Strengthening the Case for the Yield Curve as a Predictor of U.S. Recessions

Michael J. Dueker

What is the message from stock and bond markets about the likelihood of a recession within the next year? Monetary policymakers must examine such questions as part of a look-at-everything approach to decision making. Policymakers need quantitative analysis of the relevance of the answers, because many touted indicators actually predict very little about the future course of the economy. A quantitative appraisal would establish whether a recession indicator provides significant signals prior to recessions and whether it gives false signals outside of recessionary periods.

Proponents of financial indicators of the economy maintain that financial markets are good aggregators of information because of broad participation in the markets. They suggest, further, that people have strong incentives to price securities as closely as possible to fundamental values. Recent research supports this argument by showing that simple financial indicators, such as interest rates and stock prices, often do better than composite indices of leading indicators in predicting economic recessions, especially at horizons beyond one quarter (Estrella and Mishkin, 1995). The predictive power of the slope of the bond yield curve, in particular, has received notice in numerous studies. This article begins with a discussion of why the yield curve slope ought to contain information about the future prospects of the economy. Estrella and Mishkin (1995) found the yield curve slope to be a useful recession predictor; in this article, I examine results from two econometric models that further test the predictive power of the yield-curve slope relative to other recession predictors such as stock prices and the Commerce Department’s index of leading indicators.

WHY DOES THE YIELD CURVE TILT BEFORE RECESSIONS?

The yield curve shows prevailing market interest rates at various maturities. The expectations hypothesis of the term structure of interest rates claims that, for any choice of holding period, the expected return is the same for any combination of bonds of different maturities one might hold in that period. In other words, the expectations hypothesis maintains that investors do not expect different returns from holding a 1-year note versus holding two successive 6-month securities. For the same 1-year holding period, they would also expect to realize identical returns from a 1-year note and from a 30-year bond bought at the same time and sold at market price one year later. Obviously, if people had perfect foresight about future short-term interest rates, holding-period returns would necessarily be equalized through arbitrage. However, uncertainty regarding future short-term interest rates causes the interest-rate term structure to deviate from the shape implied by the risk-neutral expectations hypothesis. In particular, the yield curve normally is upward-sloping, even when investors expect relatively constant short-term rates, because holders of long-term securities bear the risk that future interest rates will be higher than expected, so they require a positive risk premium in long-term bond yields. Fluctuations in interest-rate risk premiums are thought to

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In this article, the slope of the yield curve is always taken over the entire length, between three months and the benchmark 30-year rates. Some studies, especially those that need internationally comparable data, take the slope between three months and ten years.

If investors begin to suspect that a recession is near, the response of the yield curve will depend on their assessment of the magnitude and duration of the recession’s effect on short-term interest rates. Rudebusch (1995) and Haubrich and Dombrosky (1996) observe that the public anticipates that short-term interest rates will decline gradually in a recession until the economy’s performance improves. These reductions in short-term interest rates may stem from countercyclical monetary policy designed to stimulate the economy, or they may simply reflect low real rates of return during the recession. In either case, the anticipated severity and duration of the recession will strongly influence the expected path of short-term interest rates, which will show up in the shape of the yield curve.

Figure 1 shows that a 2-year moving average of the change in the 3-month Treasury bill rate generally conforms with the notion that short-term rates fall in recessions. According to the expectations hypothesis, when investors expect a recession, they believe that long-term interest rates should fall immediately in order to equalize future expected holding-period returns. Current rates on securities with different maturities will decline by different amounts, depending on how much of their repayment (in present value terms) will take place during the period of low short-term interest rates. Today’s 1-year rate might not change at all if the period of low short-term rates is not expected to start for a year, whereas today’s 7-year rate might be the most affected if investors expect short-term rates to go down in a year and stay down for three years. Under this theory, if long-term rates, such as the 7-year rate, did not fall today, then the expected holding-period return over the next four years would be higher for those who held long-term bonds than for those who held a succession of short-term bonds throughout the period. Therefore, we expect the yield curve to tilt down today if recession looms.

