REVIEW

JANUARY/FEBRUARY 1998

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What Has Become of the "Stability-Through-Inflation" Argument?

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The purpose of this article is to examine the status of a well known argument for a positive rate of steady state inflation. The original argument, by Vickrey (1954) and Phelps (1972), suggests that the economy is more able to dampen shocks, such as fluctuations in the real rate of return on risky capital investment, when inflation rates are positive. The notion is that the conditions required for stability are sturdier at higher rates of inflation, and thus at higher nominal rates of interest, making the economy as a whole less vulnerable to stochastic shocks. This has sometimes been forwarded as an argument for *stability through inflation*. Many economists have no doubt come away from this literature thinking, albeit probably vaguely, that *a little inflation is a good thing*. Echoes of this hypothesis can be heard today as many economists wonder aloud whether monetary policymakers should proceed to lower inflation rates.

Our view is that the "stability-through-inflation" argument has not stood the test of time very well. On the one hand, the rational expectations revolution destroyed much of the argument's foundation by insisting on agents that adjust their expectations very rapidly and hence immediately violate the stability condition derived in the previous literature. The down side of the rational expectations innovation was that it left theorists arguing that a likely outcome was a stationary equilibrium on the high-inflation side of the Laffer curve. On the other hand, more sophisticated treatments of the adaptive expectations hypothesis, mostly appearing in the recent literature on learning in macroeconomic models, have found less tendency toward instability at low inflation rates. They suggest, instead, that relatively low inflation rates were associated with stability in the learning dynamics, and relatively high inflation rates were associated with instability in the learning dynamics. In these systems, higher inflation is often more variable inflation. In either case—the rational expectations case or the learning case—the notion that low rates of inflation can generate instability is, if not altogether absent, at least much less likely. Thus we conclude that the stability-through-inflation argument is effectively defunct.

We proceed by summarizing the original stability-through-inflation arguments and the grounds on which they were criticized in the literature. We then summarize the rational expectations solution. The subsequent portion of the paper then turns to a discussion of the behavior of the model under what we regard as more sophisticated treatments of the adaptive expectations hypothesis. In the second half of the paper, we turn to recasting the argument in a version of the model with fixed real government deficits financed by seignorage revenue. The final section provides a summary.

CAGAN'S FRAMEWORK

We frame our discussion in the guise of a Cagan model:

$$(1)\quad \frac{H_t}{P_t} = S(\beta_t)\ ,$$

$$(2)\ H_t = \theta H_{t-1} + \xi P_t,$$

and

$$(3)\ FP_{t+1} = \beta_t P_t,$$

where H_t is high-powered money at time t, P_t is the price level at time t, F is an operator representing the forecast made at time t for the price at time $t+1$, β_t is the expected gross rate of inflation at time t, and $S(\cdot)$ is a continuous money demand function in expected gross inflation. We assume that $S'(\cdot) < 0$ and $S''(\cdot) \geq 0\ \forall\beta$ throughout. Equation 2 is a version of the government budget constraint in which we allow two parameter configurations. The first is $\{\theta > 1, \xi = 0\}$, so that the stock of high-powered money grows at a constant rate and the revenue to the government from the inflation tax is endogenous. We will call this the constant-money-growth model. The second parameter configuration is $\{\theta = 1, \xi > 0\}$, so that we can envision the government choosing H_t to fix the real revenue from money creation at ξ each period. In this case, ξ is the fixed real government deficit, and we refer to this as the fixed-deficit model. The model is not closed until an assumption is made about how expectations are formed, and how this assumption is made has wide-ranging implications for the conclusions we will draw from this model. An equilibrium is a strictly positive sequence for money and prices such that the assumption concerning expectations formation always holds.

