Alberto Trejos is an assistant professor at Northwestern University. Randall Wright is professor of economics at the University of Pennsylvania, and consultant to the Federal Reserve Banks of Cleveland and Minneapolis.

Search-Theoretic Models of International Currency

Alberto Trejos and Randall Wright

Search-based theories of the exchange process provide economists with a way of formalizing the microfoundations of monetary economics and to discuss a variety of issues in monetary theory and policy in a new light.[1] Many questions that cannot even be formulated in more traditional monetary models can be profitably analyzed in these new models. In this essay, we propose to review recent developments using search-based models to study some international monetary issues.

There seems little doubt that international monetary economics is of central importance today. Of the types of questions one would ultimately like to answer, consider these:

- Should Europe adopt a common currency?
- If so, should individual countries continue to issue local currencies?
- If a state or province separates from a nation (as Quebec periodically discusses in Canada), how should it design its monetary system?
- Should more Latin American countries adopt currency boards, where each unit of local currency is backed by central bank reserves in U.S. dollars (as in Argentina)?
- Should more Latin American countries simply abandon their local currency and switch to U.S. dollars (as Panama did)?

Though we do not claim to be ready to provide definitive answers to all of these important questions at this time, we do think that search-theoretic models are in principle well suited to address these types of issues. More traditional theory, including models that include cash-in-advance or money-in-the-utility-function type assumptions, are clearly ill suited in this regard. They are ill suited because too much is decided by assumption. In particular, the answer to the question, "Which monies circulate in which countries?" is determined at the outset by the modeler when he chooses which money to put in which cash-in-advance constraint or in which utility function. In contrast, search models are designed to determine *endogenously* which monies circulate where.

We begin by reviewing a version of the model in Matsuyama, Kiyotaki, and Matsui (1993). Basically, it can be described as follows. There are two countries, each of which issues its own currency. There is a meeting or matching technology that describes the frequency with which a given individual interacts with other individuals, both locals and foreigners (it is assumed that one interacts with individuals from one's own country more often than a foreigner interacts with these same individuals). When two agents meet, one with money and the other with real output for sale, they have to decide whether to trade. Parameter values, as well as expectations regarding other agents' behavior, jointly determine this decision and thereby determine the realm of circulation for each currency. Potentially there are three dis-

[1] See, for example, in addition to the work discussed below, Kiyotaki and Wright (1989, 1991, and 1993), Aiyagari and Wallace (1991 and 1992), Williamson and Wright (1994), Trejos (forthcoming) and Ritter (1995).

tinct types of equilibria—or *regimes*—in which we are interested, where none, one, or both of the currencies circulate internationally.

This model allows one to answer questions like the following:

- What features of a country make it possible, or likely, for its currency to circulate internationally?
- How and when can local currencies survive in the presence of a universally accepted international currency?
- Does an international currency emerge naturally as economies become integrated?
- What are the costs and benefits to a country of having its currency serve as an international medium of exchange?
- An extension of this model in Zhou (1994) additionally allows one to study the issue of when agents would want to exchange currencies in such a world.

There are shortcomings with the models mentioned in the previous paragraph. Perhaps the most obvious is that in these models, as in all of the first-generation search-based models of money, every exchange is assumed to be a one-for-one trade. This simplifying assumption makes it possible to discuss the process of exchange—and, in particular, which objects circulate as media of exchange among which agents—without tackling the determination of the relative values of these objects. Unfortunately, however, it obviously also makes it impossible to talk about prices or exchange rates. Therefore, we also present the extension of the model in Trejos and Wright (1995b) designed to endogenize prices using bargaining theory.[2]

This extension allows us to raise a whole range of new issues, including the following:

- How does the fact that a currency circulates internationally affect its purchasing power at home?
- Where does an international currency purchase more—at home or abroad?
- What are the effects on seigniorage and welfare in each country when one money becomes an international currency?
- How are policies designed to maximize either seigniorage or welfare affected by concerns of currency substitution?
- How are national monetary policies connected, and what is the scope for international cooperation?

In the next section, we outline the basic assumptions on which the model is built. The third section presents the indivisible output version of the model in Matsuyama, Kiyotaki, and Matsui (1993). We then present the divisible output model with bargaining in Trejos and Wright (1995b), followed by a discussion of some policy implications. The final section presents some brief conclusions.

THE BASIC MODEL

The economy consists of two countries, labeled $i = 1, 2$. One's country is important in that it determines the frequency with which one interacts with other agents. Individuals interact, or meet, bilaterally according to a random matching process in continuous time, and $\hat{\alpha}_{ij}$ denotes the Poisson arrival rate at which a citizen of Country i meets citizens of Country j. We assume that $\hat{\alpha}_{ii} \geq \hat{\alpha}_{ji}$ *for* $j \neq i$. This simply says that, for example, a Mexican meets Mexicans more frequently than an American meets Mexicans.

Each country starts with a large number of citizens, and the fraction of individuals from Country i is N_i with $N_1 + N_2 = 1$. Thereafter, both populations grow at the

[2] Bargaining was first introduced into (one-country) search models of money in Trejos and Wright (1995a) and Shi (1995b). Other recent applications include Aiyagari et al. (1996), Trejos (1994), and Shi (1995a).

same rate $\gamma \geq 0$. The population sizes and the meeting technology parameters are not independent because we have the identity $N_1\hat{\alpha}_{12} = N_2\hat{\alpha}_{21}$ (both sides of the equality give the total number of international meetings per unit time). Here we take $\hat{\alpha}_{ij}$ as primitive and let the populations be free to satisfy this identity. A special case is the specification actually used in Matsuyama, Kiyotaki, and Matsui (1993), which has $\hat{\alpha}_{ii} = \beta N_1$ *and* $\hat{\alpha}_{ij} = \beta^* N_j$, *with* $\beta^* < \beta$. Thus, arrival rates are proportional to country size, but they are smaller across countries than they are within countries. As β^* gets closer to β, the two countries become more integrated.[3]

Agents are distinguished not only by their country and date of birth, but also by their tastes and technologies. As in the typical monetary search model, we need to adopt some notion of specialization. Here, for simplicity, it is assumed that there are $K \geq 3$ goods and the population of each country contains equal numbers of K types, where each Type k consumes only Good k and produces only Good k +1 (modulo K; that is, Type K produces Good 1). If we assume that meetings are random with respect to consumption-production type, then $\alpha_{ij} = \hat{\alpha}_{ij} / K$ is the rate at which a citizen of Country i meets a citizen of Country j who consumes the good he produces; it is also the rate at which the citizen of Country i meets a citizen of Country j who produces the good he consumes.

There is no centralized market or auctioneer in this model: All trade is bilateral and quid pro quo. The large number of agents rules out private credit because the probability of meeting a particular individual a second time is zero. Our specification for tastes and technologies rules out direct barter. We also make the assumption that goods are nonstorable, which rules out commodity money. Hence, trade requires the use of some form of fiat money as a medium of exchange. However—and this is the crucial point—we do not impose that any particular currency plays this role in any particular transaction. This is what distinguishes the class of models under consideration from ones with particular cash-in-advance or money-in-the-utility-function assumptions imposed exogenously.[4]

Fiat money is introduced by the government of Country j issuing one unit of Currency j to some fraction M_j (0,1) of its newborn citizens, at each point in time, in exchange for some amount of real output (how much real output depends on what assumptions we make). Issuing money in this way yields a continuous flow of seigniorage, although notice that the only time private agents interact with the government is when they first enter the economy. In the special case where $\gamma = 0$, the government simply issues an input of currency to some fraction of its citizens at the initial date and then shuts down.