Historical experience shows that on several occasions prior to recessions, long-term interest rates dipped below prevailing short-term rates, a phenomenon known as an inverted yield curve. Figure 2 illustrates episodes when the gap between the 30-year bond rate and the 3-month T-bill rate became negative—that is, the yield curve became inverted. Since 1960 the yield curve has become inverted prior to all five recessions. The extent to which the yield curve is tilted away from its normal “slope” is identified by many researchers as a valuable indicator of recessions. Of course, the yield curve does not have to become inverted to signal that recession is imminent; it may simply flatten relative to normal.

QUANTIFYING THE YIELD CURVE SLOPE AS A PREDICTOR OF RECESSIONS

Estrella and Mishkin (1995) analyze candidate recession predictors and conclude that the yield-curve slope is the single most powerful predictor of recessions. They use a probit model to predict a recession dummy variable, \( R_t \), where

\[
R_t = \begin{cases} 
1 & \text{if the economy is in recession in period } t \\
0 & \text{otherwise.}
\end{cases}
\]

The decision to forecast a recession dummy, rather than output growth, has a purpose. A goodness-of-fit measure for a model of output growth would mix information on the predictability of the strengths of recoveries and expansions with information on the timing of recessions. The recession dummy variable, in contrast, isolates the accuracy with which one can predict the date of the onset and the expected length of recessions. With a recession dummy dependent variable, Estrella and Mishkin (1995) use the probit, a standard econometric model in
which the probability of recession at time \( t \), with a forecast horizon of \( k \) periods, is described by the following equation:

\[
\text{Prob}\left( R_t = 1 \right) = \Phi \left( c_0 + c_1 X_{t-k} \right),
\]

where \( \Phi(\cdot) \) is the cumulative standard normal density function, and \( X \) is the set of explanatory variables used to forecast recessions.

The log-likelihood function for a probit model, following from Equation 1, is

\[
L = \sum R_t \ln \text{Prob}\left( R_t = 1 \mid X_{t-k} \right) + (1 - R_t) \ln \text{Prob}\left( R_t = 0 \mid X_{t-k} \right).
\]

As a measure of fit for probit models, Estrella (1995) proposed a pseudo-\( R^2 \) in which the log-likelihood of a model, \( L_u \), is compared with the log-likelihood of a nested model, \( L_c \), that by construction must have a lower likelihood value:

\[
\text{pseudo-}R^2 = 1 - \left( \frac{L_c}{L_u} \right)^{-\frac{2}{n}}.
\]

While many functions of the log-likelihoods are monotonic between zero and one, and could thus serve as measures of fit, Estrella argues that Equation 3 corresponds well with the standard \( R^2 \) function from linear regressions.

Within this framework, Estrella and Mishkin (1995) obtain a quantitative measure of fit with the pseudo-\( R^2 \) and demonstrate that, among the variables they study, the yield-curve slope is the single-most powerful predictor, \( X \), of recessions for forecast horizons, \( k \), beyond one quarter.\(^3\) In this article, we examine probit results (Equation 1) for five explanatory variables: the change in the Commerce Department's index of leading indicators (lead); real M2 growth (money); the percentage spread between the 6-month commercial paper and 6-month Treasury bill rates (spread); the percentage change in the Standard and Poor's 500 index of stock prices (stock); and the percentage difference between the yields on 30-year Treasury bonds and 3-month T-bills (yield curve). Friedman and Kuttner (1993) tout the quality spread between commercial paper and the Treasury bill rates as a predictor of recessions. I use monthly data from January 1959 to May 1995 (437 observations). The recession binary variable follows from business-cycle turning points determined by the National Bureau of Economic Research (NBER). The exact construction of the explanatory variables is given in the appendix.

\(3\) The explanatory variables studied by Estrella and Mishkin (1995) are: the spread between the 10-year bond rate and the 3-month TBill rate (the slope of the yield curve); the spread between the 6-month commercial paper rate and the corresponding TBill rate; stock price indices; monetary aggregates; indexes of leading indicators; and macroeconomic indicators, such as the purchasing managers' survey, housing starts and the trade-weighted exchange rate.
Table 1 contains the pseudo-$R^2$ for these five variables at 3-month, 6-month, 9-month and 12-month forecasting horizons. The log-likelihood values are in parentheses. The baseline log-likelihood value, $L_c$, is from a model that contains only a constant, which implies a constant probability of recession each month.

