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Inflation, Financial Markets, and Capital Formation

Sangmok Choi, Bruce D. Smith, and John H. Boyd

A consensus among economists seems to be that high rates of inflation cause "problems," not just for some individuals, but for aggregate economic performance. There is much less agreement about what these problems are and how they arise. We propose to explain how inflation adversely affects an economy by arguing that high inflation rates tend to exacerbate a number of financial market frictions. In doing so, inflation interferes with the provision of investment capital, as well as its allocation.[1] Such interference is then detrimental to long-run capital formation and to real activity. Moreover, high enough rates of inflation are typically accompanied by highly variable inflation and by variability in rates of return to saving on all kinds of financial instruments. We argue that, by exacerbating various financial market frictions, high enough rates of inflation force investors' returns to display this kind of variability. It seems difficult then to prevent the resulting variability in returns from being transmitted into real activity.

Unfortunately, for our understanding of these phenomena, the effects of permanent increases in the inflation rate for long-run activity seem to be quite complicated and to depend strongly on the initial level of the inflation rate. For example, Bullard and Keating (forthcoming) find that a permanent, policy-induced increase in the rate of inflation *raises* the long-run level of real activity for economies whose initial rate of inflation is relatively low. For economies experiencing moderate initial rates of inflation, the same kind of change in inflation seems to have no significant effect on long-run real activity. However, for economies whose initial inflation rates are fairly high, further increases in inflation significantly *reduce* the long-run level of output. Any successful theory of how inflation affects real activity must account for these nonmonotonicities.

Along the same lines, Bruno and Easterly (1995) demonstrate that a number of economies have experienced sustained inflations of 20 percent to 30 percent without suffering any apparently major adverse consequences. However, once the rate of inflation exceeds some critical level (which Bruno and Easterly estimate to be about 40 percent), significant declines occur in the level of real activity. This seems consistent with the results of Bullard and Keating.

Evidence is also accumulating that inflation adversely affects the allocative function of capital markets, depressing the level of activity in those markets and reducing investors' rates of return. Again, however, these effects seem highly nonlinear. In a cross-sectional analysis, for example, Boyd, Levine, and Smith (1995) divide countries into quartiles according to their average rates of inflation. The lowest inflation quartile has the highest level of financial market activity, and the highest inflation quartile has the lowest level of financial market activity. However, the two middle quartiles display only very minor differences. Thus for the financial system, as for real activity, there seem to be threshold effects associated with the inflation rate.

Moreover, as we will show, high rates of inflation tend to depress the real returns equity-holders receive and to increase their variability. In Korea and Taiwan, there were fairly pronounced jumps in the

[1] This explanation has been articulated in a number of recent papers. See, for example, Azariadis and Smith (forthcoming), Boyd and Smith (forthcoming), and Schreft and Smith (forthcoming and 1994).

rate of inflation in 1988 and 1989, respectively. In each country, before those dates, inflation's effects on rates of return to equity, rate of return volatility, and transactions volume appear to be insignificant. After the dates in question, these effects are generally highly significant. Thus it seems possible that—to adversely affect the financial system—inflation must be "high enough."

Why does inflation affect financial markets and real activity this way? We produce a theoretical model in which—consistent with the evidence—higher inflation reduces the rate of return received by savers in all financial markets. By itself this effect might be enough to reduce savings and hence the availability of investment capital. However, we do not believe that this explanation by itself is very plausible, for two reasons. First, to explain the nonmonotonicities we have noted, the savings function would have to bend backward. Little or no empirical evidence exists to support this notion. Second, almost all empirical evidence suggests that savings is not sufficiently sensitive to rates of return to make this a plausible mechanism for inflation to have large effects. Thus an alternative mechanism is needed.

We present a model in which inflation reduces real returns to savings and, via this mechanism, exacerbates an informational friction afflicting the financial system. The particular friction modeled is an adverse selection problem in capital markets. However, the specific friction seems not to be central to the results we obtain.[2] What is central is that the severity of the financial market friction is endogenous and varies positively with the rate of inflation.

In this specific model, higher rates of inflation reduce savers' real rates of return and lower the real rates of interest that borrowers pay. By itself, this effect makes more people want to be borrowers and fewer people want to be savers. However, people who were not initially getting credit represent "lower quality borrowers" or, in other words, higher default risks. Investors will be uninterested in making more loans to lower quality borrowers at lower rates of interest and therefore must do something to keep them from seeking external finance. The specific response here is that markets ration credit, and more severe rationing accompanies higher inflation. This rationing then limits the availability of investment capital and reduces the long-run level of real activity. In addition, when credit rationing is sufficiently severe, it induces endogenously arising volatility in rates of return to savings. This volatility must be transmitted to real activity and, hence, to the rate of inflation. Variable inflation therefore necessarily accompanies high enough rates of inflation, as we observe in practice.

This story, of course, does not explain why these effects are strongest at high—and not at low—rates of inflation. The explanation for this lies in the fact that—at low rates of inflation—our analysis suggests that credit market frictions are potentially innocuous. Thus at low rates of inflation, credit rationing might not emerge at all, and none of the mechanisms mentioned in the previous paragraph would be operative. In this case our economy would act as if it had no financial market frictions. When this occurs, our model possesses a standard Mundell-Tobin effect that makes higher inflation lead to *higher* long-run levels of real activity.[3] However, once inflation exceeds a certain critical level, credit rationing must be observed, and higher rates of inflation can have the adverse consequences noted above.

Finally, our analysis suggests that a certain kind of "development trap" phenomenon is ubiquitous, particularly at relatively high rates of inflation.[4] We often observe that economies whose performance looks fairly similar at some point in time—like Argentina and Canada circa 1940—strongly diverge in terms of their subsequent development. Although this is clearly often because of differences in government policies, presumably many governments confront similar policy options. One would thus like to know whether intrinsically similar economies can experience divergent economic performance for purely endogenous reasons.

[2] The same phenomena we report here occur in the presence of a costly state verification problem (Boyd and Smith forthcoming), or in a model where spatial separation and limited communication affect the financial system (Schreft and Smith forthcoming and 1994).

[3] In particular, in the absence of financial market frictions, our model reduces to one in which higher rates of inflation (easier monetary policy) stimulates real output growth. This occurs in a variety of monetary growth models: see Mundell (1965); Tobin (1965); Diamond (1965); or especially Azariadis (1993) (for an exposition); Sidrauski (1967); and Shell, Sidrauski, and Stiglitz (1969).

[4] See Azariadis and Drazen (1990) for one of the original theoretical expositions of development traps.

The answer in models with financial market frictions is that this can occur fairly easily: When the severity of an economy's financial market frictions is endogenous, it is possible that—for endogenous reasons—the friction is perceived to be more or less severe. If it is perceived to be more (less) severe, financial markets provide less (more) investment capital. The result is a reduced (enhanced) level of real economic performance. This validates the original perception that the friction was (was not) severe. Thus, as we show, development trapsshould be expected to be quite common.

The remainder of the article proceeds as follows: In the first section we lay out a theoretical model that illustrates the arguments just given, while in the second section we describe an equilibrium of the model. In the third section we discuss how inflation affects the level of real activity when the financial market friction is not operative, while in the fourth section we take up the same issue when it is. In the fifth section we examine when the friction will or will not be operative and derive the theoretical implications we have already discussed. In the sixth section we show that an array of empirical evidence supports these implications. In the final section we offer our conclusions.

A SIMPLE ILLUSTRATIVE MODEL

The purpose of this section is to present a model that illustrates how inflation interacts with a particular financial market friction. This friction is purposely kept very simple in order to highlight the economic mechanisms at work. Later we will argue that these mechanisms are operative very generally in economies where financial markets are characterized by informational asymmetries.

The Environment

The economy is populated by an infinite sequence of two period lived, overlapping generations. Each generation is identical in its size and composition. We describe the latter below and index time periods by $t = 0, 1, ...$

At each date, a single final commodity is produced via a technology that utilizes homogeneous physical capital and labor as inputs. An individual producer employing K_t units of capital and N_t units of labor at t produces $F(K_t,N_t)$ units of final output. For purposes of exposition, we will assume that F has the constant elasticity of substitution form

$$(1) \qquad F(K,N) = [aK^{\rho}+bN^{\rho}]^{1/\rho},$$

and we will assume throughout that $\rho < 0$ holds.[5] Defining $k \equiv K/N$ to be the capital-labor ratio, it will often be convenient to work with the intensive production function $f(k) \equiv F(k,1)$. Clearly, here

$$(1') \qquad f(k) = [ak^{\rho}+b].$$

Finally, to keep matters notationally simple, we assume that capital depreciates completely in one period.[6]

Each generation consists of two types of agents. Type 1 agents—who constitute a fraction λ (0,1) of the population—are endowed with one unit of labor when young and are retired when old. We assume that all young-period labor is supplied inelastically. In addition, type 1 agents have access to a linear technology for storing consumption goods whereby one unit stored at t yields $x > 0$ units of consumption at $t + 1$.

Type 2 agents represent a fraction $1 - \lambda$ of each generation. These agents supply one unit of labor inelastically when old and have no young-period labor endowment.[7] In addition, type 2 agents have no access to the technology for storing goods. They do, on the other hand, have access to a technology that converts one unit of final output at t into one unit of capital at $t + 1$. Only type 2 agents have access to this technology.

We imagine that any agent who owns capital at t can operate the final goods production process at that date. Thus type 2 agents are producers in old age. It entails

5 If $\rho \geq 0$, our analysis is a special case of that in Azariadis and Smith (forthcoming). We therefore restrict attention here to $\rho < 0$. The assumption that $\rho < 0$ holds implies that the elasticity of substitution between capital and labor is less than unity. Empirical evidence supports such a supposition.

6 It is easy to verify that this assumption implies no real loss of generality.

7 This assumption implies that all capital investment must be externally financed, as will soon be apparent. This provides the link between financial market conditions and capital formation that is at the heart of our analysis.

no loss of generality to assume that all such agents run the production process and work for themselves in their second period.

With respect to agents' objective functions, it is simplest to assume that all agents care only about old-age consumption and that they are risk neutral.[8] These assumptions are easily relaxed.

The central feature of the analysis is the presence of an informational friction affecting the financing of capital investments. In particular, we assume that each agent is privately informed about his own type. We also assume that nonmarket activities, such as goods storage, are unobservable, while all market transactions are publicly observed. Thus, to emphasize, an agent's type and storage activity are private information, while all market transactions—in both labor and credit markets—are common knowledge. This set of assumptions is intended to keep the informational asymmetry in our model very simple: Since type 2 agents cannot work when young, they cannot credibly claim to be type 1. However, type 1 agents might claim to be type 2 when young. We now describe what happens if they do so.

If a type 1 agent wishes to claim to be type 2, he cannot work when young, and he must borrow the same amount as type 2 agents do. Since type 1 agents are incapable of producing physical capital, it will ultimately be discovered that they have misrepresented their type. To avoid punishment, we assume that a dissembling type 1 agent absconds with his loan, becoming autarkic and financing old-age consumption by storing the proceeds of his borrowing. Dissembling type 1 agents never repay loans. Notice, however, that since type 2 agents cannot store goods, they will never choose to abscond, and hence they always repay their loans.[9] Obviously, lenders will want to avoid making loans to dissembling type 1 agents. How they do so is the subject of the section on equilibrium conditions in financial markets.

It remains to describe the initial conditions of our economy. At $t = 0$ there is an initial old generation where each agent is endowed with one unit of labor (supplied inelastically) and with $K_0 > 0$ units of capital. No other agents are endowed at any date with capital or consumption goods.

Trading

Three kinds of transactions occur in this economy. First, final goods and services are bought and sold competitively. We let p_t denote the dollar price at t of a unit of final output. Second, producers hire the labor of young type 1 agents in a competitive labor market, paying the real wage rate w_t at t. And third, young (nondissembling) type 1 workers save their entire labor income, which they supply inelastically in capital markets, thereby acquiring claims on type 2 agents—and possibly on some dissembling type 1 agents—and claims on the government, such as money or national debt. The model we present here is not rich enough to capture any distinction between different types of financial claims, such as debt or equity.[10] We thus think of young agents as simply acquiring a generalized claim against producers of capital. It entails no loss of generality to think of financial market activity as being intermediated, say through banks, mutual funds, or pension funds. We assume there is free entry into the activity of intermediation. We also let R_{t+1} be the real gross rate of return earned by intermediaries between t and $t + 1$ on (nondefaulted) investments, and we let r_{t+1} be the real gross rate of return earned by young savers. After describing government policy, we return to a description of equilibrium conditions in these markets.