To keep things tractable, we make the following assumptions. First, we assume that an agent holding a unit of currency always spends it all at once, which could obviously be guaranteed if we simply say that the monetary object is indivisible. This implies that no one holding currency ever holds less than one unit. Second, we assume that, except for the newborn, no agent can produce until after he consumes. This implies that two agents with currency do not trade with each other, and no one ever holds more than one unit of money. Hence at each point in time there will be some agents with one unit of money each, called *buyers*, and a disjoint group with no money, called *sellers*. Let the fraction of buyers from Country i with Currency j be denoted m_{ij}, and the fraction of sellers from Country i be denoted $m_{i0} = 1 - m_{i1} - m_{i2}$. Then the vector $m = (m_{ij})$ completely describes the asset distribution across agents.

What happens when a buyer and seller meet depends on which version of the model we consider. In Matsuyama, Kiyotaki, and Matsui (1993), output is indivisible and the same across the two countries. Thus if you find a seller who can produce the right type of output, there is no question that you always want to trade your money for his one indivisible good, and, in particular, you do not care

[3] In this model one does not physically travel between one country and another, nor does one choose in any other way to interact with foreigners instead of fellow citizens. It is simply that you sometimes meet foreigners in your daily routine. Imagine, for example, a town on the border populated by both Mexicans and Americans. Americans trade with fellow Americans and with Mexicans, although perhaps less frequently with the latter and vice versa. What we are interested in here is the monetary nature of these interactions; that is, which currencies get traded by whom?

[4] The simplifying assumptions in the text allow us to focus exclusively on the use of fiat currencies as media of exchange, but in principle they can all be relaxed. For examples of monetary search models with credit, see Hendry (1992) or Shi (1995a); for examples with some direct barter, see Kiyotaki and Wright (1991 and 1993) or Burdett et al. (1995); and for examples with commodity money, see Kiyotaki and Wright (1989), Aiyagari and Wallace (1991), or Li (1995).

about the seller's nationality. In Zhou (1994), output is indivisible but differs across the two countries, and consumers have tastes that fluctuate randomly, which means they generally *do* care about the seller's nationality. In Trejos and Wright (1995b), a unit of output is the same across countries, but output is perfectly divisible and the amount of output that you get for your money needs to be negotiated. This means that you might care about the nationality of the seller if the (endogenous) prices differ across countries.

In any case, the important thing from our vantage is whether a buyer from Country *h* with Currency *i* trades with a citizen of Country *j* because this determines the steady-state distribution of assets across the population. In this article, we consider only equilibria where trade always occurs if a buyer with Currency *i* meets a seller from Country *i* (pesos definitely circulate in Mexico and dollars definitely circulate in the United States). Then the key issue is whether trade occurs when a seller meets a buyer with foreign currency—that is, whether exchange occurs when a buyer with Currency *i* meets a seller from Country $j \neq i$.

We distinguish the different types of possible equilibria—or different *regimes*—as follows. Let $\lambda_j = 1$ if within Country *j* we see Currency $i \neq j$ in circulation, in which case we call Currency *i* an *international currency*; and let $\lambda_j = 0$ if in Country *j* we do not see Currency *i* in circulation. Then a regime is a list of values for $\lambda = (\lambda_1, \lambda_2)$. The four possible regimes are $\lambda = (0,0)$, (1,1), (0,1), and (1,0). In the first case there is no international currency; in the second case both currencies are international; and in the final two cases only one currency is international. Because the last two are mirror images, we focus for now only on the latter, $\lambda = (1,0)$. Hence there are three cases to consider, with either 0, 1, or 2 international monies.[5]

Given a regime, one can determine the steady-state values of m_{ij}, independent of whether output is divisible, as follows. First, note that $\lambda_i = 0$ implies $m_{ij} = 0$ (if Americans never trade for pesos then they never hold pesos). Second, note that trades between a buyer and seller of the same nationality do not alter *m* because they leave the aggregate distribution unchanged. These considerations imply that the steady-state equations are

$$(1) \quad \alpha_{ij} m_{i0} m_{ji} - \alpha_{ij} m_{ii} m_{j0} \lambda_j + \gamma (M_i - m_{ii}) = 0,$$

$$(2) \quad \alpha_{ij} m_{i0} m_{jj} \lambda_i - \alpha_{ij} m_{ij} m_{j0} - \gamma m_{ij} = 0,$$

for $i \neq j$. Given λ and the identity $m_{i0} = 1 - m_{ii} - m_{ij}$, these can be solved for the steady-state asset distribution *m*.

Consider equation 1 (equation 2 has a similar interpretation). The first term says that m_{ii} increases when a seller from Country *i* meets a buyer from Country *j* with Currency *i* (given the maintained assumption that when a seller from Country *i* meets a buyer with Currency *i*, they always trade). The second term says that m_{ii} decreases when a buyer from Country *i* with Currency *i* meets a seller from Country *j* if they trade (that is, if $\lambda_j = 1$). The final term says that m_{ii} increases when the fraction of the newborn agents in Country *i* who receive currency, M_i, exceeds m_{ii}. Steady state requires the net change to be zero.

Given the asset distribution, the idea is to use dynamic programming to solve the individual decision problems concerning when to trade. The next section considers the model with indivisible output, where every trade is a one-for-one swap. The section after that considers the case where output is divisible and agents bargain.

THE MODEL WITH INDIVISIBLE OUTPUT

Here we analyze the model with indivisible output. This is a useful first step, in that it allows us to determine the circumstances under which the different currencies will be used in transactions between different agents without simultaneously solving the bargaining problems facing these agents. For simplicity, we are interested here only in steady-state equilibria, where the asset distribution and trading strategies are constant with respect to

[5] In the model in Matsuyama, Kiyotaki, and Matsui (1993), these are the only possible regimes, except for possibly equilibria where at most one of the monies has value or equilibria that are the same except for relabeling in the sense that we call American money *pesos* and Mexican money *dollars.* In the model in Trejos and Wright (1995b), there are other candidate regimes, where whether Seller *j* takes Currency *i* depends on who the buyer is (say, Mexican sellers take dollars from Americans but not from other Mexicans); but these candidate regimes can never be equilibria under relatively weak restrictions.

time. We also restrict attention to pure-strategy equilibria.

Let the utility from consuming a unit of one's consumption good be U and the disutility of producing one's production good be C, where $0 \leq C < U$. Let V_{ij} denote the expected lifetime utility for a buyer from Country i with Currency j, and V_{i0} expected lifetime utility for a seller from Country i with no money. Let $V = (V_{ij})$. We call these V_{ij}'s the *value functions*. They satisfy the following flow versions of the Bellman equations from dynamic programming,

$$(3)\quad rV_{i0} = (\alpha_{ii}m_{ii} + \alpha_{ij}m_{ji})(V_{ii} - V_{i0} - C) + (\alpha_{ii}m_{ij} + \alpha_{ij}m_{jj})\lambda_i(V_{ij} - V_{i0} - C)$$

$$(4)\quad rV_{ii} = (\alpha_{ii}m_{i0} + \alpha_{ij}m_{j0}\lambda_j)(U + V_{i0} - V_{ii})$$

$$(5)\quad rV_{ij} = (\alpha_{ii}m_{i0}\lambda_i + \alpha_{ij}m_{j0})(U + V_{i0} - V_{ij}),$$

where r is the rate of time preference. The value functions are indexed by the agent's nationality, i, but not by his (consumption/production specialization) type, k, because we will only consider equilibria that are symmetric in the sense that all types use the same strategy and receive the same payoff in equilibrium.