Table 1 compares, at the six-month forecast horizon, the recession probabilities from the model with the yield-curve slope and the constant-probability model that has likelihood $L_c = -197.2$. The pseudo-$R^2$ is based on the difference in fit between these two models. The chart shows predictions of the recessionary state for that date, based on information that was available six months earlier. It is therefore interesting to see whether the model accurately predicted recessions six months ahead of time. Consistent with the conventional wisdom that most recessions arrive earlier or later than the date predicted by most professional forecasters, Figure 3 shows that only in the 1980 and 1981 recessions was the forecast probability of recession clearly above 50 percent at the onset. Moreover, in the relatively severe 1974 recession, the six-month forecast probability of being in a recession never reached 75 percent, demonstrating that the recession was always viewed as transitory, when actually it proved to be protracted. Similarly, in the midst of the 1981 recession, the forecast probability of recession fell below 50 percent by 1982 and kept falling, even as the recession continued through November. Hence, although the yield curve slope model surpasses all the others, it still does not absolutely predict either the onset or duration of recessions.

**THE PREDICTIVE POWER OF A DYNAMIC MODEL**

One drawback of the probit model of Equation 1 that is typical of many limited-dependent-variable econometric models is the lack of a dynamic structure for applications to time series data such as the recession variable, $R_t$. The recession predictor, $X_t$, is a time series variable with its own autocorrelation structure, but the model uses no information contained in the autocorrelation structure of the dependent variable to form predictions. Univariate time-series modeling of macroeconomic data has clearly demonstrated the relevance of a variable’s own history in generating forecasts.

To demonstrate the statistical importance of the dependent variable’s
own autocorrelations, consider the assumptions behind a probit model. Observable variables, \( X \), and an unobserved, mean-zero random shock, \( u \), determine the value of the binary dependent variable by way of an unobservable latent variable, \( y^* \), that is assumed to be a linear function of the explanatory variables and a mean-zero, normally-distributed shock, \( u \), such that

\[
y^*_t = -c_0 - c_1 X_{t-k} + u_t.
\]

By convention, the binary recession variable is equal to 1 if \( y^* \leq 0 \) and zero if \( y^* > 0 \), which implies that

\[
R_t = 1, \text{ if } u_t \leq c_0 + c_1 X_{t-k}, \text{ and } R_t = 0, \text{ if } u_t > c_0 + c_1 X_{t-k}.
\]

In the probit model, the random shocks, \( u \), are assumed to be independent, identically distributed, normal random variables with a mean of zero. For many time-series applications, however, it is implausible to assume that the conditional mean of \( u \), if we condition only on \( X_{t-k} \), without reference to whether the economy has actually been in recession in recent periods. The obstacle to applying traditional time-series techniques to address possible serial correlation in \( u \), is that \( y^* \), and therefore \( u \), are unobserved variables, so we cannot apply an autoregressive moving-average (ARMA) filter, for example, to \( u \). The solution proposed here is to remove serial correlation in \( u \) by conditioning explicitly on the recent history of \( R \), as well as \( X \).

Adding a lag of \( R \) to the specification enhances the plausibility of the assumption that \( u \) has a mean of zero, conditional on information available through time \( t-k \):

\[
\text{Prob}(R_t = 1) = \Phi(c_0 + c_1 X_{t-k} + c_2 R_{t-k}).
\]

The specification of Equation 6 is the probit analogue of adding a lagged dependent variable to a linear regression model. I ran Equation 6 for each candidate recession predictor. The unrestricted model gives \( L_u \). The restricted model, with \( c_1 = 0 \), gives \( L_c \). Note that in Equation 1 and in Estrella and Mishkin (1995), \( L_u \) comes from a model with \( c_2 = 0 \), and \( L_c \) is from the model with \( c_1 = c_2 = 0 \). Hence, Equation 6 imposes a higher standard at which a zero pseudo-R\(^2\) begins, because the recession predictor must now add to the fit provided by the lagged dependent variable. I repeat the exercise for different forecasting horizons, \( k = 3, 6, 9, \) and 12 months. Three months is probably a minimum recognition lag time for recessions. It would clearly not be reasonable to include last month’s value of the recession binary variable as an explanatory variable, because it takes more time to recognize that the economy has entered a recession. At three months, the NBER may not have officially designated and announced a business cycle turning point, but people have acquired enough other information to infer with a reasonable degree of accuracy whether the economy is in a recession or not. If three months seems less than the minimum recognition lag time for recessions, then one can concentrate on the results for 6, 9, and 12 months.