The Government

We let M_t denote the outstanding per capita money supply at t. At $t = 0$ the initial old agents are endowed with the initial per capita money supply, $M_{-1} > 0$. Thereafter, the money supply evolves according to

$$(2) \qquad M_{t+1} = \sigma M_t,$$

where $\sigma > 0$ is the exogenously given gross

[8] Risk neutrality implies that there are no potential gains from the use of lotteries in the presence of private information.

[9] The hallmark of models of credit rationing based on adverse selection or moral hazard is that different agents have different probabilities of loan repayment and hence regard the interest rate dimensions of a loan contract differently. See, for instance, Stiglitz and Weiss (1981) or Bencivenga and Smith (1993). Ours is the simplest possible version of such a scenario: Type 2 agents repay loans with probability one, while type 1 agents default with the same probability. Matters are somewhat different in models of credit rationing based on a costly state verification problem in financial markets. See, for instance, Williamson (1986 and 1987) and Labadie (1995). We will discuss such models briefly in the conclusion.

[10] For models of informational frictions that do generate debt and equity claims, see Boot and Thakor (1993), Dewatripont and Tirole (1994), Chang (1986), or Boyd and Smith (1995a and b).

rate of money creation. We assume that the government makes a once and for all choice of σ at $t = 0$: In steady-state equilibria the (gross) rate of inflation will equal σ.

Our ultimate purpose is to examine how different choices of σ affect financial markets and, through this channel, capital formation. To make our results as stark as possible, we assume that the government uses the proceeds of money creation to finance a subsidy to private capital formation. It should then be transparent that any adverse effects of inflation are a result of the presence of inflation alone and not what the revenue from the inflation tax is used for. More specifically, then, we assume that any monetary injections (withdrawals) occur via lump-sum transfers to young agents claiming to be type 2. Genuine type 2 agents will use these transfers entirely to invest in capital; hence government policy here consists of a capital subsidy program financed by printing money. If we let τ_t denote the real value of the transfer received by young type 2 agents at t, and we let μ_t $[0,1]$ denote the fraction of dissembling type 1 agents in the time t population, then the government budget constraint implies that the real value of transfers, per capita, equals the real value, per capita, of seigniorage revenue. Thus

$$(3) \qquad [(1-\lambda)+\lambda\mu_t]\tau_t = (M_t - M_{t-1})/p_t$$

must hold at all dates. If we let $m_t \equiv M_t/p_t$ denote time t real balances, equations 2 and 3 imply that

$$(3') \qquad [(1-\lambda)+\lambda\mu_t]\tau_t = [(\sigma-1)/\sigma]m_t.$$

EQUILIBRIUM CONDITIONS

Factor Markets

Let b_t denote the real value of borrowing by young type 2 agents at t. These agents also receive a transfer τ_t. All resources obtained by these individuals are used to fund capital investments at t. Each old producer at $t+1$ will hence have the capital stock

$$(4) \qquad K_{t+1} = b_t + \tau_t.$$

Let L_t denote the quantity of young labor hired by a representative producer at t. Each such producer combines this with his own unit of labor to obtain $N_t = L_t + 1$ units of labor services. Then the producer's profits, net of loan repayments, are $F(K_t,L_t+1) - w_tL_t - R_tb_{t-1}$ since an interest obligation of R_tb_{t-1} was incurred at $t-1$. Producers wish to maximize old-period income. At t, b_{t-1} is given by past credit extensions, so that the only remaining choice variable is L_t. Profits are maximized when

$$(5) \qquad w_t = F_2(K_t,L_t+1) = F_2(K_t/N_t,1)$$
$$= f(k_t) - k_tf'(k_t) = b(ak_t^\rho + b)^{(1-\rho)/\rho}$$
$$\equiv w(k_t)$$

where $k_t \equiv K_t/N_t$ is the capital labor ratio. Equation 5 asserts the standard result that labor earns its marginal product.

For future reference, it will be useful to have an expression for the consumption, c_t^2, of old type 2 agents at t. Clearly

$$(6) \qquad c_t^2 = F_1(\,)K_t + F_2(\,)(L_t+1) - w_tL_t - R_tb_{t-1}$$
$$= [F_1(\,) - R_t]b_{t-1} + w_t + F_1(\,)\tau_{t-1}.$$

The first equality in equation 6 follows from Euler's law, while the second follows from equations 3, 4, and 5. Equation 6 asserts that old producers have income equal to the marginal product of their own labor, plus the value of the capital paid for through transfer payments $[F_1(\,)\tau_{t-1}]$, plus the net income obtained from capital attained through borrowing $[(F_1(\,) - R_t)b_{t-1}]$.

Financial Markets

Intermediaries face a fairly standard adverse selection problem in financial markets.[11] If they lend to a dissembling type 1 agent, the loan will not be repaid.[12] Hence it is desirable not to lend to these agents, but at the same time such agents cannot be identified ex ante. Intermedi-

[11] For a canonical adverse selection model, see Rothschild and Stiglitz (1976).

[12] It is easy to verify that nondissembling type 1 agents will not wish to borrow if $R_{t+1} \geq \max(r_{t+1},x)$. This condition will hold in equilibrium.

aries will hence structure financial contracts to deter type 1 agents from dissembling or, in other words, to induce self-selection (only type 2 agents choose to accept funding).

Using typical assumptions in economies with adverse selection, we assume that intermediaries announce financial contracts consisting of a loan quantity b_t, and an interest rate (or return to the intermediary) of R_{t+1}. Each intermediary announces such a contract, taking the contracts offered by other intermediaries as given. Hence we seek a Nash equilibrium set of financial contracts. On the deposit side we assume that intermediaries behave competitively (that is, each intermediary assumes it can raise all the funds it wants at the going rate of return on savings r_{t+1}).

One objective of intermediaries is to induce self-selection. This requires that type 1 agents prefer to work when young and save their young-period income rather than to borrow b_t, receive the transfer τ_t, and abscond. If they work when young and save the proceeds, their utility is $r_{t+1}w_t$. If they borrow b_t, obtain the transfer τ_t, and abscond, their utility is $x(b_t+\tau_t)$. Hence self-selection requires that

$$(7) \quad r_{t+1}w_t \geq x(b_t+\tau_t).$$

Standard arguments[13] establish that equation 7 holds in any Nash equilibrium and that all type 1 agents are deterred from dissembling. Hence $\mu_t = 0$ holds at all dates.

In addition, since there is free entry into intermediation, all intermediaries must earn zero profits in equilibrium. Since $\mu_t = 0$, this simply requires that

$$(8) \quad R_{t+1} = r_{t+1}.$$

An equilibrium in financial markets now requires that five conditions be satisfied. First, given r_{t+1} and τ_t, the quantity of funds obtained in the marketplace by each type 2 agent must satisfy equation 7. Second, equation 8 must hold. Third, sources and uses of funds must be equal. Sources of funds at each date are simply the savings of young type 1 agents, which in per capita terms are λw_t. Uses of funds are loans to borrowers [$(1-\lambda)b_t$ per capita], plus real balances (m_t per capita), plus per capita storage (s_t). Thus equality between sources and uses of funds obtains if and only if

$$(9) \quad \lambda w(k_t) = (1-\lambda)b_t + m_t + s_t.$$

The fourth condition is that type 2 agents will be willing to borrow if and only if

$$(10) \quad F_1(K_t,N_t) = F_1(K_t/N_t,1) = f'(k_t) = a[a+bk^{-\rho}]^{(1-\rho)/\rho} \geq R_{t+1} = r_{t+1}$$

holds[14]. Equation 10 implies that type 2 agents perceive nonnegative profits from borrowing. And finally, type 1 agents are willing to supply funds to intermediaries if and only if the return they receive is at least as large as the return available on alternative savings instruments (money and storage). This requires that

$$(11a) \quad r_{t+1} \geq p_t/p_{t+1}$$

$$(11b) \quad r_{t+1} \geq x.$$

We will want agents to hold money in equilibrium. Hence equation 11a must always hold with equality. We will assume that equation 11b is a strict inequality; hence in equilibrium $s_t = 0$ (storage is dominated in rate of return). Equation 11b is validated, at least near steady states, by the assumption that

$$(a1) \quad 1/x > \sigma.$$

We will henceforth impose equation a1.[15]

Some Implications

We now know that in equilibrium all young type 1 agents supply their labor to producers. Hence labor market clearing requires that the per capita labor demand of producers [$(1-\lambda)L_t$] equals the per capita

[13] See Rothschild and Stiglitz (1976), or in this specific context, Azariadis and Smith (forthcoming).

[14] See equation 6.

[15] An additional requirement of equilibrium is that intermediaries perceive no incentive to "pool" dissembling type 1 agents with type 2 agents and to charge an interest rate that compensates for the defaults by dissembling type 1 agents. Azariadis and Smith (forthcoming) show that there is no such incentive if $f'(k_{t+1}) \leq r_{t+1}/(1-\lambda)$ holds for all t.

labor supply of young type 1 agents (λ). Therefore,

(12) $L_t = \lambda/(1 - \lambda)$.

It is an immediate implication that $N_t = 1 + L_t = 1/(1 - \lambda)$ and that

(13) $k_t \equiv K_t/N_t = (1 - \lambda)K_t$.

In addition, under equation a1, $s_t = 0$ holds, so that equation 9 becomes

(9′) $(1 - \lambda)b_t = \lambda w(k_t) - m_t$.

Now note that $k_{t+1} = (1 - \lambda)K_{t+1} = (1 - \lambda)(b_t + \tau_t)$. Using this fact in equation 9′, we obtain

(9″) $k_{t+1} = \lambda w(k_t) - m_t - (1 - \lambda)\tau_t$.

Finally, we know that $\mu_t = 0$. Using this fact in equation 1′ and substituting the result into equation 9″ yields

(14) $k_{t+1} = \lambda w(k_t) - m_t/\sigma$.

It is also possible to derive some further implications from the preceding discussion. Equations 10 and 11a at equality, imply that

(15) $f'(k_{t+1}) \geq r_{t+1} = p_t/p_{t+1}$.

We can now use the identity $p_t/p_{t+1} \equiv (M_{t+1}/p_{t+1})(p_t/M_t)(M_t/M_{t+1}) \equiv m_{t+1}/\sigma m_t$ to write equation 15 as

(15′) $f'(k_{t+1}) \geq m_{t+1}/\sigma m_t$.

Finally, equation 7 must hold in equilibrium. Substituting equation 4 into equation 7, and using $K_{t+1} = k_{t+1}/(1 - \lambda)$, we obtain the equivalent condition

(16) $r_{t+1} w(k_t) \geq x k_{t+1}/(1-\lambda)$.

Equation 11a also implies an alternative form of equation 16:

(16′) $[m_{t+1}/\sigma m_t] w(k_t) \geq x k_{t+1}/(1-\lambda)$.

We can now reduce our search for an equilibrium to the problem of finding a sequence $\{k_t, m_t\}$ that satisfies equations 14, 15′, and 16′ at all dates, with $k_0 > 0$ given as an initial condition. We now make an additional comment. If equation 15′ is a strict inequality at any date, young type 2 agents perceive positive profits to be made from borrowing and hence will want to borrow an arbitrarily large amount. Because this is not possible, if equation 15′ is a strict inequality, their borrowing must be constrained. The relevant constraint is equation 7. In this case equation 7 at equality determines b_t, and equation 16′ will hold with equality. In equilibrium, at least one of the conditions (equations 15′ or 16′) must thus hold with equality. If equation 15′ is an equality, the equilibrium coincides with standard equilibria that obtain in similar economies with no informational asymmetries.[16] In this case we say the equilibrium is Walrasian. If equation 15′ holds as a strict inequality, then equation 16′ is an equality. We refer to this situation as credit rationing.

