Resolving the Liquidity Effect

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"Resolving: To separate into constituent or elementary parts" 
(The Macquarie Dictionary)

The effect on interest rates of a change in monetary policy has long been an important topic in monetary economics, and there is now a large body of literature that has studied the existence and magnitude of any such effect. Strong conclusions have emerged, and yet, little is available by way of work that attempts to account for the diversity of conclusions. This article aims to fill some of this gap. As the title suggests, it does this by separating out the basic elements of the arguments that lead to the recorded conclusions. In later sections, these are enumerated and discussed. The first section of the article sets out the framework underlying existing studies, followed by an examination of whether the proper object of investigation is a single relationship or a complete system. We come down in favor of the systems viewpoint. Even then, there are many other factors that can account for a diversity of outcomes, and section three is devoted to a consideration of these, ranging from issues of measurement to the sample of data selected for the empirical work. The fourth section explores the inter-relationship of monetary policy and the term structure, while the final section presents some conclusions.

The Basic Model

Although there has been some dissent over the years, mainly from those believing that excess money balances have a powerful direct influence on expenditures, conventional wisdom on the transmission mechanism of monetary policy has been that the effects are felt via interest rates. A very stylized view of this mechanism is available from the money demand and supply relations, which are either explicit or implicit in most models:

\[ m_t^d = \alpha_t + \beta r_t + \epsilon_t^d \]
\[ m_t^s = \beta_t + \beta r_t + \epsilon_t^s \]
\[ m_t^d = m_t^s, \]

where \( d \) indicates demand, \( s \) supply, \( m_t \) is the log of nominal money, \( r_t \) is the nominal interest rate, while \( \epsilon_t^d \) and \( \epsilon_t^s \) are mutually uncorrelated demand and supply shocks. In the textbook treatment of this model, \( r_t \), responds to shifts in the money supply, engineered by varying \( \beta_t \), and the relation \( dr_t/\beta_t = (\alpha_t^d - \beta_t)^{-1} \) means that the interest rate decreases when money supply increases, provided \( \alpha_t^d < 0 \) and \( \beta_t \leq -\alpha_t^d \). This negative reaction of the interest rate to a rise in money supply is termed the liquidity effect.

When there is a random variable attached to money supply, a change in \( \beta_t \) can be thought of as a movement in the expected value of \( \beta_t + \epsilon_t^s \), and the money supply shock might simply be re-labeled \( \epsilon_t^s \), with the conceptual experiment performed by changing the expected value of \( \epsilon_t^s \) from \( \beta_t \) to a new value. Since, mathematically, there is no difference between the response to a change in \( \epsilon_t^s \) or a change in the expected value of \( \epsilon_t^s \), we will henceforth concentrate upon describing the effects of a change in \( \epsilon_t^s \). Such an orientation is now standard in the literature and will be adopted here, so that the liquidity effect will focus upon the simulated response of interest rates to a money supply shock, setting all other shocks to zero.

The above model is static and implies that all adjustments are instantaneous. To make it dynamic, one might augment each equation in equations 1 and 2 with lagged
values in \( m \) and \( r \) to produce

\[
(3) \quad m_t = \alpha_1 + \alpha_2 r_t + \beta_{m} (L) m_t + \beta_{p} (L) p_t + \epsilon_t^m
\]

\[
(4) \quad r_t = \alpha_3 + \alpha_4 r_t + \alpha_5 p_t
\]

with \( \beta_{m} (L) \) being polynomials in the lag operator of the form \( \beta_{m} (L) = \sum_{i=0}^{\infty} \beta_{m,i} L^i \). There is now a distinction to be made between impact effects and the responses over time. In general, one can solve these equations to produce a moving-average representation for interest rates:

\[
(5) \quad m_t = \alpha_1 + \alpha_2 r_t + \alpha_3 p_t + \alpha_4 y_t + \alpha_5 \epsilon_t^m
\]

\[
(6) \quad r_t = \gamma_1 + \gamma_2 m_t + \gamma_3 p_t + \gamma_4 y_t + \epsilon_t^r
\]

where \( C_i (L) = (c_{0,i} + c_{1,i} L + \ldots) \), and the impact effect will be \( c_{0,i} = (\alpha_2 - \beta_2)^{-1} \) while the effects over time are measured from the impulse responses \( c_{1,i} \).

In the framework just described, strong restrictions have been placed upon both the demand and supply functions of money, as the demand for money would also be expected to depend, inter alia, on the level of income (or wealth) and the price level, while the supply of money depends upon the "reaction function" of the authorities. In the scenario described by equation 2, the reaction function depends solely upon the current level of the interest rate, whereas one might expect that current developments in the price level, exchange rates, output, and so on would also play a role. Thus, ignoring dynamics for the moment, equations 1 and 2 might become

\[
(7) \quad \frac{\partial r_t}{\partial \epsilon_t^m} = \gamma_1 m_t / \partial \epsilon_t^m + \gamma_2 p_t / \partial \epsilon_t^m + \gamma_3 y_t / \partial \epsilon_t^m
\]

and this depends upon more parameters than just \( \alpha_2 \) and \( \beta_2 \); as \( r_t \) could change either directly, or indirectly through variations in \( p_t \) and \( y_t \). To evaluate the full effect, therefore, requires us to consider the complete system formed from \( m_t, r_t, p_t, y_t \) (and whatever other variables are important to money demand and supply). It is now no longer sufficient to focus just upon the interest elasticity of the demand and supply of money.

In practice, the relations in equation 5 will also exhibit dynamics, possibly with lagged values of all the variables appearing on the right-hand side of each function. If we collect the variables that are regarded as being part of the system in an \( n \times 1 \) vector \( z_t \), we could write the supply and demand functions as

\[
(8) \quad m_t = \alpha_1 + \alpha_2 r_t + \alpha_3 p_t + \alpha_4 y_t + \alpha_5 \epsilon_t^m
\]

\[
(9) \quad r_t = \gamma_1 + \gamma_2 m_t + \gamma_3 p_t + \gamma_4 y_t + \epsilon_t^r
\]

More generally, the whole system might be written as

\[
(10) \quad B_v z_{t-1} = B_1 z_{t-1} + \ldots B_p z_{t-p} + \epsilon_t^v
\]

Pre-multiplying equation 10 by \( B_1 \) yields the "reduced-form" vector autoregression (VAR) representation for \( z_t \):

\[
(11) \quad z_t = B_0 z_{t-1} + \ldots + B_3 z_{t-3} + B_3^1 \epsilon_t^v = A(L) z_{t-1} + \epsilon_t^v
\]

and solving for \( r_t \) gives us a moving-average representation of the interest rate of the form

\[
(12) \quad r_t = D_0 (L) \epsilon_t^r
\]

where \( z_t \) are the elements in \( z_t \) excluding \( m_t \) and \( \epsilon_t^r = \epsilon_t^v \) is the money supply shock.\(^1\) Note that there are two decompositions presented here; one involving the "reduced form" shocks \( \epsilon_t^v \) from the VAR in equation 11, and one involving the "structural" shocks \( \epsilon_t^s \) from equation 10.

Questions over the existence and magnitude of the liquidity effect are seen to hinge critically upon the measurement of the parameters in the "structural relations." In particular, to isolate the money supply shock, it is necessary that one be able to estimate both the contemporaneous effects, \( \alpha_2, \gamma_2, \) and the

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1 From now on we will identify structural equation errors according to the variable taken to appear on the left-hand side of the equation. This has the advantage of freeing up the choice of whether it is the interest rate or money that should be the dependent variable in a demand or supply equation. Hence, \( \epsilon_t^m \) is the error in the structural equation that has \( m_t \) on the left-hand side, and this might be either demand or supply, depending upon the context. For example, Gordon and Lerner (1994) choose to normalize the demand equation with money and the supply equation with the interest rate.
nature of the dynamic relationships. For example, if the terms $B_n(L)r_t$ were omitted from equation 3, the identified supply shock would actually be $B_n(L)r_t + e_t$, and so the computed impulse responses would be incorrect. It is no wonder then that much of the controversy about the presence and nature of the liquidity effect really comes down to estimation issues.

