indicates clearly the links between the two countries, through the goods, money and bond markets. The first step in solving for aggregate demand in each country is the derivation of output in country i as an explicit function of output in country j and other exogenous variables.[16] The result of this process for country 1 is given below:

$$
\begin{aligned}
(38)\quad y_{1,t} = {} & \left[\frac{[(1-\epsilon_g) - (1-\epsilon)c]2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta) + \lambda\phi}\right] g_{1,t} \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{1,t} + \left[\frac{(1-\epsilon)2c\theta\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^e_{1,t} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] y_{2,t}\,\tilde{p}_t \\
& + \left[\frac{(\epsilon_g - \epsilon c)2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] g_{2,t}\,\tilde{p}_t \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{2,t}\,\tilde{p}_t + \left[\frac{2\epsilon\phi\theta}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^e_{2,t}\,\tilde{p}_t \\
& + \left[\frac{\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] b_{m,t} \\
& + \left[\frac{2c\theta(1 - 2\epsilon)}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{1,t-1}(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) \\
& + \left[\frac{\lambda\phi - c\theta}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right] r_{1,t-1} b_{m,t-1}
\end{aligned}
$$

Equation (38) can be used to determine the direct and feedback demand linkages between the two countries, a process which is lost when one begins with a reduced form demand equation.

The coefficients on the exogenous variables in equation (38) show the direct effects of these variables on aggregate demand in country 1. For example, real government expenditures in country 2 directly affect country 1, to the extent that they increase country 1's exports, and to the extent that they increase the world interest rate. The export effect is itself comprised of two parts: a positive effect due to the increase in spending on imports by the government in country 2, as measured by $\epsilon_g$, and a negative effect

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[16] See Appendix A for the derivations.

resulting from a decline in private spending on imports in country 2 which occurs given that the increase in government spending was financed by an increase in taxes.[17] This latter effect is measured by $\epsilon c$. Thus, the overall direct impact of an increase in government spending by country 2 on aggregate demand in country 1 is indeterminate.

The term $y_{2t}$ in equation (38) captures the feedback effects of the exogenous variables on output in country 1. A change in any of the exogenous or predetermined variables in equation (38) not only has a direct effect on output in country 1 through the linkages between the goods, money and bond markets in the two countries but also has a feedback effect on aggregate demand in country 1 resulting from the effect on aggregate demand in country 2. Returning to the example of an increase in government spending by country 2, this not only directly affects country 1, as explained above, but also affects country 1 through its impact on demand in country 2. An increase in spending by the government in country 2 will increase income in country 2 which in turn will have spillover effects on country 1 by increasing trade and the world interest rate.

Thus, there are two channels of influence through which an exogenous variable affects aggregate demand in country i: a direct one through the initial effects on the markets in country i and an indirect one through the impact on output in country j which in turn works through the market linkages to affect demand in country i. This indirect, or feedback effect will be negative if $\lambda \phi > c \theta$.

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[17] Since the government's budget constraint given in equation (11) must be met, an increase in $g_{2t}$ holding $b_{2t}$ constant implies that taxes, $t_{2t}$ are increased.

Solving for output in country 2 as an explicit function of output in country 1, and substituting the resultant equation into equation (38), is the final step in the solution for aggregate demand for country 1. The aggregate demand equation for country 1 which results from this procedure is

$$(39)\quad \begin{aligned} y_{1,t} &= A_1\, g_{1,t} + A_2\, b_{1,t} + A_3\, \pi^e_{1,t} + A_4\, g_{2,t}\tilde{p}_t + A_5\, b_{2,t}\tilde{p}_t + A_6\, \pi^e_{2,t}\tilde{p}_t \\ &+ A_7\, b_{m,t} + A_8\, r_{1,t-1}(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1}) \\ &+ A_9\, r_{1,t-1}\frac{1}{2}b_{m,t-1} \end{aligned}$$

Likewise, the final form of the aggregate demand equation for country 2 is

$$(40)\quad \begin{aligned} y_{2,t} &= A_1\, g_{2,t} + A_2\, b_{2,t} + A_3\, \pi^e_{2,t} + A_4\, g_{1,t}\left(\frac{1}{\tilde{p}_t}\right) + A_5\, b_{1,t}\left(\frac{1}{\tilde{p}_t}\right) \\ &+ A_6\, \pi^e_{1,t}\left(\frac{1}{\tilde{p}_t}\right) + A_7\, b_{m,t}\left(\frac{1}{\tilde{p}_t}\right) + A_8\, r_{2,t-1}\left(b_{12,t-1}\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1}\right) \\ &+ A_9\, r_{1,t-1}\, \frac{1}{2}\, b_{m,t-1}\left(\frac{1}{\tilde{p}_{t-1}}\right) \end{aligned}$$

where the coefficients $A_1$ - $A_9$ are given in Table V.

Using equations (5b) and (18b), the aggregate demand equations (39) and (40) can then be rewritten as

$$(41)\quad \begin{aligned} y_{1,t} &= A_1\, g_{1,t} + A_2\, b_{1,t} + A_3\, \pi^e_{1,t} + A_4\, g_{2,t}\tilde{p}_t + A_5\, b_{2,t}\tilde{p}_t + A_6\, \pi^e_{2,t}\tilde{p}_t \\ &+ A_7\, b_{m,t} + i_{t-1}Y_1 - \pi_{1,t-1}Y_1 \end{aligned}$$

$$(42) \qquad y_{2,t} = A_1\, g_{2,t} + A_2\, b_{2,t} + A_3\, \pi^e_{2,t} + A_4\, g_{1,t}\left(\frac{1}{\tilde{p}_t}\right) + A_5\, b_{1,t}\left(\frac{1}{\tilde{p}_t}\right)$$
$$+ A_6\, \pi^e_{1,t}\left(\frac{1}{\tilde{p}_t}\right) + A_7\, b_{m,t}\left(\frac{1}{\tilde{p}_t}\right) + i_{t-1}\mathbf{Y}_2 - \pi_{2,t-1}\mathbf{Y}_2$$

where:

$$\mathbf{Y}_1 = A_8(b_{21,t-1}\,\tilde{p}_{t-1} - b_{12,t-1} + A_9\frac{1}{2}b_{m,t-1}$$
$$\mathbf{Y}_2 = A_8(b_{12,t-1}\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1}) + A_9\frac{1}{2}b_{m,t-1}(\frac{1}{\tilde{p}_{t-1}})$$

The slope of each country's aggregate demand curve ($-1/Y_i$) in period t depends on the next debtor/creditor status of each country in period t-1. Given that this is a two country model, both countries can not be net creditors. Furthermore, given the existence of a common independent central bank, one country must be a net debtor.[18] When country 1 is not a net debtor, the amount of country 2's bonds held by the residents of country 1 must be greater than the amount of country 1's bonds held by the residents of country 2 ($b_{21}\tilde{p} > b_{12}$). Likewise, when country 2 is not a net debtor, the

[18] In this system the sum of the balance of payment accounts of two countries does not equal zero, but instead equals the central bank's holdings of bonds less its real interest earnings from last period's holdings of bonds. This can be shown as follows:

$$\begin{aligned} bop_{1,t} + bop_{2,t} &= nx_{1,t} + r_{1,t-1}(b_{21,t-1}\tilde{p}_{t,1} - b_{12,t-1} - b_{1m,t-1}) \\ &\quad + b_{12,t} + b_{1m,t} - b_{21,t}\tilde{p}_t + nx_{2,t}\tilde{p}_t \\ &\quad + r_{1,t-1}\left(b_{12,t-1}\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1} - b_{2m,t-2}\frac{1}{\tilde{p}_{t-1}}\right)\tilde{p}_{t-1} \\ &\quad + b_{21,t}\tilde{p}_t + b_{2m,t} - b_{12,t} \\ &= b_{1m,t} + b_{2m,t} - r_{1,t-1}(b_{1m,t-1} + b_{2m,t-1}) \\ &= b_{m,t} - r_{1,t-1}b_{m,t-1} \end{aligned}$$

amount of country 1's bonds held by the residents of country 2 must be greater than the amount of country 2's bonds held by the residents of country 1 ($b_{12} > b_{21}\tilde{p}$). If both countries are net debtors, the absolute value of the difference between the cross border bond holdings of country 1 and country 2 is restricted to be less than half of the bond holdings of the central bank ($|b_{21} - b_{12}| < 1/2\ b_m$).

These conditions indicate that the ability of one country alone to be a net borrower is constrained by the willingness of the residents of the other country to be lenders. This indicates that even if the central bank is unwilling to finance the expansionary policies of a country, it can still pursue these policies as long as the residents of the other country are willing to lend to it. That is, given the lack of capital controls between the two countries, as long as the residents of one country are willing to finance the deficit spending of the other, neither the central bank nor the other government will be able to prevent or constrain the expansionary policies of one government. The central bank will still be able to constrain one government to the extent that it is willing to use restrictive monetary policies which will affect both countries. There are several issues to consider in this regard. The first is the relative impact of restrictive monetary polices on both countries, versus the impact of the expansionary fiscal policies on both countries. The second is the relative impact of the two policies on the country pursuing expansionary fiscal policies. A third issue is the willingness of the "errant" country to challenge the central bank and to engender the disdain of its neighbors. All of these factors are likely to be significant in determining policy outcomes. The first two issues are considered further below.[19]

In order for a country's aggregate demand curve to be downward sloping, it is necessary that $\lambda\phi > 2c\theta$. This is the same condition which indicates that the feedback effect in equation 38 is negative.[20] This condition is also useful in determining the signs of some of the coefficients in the aggregate demand equations. These coefficients (see Table V) indicate the overall effect of changes in the policy and exogenous variables on aggregate demand in each country.

The direct, feedback and overall effects of an increase in the variables in the aggregate demand equation for country i are given in Table VI. An increase in real expenditures by country 1's government, financed by taxes, directly increases aggregate demand in country 1, as shown in Figure 1. The increase in government expenditures has a spillover effect on country 2, increasing its aggregate demand, through an increase in exports by country 2 to country 1. This increase in aggregate demand in country 2 has a net crowding out effect, and thus contracts aggregate demand in country 1. Although this feedback effect is negative, it is not strong enough to offset the initial increase in aggregate demand in country 1. The increase in

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[19] Another issue which is outside the realm of this model is whether such expansionary policies will cause the residents of the two countries to view the bonds of the expansionary government as risky, and require a risk premium, which will drive a wedge between interest rates on the bonds. Such a premium depends on the belief that the government in question is likely to default on its debt.

[20] Even with this condition, it is possible that when one country is a net debtor and the other a net creditor, the aggregate demand curve for the net debtor country could be upward sloping. This result does not change the comparative static analysis in the following section. To simplify the discussion, it is assumed that both countries' aggregate demand curves are downward sloping.

country 1's aggregate demand also feeds back into country 2's aggregate demand, causing a diminishing of the original spillover effect (Figure 1). In both countries, the overall effect of a tax financed increase in country 1's government expenditures is an increase in aggregate demand.

An increase in real expenditures by country 1's government, financed through bond issues, directly increases aggregate demand in country 1 and has a positive spillover effect on country 2 (Figure 2). As with tax financed expenditures the resultant increase in income in one country has a negative feedback effect on aggregate demand in the other country. For country 1, the direct effect is stronger than the feedback effect resulting in an increase in aggregate demand. For country 2, the overall effect is indeterminate.

An increase in the real bond holdings of the central bank increases aggregate demand in both countries. This comes directly from the increase in real balances' shifting out the LM curve. In both countries, the feedback effect is negative but not strong enough to offset the direct effect.

## IV. EQUILIBRIUM INFLATION AND OUTPUT

The equilibrium inflation and output equations for country 1 and country 2 are given in Tables VII and VIII. These were derived using the aggregate demand and supply equations discussed in Section V.[21]

### Comparative Statics

In the standard closed economy model an increase in government spending increases aggregate demand and so has a positive effect on output and prices. An increase in bonds issued by the government causes a contraction in

[21] See Appendix B for these derivations.

aggregate demand by pushing up interest rates and thereby decreasing investment. Thus output and prices decline. The overall effect of a bond financed increase in government spending on output and prices is generally positive; the crowding out of investment which occurs is not complete. Expansionary monetary policy carried out through open market purchases increases output and inflation in the closed economy, by shifting out the LM curve and thus increasing aggregate demand.

In a model of a large open economy with perfect capital mobility and fixed exchange rates, both the fiscal and monetary policy actions of a country also have an effect on the rest of the world. Expansionary fiscal policy, regardless of how it is financed, has an ambiguous effect on output and prices abroad. The increased spending at home increases aggregate demand abroad through trade effects. At the same time, however, the increase in the world interest rate dampens demand overseas. A bond financed fiscal expansion will increase the magnitude of the latter effect. In large open economy models it is uncertain as to whether the trade effect or the interest rate effect dominates. Buiter (1988) states that if the countries are of similar size, then a fiscal expansion at home will always have a positive effect on world output and inflation. In the large country case, a monetary expansion at home has a positive effect on output and inflation abroad.

As noted previously, standard open economy models focus on the demand side of the economy. Even where a supply side is modelled, these models ignore connections between the demand and supply side of economies. In the model developed in this paper countries are linked on the demand side through the goods market and assets market. Thus, policies which affect aggregate demand in one country lead to spillover effects on demand in the other country. Because of the market linkages, changes in inflation in one country affect the other country. This link comes through the supply side of the economies. Thus, a change in inflation in one country will lead to spillover effects on supply in the other country.

The comparative statics for the model of a monetary union developed in this chapter are given in Tables IX and X. The first column in each table gives the comparative statics which result from focusing only on demand effects.[22] Tax financed expansionary fiscal policy will increase output and inflation in both countries. The increase at home will be larger than the increase abroad. Bond financed fiscal policy will increase output and inflation at home, but will decrease output and inflation abroad. Expansionary monetary policy will increase output and inflation in both countries. These results may be weakened or reversed through the inclusion of supply effects (see column three in Table IX and X). Determining the conditions under which supply effects weaken or offset demand effects requires an analysis of the spillover demand effects and the magnitude of the supply effects.

**Determining the Comparative Statics**

For every policy variable in the equilibrium output and inflation equations there are two separate effects that determine the overall effect of a change in that variable on output and inflation.[23] A change in fiscal or

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[22] The analysis for equilibrium output and inflation, in this section, considers only the overall change in demand, and does not look at the component effects.

[23] The analysis concentrates on the effects as given by the numerators of the coefficients on the variables in the output and inflation equations. There (continued...)

monetary policy has a direct effect on aggregate demand, resulting in a change in a country's inflation rate. The feedback effect captures the subsequent impact of this change in inflation on aggregate supply in the other country.[24] In most cases the aggregate supply effects reinforce the inflation effects of the change in aggregate demand, but weaken the output effects.

There are two potential sources for indeterminacy in signing the comparative static effects. First, as noted in Table V, the overall demand effect on a country of bond financed fiscal policy changes by the other country is indeterminate, which in turn presents a source of ambiguity in determining the equilibrium output and inflation effects. Second, as noted above, in many cases the feedback effects work to offset the effect of a change in aggregate demand on output. The overall effect depends on the direction of the changes in aggregate demand and aggregate supply and the magnitudes of these two changes.

An important determinant of the effects of policies on output and inflation is the steepness of a country's aggregate demand curve. The flatter the slope of the aggregate demand curve the larger is the effect of a shift in aggregate supply on output and the smaller is the effect of a shift in aggregate supply on inflation. The slope of the aggregate demand curve -

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[23](...continued)
are a two reasons for this emphasis. First, given that $\gamma>1/2$, and $Y_1$ and $Y_2$ are both positive, then $\Omega_i$ is positive. Therefore, determining the comparative static effects of a change in one of the exogenous variables is equivalent to determining the sign of the numerator of the exogenous variable in question. Also, it is the behavioral parameters in the numerator which are driving the aggregate demand and supply effects.

[24] As shown by equations (36) and (37), an increase in inflation in country i will decrease aggregate supply in country j.

$(1/Y_i)$, is determined by last period's current account balance of a country. If a country was a net debtor (net creditor) last period its aggregate demand curve will be relatively steep (flat). An increase in inflation reduces the real interest earnings on private holdings of bonds, thereby reducing disposable income (equations 9 and 22). An increase in inflation also reduces the government's real interest payments on bonds, thereby reducing the government deficit (equations 11 and 24). Since the aggregate demand curve for a country is downward sloping regardless of whether it is a net creditor or a net debtor, an increase in inflation produces a net negative effect on output (the output effect of the inflation benefit to the government can not fully offset the output effect of the inflation cost to the private sector). As expected, a net creditor nation is hurt more by inflation than a net debtor nation, which is reflected in the flatter aggregate demand curve for a net creditor and the steeper aggregate demand curve for a net debtor.

The importance of the slopes of the aggregate demand curves for determining the comparative statics can be illustrated by examining the two types of policy effects which are present in the model: 1) expansionary policies which have positive effects on aggregate demand in both countries; and, 2) expansionary policies which have positive effects on aggregate demand at home, but negative spillover effects on aggregate demand.

Policies which fall into the first category initially increase inflation abroad which causes a decline in aggregate supply at home. The supply effect strengthens the positive own demand effect on inflation but weakens the demand effect on output. As shown in Figure 4, if a country is a net creditor the supply effect is more likely to offset the demand effect on output. Whereas, if a country is a net debtor the supply effect is less likely to offset the demand effect on output. This follows from the fact that inflation is less harmful (in output terms) to a net debtor than to a net creditor.

Policies which fall into the second category lead to a reduction in inflation abroad (due to negative demand spillovers) which causes an increase in aggregate supply at home. Since the home demand effect is positive, the supply effect strengthens the demand effect on output but weakens the demand effect on inflation. In this case, as shown in Figure 5, a net debtor country is more likely to experience an overall decrease in inflation than a net creditor. However, the output effect is larger for a net creditor than for a net debtor. This result follows from the greater benefit of a reduction in inflation to a net creditor than to a net debtor.