One difference between Table 1 and Table 2 is that \( L_c \) varies with the forecast horizon in Table 2. At the 3-month and 6-month horizons, \( L_c \) is much higher than the -197.2 obtained in the constant-probability model in Table 1. The forecasting
power of the lagged dependent variable worsens at longer horizons, however, so the log-likelihood at which the recession predictor begins to have a non-zero pseudo-$R^2$ decreases. For this reason, the pseudo-$R^2$ of the slope of the yield curve actually increases up to the nine-month horizon, even though the log-likelihood decreases. As the input, $X_{t-k}$, to Equation 6, the slope of the yield curve dominates the other recession predictors at all forecast horizons, not merely the longer ones. In particular, the index of leading indicators does not have a high $R^2$ at short horizons in the presence of the lagged dependent variable in Table 2, whereas the $R^2$ was high at short horizons in Table 1. This difference suggests that the explanatory power of the index of leading indicators largely overlaps with that of the lagged dependent variable. The explanatory power of the slope of the yield curve, in contrast, appears to complement the lagged dependent variable. Thus, rather than weakening the importance of the yield-curve slope as a recession predictor, the addition of the lagged dependent variable has buttressed the dominance of the yield curve as a recession predictor.

Figure 4 contains two sets of recession probabilities—one from the baseline model that uses the lagged dependent variable, and the other from the model that uses both the lagged dependent variable and the slope of the yield curve. Here the pseudo-$R^2$ exceeds zero only to the extent that a model's explanatory variable adds to the fit provided by the lagged dependent variable. Here the pseudo-$R^2$ exceeds zero only to the extent that a model's explanatory variable adds to the fit provided by the lagged dependent variable. In contrast to Figure 3, Figure 4 shows that the model predicts the severity and duration of the 1974 recession after lapsing briefly toward the beginning of the recession. The model with the lagged dependent variable better captures the duration of recessions but still fails to foresee, with six months' notice and at least 50 percent probability, the onset of recessions in 1960, 1970, 1980 and 1990. On the other hand, the lagged dependent variable helps the model foresee the importance of the recessions in 1970 and 1990 better in Figure 4 than in Figure 3.

### Table 2

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k=3$</td>
</tr>
<tr>
<td>Yield curve</td>
<td>.163</td>
</tr>
<tr>
<td>Lead</td>
<td>.069</td>
</tr>
<tr>
<td>Money</td>
<td>.058</td>
</tr>
<tr>
<td>Stock</td>
<td>.046</td>
</tr>
<tr>
<td>Spread</td>
<td>.020</td>
</tr>
<tr>
<td>Log-like. $L_c$</td>
<td>(-112.3)</td>
</tr>
</tbody>
</table>

Log-likelihood values are in parentheses.

### A Richer Time-Series Approach

The probit model with Markov switching further challenges the slope of the yield curve to make a forecasting contribution, because $L_c$ is higher than in Table 2, making it more difficult to achieve...
COEFFICIENT VARIATION VIA MARKOV SWITCHING IN THE PROBIT MODEL

With time-series data, one might also question the statistical assumption of independent, identically-distributed random shocks, in addition to the assumption of zero conditional means. One way to make the independence assumption more plausible is to allow for some time variation—that is, time-varying coefficients—in the structure of the model. For example, if the relationship between recessions and the slope of the yield curve were to undergo random fluctuations, these could cause a failure of the independence assumption, if left unmodeled.