**ESTIMATION METHODS**

**Single-Equation Estimation Methods**

In a single-equation method, an attempt is made to directly estimate the terms of $C_n(L)$ in equation 12. Early studies, summarized in Thornton (1988), absorbed $C_n(L)e_t^\pi$ into the error term, and then proceeded to measure $e_t^\pi$ by regressing $\ln r_t$ against lagged values of $\ln m_t$, $\ln y_t$, and $\ln p_t$, and so on. However, such a regression does not produce an estimate of $e_t^\pi$ in general, but rather the reduced-form error $e_t^\pi$. The two will coincide only if there are no contemporaneous effects of any variables upon money. Hence, the methodology involves strong assumptions. A further problem is that the error term in the regression of $r_t$ on $\hat{\epsilon}_t^\pi$ cannot be uncorrelated with $e_t^\pi$ unless all the shocks are uncorrelated. This assumption seems most problematic if the system has been under-specified, either in terms of lag length or the number of variables taken to constitute it. Failure to account for these effects will lead to biases in the estimated coefficients. A different complication is the fact that residuals replace $\hat{\epsilon}_t^\pi$ in the estimated relation. Because one is estimating the coefficients of lagged values of $\hat{\epsilon}_t^\pi$, the situation is that analyzed in Pagan (1984), where it is shown that the estimated standard errors are understated.

A related single-equation approach which focuses on estimating the impact response $c_{0t}$ is that of Mishkin (1981, 1982). He inverted the money-demand equation as in equation 6 and took expectations with respect to some assumed information set $\eta_{-1}$ to produce

\begin{equation}
E(r_t|\eta_{-1}) = \gamma_1 + \gamma_2 E(m_t|\eta_{-1}) + \gamma_3 E(p_t|\eta_{-1}) + \gamma_4 E(y_t|\eta_{-1}).
\end{equation}

Subtracting equation 13 from equation 6 then yields a relation among the reduced-form errors:

\begin{equation}
e_t^\pi = \gamma_2 \pi_t^\pi + \gamma_3 \pi_t^\pi + \gamma_4 \pi_t^\pi + \pi_t^\pi.
\end{equation}

Effectively, one is attempting to estimate the parameters of a money-demand function. However, one might query whether this is a satisfactory method for doing so. First, $\pi_t^\pi$ only measures the money supply shock if there are no contemporaneous effects of $p_t$ or $r_t$ on money supply (a restriction explicitly recognized by Mishkin). Second, $\pi_t^\pi$, $\pi_t^\pi$ and so on are correlated with $\pi_t^\pi$ in general, since, from equation 11, $\pi_t^\pi = B_0^\pi \pi_t^\pi$ will be a function of $\pi_t^\pi$. Finally, it is necessary that precise estimates of $\pi_t^\pi$ be extracted, and this necessitates making the set of conditioning variables large enough to completely describe the money supply relation.

The two methods just described will be referred to as single-equation procedures and designated as SING1 and SING2, respectively.

**Systems Methods**

Simultaneous-equation estimation methods address the issue of how to estimate the parameters of a system such as those in equation 10. However, some assumptions have to be made about the nature of the system if consistent estimates are to be obtained, and a number of approaches have emerged in this regard. Each approach is in evidence in the literature on the liquidity effect and involves some constraint upon the covariance matrix of the errors $\epsilon_t^\pi$ and/or the parameters in the matrices $B_0, B_1, \ldots$. Table 1 summarizes the four main approaches in this context.

In the Cowles Commission methodology, $\text{cov}(\epsilon_t^\pi)$ was left unrestricted, but the $B_j(j \geq 0)$ was restricted. Of course, the Cowles Commission methodology recognized other possibilities, which effectively corresponded to the other approaches documented in Table 1, but these were rarely implemented. See, for example, Koopmans, Rubin and Leipnik (1950).
Table 1

Restrictions on Equation 10 Used in Different Systems Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>$B_0$</th>
<th>$B_j (j \geq 1)$</th>
<th>$\text{cov}(\epsilon_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cowles Commission SEM</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR (SYS1)</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>VAR (SYS2)</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>VAR (SYS2 + SYS3)</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

inmacroeconometric work. Having decided that no elements in $B_j (j \geq 1)$ could be restricted, that is, all lagged values appear in every equation, Sims was forced to adopt two other assumptions to estimate $B_0$.

First, he proposed that the structural errors $\epsilon_j$ have a diagonal covariance matrix, that is, they were uncorrelated, so that a money-supply shock could be regarded as independent of a money-demand shock. Second, he chose to make $B_0$ lower triangular. Together, these assumptions produced a Wold causal ordering, and that terminology is one frequently used in the literature. Thus, the ordering $[m, p, y, r]$ means that $m$ is determined; $m$ depends only on lagged values of $m_{t-1}, p_t, y_t$, and $r_t$. The next variable in the ordering depends on contemporaneous values of the previous variables in the ordering and lagged values of itself and the remaining variables; for example, $p_t$ depends on $m_t$ and lagged values of $p_t$, $y_t$, and $r_t$. An alternative way of expressing the implications of these assumptions is that the simultaneous system in equation 10 has been transformed to one that is recursive, making OLS the appropriate estimator of the unknown parameters in $B_0$. It is rather unclear why this set of assumptions is viewed as any more credible than those proposed by the Cowles Commission. Indeed, if Sims' assumptions are invalid, inconsistent estimates of the contemporaneous impact of the variables will result, just as they would be obtained if the exclusion restrictions adopted by the Cowles Commission were incorrect.

One important difference to the Cowles Commission framework is that the latter generally works with over-identified systems, that is, more restrictions were placed upon the $B_j$'s than were needed to exactly identify the parameters. The assumption of a recursive model exactly identifies the parameters of the system and, hence, imposes no testable restrictions on the VAR. One might therefore categorize the differences as simply amounting to whether one wants to work with an exactly identified system or not.

The Wold ordering technique seems to be very popular in the literature on the liquidity effect, being used by Leeper and Gordon (1992), Eichenbaum (1992), Christiano and Eichenbaum (1992), Sims (1992) and Eichenbaum and Evans (1992), inter alios. This method will be denoted as SYS1 in what follows. For a given set of variables, authors utilizing the SYS1 approach often experiment with many different orderings, and seem to select between these observationally equivalent structures according to some prior belief about the signs and persistence of selected impulse responses computed from the system. For example, Eichenbaum criticizes the ordering adopted by Sims (1992), in which the interest rate is taken as pre-determined, on the grounds that a monetary expansion, brought about by a decrease in $\epsilon_1$, produces persistent negative effects upon prices. Actually, this modus operandi is quite similar to the approach taken by researchers within the Cowles Commission tradition, in the sense that the validity of their estimates was often analyzed by the simulation properties of the models, that is, the dynamic responses of endogenous variables to selected exogenous variables.

Of course, there are intermediate positions. The order condition for identification requires that the number of unknown parameters in $B_0$ must not exceed $n(n+1)/2 - n$, and these might be distributed throughout $B_0$ rather than being placed so as to make it triangular. This method is often referred to as a structural VAR (SVAR) approach, in the sense that while no restrictions are imposed upon the dynamics via $B_j (j \geq 1)$, non-triangular restrictions are imposed on $B_0$. We will designate this as the SYS2 method. In the liquidity literature, the main representative of an SYS2 structure is Gordon and Leeper (1994), who work with a system of seven variables $[m, r, u, y, p, r_{10}, \epsilon_1]$, where $u$ is the unemployment rate, $r_{10}$ is the 10-year bond...
rate, and \( cp \) is the log of the commodity price index. The system is taken to be recursive except for money demand and supply which have the form

\[
m_i = \alpha_i + \alpha_2 r_i + \alpha_3 p_i + \alpha_4 y_i + B_{mc}(L)z_i + \varepsilon_i
\]

\[
r_i = \gamma_1 + \gamma_2 m_i + \gamma_3 f_i + \gamma_4 cp_i + B_{z}(L)z_i + \varepsilon_i,
\]

respectively.