In the foreign country aggregate demand decreases due to the spillover effect, and given the increase in inflation in the home country, foreign aggregate supply will also decrease, Figure 6. This shift in aggregate supply reinforces the demand induced decrease in output but works to counteract the demand induced decrease in inflation. The overall effect on output will be greater if the foreign country is a net creditor. Inflation is more likely to increase if the foreign country is a net debtor.

Policies pursued by one country will have differing effects on the two countries due to differences in the demand effects in the countries and due to differences in the past behavior of the two countries. The more similar the past policies of the two countries, the more similar the current account balances of the two and thus the more similar the slopes of their aggregate demand curves. If the countries are symmetric[25] the two aggregate demand curves have the same slope. In this case an expansionary policy pursued by

[25] Symmetry arises if $b_{12,t-1} = b_{21,t-1}$.

country 1 has the same effect on country 2 as an expansionary policy pursued by country 2 has on country 1. If the countries are not symmetric this does not hold. For example, if country 1 is a net creditor and country 2 is a net debtor, an expansionary fiscal policy pursued by country 1 may increase output in country 2, but an expansionary fiscal policy pursued country 2 may decrease output in country 1. In general, policies which increase aggregate demand in both countries are less likely to have an overall positive output spillover if the country pursuing the policies is a net debtor and the other country is a net creditor.

**Policy Effects: Country 1 is a Net Debtor, Country 2 is a Net Creditor[26]**

A tax financed increase in government spending in country 1 has a positive effect on its own aggregate demand ($A_1 > 0$), and the increase in spending also raises aggregate demand in country 2 ($A_4 > 0$). This policy fits into category 1, given above. The shift in aggregate demand in country 1 increases its inflation rate which reduces aggregate supply in country 2. Likewise, inflation in country 2 increases which reduces aggregate supply in country 1. Therefore, in both countries, the effect on inflation is compounded but the effect on output is reduced, Figure 7.

Since country 1 is a net debtor the slope of its aggregate demand curve is relatively steep which acts to limit the effect on output of a reduction in aggregate supply. Thus, the expansionary fiscal policy increases inflation and output in country 1. The output effect on country 2 is less likely to be

[26] The analysis in this section concentrates on the effects of changes in policies by country 1 on the two countries. The analysis is the same with respect to changes in country 2's policy variables. See appendix D for the derivation of the comparative static results given in this section.

positive. Given that $A_4<A_1$, aggregate demand increases less in country 2 than in country 1. Thus, the direct effect on output is smaller. Also, since country 2 is a net creditor, the slope of its aggregate demand curve is relatively flat (inflation is more harmful to country 2 than it is to country 1). The decrease in aggregate supply exacerbates inflation and so causes a large (relative to country 1) reduction in output. It is possible that the negative supply effect on output more than offsets the positive demand effect on output, and output decreases in country 2. Given that inflation definitely increases, expansionary fiscal policy undertaken by country 1 can lead to stagflation in country 2. The greater the asymmetry between the countries, the greater the differences in the output effects.

A bond financed fiscal expansion by country 1 has a positive effect on aggregate demand in country 1, but the spillover effect on demand in country 2 may be positive or negative. If aggregate demand in country 2 increases ($A_4+A_5>0$) then the effects on the two countries will be similar to a tax financed fiscal expansion. Output and inflation in country 1 will increase. In country 2 inflation will increase but output may decrease. The more asymmetric the two countries the more likely that the fiscal expansion in country 1 will decrease output in country 2.

If aggregate demand in country 2 decreases as a result of the fiscal expansion in country 1 ($A_4+A_5<0$), this will bring about a decline in inflation in country 2 which increases aggregate supply in country 1. Since the fiscal expansion had a positive demand effect in country 1, the shift in aggregate supply strengthens the effect on output but weakens the effect on inflation, Figure 8. Overall, output in country 1 increases, and since the supply effect is not strong enough to offset the demand effect on inflation, inflation also increases. In country 2 there is a decrease in aggregate demand and, due to the increase in inflation in country 1, a decrease in aggregate supply. Both of these shifts work to reduce output but the demand shift decreases inflation while the supply shift works to offset that decrease. As with country 1, the demand effect is stronger than the supply effect and so inflation falls. Consequently, it is possible for a fiscal expansion by country 1 to increase output and inflation in country 1, while having the opposite effect on inflation and output in country 2.

Expansionary monetary policy results in an increase in real balances, and has an identical effect on aggregate demand in the two countries. The increase in aggregate demand in country 1 increases its inflation rate which causes a decrease in aggregate supply in country 2. Likewise, the increase in aggregate demand in country 2 increases its inflation rate which causes a decrease in aggregate supply in country 1. Thus, the aggregate supply effects further increase inflation in both countries, but work to offset the demand induced increases in output in both countries. In both countries the overall effect is an increase in inflation and output. However, the increase in inflation and output is greater in country 1 than in country 2, as shown in Figure 9. This result follows from the fact that country 1 is a net debtor and country 2 is a net creditor. The slope of the aggregate demand curve in country 1 is steep relative to that in country 2. Thus, as explained above, a shift in aggregate supply has a relatively greater effect on inflation in country 1 and a relatively greater effect on output in country 2. Since the supply effect and demand effect both lead to an increase in inflation, the overall inflation rate is higher in country 1. Since the change in supply reduces the demand effect on output, the overall change in output is greater in country 1.

**Policy Effects: Country 1 is a Net Creditor, Country 2 is a Net Debtor**

A tax financed increase in government spending in country 1 increases output and inflation in country 1. Since country 1 is a net creditor the slope of its aggregate demand curve is relatively flat which acts to limit the effect on inflation of a reduction in aggregate supply, but increases the effect on output, as shown in Figure 10. Thus, the output and inflation effects, although positive are smaller than in the case where country 1 is a net debtor. Given that country 2 is a net debtor the slope of its aggregate demand curve is relatively steep. The steepness of the slope limits the effect of a reduction in aggregate supply on output. Thus, expansionary fiscal policy conducted by country 1 has a positive effect on output and inflation in country 2. These effects are greater than in the case where country 2 is a net creditor.

The effects of a bond financed fiscal expansion by country 1 depend on whether the demand spillover effect on country 2 is positive or negative. If aggregate demand in country 2 increases ($A_4+A_5>0$) then, as in the case where country 1 is a net debtor and country 2 a net creditor, the effects on the two countries will be similar to a tax financed fiscal expansion. Output and inflation in both countries will increase, with the effects on country 1 being greater in the case where country 1 is a net debtor and country 2 a net creditor, and the effects on country 2 are smaller in this case.

If aggregate demand in country 2 decreases as a result of the fiscal expansion in country 1 ($A_4+A_5<0$) both aggregate demand and aggregate supply in country 1 increase as shown in Figure 11. Both of these increases in turn increase output, and since the supply effect is not strong enough to offset the demand effect on inflation, inflation also increases. In country 2, the decline in aggregate demand and the decline in aggregate supply both work to reduce output. The demand shift decreases inflation while the supply shift works to offset that decrease. The overall effect on inflation is indeterminate. The effects on country 1 are once again greater than in the case where country 1 is a net debtor and country 2 a net creditor, while the effects on country 2 are smaller.

Expansionary monetary policy results in an increase in inflation and output in both countries. Although the nature of the effects are the same as in the case where country 1 is a net debtor and country 2 a net creditor, the magnitudes are reversed. When country 1 is a net creditor and country 2 is a net debtor, the increase in inflation and output is greater in country 2 than in country 1 as shown in Figure 9.

In sum, if a country is a net debtor any expansionary policies which it enacts or which are undertaken by the central bank have a greater effect on its inflation rate and level of output than if it is a net creditor. If a country is a net debtor expansionary policies undertaken by the other country are more likely to have positive effects on its output and inflation rate than if it is a net creditor.

These results differ from the standard open economy models due not only to the inclusion of supply effects, but also the addition of interest earnings in the aggregate demand equations for the two countries. The inclusion of supply effects are important because they may work to offset the effects of a change in demand on output and/or inflation. Therefore, the supply effects can change the results one would obtain by solely concentrating on the demand side of economies. The addition of interest earnings in the aggregate demand equations is the means by which the net debtor/creditor status of a country affects the model. As shown above, this status determines the steepness/flatness of the slope of the demand curve which weakens or strengthen the effects of a shift in aggregate supply on output and inflation.

## V. CONCLUSION

This paper develops a two-country model of a monetary union. In order to depict more fully the nature of the linkages between the countries and the central bank, the model does not begin with reduced form equations, but derives these equations based on a structural model of goods market, bond market and money market interaction. Furthermore, the model developed in this paper does not make the common assumption that output is demand determined, but instead also develops a supply side of the economy to capture both the demand and supply effects on output and inflation. A monetary union introduces linkages between the two countries so that fiscal policy pursued by one country or monetary policy pursued by the central bank affects both aggregate supply and demand in the two countries. The overall effect of a policy on each country's output and inflation rate depends not only on who pursued the policy (a country's own fiscal policy has a greater effect on its aggregate demand than does fiscal policy pursued by the other country) but also on last period's current account balance of each country. In this way present policies affect both countries through aggregate demand and aggregate supply links between the countries, but past policies through their effect on the current account balances of each country also determine the overall effect of current policies on output and inflation in each country.

The results of this paper suggest that asymmetries in past policies, reflected in asymmetries in the current account balances of countries, and the continuation of asymmetric fiscal policies can be a source of friction among the countries in a monetary union. Looking at the countries that will comprise the European Monetary Union, it is clear that such asymmetries do exist. In the 1980s France, Italy and Greece had persistent current account deficits, while the Federal Republic of Germany, the Netherlands, Belgium and Luxembourg had persistent current account surpluses. The United Kingdom started the decade with a current account surplus (due to its oil exports), but since 1986 it has run current account deficits. In 1989 the current account deficit of the United Kingdom was equal to 4.1 percent of its GDP, while the current account surplus of the Federal Republic of Germany was equal to 4.4 percent of its GDP.

There is evidence that the creation of the European Monetary Union will increase these asymmetries. Artis and Bayoumi (1991) found that the increase in capital integration in the world economy in the 1980s corresponded with growing capital account imbalances. An increase in capital mobility reduces the external constraints on borrowing. Thus, the European Monetary Union, which is to be characterized by full capital mobility, is likely to increase the level and persistence of current account imbalances among its member countries. Differences in preferences for consumption versus saving among countries are more easily maintained when countries only need to concern themselves with a solvency constraint and not an external constraint.

The findings of this paper indicate that fiscal policy convergence and the maintenance of convergence through restrictions on fiscal policies may be necessary to ensure that the monetary union functions smoothly. Restricting fiscal policies, however, may not produce optimal results in a monetary union in which the member countries have dissimilar economies. Given the small degree of labor mobility within the EC and the lack of an automatic redistribution system constraints on fiscal policies may hinder adjustment to asymmetric shocks.[27] Bayoumi and Eichengreen (1992) find that asymmetric shocks are more prevalent among the member countries of the EC than the regions in the U.S. Furthermore they find that the adjustment to such shocks in faster in the U.S. which they conclude is likely due to the higher degree of labor mobility in the U.S. than in Europe.

If fiscal policy restrictions are necessary to ensure the smooth functioning of a monetary union, but restrictions hinder adjustment to economic shocks the solution may be to create a "two-speed" Europe. A core group of EC countries[28] that experience similar shocks would move forward to form a monetary union, while the other countries would join only when their economies become similar to those of the core. Another solution is to increase the budgetary powers of the European Community government so that it can use taxes and transfer payments to ease the burden of asymmetric shocks.

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[27] This point has been made by several economists including DeGrauwe (1992) and Feldstein (1992). It is possible that the Maastricht treaty will be interpreted as allowing for deviations in fiscal policies to react to asymmetric shocks. This, however, will depend on the willingness of the Council, and therefore, the member countries to tolerate such deviations.

[28] France, Germany and the Benelux countries are usually suggested as comprising this core.

However, given the increasing emphasis on subsidiarity this latter approach is unlikely to be adopted.

More research is needed to examine the macroeconomic costs and benefits of monetary union to the members of the European Community. The analysis in this paper focusses on the impact effects of policies. Two issues which deserve further study are the possibility of strategic behavior on the part of the policy makers in a monetary union and the long-run policy effects. Further research should explore the long run effects on member countries of allowing independent fiscal policies or of restricting fiscal policies. The advantage of the model developed in this paper is that it can be used to address both of these issues.[29]

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[29] For example, Pollard (1992) uses the model developed in this paper to examine policy effects in a strategic setting.

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Table I: Notation

Variables:

$a$ = real private domestic absorption

$b^p$ = real private bond holdings

$b$ = real bond issues by the government

$b_m$ = real central bank bond holdings

$g$ = real government spending

$\iota$ = nominal world interest rate

$i = 1 + \iota$

$m$ = real balances

$nx$ = real net exports

$p$ = domestic price index

$p^c$ = consumer price index

$R$ = real interest rate

$r = 1 + R$

$R^e$ = expected real interest rate

$t$ = real lump sum taxes

$y$ = real output

$y^d$ = real disposable income

$\bar{y}$ = optimal or natural level of real output.

$\pi$ = consumer price inflation

$\pi^e$ = expected consumer price inflation

$d$ = government budget deficit

Parameters:

$c$ = marginal propensity to consume

$\epsilon$ = private marginal propensity to import

$\epsilon_g$ = government marginal propensity to import

$\lambda$ = income sensitivity of money demand

$\theta$ = interest sensitivity of money demand

$\phi$ = interest sensitivity of domestic absorption

$\alpha$ = sensitivity of domestic price inflation to deviations of output from its natural level

$\gamma$ = weight attached to domestic prices in the consumer price index

Table II: Equations Underlying Country 1's Economy

*Supply*:

(1) $$\frac{p_{1,t}-p_{1,t-1}}{p_{1,t-1}} = \alpha \frac{y_{1,t}-\bar{y}}{\bar{y}} + \frac{E_{t-1}p_{1,t}-p_{1,t-1}}{p_{1,t-1}}, \qquad \alpha > 0$$

(2) $$p^c_{1,t} = \gamma p_{1,t} + (1-\gamma)p_{2,t}, \qquad \frac{1}{2} < \gamma < 1$$

(3) $$\pi_{1,t-1} \equiv \frac{p^c_{1,t}-p^c_{1,t-1}}{p^c_{1,t-1}}$$

*Demand*:

(4) $$R^e_{1,t} \equiv \iota_t - \pi^e_{1,t}$$

(5*a*) $$R_{1,t} \equiv \iota_t - \pi_{1,t}$$ (5*b*) $$r_{1,t} \equiv i_t - \pi_{1,t}$$

(6) $$\tilde{p}_t \equiv \left(\frac{p^c_{2,t}}{p^c_{1,t}}\right)$$

(7) $$y_{1,t} = a_{1,t} + g_{1,t} + nx_{1,t}$$

(8) $$a_{1,t} = cy^d_{1,t} - \phi R^e_{1,t}, \qquad \frac{1}{2} < c < 1, \qquad 0 < \phi < 1$$

(9) $$y^d_{1,t} = y_{1,t} + r_{1,t-1}\, b^p_{1,t-1} - t_{1,t}$$

(10) $$nx_{1,t} = \epsilon a_{2,t}\tilde{p}_t + \epsilon_g g_{2,t}\tilde{p}_t - \epsilon a_{1,t} - \epsilon_g g_{1,t},$$
$$0 < \epsilon < \frac{1}{2}, \quad \epsilon_g \le \epsilon,$$

(11) $$g_{1,t} = t_{1,t} - r_{1,t-1}b_{1,t-1} + b_{1,t}$$

(12) $$m_{1,t} = \lambda y^d_{1,t} - \theta\iota_t, \qquad 0 < \lambda \le 1, \qquad 0 < \theta < 1$$

(13) $$b^p_{1,t} = y^d_{1,t} - a_{1,t}$$

## Table III: Equations Underlying Country 2's Economy

*Supply*:

(14) $$\frac{p_{2,t}-p_{2,t-1}}{p_{2,t-1}} = \alpha \frac{y_{2,t}-\frac{\bar{y}}{\tilde{p}_t}}{\frac{\bar{y}}{\tilde{p}_t}} + \frac{E_{t-1}p_{2,t}-p_{2,t-1}}{p_{2,t-1}}, \qquad \alpha > 0$$

(15) $$p^c_{2,t} = \gamma p_{2,t} + (1-\gamma)p_{1,t}, \qquad \frac{1}{2} < \gamma < 1$$

(16) $$\pi_{2,t-1} \equiv \frac{p^c_{2,t}-p^c_{2,t-1}}{p^c_{2,t-1}}$$

*Demand*:

(17) $$R^e_{2,t} \equiv \iota_t - \pi^e_{2,t}$$

(18*a*) $$R_{2,t} \equiv \iota_t - \pi_{2,t}$$ (18*b*) $$r_{2,t} \equiv i_t - \pi_{2,t}$$

(19) $$\frac{1}{\tilde{p}_t} \equiv \frac{p^c_{1,t}}{p^c_{2,t}}$$

(20) $$y_{2,t} = a_{2,t} + g_{2,t} + nx_{2,t}$$

(21) $$a_{2,t} = cy^d_{2,t} - \phi R^e_{2,t}, \qquad \frac{1}{2} < c < 1, \quad 0 < \phi < 1$$

(22) $$y^d_{2,t} = y_{2,t} + r_{2,t-1}\, b^p_{2,t-1} - t_{2,t}$$

(23) $$nx_{2,t} = \epsilon a_{1,t}\frac{1}{\tilde{p}_t} + \epsilon_g g_{1,t}\frac{1}{\tilde{p}_t} - \epsilon a_{2,t} - \epsilon_g g_{2,t}$$

$$0 < \epsilon < \frac{1}{2}, \quad \epsilon_g \le \epsilon$$

(24) $$g_{2,t} = t_{2,t} - r_{2,t-1}b_{2,t-1} + b_{2,t}$$

(25) $$m_{2,t} = \lambda y^d_{2,t} - \theta\iota_t \qquad 0 < \lambda \le 1, \quad 0 < \theta < 1$$

(26) $$b^p_{2,t} = y^d_{2,t} - a_{2,t}$$

Table IV: Equilibrium Conditions[1]

*Bond Market*:

(28) $b_{1,t} = b_{11,t} + b_{12,t} + b_{1m,t}$

(29) $b_{2,t} = b_{21,t} + b_{22,t} + b_{2m,t}$

(30) $b_t = b_{1,t} + b_{2,t}\,\tilde{p}_t$

*Money Market*:

(31) $m_t = m^d_{1,t} + m^d_{2,t}\,\tilde{p}_t$

(32) $m_t = b_{1m,t} + b_{2m,t}\,\tilde{p}_t = b_{m,t}$

(33) $b_{1m,t} = \frac{1}{2}\,m_t = \frac{1}{2}\,b_{m,t}$

(34) $b_{2m,t} = \frac{1}{2}\,m_t\,\frac{1}{\tilde{p}_t} = \frac{1}{2}\,b_{m,t}\,\frac{1}{\tilde{p}_t}$

*Goods Market*:

(35) $y^d_{i,t} - a_{i,t} - d_{i,t} = ca_{i,t} \qquad i = 1,\ 2$

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[1] All world variables are deflated by country 1's price index. This is done for simplicity. It is also possible to use the world consumer price index (a weighted average of the two consumer price indices) to deflate world variables.