We note for future use that in the case of a constant-money-growth model, the actual law of motion for prices is given by

$$(4)\ P_t = \frac{\theta S(\beta_{t-1})}{S(\beta_t)} P_{t-1}.$$

In the case of a fixed-deficit model, the actual law of motion for prices is given by

$$(5)\ P_t = \frac{S(\beta_{t-1})}{S(\beta_t) - \xi} P_{t-1}.$$

Equations 1-3 can be given a general equilibrium interpretation. In particular, these equations arise in a simple overlapping-generations model with fiat money, where the technology is such that one unit of labor produces one unit of the good. In that case, the function $S(\cdot)$ is an aggregate excess demand function and is thus continuous. The assumption that $S'(\cdot) < 0$ is a gross substitutes assumption in such an economy. If preferences are time-separable logarithmic, the function $S(\cdot)$ is linear in β. Any choice of a continuous function for $S(\cdot)$ can be mapped into a set of well-defined preferences for the agents in the overlapping generations economy. Finally, in the overlapping generations economy, Equation 2 can be viewed as a government budget constraint. In the constant-money-growth specification, government revenue is endogenous, while in the fixed-deficit specification, the money supply is chosen to produce a fixed stream of revenue.[1]

THE CONSTANT MONEY GROWTH MODEL

Adaptive Expectations

We first illustrate the argument for stability through inflation posed by Vickrey (1954) and Phelps (1972). We accordingly close the model with an adaptive expectations assumption. We derive the stability condition for a general demand function, although the early arguments were often posed in terms of a specific function, a Cagan demand schedule given by

$$(6)\ S(\beta_t) = e^{\alpha(1-\beta_t)},$$

where $\alpha > 0$ is the semi-log slope of the demand function.

The adaptive expectations hypothesis is given by

$$(7)\ \beta_t = \beta_{t-1} + \gamma\left[\frac{\theta S(\beta_{t-2})}{S(\beta_{t-1})} - \beta_{t-1}\right],$$

where $\gamma > 0$. The agents in the economy update their expectations by multiplying the previous expectational error by a constant. Here we have made use of the fact that

$$\frac{\theta S(\beta_{t-2})}{S(\beta_{t-1})}$$

[1] For more detail on the overlapping-generations interpretation, see the exposition in Sargent (1987).

is the actual gross rate of inflation in this economy. We will call the constant γ the *gain*. Equation 7 defines a dynamic system in one variable. One can verify by inspection that $\beta = \theta$ is a steady state of this system. A second steady state could exist at a stationary value of β such that $S(\beta) = 0$, and we will denote this steady state by $\beta = \bar{\beta}$. We assume throughout that $\beta < \bar{\beta}$ — that is, that the rate of currency creation is not so high as to cause agents to cease holding currency altogether. We will call the steady state at $\beta = \theta$ the monetary steady state and the steady state at $\beta = \bar{\beta}$ the nonmonetary steady state to reflect the fact that no currency is held in the latter situation. Since we assume that $S'(\cdot) < 0$, the nonmonetary steady state is unique. It may be the case that money demand merely approaches zero asymptotically, as with the Cagan schedule, and here we simply think in terms of $\bar{\beta} \longrightarrow \infty$.[2]

Equation 7 implies a first-order system which can be written as

$$(8)\ \beta_t = \beta_{t-1} + \gamma\left[\frac{\theta S(\beta_{t-2})}{S(\beta_{t-1})} - \beta_{t-1}\right],$$

$$(9)\ \beta_{t-1} = \beta_{t-1},$$

so that, defining $z_t = [\beta_t, \beta_{t-1}]'$ and $G(\cdot)$ by the right-hand side of these equations, we can write $z_t = G(z_{t-1})$. We can study stability by linearizing this system at either one of the steady states, which we denote generically by $z^* = [\beta, \beta]$. Linearization results in the Jacobian matrix

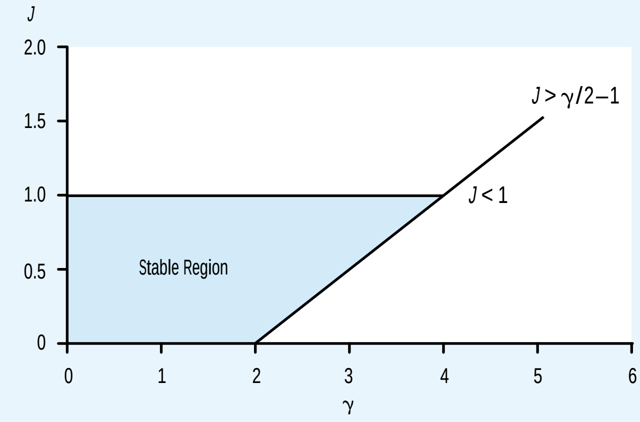
$$(10)\ DG(z^*) = \begin{bmatrix} 1-\gamma+J & -J \\ 1 & 0 \end{bmatrix},$$

where

$$J = \frac{-\gamma\theta S'(\beta)}{S(\beta)}.$$

A study of the characteristic equation of this matrix leads to the conclusion that stability of the system in the vicinity of a steady state is governed by two conditions.[3] The first condition is that $J < 1$, and the second condition is that the gain γ is not greater than 1.