An alternative way for reducing the number of unknown parameters in a SVAR is to impose restrictions between the elements of \( B_1 \) and \( B_0 \), a strategy we will refer to as SYS3. These constraints arise from the belief that certain multipliers in the system have known long-run values. Shapiro and Watson (1988) provide a general treatment of re-parameterizations for studying models that have the SYS3 nature, and they show that such strategies free up some of the elements in \( B_0 \) to be used as instruments. To illustrate this, consider the simple bivariate system

\[
z_{it} = b_{12} z_{it-1} + b_{21} z_{it-1} + \varepsilon_{it}
\]

(16)

\[
z_{2i} = b_{21} z_{it} + b_{22} z_{it-1} + \varepsilon_{2i}
\]

(17)

If this was a traditional system, \( b_{12} \) and \( b_{21} \) are not identifiable. However, if one imposes the restriction that \( E(\varepsilon_{1i}, \varepsilon_{2i}) = 0 \), one of them is estimable. Now, let us consider the long-run response of \( z_{1i} \) to \( \varepsilon_{2i} \), which is \( (b_{12} + b_{21})/ \{1 - (b_{12} + b_{11})(1 - b_{22}) - (b_{12} + b_{21})\} \). If this response is set to zero, then \( b_{12} = -b_{21} \) and equation 16 becomes

\[
z_{1i} = b_{11} z_{it} + b_{12} z_{it-1} + \varepsilon_{1i}
\]

(18)

and so \( b_{12} \) can be estimated consistently by using \( z_{2i-1} \) as an instrument for \( \Delta z_{1i} \). Hence, this procedure in SVAR work is identical to the long-recognized possibility of estimating \( B_0 \) by imposing restrictions (other than exclusion ones) upon the parameters of a simultaneous equations system.

The argument generalizes to a system of the form

\[
B_0 z_{1i} = B_1 z_{1i-1} + \ldots + B_p z_{1i-p} + \varepsilon_{1i}
\]

(19)

in the following way. Let the long-run multipliers of a change in \( z_i \) to \( \varepsilon_i \) be \( (B_0 - B_1 - \ldots - B_p)^{-1} = \text{adj}(B_0 - B_1 - \ldots - B_p)/\det(B_0 - B_1 - \ldots - B_p) \). Suppose that one of the long-run multipliers is zero, say the \((i,j)\)'th. Then \( \text{adj}(B_0 - B_1 - \ldots - B_p)_{ij} = 0 \) and this imposes some restrictions between the parameters in \( B_0 \) and those in \( B_1 , \ldots , B_p \). To illustrate the impact of this, consider estimating the first equations

\[
z_{it} = \sum_{k=2}^{n} B_{1k} z_{kt} + \sum_{k=1}^{n} \sum_{k=1}^{n} B_{ik} z_{kt-1} + \varepsilon_{1i}
\]

(20)

simplified by setting \( n = 2, p = 2 \) to get

\[
z_{it} = b_{12} z_{it} + b_{11} z_{it-1} + b_{21} z_{it-2} + \varepsilon_{1i}
\]

(21)

Now, the long-run multiplier being zero will generate a restriction that \( \phi(b_{12}, b_{11}, b_{21}, b_{22}^2) = 0 \), and we should be able to write \( b_{12} = \phi(b_{12}, b_{11}, b_{21}, b_{22}^2) \) so that the equation reduces to

\[
z_{it} = b_{12} z_{it} + \phi(z_{it-1} + b_{11} z_{it-2} + \varepsilon_{1i}
\]

(22)

This restriction frees up an instrument for \( z_{2i} \) among \( z_{1i}, z_{2i}, z_{1i-1} \) and \( z_{2i-1} \) since \( \phi \) is known once the other parameters are given. Consequently, provided the long-run restriction actually involves the parameters of interest (which may not happen as it is \( \text{adj}(B_0 - B_1 - \ldots - B_p)_{ij} \) which equals zero), one can estimate \( b_{12} \) using as instruments \( z_{1i}, \ldots , z_{2i-2} \).

In the liquidity literature, the SYS3 approach has been applied by Lastrapes and Selgin (1994), while Gali (1992) uses ideas from both the SYS2 and SYS3 approaches.

As is evident from the preceding discussion, there have been many proposals about how to estimate the parameters of the simultaneous system. In all instances, certain moment conditions are used, and so the estimators can be given instrumental variable (IV) interpretations, in which pre-determined variables in the system are used as instruments. In the Cowles approach, it is necessary that the pre-determined variables excluded from an equation be uncorrelated with the equation's error term while, in the recursive systems approach, the structural equation errors need to be uncorrelated with one another as well as any right-hand side endogenous variables.
When the number of unknown parameters equals the number of moment conditions, as in a recursive VAR, it is impossible to test the validity of such restrictions, and it becomes simply an act of faith that they are valid. If the assumption is wrong, then it would be expected that there will be biases in the estimates of the parameters. For example, observe that a liquidity effect may require that the demand-interest elasticity be negative. In the event that a liquidity effect is not found, one might ask: What is problematic about the implicit demand function being estimated? Given that we are concerned with a simultaneous-equation system, the most likely explanation would be bias due to the simultaneity. For example, if the system is ordered recursively as \( m, p, r, y \), but \( m \) is not predetermined for \( r \), then the OLS estimator of the contemporaneous liquidity effect will be biased away from a true negative value and might even produce a positive value. Hence, it is hard to know whether any lack of evidence for a liquidity effect is due to the actual state of the world or estimation/identification difficulties. Accordingly, it seems that there is always going to be an element of indeterminacy in a study of the existence of the liquidity effect.

Another estimation issue concerns the usefulness of the available instruments. In particular, it is important that the instruments are correlated with their respective endogenous variables. When instruments \( X_0 \) are in a structural equation already, it is the correlation of the complete set of instruments \( X \) with the endogenous variable, after partialling out \( X_0 \), that is important. It may be that the raw correlation is high while the partial correlation is very low. Studies by Staiger and Stock (1993), Pagan and Jung (1993), Kocherlakota (1990) and Nelson and Startz (1990) have all concluded that there can be large biases in the estimators of the parameters attached to the endogenous variables if the partial instrument correlation is weak, for example, \(< 0.2\). Thus, it is important that this quantity be examined. In the simple SYS3 example constructed above, the correlation between the instrument and regressor is determined by the magnitude of the autocorrelation in \( z_2 \). As the autoregressive root tends to unity, one would get worse estimates of \( h_{12} \). This problem has been studied by Sarte (1994) and, in the context of the liquidity effect, Pagan and Robertson (1995).

**EXAMINING THE STUDIES**

Table 2 presents a summary of some of the evidence on the liquidity effect for studies using monthly or quarterly data. Perhaps the most striking characteristic is the fact that early failure to detect a liquidity effect (largely based on single-equation methods) has been replaced by a conclusion that there generally is a liquidity effect when inferences are based on systems methods. Although this is a comforting outcome, the transition needs to be analyzed carefully, to ensure that the observed relation is in fact robust to any assumptions made in order to identify it. Four concerns can be distinguished, involving how sensitive the conclusion is to:

1. different definitions of the monetary stance;
2. different models;
3. different estimation procedures and restrictions; and
4. different data samples.

In what follows, we examine these issues using monthly data. Descriptions of the data are contained in the appendix. The money, price and output series are measured in logs and are seasonally adjusted. Three sample periods have been chosen. The longest, from 1959:1-1993:12, was fitted with a 14th-order VAR, while the shortest runs from 1982:12-1993:12 and has a sixth-order VAR. An intermediate period of 1974:1-1993:12 with an eighth-order VAR was selected to roughly coincide with the period of flexible exchange rates. These choices also reflect those adopted in the literature. Equation-by-equation and system diagnostic tests (not reported) indicated the absence of residual autocorrelation, but found autoregressive conditional heteroskedasticity (ARCH) and some non-normality, particularly in the money and interest rate equation residuals estimated over longer sample periods. The ARCH effect was less evident in models using post-1982 data.