Table VI:
Effect of Changes in Policy Variables on Aggregate Demand in Country i

| | Direct Effect | Feedback Effect | Overall Effect |
|---|---|---|---|
| Increase in tax financed government expenditures by country i | + | - | + |
| Increase in bond financed government expenditures by country i | + | - | + |
| Increase in tax financed government expenditures by country j | + | - | + |
| Increase in bond financed government expenditures by country j | + | - | ± |
| Increase in real balances | + | - | + |

Table V: Coefficients in the Aggregate Demand Equation

$$A_1 = 1 - \frac{e_g}{1-c+2ce} > 0$$

$$A_2 = \frac{2\theta c(c-1+e-2ce) + \lambda\phi(1-2c+4ce)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}$$

$$A_3 = \frac{2\phi\theta(c-1+e-2ce)+\lambda\phi^2(2e-1)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} > 0$$

$$A_4 = \frac{e_g}{1-c+2ce} > 0$$

$$A_5 = \frac{\lambda\phi-2ce\theta}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} < 0$$

$$A_6 = \frac{\phi(\lambda\phi-2e\lambda\phi-2e\theta)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)}$$

$$A_7 = \frac{\phi}{2(\theta-c\theta+\lambda\phi)} > 0$$

$$A_8 = \frac{c(1-2e)}{1-c+2ce} > 0$$

$$A_9 = \frac{c\theta-\lambda\phi}{c\theta-\theta-\lambda\phi} > 0$$

$$A_1+A_2 = \frac{2\theta(1-c)(1-e_g) + 2\theta ce + \lambda\phi(1-2e_g)}{2(1-c+2ce)(\theta-c\theta+\lambda\phi)} > 0$$

$$A_4+A_5 = \frac{2e_g\theta + 2c\theta(e-e_g) - \lambda\phi(1-2e_g)}{2(1-c+2ce)(\theta-c\theta+\lambda\phi)}$$

Table VII: Equilibrium Inflation

$$\pi_{1,t-1} = \left[ \frac{A_1[\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_4\alpha\gamma_2\bar{y}}{\Omega_1} \right] g_{1,t}$$

$$+ \left[ \frac{A_2[\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_5\alpha\gamma_2\bar{y}}{\Omega_1} \right] b_{1,t}$$

$$+ \left[ \frac{A_3[\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_6\alpha\gamma_2\bar{y}}{\Omega_1} \right] \pi^e_{1,t}$$

$$+ \left[ \frac{A_4[\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_1\alpha\gamma_2\bar{y}}{\Omega_1} \right] g_{2,t}\tilde{p}_t$$

$$+ \left[ \frac{A_5[\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_2\alpha\gamma_2\bar{y}}{\Omega_1} \right] b_{2,t}\tilde{p}_t$$

$$+ \left[ \frac{A_6[\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_3\alpha\gamma_2\bar{y}}{\Omega_1} \right] \pi^e_{2,t}\tilde{p}_t$$

$$+ \left[ \frac{\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y} + \alpha\gamma_2\bar{y}}{\Omega_1} \right] A_7 b_{m,t}$$

$$+ \left[ \frac{\alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha Y_1\gamma_1\bar{y} + \alpha Y_2\tilde{p}_t\gamma_2\bar{y}}{\Omega_1} \right] i_{t-1}$$

$$+ \left[ \frac{\bar{y}(\alpha\gamma_3 Y_2\tilde{p}_t + \bar{y})}{\Omega_1} \right] \pi^e_{1,t-1}$$

$$- \left[ \frac{\alpha\bar{y}\,\gamma_2 Y_2\tilde{p}_t}{\Omega_1} \right] \pi^e_{2,t-1}$$

$$- \left[ \frac{\alpha\bar{y}[\alpha Y_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\bar{y} + \gamma_2\bar{y}]}{\Omega_1} \right]$$

$$\Omega_1 = \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1 Y_1 + \gamma_3 Y_2\tilde{p}_t) + \bar{y}^2$$

$$\pi_{2,t-1} = \left[ \frac{A_1 \left( \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_4 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{2,t}$$

$$+ \left[ \frac{A_2 \left( \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_5 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{2,t}$$

$$+ \left[ \frac{A_3 \left( \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_6 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{2,t}$$

$$+ \left[ \frac{A_4 \left( \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_1 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{1,t}$$

$$+ \left[ \frac{A_5 \left( \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_2 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[ \frac{A_6 \left( \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} \right) + A_3 \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[ \frac{\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\gamma_3 \frac{\bar{y}}{\tilde{p}_t} + \alpha\gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] A_7 b_{m,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[ \frac{\alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha \Upsilon_2 \gamma_3 \frac{\bar{y}}{\tilde{p}_t} + \alpha \Upsilon_1 \frac{1}{\tilde{p}_t} \gamma_4 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] i_{t-1}$$

$$- \left[ \frac{\frac{\bar{y}}{\tilde{p}_t} \left( \alpha\gamma_4 \Upsilon_1 \frac{1}{\tilde{p}_t} + \frac{\bar{y}}{\tilde{p}_t} \right)}{\Omega_2} \right] \pi^e_{1,t-1}$$

$$+ \left[ \frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{2,t-1}$$

$$- \left[ \frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \left( \alpha \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \frac{\bar{y}}{\tilde{p}_t} (\gamma_3 + \gamma_4) \right)}{\Omega_2} \right]$$

$$\Omega_2 = \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha \frac{\bar{y}}{\tilde{p}_t} \left( \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} + \gamma_3 \Upsilon_2 \right) + \frac{\bar{y}^2}{\tilde{p}_t^2}$$

Table VIII: Equilibrium Output

$$y_{1,t} = \left[\frac{A_1[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_4\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] g_{1,t}$$

$$+ \left[\frac{A_2[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_5\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] b_{1,t}$$

$$+ \left[\frac{A_3[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_6\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{1,t}$$

$$+ \left[\frac{A_4[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_1\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] g_{2,t}\tilde{p}_t$$

$$+ \left[\frac{A_5[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_2\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] b_{2,t}\tilde{p}_t$$

$$+ \left[\frac{A_6[\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_3\alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] \pi^e_{2,t}\tilde{p}_t$$

$$+ \left[\frac{\alpha\gamma_3\Upsilon_2\tilde{p}_t\bar{y} + \bar{y}^2 - \alpha\Upsilon_1\gamma_2\bar{y}}{\Omega_1}\right] A_7 b_{m,t}$$

$$+ \left[\frac{\alpha\Upsilon_1\Upsilon_2\tilde{p}_t\bar{y}(\gamma_3-\gamma_2) + \Upsilon_1\bar{y}^2}{\Omega_1}\right] i_{t-1}$$

$$- \left[\frac{\bar{y}\,\Upsilon_1\,(\alpha\gamma_3\Upsilon_2\tilde{p}_t + \bar{y})}{\Omega_1}\right] \pi^e_{1,t-1}$$

$$+ \left[\frac{\alpha\bar{y}\gamma_2\Upsilon_1\Upsilon_2\tilde{p}_t}{\Omega_1}\right] \pi^e_{2,t-1}$$

$$+ \frac{\alpha\Upsilon_1\bar{y}[\alpha\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \bar{y}(\gamma_1+\gamma_2)]}{\Omega_1}$$

Table VIII (continued)

$$y_{2,t} = \left[ \frac{A_1 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}^2} \right) - A_4 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{2,t}$$

$$+ \left[ \frac{A_2 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_5 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{2,t}$$

$$+ \left[ \frac{A_3 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_6 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{2,t}$$

$$+ \left[ \frac{A_4 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_1 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[ \frac{A_5 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} \right) - A_2 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[ \frac{A_6 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}^2} \right) - A_3 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{1,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[ \frac{\alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t^2} - \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] A_7 b_{m,t} \frac{1}{\tilde{p}_t}$$

$$+ \left[ \frac{\alpha \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t} (\gamma_1 - \gamma_4) + \Upsilon_2 \frac{\bar{y}^2}{\tilde{p}_t^2}}{\Omega_2} \right] i_{t-1}$$

$$+ \left[ \frac{\frac{\bar{y}}{\tilde{p}_t} \Upsilon_2 \left( \alpha \gamma_4 \Upsilon_1 \frac{1}{\tilde{p}_t} + \frac{\bar{y}}{\tilde{p}_t} \right)}{\Omega_2} \right] \pi^e_{1,t-1}$$

$$- \left[ \frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2}{\Omega_2} \right] \pi^e_{2,t-1}$$

$$+ \frac{\alpha \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t} \left( \alpha \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1 \gamma_3 - \gamma_2 \gamma_4) - \frac{\bar{y}}{\tilde{p}_t} (\gamma_3 + \gamma_4) \right)}{\Omega_2}$$

*Figure 6a*

**Net Debtor Effects with Negative Demand Spillovers (Foreign Country)**

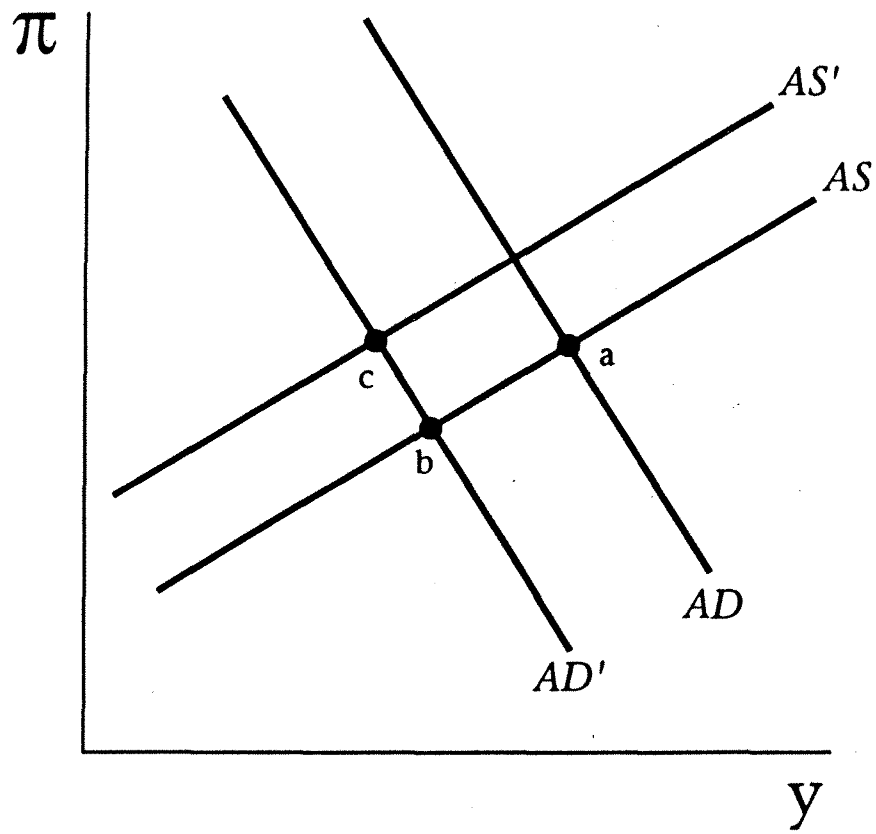



*Figure 6b*

**Net Creditor Effects with Negative Demand Spillovers (Foreign Country)**

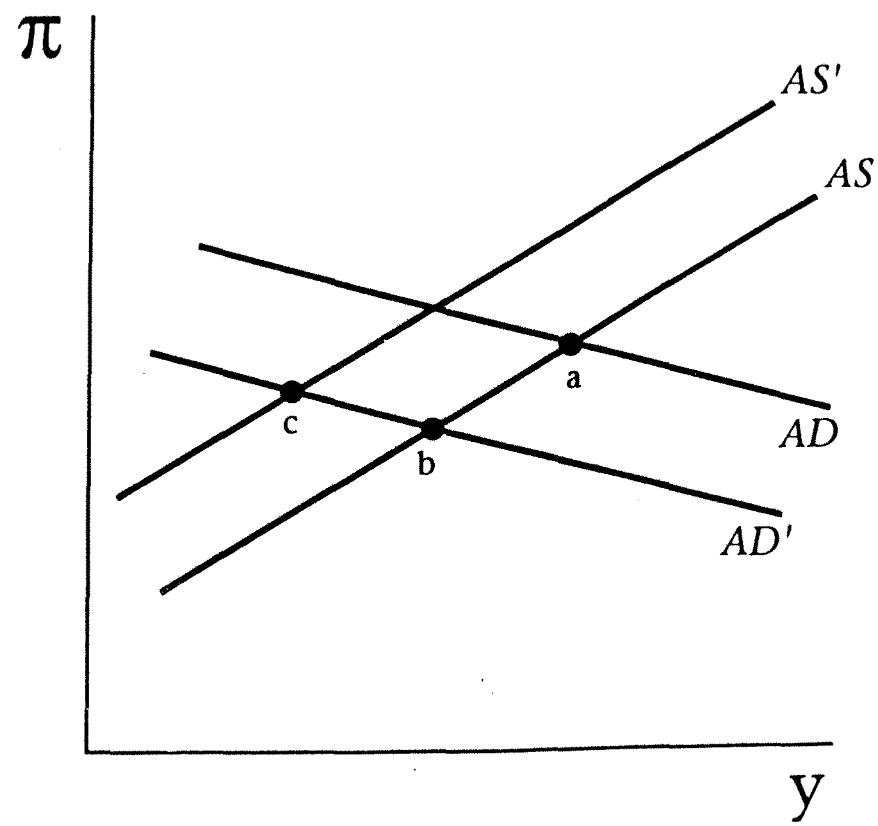
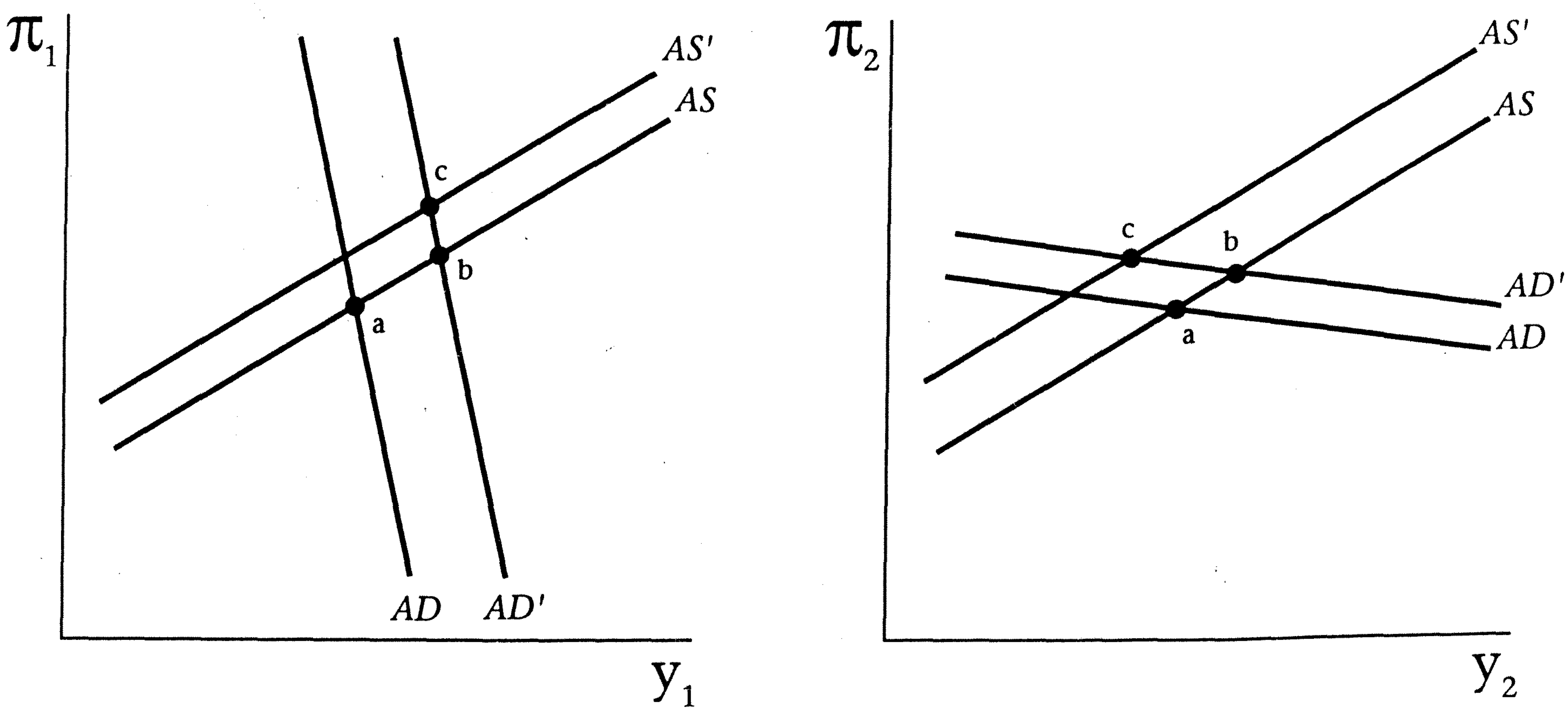