WALRASIAN EQUILIBRIA

We now describe sequences that satisfy equations 14 and 15′ at equality. For the present we do not impose equation 16′: This amounts to assuming that agents' types are publicly observed. In the section on the endogeneity of financial market frictions, we ask when such sequences will also satisfy equation 16 or, in other words, when Walrasian resource allocations can be sustained even in the presence of the informational asymmetry. We begin with steady-state equilibria, and then briefly describe the nature of equilibrium paths that approach the steady state. Because the material in this section is quite standard,[17] we attempt to present it fairly concisely.

Steady States

In a steady state k_t and m_t are constant. Hence equation 15′ at equality reduces to

[16] See, for example, Diamond (1965), Tirole (1985), or Azariadis (1993, chapter 26.2).

[17] See, for instance, Diamond (1965), Tirole (1985), or Azariadis (1993, chapter 26.2).

(17) $f'(k) = 1/\sigma = p_t/p_{t+1}$,

while equation 14 becomes

(18) $m = \sigma[\lambda w(k) - k]$.

It is immediately apparent from equation 17 that *increases* in the rate of money growth (and inflation), σ, increase the steady-state capital-labor ratio, per capita output, and productivity of labor. This is true for all rates of money growth satisfying equation a1. Because the empirical evidence cited in the introduction strongly suggests that higher inflation can lead to higher long-run levels of real activity *only* if initial rates of inflation are relatively low, it is clear that our model cannot confront the whole array of empirical experience in the absence of the informational asymmetry.

For future reference, it will be convenient to give an explicit form for the capital stock (or variables related to it) as a function of the money growth rate. To this end we define the variable

(19) $z_t \equiv (b/a)k_t^{-\rho} \equiv w(k_t) \,/\, k_t f'(k_t)$.

It is readily verified that z_t is simply the ratio of labor's share to capital's share: The assumption that $\rho < 0$ implies that z_t is an increasing function of k_t. Hence movements in z_t simply reflect similar movements in k_t.

It is easy to check that $f'(k_t) = a^{1/\rho}[1+(b/a)k_t^{-\rho}]^{(1-\rho)/\rho} \equiv a^{1/\rho}(1+z_t)^{(1-\rho)/\rho}$. Then, if we let $z^*(\sigma)$ denote the value of z satisfying equation 17 for each σ, we have that

(20) $z^*(\sigma) = [a^{-1/\rho}(1/\sigma)]^{\rho/(1-\rho)} - 1$.

Equations 19 and 20 give the capital stock in a Walrasian steady state.

Dynamics

Equations 14 and 15′ at equality describe how the economy evolves given k_0 and m_0: the initial capital-labor ratio and initial real balances. The initial price level is endogenous, and so $m_0 \equiv M_0/p_0$ is endogenous.

It is easy to show that the monetary steady state is a saddle or, in other words, that there is only one choice of m_0 that averts a hyperinflation where money asymptotically loses all value. Thus nonhyperinflationary equilibria are determinate (only one possible equilibrium path approaching the monetary steady state exists), and it is possible to show that the steady state is necessarily approached monotonically. Walrasian equilibria therefore *cannot* display economic fluctuations in output, real returns to investors, or the rate of inflation.

Summary

Walrasian equilibria are unique. Growth traps are therefore impossible. Moreover, Walrasian equilibria do not display economic fluctuations. Finally, Walrasian equilibria have the feature that increases in the long-run rate of inflation lead to higher long-run levels of real activity and productivity.

EQUILIBRIA WITH CREDIT RATIONING

In this section we investigate sequences $\{k_t, m_t\}$ that satisfy equations 14 and 16′ at equality at all dates. In the section on the endogeneity of financial market frictions, we then examine when a Walrasian equilibrium or an equilibrium with credit rationing will actually obtain. As before, we begin with steady-state equilibria.

Steady States

When k_t and m_t are constant, equation 16′ at equality implies that the steady-state capital-labor ratio satisfies

(21) $w(k)/k = x\sigma/(1-\lambda)$.

Equation 21 says that the capital stock is determined by how financial markets control borrowing to induce self-selection. The rate of inflation matters because it affects the rate of return that nondissembling type 1 agents receive on their savings. As inflation rises, this return falls,[18] with the consequence that the utility of working and saving declines. To prevent type 1 agents from dissembling, the utility of doing so must also fall. Equation 21 describes the consequences for the per capita capital stock.

It will be convenient to transform equation 21 as follows: First note that it can be written as

(21′) $[w(k)/kf'(k)]f'(k) = x\sigma/(1-\lambda)$.

Second, given equation 19 and our previous observations about $f'(k)$, it is easy to verify that $[w(k)/kf'(k)]f'(k) = a^{1/\rho}z(1+z)^{(1-\rho)/\rho}$. This observation allows us to rewrite the equilibrium condition equation 21′ as

(22) $a^{-1/\rho}[x\sigma/(1-\lambda)]$
$= z(1+z)^{(1-\rho)/\rho} \equiv H(z)$.

Equation 22 determines the steady-state equilibrium value(s) of z as a function of the long-run inflation rate σ. Equation 19 then gives the steady-state per capita capital stock. Steady-state real balances are determined from equation 14 with k and m constant:

(23) $m = \sigma[\lambda w(k) - k]$.

Equation 21 permits us to rewrite equation 23 as

(23′) $m = \sigma k\{[x\lambda\sigma/(1-\lambda)] - 1\}$.

For future reference, it will be convenient to define the function $A(\sigma)$ by

(24) $A(\sigma) \equiv [x\lambda/(1-\lambda)]\sigma$.

We can now state our first result.

RESULT 1. *Define* $\hat{\sigma}$ *by*

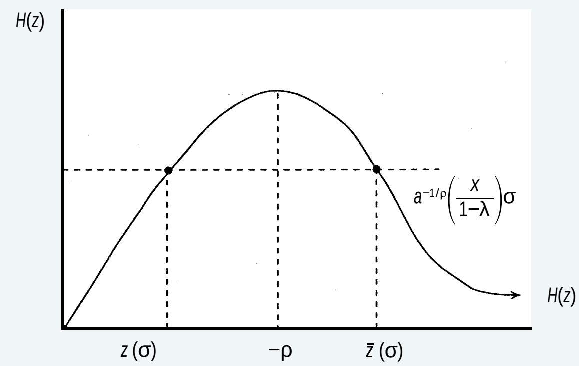
(2) $\hat{\sigma} \equiv -\rho(1-\rho)^{(1-\rho)/\rho}[(1-5\lambda)/x]a^{1/\rho}$.

Then if $\sigma \le \hat{\sigma}$*, there exists a solution to equation 22. If, in addition,*

(a2) $A(\sigma) > 1$

Figure 1

Determination of z Under Credit Rationing



all solutions to equation 22 yield positive levels of real balances.

Result 1 is proved in the Appendix.

As the Appendix establishes, the function $H(z)$ defined in equation 22 has the configuration depicted in Figure 1. In particular,

$$H(0)=0=1_{z\to\infty} H(z)$$

and H attains a unique maximum at $z = -\rho$. Thus, if

(26) $H(-\rho) \ge a^{-1/\rho}[x\sigma/(1-\lambda)]$

equation 22 has a solution, which is depicted in Figure 1. If $\sigma < \hat{\sigma}$ where

$\hat{\sigma} = [(1-\lambda)/x]a^{1/\rho}H(-\rho)$,

there will be exactly two solutions to equation 22 that are denoted by $\underline{z}(\sigma)$ and $\bar{z}(\sigma)$ in Figure 1.

The conditions $A(\sigma) > 1$ and $\sigma \le \hat{\sigma}$ are equivalent to

(a3) $(1-\lambda)/x\lambda < \sigma \le \hat{\sigma}$.

We henceforth assume that equation a3 holds. We also assume that

(a4) $1/x \ge \hat{\sigma}$

so that equation a3 implies satisfaction of equation a1.[19]

[18] In this analysis, inflation is inversely related to the return on real balances and hence to the return on savings. However, the intuition underlying our results is not dependent on real balances earning the same real return as other savings instruments. Higher inflation will also reduce the return on savings in economies where nominal interest rate ceilings bind or where binding reserve requirements subject intermediaries to inflationary taxation. Binding interest rate ceilings and reserve requirements are very common in developing countries and are hardly unknown in the United States. Finally, our empirical results do support the notion that higher inflation does reduce the real returns received by investors (see the section on some empirical evidence).

[19] Clearly $1/x > (1-\lambda)/\lambda x$ can hold only if $\lambda > 0.5$. Equation a4 obviously implies this.

Figure 2

The Consequences of Higher Inflation

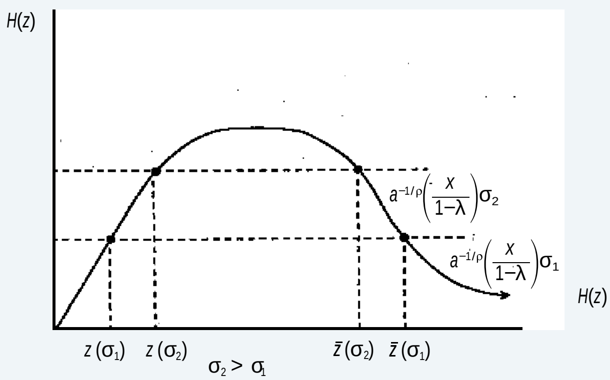
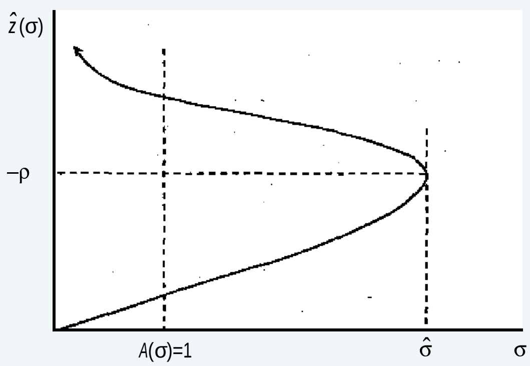


Figure 3

Inflation and its Consequences Under Credit Rationing



The Effects of Higher Inflation

The consequences of an increase in the steady-state inflation rate are depicted in Figure 2. Evidently, an increase in σ raises $z(\sigma)$ and reduces $\bar{z}(\sigma)$ or, in other words

$$(2\ )\qquad \underline{z}'(\sigma) > 0 > \bar{z}'(\sigma)$$

holds. The same statements apply to k. Hence, in the low- (high-) capital-stock steady state, an increase in the inflation rate raises (lowers) the steady-state capital stock. These effects occur because an increase in σ reduces the steady-state return on savings. Other things equal, this lowers the utility of honest type 1 agents and would cause them to misrepresent their type. To preserve self-selection $w(k)$ must rise relative to $(b+\tau) = k/(1-\lambda)$. In the low- (high-) capital-stock steady state, this requires that k rise (fall). Thus higher inflation exacerbates informational asymmetries, with implications for the capital stock that are adverse in the high-capital-stock steady state.

Figure 3 depicts the solutions to equation 22 as a function of σ, where we denote by $\hat{z}(\sigma)$ any solutions to that equation. Evidently, there can be no solution to equation 22 if the government sets σ above $\hat{\sigma}$. For σ satisfying equation a3, clearly we have

$$(2\ )\qquad \underline{z}(\sigma) < -\rho < \bar{z}(\sigma).$$

Evidently, when $\sigma \le \hat{\sigma}$, there are two solutions to equation 22. This multiplicity of candidate equilibria derives from the way that financial markets respond to the presence of the adverse selection problem. To induce self-selection at any given value of σ, $w(k)$ and $(b+\tau)$ must be linked. One way that self-selection can occur is for $w(k)$ and $(b+\tau)$ both to be low; this requires that z be low. Alternatively, $w(k)$ and $(b+\tau)$ can both be relatively high; this requires that z be high. The possibility that there is more than one way for financial markets to address an informational asymmetry has a generality beyond this particular model, as shown by Boyd and Smith (forthcoming) or Schreft and Smith (forthcoming and 1994).