The Bellman equations are interpreted as follows. Equation 3 says that the flow value of being a seller from Country i is the rate at which one meets local or foreign buyers with Currency i multiplied by the gain from trade in such a meeting, plus the rate at which one meets local or foreign buyers with Currency j multiplied by the gain from trade if such a meeting results in trade (that is, if $\lambda_i = 1$). Equation 4 says that the flow value to holding domestic money is the rate at which one meets local sellers (who always take local currency) or foreign sellers who take local currency (which they do if $\lambda_i = 1$) multiplied by the gain from trading. Equation 5 has a similar interpretation.[6]

Whether exchange occurs in a meeting depends here exclusively on the seller because the buyer wants to trade in every meeting where the seller can produce his desired consumption good—he gets an indivisible unit of output in every trade. Recall that here we look only for equilibria where sellers always want to trade for local currency, although it still has to be checked that this is rational. The interesting issue is whether a seller wants to trade for foreign currency. Whether he wants to trade for foreign money depends on the following incentive condition:

$$(6)\quad V_{ij} - C > V_{i0} \qquad \lambda_i = 1.$$

$$(7)\quad V_{ij} - C < V_{i0} \qquad \lambda_i = 0.$$

That is, a seller accepts foreign money just in case the value of holding foreign money minus production costs exceeds the value of remaining a seller (and waiting for local money).

An equilibrium here can be defined as a list (λ, m, V) satisfying equations 1–7. For each possible regime we will solve equations 1–5 and check equations 6–7; if equations 6–7 are satisfied, this regime constitutes an equilibrium. In this way we can determine the parameter values that allow the existence of each type of equilibrium. For simplicity, and to facilitate comparison with Matsuyama, Kiyotaki, and Matsui (1993), we consider here only the limiting case where $C \rightarrow 0$.

NO INTERNATIONAL MONEY

Consider first the regime with no international money, and, therefore, no international trade: $\lambda = (0,0)$. In this case equations 1–2 imply $m_{ii} = M_i$ and $m_{ij} = 0$ for $j \neq i$. To see when this regime actually constitutes an equilibrium, insert m and $\lambda = 0$ into equations 3–5 to yield

$$(8)\quad rV_{i0} = \alpha_{ii}M_i(V_{ii} - V_{i0})$$

$$(9)\quad rV_{ii} = \alpha_{ii}(1 - M_i)(U + V_{i0} - V_{ii})$$

$$(10)\quad rV_{ij} = \alpha_{ij}(1 - M_j)(U + V_{i0} - V_{ij}).$$

Note that in this regime, citizens of Country i never actually hold Currency $j \neq i$, but V_{ij} gives the value they would achieve *if* they were to accept Currency j.

[6] One can prove that there are no currency exchanges in pure-strategy equilibrium; that is, there is no trade when an agent with Currency 1 meets an agent with Currency 2.

We need to check two things: the maintained hypothesis that agents always accept local currency and the incentive conditions in equations 6–7 that say, in this case, that they do not accept foreign currency. The former is satisfied for all parameter values in this case. Hence it remains only to check $V_{ij} \leq V_{i0}$. Simple algebra implies that this holds if and only if

$$(11) \quad \alpha_{ij}(1 - M_j) \leq \frac{\alpha_{ii}^2 M_i(1 - M_i)}{r + \alpha_{ii}}.$$

One thing we can conclude from this analysis is that for this regime to constitute an equilibrium we require α_{ij} to be small relative to α_{ii}. If one meets foreigners frequently enough, then it is not rational to reject foreign money even if you expect local sellers will reject it. Hence a very open economy is not likely to settle on an equilibrium where foreign money does not circulate. Another thing we can conclude is that for this regime to constitute an equilibrium, we require M_j to be big. If there are few foreign buyers and many foreign sellers, then it is easier to spend foreign money, and so you should accept it even if you expect local sellers will reject it. Other parameters have similarly reasonable effects.

TWO INTERNATIONAL MONIES

Now turn to the regime with two international currencies, $\lambda = (1,1)$. The first thing to do is to solve equations 1–2 for m_{ij}. Routine algebra yields the steady state for citizens of Country 1, which can be described by

$$(12) \quad m_{10} = 1 - \frac{(\gamma + \alpha_{21})M_1 + \alpha_{12}M_2}{\gamma + \alpha_{12} + \alpha_{21}}$$

$$(13) \quad m_{11} = M_1 \frac{\gamma(\gamma + \alpha_{12} + \alpha_{21}) + \alpha_{21}[(\gamma + \alpha_{21})(1 - M_1) + \alpha_{12}(1 - M_2)]}{(\gamma + \alpha_{12} + \alpha_{21})[\gamma + \alpha_{21}(1 - M_1) + \alpha_{12}(1 - M_2)]}.$$

The steady state for citizens of Country 2 is described by reversing the subscripts. Notice that m_{i0} is decreasing in both M_1 and M_2.

The value functions now satisfy:

$$(14) \quad rV_{i0} = (\alpha_{ii}m_{ii} + \alpha_{ij}m_{ji})(V_{ii} - V_{i0}) + (\alpha_{ii}m_{ij} + \alpha_{ij}m_{jj})(V_{ij} - V_{i0})$$

$$(15) \quad rV_{ii} = (\alpha_{ii}m_{i0} + \alpha_{ij}m_{j0})(U + V_{i0} - V_{ii})$$

$$(16) \quad rV_{ij} = (\alpha_{ii}m_{i0} + \alpha_{ij}m_{j0})(U + V_{i0} - V_{ij}).$$

These equations imply that $V_{ii} = V_{ij}$. That is, buyers from either country are indifferent between holding Currency i and Currency $j \neq i$. So the currencies are perfect substitutes. It also follows that $V_{ii} > V_{i0}$, and so equations 6–7 are satisfied for all parameter values. Hence, this regime always constitutes an equilibrium. The intuition is that if agents believe the two currencies will be accepted by all sellers then it is always rational for them to accept both currencies themselves.

ONE INTERNATIONAL MONEY

We now turn to the regime where citizens of Country 1 accept Currency 2, but not vice versa, $\lambda = (1,0)$. One can solve equations 1–2 for $m_{11} = M_1$ and $m_{21} = 0$. The distribution of Currency 2 holdings is given by:

$$(17) \quad m_{12} = \frac{\alpha_{12}(1 - M_1)M_2}{\gamma + \alpha_{12} + \alpha_{21}(1 - M_1)},$$

$$(18) \quad m_{22} = \frac{(\gamma + \alpha_{12})M_2}{\gamma + \alpha_{12} + \alpha_{21}(1 - M_1)}.$$

From these one can show that, for given values of M_i, m_{10} is lower and m_{20} higher in this regime than in the other regimes. Also, m_{i0} is decreasing in both M_1 *and* M_2.

The value functions for agents from Country 1 satisfy

$$(19) \quad rV_{10} = \alpha_{11}m_{11}(V_{11} - V_{10}) + (\alpha_{11}m_{12} + \alpha_{12}m_{22})(V_{12} - V_{10}),$$

$$(20) \quad rV_{11} = \alpha_{11}m_{10}(U + V_{10} - V_{11})$$

$$(21) \quad rV_{12} = (\alpha_{11}m_{10} + \alpha_{12}m_{20})(U + V_{10} - V_{12}),$$

and for agents from Country 2 they satisfy

$$(22)\quad rV_{20} = (\alpha_{21}m_{12} + \alpha_{22}m_{22})(V_{22} - V_{20})$$

$$(23)\quad rV_{21} = \alpha_{21}m_{10}(U + V_{20} - V_{21})$$

$$(24)\quad rV_{22} = (\alpha_{21}m_{10} + \alpha_{22}m_{20})(U + V_{20} - V_{22}).$$

One thing that follows immediately from these equations is that $V_{i2} > V_{i0}$, so citizens from both countries are willing to accept Currency 2.