*Figure 7*

**Tax Financed Increase in Government Spending by Country 1**




**Country 1 (Net Debtor)** **Country 2 (Net Creditor)**

*Figure 8*

**Bond Financed Increase in Government Spending by Country 1**

$(A_4 + A_5) < 0$

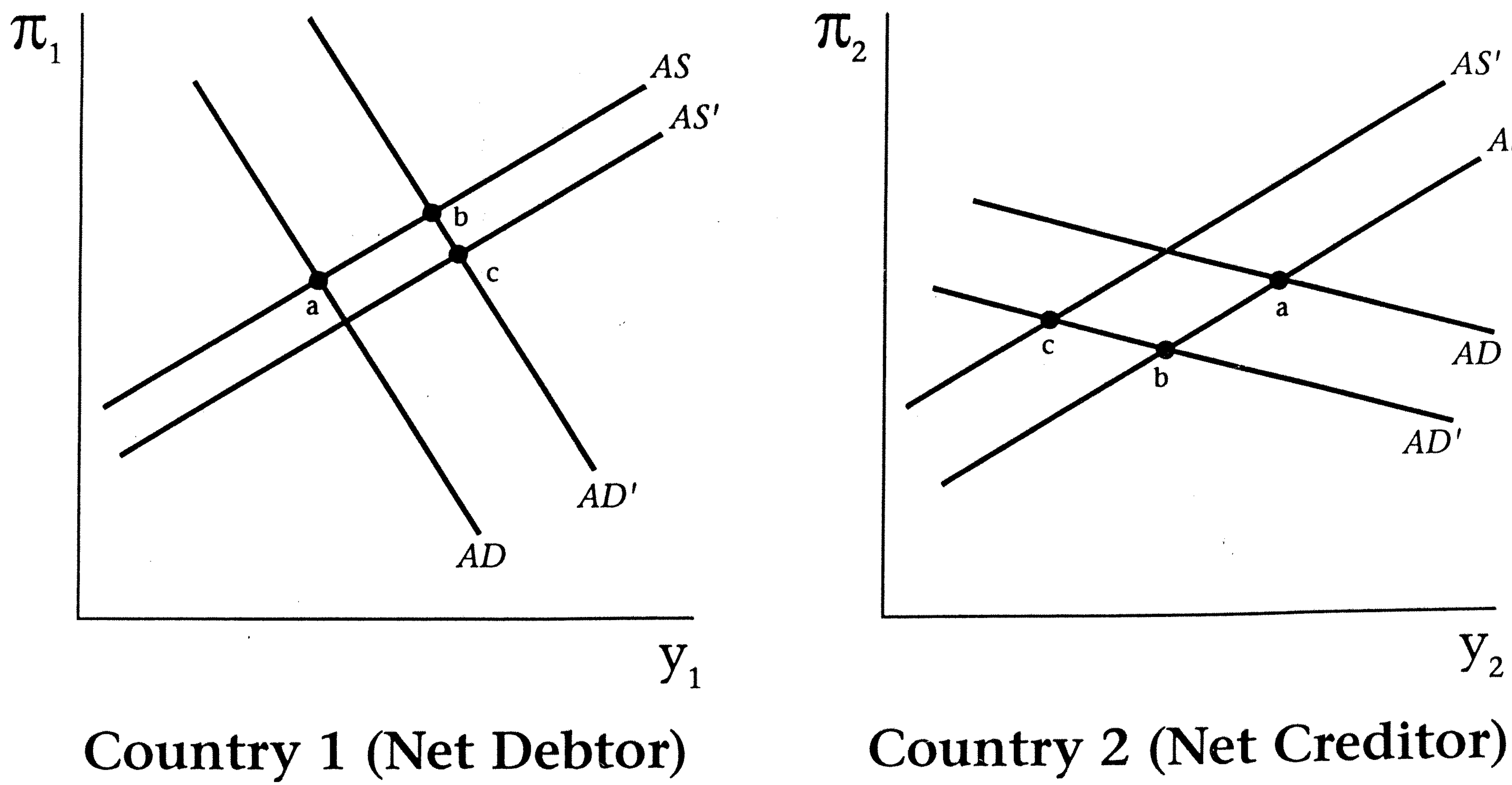


*Figure 9*

**Expansionary Monetary Policy**

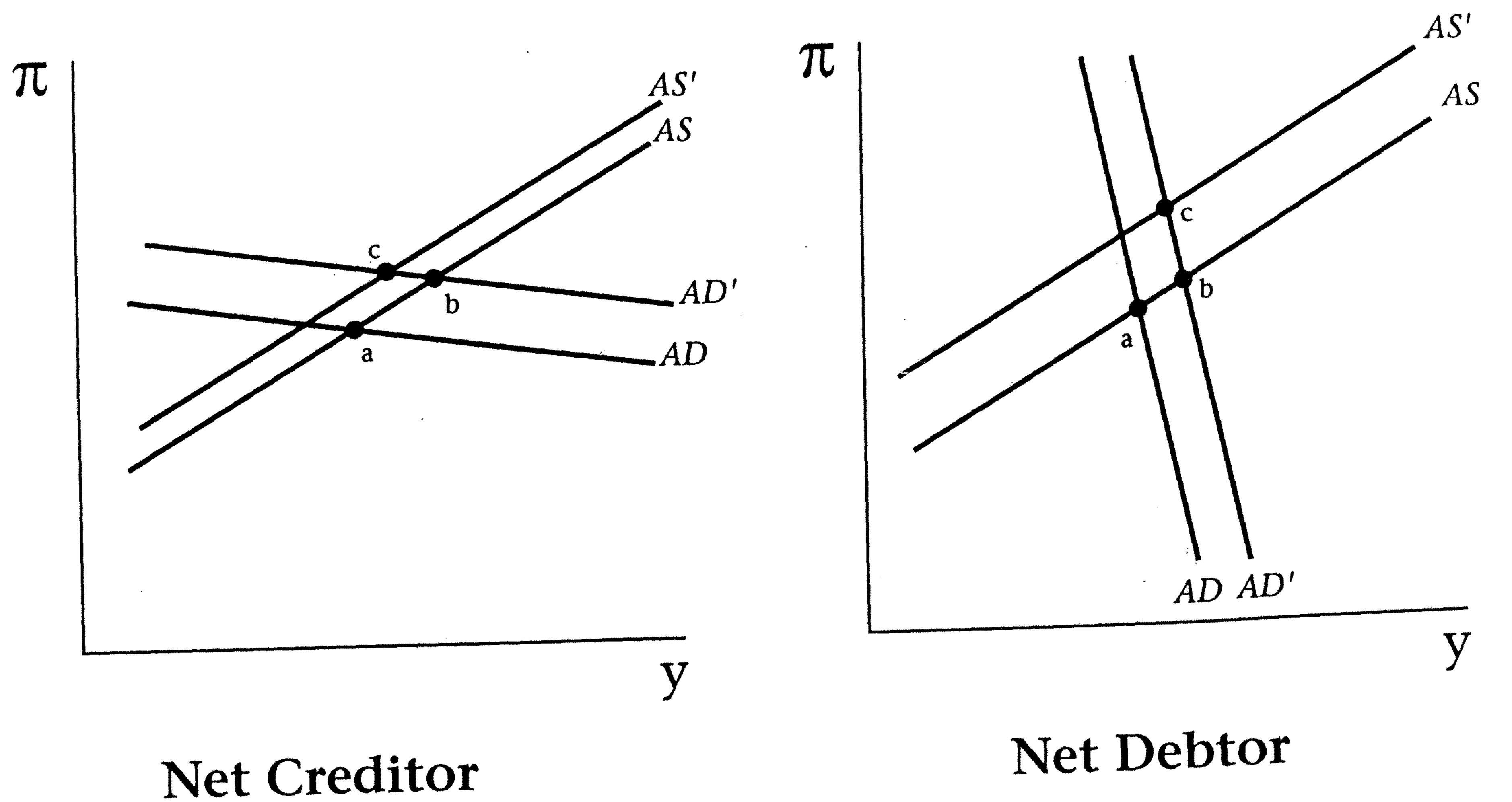


*Figure 10*

**Tax Financed Increase in Government Spending by Country 1**

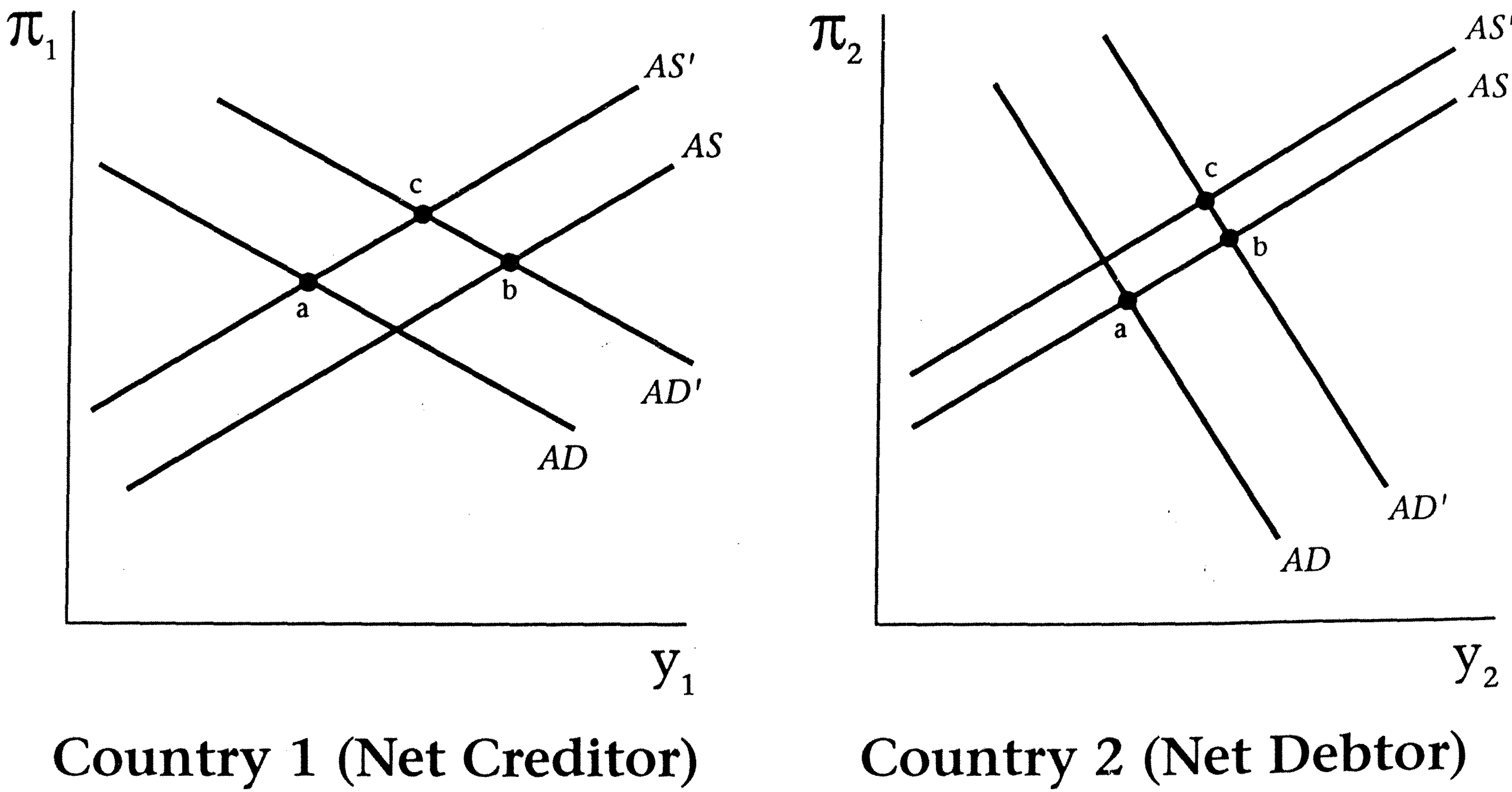
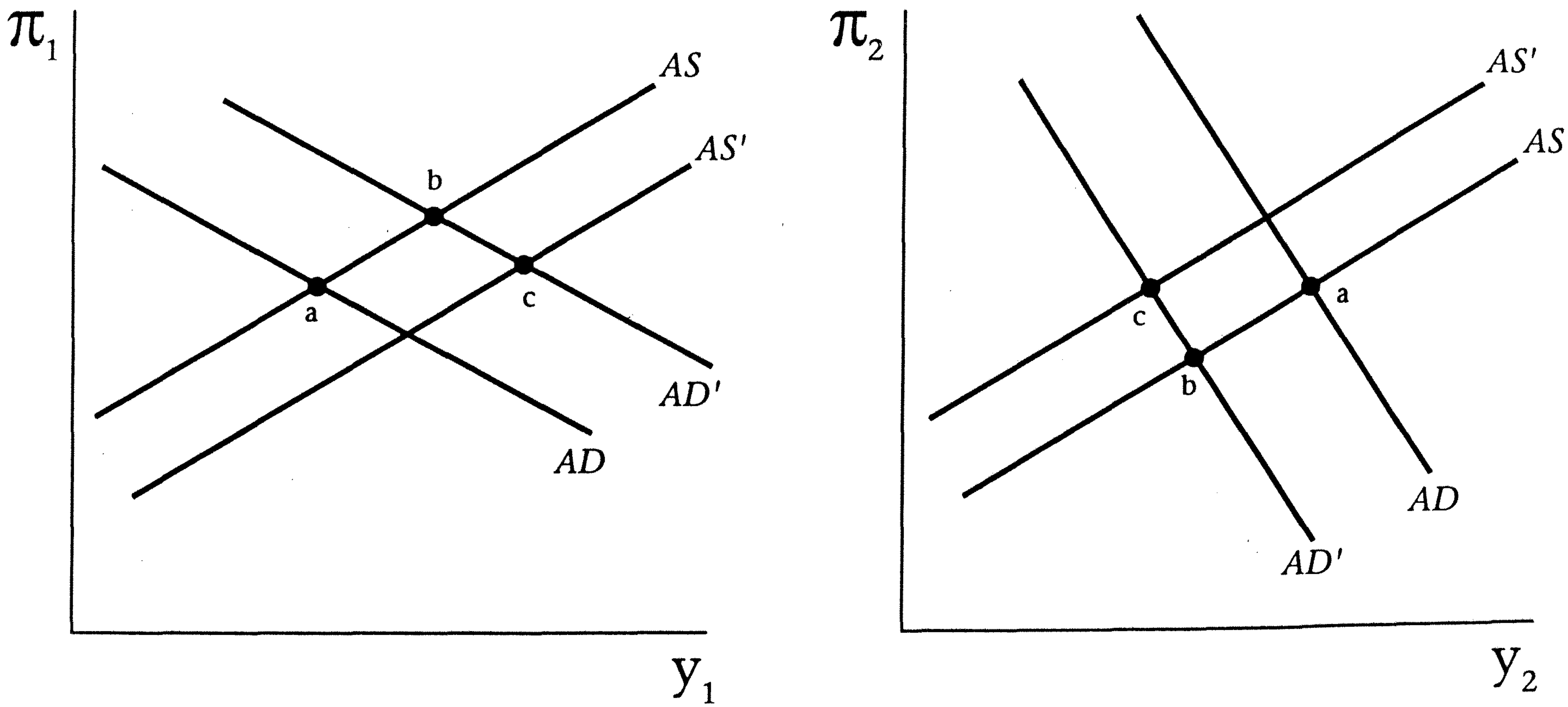


*Figure 11*

**Bond Financed Increase in Government Spending by Country 1**

$(A_4 + A_5) < 0$




**Country 1 (Net Creditor)**

**Country 2 (Net Debtor)**

# APPENDIX A

## SOLVING FOR AGGREGATE SUPPLY AND DEMAND

This appendix uses the equations in Tables I and II to derive the solutions for aggregate supply and aggregate demand for country 1, as given by equations (36) and (39), respectively, in the text. The derivations of the aggregate supply and demand equations for country 2, are discussed where they differ from those for country 1.