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5 Some have attempted to control for simultaneity by choosing data periods and intervals in which \( m \) can be reasonably regarded as predetermined, for example, by using weekly data in the log level reserve accounting regime—see for example, Cochrane (1989).
A crucial question is whether changing the definition of money has been important. Here, it would seem as if the answer is yes. The consensus from Table 2 is that for single-equation and recursive models, defining money as M0 or M1 does not result in a liquidity effect, while finer measures such as nonborrowed reserves, NBR, or the ratio of nonborrowed to total reserves, NBRX, do. Nevertheless, one should dig a little deeper into the issue of measuring monetary action. Remember from equations 1 and 2 that we are concerned with the response of interest rates to a shift in the intercept of the money supply equation, and this was measured by computing the impulse response of interest rates to the money supply structural errors. Hence, if one could identify a series corresponding to shifts in the intercept over time, that would constitute the basis for an appropriate way to measure the monetary stance. Such series have been constructed by Romer and Romer (1989) and Boschen and Mills (1993). Eichenbaum and Evans (1992) have shown that there is a strong liquidity effect when the first of these measures is used.

For recursive models, a money-supply or M-rule interpretation implies that shocks to the money-supply equation are identified with monetary policy. For example, one might assume an ordering such that money is predetermined for the interest rate (and possibly other variables as well) and use the error from the money equation and the estimated dynamics to derive the impulse responses of the interest rate. Ignoring the dynamics, this amounts to assuming that the supply function of money is perfectly inelastic with respect to the interest rate. A different strategy employed by Sims (1992), and Bernanke and Blinder (1992), is to order the VAR such that the interest rate is predetermined for money and to treat shocks to the interest rate equation as the monetary policy indicator. This yields an interest rate or R-rule interpretation, since, ignoring the dynamics, this is equivalent to assuming that the supply function is perfectly elastic with respect to interest rates. Empirically, defining money as M0 or M1 does not result in a liquidity effect in a recursive VAR under M-rule interpretations, while using NBR or NBRX does yield a liquidity effect for either M-rule or R-rule identification schemes. For example, Figure 1 presents the implied interest rate responses to a one-unit monetary expansion under an M-rule (an increase in e^t) for various measures of money, and two alternative
orderings of a four-variable VAR of \( m, r, y \) and \( p \), where \( r \) is measured by the federal funds rate, \( FF \), \( p \) is measured by the log of the consumer price index, \( P \), and \( y \) is measured by the log of the industrial production index, \( Y \). The VAR is fit to the sample 1959:01-1993:12, and the recursive models parallel some of those reported in Christiano and Eichenbaum (1992).

It is not sufficient, however, to simply concentrate upon the impulse response functions relating to interest rates and money, as it is possible that a model producing a plausible liquidity effect also creates implausible effects of monetary policy upon other variables in the system. This was Eichenbaum's (1992) objection to Sims' work. Sims pointed out that there was a "price puzzle" generated from a simple four-variable model based on \( M_1 \), since an expansionary monetary action (in his case, an R-rule contraction in \( \epsilon_r \)) led to a persistent fall in the price level. Eichenbaum's proposed solution to this was to replace \( M_1 \) or \( M_0 \) with \( NBR \), and to place \( P \) and \( Y \) prior to money and interest rates in the ordering, so that the Federal Reserve's M-rule responds contemporaneously to price and output variables, but not interest rates. Eichenbaum reports a small positive response to expansionary monetary policy in this case. Earlier, Thornton (1988), in a single-equation analysis, observed that \( NBR \) was the only measure of money which displayed evidence of a liquidity effect. Thornton's conclusion has been reiterated by Christiano and Eichenbaum (1992) in a systems context (see Figure 1). Subsequently, Strongin (1992) has suggested that the ratio of \( NBR \) to total reserves, \( TR \), denoted \( NBRX \), is the best monetary measure, and Eichenbaum and Evans (1992) have adopted \( NBRX \) in their work on exchange rates.\(^5\)

Figure 2 presents the impulse responses of \( P, Y \) and \( FF \) to monetary shocks in VARs ordered as \( \{Y, P, NBR, FF, TR\}, \{Y, P, NBRX, FF\} \) and \( \{Y, P, NBR, FF\} \), respectively. In contrast to the finding in Eichenbaum (1992), it is apparent that the price puzzle is still present regardless of which monetary measure is adopted, although in all cases the estimated responses are relatively small.\(^7\) The difference between these and the Eichenbaum results can be explained by noting that Eichenbaum used a slightly different sample period (1965:01-1990:01). Computing impulse responses from a VAR fit to this sub-sample does produce impulse responses very similar to those he reports. Hence, it seems as if the estimated price-impulse responses are unstable, at least if \( NBR \) or \( NBRX \) are used to measure monetary actions. We examine the issues of model stability and the precision of the point estimates in more detail further in this article.

Perhaps the most controversial issue with the use of nonborrowed reserves is whether it constitutes an effective way of measuring monetary policy. The variable \( NBRX \) is very highly negatively correlated with borrowed reserves \( BR \) (-0.82 over the period 1959:01-1993:12), raising the question about how the latter should be treated. Suppose that total

\(^5\) Strongin (1992) actually used the ratio of \( NBR \) to \( TR \).

\(^7\) These results are also quite robust to reversing the ordering of \( Y \) and \( P \) at the top of the recursion.
reserves, \( TR = NBR + BR \), showed no variation. Then, if \( BR \) has a positive relation to \( FF \), \( NBR \) must be negatively related to \( FF \). A model of this sort was constructed by Gilles and others (1993). They effectively fix the total demand for reserves by making it depend upon real factors exogenous to the monetary sector, and then add a “discount window” function in which the supply of borrowed reserves is a positive function of \( FF \). Hence, they concluded that the observed negative relation between \( NBR \) and \( FF \) simply reflects the way that the Federal Reserve has operated the discount window. The import of this model is not entirely clear because it makes the supply of \( BR \) a function of \( FF \), whereas the data indicates that the relation is between \( BR \) and the spread between the Federal funds and the discount rate, \( RD \)—that is, \( SPRD = FF - RD \) (see Mishkin, 1992), and therefore, \( BR \) is not a function of \( FF \) alone. Indeed, statistically, it would not make sense to relate \( BR \) solely to \( FF \); as the latter is best described as an integrated process while the former is not. This is evidenced by augmented Dickey-Fuller (with 12 lags) tests of \(-1.88 (FF) \) and \(-3.47 (BR) \), as compared to a 5 percent critical value of \(-2.86 \).

What is in dispute here is the degree of substitutability of \( NBR \) and \( BR \). With zero substitutability, \( NBR \) would appear to summarize monetary policy quite well. But if there was perfect substitutability, total reserves would be a better measure, and, with the exception of the study by Gordon and Leeper (1994), this does not seem to result in a liquidity effect, all responses being quite similar to those from \( M0 \) or \( M1 \). An attempt to allow for non-zero substitutability might be to incorporate demand and supply functions for both \( NBR \) and \( BR \) into the analysis. A variant of this idea would be to include both \( NBR \) and total reserves (\( TR \)) in the VAR, and this has been done by Christiano and others (1994). Doing so produces more reasonable price and income responses than the \( [Y, P, NBR, FF] \) model, and broadly similar responses to those from the \( [Y, P, NBR, FF] \) model (Figures 2a and 2b), although the price effect is still negative for a long period of time. There is also some increase in the magnitude of the liquidity effect, and it is less persistent than for the model \( [Y, P, NBR, FF] \) (Figure 2c).  

The result that neither of the \( NBRX \) or \( NBR/TR \) formulations are capable of completely eliminating the price puzzle is consistent with the view of Sims (1992) that the main source of the price puzzle is the
absence of some pre-determined inflation indicator variable in the Fed's policy response function. This implies that the model should be extended to include variables other than just money, interest rates, output and the general price level. This line of argument is taken up in the next sub-section, which deals with the issue of using alternative model formulations.