Hence, to show when this regime constitutes an equilibrium, we need to check the incentive constraints $V_{11} \geq V_{10}$ (to check that it is rational for Country 1 sellers to accept Currency 1) and that $V_{21} \leq V_{20}$ (to check that it is rational for Country 2 sellers to reject Currency 1). The first condition is satisfied if and only if

$$(25)\quad \frac{\alpha_{11}m_{10}}{\alpha_{12}m_{20}} \geq \frac{\alpha_{11}m_{12}+\alpha_{12}m_{22}}{r+\alpha_{11}m_{10}+\alpha_{12}m_{20}};$$

the second is satisfied if and only if

$$(26)\quad \frac{\alpha_{21}m_{10}}{\alpha_{22}m_{20}} \leq \frac{\alpha_{21}m_{12}+\alpha_{22}m_{22}}{r+\alpha_{21}m_{10}+\alpha_{22}m_{20}}.$$

These can be interpreted as saying that it is relatively easy for a buyer from Country 1 to find a Country 1 seller, but relatively hard for a Country 2 buyer to find a Country 1 seller.

MODEL SUMMARY

Here we summarize what has been learned from the preceding analysis. Given our maintained assumptions, there are three qualitatively different types of equilibria. There is at most one equilibrium of each type (that is, each λ implies a unique m and V). Properties of the different regimes include:

1. $\lambda = (0,0)$ (dollars circulate only in America, pesos only in Mexico) is an equilibrium if and only if equation 11 holds, which, in particular, is true when α_{ij} is small relative to α_{ii}.

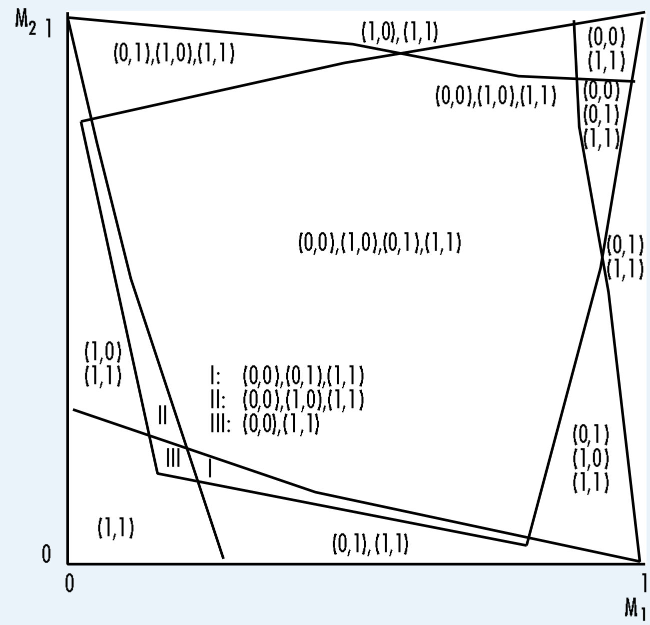
2. $\lambda = (1,1)$ (dollars and pesos both circulate in both countries) is always an equilibrium, and in this regime $V_{i1} = V_{i2}$ (dollars and pesos are perfect substitutes);

3. $\lambda = (1,0)$ (dollars circulate in both countries, pesos only in Mexico) is an equilibrium if and only if equations 25–26 hold.

Figure 1a

Regions in (M_1,M_2) Space where the Different Equilibria Exist Fixed Price Model

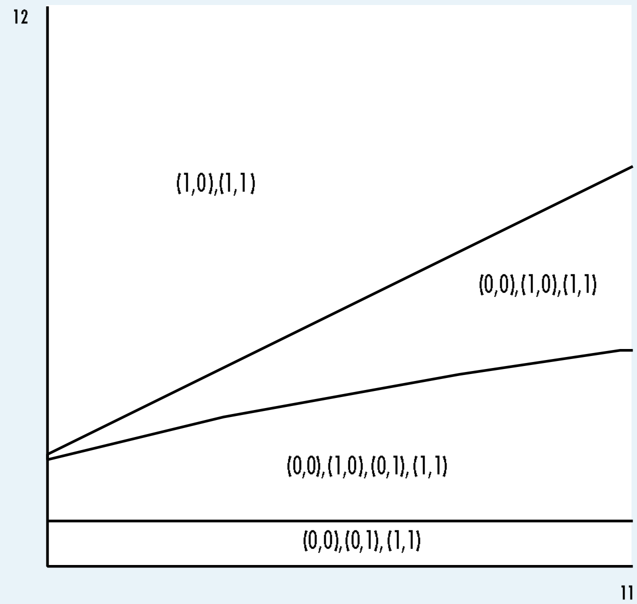
For $\alpha_{11} = 1, \alpha_{12} = 0.15, \alpha_{21} = 0.15, \alpha_{22} = 1$, r = 0.1, γ = 0.02



In Figure 1a we examine the set of equilibria for different values of M_1 and M_2, holding the other parameters fixed. If a region in the figure contains the label (λ_1,λ_2), then regime $\lambda = (\lambda_1,\lambda_2)$ is an equilibrium in this region. Notice that equilibrium $\lambda = (1,1)$ exists for all M_1 and M_2. Notice also that equilibrium $\lambda = (0,0)$ exists only as long as M_1 *and* M_2 are neither too big nor too small. When Currency i becomes too scarce or too abundant, citi-

Figure 1b

Regions in (α_{11},α_{12}) Space where the Different Equilibria Exist
Fixed Price Model
For $M_1 = 0.75, M_2 = 0.75, \alpha_{21} = 0.1, \alpha_{22} = 2.5, r = 0.1, \gamma = 0.02$



zens from Country *i* find that either buying or selling locally is too difficult and find it too tempting to take foreign money that can be spent easily. The figure also tells us about the existence regions for equilibria $\lambda = (1,0)$ and $\lambda = (0,1)$; Currency 1 will be a unique international currency as long as it is not too abundant and Currency 2 is not too scarce.

In Figure 1b we examine different values of α_{11} and α_{12}, holding the other parameters fixed. We see that as Country 1 becomes more open (α_{12} is large relative to α_{11}), it is more difficult to sustain an equilibrium where $\lambda_1 = 0$, or where $\lambda_2 = 1$. The opposite implications occur as local trade in that country becomes easier (that is, as α_{11} becomes larger).

For many parameter values there exist multiple equilibria. Hence, which regime we end up in depends to a large degree on expectations. At the same time, fundamentals—that is, preferences, technology, and the values of M_1 and M_2—exert an important influence. At least in some cases, a particular regime cannot be an equilibrium unless these fundamentals take on the right values.

An interesting extension is the one analyzed by Zhou (1994), who considers a version of the model where agents differentiate between home and foreign goods. A given agent gets utility, *u*, from his preferred good and $\upsilon < u$ from the other good, where his preferred good fluctuates between domestic and imported, and back again, according to Poisson processes. Agents may consume only preferred goods or some of both goods in equilibrium, depending on parameter values and on the type of equilibrium. In this model there can also be currency exchanges in pure strategy equilibria. For example, when a Mexican holding a peso but having a taste for American goods meets an American holding a dollar but having a taste for Mexican goods, there is an incentive for them to swap monies in any regime other than $\lambda = (1,1)$ (because in that regime the currencies are perfect substitutes).

In any case, one shortcoming of the model as it stands is that every trade is a one-for-one swap. This of course makes it impossible to discuss the way nominal prices and exchange rates depend on various components of the model. The extension in the next section remedies this.

THE MODEL WITH DIVISIBLE OUTPUT

We now turn to the model with divisible output. It is still assumed that when buyers spend their money, they spend it all, but now the amount of output they get for their money will be determined endogenously. When a seller produces q units of output for a buyer (of the right type), the latter enjoys utility, $u(q)$, while the former suffers disutility, $c(q)$. With no loss in generality, we can normalize $c(q) = q$, as long as we also renormalize $u(q)$. This simply means that agents bargain in terms of utils and not physical units of output. We assume that $u(0) = 0$, $u'(0) > 1$, $u'(q) > 0$ for all $q > 0$, $u''(q) < 0$ for all $q > 0$, and there exists $\hat{q} > 0$ such that $u(\hat{q}) = \hat{q}$.