### Solving for Aggregate Supply

Lagging equation (2) yields:

$$(A1)\qquad p^c_{1,t-1} = \gamma p_{1,t-1} + (1-\gamma)p_{2,t-1}$$

Substituting equations (2) and (A1) into equation (3) yields:

$$(A2)\qquad \pi_{1,t-1} = \frac{\gamma(p_{1,t}-p_{1,t-1})}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}} + \frac{(1-\gamma)(p_{2,t}-p_{2,t-1})}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}$$

which can be rewritten as:

$$(A3)\qquad \pi_{1,t-1} = \left(\frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\frac{p_{1,t}-p_{1,t-1}}{p_{1,t-1}}\right) + \left(\frac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}\right)\left(\frac{p_{2,t}-p_{2,t-1}}{p_{2,t-1}}\right)$$

Rewriting equation (14) as:[30]

[30] This is done so that in the aggregate supply equation for country 1, all real variables will be measured in the same units.

$$(A4) \quad \frac{p_{2,t} - p_{2,t-1}}{p_{2,t-1}} = \alpha \left( \frac{y_{2,t}\,\tilde{p}_t - \bar{y}}{\bar{y}} \right) + \frac{E_{t-1}p_{2,t} - p_{2,t-1}}{p_{2,t-1}}$$

Substituting equations (1) and (A4) into equation (A3) yields:

$$(A5) \quad \pi_{1,t-1} = \left( \frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1} + (1-\gamma) p_{2,t-1}} \right) \left[ \alpha \, \frac{y_{1,t} - \bar{y}}{\bar{y}} + \frac{E_{t-1}p_{1,t} - p_{1,t-1}}{p_{1,t-1}} \right] + \left( \frac{(1-\gamma) p_{2,t-1}}{\gamma p_{1,t-1} + (1-\gamma) p_{2,t-1}} \right) \left[ \alpha \, \frac{y_{2,t}\,\tilde{p}_t - \bar{y}}{\bar{y}} + \frac{E_{t-1}p_{2,t} - p_{2,t-1}}{p_{2,t-1}} \right]$$

Next, solving equations (2) and (15) for $p_{1,t}$ and $p_{2,t}$, respectively:

$$p_{1,t} = \frac{p_{1,t}^c}{\gamma} - \frac{(1-\gamma) p_{2,t}}{\gamma}, \qquad p_{2,t} = \frac{p_{2,t}^c}{\gamma} - \frac{(1-\gamma) p_{1,t}}{\gamma}$$

which after some algebra yields:

$$(A6) \quad p_{1,t} = \frac{\gamma}{2\gamma-1} p_{1,t}^c - \frac{1-\gamma}{2\gamma-1} p_{2,t}^c, \qquad (A7) \quad p_{2,t} = \frac{\gamma}{2\gamma-1} p_{2,t}^c - \frac{1-\gamma}{2\gamma-1} p_{1,t}^c$$

Taking expectations at t-1 of equations (A6) and (A7) gives:

$$(A8) \quad E_{t-1}p_{1,t} = \frac{\gamma}{2\gamma-1} E_{t-1}p_{1,t}^c - \frac{1-\gamma}{2\gamma-1} E_{t-1}p_{2,t}^c$$

$$(A9) \quad E_{t-1}p_{2,t} = \frac{\gamma}{2\gamma-1} E_{t-1}p_{2,t}^c - \frac{1-\gamma}{2\gamma-1} E_{t-1}p_{1,t}^c$$

Lagging equations (A6) and (A7), and using equations (A8) and (A9), it is possible to rewrite:

$$\frac{E_{t-1}p_{1,t} - p_{1,t-1}}{p_{1,t-1}} \quad \text{and,} \quad \frac{E_{t-1}p_{2,t} - p_{2,t-1}}{p_{2,t-1}}$$

as:

$$\pi^e_{1,t-1}\left(\frac{\gamma p^c_{1,t-1}}{\gamma p^c_{1,t-1}-(1-\gamma)p^c_{2,t-1}}\right)-\pi^e_{2,t-1}\left(\frac{(1-\gamma)p^c_{2,t-1}}{\gamma p^c_{1,t-1}-(1-\gamma)p^c_{2,t-1}}\right)$$

and:

$$\pi^e_{2,t-1}\left(\frac{\gamma p^c_{2,t-1}}{\gamma p^c_{2,t-1}-(1-\gamma)p^c_{1,t-1}}\right)-\pi^e_{1,t-1}\left(\frac{(1-\gamma)p^c_{1,t-1}}{\gamma p^c_{2,t-1}-(1-\gamma)p^c_{1,t-1}}\right)$$

respectively.

Making these substitutions into equation (A5) and solving for $y_{1,t}$, yields:

$$(A10)\quad y_{1,t}=\frac{\gamma_1+\gamma_2}{\gamma_1}\bar{y}-\frac{\bar{y}}{\alpha\gamma_1}\left(\gamma_1\tilde{\gamma}^c_1-\gamma_2\tilde{\gamma}^c_4\right)\pi^e_{1,t-1}+\frac{\bar{y}}{\alpha\gamma_1}\left(\gamma_1\tilde{\gamma}^c_2-\gamma_2\tilde{\gamma}^c_3\right)\pi^e_{2,t-1}$$
$$+\frac{\bar{y}}{\alpha\gamma_1}\pi_{1,t-1}-\frac{\gamma_2}{\gamma_1}y_{2,t}\tilde{p}_t$$

where:

$$\gamma_1=\frac{\gamma p_{1,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}},\qquad \gamma_2=\frac{(1-\gamma)p_{2,t-1}}{\gamma p_{1,t-1}+(1-\gamma)p_{2,t-1}}$$

$$\tilde{\gamma}^c_1=\frac{\gamma p^c_{1,t-1}}{\gamma p^c_{1,t-1}-(1-\gamma)p^c_{2,t-1}},\qquad \tilde{\gamma}^c_2=\frac{(1-\gamma)p^c_{2,t-1}}{\gamma p^c_{1,t-1}-(1-\gamma)p^c_{2,t-1}}$$

$$\tilde{\gamma}^c_3=\frac{\gamma p^c_{2,t-1}}{\gamma p^c_{2,t-1}-(1-\gamma)p^c_{1,t-1}},\qquad \tilde{\gamma}^c_4=\frac{(1-\gamma)p^c_{1,t-1}}{\gamma p^c_{2,t-1}-(1-\gamma)p^c_{1,t-1}}$$

Following the same procedure for country 2 yields:

$$(A11)\quad y_{2,t} = \frac{\gamma_3 + \gamma_4}{\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right) - \frac{1}{\alpha\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right)\left(\gamma_3\tilde{\gamma}_3^c - \gamma_4\tilde{\gamma}_2^c\right)\pi^e_{2,t-1}$$

$$+ \frac{1}{\alpha\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right)\left(\gamma_3\tilde{\gamma}_4^c - \gamma_4\tilde{\gamma}_1^c\right)\pi^e_{1,t-1} + \frac{1}{\alpha\gamma_3}\left(\frac{\bar{y}}{\tilde{p}_t}\right)\pi_{2,t-1}$$

$$- \frac{\gamma_4}{\gamma_3}\left(\frac{y_{1,t}}{\tilde{p}_t}\right)$$

where:

$$\gamma_3 = \frac{\gamma p_{2,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}, \qquad \gamma_4 = \frac{(1-\gamma)p_{1,t-1}}{\gamma p_{2,t-1} + (1-\gamma)p_{1,t-1}}$$

Substituting equation (A11) into equation (A10):

$$(A12)\quad y_{1,t} = \left(-\frac{\gamma_2}{\gamma_1}\right)\left[\frac{\gamma_3 + \gamma_4}{\gamma_3}\bar{y} - \frac{\bar{y}}{\alpha}\left(\frac{\gamma_3\tilde{\gamma}_3^c - \gamma_4\tilde{\gamma}_2^c}{\gamma_3}\right)\pi^e_{2,t-1} + \frac{\bar{y}}{\alpha}\left(\frac{\gamma_3\tilde{\gamma}_4^c - \gamma_4\tilde{\gamma}_1^c}{\gamma_3}\right)\pi^e_{1,t-1}\right]$$

$$+ \left(-\frac{\gamma_2}{\gamma_1}\right)\left[\frac{\bar{y}}{\alpha\gamma_3}\pi_{2,t-1} - \frac{\gamma_4}{\gamma_3}y_{1,t}\right] + \frac{\gamma_1 + \gamma_2}{\gamma_1}\bar{y} - \frac{\bar{y}}{\alpha}\left(\frac{\gamma_1\tilde{\gamma}_1^c - \gamma_2\tilde{\gamma}_4^c}{\gamma_1}\right)\pi^e_{1,t-1}$$

$$+ \frac{\bar{y}}{\alpha}\left(\frac{\gamma_1\tilde{\gamma}_2^c - \gamma_2\tilde{\gamma}_3^c}{\gamma_1}\right)\pi^e_{2,t-1} + \frac{\bar{y}}{\alpha\gamma_1}\pi_{1,t-1}$$

and solving for $y_{1t}$ gives:

$$(A13)\quad y_{1,t} = \bar{y}\left[1 + \frac{\gamma_3}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\pi_{1,t-1} - \frac{\tilde{\gamma}_1^c}{\alpha}\pi^e_{1,t-1}\right]$$

$$- \bar{y}\left[\frac{\gamma_2}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\pi_{2,t-1} - \frac{\tilde{\gamma}_2^c}{\alpha}\pi^e_{2,t-1}\right]$$

It can be shown that:

$$\tilde{\gamma}_1^c = \frac{\gamma_3}{\gamma_1\gamma_3 - \gamma_2\gamma_4}, \qquad \tilde{\gamma}_2^c = \frac{\gamma_2}{\gamma_1\gamma_3 - \gamma_2\gamma_4}$$

Thus, the aggregate supply curve for country 1 can be written as:

$$(A14) \qquad y_{1,t} = \bar{y}\left[1 + \frac{\gamma_3}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\,(\pi_{1,t-1} - \pi^e_{1,t-1})\right]$$

$$- \bar{y}\left[\frac{\gamma_2}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,(\pi_{2,t-1} - \pi^e_{2,t-1})\right]$$

To solve for aggregate supply in country 2, substitute equation (A10) into equation (A11):

$$(A15) \qquad y_{2,t} = \left(-\frac{\gamma_4}{\gamma_3}\right)\left[\frac{\gamma_1 + \gamma_2}{\gamma_1}\,\frac{\bar{y}}{\tilde{p}_t} - \frac{\bar{y}}{\tilde{p}_t\alpha}\left(\left[\frac{\gamma_2\tilde{\gamma}^c_4}{\gamma_1} - \tilde{\gamma}^c_1\right]\pi^e_{1,t-1} + \left[\tilde{\gamma}^c_2 - \frac{\gamma_2\tilde{\gamma}^c_3}{\gamma_1}\right]\pi^e_{2,t-1}\right)\right]$$

$$+ \left(-\frac{\gamma_4}{\gamma_3}\right)\left[\frac{\bar{y}}{\tilde{p}_t\alpha\gamma_1}\,\pi_{1,t-1} - \frac{\gamma_2}{\gamma_1}\,y_{2,t}\right] + \frac{\gamma_3 + \gamma_4}{\gamma_3}\,\frac{\bar{y}}{\tilde{p}_t}$$

$$- \frac{\bar{y}}{\tilde{p}_t\alpha}\left(\frac{\gamma_4\tilde{\gamma}^c_2}{\gamma_3} - \tilde{\gamma}^c_3\right)\pi^e_{2,t-1} - \frac{\bar{y}}{\tilde{p}_t\alpha}\left(\tilde{\gamma}^c_4 - \frac{\gamma_4\tilde{\gamma}^c_1}{\gamma_3}\right)\pi^e_{1,t-1}$$

$$+ \frac{\bar{y}}{\tilde{p}_t\alpha\gamma_3}\,\pi_{2,t-1}$$

and solving for $y_{2t}$ gives:

$$(A16) \qquad y_{2,t} = \frac{\bar{y}}{\tilde{p}_t}\left[1 + \frac{\gamma_1}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\,\pi_{2,t-1} - \frac{\tilde{\gamma}^c_3}{\alpha}\left(\frac{2\gamma-1}{\gamma}\right)\pi^e_{2,t-1}\right]$$

$$- \frac{\bar{y}}{\tilde{p}_t}\left[\frac{\gamma_4}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\,\pi_{1,t-1} - \frac{\tilde{\gamma}^c_4}{\alpha}\left(\frac{2\gamma-1}{\gamma}\right)\pi^e_{1,t-1}\right]$$

It can be shown that

$$\tilde{\gamma}^c_3 = \frac{\gamma_1}{\gamma_1\gamma_3-\gamma_2\gamma_4}, \qquad \tilde{\gamma}^c_4 = \frac{\gamma_4}{\gamma_1\gamma_3-\gamma_2\gamma_4}$$

Thus, the aggregate supply curve for country 2 can be written as:

$$y_{2,t} = \frac{\bar{y}}{\tilde{p}_t}\left[1 + \frac{\gamma_1}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}\;(\pi_{2,t-1} - \pi^e_{2,t-1})\right]$$

$$- \frac{\bar{y}}{\tilde{p}_t}\left[\frac{\gamma_4}{\alpha(\gamma_1\gamma_3 - \gamma_2\gamma_4)}\;(\pi_{1,t-1} - \pi^e_{1,t-1})\right]$$

Solving for Aggregate Demand

Adding equations (12) and (25) yields:

$$(A17) \quad m_{1,t} + m_{2,t}\tilde{p}_t = \lambda(y^d_{1,t} + y^d_{2,t}\tilde{p}_t) - 2\theta\iota_t$$

Substituting equation (31) into equation (A17) and solving for $\iota_t$:

$$(A18) \quad \iota_t = \frac{\lambda}{2\theta}(y^d_{1,t} + y^d_{2,t}\tilde{p}_t) - \frac{1}{2\theta}b_{m,t}$$

Substituting equation (4) into equation (8), and equation (17) into equation (21) yields:

$$(A19) \quad a_{1,t} = cy^d_{1,t} - \phi\iota_t + \phi\pi^e_{1,t}$$

$$(A20) \quad a_{2,t} = cy^d_{2,t} - \phi\iota_t + \phi\pi^e_{2,t}$$

Substituting equation (A18) into (A19):

$$(A21) \quad a_{1,t} = cy^d_{1,t} - \frac{\lambda\phi}{2\theta}(y^d_{1,t} + y^d_{2,t}\tilde{p}_t) + \frac{\phi}{2\theta}b_{m,t} + \phi\pi^e_{1,t}$$

and substituting equation (A18) into (A20):

$$(A22) \quad a_{2,t} = cy^d_{2,t} - \frac{\lambda\phi}{2\theta}(y^d_{2,t} + y^d_{1,t}\frac{1}{\tilde{p}_t}) + \frac{\phi}{2\theta}b_{m,t}\frac{1}{\tilde{p}_t} + \phi\pi^e_{2,t}$$

Substituting equation (10) into equation (7):

$$(A23) \quad y_{1,t} = (1-\epsilon)a_{1,t} + (1-\epsilon_g)g_{1,t} + \epsilon a_{2,t}\tilde{p}_t + \epsilon_g g_{2,t}\tilde{p}_t + \eta$$

Substituting equations (A21) and (A22) into equation (A23):

$$(A24) \quad y_{1,t} = \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right] y_{1,t}^d - \left[\frac{\lambda\phi}{2\theta} - \epsilon c\right] y_{2,t}^d \tilde{p}_t + \frac{\phi}{2\theta}b_{m,t} + (1-\epsilon)\phi\pi_{1,t}^e + \epsilon\phi\pi_{2,t}^e\tilde{p}_t + (1-\epsilon_g)g_{1,t} + \epsilon_g g_{2,t}\tilde{p}_t$$

Substituting equations (9) and (22) into equation (A24):

$$(A25) \quad y_{1,t} = \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right]\left(y_{1,t} + r_{1,t-1}b_{1,t-1}^p - t_{1,t}\right) - \left[\frac{\lambda\phi}{2\theta} - \epsilon c\right]\left(y_{2,t} + r_{2,t-1}b_{2,t-1}^p - t_{2,t}\right)\tilde{p}_t + \frac{\phi}{2\theta}b_{m,t} + (1-\epsilon)\phi\pi_{1,t}^e + \epsilon\phi\pi_{2,t}^e\tilde{p}_t + (1-\epsilon_g)g_{1,t} + \epsilon_g g_{2,t}\tilde{p}_t$$

Solving equation (11) for $t_{1t}$ and equation (24) for $t_{2t}$, and substituting the resulting equations into equation (A25) yields:

$$(A26) \quad y_{1,t} = \left[(1-\epsilon)c - \frac{\lambda\phi}{2\theta}\right]\left(y_{1,t} + r_{1,t-1}b_{1,t-1}^p - g_{1,t} - r_{1,t-1}b_{1,t-1} + b_{1,t}\right) - \left[\frac{\lambda\phi}{2\theta} - \epsilon c\right]\left(y_{2,t} + r_{2,t-1}b_{2,t-1}^p - g_{2,t} - r_{2,t-1}b_{2,t-1} + b_{2,t}\right)\tilde{p}_t + \frac{\phi}{2\theta}b_{m,t} + (1-\epsilon)\phi\pi_{1,t}^e + \epsilon\phi\pi_{2,t}^e\tilde{p}_t + (1-\epsilon_g)g_{1,t} + \epsilon_g g_{2,t}\tilde{p}_t$$

Solving equation (A26) for $y_{1t}$ gives country 1's aggregate demand as a function of aggregate demand in country 2:

$$\text{(A27)} \quad y_{1,t} = \left[\frac{[(1-\epsilon_g) - (1-\epsilon)c]2\theta + \lambda\phi}{(1-(1-\epsilon)c)2\theta) + \lambda\phi}\right] g_{1,t}$$
$$+ \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] r_{1,t-1}\ (b^p_{1,t-1} - b_{1,t-1})$$
$$+ \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] b_{1,t} + \left[\frac{(1-\epsilon)2c\theta\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^e_{1,t}$$
$$+ \left[\frac{2c\epsilon\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] y_{2,t}\tilde{p}_t$$
$$+ \left[\frac{(\epsilon_g - \epsilon c)2\theta + \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] g_{2,t}\tilde{p}_t$$
$$+ \left[\frac{2c\epsilon\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] r_{2,t-1}\ (b^p_{2,t-1} - b_{2,t-1})\tilde{p}_t$$
$$+ \left[\frac{2c\epsilon\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] b_{2,t}\tilde{p}_t + \left[\frac{2\epsilon\phi\theta}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^e_{2,t}\tilde{p}_t$$
$$+ \left[\frac{\phi}{2\theta - (1-\epsilon)2c\theta + \lambda\phi}\right] b_{m,t}$$

equivalently, solving for $y_{2t}$ in terms of $y_{1t}$ yields:

$$\text{(A28)} \quad y_{2,t} = \left[\frac{[(1-\epsilon_g) - (1-\epsilon)c]2\theta + \lambda\phi}{(1-(1-\epsilon)c)2\theta) + \lambda\phi}\right] g_{2,t}$$
$$+ \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] r_{2,t-1}\ (b^p_{2,t-1} - b_{2,t-1})$$
$$+ \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] b_{2,t} + \left[\frac{(1-\epsilon)2c\theta\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^e_{2,t}$$
$$+ \left[\frac{2c\epsilon\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] y_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{(\epsilon_g - c\epsilon)2\theta + \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] g_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{2c\epsilon\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] r_{1,t-1}\ (b^p_{1,t-1} - b_{1,t-1})\ \frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{2c\epsilon\theta - \lambda\phi}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] b_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{2\epsilon\phi\theta}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right] \pi^e_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{\phi}{2\theta - (1-\epsilon)2c\theta + \lambda\phi}\right] b_{m,t}\frac{1}{\tilde{p}_t}$$