Of particular interest in this context is the possibility that an increase in the long-run rate of inflation can reduce the long-run capital stock, real activity, and productivity. Such consequences are often observed empirically when inflation increases, particularly when the initial rate of inflation is relatively high. This outcome is observed in the high-capital-stock steady state. We now want to know which, if either, steady state can be approached under credit rationing.

Dynamics

Given an initial capital-labor ratio, k_0, and an initial level of real balances, m_0, equations 14 and 16′ at equality govern the subsequent evolution of k_t and m_t. The Appendix establishes our second result.

RESULT 2. *(a) The low-capital-stock steady state is a saddle. All $\{k_t,m_t\}$ sequences approaching it do so monotonically. (b) The high-capital-stock steady state is a sink if $\hat{\sigma}$ is not too large.*

Result 2a implies that both the high- and the low-capital-stock steady states can potentially be approached. To approach the low-capital-stock steady state, m_0 must be chosen to lie on a "saddle path;" that is, there is a unique choice of m_0 that allows the economy to approach the low-capital-stock steady state.

Result 2b implies that, for some open set of values of k_0, there is a whole interval of choices of m_0 that allow the high-capital-stock steady state to be approached. The requirement of avoiding a hyperinflation thus no longer implies what m_0 must be. Monetary equilibria have become *indeterminate.* A continuum of possible equilibrium values of m_0 exist and hence so does a continuum of possible equilibrium paths approaching the high-capital-stock steady state. This is a consequence of the informational friction afflicting capital markets.

Not only is the informational asymmetry a source of indeterminacy, it is a potential source of endogenous economic volatility as well. We now establish that such volatility *must* be observed near the high-capital-stock steady state whenever the rate of inflation is sufficiently high. At high rates of inflation, the economy must thus pay a price to avoid the low-capital-stock steady state: This price is the existence of endogenous volatility in real activity, inflation, and asset returns.

RESULT 3. *Suppose that σ is sufficiently close to $\hat{\sigma}$. Then all paths approaching the high-capital-stock steady state do so nonmonotonically.*

Result 3 is proved in the Appendix.

Summary

When financial market frictions bind, there can be two steady-state equilibria differing in their levels of real development. Both steady states can potentially be approached. A continuum of paths approaching the high-capital-stock steady state exists so that the operation of financial markets creates an indeterminacy. If the steady-state inflation rate is high enough, all such paths display endogenously arising volatility in real activity, real returns, and inflation. In this sense high inflation also engenders variable inflation.

THE ENDOGENEITY OF FINANCIAL MARKET FRICTIONS

In the section on Walrasian equilibria, we described equilibria under the assumption that information about borrower type is publicly available. In the section on equilibria with credit rationing, we described candidate equilibria under the assumption that equation 16′ holds as an equality. In this section we ask when equation 16′ will and will not be an equality in equilibrium. When it is, credit rationing will occur. When it is not, self-selection occurs even with Walrasian allocations. In the former situation, financial market frictions are severe enough to affect the allocation of investment capital for entirely endogenous reasons. In the latter situation, it transpires—again for entirely endogenous reasons—that financial market frictions are not severe enough to affect allocations. One of our main results is that when the steady-state inflation rate is high enough, financial market frictions *must* matter and credit rationing *must* occur. Thus high enough rates of inflation imply that market frictions must adversely affect the extension of credit and capital formation as well.

When Are Walrasian Allocations Consistent With Self-Selection?

When do candidate Walrasian equilibria (sequences $\{k_t,m_t\}$ satisfying equations

Figure 4a

Multiple Steady States

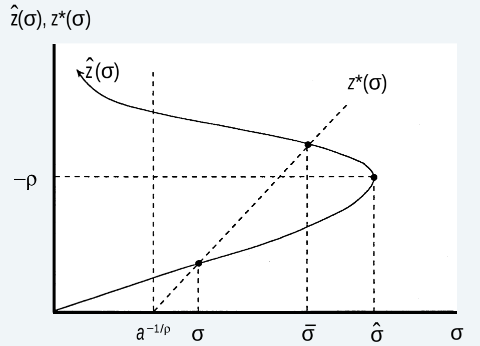
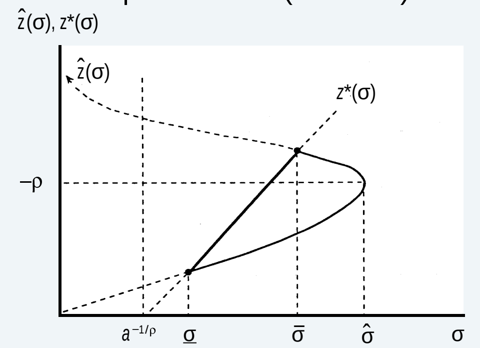


Figure 4b

The Steady-State Equilibrium Correspondence (Case 1)



14 and 15′ at equality) also satisfy equation 16′? For simplicity of exposition, we focus our discussion on steady states.

Walrasian steady states satisfy equation 16′ when equation 17 holds and when the implied value of k satisfies

$$(29)\quad [w(k)/kf'(k)]f'(k) \geq x\sigma/(1-\lambda).$$

We have already established that $[w(k)/kf'(k)] = a^{1/\rho}z(1+z)^{(1-\rho)/\rho}$; hence equation 29 is equivalent to

$$(30)\quad H[z^*(\sigma)] \geq a^{-1/\rho}[x\sigma/(1-\lambda)].$$

We now demonstrate our fourth result.

RESULT 4. *Equation 30 is satisfied if and only if* $\underline{z}(\sigma) \leq z^*(\sigma) \leq \bar{z}(\sigma)$ *holds.*

Result 4 is proved in the Appendix. The result asserts that Walrasian allocations are consistent with self-selection if and only if the steady-state value of z under full information lies between the values of z solving equation 16′ at equality. When this condition is satisfied, the Walrasian allocation continues to constitute an equilibrium, even in the presence of the informational asymmetry. Endogenous factors allow the friction to be sufficiently mild that it does not affect the allocation of investment capital. Thus, when $z^*(\sigma) \in [\underline{z}(\sigma), \bar{z}(\sigma)]$, Walrasian allocations are equilibrium allocations. When $z^*(\sigma) \notin [\underline{z}(\sigma), \bar{z}(\sigma)]$, Walrasian allocations are inconsistent with self-selection and do not constitute legitimate equilibria.

Credit Rationing

We now ask the opposite question: When do solutions to equation 16′ at equality satisfy equation 15′? Since $f'(k) = a^{1/\rho}(1+z)^{(1-\rho)/\rho}$, clearly they do so if and only if

$$(31)\quad \hat{z}(\sigma) \leq z^*(\sigma).$$

In particular, equation 31 asserts that credit can be rationed if and only if the solution to equation 22 yields a lower capital stock than would obtain under a Walrasian allocation. This observation has the following implication: The smaller (larger) solution to equation 16′ at equality is an equilibrium if and only if $\underline{z}(\sigma)[\bar{z}(\sigma)] \leq z^*(\sigma)$. We now put all these facts together.

The Steady-State Equilibrium Correspondence

Here we describe the full set of steady-state equilibria for each potential choice of σ. We begin by depicting $z^*(\sigma)$ and $\hat{z}(\sigma)$ simultaneously in Figure 4a and b. It is easy to check that $z^*(\sigma)$ is an increasing function, and that $z^*(a^{-1/\rho}) = 0$. Combining this with our previous results about the correspondence $\hat{z}(\sigma)$, it follows that there are three possible configurations of the steady-state equilibrium correspondence.

We now briefly discuss each case. The first case is the one of primary interest to us.

Case 1. Suppose that

$$(3\ )\qquad z^{*}(\hat{\sigma}) > -\rho.$$

Then we have the configuration depicted in Figure 4a.[20] The loci $z^{*}(\sigma)$ and $\hat{z}(\sigma)$ intersect twice at $\underline{\sigma}$ and $\overline{\sigma}$.[21]

For $\sigma < \underline{\sigma}, z^{*}(\sigma) < \mathbf{z}(\sigma)$ holds. Hence neither the Walrasian situation nor the credit rationing situation constitutes a legitimate equilibrium. Then if $\sigma < \underline{\sigma}$, there are *no* monetary steady states.

For $\sigma \quad [\underline{\sigma}, \overline{\sigma}], z^{*}(\sigma) \quad [\underline{z}(\sigma), \overline{z}(\sigma)]$ holds. It follows that the Walrasian steady state is consistent with self-selection whenever $\sigma \quad [\underline{\sigma}, \overline{\sigma}]$ and hence is a true steady-state equilibrium. At the same time, $\underline{z}(\sigma) \leq z^{*}(\sigma)$ also holds. Thus $\underline{z}(\sigma)$ is a legitimate steady state with credit rationing. Clearly, $\overline{z}(\sigma) > z^{*}(\sigma)$ holds for all $\sigma \quad [\underline{\sigma}, \overline{\sigma}]$ and hence $\overline{z}(\sigma)$ is *not* a legitimate steady state for $\sigma < \overline{\sigma}$. Thus, for $\sigma \quad [\underline{\sigma}, \overline{\sigma}]$ exactly two steady-state equilibria exist: one with credit rationing and one without. Our previous results imply that both steady states are saddles and hence that both can potentially be approached.[22] If credit rationing arises, the result will be that the capital stock is depressed. The capital stock is low, and therefore $w(k)$ must be low relative to $(b + \tau) = k/(1 - \lambda)$. This forces intermediaries to ration credit to induce self-selection. Credit rationing can thus arise for fully endogenous reasons.

Suppose that two intrinsically identical economies[23] with $\sigma \quad [\underline{\sigma}, \overline{\sigma}]$ land in different steady states. The economy with a low capital stock will experience credit rationing, while that with a high capital stock does not. Thus the better-developed economy will appear to have a better functioning financial system, as in fact it does. However, the inefficient functioning of capital markets in the poorer economy is a purely endogenous outcome.

When $\sigma > \overline{\sigma}$ holds, $z^{*}(\sigma) < \overline{z}(\sigma)$ holds as well. Hence Walrasian outcomes are no longer consistent with self-selection and they cannot be equilibria. Thus, when steady-state inflation exceeds a critical level $(\overline{\sigma})$ informational frictions *must* interfere with the operation of capital markets.

Since $\mathbf{z}^{*}(\sigma) > \overline{z}(\sigma)$ for all $\sigma > \overline{\sigma}$, both $\overline{z}(\sigma)$ and $\underline{z}(\sigma)$ constitute legitimate equilibria with credit rationing. For $\sigma \quad (\overline{\sigma}, \hat{\sigma})$, there are thus again two steady-state equilibria. Our previous results indicate that one is a sink and one a saddle; hence both can potentially be approached. In the high- (low-) capital-stock steady state, credit rationing appears to be less (more) severe.

To summarize, in this case for $\sigma \quad (\underline{\sigma}, \hat{\sigma})$ potentially two steady-state equilibria exist. In one credit market frictions are relatively severe; in the other they are less so.

We have thus far not insisted that a steady-state equilibrium have a positive level of real balances. Keeping this condition in mind, we present our fifth result.

RESULT 5. *Suppose that* $A(\sigma) > 1$ *holds. Then any steady state has positive real balances.*

Result 5 is proved in the Appendix.[24]

In this case, then, the steady-state equilibrium correspondence is given by the solid locus in Figure 4b. For $\sigma \leq \overline{\sigma}$, the steady-state equilibrium value of z, and hence of k, increases with σ. Thus, *for low initial rates of inflation,* increases in σ result in higher steady-state capital stocks and output levels (*unless* increases in σ result in a shift from a Walrasian equilibrium to an equilibrium with credit rationing). However, for $\sigma > \overline{\sigma}$, equilibria lying along the upper branch of this locus will have z (and hence k) *decreasing* as σ increases. Thus, at high initial inflation rates, increases in σ can reduce long-run output levels. This situation is very consistent with the empirical evidence reviewed in the introduction.[25]

Case 2. (Figure 5a).