We need to appeal to some form of bilateral bargaining theory to determine q. For example, one could use the generalized Nash bargaining solution. This says that when a buyer from Country i with Currency j meets a seller from Country h, if they trade at all, the quantity q solves the following problem:

(27)
$$\max[u(q) + V_{i0} - V_{ij}]^{\theta}[-q + V_{hj} - V_{ho}]^{1-\theta}$$
$$\text{subject to } u(q) + V_{i0} - V_{ij} \geq 0,$$
$$\text{and } -q + V_{hj} - V_{h0} \geq 0.$$

In this problem, q maximizes the so-called Nash product, which is the payoff to the buyer, $u(q) + V_{i0}$, minus his threat point, V_{ij}, all to the power θ, times the payoff to the seller, $-q + V_{hj}$, minus his threat point, V_{h0}, all to the power $1-\theta$. The payoffs are what the agents get if they trade, the threat points are what they get if they do not trade, and θ represents the relative bargaining power in the hands of the buyer. The constraints say that agents must get a greater payoff from trading than from not trading.[7]

For simplicity, we assume here that the buyer has all the bargaining power: $\theta = 1$. This effectively means that he can make a take it or leave it offer to the seller. Two things follow immediately from this. First, assuming that the buyer wants to trade, he offers to exchange his money for the quantity that makes the seller indifferent between accepting and rejecting (that is, the second constraint will bind). Second, this implies that the seller never gets any of the gains from trade, and therefore $V_{i0} = 0$. Hence when a buyer from Country i with Currency j meets a seller from Country h, if they trade at all, then the quantity is given by

$$(28) \quad q_{hj} = V_{hj}.$$

Note that this quantity depends on the nationality of the seller and the currency being used, but not on the nationality of the buyer. Also note that the nominal price of output in Country h in terms of Currency j is $p_{hj} = 1/q_{hj}$.

By construction, the seller always wants to trade at the take-it-or-leave-it offer. It is possible, however, that a buyer may prefer to not trade at these terms. For example, although an American seller will always take pesos at some price, a buyer with pesos may prefer to wait until he meets a Mexican seller if the seller is expected to be willing to agree to a sufficiently better price. A buyer with Currency j is willing to trade with a seller from Country i if and only if the utility he derives from consuming q_{ij} exceeds the value to keeping his money and spending it elsewhere. Hence the following incentive condition must be satisfied if regime λ is to constitute an equilibrium:

$$(29) \quad u(q_{ij}) > V_{jj} \qquad \lambda_i = 1$$

$$(30) \quad u(q_{ij}) < V_{jj} \qquad \lambda_i = 0.$$

The value functions for buyers satisfy

$$(31) \quad rV_{ii} = \alpha_{ii}m_{i0}[u(q_{ii}) - V_{ii}] + \alpha_{ij}m_{j0}\lambda_j[u(q_{ji}) - V_{ii}]$$

$$(32) \quad rV_{ij} = \alpha_{ii}m_{i0}\text{-}\lambda_i[u(q_{ij}) - V_{ij}] + \alpha_{ij}m_{j0}[u(q_{jj}) - v_{ij}],$$

for $j \neq i$. We write equations 31 and 32 in this way to facilitate comparison with the model in the previous section; in particular, see equations 4 and 5. However, strictly speaking, equations 31 and 32 are not exactly correct without some modification in the framework, for the following reason. Consider, for example, the case $\lambda_j = 0$, which says that Currency i does not circulate in Country j. Then equation 31 says that a buyer from Country i with Currency i who meets a seller from Country j cannot trade. But, given the bargaining rules, he always *could* offer to trade and get $V_{ji} > 0$ (the amount a seller from Country j is willing to give for Currency i), and in principle we should allow him that option. In fact, one can show that even if agents believe that $\lambda_j = 0$, a buyer from Country i with Currency i who meets a

[7] See Osborne and Rubinstein (1990) for an extended discussion on Nash bargaining and its relation to explicit strategic bargaining games.

seller from Country j *will* offer to trade.[8] This means that $\lambda_j = 0$ can never be an equilibrium in the model as it stands.

However, this result is really just an artifact of our simplifying assumptions that the buyer gets to make a take it or leave it offer and there is no possibility of direct barter. Relaxing either of these allows $\lambda_j = 0$ to be an equilibrium for some parameter values, although it also complicates the model significantly. We can maintain the simplicity of the present set up and still generate equilibria with $\lambda_j = 0$ in another (albeit perhaps less satisfactory) way, as follows. First, expand the bargaining game so that a seller has to decide whether to enter negotiations with a buyer before the buyer gets to make his take it or leave it offer. Then, given certain additional technical assumptions, it will be an equilibrium for the seller to refuse to trade given that he believes $\lambda_j = 0$. In this case, equations 31 and 32 do describe the value functions as functions of λ. See Trejos and Wright (1995b), for details.

Define the vectors $q = (q_{ij})$, $V = (V_{ij})$, and $m = (m_{ij})$. Then an *equilibrium* is now a list (λ, m, V, q) satisfying equations 1–2 and equations 28–32. Henceforth we ignore the value functions because they are redundant by virtue of the equilibrium condition $V_{ij} = q_{ij}$. Therefore, a steady-state equilibrium is completely characterized by the regime λ, the asset distribution m, and the values of the currencies q.

NO INTERNATIONAL MONEY

Consider first $\lambda = (0,0)$. First, note that m does not depend on whether output is divisible, so we already know the asset distribution from the previous section (this is, of course, true in any regime). Now equations 31–32 can be rearranged to show that prices satisfy

$$(33)\quad q_{ii} = \frac{\alpha_{ii} m_{i0}}{r + \alpha_{ii} m_{i0}} u(q_{ii})$$

$$(34)\quad q_{ij} = \frac{\alpha_{ij} m_{j0}}{r + \alpha_{ij} m_{j0}} u(q_{jj}),$$

for $j \neq i$. In this regime, no transactions actually occur at q_{ij} *for* $j \neq i$, but q_{ij} but tells us how much one could get with Currency j from Country i sellers.

It is not hard to show that, as long as this equilibrium exists, q_{ii} *and* q_{ji} are both decreasing in M_i, and are independent of M_j for $j \neq i$. Also, $q_{11} > q_{22}$ if and only if $\alpha_{11}(1 - M_1) > \alpha_{22}(1 - M_2)$. Because one unit of Currency i buys q_{ii} units of real output, we can say that one unit of Currency 1 is worth $e = q_{11}/q_{22} = p_{22}/p_{11}$units of Currency 2. This is the exchange rate that would be implied by purchasing power parity.[9] The exchange rate e falls with M_1 and rises with M_2. Also, e rises with α_{11} and falls with α_{22}. Of course, international differences in utility and production functions would also influence e. We have kept these the same across countries purely for notational simplicity.

To see when this regime actually constitutes an equilibrium, we must check the maintained hypothesis that Currency i circulates in Country i and the incentive conditions in equations 29–30. The former can easily be seen to hold for all parameter values. The latter, which says that individuals with Currency j do not want to buy from sellers from Country i, holds if and only if $u(q_{ij}) \leq q_{jj}$, for $i \neq j$. Using equations 33–34, we can rewrite this inequality as

$$(35)\quad u\left[\frac{\alpha_{ij}(r + \alpha_{jj} m_{j0})}{\alpha_{jj}(r + \alpha_{ij} m_{j0})} q_{jj}\right] \leq q_{jj},$$

which is satisfied if and only if α_{ij} is small compared with α_{jj}. The intuition is similar to the model with indivisible output.