One simple yet tractable model allows the coefficients to change values according to an unobserved binary state variable, $S_t$, which follows a Markov process:

$$ S_t \in \{0,1\}; $$
$$ c_{it} = c_i(S_t); $$
$$ \text{Prob}(S_t = 1 | S_{t-1} = 1) = q; \text{ and} $$
$$ \text{Prob}(S_t = 0 | S_{t-1} = 0) = p. $$

In this way, the coefficients take on either of two values, depending on the value of the state variable, changing the magnitude of the shock needed to induce a recession:

$$ R_t = 1, \text{ if } u_t \leq c_0 (S_t) + c_1 (S_t) X_{t-k} + c_2 (S_t) R_{t-k}, \text{ and} $$
$$ R_t = 0, \text{ if } u_t > c_0 (S_t) + c_1 (S_t) X_{t-k} + c_2 (S_t) R_{t-k}. $$

The transition probabilities, $p$ and $q$, indicate the persistence of the states and determine the unconditional probability of the state $S_t = 0$ to be $(1 - q)/(2 - p - q)$. In the estimation, Bayes' Rule is used to obtain filtered probabilities of the states in order to integrate out the unobserved states and evaluate the likelihood function, as in Hamilton (1990):

$$ \text{Prob}(S_t = 0 | R_t = 0) $$

$$ = \frac{1}{\sum_{s=0}^{1} \text{Prob}(S_t = s | X_{t-1}, R_{t-k}) \text{Prob}(R_t = 0 | S_t = s)}; $$

$$ \left( \text{Prob}(S_t = 0 | X_{t-k} R_{t-k}) \text{Prob}(S_t = 1 | X_{t-k} S_{t-k}) \right)^\prime $$

$$ = G^k \left( \text{Prob}(S_{t-k} = 0 | X_{t-k} R_{t-k}) \text{Prob}(S_{t-k} = 1 | X_{t-k} R_{t-k}) \right)^\prime, $$

where $G$ is the transition matrix of the Markov state variable.

The function maximized is then

$$ \sum_{i=1}^{T} R_i \ln \left( \sum_{s=0}^{1} \text{Prob}(S_i = s | X_{i-k}, R_{i-k}) \text{Prob}(R_i = 1 | S_i = s) \right) $$

$$ + (1 - R_i) \ln \left( \sum_{s=0}^{1} \text{Prob}(S_i = s | X_{i-k}, R_{i-k}) \text{Prob}(R_i = 0 | S_i = s) \right). $$
Now the predictor variable has to show explanatory power above and beyond the predictions of not only the lagged dependent variable, but also the time-series Markov process governing the state variable. The Markov process helps forecast, because a shifting \( c_s(S_s) \), for example, would help raise the likelihood value relative to a stationary \( c_s \). Results from estimating the Markov-switching probit models in Table 3 show that the slope of the yield curve still dominates the other candidate predictor variables and attains significant pseudo-\( R^2 \) values.

The difference in log-likelihood values between entries in Table 3 and Table 2 shows the gain from allowing Markov switching in the coefficients, at the cost of adding five parameters to the models. Likelihood ratio statistics do not obey their usual chi-square limiting distributions, however, because the transition probabilities are not identified under the null of no Markov switching. Therefore, we do not present a formal test of the significance of Markov switching, but it is reasonable to believe that large likelihood ratio test statistics, such as 31.2 for the yield curve at the 12-month horizon, are significant. Generally, the Markov switching appears to be most relevant at longer horizons. At horizons less than one year, the addition of Markov-switching coefficient variation is only of marginal benefit in the log-likelihoods. One possible explanation—not easily proven—for less temporal stability in the coefficients at a one-year horizon is that the degree of 12-month serial correlation in the business cycle varies over time more than the degree of 3-month serial correlation.

Figure 5 plots the six-month-ahead recession probabilities for the models with Markov switching, both with and without the slope of the yield curve. Relative to Figures 3 and 4, the model with Markov switching does a better job at predicting the length of recessions, once it recognizes that a recession is under way. As for forecasting the onset of recessions, the model with Markov switching does reasonably well for the “major” recessions: 1974,

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Forecast Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( k=3 )</td>
</tr>
<tr>
<td>Yield curve</td>
<td>.159</td>
</tr>
<tr>
<td>Lead</td>
<td>.086</td>
</tr>
<tr>
<td>Money</td>
<td>.067</td>
</tr>
<tr>
<td>Stock</td>
<td>.043</td>
</tr>
<tr>
<td>Spread</td>
<td>.063</td>
</tr>
<tr>
<td>Log-like.</td>
<td>Lc</td>
</tr>
</tbody>
</table>

Log-likelihood values are in parentheses.