In this case $z^{*}(\sigma)$ and $\hat{z}(\sigma)$ (generically) have two intersections, as they did previously. In addition, for $\sigma < \underline{\sigma}$ there are *no* steady-state equilibria, as in Case 1. Similarly, for $\sigma \quad [\underline{\sigma}, \overline{\sigma}]$ there are two

[20] Equation 32 holds if and only if equation A19 holds, as established in the Appendix section on the existence of steady-state equilibria. Thus A19 gives a primitive condition under which Case 1 obtains.

[21] The Appendix section that covers Result 6 proves that there are at most two intersections and that there are exactly two intersections in this particular case.

[22] The existence of two saddles is possible because dynamical equilibria follow different laws of motion depending on whether a Walrasian regime or a regime of credit rationing pertains.

[23] Except, possibly, for their initial capital stocks.

[24] Strictly speaking, in any steady state with credit rationing, it is necessary that intermediaries perceive no arbitrage opportunities associated with "pooling" type 2 and dissembling type 1 agents (see footnote 11). The Appendix establishes that intermediaries perceive no such incentive, for any value of $\sigma \quad [\underline{\sigma}, \overline{\sigma}]$, so long as $-\rho/x \leq \overline{\sigma}^2$ and $\sigma^2 \geq (1 - \lambda)\hat{\sigma}^2$ both are satisfied.

[25] For both outcomes to be consistent with positive levels of real balances, it is necessary that $(1-\lambda)/x\lambda < \overline{\sigma}$ holds. The Appendix establishes that $(1-\lambda)/x\lambda < \overline{\sigma}$ holds if either equations A27 or A28 and A29 are satisfied.

Figure 5a

Multiple Steady States

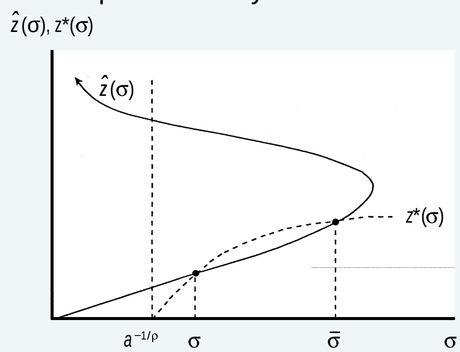
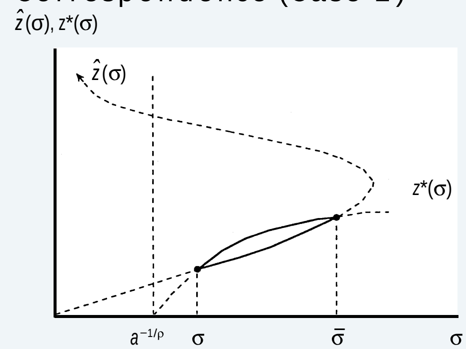


Figure 5b

The Steady-State Equilibrium Correspondence (Case 2)



steady-state equilibria, just as in Case 1. However, here for $\sigma \quad [\bar{\sigma},\hat{\sigma}], z(\sigma)>z^*(\sigma)$ holds, so that neither the Walrasian nor the credit rationing allocations are legitimate steady states. Hence steady-state equilibria exist if and only if $\sigma \quad [\sigma,\bar{\sigma}]$.

The steady-state equilibrium correspondence for Case 2 is depicted in Figure 5b. In this case no branch of the correspondence exists for which *z* (and *k*) are decreasing in σ. Thus this case cannot easily capture the empirical observations cited in the introduction.

Case 3.

Here $\underline{z}(\sigma)>z^*(\sigma)$ holds for all σ. It follows that there are *no* steady-state equilibria for any value of σ.

Discussion

Of the various possible configurations of the steady-state equilibrium correspondence, only that in Case 1 seems like it can easily confront empirical findings like those of Bullard and Keating (forthcoming) and Bruno and Easterly (1995). We therefore regard this as the most interesting case and explore it somewhat further.

As shown in Result 3, some critical value (σ_c) of the money growth rate exists, with $\sigma_c<\hat{\sigma}$ such thatforall $\sigma>\max\{\bar{\sigma}, \sigma_c\}$, equilibrium paths approaching the high-capital-stock steady state necessarily display endogenous oscillation. Then, in particular, if $(1-\lambda)/x\lambda<\bar{\sigma}$ (see the Appendix), there are three distinct possibilities:

- $\sigma \in [\max\{\underline{\sigma}, (1-\lambda)/x\lambda\}, \bar{\sigma}]$. Here one steady-state equilibrium exists with credit rationing and one exists without. Paths approaching both steady states do so monotonically. Increases in σ (within this interval) raise the capital stock in each steady state.[26]
- $\sigma \in (\bar{\sigma}, \sigma_c)$.[27] Here there are two steady-state equilibria, each displaying credit rationing. Dynamical equilibrium paths approaching each steady state may do so monotonically. In the higher of the steady states, increases in the steady-state inflation rate are detrimental to capital formation and the long-run level of real activity.
- $\sigma \in (\sigma_c, \hat{\sigma})$. Here there continue to be two steady-state equilibria with credit rationing $(i \;\; \sigma<\hat{\sigma})$. Now equilibrium paths approaching the high-capital-stock steady state necessarily display endogenous fluctuations. This is the price paid for avoiding convergence to the low-capital-stock steady state. Moreover, if low levels of real activity are to be avoided, high rates of money growth induce volatility in all economic variables, including the inflation rate. High rates of inflation are then associated with variable rates of inflation.

An Example

We now present a set of parameter values satisfying equations a4, A19 (implying

[26] However, increases in σ can still result in a reduction in the steady-state capital stock if they induce transitions from the Walrasian to the credit rationing regime. The current analysis provides no guidance as to when such transitions might or might not occur.

[27] Obviously we are assuming here that $\sigma_c > \bar{\sigma}$.

Table 1

Regressions from Stock Market Data*: United States

(1)

$$RV_t^{\dagger} = .00 + .01\,V(t) + 2.9\,GIP(t) - .05\,INF(t)$$
$$\quad (.01)(.003)^{\ddagger} \quad (3.8) \qquad (.02)^{\ddagger}$$

$R^2 = .03$, $DW = 1.97$, $Q(60) = 77.7$

(2)

$$RR_t = -.01 - .01\,RR(t-1) - .35\,V(t) + .10\,RRAT(t) - 2.9\,INF(t)$$
$$\quad (.20)(.05) \qquad (.10)^{\ddagger} \quad (.12) \qquad (1.1)^{\ddagger}$$

$R^2 = .09$, $Q(60) = 56.2$

(3)

$$NR_t = -.01 - .01\,NR(t-1) - .34\,V(t) + .10\,RRAT(t) - 1.9\,INF(t)$$
$$\quad (.20)\ (.05) \qquad (.10)^{\ddagger} \quad (.13) \qquad (1.1)^{\ddagger}$$

$R^2 = .06$, $Q(60) = 56.4$

(4)

$$V_t = .01 + .19\,RRAT(t) - 2.87\,GIP(t) + 2.26\,INF(t)$$
$$\quad (.1) \quad (.08)^{\ddagger} \qquad (37.6) \qquad (.73)^{\ddagger}$$

$R^2 = .02$, $DW = 1.99$, $Q(60) = 71.5$

* Monthly, 1958-93.

† Denotes that a Cochrane-Orcutt procedure has been employed.

‡ Denotes significance at the 5 percent level or higher.

Standard errors are in parentheses.

DW: Durbin-Watson statistic

Q: Ljung-Box Q statistic

that we have a Case 1 economy), A24′, A26′ (implying that intermediaries have no incentive to pool different agent types in any steady-state equilibrium), and A27 [implying that $(1-\lambda)/\lambda x < \bar{\sigma}$]. One set of parameter values satisfying these conditions is given by $\hat{\sigma} = 2$, $\rho = -1$, $x = 1/32$, $\lambda = 63/64$, and $a = 1/16$. For these parameter values, equation a3 reduces to σ (0.508, 2). These parameter values imply, parenthetically, that the government can allow the money supply to grow as rapidly as 100 percent per period, or could contract the money supply by as much as 49 percent per period. They also imply an empirically plausible elasticity of substitution between capital and labor of 0.5. It is also easy to check that, for all $\sigma > (1-\lambda)/\lambda x$, the high-capital-stock steady state has labor's share exceeding capital's share, as is true empirically.

SOME EMPIRICAL EVIDENCE

The theoretical analysis of the previous sections yields several predictions that can be tested empirically.

1. Increases in the steady-state rate of inflation reduce the real returns investors receive.
2. Such increases can lead to greater inflation variability and also to greater variability in the returns on all assets.
3. Higher long-run rates of inflation raise steady-state output levels for economies whose rate of inflation is initially low enough.[28] For economies with initially high rates of inflation ($\sigma \geq \bar{\sigma}$) further increases in inflation must reduce long-run output levels, unless the economy is in a development trap.
4. When higher inflation is detrimental to long-run output levels, inflation adversely affects the level of activity in financial markets.

As we have noted, many of these results are empirically well-supported in the existing literature. For example, it is well-known that higher rates of inflation are typically accompanied by greater inflation variability, as shown in Friedman (1992). Similarly, the third implication listed above is consistent with the empirical evidence presented by Bullard and Keating (forthcoming) and Bruno and Easterly (1995), which we summarized in the introduction. We now address evidence for the remaining propositions.

Table 1 presents the results of four regressions using stock market data for the United States over the period 1958–93.[29] The dependent variables are the growth rate of the real value of transactions on the New York Stock Exchange (RV); real returns on the Standard & Poor's 500 Index, inclusive of dividends (RR); nominal returns on the Standard & Poor's Index, inclusive of dividends (NR); and the standard deviation of daily returns on the Standard & Poor's Index (V). The explana-

[28] As we have seen, this is true along either branch of the steady-state equilibrium correspondence if $\sigma < \bar{\sigma}$. The statement in the text does require some qualification, though. In particular, as noted above, if higher inflation causes the economy to shift from the Walrasian to the credit rationed equilibrium for $\sigma < \bar{\sigma}$, then an increase in the inflation rate can cause long-run output to fall.

[29] Data sources are listed in the Appendix.

Figure 6

Ratio of the Value of Stock Market Transactions to GDP: United States

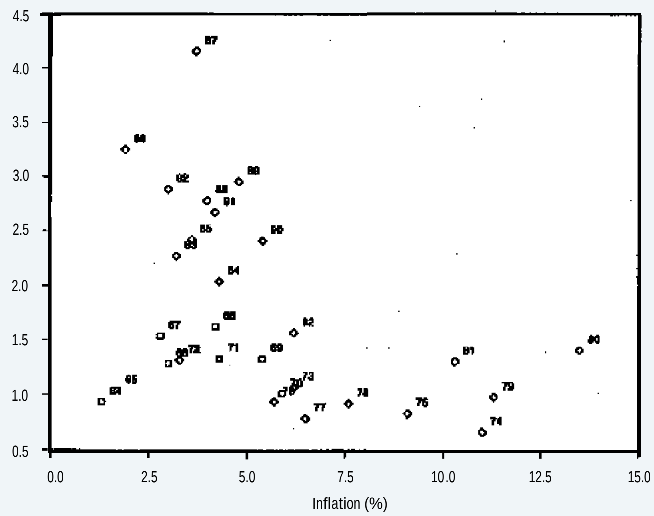


Figure 7

Ratio of the Value of Stock Market Transactions to GDP: Chile

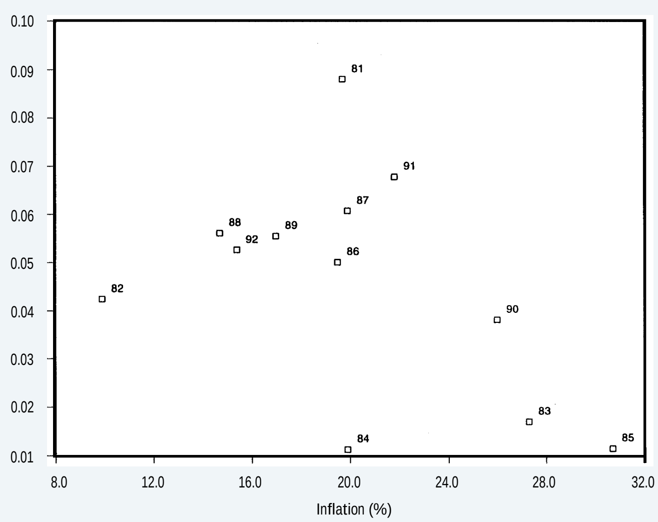


Table 2

Regressions from Stock Market Data*: Chile

(1)

$$RV_t = .00 + .75\,GIP(t) - .30\,INF(t)$$
$$\quad (.02)\ (.20)^\dagger \quad (1.5)$$
$R^2 = .10$, $DW = 2.24$, $Q(33) = 25.5$

(2)

$$RR_t = .00 + .22\,RR(t{-}1) - .02\,RRAT(t) - 2.56\,INF(t)$$
$$\quad (.01)\ (.08)^\dagger \quad (.007)^\dagger \quad (.68)^\dagger$$
$R^2 = .19$, $Q(33) = 19.9$

(3)

$$NR_t = .00 + .17\,NR(t{-}1) - .02\,RRAT(t) - 1.30\,INF(t)$$
$$\quad (.01)(.09)^\dagger \quad (.007)^\dagger \quad (.66)^\dagger$$
$R^2 = .09$, $Q(33) = 19.7$

* Monthly, 1981-91.