TWO INTERNATIONAL MONIES

Now turn to the regime with two international currencies, $\lambda = (1,1)$. One can show that in this regime $q_{ii} = q_{ij} = Q_i$; That is, the two currencies are perfect substitutes in the sense that they purchase the same amount from a given seller. If there were a market in which agents

[8] We thank Rao Aiyagari for pointing this out to us.

[9] Suppose one peso buys q_{11} units of output in Mexico and one dollar buys q_{22} units of output in America, and suppose that there is a market in which currencies can be traded at the nominal exchange rate e (that is, one peso buys e dollars). Then one peso could be used to buy eq_{22} units of output in America using dollars. Purchasing power parity holds if a peso buys the same amount directly at home and using dollars abroad: $q_{11}eq_{22}$.

could trade the two currencies, the market clearing price would have to be 1. But the purchasing power parity exchange rate, as defined above, is given by $e = Q_1/Q_2$. In general e differs from unity in this regime: Even though quantities do not depend on which currency the buyer is using, they do depend on the nationality of the seller.

One can also show that Q_1 and Q_2 are both decreasing in M_1 and M_2. Thus an increase in either money supply increases the price level in both countries. Also, as long as the two countries are not too different in terms of α_{ij} and M_j, the effect of a change in M_1 is stronger in Country 1 than in Country 2 and vice versa. This means that the purchasing power parity exchange rate $e = Q_1/Q_2$ is decreasing in M_1 and increasing in M_2.

To see when this regime constitutes an equilibrium, we need to check the incentive conditions in equations 29–30. These hold in this regime if and only if $u(Q_i) \geq Q_j$, $j \neq i$. If the countries are symmetric, in the sense that $M_1 = M_2$, $\alpha_{ii} = \alpha_{jj}$ and $\alpha_{ij} = \alpha_{ji}$, these conditions always hold and the equilibrium with two international currencies always exists. But if they are sufficiently dissimilar, then the incentive conditions will be violated and $\lambda = (1,1)$ is not an equilibrium. The model is very different in this regard from the one with indivisible output. In that model, equilibrium $\lambda = (1,1)$ exists for all parameter values. Here, even if the two countries' currencies are perfect substitutes, the two nationalities of sellers are not. For example, if α_{jj} is low relative to α_{ii} then Country j sellers do not give much for (either) currency compared with Country i sellers, and Country i buyers may prefer to hold on to their cash rather than spend it in Country j.

ONE INTERNATIONAL MONEY

We now turn to the regime where citizens of Country 1 accept Currency 2, but not vice versa, $\lambda = (1,0)$. The purchasing power of the national currency is given by

$$(36)\quad q_{11} = \frac{\alpha_{11}m_{10}}{r+\alpha_{11}m_{10}}u(q_{11})$$

$$(37)\quad q_{21} = \frac{\alpha_{21}m_{10}}{r+\alpha_{21}m_{10}}u(q_{11}),$$

which are qualitatively the same as in the $\lambda = (0,0)$ regime. Because m_{10} is lower in this regime than in $\lambda = (0,0)$, q_{11} and q_{21} are lower here. Intuitively, the influx of foreign money inflates prices denominated in the domestic currency. Also, q_{11} and q_{21} are decreasing in both M_1 and M_2 because m_{10} is decreasing in both M_1 and M_2.

The purchasing power of the international currency depends on the seller, as given by

$$(38)\quad q_{12} = \frac{\alpha_{11}m_{10}u(q_{12}) + \alpha_{12}m_{20}u(q_{22})}{r + \alpha_{11}m_{10} + \alpha_{12}m_{20}}$$

$$(39)\quad q_{22} = \frac{\alpha_{21}m_{10}u(q_{12}) + \alpha_{22}m_{20}u(q_{22})}{r + \alpha_{21}m_{10} + \alpha_{22}m_{20}}.$$

There is a unique nonzero solution for (q_{12}, q_{22}) and both are decreasing in M_1 and M_2. Since m_{10} is lower and m_{20} higher in this regime, q_{12} is lower here than in the $\lambda = (1,1)$ regime. Moreover, $q_{12} > q_{11}$, with the difference increasing in $M_1 - M_2$. Hence, citizens of Country 1 value the international currency more than their own domestic currency. As long as the countries are not too dissimilar, one can also show that $q_{22} > q_{12}$.

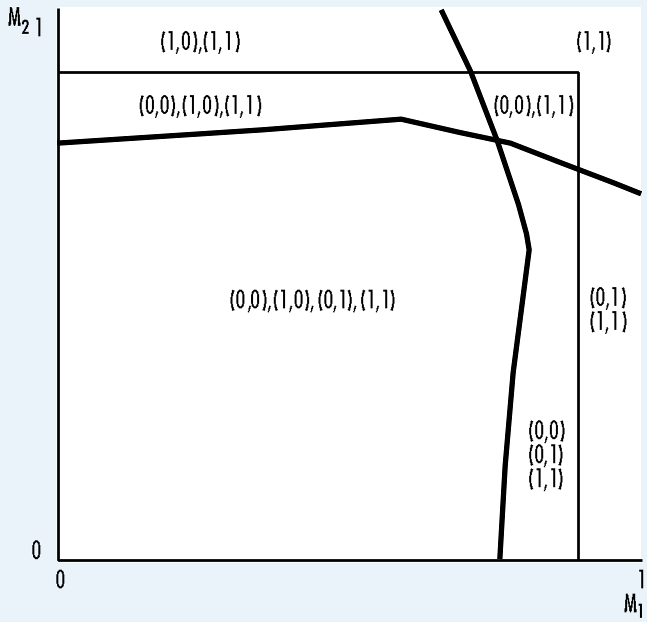
We still need to check when this equilibrium exists. The maintained hypothesis that Currency i always circulates in Country i holds if and only if $u(q_{22}) \geq q_{12}$.[10] Then equations 29–30 require $u(q_{12}) \geq q_{22}$, so Country 2 buyers with Currency 2 buy from Country 1 sellers, and $u(q_{21}) \leq q_{11}$, so that Country 1 buyers with Currency 1 do not buy from Country 2 sellers. Each of these conditions looks qualitatively like one that has been encountered earlier and holds under similar circumstances; however, because q_{ij} differs across regimes, the parameter values for which the equilibria exist are quantitatively different.

[10] This condition, which says that agents from Country 1 with Currency 2 buy from Country 2 sellers, can bind for some parameter values in this regime. If this condition were violated, then Mexicans, for example, would sell goods for dollars and then spend these dollars on Mexican sellers, but not on American sellers.

Figure 2a

Regions in (M_1,M_2) Space where the Different Equilibria Exist

For $\alpha_{11} = 1, \alpha_{12} = 0.15, \alpha_{21} = 0.15, \alpha_{22} = 1, r = 0.1, \gamma = 0.02$



DIVISIBLE OUTPUT SUMMARY

As in the indivisible output model, there are three qualitatively different regimes and at most one equilibrium of each type (that is, each λ implies a unique m and q). Other things being equal, the value of a currency is higher if it circulates internationally and lower if the other money circulates internationally. Other properties of the different regimes include the following:

1. $\lambda = (0,0)$ (dollars circulate only in America, pesos only in Mexico) is an equilibrium if and only if α_{ij} is small relative to α_{ii}; in this regime, q_{ii} is decreasing in M_i and independent of M_j.

2. $\lambda = (1,1)$ (dollars and pesos both circulate in both countries) is an equilibrium if and only if the two countries are not too dissimilar; in this regime, $q_{11} = q_{12} = Q_1$ and $q_{21} = q_{22} = Q_2$ (dollars and pesos are perfect substitutes), Q_i is decreasing in M_i and M_j, and $e = Q_1/Q_2$ and $e = Q_1/Q_2$ is decreasing in M_1 and increasing in M_2.