Model Variation

One explanation for the range of conclusions regarding the liquidity effect arises from the non-uniqueness of models. We have already alluded to this when discussing recursive versus non-recursive systems, and even within a given causal framework models can vary, as reflected in the ordering or set of variables taken as constituting the system. Too small a set of variables implies misspecified relations, which can affect estimates of both contemporaneous and dynamic responses. Because there is a cost to making the list of variables too large, it is imperative that theoretical ideas and past research are used to indicate what variables are likely to be of major importance. For example, Sims (1992) and Christiano and others (1994) extend the NBR/TR formulation to include a measure of commodity prices. In particular, they consider the M-rule ordering \([Y, P, CP, NBR, FF, TR]\), where CP is a commodity price index. Thus, output, the general price level and commodity prices are taken as predetermined in setting policy. Estimating their model using the monthly data, we find that the \(Y\) response is initially negative, but then persistently positive after a few months, while the \(P\) responses are now persistently positive (Figure 3a) and the liquidity effect lasts approximately seven months (Figure 3b). It seems that including additional variables in the policy setting rule goes some way to eliminating the anomalous price effects that were obtained using simpler models.

Another possible model variation is to allow for interaction with the foreign sector. Open economy models, for example, McKibbin and Sachs (1991), emphasize the determinants of the size of the liquidity effect in the following quotation:

"If the effect of the exchange rate on domestic demand is large (through the effect on the trade balance), and if the effect of domestic demand on money demand is large (through the income elasticity of demand), and if the home currency depreciation causes a rapid rise in domestic prices, then it can be shown that home nominal interest rates will tend to rise after the money expansion... But if one or all of these three channels are weak, then domestic nominal interest rates will tend to fall after the money expansion... ."

Using the MSG model, their simulations show a strong liquidity effect for the United States but a weak one for Japan, even though
the interest elasticity of demand in both countries is assumed to be the same.

It is clear from such studies that there is a need to allow for an exchange rate \( e \), offset. Introducing an exchange rate also demands the addition of a foreign interest rate \( r_f \), to allow for the possibility of uncovered interest parity, that is, \( e = r - r_f \). Within a recursive system, \( r_f \) would need to appear as the first variable and \( e \) will appear after \( r \). Eichenbaum and Evans (1992) and Sims (1992) contain results which suggest that the conclusions reached with systems excluding \( e \) and \( r_f \) remain valid, although the magnitude of any effects differ. Using the trade-weighted exchange rate, \( ER \), a weighted foreign interest rate series, \( RE \), and an ordering \([RF, Y, P, CP, NBR, FF, ER, TR] \), later referred to as the exchange rate model (ER), we find that the liquidity effect is reduced slightly from that observed for the “commodity price” \( CP \) formulation \([Y, P, CP, NBR, FF, TR] \) (Figure 4a). There are greater qualitative differences for the price responses. Figure 4b shows these for the CP and ER models. Unlike the situation for the full sample, there is a perverse price response with the CP model that is largely corrected by the ER model, pointing to the fact that the long-run responses can be very different as models change, even though the short-run responses are similar. In contrast, the estimated short- and long-run responses of \( Y \) are similar in both the CP and ER models, as shown in Figure 4c.

The question of how to choose between alternative models is a vexed one. As mentioned previously, most analyses seem to concentrate upon how closely multipliers correspond to prior conceptions. This seems to be a restrictive viewpoint. Structural relations have been estimated in getting the multipliers and it seems appropriate that one should examine how plausible the estimates of these parameters are. In particular, the nature of the liquidity effect directs us to the demand for money function, and we would expect that it should feature negative interest, positive income and (probably) positively signed price elasticities. A full set of structural coefficient estimates for the CP and ER models is presented below. The CP model results for the periods 1959:01-1993:12 and 1974:1-1993:12 are presented in equations 23 and 24, respectively, and the ER model results for the sub-period (1982:12-1993:12) are in equation 25. Note that because money is ordered immediately prior to the interest rate in the CP and ER model, the initial impulse response of the interest rate to money shocks...
is simply given by the magnitude of the interest elasticity of demand for money. This follows directly from equation 7 as the stated recursive structure has \( \partial r / \partial e_t = \partial y_t / \partial e_t = 0 \). More generally, however, it is clear that it would be possible for the liquidity effect to "exist" and yet for all of the parameter estimates in the demand function to be incorrectly signed.

\[
(23) \quad P = .0186Y
\]
\[
CP = .43P + .25Y
\]
\[
NBR = -.007CP - .39P - .25Y
\]
\[
FF = 8.08Y - 5.42P - 2.62CP
\]
\[
-12.41NBR
\]
\[
TR = -.02Y + .36C + .012CP
\]
\[
+.423NBR + .006FF
\]

\[
(24) \quad P = .038Y
\]
\[
CP = -.42P + .40Y
\]
\[
NBR = -.04CP - .29P - .41Y
\]
\[
FF = 11.73Y + 19.20P - 1.73CP
\]
\[
-17.07NBR
\]
\[
TR = .005Y + .6P - .009CP
\]
\[
+.41NBR + .006FF
\]

\[
(25) \quad P = .002RF + .055Y
\]
\[
CP = -.12RF + .212Y - .130P
\]
\[
NBR = -.008RF - .492Y
\]
\[
+ .171P + .049CP
\]
\[
FF = .401RF + 11.732Y + 7.332P
\]
\[
-2.052CP - 14.952NBR
\]
\[
TR = .005RF + .037Y + .4192P
\]
\[
-.006CP + .410NBR
\]
\[
+ .005FF + .012ER
\]

With the possible exception of the \( P \) variable in the demand for money function (\( FF \)) of the CP model estimated over the period 1959:01-1993:12, the estimated structural relations are what would be expected, with prices responding in a procyclical way, monetary policy (in terms of real NBR movements) reacting negatively to expansions in prices and output, and a demand for money function that has positive income and negative interest rate effects. Interpretation of the equation for TR is harder, but it is interesting in that it shows that changes in NBR are only partially reflected in TR, which can be interpreted as indicating that there is substitutability between NBR and BR.

Perhaps the main use of the idea that one should think of the issue in structural terms is that it forces one to think carefully about the complete specification of the system, and such considerations suggest that there may be problems in modeling the data with particular choices of the set of variables. For example, suppose M2 is used as the measure of money. Then, for such a broad measure of money, one really needs to have another interest rate in the system to capture the fact that a large component of the assets making up M2 are interest-bearing. If the dependent variable in the (inverted) demand for money function is taken to be the three-month T-bill rate, \( R3 \), then we might take the federal funds rate as proxying the rate of return on M2 assets. For a VAR ordered as \( \{ P, Y, M2, FF, R3 \} \) we find that the estimated implied demand for money function appears relatively stable based on a CUSUM test, and a liquidity effect is observed. But the demand relation is quite unstable if FF and/or its lags are omitted from the VAR. Hence, a VAR only in the variables \( \{ P, Y, M2, R3 \} \) would appear to be a poor choice. More generally, given the large body of literature that has evolved pointing to the instability of U.S. money demand functions, the fact that estimated parameters of a demand for money function are fundamental to any conclusion regarding the liquidity effect has to be cause for concern. Even if the menu of variables seems complete, it still may be that the relationship between them is unstable, or the use of linear models inappropriate, for some measures of money and interest rates, and for some sample periods.