† Denotes significance at the 5 percent level or higher.

Standard errors are in parentheses.

DW: Durbin-Watson statistic

Q: Ljung-Box Q statistic

tory variable of interest is the rate of inflation in the Consumer Price Index (INF). Other explanatory variables are also employed. However, the results appear to be quite robust to the inclusion of other explanatory variables. These regressions were selected as being representative of a much larger set that we estimated. Finally, all data are reported as deviations from their sample means,[30] and pass standard stationarity tests in that form. In some regressions we corrected for serial correlation using a Cochrane-Orcutt procedure.

As is apparent from Table 1, higher rates of inflation significantly reduce the growth rate of stock market transactions. As predicted by theory, higher inflation thus attenuates financial market activity. In addition, as the inflation rate rises, the real return received by investors falls significantly.[31] Indeed, over this time period even nominal returns to investors appear to be negatively associated with inflation. Finally, higher inflation increases the volatility of stock returns. All of these results are consistent with the predictions of our model.

Figure 6 depicts the ratio of the value of stock market transactions (on the New York Stock Exchange) to gross domestic product (GDP), plotted against the rate of inflation. Apparently, higher inflation rates also tend to reduce the level of financial market activity using this particular measure.

30 We also ran the regressions reported without removing the sample means. This led to no differences in results.

Table 3

Regressions from Stock Market Data*: Korea

A. 1982-87

(1)

$$RV_t = .01 + .12\,V(t) - .008\,GIP(t) - .12\,INF(t)$$
$\quad (.05)(.13) \quad (.008) \quad (.1)$

$R^2 = .04$, $DW = 2.3$, $Q(24) = 33.2$

(2)

$$RR_t = .00 - .17\,RR(t-1) + .25\,V(t) - 1.69\,RRAT(t) - 2.32\,INF(t)$$
$\quad (.72)(.12) \quad (1.9) \quad (4.50) \quad (4.50)$

$R^2 = .03$, $Q(24) = 17.6$

(3)

$$V_t^{\dagger} = .04 + .29\,RRAT(t) - .01\,GIP(t) - .11\,INF(t)$$
$\quad (.11)\ (.42) \quad (.005)^{\ddagger} \quad (.42)$

$R^2 = .39$, $DW = 2.34$, $Q(24) = 15.6$

B. 1988-94

(1)

$$RV_t = .00 + .34\,V(t) + .02\,GIP(t) - .28\,INF(t)$$
$\quad (.07)(.16)^{\ddagger} \quad (.01)^{\ddagger} \quad (.13)^{\ddagger}$

$R^2 = .11$, $DW = 2.15$, $Q(27) = 29.4$

(2)

$$RR_t = -.19 - .18\,RR(t-1) + 1.9\,V(t) - 6.5\,RRAT(t) - 9.77\,INF(t)$$
$\quad (.77)(.11) \quad (1.74) \quad (3.55)^{\ddagger} \quad (3.86)^{\ddagger}$

$R^2 = .11$, $Q(27) = 34.8$

(3)

$$NR_t = -.23 - .23\,NR(t-1) + 2.5\,V(t) - 5.43\,RRAT(t) - 7.11\,INF(t)$$
$\quad (.73)(.11)^{\ddagger} \quad (1.72) \quad (3.36) \quad (3.63)^{\ddagger}$

$R^2 = .10$, $Q(27) = 32.1$

(4)

$$V_t^{\dagger} = -.01 + .33\,RRAT(t) - .01\,GIP(t) + .40\,INF(t)$$
$\quad (.04)\ (.36) \quad (.006)^{\ddagger} \quad (.38)$

$R^2 = .08$, $DW = 1.95$, $Q(24) = 33.0$

* Monthly.

† Denotes that a Cochrane-Orcutt procedure has been employed.

‡ Denotes significance at the 5 percent level or higher.

Standard errors are in parentheses.

DW: Durbin-Watson statistic

Q: Ljung-Box Q statistic

Table 4

Regressions from Stock Market Data*: Taiwan

A. 1983-88

(1)

$$RV_t = -.02 + 1.1\,GIP(t) + 9.1\,INF(t)$$
$\quad (.05) \quad (.52)^{\dagger} \quad (6.5)$

$R^2 = .08$, $DW = 1.8$, $Q(24) = 13.4$

(2)

$$RR_t = .00 + .23\,RR(t-1) - .01\,RRAT(t) - 7.3\,INF(t)$$
$\quad (.02)(.13)^{\dagger} \quad (.01) \quad (8.5)$

$R^2 = .18$, $Q(24) = 19.6$

B. 1988-94

(1)

$$RV_t^{\ddagger} = -.03 + .5\,GIP(t) - 7.1\,INF(t)$$
$\quad (.05)(.50) \quad (4.0)^{\dagger}$

$R^2 = .11$, $DW = 1.88$, $Q(21) = 14.1$

(2)

$$RR_t = .00 + .36\,RR(t-1) - .01\,RRAT(t) - 18.7\,INF(t)$$
$\quad (.01)(.11)^{\dagger} \quad (.01) \quad (10.1)^{\dagger}$

$R^2 = .21$, $Q(24) = 26.8$

(3)

$$NR_t = .00 + .35\,NR(t-1) - .01\,RRAT(t) - 17.8\,INF(t)$$
$\quad (.01)\ (.11)^{\dagger} \quad (.01) \quad (10.1)^{\dagger}$

$R^2 = .17$, $Q(24) = 25.4$

* Monthly.

† Denotes significance at the 5 percent level or higher.

‡ Denotes that a Cochrane-Orcutt procedure has been employed.

Standard errors are in parentheses.

DW: Durbin-Watson statistic

Q: Ljung-Box Q statistic

Table 2 reports the results of estimating similar regressions using stock market data from Chile. Here RV represents the growth rate of the real value of stock market transactions on the Santiago Stock Exchange, and RRAT is the real interest rate on 30–89 day bank deposits. A lack of daily data prevents us from examining the volatility of stock market returns. As in the case of the United States, we see that higher rates of inflation significantly reduce investors' real and nominal rates of return on the stock exchange. The point estimate suggests that higher inflation also depresses the growth rate of market transactions, although here the

[31] Boudoukh and Richardson (1993), using a much longer time series, also find that higher rates of inflation have reduced real stock market returns in the United States and in the United Kingdom.

Figure 8

Ratio of the Value of Stock Market Transactions to GDP: Korea

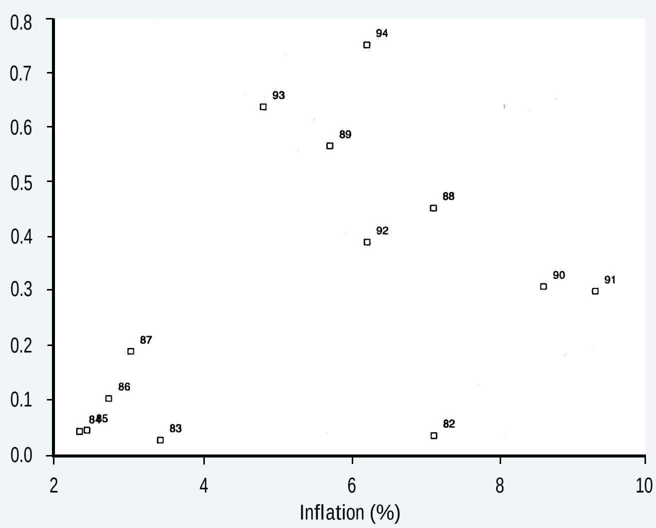
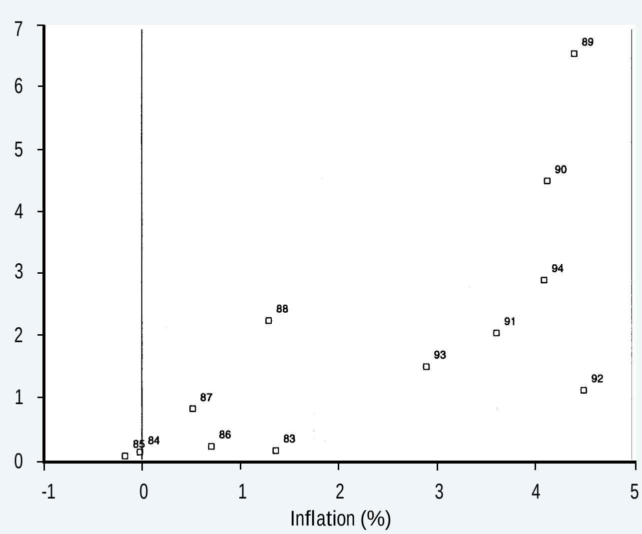


Figure 9

Ratio of the Value of Stock Market Transactions to GDP: Taiwan



point estimate is not significantly different from zero.

Figure 7 depicts the ratio of the value of stock market transactions to GDP for Chile, plotted against its rate of inflation. Again we perceive a negative relationship, particularly if the one single-digit inflation year (1982) is excluded as an outlier.

Tables 3 and 4 report analogous regression results for Korea and Taiwan. Here we proceed somewhat differently, since both Korea and Taiwan experienced fairly pronounced jumps in their rates of inflation in 1988 and 1989, respectively. In particular, in Korea the average monthly inflation rate was 0.27 percent over the period 1982–87, while from 1988–94 it was 0.54 percent. In Taiwan, the average monthly rate of inflation over the period 1983–88 was 0.07 percent, but jumped to 0.33 percent from 1989–93. These increases are apparent in Figures 8 and 9, respectively.

In view of these marked changes in the inflation rate, we proceeded as follows. For each country we divided the sample and ran regressions analogous to those reported above. For Korea the results are reported in Table 3. Over the low inflation period (1982–87), inflation has no significant effects on the real return on equity, its volatility, or on the growth rate of stock market transactions. However, during the period of higher inflation, increases in the rate of inflation lead to statistically significant reductions in the growth rate of transactions and the real and nominal return on equity. With respect to the volatility of market returns, our point estimates again suggest that inflation leads to higher volatility, but the inflation coefficient is not significantly different from zero.

Figure 8 represents Korea's ratio of the value of stock market transactions to GDP and its rate of inflation. Clearly, in the higher inflation period of 1988–93, the negative relationship between market activity and the rate of inflation is highly pronounced. This is not the case for the low-inflation period 1982–87. Here then we see some evidence for threshold effects: Inflation seems to have significant adverse consequences only after it exceeds some critical level.