3. $\lambda = (1,0)$ (dollars circulate in both countries, pesos only in Mexico) exists if a combination of the above conditions hold. In this regime, q_{ij} is decreasing in M_i and M_j for all i, j, $q_{12} \geq q_{11}$ (Mexicans value dollars more than Mexicans value pesos), and $q_{22} > q_{12}$ (Americans value dollars more than Mexicans value dollars) at least if the two countries are not too dissimilar.

In Figure 2a we examine different values of M_1 and M_2, holding the other parameters fixed. Notice that the regime $\lambda = (1,1)$ exists for all M_1 and M_2 in this example, although this is not true in general. Also, because α_{ij} is relatively small, equilibrium $\lambda = (0,0)$ exists for all but very high values of M_1 and M_2. Equilibria where only one currency circulates internationally exist only if the other currency is not too abundant. For example, $\lambda = (1,0)$ exists only if M_1 is not too big. In Figure 2b we examine variation in α_{11} and α_{12}. Now equilibrium $\lambda = (1,1)$ exists in only a small region of the figure. Equilibrium $\lambda = (0,0)$ exists if and only if α_{12} is below some cutoff. Also notice that as either α_{11} or α_{12} increases, it becomes more likely that equilibrium $\lambda = (1,0)$ exists. This is because as Country 1 increases either its internal economic activity or its openness, sellers in Country 1 value money more highly and this makes Country 2 buyers more willing to deal with them.

POLICY

In this section we analyze the effects of changes in (M_1, M_2) in the model with divisible output and endogenize (M_1, M_2) by modeling the objective functions of the two governments and the rules for their strategic interaction. Because for some questions analytical results are difficult to derive, we analyze numerically an example with $u(q) = \sqrt{q}$. The features that we highlight are ones that we can either prove analytically to hold in general or those which seem to be robust to alternative parameterizations in examples.

Because Government i is assumed to purchase goods from a fraction M_i of its newborn citizens, per capita seigniorage revenue in real terms is given by $S_i = \gamma M_i q_{ii}$. One can show that S_1 first increases and then decreases with M_1. Also, given that multiple equilibria exist, S_1 is greater when Currency 1 is international than when it is not and lower when Currency 2 is international than when it is not. Finally, given M_2, S_1 is maximized at different levels of M_1 in the different regimes, depending on the extent to which Country 1 is able to export inflation abroad.

We define welfare as the average (steady state) utility of private citizens:

$$W_i = m_{i0}V_{i0} + m_{i1}V_{i1} + m_{i2}V_{i2}. \tag{40}$$

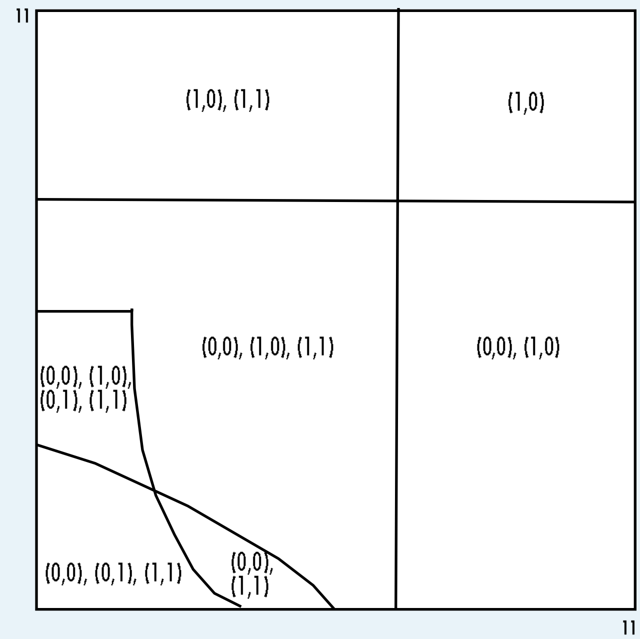
Which regime yields the highest welfare depends on M_1 and M_2. For instance, if M_1 is very low, then W_1 is highest in regime $\lambda = (1,0)$, where Currency 2 is accepted in Country 1 but not vice versa, because this makes trade easier within the host country. Also, $\lambda = (1,1)$ may or may not dominate $\lambda = (0,0)$. For instance, if M_2 is very low or very high, then W_1 is highest in regime $\lambda = (0,0)$.

The next step is to endogenize (M_1, M_2). We assume that governments take the regime as given and restrict their choices to policies that allow for the existence of that regime as an equilibrium (that is, we ignore policies aimed at changing a currency's realm of circulation).[11] We also restrict attention to policies that do not change over time and to steady-state comparisons.

We look for Nash equilibria when each government seeks to maximize the welfare of its own citizens, denoted (M_1^W, M_2^W). In other words, W_1 is maximized at M_1^W when M_2^W is taken as given and vice versa. We also look for Nash equilibria when each government seeks to maximize seigniorage, denoted (M_1^S, M_2^S). Given that they seek to maximize seigniorage, we also consider the possibility of international policy coordination by letting the governments choose

Figure 2b

Regions in $(\boldsymbol{\alpha}_{11}, \boldsymbol{\alpha}_{12})$ Space where the Different Equilibria Exist



policies jointly. One way to do this is to assume that seigniorage is freely transferable across countries, in which case they maximize $S_1 + S_2$. We denote this outcome by (M_1^T, M_2^T). Or we can assume seigniorage is nontransferable, in which case we use the bargaining solution that chooses (M_1^N, M_2^N) to solve max $[S_1 - S_1^S][S_2 - S_2^S]$, where S_j^S is seigniorage in Country j when policy is given by (M_1^S, M_2^S).

We can summarize our findings as follows. First, if foreign money circulates in Country i, independently of whether Currency i circulates abroad, then the welfare maximizing level of M_i is lower than the level that maximizes seigniorage. However, when foreign money does not circulate in Country i, welfare and seigniorage are maximized at the same value of M_i.[12] Second, starting from the Nash equilibrium where governments maximize seigniorage, reducing the amount of money in both countries increases welfare in both countries. Third, and more interestingly, reducing the amount of money in both countries increases seigniorage for both

[11] There may actually be very interesting policies, but we leave such a discussion to future research.

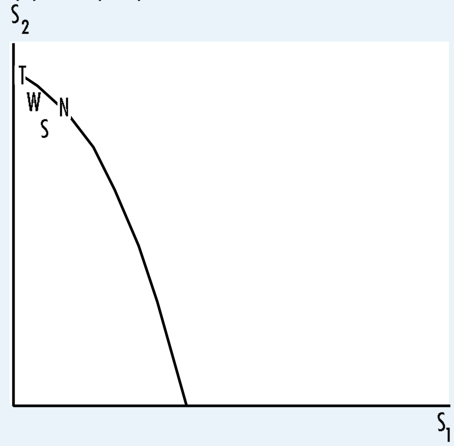
[12] This can be explained as follows. Welfare in Country 1, for example, is $W_1 = m_{11} V_{11} + m_{12} V_{12}$, whereas seigniorage is $S_1 = \gamma M_1 V_{11}$. When no foreign money is held at home ($m_{12} = 0$), m_{11} is proportional to M_1 and maximizing seigniorage is the same as maximizing welfare. This result is not particularly robust and does not hold in generalized versions of the model that include barter or other bargaining solutions.