Figure 1



Note: Local stability for the constant money growth model with adaptive expectations. The quantity J must be less than one, and the gain γ cannot be too large.

This situation is depicted in Figure 1, which plots the value of J against the value of γ.

The condition on the gain parameter γ simply says that if there is too much emphasis on the expectational error in the expectations adjustment process, the dynamics of the system will be locally unstable, regardless of any other parameter values. According to the figure, this situation occurs only with relatively large values of γ, in particular values that are greater than unity. While this result is informative, we normally want to think of expectations as being adjusted by some fraction of the most recent expectational error. If we restrict the gain to be less than one, then the condition for local stability reduces to $J < 1$. With this restriction and a Cagan schedule, the local stability condition at the monetary steady state would become $\alpha\gamma\theta < 1$.[4] When the condition $J < 1$ is satisfied, the steady state will be stable so long as one starts the system close to the steady state. When the condition is violated, one of two dynamics may be observed. The system may simply diverge, or it may settle into a cycle. Which of these possibilities occurs depends, in a complicated way, on the second and third derivatives of the money-demand schedule.[5]

We now comment on the condition $J < 1$. We begin with the monetary steady state, $\beta = \theta$. We first note that high rates of money creation, $\theta \gg 1$ imply instability.

[2] In some sense there will be no second equilibrium in the model in this case. Bruno and Fischer (1990) interpret a related model with bonds and a Cagan money-demand schedule. In their interpretation, a constant-money-growth policy is superior to a constant-deficit policy because, in the former case, the nonmonetary steady state would not exist.

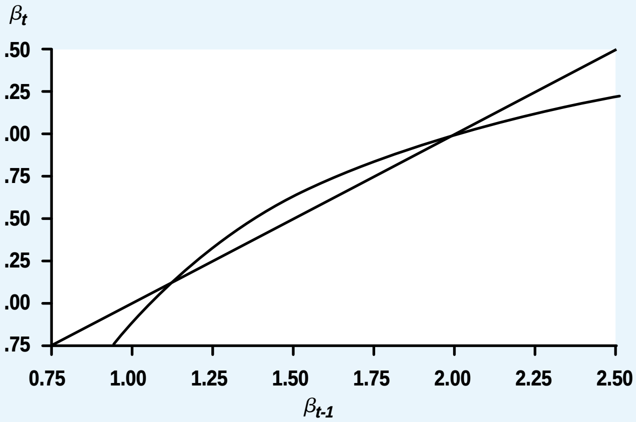
[3] We analyze the characteristic equation according to the conditions laid out in Baumol (1959).

[4] Cagan (1956) studied the stability of a related continuous time system under adaptive expectations using what has become known as the Cagan demand schedule. The stability condition in that case is $\alpha\gamma < 1$.

[5] For a more detailed analysis of dynamics of this sort, see Bullard (1994).

Figure 2

Steady States Under Perfect Foresight



Linear demand with $\lambda = 0.5$, $\theta = 1.1$

Note: Steady states under perfect foresight. A plot of the function describing the system, the curved line in the graph, crosses the 45 degree line, the straight line in the graph, at two points. The lower of these two is the monetary steady state, where the rate of inflation is equal to the rate of money growth. The dynamics, which can be read off the graph, indicate that the monetary steady state is unstable in the perfect foresight dynamics.

Since the steady-state rate of inflation is the constant rate of money creation in the monetary steady state of this model, arguments concerning low values of inflation must involve relatively low values of θ. We note that higher values of the gain parameter γ also imply instability. But the argument for stability through inflation involved the semi-log slope of the demand function. The argument by Vickrey (1954) and Phelps (1972) suggested that α should be viewed as itself depending on inflation, and that it would rise dramatically as inflation fell. In particular, $\lim_{\beta \longrightarrow 1} \alpha = +\infty$. Thus the condition would be violated at rates of inflation sufficient to drive the nominal interest rate to zero.