Table 4 repeats the same regression procedure for Taiwan, but lack of daily data prevents us from constructing a volatility of returns measure. Here we see a similar pattern to that for Korea. During the period of low inflation (1983–88), the

Table 5

Comparisons Across Countries: Simple Correlations of Market and Macrovariables with the Inflation Rate (INF)

	Country / Period					
Variable	U.S. 1958-93	Chile 1981-91	Korea* 1982-87	Korea† 1988-94	Taiwan‡ 1983-88	Taiwan§ 1989-93
RV	–.06	.02	–.12	–.19	.12	–.20
RR	–.25	–.15	–.06	–.20	.12	–.23
V	.05	—	.09	.12	—	—
GIP	.02	.10	–.14	.03	–.12	–.24
RRAT	–.70	–.67	–.95	–.91	–.99	–.99

* Average monthly inflation rate 0.27 percent.

† Average monthly inflation rate 0.54 percent.

‡ Average monthly inflation rate 0.07 percent.

§ Average monthly inflation rate 0.33 percent.

effect of inflation on the growth rate of real stock market activity is insignificant and similarly for the real returns on equity. However, in the period of high inflation, both the growth rate of real equity market activity and the real returns on equity were adversely effected by inflation in a statistically significant way. Nominal equity returns are negatively associated with inflation, with a *t*-value of about 1.6. Here we see further evidence that inflation may be detrimental only after it exceeds some threshold level.

Figure 9 displays Taiwan's value of stock market transactions to GDP ratio, as well as its inflation rate. Clearly, this measure does not suggest that inflation has been detrimental to the level of equity market activity.

Table 5 shows simple correlations of the financial variables with the inflation rate for each of the countries and subperiods. These results are quite consistent with the regression results. On the whole, this empirical evidence seems to support our model's predictions. We have even seen evidence that inflation's adverse consequences may only be observed if the rate of inflation is sufficiently high.

CONCLUSIONS

Both our theoretical analysis and our empirical evidence indicate that higher rates of inflation tend to reduce the real rates of return received by savers in a variety of markets.[32] When credit is rationed, this reduction in returns worsens informational frictions that interfere with the operation of the financial system. Once inflation exceeds a certain critical rate, a potential consequence is that the financial system provides less investment capital, resulting in reduced capital formation and long-run levels of real activity. Such forces need not operate at low rates of inflation, providing an explanation of why the consequences of higher inflation seem to be so much more severe once inflation exceeds some threshold level.

In addition, high enough rates of inflation force endogenously arising economic volatility to be observed. Thus, as we observe, high inflation induces inflation variability and variability in rates of return on all savings instruments. Theory predicts that this volatility should be transmitted to real activity as well.

Obviously, these results have been obtained in the context of a highly stylized and simplified model of the financial system. How general are they? We believe they are quite general. Boyd and Smith (forthcoming) produce a model of a financial system that is subject to a costly state verification problem, one where investors provide some internal financing of their own investment projects. Again, two monetary steady-state equilibria exist and both

[32] Further evidence on this point appears in Boyd, Levine, and Smith (1995).

can potentially be approached. Thus development trap phenomena arise. In the steady state with higher levels of real activity, higher inflation interferes with the provision of internal finance, thereby exacerbating the costly state verification problem. As a result, greater inflation reduces the long-run level of real activity, the level of financial market activity, and real returns to savers. Moreover, as is the case here, high enough rates of inflation force endogenously generated economic volatility to emerge. And, interestingly, Boyd and Smith (forthcoming) obtain a result that is not available here: Once inflation exceeds a critical level, it is possible that only the low-activity steady state can be approached. Inflation rates exceeding this level can then force the kinds of crises discussed by Bruno and Easterly (1995). Related results are obtained by Schreft and Smith (forthcoming and 1994) in models where financial market frictions take the form of limited communication, as in Townsend (1987) and Champ, Smith, and Williamson (forthcoming).

A shortcoming of all of the models mentioned—including ours—is that they do not give rise to distinct and/or interesting roles for debt and equity markets. Empirical evidence suggests that both kinds of markets are adversely affected by high inflation.[33] This is a natural topic for future investigation.

[33] See Boyd, Levine, and Smith (1995).

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Appendix

PROOF OF RESULT 1

It will be useful to begin by describing some properties of the function $H(z) \equiv z(1+z)^{(1-\rho)/\rho}$. Clearly, $H(0) = 0$, and

$$(A1) \qquad \lim_{z\to\infty} H(z) = 0$$

is established by an application of L'Hopital's rule. Moreover, clearly,

$$(A2) \qquad zH'(z)/H(z) = 1 + \left[(1-\rho)/\rho\right]\left[z/(1+z)\right].$$

Thus $H'(z) \geq (<)\ 0$ holds if and only if $z \geq (<) -\rho$.

It follows from these observations that equation 22 has a solution if and only if equation 26 holds. This is readily verified to be equivalent to $\sigma \geq \hat{\sigma}$, with $\hat{\sigma}$ defined by equation 25.

When equation 22 has a solution, the associated value of k can be obtained from equation 19. Equation 23′ then produces m. Evidently, m is positive if and only if $A(\sigma) > 1$.

PROOF OF RESULT 2

Using equation 14 to replace k_{t+1} in equation 16′ gives the relation

$$(A3) \quad m_{t+1} = A(\sigma)m_t - \left[x/(1-\lambda)\right]m_t^2/w(k_t).$$

Equations 14 and A3 govern the evolution of the sequence $\{k_t, m_t\}$. Near a steady state this evolution is described by a linear approximation of these two equations. Letting (k,m) denote any pair of steady-state equilibrium values for the capital-labor ratio and real balances, this linear approximation is given by $(k_{t+1}-k, m_{t+1}-m)' = J(k_t-k, m_t-m)'$, where J is the Jacobian matrix

$$J = \begin{bmatrix} \lambda w'(k) & -1/\sigma \\ \left[(1-\lambda)/x\right]\left[A(\sigma)-1\right]^2 w'(k) & 2-A(\sigma) \end{bmatrix}.$$

Let $T(\sigma)$ and $D(\sigma)$ denote the trace and determinant, respectively, of J, where we explicitly denote their dependence on σ.

It is well-known from Azariadis (1993, chapter 6.4) that a steady state is a saddle if $T(\sigma) > 1 + D(\sigma)$. A steady state is a sink if $|D(\sigma)| < 1$ and $-1 - D(\sigma) < T(\sigma) < 1 + D(\sigma)$.

We now state the following preliminary result.

LEMMA 1. *At the low (high) capital stock steady state,* $D(\sigma) > (<)\ 1$ *holds.*

Proof. It is straightforward to show that

$$(A4) \qquad D(\sigma) = (1-\rho)/\left[1+\hat{z}(\sigma)\right].$$

Since $z(\sigma) < -\rho\ \left[\bar{z}(\sigma) > -\rho\right], D(\sigma) > (<) 1$ holds at the low- (high-) capital-stock steady state.

It is now possible to demonstrate the following.

LEMMA 2. *At the low- (high-) capital-stock steady state,* $T(\sigma) > (<)\ 1 + D(\sigma)$ *holds.*

Proof. As is readily verified,

$$(A5) \qquad T(\sigma) = 2 - A(\sigma) + A(\sigma)D(\sigma).$$

Thus $T(\sigma) > (<)\ 1 + D(\sigma)$ holds if and only if

$$(A6) \qquad [A(\sigma) - 1]D(\sigma) > (<)\ [A(\sigma) - 1].$$

$A(\sigma) > 1$ and Lemma 1 implies that $> (<)$ holds at the low- (high-) capital-stock steady state.

Lemma 2 implies that the low-capital-stock steady state is a saddle. Paths approaching it necessarily do so monotonically if $T(\sigma) > 0$ at that steady state. But $T(\sigma) > 0$ follows from equation A5 and Lemma1. Thus Result 2(a) is established.

Lemmas 1 and 2 also imply that the high-capital-stock steady state is a sink if

$$(A7) \qquad T(\sigma) > -1 - D(\sigma)$$

holds at that steady state. By equation A5, equation A7 is equivalent to

Appendix

(A7′) $D(\sigma) > [A(\sigma) - 3]/[A(\sigma) + 1]$.

Equation A4 implies that equation A7′ necessarily holds if $3 \geq A(\sigma)$, which in turn is implied by $3 \geq A(\hat{\sigma})$ The high-capital-stock steady state is thus a sink, if $\hat{\sigma}$ is not too large.

PROOF OF RESULT 3

It is well-known that paths approaching a steady state do so nonmonotonically if $T(\sigma)^2 < 4D(\sigma)$ holds, as in Azariadis (1993, chapter 6.4). We first establish the following.

LEMMA 3. *$T(\sigma) > 0$ holds if σ is sufficiently close to $\hat{\sigma}$.*

Proof. From equation A5, $T(\sigma) > 0$ holds if

(A8) $2 > A(\sigma)[1 - D(\sigma)]$

where at the high-capital-stock steady state

(A9) $D(\sigma) = (1 - \rho) / [1+\bar{z}(\sigma)]$.

Thus $\lim_{\sigma \uparrow \bar{\sigma}} D(\sigma) = 1$.

It follows that equation A8 necessarily holds for large enough values of σ.

Lemma 3 implies that $T(\sigma)^2 < 4D(\sigma)$ holds for large enough σ if and only if

(A10) $T(\sigma) < 2\sqrt{D(\sigma)}$.

Substituting equation A5 into equation A10 and rearranging terms yields the equivalent condition

(A10′) $2[1-\sqrt{D(\sigma)}]$

$$< A(\sigma)[1-\sqrt{D(\sigma)}][1+\sqrt{D(\sigma)}]$$

or, since $D(\sigma) < 1$,

(A11) $[2-A(\sigma)]/A(\sigma) < \sqrt{D(\sigma)}$.

We now show that equation A11 holds for $\sigma = \hat{\sigma}$ and hence by continuity, that it holds for σ sufficiently near $\hat{\sigma}$. In particular,

$$\lim_{\sigma \uparrow \bar{\sigma}} [2 - A(\sigma)] / A(\sigma) =$$

$$\{2-[x\lambda / (1 - \lambda)]\hat{\sigma}\}/[x\lambda / (1 - \lambda)\hat{\sigma}]$$

while

$$\lim_{\sigma \uparrow \bar{\sigma}} \sqrt{D(\sigma)} = 1.$$

Thus equation A11 holds for σ near $\hat{\sigma}$ if

(A12) $2-[x\lambda / (1-\lambda)]\hat{\sigma} < [x\lambda / (1-\lambda)]\hat{\sigma}$.

But equation A12 is implied by $A(\hat{\sigma})>1$. This establishes the result.

PROOF OF RESULT 4

$z(\sigma)$ and $\bar{z}(\sigma)$ both satisfy

$H(z) = a^{-1/\rho}[x\sigma/(1 - \lambda)]$.

Thus $z^*(\sigma)$ satisfies equation 30 if and only if $H[z^*(\sigma)] \geq H[\underline{z}(\sigma)]$ holds. As is apparent from Figure1, this will be the case if and only if $\underline{z}(\sigma) \leq z^*(\sigma) \leq \bar{z}(\sigma)$.

RESULT 6

$z^*(\sigma)$ intersects $\hat{z}(\sigma)$ at most twice.

Proof. $z^*(\sigma)$ satisfies

(A13) $[1 + z^*(\sigma)]^{(1-\rho)/\rho} \equiv a^{-1/\rho}/\sigma$.

Multiplying both sides of equation A13 by $z^*(\sigma)$ gives the equivalent condition

(A13′) $H[z^*(\sigma)] = a^{-1/\rho} z^*(\sigma)/\sigma$.