Next note the following:

$$(A29)\quad r_{2,t-1}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_t = \frac{i_{t-1}}{1+\pi_{2,t-1}}(b^{p}_{2,t-1} - b_{2,t-1})\tilde{p}_t$$

$$= (i_{t-1})\left(\frac{p^{c}_{2,t-1}}{p^{c}_{2,t}}\right)\left(\frac{p^{c}_{2,t}}{p^{c}_{1,t}}\right)(b^{p}_{2,t-1} - b_{2,t-1})$$

$$= (i_{t-1})\left(\frac{p^{c}_{2,t-1}}{p^{c}_{2,t}}\right)\left(\frac{p^{c}_{2,t}}{p^{c}_{1,t}}\right)\left(\frac{p^{c}_{1,t-1}}{p^{c}_{1,t-1}}\right)(b^{p}_{2,t-1} - b_{2,t-1})$$

$$= (i_{t-1})\left(\frac{p^{c}_{2,t-1}}{p^{c}_{1,t-1}}\right)\left(\frac{p^{c}_{1,t-1}}{p^{c}_{1,t}}\right)(b^{p}_{2,t-1} - b_{2,t-1})$$

$$= r_{1,t-1}\,(b^{p}_{2,t-1} - b_{2,t-1})\,\tilde{p}_{t-1}$$

Furthermore,

$$(A30)\quad b^{p}_{1,t-1} - b_{1,t-1} = b_{21,t-1}\,\tilde{p}_{t-1} - b_{12,t-1} - \frac{1}{2}\,b_{m,t-1}$$

$$(A31)\quad b^{p}_{2,t-1} - b_{2,t-1} = b_{12,t-1}\,\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1} - \frac{1}{2}b_{m,t-1}\,\frac{1}{\tilde{p}_{t-1}}$$

Using equations A(29) - A(31), equations A(27) and A(28) can be rewritten as:

$$\begin{aligned}
(A32)\quad y_{1,t} = {} & \left[\frac{[(1-\epsilon_g) - (1-\epsilon)c]2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta) + \lambda\phi}\right]g_{1,t} \\
& + \left[\frac{2c\theta(1-2\epsilon)}{(1-(1-\epsilon)c)2\theta+\lambda\phi}\right]r_{1,t-1}\,(b_{21,t-1}\,\tilde{p}_{t-1} - b_{12,t-1}) \\
& + \left[\frac{(1-\epsilon)2c\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right]b_{1,t} + \left[\frac{(1-\epsilon)2c\theta\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right]\pi^{e}_{1,t} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right]y_{2,t}\tilde{p}_t \\
& + \left[\frac{(\epsilon_g - \epsilon c)2\theta + \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right]g_{2,t}\tilde{p}_t \\
& + \left[\frac{\lambda\phi - c\theta}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right]r_{1,t-1}\,b_{m,t-1} \\
& + \left[\frac{2c\epsilon\theta - \lambda\phi}{(1 - (1-\epsilon)c)2\theta + \lambda\phi}\right]b_{2,t}\tilde{p}_t + \left[\frac{2\epsilon\phi\theta}{(1-(1-\epsilon)c)2\theta + \lambda\phi}\right]\pi^{e}_{2,t}\tilde{p}_t \\
& + \left[\frac{\phi}{2\theta - (1-\epsilon)2c\theta + \lambda\phi}\right]b_{m,t}
\end{aligned}$$

$$\text{(A33)}\quad y_{2,t} = \left[\frac{[(1-e_g) - (1-e)c]2\theta + \lambda\phi}{(1 - (1-e)c)2\theta) + \lambda\phi}\right] g_{2,t}$$
$$+ \left[\frac{2c\theta(1-2e)}{(1-(1-e)c)2\theta + \lambda\phi}\right] r_{1,t-1}(b_{12,t-1}\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1})$$
$$+ \left[\frac{(1-e)2c\theta - \lambda\phi}{(1 - (1-e)c)2\theta + \lambda\phi}\right] b_{2,t} + \left[\frac{(1-e)2c\theta\phi}{(1 - (1-e)c)2\theta + \lambda\phi}\right]\pi^e_{2,t}$$
$$+ \left[\frac{2ce\theta - \lambda\phi}{(1 - (1-e)c)2\theta + \lambda\phi}\right] y_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{(e_g - ce)2\theta + \lambda\phi}{(1 - (1-e)c)2\theta + \lambda\phi}\right] g_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{\lambda\phi - c\theta}{(1-(1-e)c)2\theta + \lambda\phi}\right] r_{1,t-1}\, b_{m,t-1}\frac{1}{\tilde{p}_t - 1}$$
$$+ \left[\frac{2ce\theta - \lambda\phi}{(1 - (1-e)c)2\theta + \lambda\phi}\right] b_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{2e\phi\theta}{(1 - (1-e)c)2\theta + \lambda\phi}\right]\pi^e_{1,t}\frac{1}{\tilde{p}_t}$$
$$+ \left[\frac{\phi}{2\theta - (1-e)2c\theta + \lambda\phi}\right] b_{m,t}\frac{1}{\tilde{p}_t}$$

Substituting equation (A32) into equation (A33) yields:

$$\text{(A34)}\quad y_{1,t} = \left[1 - \frac{2e_g\theta}{\Lambda}\right] g_{1,t} + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{1,t} + \frac{(1-e)2\phi\theta}{\Lambda}\pi^e_{1,t}$$
$$+ \frac{2ec\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{2,t}\tilde{p}_t + \frac{2e\phi\theta}{\Lambda}\pi^e_{2,t}\tilde{p}_t$$
$$+ \frac{\phi}{\Lambda} b_{m,t} + \frac{(1-2e)2c\theta}{\Lambda} r_{1,t-1}(b_{21,t-1}\tilde{p}_{t-1} - b_{12,t-1})$$
$$+ \frac{\lambda\phi - c\theta}{\Lambda} r_{1,t-1} b_{m,t-1}$$
$$+ \left[\frac{2e\phi\theta - \lambda\phi}{\Lambda}\right] \{\left(1 - \frac{\beta e_g}{\Lambda}\right) g_{2,t}\tilde{p}_t + \frac{(1-e)2c\theta - \lambda\phi}{\Lambda} b_{2,t}\tilde{p}_t$$
$$+ \frac{(1-e)2\phi\theta}{\Lambda}\pi^e_{2,t}\tilde{p}_t + \frac{2ce\theta - \lambda\phi}{\Lambda} y_{1,t} + \frac{(e_g - ec)2\theta + \lambda\phi}{\Lambda} g_{1,t}$$
$$+ \frac{2ce\theta}{\Lambda} b_{1,t} + \frac{2ce\theta}{\Lambda}\pi^e_{1,t} + \frac{\phi}{\Lambda} b_{m,t}$$
$$+ \frac{(1-2e)2c\theta}{\Lambda} r_{1,t-1}(b_{12,t-1}\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1})$$
$$+ \frac{\lambda\phi - c\theta}{\Lambda} r_{1,t-1} b_{m,t-1}\frac{1}{\tilde{p}_{t-1}}\}$$

where:

$$\Lambda \equiv (1 - (1-\epsilon)c)2\theta + \lambda\phi$$

Combining terms and solving for $y_{1t}$ gives:

$$\begin{aligned}
(A35) \quad y_{1,t} = & \left[ \left(1 - \frac{2\epsilon_g\theta}{\Lambda}\right) + V\left(\frac{(\epsilon_g - \epsilon c)2\theta + \lambda\phi}{\Lambda}\right) \right]\frac{1}{\Psi}\, g_{1,t} \\
& + \left[ \frac{(1-\epsilon)2c\theta - \lambda\phi}{\Lambda} + V^2 \right]\frac{1}{\Psi}\, b_{1,t} \\
& + \left[ \frac{(1-\epsilon)2\phi\theta}{\Lambda} + V\,\frac{2\epsilon\phi\theta}{\Lambda} \right]\frac{1}{\Psi}\, \pi^e_{1,t} \\
& + \left[ V\left(1 + \frac{(1-\epsilon)2c\theta - \lambda\phi}{\Lambda}\right) \right]\frac{1}{\Psi}\, b_{2,t}\tilde{p}_t \\
& + \left[ \frac{(\epsilon_g - \epsilon c)2\theta + \lambda\phi}{\Lambda} + V\left(1 - \frac{2\epsilon_g\theta}{\Lambda}\right) \right]\frac{1}{\Psi}\, g_{2,t}\tilde{p}_t \\
& + \left[ \frac{2\epsilon\phi\theta}{\Lambda} + V\,\frac{(1-\epsilon)2\phi\theta}{\Lambda} \right]\frac{1}{\Psi}\pi^e_{2,t}\tilde{p}_t + \left[ \frac{\phi}{\Lambda}(1 + V) \right]\frac{1}{\Psi}\, b_{m,t} \\
& + \left[ \frac{(1-2\epsilon)2c\theta}{\Lambda} - V\,\frac{(1-2\epsilon)2c\theta}{\Lambda} \right]\frac{1}{\Psi}\, r_{1,t-1}(b_{21,t-1}\,\tilde{p}_{1-1} - b_{12,t-1}) \\
& + \left[ \frac{\lambda\phi - c\theta}{\Lambda} + V\,\frac{\lambda\phi - c\theta}{\Lambda} \right]\frac{1}{\Psi}\, r_{1,t-1}b_{m,t-1}
\end{aligned}$$

where:

$$V = \frac{2c\epsilon\theta - \lambda\phi}{\Lambda}$$

$$\Psi = \frac{\Lambda^2 - (2c\epsilon\theta - \lambda\phi)^2}{\Lambda^2}$$

Which can then be simplified to give:

$$(A36)\quad y_{1,t} = \left(1 - \frac{e_g}{1-c+2ce}\right) g_{1,t}$$
$$+ \frac{2\theta c(c-1+e-2ce) + \lambda\phi(1-2c+4ce)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} b_{1,t}$$
$$+ \frac{2\phi\theta(c-1+e-2ce)+\lambda\phi^2(2e-1)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} \pi^e_{1,t}$$
$$+ \frac{e_g}{1-c+2ce} g_{2,t}\tilde{p}_t + \frac{\lambda\phi-2ce\theta}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} b_{2,t}\tilde{p}_t$$
$$+ \frac{\phi(\lambda\phi-2e\lambda\phi-2e\theta)}{2(1-c+2ce)(c\theta-\theta-\lambda\phi)} \pi^e_{2,t}\tilde{p}_t + \frac{\phi}{2(\theta-c\theta+\lambda\phi)} b_{m,t}$$
$$+ \frac{c(1-2e)}{1-c+2ce} r_{1,t-1}(b^p_{1,t-1} - b_{1,t-1})$$
$$+ \frac{\lambda\phi-c\theta}{\theta-c\theta+\lambda\phi} r_{1,t-1}\frac{1}{2}b_{m,t-1}$$

The solution process for $y_{2,t}$ is the same as for $y_{1,t}$.

# APPENDIX B

## SOLVING FOR EQUILIBRIUM INFLATION AND OUTPUT

This appendix uses the Aggregate Supply and Demand equations for country 1 and country 2 to solve for the each country's equilibrium inflation rate and output level as given by Tables VIII and IX.

Country 1's and country 2's aggregate supply equations are as follows:

$$(B1) \quad y_{1t} = \bar{y} + \frac{\bar{y}}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}[\gamma_3(\pi_{1t-1} - \pi^e_{1t-1}) - \gamma_2(\pi_{2t-1}-\pi^e_{2t-1})]$$

$$(B2) \quad y_{2t} = \frac{\bar{y}}{\tilde{p}} + \frac{\bar{y}/\tilde{p}}{\alpha(\gamma_1\gamma_3+\gamma_2\gamma_4)}[\gamma_1(\pi_{2t-1}-\pi^e_{2t-1}) - \gamma_4(\pi_{1t-1}-\pi^e_{1t-1})]$$

where the coefficients:

$$\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4$$

are defined in the text.

The aggregate demand equations for country 1 and country 2 are:

$$(B3) \quad y_{1,t} = A_1\, g_{1,t} + A_2\, b_{1,t} + A_3\, \pi^e_{1,t} + A_4\, g_{2,t}\tilde{p}_t + A_5\, b_{2,t}\tilde{p}_t + A_6\, \pi^e_{2,t}\tilde{p}_t + A_7\, b_{m,t} + i_{t-1}\, Y_1 - \pi_{1,t-1}Y_1$$

$$(B4) \quad y_{2,t} = A_1\, g_{2,t} + A_2\, b_{2,t} + A_3\, \pi^e_{2,t} + A_4\, g_{1,t}\frac{1}{\tilde{p}_t} + A_5\, b_{1,t}\frac{1}{\tilde{p}_t} + A_6\, \pi^e_{1,t}\frac{1}{\tilde{p}_t} + A_7\, b_{m,t}\frac{1}{\tilde{p}_t} + i_{t-1}Y_2 - \pi_{2,t-1}Y_2$$

where the coefficients $A_1$ to $A_{10}$ are defined in Table V, and:

$$\Upsilon_1 = A_8 (b_{21,t-1}\,\tilde{p}_{t-1} - b_{12,t-1}) + A_9 \frac{1}{2} b_{m,t-1}$$

$$\Upsilon_2 = A_8 (b_{12,t-1}\,\frac{1}{\tilde{p}_{t-1}} - b_{21,t-1}) + A_9 \frac{1}{2} b_{m,t-1}\left(\frac{1}{\tilde{p}_{t-1}}\right)$$

Substituting equation (B3) into (B1) gives $\pi_{1,t-1}$ as a function of the pre-determined, exogenous and policy variables, and $\pi_{2,t-1}$:

$$(B5) \quad \pi_{1,t-1} = \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}} \left( A_1 g_{1,t} + A_2 b_{1,t} + A_3 \pi^e_{1,t} + A_4 g_{2,t}\tilde{p}_t \right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}} \left( A_5 b_{2,t}\tilde{p}_t + A_6 \pi^e_{2,t}\tilde{p}_t + A_7 b_{m,t} \right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}} \left( \Upsilon_1 i_{t-1} - \bar{y} \right)$$
$$+ \frac{\bar{y}}{\Upsilon_1\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\bar{y}} \left[ \gamma_3 \pi^e_{1,t-1} + \gamma_2(\pi_{2,t-1} - \pi^e_{2,t-1}) \right]$$

Likewise, substituting equation (B4) into (B2) gives $\pi_{2,t-1}$ as a function of the pre-determined, exogenous and policy variables, and $\pi_{1,t-1}$:

$$(B6) \quad \pi_{2,t-1} = \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\bar{y}} \left( A_1 g_{2,t}\tilde{p}_t + A_2 b_{2,t}\tilde{p}_t + A_3 \pi^e_{2,t}\tilde{p}_t \right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\bar{y}} \left( A_4 g_{1,t} + A_5 b_{1,t} + A_6 \pi^e_{1,t} + A_7 b_{m,t} \right)$$
$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\bar{y}} \left( \Upsilon_2 i_{t-1} - \bar{y} \right)$$
$$+ \frac{\bar{y}}{\Upsilon_2\tilde{p}_t\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\bar{y}} \left[ \gamma_1 \pi^e_{2,t-1} + \gamma_4(\pi_{1,t-1} - \pi^e_{1,t-1}) \right]$$

Substituting (B6) into (B5) and solving for $\pi_{1,t-1}$ gives equilibrium inflation in country 1, with all variables deflated by country 1's price index. This is the $\pi_{1,t-1}$ equation in Table VII in the text. Substituting this equation into (B3) and solving for $y_{1,t}$ gives

$$
\begin{aligned}
(B7)\quad y_{1,t} = {} & \left[\frac{A_1\,[\Omega_1 - \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - Y_1 \alpha \gamma_1 \bar{y}] - A_4 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}\right] g_{1,t} \\
& + \left[\frac{A_2\,[\Omega_1 - \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - Y_1 \alpha \gamma_1 \bar{y}] - A_5 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}\right] b_{1,t} \\
& + \left[\frac{A_3\,[\Omega_1 - \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha Y_1 \gamma_1 \bar{y}] - A_6 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}\right] \pi^e_{1,t} \\
& + \left[\frac{A_4\,[\Omega_1 - \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha Y_1 \gamma_1 \bar{y}] - A_1 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}\right] g_{2,t}\tilde{p}_t \\
& + \left[\frac{A_5\,[\Omega_1 - \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha Y_1 \gamma_1 \bar{y}] - A_2 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}\right] b_{2,t}\tilde{p}_t \\
& + \left[\frac{A_6\,[\Omega_1 - \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha Y_1 \gamma_1 \bar{y}] - A_3 \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}\right] \pi^e_{2,t}\tilde{p}_t \\
& + \left[\frac{\Omega_1 - \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha Y_1 \gamma_1 \bar{y} - \alpha Y_1 \gamma_2 \bar{y}}{\Omega_1}\right] A_7 b_{m,t} \\
& + \left[\frac{Y_1\Omega_1 - \alpha^2 Y_1^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha Y_1^2 \gamma_1 \bar{y} - \alpha Y_1 Y_2 \tilde{p}_t \gamma_2 \bar{y}}{\Omega_1}\right] i_{t-1} \\
& - \left[\frac{Y_1 \bar{y}\,(\alpha\gamma_3 Y_2 \tilde{p}_t + \bar{y})}{\Omega_1}\right] \pi^e_{1,t-1} \\
& - \left[\frac{\alpha Y_1 Y_2 \tilde{p}_t \bar{y} \gamma_2}{\Omega_1}\right] \pi^e_{2,t-1} \\
& + \frac{\alpha Y_1 \bar{y}(\alpha Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \bar{y}(\gamma_1 + \gamma_2))}{\Omega_1}
\end{aligned}
$$

where:

$$\Omega_1 = \alpha^2 Y_1 Y_2 \tilde{p}_t(\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha\bar{y}(\gamma_1 Y_1 + \gamma_3 Y_2 \tilde{p}_t) + \bar{y}^2$$

Which can be simplified to arrive at the equilibrium output equation for country 1, as given by Table VIII in the text.