This can happen. An example of stability through inflation under adaptive expectations can be constructed for the Lucas (1994) money demand schedule,

$$(11)\quad S(\beta_t) = (\beta_t - 1)^{-\eta},$$

where $\eta \epsilon (0, 1)$. In this case, the condition $J < 1$ amounts to

$$\frac{\eta\gamma\theta}{\theta - 1} < 1.$$

The limit of this expression as $\theta \longrightarrow 1$ is $+\infty$, so the stability condition is always violated at sufficiently low rates of inflation.

The argument that $\alpha\gamma$ is more likely to be greater than one at low rates of inflation may be countered with the argument that γ is likely to be higher at high rates of inflation and lower at low rates of inflation. It seems plausible that people revise expectations more rapidly the higher the inflation rate, so that the increase in α at low rates of inflation may be offset by a corresponding reduction in γ. Later in this section, we will see that this type of phenomenon is in fact possible under alternative expectational assumptions.

The nonmonetary steady state will never be locally stable based on the condition $J < 1$, because while $S'(\bar{\beta})$ is finite, $S(\bar{\beta}) = 0$ and thus we expect the condition to be violated. The model with the Cagan demand schedule does not possess a finite value for $\bar{\beta}$ and thus we cannot evaluate that model at $\bar{\beta}$.

Rational Expectations

The argument for stability through inflation is largely moot if one assumes rational expectations, but at the same time the results under rational expectations are unsettling. Here we close the model 1-3 with a perfect foresight assumption:

$$(12)\quad \beta_t = \frac{P_{t+1}}{P_t},$$

so that Equations 1-2 imply the first-order difference equation,

$$(13)\quad \beta_t = S^{-1}\left[\frac{\theta S(\beta_{t-1})}{\beta_{t-1}}\right].$$

The equation has steady states at $\beta = \theta$ and $\beta = \bar{\beta}$. We plot this function qualitatively in Figure 2 for the case of a linear demand schedule, $1 - \lambda\beta_t$, with $\lambda \epsilon (0, 1)$.

There is an infinity of equilibrium sequences indexed by β_0. The monetary steady state is the asymptotic outcome only if $\beta_0 = \theta$. All other equilibrium sequences have the property that $\lim_{t \longrightarrow \infty} \beta_t = \bar{\beta}$.

The model is silent about which of these equilibrium sequences will actually be achieved, but many have interpreted this result as meaning that the monetary steady state is unstable under the rational expectations dynamics. That is, the nonmonetary steady state is locally stable, unless $\bar{\beta} \longrightarrow \infty$, in which case the inflation rate accelerates forever and real balances fall to zero asymptotically.

Thus, there is little to argue about the slope of the demand schedule in the perfect foresight case. Here the monetary steady state of the system is unstable regardless of parameter values. We note that the high-inflation stationary equilibrium at $\bar{\beta}$ is a Pareto inferior outcome in the general equilibrium interpretation of the model.

Least Squares Learning

Since the rational expectations dynamics do not seem entirely sensible, some authors have tried to improve on the adaptive expectations hypothesis by employing a learning assumption. Bullard (1994) has analyzed the constant-money-growth model under least squares learning by using methods introduced by Marcet and Sargent (1989). The model is closed by assuming that

$$(14)\quad \beta_t = \left[\sum_{s=1}^{t-1} P_{s-1}^2\right]^{-1}\left[\sum_{s=1}^{t-1} P_{s-1}P_s\right],$$

so that agents form expectations by calculating a first-order autoregression on past prices. The least squares formula can be written recursively as

$$(15)\quad \beta_t = \beta_{t-1} + C_{t-1}^{-1}P_{t-2}\left[P_{t-1} - P_{t-2}\beta_{t-1}\right],$$

where

$$C_{t-1} = \sum_{s=1}^{t-1} P_{s-1}^2 .$$

If the recursive rewrite is combined with the actual law of motion for prices in this system, we obtain

$$(16)\quad \beta_t = \beta_{t-1} + \gamma_t\left[\frac{\theta S(\beta_{t-2})}{S(\beta_{t-1})} - \beta_{t-1}\right],$$

where

$$\gamma_t = P_{t-2}^2\left[\sum_{s=1}^{t-1} P_{s-1}^2\right]^{-1} .$$

Thus, this system differs from the adaptive expectations scheme in that the gain parameter varies between zero and one. This situation creates a dynamic system in three variables, β_t, β_{t-1}, and γ_t and if the linearized system is evaluated at the steady state, there are three associated eigenvalues.