**Different Estimation Methods and Restrictions**

How much do systems methods contribute to the analysis of the liquidity effect? Potentially, a good deal. As previously mentioned in the discussion on single-equation estimation procedures, the estimates made of the monetary stance are ideally the structural rather than reduced-form errors, and so a regression of \( r \) upon a distributed lag of these values could produce quite different results. Only if the monetary policy variable...
For example, Gordon and Leeper estimate that the structural model of seven variables, \( z \), the money-supply disturbance from a structural model, is necessary to proceed in some other way. If one wishes to estimate equations 8 and 9, excluding the supply-equation residuals, reveals that the correlation between the demand- and supply-equation residuals is not zero (if money is measured by \( M_2 \), the correlation becomes \(-0.79\)). Also, the excess demand elasticity stems in part from the presence of the liquidity effect. The existence of the liquidity effect hinges upon the signs and magnitudes of both the demand and the supply elasticity, and there are a number of issues in this regard. First, the precision of estimation of the demand elasticity stems in part from the use of the residual of the supply equation as an additional instrument, and the structural residuals are only valid instruments if \( E(\bar{\epsilon}_1^*\epsilon_2^*) = 0 \). In this instance, the assumption may be checked as the system is over-identified—that is, there are more instruments among \( X_i \) than are needed to estimate the parameters. Using the parameter estimates from doing IV with \( X_i \), only, that is, excluding the supply-equation residuals, reveals that the correlation between the demand- and supply-equation residuals is \(-0.39\), which is significantly different from zero (if money is measured by \( M_2 \), the correlation becomes \(-0.79\)). Also, the excess instruments in \( X_i \) contribute little to the prediction of \( TR \) in the supply equation. The F-test of the hypothesis that they do not enter the first-stage \( TR \) regression yields a value of only 1.49, compared to a 10 percent critical value of 2.18. The presence of weak instruments means that the elasticity estimates may be severely biased. Finally, as Gordon and Leeper acknowledge, R10 is probably not a valid instrument for \( FF \) in the demand equation.

\[
(26) \quad m_t = \alpha_1 + \alpha_2 r_t + \alpha_3 p_t + \alpha_4 y_t + B_m(L) z_t + \epsilon_t
\]

respectively, with \( E(\bar{\epsilon}_1^*\epsilon_2^*) = 0 \). The rest of the system is taken to be recursive, ordered as \( \{u_t, y_t, p_t, r_{t0}, \epsilon_t\} \). Because these variables are predetermined for \( m_t \) and \( r_t \), \( X_i = \{1, u_t, y_t, p_t, r_{t0}, \epsilon_t, j \geq 0\} \) provide a valid set of instrumental variables for \( r_t \) in the money-demand equation, and for \( m_t \) in the money-supply equation. They estimate equation 26 subject to \( E(\bar{\epsilon}_1^*\epsilon_2^*) = 0 \) via FIML, using a six-lag VAR, and monthly data from 1982:12 to 1992:04. Pagan and Robertson (1993) extend the sample period to 1993:12, giving \( T = 127 \) observations, and focus on the results for \( m_t = TR \) and \( r_t = FF \).

The existence of the liquidity effect is determined solely by past quantities will the two coincide. In terms of the recursive VAR, a single-equation approach corresponds to a case in which the monetary variable is ordered first, whereas the systems approach generally has money appearing later in the ordering. However, it turns out that the conclusions reached concerning the liquidity effect do not differ greatly because of this modification, as evidenced by the close correspondence of the distributed lag coefficients from the regression of \( FF \) against 36 lags of \( \bar{\epsilon}_1^* \) in Table 3, in which \( \bar{\epsilon}_1^* \) is alternatively measured as the structural errors from the two orderings \( \{NBR, P, Y, FF\} \) and \( \{P, Y, NBR, FF\} \). Apparently the conclusions reached concerning the liquidity effect do not differ greatly because of this modification, as evidenced by the close correspondence of the distributed lag coefficients from the regression of \( FF \) against 36 lags of \( \bar{\epsilon}_1^* \) in Table 3, in which \( \bar{\epsilon}_1^* \) is alternatively measured as the structural errors from the two orderings \( \{NBR, P, Y, FF\} \) and \( \{P, Y, NBR, FF\} \). Apparently the conclusions reached concerning the liquidity effect do not differ greatly because of this modification, as evidenced by the close correspondence of the distributed lag coefficients from the regression of \( FF \) against 36 lags of \( \bar{\epsilon}_1^* \) in Table 3.

### Table 3: Impulse Responses of FF to NBR

<table>
<thead>
<tr>
<th>(NBR, P, Y, FF)</th>
<th>(P, Y, NBR, FF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{35} )</td>
<td>(-15.39)</td>
</tr>
<tr>
<td>( c_{34} )</td>
<td>(-14.56)</td>
</tr>
<tr>
<td>( c_{33} )</td>
<td>(-25.70)</td>
</tr>
<tr>
<td>( c_{32} )</td>
<td>(-20.32)</td>
</tr>
<tr>
<td>( c_{31} )</td>
<td>(-24.58)</td>
</tr>
<tr>
<td>( c_{30} )</td>
<td>(-18.48)</td>
</tr>
<tr>
<td>( c_{29} )</td>
<td>(-23.56)</td>
</tr>
<tr>
<td>( c_{28} )</td>
<td>(-18.28)</td>
</tr>
<tr>
<td>( c_{27} )</td>
<td>(-25.62)</td>
</tr>
<tr>
<td>( c_{26} )</td>
<td>(-19.92)</td>
</tr>
</tbody>
</table>
Estimates of Gordon and Leeper Demand and Supply Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Demand (TR)</td>
<td>Supply (FF)</td>
<td>Demand (TR)</td>
</tr>
<tr>
<td>FF</td>
<td>-0.028</td>
<td>-0.026</td>
<td>-0.0099</td>
</tr>
<tr>
<td></td>
<td>(.014)</td>
<td>(.013)</td>
<td>(.010)</td>
</tr>
<tr>
<td>TR</td>
<td>30.28</td>
<td>22.578</td>
<td>23.529</td>
</tr>
<tr>
<td></td>
<td>(10.95)</td>
<td>(9.987)</td>
<td>(13.629)</td>
</tr>
<tr>
<td>P</td>
<td>0.958</td>
<td>1.007</td>
<td>0.796</td>
</tr>
<tr>
<td></td>
<td>(.752)</td>
<td>(.603)</td>
<td>(.466)</td>
</tr>
<tr>
<td>Y</td>
<td>0.495</td>
<td>0.650</td>
<td>0.353</td>
</tr>
<tr>
<td></td>
<td>(.315)</td>
<td>(.311)</td>
<td>(.237)</td>
</tr>
<tr>
<td>RT0</td>
<td>0.456</td>
<td>0.341</td>
<td>0.342</td>
</tr>
<tr>
<td></td>
<td>(.101)</td>
<td>(.099)</td>
<td>(.010)</td>
</tr>
<tr>
<td>CP</td>
<td>1.893</td>
<td>1.879</td>
<td>1.890</td>
</tr>
<tr>
<td></td>
<td>(1.326)</td>
<td>(1.410)</td>
<td>(1.418)</td>
</tr>
<tr>
<td>var(εt)</td>
<td>6.73e-5</td>
<td>6.61e-5</td>
<td>4.62e-5</td>
</tr>
<tr>
<td></td>
<td>5.04e-2</td>
<td>4.98e-2</td>
<td>5.31e-2</td>
</tr>
<tr>
<td>corr(εt, εt)</td>
<td>0</td>
<td>0</td>
<td>-0.385</td>
</tr>
<tr>
<td>over-id test</td>
<td>p value</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>p value</td>
<td>0.41</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Comparing the IV and FIML results reported in Table 4, we see there is a close correspondence between the IV and FIML estimates of the supply equation. In contrast, the IV demand elasticity estimate is much larger than the corresponding FIML estimate (−0.01 vs. −0.026) and is no longer significantly negative. A negative correlation between the structural errors would be expected to produce a negative bias in the FIML estimator of the demand elasticity, and this leads to a smaller magnitude for the liquidity effect for a given supply elasticity estimate. The inconsistency will be proportional to the actual correlation between $\epsilon''$ and $\epsilon'$ when $E(X', \epsilon'') = 0$. Against this, the supply elasticity estimate itself may be biased due to weak instruments. The net outcome of these two effects is indeterminate but does cast some doubt on whether the liquidity effect uncovered by Gordon and Leeper is a real one.