In order to calculate country 2's equilibrium inflation rate, with all the variables deflated by country 2's consumer price index, it is necessary to multiply both sides of equation (B1) by $(1/\tilde{p})$:

$$(B8) \quad \frac{y_{1,t}}{\tilde{p}_t} = \frac{\bar{y}}{\tilde{p}} + \frac{\bar{y}/\tilde{p}}{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}[\gamma_3(\pi_{1t-1}-\pi^e_{1t-1}) - \gamma_2(\pi_{2t-1}-\pi^e_{2t-1})]$$

Now, substituting (B3) into (B8) to solve for $\pi_{1,t-1}$ as a function of the pre-determined, exogenous and policy variables, and $\pi_{2,t-1}$, yields:

$$\begin{aligned}(B9) \quad \pi_{1,t-1} = {} & \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\frac{\Upsilon_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}\left(A_1 g_{1,t}\frac{1}{\tilde{p}_t} + A_2 b_{1,t}\frac{1}{\tilde{p}_t} + A_3\pi^e_{1,t}\frac{1}{\tilde{p}_t}\right) \\ & + \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\frac{\Upsilon_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}\left(A_3\pi^e_{1,t}\frac{1}{\tilde{p}_t} + A_4 g_{2,t} + A_5 b_{2,t}\right) \\ & + \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\frac{\Upsilon_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}\left(A_6\pi^e_{2,t} + A_7 b_{m,t}\frac{1}{\tilde{p}_t} + \frac{\Upsilon_1}{\tilde{p}_t}i_{t-1} - \frac{\bar{y}}{\tilde{p}_t}\right) \\ & + \frac{\frac{\bar{y}}{\tilde{p}_t}}{\frac{\Upsilon_1}{\tilde{p}_t}\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_3\frac{\bar{y}}{\tilde{p}_t}}[\gamma_3\pi^e_{1,t-1} + \gamma_2(\pi_{2,t-1} - \pi^e_{2,t-1})]\end{aligned}$$

Likewise substituting (B4) into (B2) gives $\pi_{2,t-1}$ as a function of the pre-determined, exogenous and policy variables, and $\pi_{1,t-1}$:

$$(B10) \quad \pi_{2,t-1} = \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left(A_1 g_{2,t} + A_2 b_{2,t} + A_3 \pi^e_{2,t}\right)$$

$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left(A_4 g_{1,t}\frac{1}{\tilde{p}_t} + A_5 b_{1,t}\frac{1}{\tilde{p}_t} + A_6 \pi^e_{1,t}\frac{1}{\tilde{p}_t}\right)$$

$$+ \frac{\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4)}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left(A_7 b_{m,t}\frac{1}{\tilde{p}_t} + \Upsilon_2 i_{t-1} - \frac{\bar{y}}{\tilde{p}_t}\right)$$

$$+ \frac{\frac{\bar{y}}{\tilde{p}_t}}{\Upsilon_2\alpha(\gamma_1\gamma_3-\gamma_2\gamma_4) + \gamma_1\frac{\bar{y}}{\tilde{p}_t}}\left[\gamma_1\pi^e_{2,t-1} + \gamma_4(\pi_{1,t-1} - \pi^e_{1,t-1})\right]$$

Substituting equation (B9) into (B10) one can solve for $\pi_{2,t-1}$, the equilibrium inflation rate in country 2, as given on the next two pages:

$$(B11)\quad \pi_{2,t-1} = \left[\frac{A_1\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right] + A_4\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\bar{y}^2}{\tilde{p}_t^2}}\right] g_{2,t}$$

$$+ \left[\frac{A_2\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right] + A_5\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\bar{y}^2}{\tilde{p}_t^2}}\right] b_{2,t}$$

$$+ \left[\frac{A_3\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right] + A_6\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\bar{y}^2}{\tilde{p}_t^2}}\right] \pi^e_{2,t}$$

$$+ \left[\frac{A_4\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right] + A_1\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\bar{y}^2}{\tilde{p}_t^2}}\right] g_{1,t}\frac{1}{\tilde{p}_t}$$

$$+ \left[\frac{A_5\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right] + A_2\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\bar{y}^2}{\tilde{p}_t^2}}\right] b_{1,t}\frac{1}{\tilde{p}_t}$$

$$+ \left[\frac{A_6\left[\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t}\right] + A_3\alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\bar{y}^2}{\tilde{p}_t^2}}\right] \pi^e_{1,t}\frac{1}{\tilde{p}_t}$$

$$+ \left[\frac{\left(\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_3\frac{\bar{y}}{\tilde{p}_t} + \alpha\gamma_4\frac{\bar{y}}{\tilde{p}_t}\right)A_7}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\bar{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\bar{y}^2}{\tilde{p}_t^2}}\right] b_{m,t}\frac{1}{\tilde{p}_t}$$

$\pi_{2,t-1}$ continued:

$$+\left[\frac{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\Upsilon_2\gamma_3\frac{\overline{y}}{\tilde{p}_t} + \alpha\Upsilon_1\frac{1}{\tilde{p}_t}\gamma_4\frac{\overline{y}}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\overline{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\overline{y}^2}{\tilde{p}_t^2}}\right] i_{t-1}$$

$$-\left[\frac{\frac{\overline{y}}{\tilde{p}_t}\left(\alpha\gamma_4\ \Upsilon_1\frac{1}{\tilde{p}_t} + \frac{\overline{y}}{\tilde{p}_t}\right)}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2 p(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\overline{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\overline{y}^2}{\tilde{p}_t^2}}\right] \pi^e_{1,t-1}$$

$$+\left[\frac{\alpha\frac{\overline{y}}{\tilde{p}_t}\ \gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\frac{\overline{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t} + \gamma_3\Upsilon_2) + \frac{\overline{y}^2}{\tilde{p}_t^2}}\right] \pi^e_{2,t-1}$$

$$-\frac{\alpha\frac{\overline{y}}{\tilde{p}_t}\left(\alpha\Upsilon_1\frac{1}{\tilde{p}_t}(\gamma_1\gamma_3-\gamma_2\gamma_4) + \frac{\overline{y}}{\tilde{p}_t}(\gamma_3+\gamma_4)\right)}{\alpha^2\Upsilon_1\frac{1}{\tilde{p}_t}\Upsilon_2(\gamma_1\gamma_3-\gamma_2\gamma_1) + \alpha\frac{\overline{y}}{\tilde{p}_t}(\gamma_1\Upsilon_1\frac{1}{\tilde{p}_t}+\gamma_3\Upsilon_2) + \frac{\overline{y}^2}{\tilde{p}_t^2}}$$

Substituting equation (B11) into (B4) and solving for $y_{2,t}$ gives

$$
\begin{aligned}
(B12)\qquad y_{2,t} = {} & \left[ \frac{A_1 \left[\Omega_2 - \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}\right] - A_4 \alpha\gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{2,t} \\
& + \left[ \frac{A_2 \left[\Omega_2 - \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}\right] - A_5 \alpha\gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{2,t} \\
& + \left[ \frac{A_3 \left[\Omega_2 - \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}\right] - A_6 \alpha\gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{2,t} \\
& + \left[ \frac{A_4 \left[\Omega_2 - \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}\right] - A_1 \alpha\gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{1,t} \frac{1}{\tilde{p}_t} \\
& + \left[ \frac{A_5 \left[\Omega_2 - \alpha^2 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) - \alpha\gamma_3 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}\right] - A_2 \alpha\gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{1,t} \frac{1}{\tilde{p}_t} \\
& + \left[ \frac{\frac{\bar{y}}{\tilde{p}_t} \Upsilon_2 \left(\alpha\gamma_4 \Upsilon_1 \frac{1}{\tilde{p}_t} + \frac{\bar{y}}{\tilde{p}_t}\right)}{\Omega_2} \right] \pi^e_{1,t-1} \\
& - \left[ \frac{\frac{\bar{y}}{\tilde{p}_t} \alpha\gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2}{\Omega_2} \right] \pi^e_{2,t-1} \\
& - \frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \Upsilon_2 \left(\alpha \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \frac{\bar{y}}{\tilde{p}_t} (\gamma_3 + \gamma_4)\right)}{\Omega_2}
\end{aligned}
$$

where:

$$
\Omega_2 = \alpha^2 Y_1 \frac{1}{\tilde{p}_t} Y_2 (\gamma_1\gamma_3 - \gamma_2\gamma_4) + \alpha \frac{\bar{y}}{\tilde{p}_t} \left(\gamma_1 \frac{Y_1}{\tilde{p}_t} + \gamma_3 Y_2\right) + \frac{\bar{y}^2}{\tilde{p}_t^2}
$$

Making use of this definition of $\Omega_2$, equation (B12) can be simplified to equilibrium output equation for country 2

$$
\begin{aligned}
(B13) \qquad y_{2,t} = {} & \left[ \frac{A_1 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}} \right) - A_4 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{2,t} \\
& + \left[ \frac{A_2 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t} \right) - A_5 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{2,t} \\
& + \left[ \frac{A_3 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t} \right) - A_6 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{2,t} \\
& + \left[ \frac{A_4 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t} \right) - A_1 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] g_{1,t} \frac{1}{\tilde{p}_t} \\
& + \left[ \frac{A_5 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t} \right) - A_2 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] b_{1,t} \frac{1}{\tilde{p}_t} \\
& + \left[ \frac{A_6 \left( \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}} \right) - A_3 \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] \pi^e_{1,t} \frac{1}{\tilde{p}_t} \\
& + \left[ \frac{\alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \frac{\bar{y}}{\tilde{p}_t} + \frac{\bar{y}^2}{\tilde{p}_t} - \alpha \gamma_4 \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t}}{\Omega_2} \right] A_7 b_{m,t} \frac{1}{\tilde{p}_t} \\
& + \left[ \frac{\alpha \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2 \frac{\bar{y}}{\tilde{p}_t} (\gamma_1 - \gamma_4) + \Upsilon_2 \frac{\bar{y}^2}{\tilde{p}_t^2}}{\Omega_2} \right] i_{t-1} \\
& + \left[ \frac{\frac{\bar{y}}{\tilde{p}_t} \Upsilon_2 \left( \alpha \gamma_4 \Upsilon \frac{1}{\tilde{p}_t} + \frac{\bar{y}}{\tilde{p}_t} \right)}{\Omega_2} \right] \pi^e_{1,t-1} \\
& + \left[ \frac{\frac{\bar{y}}{\tilde{p}_t} \alpha \gamma_1 \Upsilon_1 \frac{1}{\tilde{p}_t} \Upsilon_2}{\Omega_2} \right] \pi^e_{2,t-1} \\
& - \frac{\alpha \frac{\bar{y}}{\tilde{p}_t} \Upsilon_2 \left( \alpha \Upsilon_1 \frac{1}{\tilde{p}_t} (\gamma_1 \gamma_3 - \gamma_2 \gamma_4) + \frac{\bar{y}}{\tilde{p}_t} (\gamma_3 + \gamma_4) \right)}{\Omega_2}
\end{aligned}
$$

# APPENDIX C

## COMPARATIVE STATICS

This appendix uses the inflation and output equations for country 1 (Tables VII and VIII), to derive the signs of the comparative statics given in Tables IX and X.

### Effect on Inflation of a Change in One of the Policy Variables

The denominator of each coefficient is $\Omega_1$. Given that $Y_2$, $Y_1$, $\alpha$, $\gamma_i$, and $\bar{y}$ are positive, the sign of the denominator depends upon the sign of the term $(\gamma_1\gamma_3 - \gamma_2\gamma_4)$. Since $\gamma > 1/2$ this term is positive. Therefore, the denominator of each coefficient is positive. Determining the sign of the coefficients on the policy variables in the inflation equation thus, becomes a matter of determining the sign of the numerators of these coefficients.

Since $(\gamma_1\gamma_3 - \gamma_2\gamma_4)>0$, the signs of the numerators of the policy variable coefficients in the equilibrium inflation equation for country 1, depend upon the signs of the aggregate demand coefficients ($A_1$ through $A_7$).

**a) Tax financed expenditures by the government of country 1:**

$$\frac{A_1[\alpha^2 Y_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y}] + A_4\alpha\gamma_2\bar{y}}{\Omega_1}$$

Since $A_1 > 0$ and $A_4 > 0$ the numerator is positive. Thus an increase in own government expenditures will increase inflation in country 1.

b) Bond financed expenditures by the government of country 1:

$$\frac{(A_1+A_2)\ [\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4)\ +\ \alpha\gamma_1\overline{y}\ ]\ +\ (A_4+A_5)\alpha\gamma_2\overline{y}}{\Omega_1}$$

$(A_1 + A_2) > 0$ but $(A_4 + A_5)$ may be positive or negative. If $(A_4 + A_5 > 0$, then the numerator is positive. If $(A_4 + A_5) < 0$ then the sign of the numerator is also positive. This result follows since $\gamma_1 > \gamma_2$ and $(A_1 + A_2) > |A_4 + A_5|$.

c) Tax financed expenditures by the government of country 2:

$$\frac{A_4[\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4)\ +\ \alpha\gamma_1\overline{y}\ ]\ +\ A_1\alpha\gamma_2\overline{y}}{\Omega_1}$$

Since $A_1 > 0$ and $A_4 > 0$ the numerator is positive. Thus an increase in government expenditures by country 2 will increase inflation in country 1.

d) Bond financed expenditures by the government of country 2:

$$\frac{(A_4+A_5)\ [\alpha^2\Upsilon_2\tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4)\ +\ \alpha\gamma_1\overline{y}\ ]\ +\ (A_1+A_2)\alpha\gamma_2\overline{y}}{\Omega_1}$$

$(A_1 + A_2) > 0$ but, as noted above, $(A_4 + A_5)$ may be positive or negative. If $(A_4 + A_5) > 0$, then the numerator is positive. If $(A_4 + A_5) < 0$ then the sign of the numerator is indeterminate.

e) Bond holdings of the central bank:

$$\frac{A_7[\alpha^2 \mathrm{Y}_2 \tilde{p}_t(\gamma_1\gamma_3-\gamma_2\gamma_4) + \alpha\gamma_1\bar{y} + \alpha\gamma_2\bar{y}]}{\Omega_1}$$

Given that $A_7 > 0$ an increase in bond holdings by the central bank will increase inflation in country 1.

Effect on Output of a Change in One of the Policy Variables

The denominator of each coefficient is $\Omega_1$. As shown above, this term is positive. Thus, determining the sign of the coefficients on the exogenous variables in the output equation becomes a matter of determining the sign of the numerators of these coefficients.

**a) Tax financed expenditures by the government of country 1:**

$$\frac{A_1[\alpha\gamma_3\mathrm{Y}_2\tilde{p}_t\bar{y} + \bar{y}^2] - A_4\alpha\mathrm{Y}_1\gamma_2\bar{y}}{\Omega_1}$$

$A_1 > 0$ and $A_4 > 0$. Given that $\gamma > ½$, $\gamma_3 > \gamma_2$. If

$$(C1) \quad \mathrm{Y}_2 > \mathrm{Y}_1$$

then the numerator is definitely positive. A necessary condition for the inequality in (C1) to hold is for country 1 to have had a larger current account deficit last period than that of country 2. A sufficient condition is that country 1 was a net debtor and country 2 was a net creditor last period.

If this the inequality given by (C1) does not hold then the conditions for the numerator to be positive can be found by rewriting the numerator as:

$$(C2) \quad A_1[\alpha\gamma_3\mathrm{Y}_2\tilde{p}_t\bar{y}+\bar{y}^2] - A_4\alpha\gamma_2\bar{y}[-\mathrm{Y}_2\tilde{p}_{t-1} + A_9 b_{m,t-1}]$$

Table IX: Effect of Changes in Policy Variables on Output in Country i

| | Direct Effect | Feedback Effect | Total Effect |
|---|---|---|---|
| Increase in tax financed government expenditures by country i | + | - | + |
| Increase in bond financed government expenditures by country i | +<br>+ | $-(A_4+A_5>0)$<br>$+(A_4+A_5<0)$ | +<br>+ |
| Increase in tax financed government expenditures by country j | + | - | ± |
| Increase in bond financed government expenditures by country j | $+(A_4+A_5>0)$<br>$-(A_4+A_5<0)$ | -<br>- | ±<br>- |
| Increase in real balances | + | - | + |

Table X: Effect of Changes in Policy Variables on Inflation in Country i

| | Direct Effect | Feedback Effect | Total Effect |
|---|---|---|---|
| Increase in tax financed government expenditures by country i | + | + | + |
| Increase in bond financed government expenditures by country i | +<br>+ | $+(A_4+A_5>0)$<br>$-(A_4+A_5<0)$ | +<br>+ |
| Increase in tax financed government expenditures by country j | + | + | + |
| Increase in bond financed government expenditures by country j | $+(A_4+A_5>0)$<br>$-(A_4+A_5<0)$ | +<br>+ | +<br>± |
| Increase in real balances | + | + | + |