One of the eigenvalues is θ^{-2}, and given our assumption that $\theta > 1$, this root is stable. Whether the remaining eigenvalues are inside the unit circle depends on whether $J < 1$, where J is now given by

$$(17)\quad J = \frac{(1-\theta^2)S'(\beta)}{\theta S(\beta)}.$$

If we employ the Cagan schedule and evaluate the condition at the monetary steady state, $J < 1$ implies that

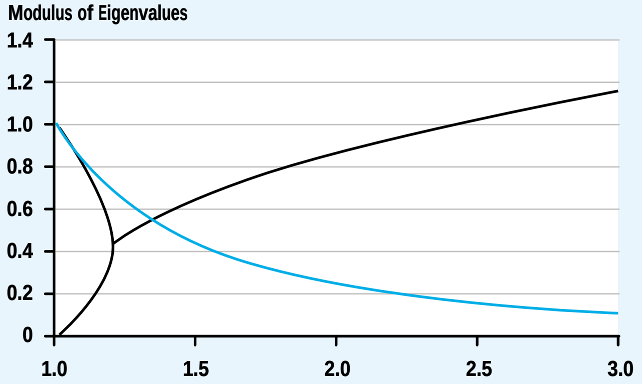
$$(18)\quad \alpha < \frac{\theta}{\theta^2 - 1} .$$

From this, we come to still a different conclusion regarding the prospects for the stability of the monetary steady state. First, the right-hand side of this inequality goes to zero as $\theta \longrightarrow \infty$. Thus we again conclude that high rates of money growth will be destabilizing. But for low rates of inflation, that is, for $\theta \longrightarrow 1$ from the positive side, the right-hand side of the inequality tends to $+\infty$. The inequality is very likely to be satisfied for low rates of money creation and hence low rates of inflation. The argument for stability through inflation was that α might become very large for low rates of inflation, and we conclude that in the least squares learning case the semilog slope would have to tend to infinity *very rapidly* at low rates of inflation if the condition for stability were to be violated. Thus, the condition in this case is far sturdier.

The increase in sturdiness can be illustrated if we return to Lucas's (1994) money demand schedule, $S(\beta_t) = (\beta_t - 1)^{-\eta}$, and compute the value of J at the monetary steady state:

Figure 3

Stability As a Function of θ, Learning Case

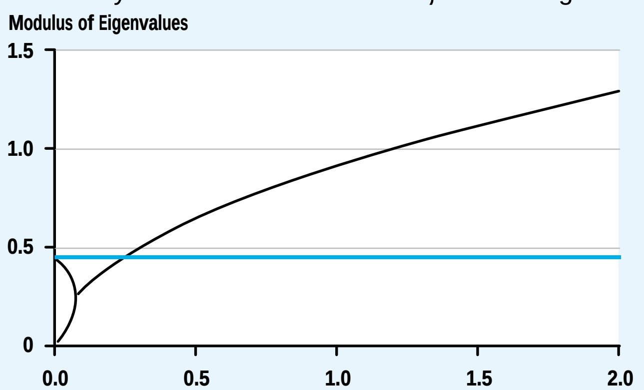


Cagan schedule, α = 0.5

Note: Stability as a function of θ under least squares learning. For low values of θ, the eigenvalues are real, distinct, and stable. As θ increases, two eigenvalues coalesce into a complex conjugate and eventually cross the unit circle.

Figure 4

Stability As a Function of α, Learning Case



Cagan schedule, θ= 1.5

Note: Stability as a function of α under least squares learning. When α is small, the eigenvalues are real, distinct, and stable. As α increases, two of the roots coalesce into a complex conjugate and eventually cross the unit circle.

$$(19)\quad J = \frac{-\eta(1-\theta^2)}{\theta(\theta-1)}.$$

Here, $\lim_{\theta \to 1} J = 2\eta$, so that stability is retained so long as $\eta < 0.5$. Lucas's (1994) preferred value for η was in fact 0.5. More broadly, a situation that was always unstable in the adaptive expectations case, regardless of parameters, is here stable for some fairly reasonable parameter configurations.