Figure 3a,b

Feasible and Equilibrium Seigniorage

For $a_{11} = 1, a_{12} = 0.15, a_{21} = 0.15, a_{22} = 1$,
$r = 0.1, g = 0.02$

(a) $\lambda = (1,0)$



(b) $\lambda = (1,1)$

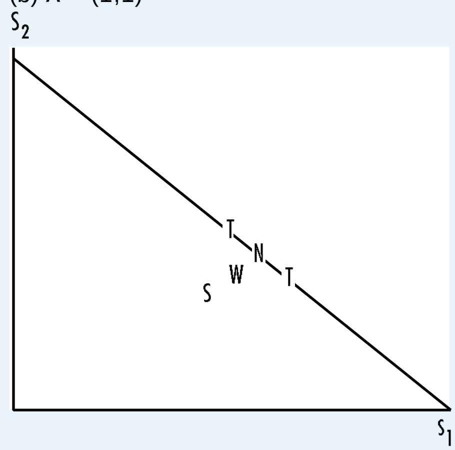
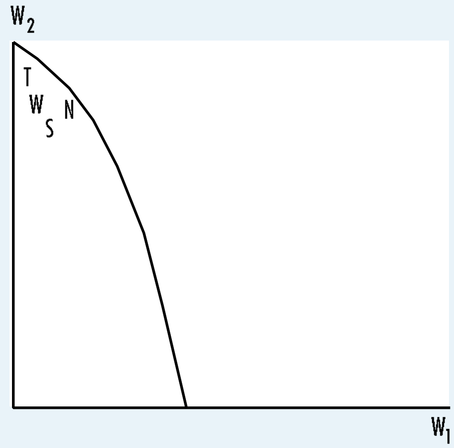


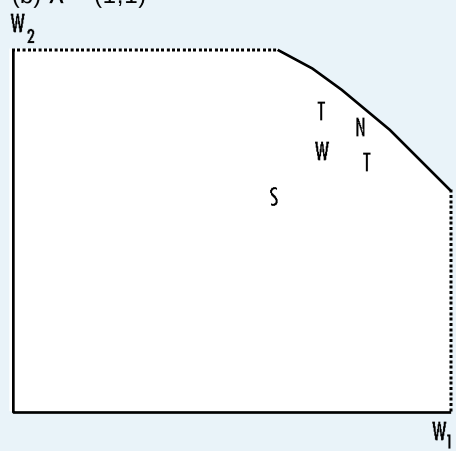
Figure 4a,b

Feasible and Equilibrium Welfare

For $a_{11} = 1, a_{12} = 0.15, a_{21} = 0.15, a_{22} = 1$,
$r = 0.1, g = 0.02$



(b) $\lambda = (1,1)$



governments. As neither government takes into account the effect it has on the other, the noncooperative equilibrium is characterized by too much money. The cooperative equilibrium (M_1^N, M_2^N) involves less money and more seigniorage.

These results hold in general. We now describe in more detail the findings from the numerical examples. We depict the frontier and the outcomes in (S_1, S_2) space, assuming the regime is $\lambda = (1,0)$ in Figure 3a and $\lambda = (1,1)$ in Figure 3b (the regime with no international currency is uninteresting for this exercise). The points labeled *W*, *S*, *T*, and *N* are the payoffs in the four scenarios: the noncooperative equilibrium between welfare maximizers, the noncooperative equilibrium between seigniorage maximizers, the cooperative solution when revenue is transferable, and the cooperative solution when revenue is nontransferable. In the transferable revenue case, the point *T* depicts the seigniorage raised in each country and not the final division of the revenue between the governments. Figures 4a and 4b show the same points in (W_1, W_2) space. Because the graphs are drawn using the same scales, one thing they illustrate is how the possible values of seigniorage and welfare vary across regimes.

The results we wish to highlight are as follows. First, though the cooperative solutions are on the frontier in (S_1, S_2) space, the noncooperative solutions are inside the frontier. This is especially so in regime $\lambda = (1,1)$, where it is easiest to export inflation. Also, Figure 3a shows that in regime $\lambda = (1,0)$, when transfers are allowed, the cooperative solution is to concentrate seigniorage collection in Country 2, where it can be done more efficiently. That is, the governments print more of the international currency and less of the national currency in addition to printing less money in total.

Figure 4 shows how the possible values of (W_1, W_2) are much higher in regime $\lambda = (1,1)$. However, each of the outcomes is inside the frontier. Also, when both governments try to maximize seigniorage, one of them may actually end up with less seigniorage than when both are trying to maximize welfare. Symmetrically, citizens in one country may be worse off in terms of welfare when both governments are trying to maximize welfare than when both governments are concerned with seigniorage, as can be seen in the figure for Country 1 in regime $\lambda = (1,0)$. Finally, in regime $\lambda = (1,0)$, the country that issues international money is better off.

FINAL REMARKS

This article has summarized some recent results in the literature on search-theoretic models of international currency. We think that models in this class will eventually prove useful for studying important issues in international monetary economics. The analysis illustrates the potential welfare gains from having one currency (or two currencies that serve as perfect substitutes). However, it also indicates that there can be a welfare loss in one country from having a unified currency if another country is pursuing a particularly bad policy from the former's point of view. It may be interesting to consider versions of the model where the two countries have different preferences or production technologies or are subject to different shocks to analyze the tradeoffs between having one currency, one central bank or both. While this may help to facilitate trade, it makes it more difficult to have independent monetary policies tailored to conditions in the individual economies. We leave this to future research.

REFERENCES

Aiyagari, S. Rao, and Neil Wallace. "Existence of Steady States with Positive Consumption in the Kiyotaki-Wright Model," *Review of Economic Studies* (October 1991), pp. 901–16.

______ and ______. "Fiat Money in the Kiyotaki-Wright Model," *Economic Theory* (Vol. 2, 1992), pp. 447–64.

______, ______, and Randall Wright. "Coexistence of Money and Interest-Bearing Securities," *Journal of Monetary Economics* (June 1996), pp. 397-419.

Burdett, Kenneth; Melvyn Coles; Nobuhiro Kiyotaki; et al. "Buyers and Sellers: Should I Stay or Should I Go?" *The American Economic Review, Papers and Proceedings* (May 1995), pp. 281–86.

Hendry, Scott. "Credit in a Search Model with Money as a Medium of Exchange." Manuscript, University of Western Ontario, (1992).

Kiyotaki, Nobuhiro, and Randall Wright. "A Contribution to the Pure Theory of Money," *Journal of Economic Theory* (April 1991), pp. 215–35.

______ and ______. "On Money as a Medium of Exchange," *Journal of Political Economy* (August 1989), pp. 927–54.

______ and ______. "A Search-Theoretic Approach to Monetary Economics," *The American Economic Review* (March 1993), pp. 63–77.

Li, Yiting. "Commodity Money Under Private Information," *Journal of Monetary Economics* (December 1995), pp. 573-92.

Matsuyama, Kiminori, Nobuhiro Kiyotaki, and Akihiko Matsui. "Toward a Theory of International Currency," *Review of Economic Studies* (April 1993), pp. 283–307.

Osborne, Martin, and Ariel Rubinstein. *Bargaining and Markets*. Academic Press (1990).

Ritter, Joseph A. "The Transition from Barter to Fiat Money," *The American Economic Review* (March 1995), pp. 134–49.

Shi, Shouyong. "Credit and Money in a Search Model with Divisible Commodities." Manuscript, University of Queens, (1995a).

______. "Money and Prices: A Model of Search and Bargaining," *Journal of Economic Theory* (December 1995b), pp. 467-96.

Trejos, Alberto. "Money, Prices and Private Information." Manuscript, Northwestern University, (1994).

______. "Qualitative Uncertainty and the Liquidity of Money," *Economic Theory (* December 1995b), pp. 467-96.

______ and Randall Wright. "Search, Bargaining, Money and Prices," *Journal of Political Economy* (February 1995a), pp. 118–41.

______ and ______. "Toward a Theory of International Currency: A Step Further." Manuscript, Northwestern University, (1995b).

Williamson, Steve, and Randall Wright. "Barter and Monetary Exchange Under Private Information," *The American Economic Review* (March 1994), pp. 104–123.

Zhou, Ruilin. "Currency Exchange in a Random Search Model." Manuscript, University of Pennsylvania, (1994).