Moreover, whenever $z^*(\sigma) = \hat{z}(\sigma)$, we have

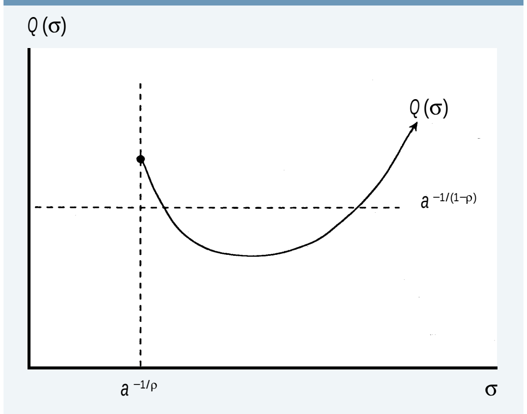
(A14) $H[z^*(\sigma)] = H[\hat{z}(\sigma)]$

$$\equiv a^{-1/\rho}[x\sigma / (1 - \lambda)].$$

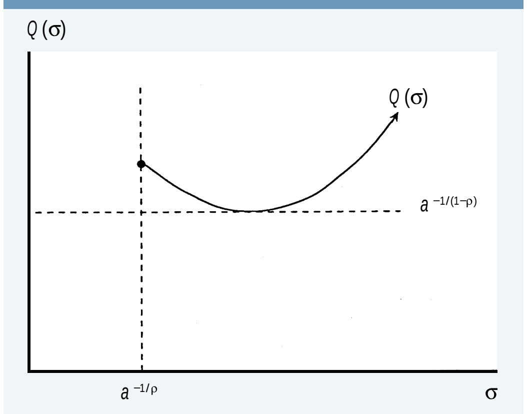
Equations A13′ and A14 imply that $z^*(\sigma) = \hat{z}(\sigma)$ if and only if

Appendix

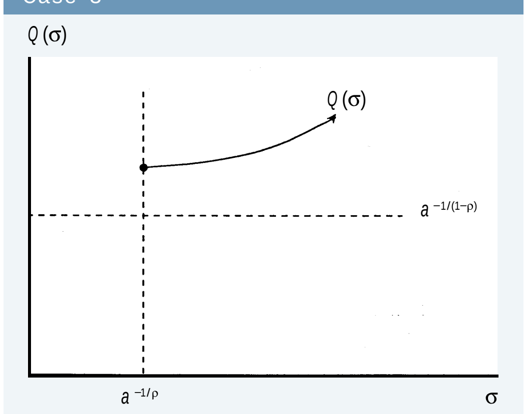
Case 1



Case 2



Case 3



(A15) $z^*(\sigma) = [x/(1-\lambda)]\sigma^2$.

Equation A15 is readily shown to be equivalent to the condition

(A15′) $a^{-1/(1-\rho)} = \sigma^{\rho/(1-\rho)} + [x/(1-\lambda)]\sigma^{(2-\rho)/(1-\rho)} \equiv Q(\sigma)$.

The function $Q(\sigma)$ is depicted in the Figure. It is readily demonstrated that $Q(a^{-1/\rho}) > a^{-1/(1-\rho)}$ and that $Q'(\sigma) \geq 0$ holds if and only if

(A16) $\sigma \geq [-\rho(1-\lambda)/x(2-\rho)]^{0.5}$.

There are therefore three possibilities.

Case 1.

Suppose that

(A17) $a^{-1/\rho} < [-\rho(1-\lambda)/x(2-\rho)]^{0.5}$

and that

(A18) $Q\{[-\rho(1-\lambda)/x(2-\rho)]^{0.5}\} < a^{-1/(1-\rho)}$.

Then equation A15′ has exactly two solutions, as shown in the Figure. It follows that $z^*(\sigma)$ intersects $\hat{z}(\sigma)$ exactly twice.

Case 2.

Suppose that equation A17 holds but that equation A18 fails. Then there is at most one intersection of $z^*(\sigma)$ and $\hat{z}(\sigma)$, as shown in the Figure.

Case 3.

Suppose that equation A17 fails. Then $Q'(\sigma) \geq 0$ holds for all $\sigma \geq a^{-1/\rho}$. There are no intersections of $z^*(\sigma)$ and $\hat{z}(\sigma)$, as the Figure shows.

These three cases exhaust the set of possibilities and establish the result.

EXISTENCE OF STEADY-STATE EQUILIBRIA

Result 6 implies that steady-state equilibria exist, in general, if and only if σ satisfies equation a3 *and* equations A17 and A18 hold. It will be useful to have a sufficient condition implying that equations A17 and A18 are satisfied. From Figure 4a,

Appendix

it is apparent that $z^*(\sigma)$ intersects $\hat{z}(\sigma)$ if $z^*(\hat{\sigma}) > -\rho$. We now describe when this condition holds.

RESULT 7. $z^*(\hat{\sigma}) > -\rho$ *holds if and only if*

$$\text{(A19)} \quad \hat{\sigma} > \left[-\rho(1-\lambda)/x\right]^{0.5}.$$

Proof. Equations 20 and 25 imply that $z^*(\sigma) > -\rho$ holds if and only if

$$\text{(A20)} \quad (a^{-1/\rho})^2\left[x/(1-\lambda)\right] < -\rho\left[(1-\rho)^{(1-\rho)/\rho}\right]^2.$$

Equation A20 is easily shown to be equivalent to equation A19.

Thus equation A19 implies the existence of multiple steady states for all σ satisfying equation a3.

PROOF OF RESULT 5

This result has already been established when credit rationing obtains. Thus we must establish that $m > 0$ holds at the Walrasian steady state. From equation 18, in a Walrasian steady state

$$\text{(A21)} \quad m = \sigma k\{\lambda[w(k)/k] - 1\}.$$

Since $z^*(\sigma) \quad \left[z(\sigma), \overline{z}(\sigma)\right]$, we also have that $w(k)/k \geq x\sigma/(1-\lambda)$. Hence $A(\sigma) > 1$ implies that $m > 0$ holds.

IMPOSSIBILITY OF POOLING

Azariadis and Smith (forthcoming) prove that there is never an incentive for an intermediary to pool type 2 and dissembling type 1 agents in a Walrasian equilibrium. They also prove that there is no such incentive under credit rationing if

$$\text{(A22)} \quad 1 \geq \sigma(1-\lambda)f'(k) = \sigma(1-\lambda)a^{1/\rho}\left[1 + \hat{z}(\sigma)\right]^{(1-\rho)/\rho}$$

or equivalently, if

$$\text{(A22}'\text{)} \quad \hat{z}(\sigma) \geq \sigma(1-\lambda)a^{1/\rho}H\left[\hat{z}(\sigma)\right] \equiv x\sigma^2.$$

Equation A22′ holds at the high-capital-stock steady state if

$$\text{(A23)} \quad \overline{z}(\sigma)/\sigma^2 \geq x.$$

Since the left-hand side of equation A23 is decreasing in σ, A23 holds for all $\sigma \leq \hat{\sigma}$ if

$$\text{(A24)} \quad \overline{z}(\hat{\sigma})/x \geq \hat{\sigma}^2.$$

or equivalently, if

$$\text{(A24}'\text{)} \quad -\rho/x \geq \hat{\sigma}^2.$$

Equation A22′ holds at the low-capital-stock steady state if

$$\text{(A25)} \quad z(\sigma)/\sigma^2 \geq x.$$

Clearly, a sufficient condition for equation A25 is that

$$\text{(A26)} \quad z(\sigma) \geq x\hat{\sigma}^2.$$

Since $z(\sigma) = \left[x/(1-\lambda)\right]\sigma^2$ it follows that

$$\text{(A26}'\text{)} \quad \sigma^2 \geq (1-\lambda)\hat{\sigma}^2$$

is sufficient for equation A22′ to hold at the low-capital-stock steady state.

To summarize, equation A22′ holds at $z(\sigma)$ and $\overline{z}(\sigma)$ for all $\sigma \quad \left[\sigma, \hat{\sigma}\right]$ if equations A24′ and A26′ hold.

RESULT 8

(a) $(1-\lambda)/x\lambda \in \left[\underline{\sigma}, \overline{\sigma}\right]$ if and only if

$$\text{(A27)} \quad \hat{\sigma} \geq -\rho(1-\rho)^{(1-\rho)/\rho} \times \lambda\left\{1 + \left[(1-\lambda)/x\lambda^2\right]\right\}^{-(1-\rho)/\rho}.$$

(b) $(1-\lambda)/x\lambda \leq \underline{\sigma}$ holds if and only if

Appendix

$$\text{(A28)}\quad (1-\lambda)/x\lambda < \left[-\rho(1-\lambda)/x(2-\rho)\right]^{0.5}$$

and

$$\text{(A29)}\quad \hat{\sigma} \le -\rho(1-\rho)^{(1-\rho)/\rho} \times \left\{1+\left[(1-\lambda)/x\lambda^2\right]\right\}^{-(1-\rho)/\rho}$$

Proof. It is easy to verify that $(1-\lambda)/\lambda x \quad [\sigma, \overline{\sigma}]$ if and only if

$$\text{(A30)}\quad Q\left[(1-\lambda)/x\lambda\right] \le a^{-1/(1-\rho)}$$

holds. Using the definition of Q, it is straightforward to show that equation A30 is equivalent to

$$\text{(A30}'\text{)}\quad -a^{1/\rho}\left[x/(1-\lambda)\right]\rho(1-\rho)^{(1-\rho)/\rho} \ge -\rho(1-\rho)^{(1-\rho)/\rho}\lambda \times \left\{1+\left[(1-\lambda)/x\lambda^2\right]\right\}^{-(1-\rho)/\rho}.$$

But, as is apparent from equation 25, equation A30′ is equivalent to equation A27.

It can be shown that $(1-\lambda)x\lambda \ge \sigma$ holds if and only if $Q'\left[(1-\lambda)/x\lambda\right] < 0$ and $Q\left[(1-\lambda)/x\lambda\right] \ge a^{-1/(1-\rho)}$ are satisfied. The former condition is equation A28, the latter is equation A29.

DATA SOURCES

1. United States

Monthly data are available over the period 1958–93.

Sources.

- RV: Growth rate of real value of transactions on the New York Stock Exchange. (New York Stock Exchange *Factbook*, various dates.)
- RR: Real returns on the Standard & Poor's 500 index, inclusive of dividend yields. (Standard & Poor's Statistics, *SBBI Yearbook*, various dates.)
- NR: Nominal returns on the Standard & Poor's 500 index, inclusive of dividend yields. (Standard & Poor's Statistics, *SBBI Yearbook*, various dates.)
- V: Standard deviation of returns on the daily Standard & Poor's index. (*SBBI Yearbook*, various dates.)
- INF: Rate of inflation in the CPI. (Bureau of Labor Statistics.)
- GIP: Growth rate of industrial production index. (Federal Reserve Industrial Production Indices.)
- RRAT: Three-month Treasury bill rate, in real terms. (*Federal Reserve Bulletin*, various dates.)

2. Chile

Sources.

All data are from the *Boletoin mensual* (Banco Central de Chile). They are available monthly from 1981–91.

- RV: Growth rate of the real value of transactions on the Santiago Stock Exchange.
- RR: Real return to equity, inclusive of dividend yields.
- NR: Nominal returns to equity, inclusive of dividend yields.
- INF: Rate of change in the CPI.
- RRAT: Real rate of interest on 30- to 89-day bank deposits.

3. Korea

All monthly data are available from 1982–94.

Sources.

- RV: Growth rate of the real value of transactions on the Korea Stock Exchange. (*Securities Statistics Monthly,* Korea Stock Exchange, various dates.)
- RR: Real returns to equity, inclusive of dividend yields. (*Securities Statistics Monthly,* Korea Stock Exchange, various dates.)
- NR: Nominal returns to equity, inclusive of dividend yields. (*Securities Statistics Monthly*, Korea Stock Exchange, various dates.)

Appendix

V: Standard deviation of daily returns. (*Securities Statistics Monthly,* Korea Stock Exchange, various dates.)

INF: Rate of growth of the CPI. (*Economic Statistics Yearbook*, Bank of Korea, various dates.)

GIP: Growth rate of industrial production. (*Economic Statistics Yearbook*, Bank of Korea, various dates.)

RRAT: Three-month corporate bill rate, in real terms.

4. Taiwan

All data are available monthly from 1983–93.

Sources.

RV: Growth rate of the real value of stock transactions in the Taiwan area. (*Financial Statistics Monthly,* Central Bank of China, various dates.)

RR: Real returns, inclusive of dividend yields. (*Financial Statistics Monthly,* Central Bank of China, various dates.)

NR: Nominal returns, inclusive of dividend yields. (*Financial Statistics Monthly*, Central Bank of China, various dates.)

INF: Growth rate of CPI. (*Monthly Statistics of the Republic of China*, Central Bank of China, various dates.)