Figures 3 and 4 plot qualitatively the modulus of the eigenvalues for this system, computed at the monetary steady state, for the Cagan money-demand schedule. In Figure 3, θ is on the horizontal axis, while in Figure 4, α is on the horizontal axis. These diagrams are unchanged in qualitative terms for different fixed levels of α (in Figure 3) and θ (in Figure 4). When the quantity $\alpha\theta^{-1}(\theta^2-1)$ is small, as it will be for low α and low rates of money creation, the system is stable, and all three roots are real and inside the unit circle. As we move to the right in the diagrams, two roots combine into a complex conjugate and cross the unit circle. When the condition $J < 1$ is satisfied and the system is started in the vicinity of the monetary steady state, the dynamics converge. If the condition is violated, either the system is locally nonconvergent or the dynamics settle into a cycle. Bullard (1994) analyzes this bifurcation in some detail and provides examples of the possible outcomes.

The nonmonetary steady state will not be locally stable under least squares learning. If we think in terms of a linear demand schedule, for instance, the condition in Equation 19 will never be met, because $S'(\bar{\beta})$ is a constant and $S(\bar{\beta}) = 0$, causing $J \longrightarrow \infty$ at this stationary equilibrium. The Cagan schedule cannot be evaluated at this steady state because $\bar{\beta} \longrightarrow \infty$ in that case.[6]

We now turn to stability-through-inflation arguments in the fixed deficit model.

THE FIXED DEFICIT MODEL

Adaptive Expectations

The fixed deficit model with adaptive expectations is given by

$$(20)\quad \beta_t = \beta_{t-1} + \gamma\left[\frac{S(\beta_{t-2})}{S(\beta_{t-1})-\xi} - \beta_{t-1}\right].$$

We look for steady states of this equation, which occur at

$$(21)\quad \frac{S(\beta)}{S(\beta)-\xi} - \beta = 0.$$

[6] We comment briefly on the case where $\theta \leq 1$. Here the monetary steady state involves either constant prices or a declining price sequence. The roots of the system are no longer given according to the previous paragraph. Instead the gain, γ_t, tends to zero at a

or

$$(22)\ \xi = \frac{(\beta - 1)S(\beta)}{\beta}.$$

If $\xi = 0$, the steady states occur at $\beta = 1$ and $\beta = \bar{\beta}$, which are the monetary and nonmonetary steady states of the constant-money-growth model in the special case where the currency stock is fixed ($\theta = 1$). This is a way of saying that the constant-money-growth model with no growth in the money supply is the same as the fixed deficit model with a zero deficit, because a zero deficit implies that the money stock is constant.

To find the steady states in the case where $\xi > 0$, we derive a Laffer curve as follows. First, find the derivative of ξ with respect to a steady state rate of expected gross inflation β as

$$(23)\ \frac{d\xi}{d\beta} = S'(\beta) - \frac{S(\beta)}{\beta} + \frac{S(\beta)}{\beta^2}.$$

This derivative is positive as $\beta \longrightarrow 1$ from the positive side, and negative as $\beta \longrightarrow \bar{\beta}$ from the left. The maximum value of the deficit can be obtained when this derivative is zero, which is when

$$(24)\ \beta(1 - \beta) = \frac{S(\beta)}{S'(\beta)}.$$

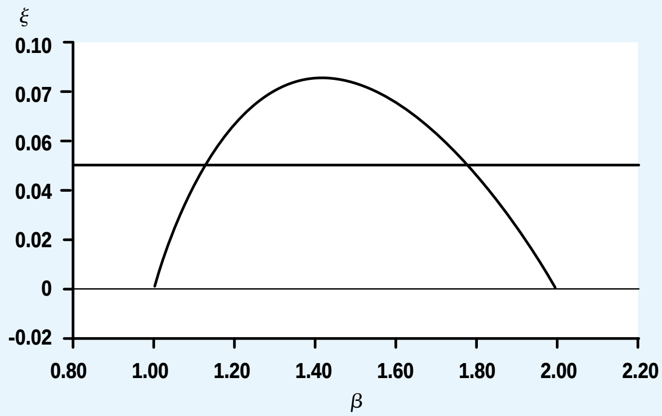
Because the right-hand side is negative, ξ_{max} must occur at a value of $\beta > 1$. Since $\beta > \bar{\beta}$ implies that the right-hand side is positive, ξ_{max} must occur at a value $\beta < \bar{\beta}$. From Figure 5 we deduce that there are always two steady states so long as $0 < \xi < \xi_{max}$. We call these the low-inflation steady state, β_l, and the high-inflation steady state, β_h. We note that $1 < \beta_l < \beta_h < \bar{\beta}$ and that money is held at both steady states.