*Figure 1*

**Effect of an Increase in Government Spending (Tax Financed) by Country 1 on Demand in both Country 1 and Country 2**

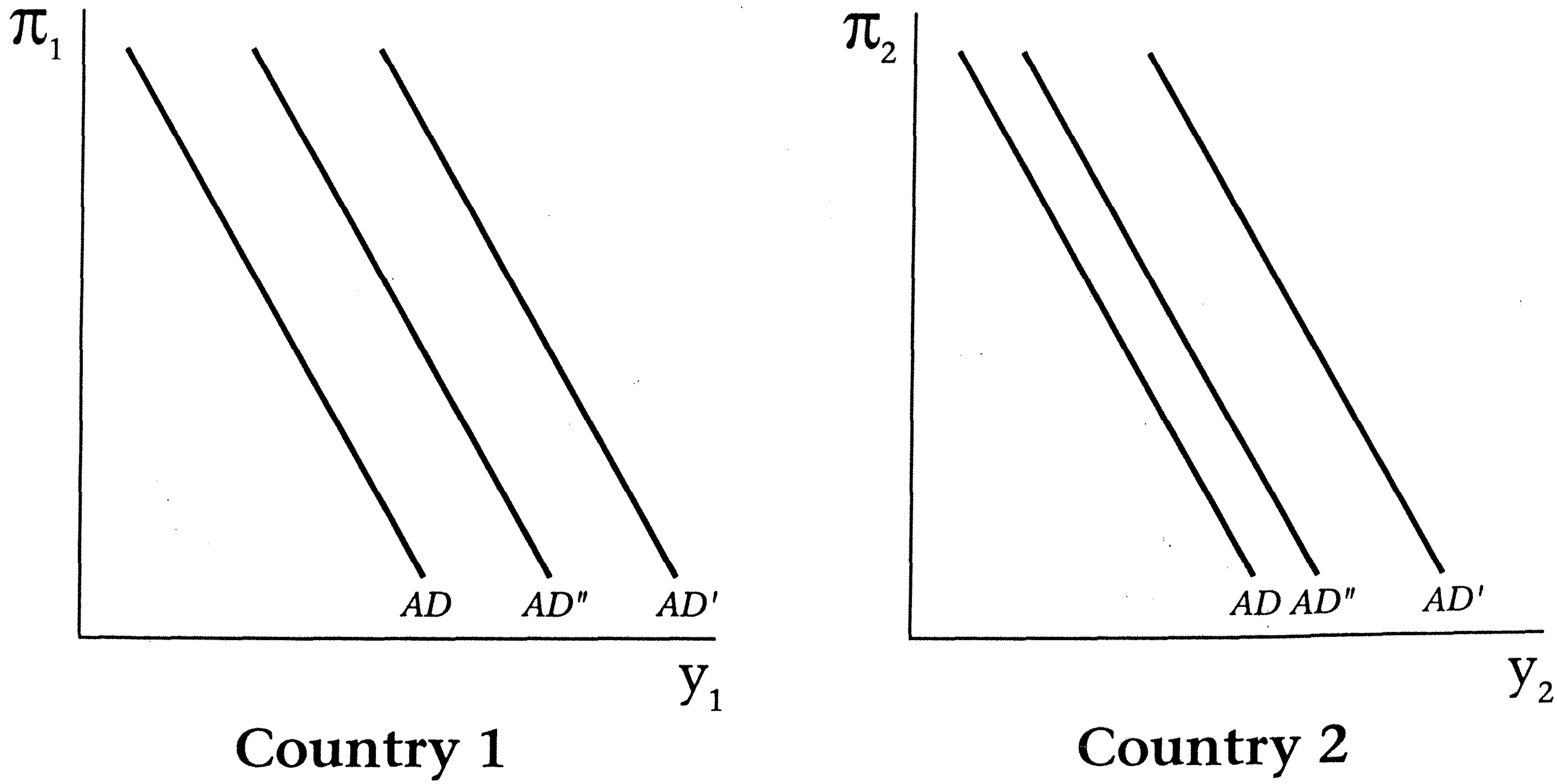
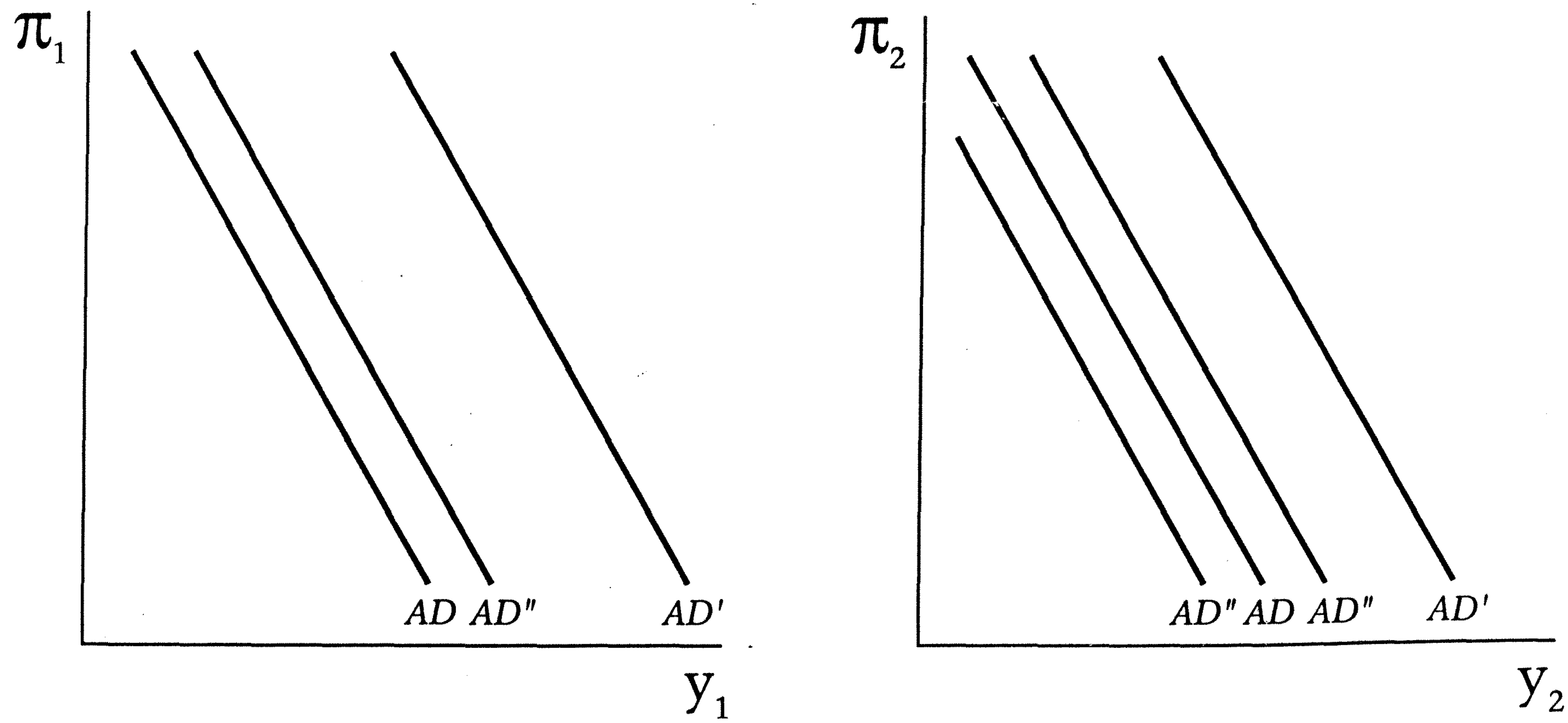


*Figure 2*

**Effect of an Increase in Government Spending (Bond Financed) by Country 1 on Demand in both Country 1 and Country 2**




**Country 1** **Country 2**

*Figure 3*

**Effect of an Increase in the Bond Holdings of the Central Bank on Demand in both Country 1 and Country 2**

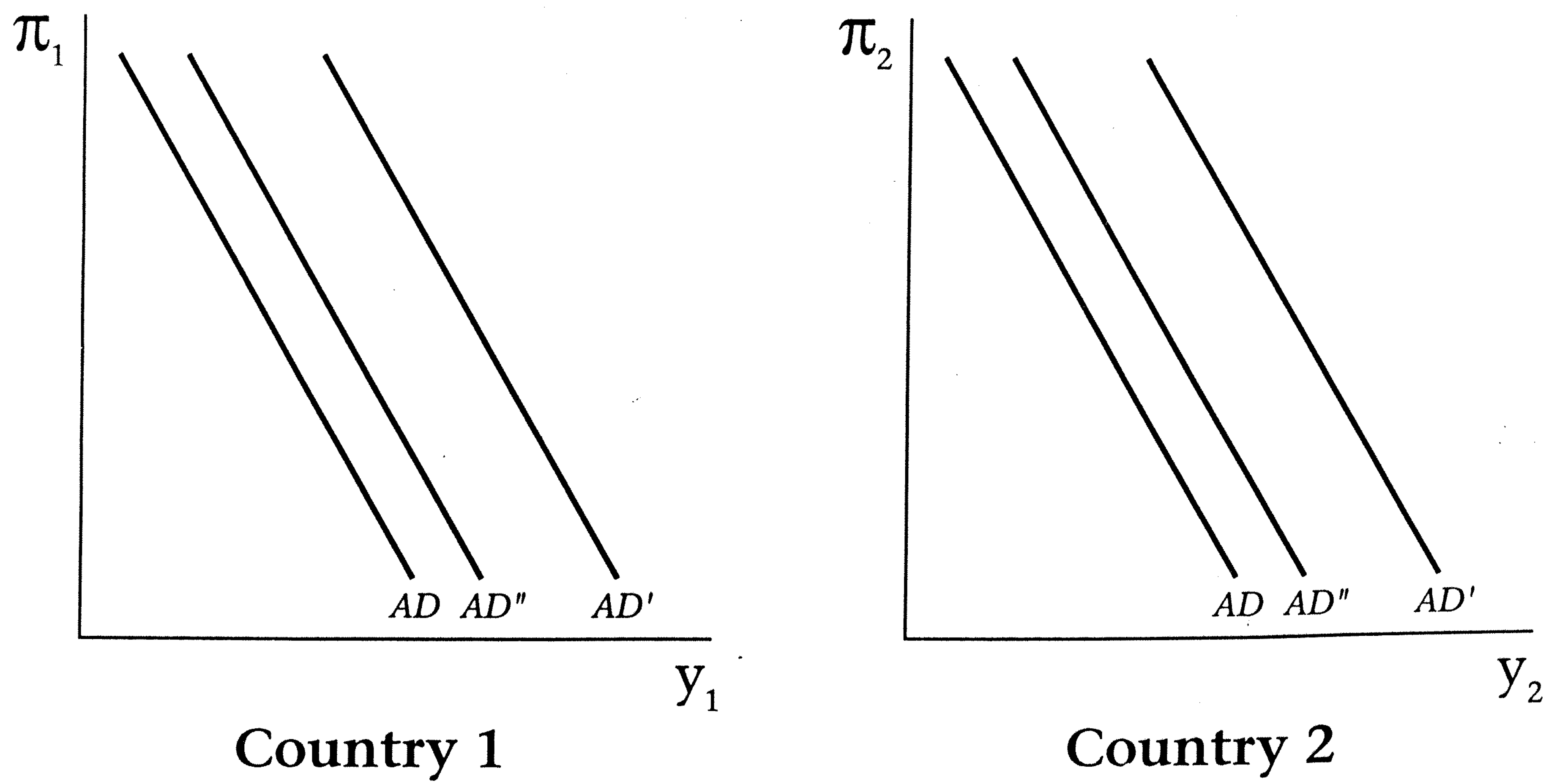


*Figure 4a*

**Net Debtor Policy Effects with Positive Demand Spillovers**

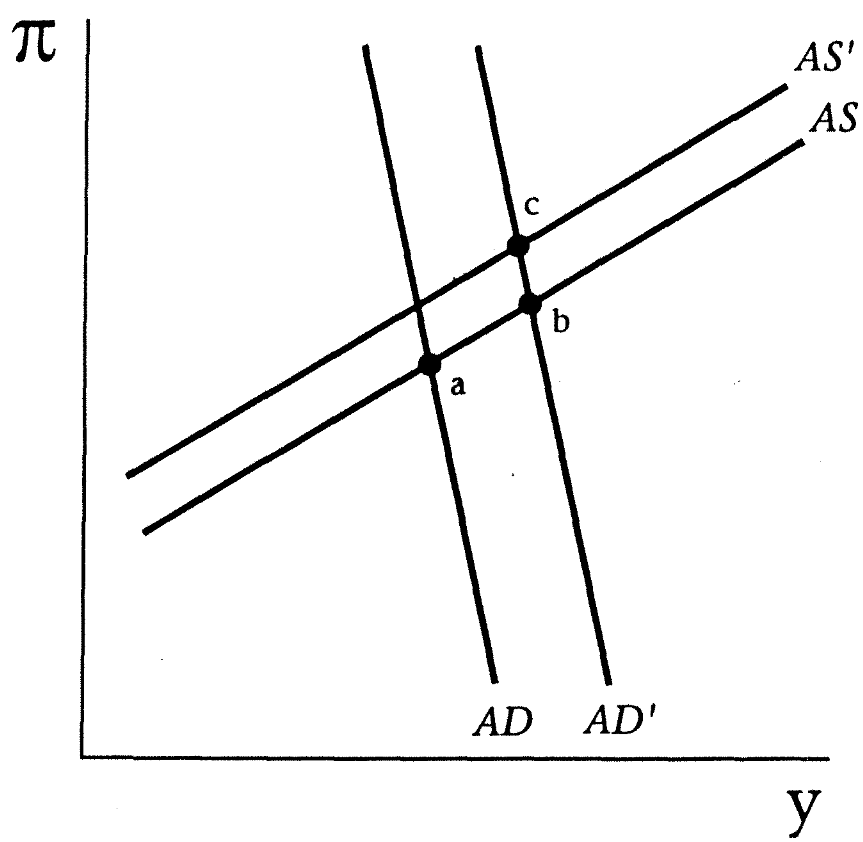



*Figure 4b*

**Net Creditor Policy Effects with Positive Demand Spillovers**

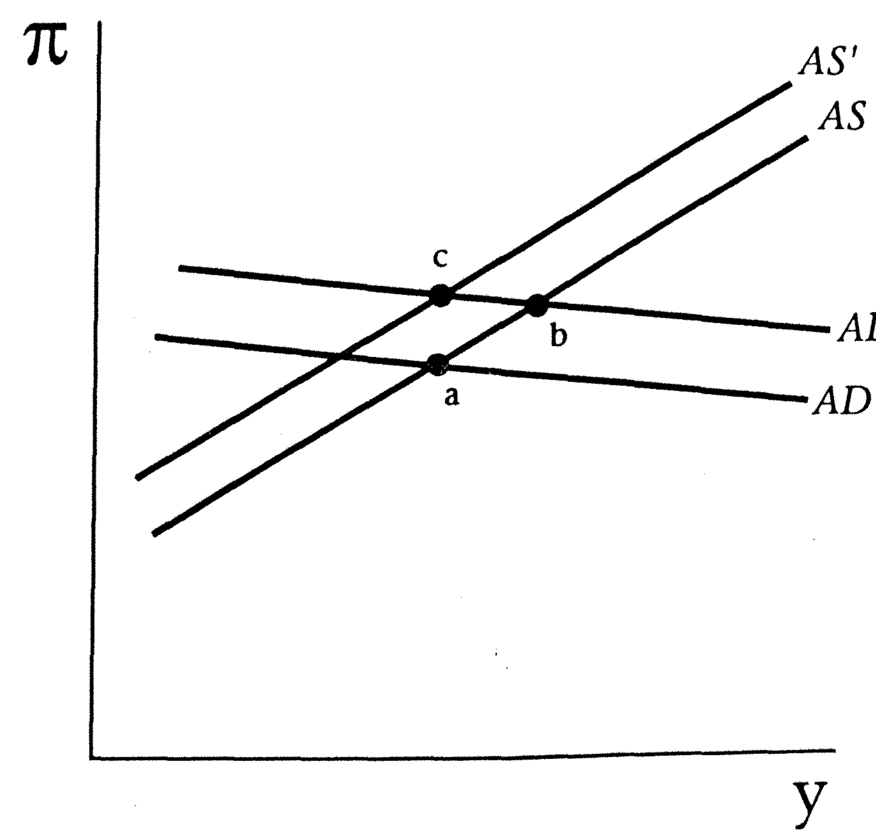


*Figure 5a*

**Net Debtor Effects with Negative Demand Spillovers (Home Country)**

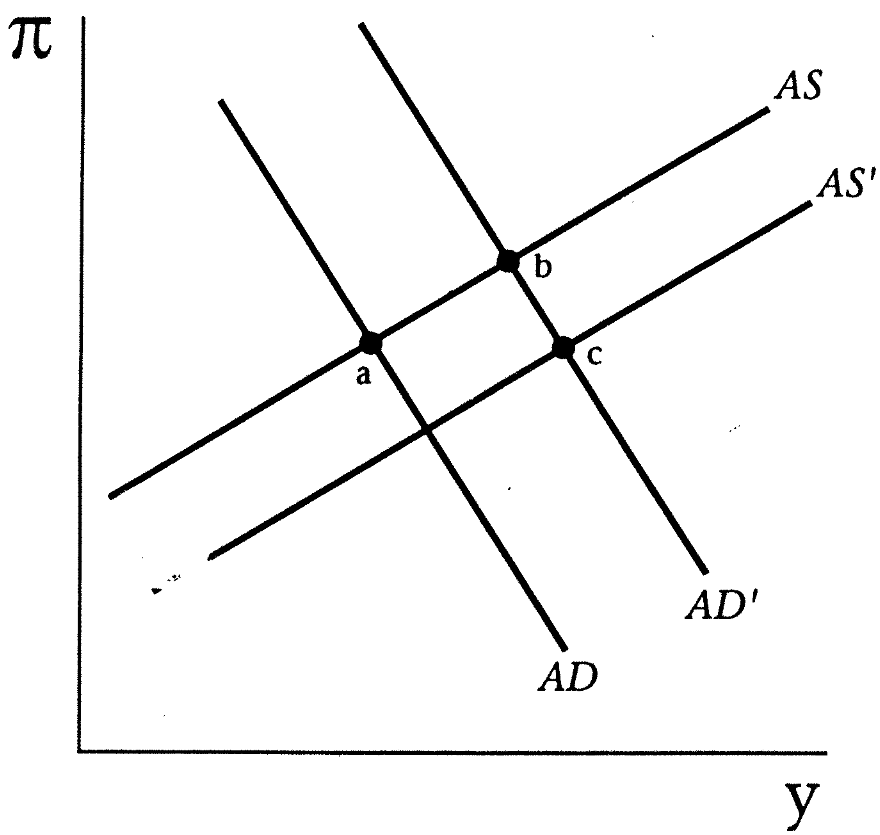



*Figure 5b*

**Net Creditor Effects with Negative Demand Spillovers (Home Country)**