Another approach to estimating equations 8 and 9 that eschews recursive assumptions is to impose some long-run restrictions upon the impact of monetary shocks. Lastrapes and Selgin (1994) and Gali (1992) impose a variety of these. Lastrapes and Selgin begin by postulating that a unit shock in the money supply causes prices to rise by a unit in the long run, that is, real money balances do not change, while there is a zero long-run impact on output and interest rates. As explained in the preceding section, when discussing the SYS3 procedure, such restrictions free up instruments that can be used to estimate the elements of $B_p$. Taking the system to be estimated as (where all lagged values are suppressed)
imposition of the long-run restrictions on
each of the equations for \( p_t, y_t \) and \( r_t \) enables the estimation of three of the \( b_{01}^{0} \).

Before further analysis, one has to consider why the system above is measured in differ-
ences, whereas most of the systems described previously are in levels. Lastrapes and Selgin
(1994) argue that the variables \( m_t, p_t, y_t \) and \( r_t \) are integrated but not cointegrated. If
equation 27 was written in levels, the error
terms must be integrated of order one \( I(1) \);
otherwise, the equations would represent
cointegrating relations among the variables.
Hence, it is appropriate to transform all the
variables by differencing. Suppose, instead,
that one proceeded to impose the long-run
restrictions upon the levels model, To make
the analysis simple, focus on the equation for
output and assume that the only right-hand
side variables are \( m_t \) and \( r_t \). Then, as pre-
viously explained in the second section, one
would be using \( m_{t-1} \) as an instrument for \( \Delta m_t \)
when the equation is re-parameterized to have
\( \Delta m_t \) and \( m_{t-1} \) as the two regressors (\( m_{t-1} \) is eliminated because its coefficient is the long-
run response of zero, leaving the only regressor as
\( \Delta m_t \)). This estimator is \( \hat{b}_{02}^{0} = \hat{B}_{02}^{0} - (T^{-1}\Sigma m_{t-1}^{0} \Delta m_{t-1}^{0})^{-1} (T^{-1}\Sigma m_{t-1}^{0} \Delta m_{t-1}^{0}) \).
If \( m_t \) is \( I(1) \), both the numerator and denominator are asymptotically random
variables, and the instrumental variables estimator converges asymptotically to a random
variable, failing to even be consistent. The use of differenced variables obviates this problem
as the new re-parameterized equation features
\( \Delta m_t \), as regressor and \( \Delta m_{t-1} \), as instrument, and
\( T^{-1}\Sigma m_{t-1}^{0} \Delta^2 m_t \) will converge to a constant.

Now, let us consider the various estimates
that might be made of the initial impulse
response of \( r_t \) to shocks in \( m_t \). To estimate this,
we need to be able to form \( B_{0}^{-1} \). Accordingly,
six restrictions need to be placed upon the
system to identify the elements in \( B_{0} \). It is
useful to draw these from one of the following
five alternatives:

1. The matrix of long-run impulse responses,
\( C(1) \), is lower triangular. This implies that
the three long-run restrictions on the impact
of money supply shocks on prices, output
and interest rates hold, as well as analogous
ones involving money demand and agreg-
gate demand shocks.

2. The three long-run money supply shock restric-
tions hold, along with \( b_{12}^{0} = b_{24}^{0} = b_{34}^{0} = 0 \).

3. Long-run restrictions on the effect of
money supply shocks on prices and output hold
(but not on interest rates), along with
\( b_{12}^{0} = b_{14}^{0} = b_{23}^{0} = b_{24}^{0} = 0 \).

4. Only the long-run restriction on the effect
of money supply shocks on prices holds,
along with \( b_{12}^{0} = b_{14}^{0} = b_{23}^{0} = b_{24}^{0} = b_{34}^{0} = 0 \).

5. There are no long-run restrictions, and \( B_{0} \)
is lower triangular, that is, the system is
recursive.

The first of these is what Lastrapes and
Selgin actually use. Most of their paper specif-
ically mentions only three long-run restric-
tions, but this fails to identify the magnitude
of the responses, and the quantitative results
they present require the extra long-run restric-
tions. As an experiment, we consider other
ways of estimating \( B_{0} \) that impose only the
long-run restrictions emphasized by Lastrapes
and Selgin, allied with various short-run
assumptions. In particular, we build up to a
recursive system \( \{ p_t, y_t, m_t, r_t \} \) by progressively
removing the long-run assumptions. Given
these choices, and with \( m \) being base money
and \( r \) the three-month T-bill rate, the impact
multipliers are, respectively, \( -63, -20, -8, 5 \)
and \( 7 \), showing that the long-run restrictions
do indeed help to identify a liquidity effect.
The magnitude of the effect is large if six
long-run restrictions are imposed, but if only
the three restrictions Lastrapes and Selgin
discuss are adopted, the magnitude is much
the same as found with simple recursive
systems featuring NBR and FE.

Clearly, there are a number of econometric
estimation issues raised by the work with
non-recursive models such as those of Gordon
and Leeper, Lastrapes and Selgin, and Galì,
and some of these are explored in detail in
Pagan and Robertson (1995). For instance, it
is shown there that the instruments implicitly
used by all three studies are very weak, and
this leads to biases in the estimated impulse
structural models in the various studies, most of the empirical models are estimated using different sample periods. There are a number of ways of examining the robustness of results from changing the sample period, some of which are considered here. First, the estimates could be sensitive to estimation over a sub-period. Examining the impulse responses for the CP and ER models when estimated only with observations from the period 1982:12-1993:12, we find that each model produces small negative initial effects on interest rates and that the largest negative effects, after three or four periods, are around one-third of what was in evidence over the period 1974:01-1993:12. Compare Figures 4a and 5a. Moreover, while the price responses are similar for both models (see Figure 5b), the income responses are perverse (see Figure 5c).

To understand why the conclusions drawn from the models fitted over the 1982:12-1993:12 sub-sample are so different, we might start by examining the underlying structural relations. As mentioned earlier, in recursive systems like the ER and CP models, the initial effect of money shocks on interest rates requires that one only examine the interest elasticity of money demand drawing our attention to the estimated money demand curves in each period. The implicit contemporaneous components of the demand equations corresponding to those in equations 24 and 25 for the 1982:12-1993:12 period are

CP: \[ FF = -1.14NBR + 12.42Y + 18.28P + 2.97CP \]

ER: \[ FF = .26NBR + 12.12Y + 11.14P + .19RF + 3.05CP \]

Over the longer period, the interest rate coefficient was strongly negative so that the estimated liquidity effect was genuine. In this shorter sample, the situation is not as clear. A comparison of the two sets of estimates points to instability in the money-demand equation. On the basis of this evidence, one would have to be skeptical about the presence of a liquidity effect, although an alternative interpretation might be that the observations from the 1982-93 decade are just uninformative about the size of the interest rate coefficient, and that a longer series of data has

**Different Data Samples**

Compounding the difficulties arising from the use of different sets of variables and response functions, raising the possibility that the observed magnitudes for the various responses are partly an artifact of the estimation procedures adopted.
managed to produce more precise estimates of that parameter.\textsuperscript{12}

To shed further light on this issue, we estimated the money-demand equation from the CP model using varying-coefficient techniques. Figure 6a presents the recursive estimate of the NBR coefficient in the FF equation. What is striking in this graph is that the magnitude of the liquidity effect increased very sharply after the change of operating procedures of the Fed in October 1979. In light of the standard errors, the evidence for a liquidity effect in pre-1979 data does not seem very convincing, and there is a suggestion that the 1982-93 decade may be closer to the pre-1979 period in what it says about liquidity effects. To assess this latter proposition, we re-estimated the NBR coefficient, but now with a moving sample window of 120 months so that the last point estimate uses data from 1983:12-1993:12. Figure 6b presents this information. It is very clear from this graph that 1979-82 is a watershed period when it comes to empirical work on the liquidity effect. If it is omitted from the data, it would be very hard to believe that the initial impact on interest rates of money supply movements is not close to zero.\textsuperscript{13}

Given the sensitivity of results to the sample period, it is desirable to investigate the uncertainty about the estimates in more detail. Here we encounter some difficulties. The presence of (near) unit roots in the data means that standard asymptotic formulae for standard errors, based on the assumption that the random variables are stationary, will be incorrect and parametric simulation methods seem to be the best approach to producing standard errors. Even then, there are problems in implementing the simulations. One of these arises from the fact that, over any period incorporating 1979-82, there is extensive ARCH in the VAR equations for interest rates and money. The dependence introduced by the ARCH errors means that one cannot assume that the shocks are i.i.d. and, therefore, simple bootstrapping methods are not strictly appropriate in this context.\textsuperscript{14} We have ignored the effects of ARCH and have determined percentile-based, 90 percent confidence intervals for the CP model by re-estimating the impulse responses from 1,000 samples of artificial data bootstrapped from the estimated CP model.\textsuperscript{15} Figures 7a-c present the computed confidence intervals for the income responses over the three sample periods of 1959:01-1993:12, 1974:01-1993:12 and 1982:12-1993:12, respectively. We find that the income responses could easily be zero for the first few periods, and are then only positive in subsequent periods for models fit using the longer samples. The corresponding results for prices are presented in Figures 7d-i, and these show that negative price responses are easily realized from a model that has positive point estimates for price responses. Finally, as Figures 7g-i show, one gets a well-defined liquidity effect over the first two periods.