### Table 4

#### Coefficient Estimates for the Markov-Switching Model of the Yield-Curve Slope
(six-month forecast horizon)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_s(S_s = 0) ) Intercept</td>
<td>-1.18 (.275)</td>
</tr>
<tr>
<td>( c_s(S_s = 1) ) Intercept</td>
<td>-0.769 (.125)</td>
</tr>
<tr>
<td>( c_1(S_s = 0) ) Yield curve</td>
<td>-2.58 (.549)</td>
</tr>
<tr>
<td>( c_1(S_s = 1) ) Yield curve</td>
<td>-.597 (.086)</td>
</tr>
<tr>
<td>( c_2(S_s = 0) ) Lagged dep.</td>
<td>2.04 (.577)</td>
</tr>
<tr>
<td>( c_2(S_s = 1) ) Lagged dep.</td>
<td>1.30 (.241)</td>
</tr>
<tr>
<td>( p = \text{Prob}(S_s = 0 \mid S_{s-1} = 0) )</td>
<td>.9898 (.0079)</td>
</tr>
<tr>
<td>( q = \text{Prob}(S_s = 1 \mid S_{s-1} = 1) )</td>
<td>.9997 (.0037)</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses.
1980, and 1981, but it does not predict the onsets of the milder recessions in 1960, 1970, and 1990. The fit does not seem “worse” after 1985, as Haubrich and Dombrosky (1996) suggest; instead, the Markov-switching model suggests that milder recessions, such as 1990, are less accurately forecast throughout the sample.

Table 4 contains the estimated coefficients and standard errors for the Markov switching model when the slope of the yield curve is the explanatory variable and the forecast horizon is six months. Equation 8 suggests that the coefficient on the yield-curve slope, $c_1$, should be negative, because steep slopes ought to indicate that recessions are less likely. Similarly, the expected sign for the coefficient on the lagged dependent variable is positive, because a recent recession ought to make a subsequent recession more likely. The estimated coefficients in Table 4 have the expected signs; they do not change signs across switches in the state variable, only magnitudes.

**CONCLUSION**

As in Estrella and Mishkin (1995), I use a recession dummy as the dependent variable to focus on the timing of recessions. The yield-curve slope remains the single best recession predictor in the examined set of variables, even under two extensions of the basic time-series probit model. The robustness of this finding strengthens the claim that the yield curve should be considered a useful recession predictor. The yield curve has two factors in its favor as a recession indicator, other than the statistical backing outlined in this article. First, it is readily observable at high frequencies and gives a signal that is easy to interpret. Second, the expectations theory for the term structure of interest rates provides a theoretical foundation for the predictive power of the yield curve.

This paper also attempts to address statistical questions about the application of probit models to time-series data. The results for recession dummies indicate the importance of allowing for dynamic serial correlation in the model. They also suggest that allowance for general coefficient variation is not particularly significant at horizons less than one year.

**REFERENCES**


### Appendix

**Construction of the Explanatory Variables**

If we denote the constant-maturity, 30-year bond rate as TB30, the Commerce Department's index of leading indicators as LI, the 6-month commercial paper rate as CP, and the 6-month Treasury bill rate as bill, then the explanatory variables found in the tables are as follows:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield curve</td>
<td>( \ln\left(\frac{1 + \text{TB30}/100}{1 + \text{bill}/100}\right) )</td>
</tr>
<tr>
<td>Lead</td>
<td>( \text{LI}<em>t - \text{LI}</em>{t-1} )</td>
</tr>
<tr>
<td>Money</td>
<td>( \ln\left(\frac{\text{M1}}{\text{P}<em>t}\right) - \ln\left(\frac{\text{M1}}{\text{P}</em>{t-1}}\right) )</td>
</tr>
<tr>
<td>Stock</td>
<td>( \ln\left(\frac{\text{S&amp;P500}<em>t}{\text{S&amp;P500}</em>{t-1}}\right) )</td>
</tr>
<tr>
<td>Spread</td>
<td>( \ln\left(\frac{1 + \text{CP}/100}{1 + \text{bill}/100}\right) )</td>
</tr>
</tbody>
</table>