We can again define $z_t = [\beta_t, \beta_{t-1}]'$ and $G(\cdot)$ based on the right hand side of Equation 22, so that $z_t = G(z_{t-1})$, and we let the two steady states be represented generically by $z^* = [\beta, \beta]'$. The relevant Jacobian matrix is then given by

$$(25)\ DG(z^*) = \begin{bmatrix} 1 - \gamma + \beta J & -J \\ 1 & 0 \end{bmatrix},$$

Figure 5

Steady States Based on a Laffer Curve



Linear demand with, $\lambda = 0.5$

Note: Steady states based on a Laffer curve. The straight line in the figure represents government revenue requirements from money creation. It intersects the Laffer curve twice, once at a relatively low rate of inflation and once at a relatively high rate of inflation, so long as $\xi < \xi_{max}$.

where

$$J = \frac{-\gamma\beta S'(\beta)}{S(\beta)}.$$

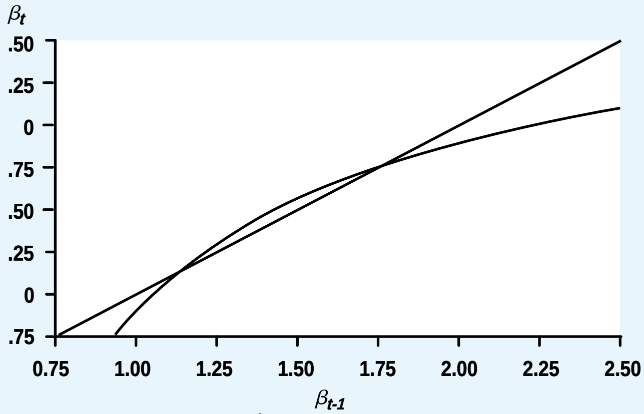
Stability in the vicinity of a steady state is governed by two conditions. The first is that $J < 1$, and the second is that the gain γ is not too large. In fact, as $\beta \longrightarrow 1$, a depiction of the local stability conditions is exactly as shown in Figure 1. For systems with more inflation, the upward-sloping line in that figure becomes flatter while still going through the point (2, 0). Again, instability arising solely from too large a gain only occurs for high values of γ, well in excess of unity. This makes intuitive sense in that it means that a small expectational error in the previous period gets translated into a large change in expectations in the current period, to such an extent that the system is destabilized. If we make the restriction $\gamma < 1$, then there is a single condition for local stability, namely $J < 1$.[7]

In considering this condition, we note that β enters as a separate term in the numerator and thus that stability will depend in part on the level of steady-state inflation. In the case of a Cagan schedule, we conclude that the condition for the stability of the low-inflation steady state

rapid rate. This can inhibit learning since learning ceases with this algorithm when $\gamma_t = 0$; that is, the system can get stuck at some point because the gain is zero and no further adjustment can be made. The stability of the system is not in question, however, and the problem of the gain tending to zero can be remedied by adding a stochastic element to the money-creation process.

Figure 6

Steady States Under Perfect Foresight



Linear demand with, $\lambda = 0.5$, $\xi = 1.1$

Note: Steady states under perfect foresight. The graph of the function describing the system intersects the 45 degree line, the straight line in the figure, at two points. The intersection involving lower inflation is unstable in the perfect foresight dynamics.

would be $\alpha\gamma\beta_l < 1$, and for the high-inflation steady state would $\alpha\gamma\beta_h < 1$. This means that, if there is any steady-state inflation rate on the low-inflation side of the Laffer curve for which the condition $J < 1$ fails, then the condition fails at all points on the high-inflation side of the Laffer curve, since the steady-state inflation rates there are higher still. Thus stability on the high-inflation side of the Laffer curve is unlikely.

The arguments for stability through inflation in the deficit model mirror those for the constant-money-growth model, since stability conditions are analogous.

Rational Expectations

Sargent and Wallace (1981, 1987) have studied this model under rational expectations. We summarize their findings here. We assume perfect foresight, which implies that equilibrium is described by the difference equation

$$(26)\ \beta_t = S^{-1}\left[\frac{S(\beta_{t-1})}{\beta_{t-1}} + \xi\right].$$

The qualitative graph is given in Figure 6.

The low-inflation steady state is unstable under the rational expectations dynamics, and the high-inflation steady state is locally stable. Again, there is little to discuss in the case of rational expectations. Marcet and Sargent (1989) note that the equilibrium at β_h involves perverse comparative statics, an increase in the deficit leading to a decrease in the steady state rate of inflation. These comparative statics are the result of the fact that the equilibrium is on the high inflation side of the Laffer curve.

Least Squares Learning

The least squares learning version has been analyzed by Marcet and Sargent (1989) in the special case that the demand function is linear. We summarize their findings here. The dynamic system is given by

$$(27)\ \beta_t = \beta_{t-1} + \gamma_t\left[\frac{S(\beta_{t-2})}{S(\beta_{t-1}) - \xi} - \beta_{t-1}\right],$$

where

$$\gamma_t = P_{t-2}^2\left[\sum_{s=1}^{t-1} P_{s-1}^2\right]^{-1}.$$

This is a three-dimensional system, and here the gain γ_t is always between zero and one. One characteristic root of the linearized system is always in the stable region. The remaining roots are the eigenvalues of the characteristic equation associated with

$$(28)\ DG(z^*) = \begin{bmatrix} \beta^{-2} + \beta J & -J \\ 1 & 0 \end{bmatrix},$$

where

$$J = \frac{-(1-\beta^{-2})\beta S'(\beta)}{S(\beta)}.$$

Marcet and Sargent (1989) report that, for the linear demand schedule, the low-inflation steady state is locally stable if and only if $J < 1$, and that the high-inflation steady state is unstable in the least squares learning dynamics. The condition that $J < 1$ is exactly the same as in the fixed money growth case except that θ is replaced by β_l. The arguments concerning stability

[7] Bruno and Fischer (1990) studied a related model under adaptive expectations and a Cagan demand schedule, but in continuous time. There the stability condition is $\alpha\gamma < 1$.

through inflation are therefore essentially the same here as in the constant-money-growth model.

CONCLUSIONS

We have analyzed the status of the stability-through-inflation argument due to Vickrey (1954) and Phelps (1972), and we have done so largely within the framework of original argument. This framework is simple and abstract. Certainly much more could be done in the context of less abstract models; we leave that task as a challenge for future research.

Other attempts have been made to get to the bottom of the stability question in versions of the Cagan model. Marimon and Sunder (1993) conducted experiments with human subjects in the fixed-deficit model with a linear demand schedule and found that the resulting dynamics nearly always converge near the low-inflation steady state. Marimon and Sunder (1993) concluded that least squares learning provides a better approximation to the behavior of their subjects than rational expectations.

Arifovic (1995) studied some of the systems outlined here in the case of genetic algorithm learning. The genetic algorithm envisions large numbers of agents trying out alternative decision rules simultaneously. Successful rules are copied more often than unsuccessful ones, and Arifovic studies the convergence properties of these systems. Her main finding is that the systems with genetic algorithm learning tend to converge to the low-inflation steady state, and that these systems sometimes converge even when the same system under least squares learning does not.

It appears that not much remains of the original argument that stability can be maintained through inflation. Under adaptive expectations, it is possible that the local stability condition is violated because of an increase in the elasticity of money demand at low or negative inflation rates. This happens, for instance, in the case of the Lucas (1994) money demand schedule. But under least squares learning, any effect of low inflation on the slope of the demand schedule can be offset by a countervailing movement in the gain. Under rational expectations, the argument for stability through inflation is turned on its head. Stability in this case is achieved only at high rates of inflation, at a socially undesirable point on the high-inflation side of the Laffer curve.

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