\textsuperscript{12} The ordering is therefore \((2, 1, 0), \text{AR}(3),\text{ARCH}(4, 2)\), which reverses Pand Q from the CP model. This has little effect. For example, for the recursive model, if one orders 3 first, the response at impact is nine rather than seven. There seems no good reason to choose one ordering over the other.

\textsuperscript{13} This is consistent with Cochrane (1988) and Gordon and Keefer (1992), who find a strong liquidity effect using single-equation, distributed techniques on data for the period 1979 to 1982, whereas similar analyses using data prior to 1979 were unable to find evidence for the liquidity effect.

\textsuperscript{14} There are many other problems that arise in computing confidence intervals which are not adequately dealt with in the literature. First, some studies use a Monte Carlo integration procedure in RATS, which assumes that VAR parameter estimators are normally distributed, and this will be incorrect in the presence of unit roots. Second, because the information presented is the whole impulse response function, the standard errors computed for any given response (say the \(k\)th step) do not capture the range of uncertainty about the whole function. Finally, the impulse responses are functions of the VAR parameters. If there are more of the former than the latter, some properties of the former must have a singular distribution. Since one sometimes sees hundreds of impulses displayed on a page, it is very likely that the distributions are singular.

\textsuperscript{15} Similar results to those reported here are obtained when the error is simulated from a \(N(0, \hat{\Sigma}_{v})\) distribution instead.
but not over the last. Notice also that, particularly for prices and output, the confidence intervals are asymmetric. This asymmetry may be due to the non-stationarity in the data. Some previous studies have assumed that the estimated coefficients can be drawn from a normal distribution, whereas it is known theoretically that they should be sampled from a skewed distribution if there are unit roots in the data. Sampling from a normal density will induce the confidence intervals to look symmetric. Lastrapes and Selgin (1994) are an exception, and they find asymmetry in their bootstrapped confidence intervals. In their case, however, we suspect the asymmetries are the result of biases in the point estimates arising from the use of first-differenced variables as instruments (see Pagan and Robertson, 1995, for details).

**The Effects on the Term Structure**

Relatively little attention has been paid to the impact of monetary policy upon the complete term structure of interest rates, despite the fact that the results will be
important to an understanding of the transmission mechanism. There is a voluminous literature on the term structure in both finance and economics which concentrates upon the slope of the term structure and the number of factors influencing it. Rarely are the factors decomposed into those that are monetary and those that are not. Cook and Hahn (1989) study the immediate changes seen in longer-term rates in response to an announced change in the federal funds rate, concluding that this effect becomes small for longer maturities. However, this does not address the question of the influence of a monetary policy change, since the federal funds rate is influenced by many factors, and we might expect them to have different influence at different points in the term structure.

One way to proceed would be to utilize the expectations theory of the term structure, which links long-term rates to the average of expected short-term rates.\[ r^L_n = n^{-1}E_t\left[\sum_{i=0}^{n-1} r_{it}\right].\]

Using the expression for \( r^L_n \) in equation 12 and taking derivatives with respect to \( e^n_t \),

\[ \frac{\partial r^L_n}{\partial e^n_t} = n^{-1} \sum_{i=0}^{n-1} e^n_{it}, \]

we can obtain the long-run responses by summing the short-term ones. For one-unit shocks to \( e^n_t \) in the CP model over the full sample period, these are \(-12.4 (n=1), -19.3 (n=4) \) and \( 3.180 (n=120) \), which are of the same order of magnitude as for the federal funds rate but of opposite sign at longer maturities.

An alternative method, which does not depend upon the expectations theory holding, is to simply add longer-term rates to the VAR and to directly compute impulse responses for various interest rates. These are presented in Figure 8 for FF, R3 and R10 using an augmented CP model ordered as \{Y, P, CP, NBR, FF, R3, R10, TR\}, and estimated over the three sample periods used in the paper. For the two longer periods, the outcomes resemble those noted by Cook and Hahn (1989), but the period 1982:12-1993:12 shows the greatest effect of monetary variations to be on the long-term rate.

**CONCLUSION**

That the Fed can influence the federal funds rate on a daily basis is scarcely debatable. What is puzzling has been the failure of these actions to show up in data. Perhaps this simply reflects the fact that most empirical

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16 In fact, this is a linearization of the precise formula, and higher-order terms in the Taylor series expansion show that the long-term rate will depend upon higher-order moments of the conditional density.
there are a number of caveats. Foremost among these are: The models do not seem to be very robust to data coming from the 1980s; The implied structural models can sometimes be implausible. The estimation procedures often rely on weak information and, for recursive models, the long-run multipliers can be contrary to a priori beliefs. How much damage these features do to the new view is an unsolved puzzle. If one encounters odd results, it is hard to know what their cause is without some underlying economic model. It may be that one can produce the observed responses within a plausible economic model as a consequence of choosing a particular calibration of it. Research in the past five years has to be credited with directing attention to the fact that analyses of the transmission mechanism require a systems perspective, but it is not clear that the recursive systems chosen for the investigation are as useful as they might be. Once unexpected results are found, the lack of a structure makes it very hard to account for them. In our view, the natural progression has to be toward non-recursive models with less profligate dynamics. The attempt to say nothing about dynamics has inevitably lead to a focus upon a set of variables that may be too narrow to capture the main interactions in an economy.

REFERENCES


Appendix

DATA AND DATA SOURCES

Except for the commodity price series the data are sourced from CITIBASE. The corresponding CITIBASE mnemonics are reported in parentheses. The data are monthly from 59:01 to 93:12. All series except interest rates and the exchange rate are seasonally adjusted.

Money (seasonally adjusted):
- \( M_2 \) (FM2) = log of \( M_2 \).
- \( M_1 \) (FM1) = log of \( M_1 \).
- \( M_0 \) (FMBASE) = log of money base (Federal Reserve Bank of St Louis definition).
- \( YR \) (FMRQA-vF6CMRE) = log of total reserves.
- \( NBR \) (FMRNBC) = log of non-borrowed reserves plus extended credit.
- \( BR \) (FMRRA - FMRNBC) = log of borrowed reserves excluding extended credit.
- \( NBRX \) = ratio of non-borrowed to total reserves (proportion).

Interest Rates (percent, not seasonally adjusted):
- \( R_10 \) (FYGT10) = 10-year Treasury Note yield.
- \( R_3 \) (FYGM3) = three-month Treasury bill yield (secondary market).
- \( FF \) (FYFF) = federal funds rate.
- \( RD \) (FYGD) = discount rate.
- \( RF \) (FWAFIT) = weighted-average foreign interest rate.

Other Series:
- \( Y \) (IP) = log of industrial production index.
- \( P \) (PUNEW) = log of consumer price index, urban.
- \( CP \) (76AXD) = log of industrial country commodity price index. From the IMF International Financial Statistics data tape.
- \( U \) (LHUR) = unemployment rate, all workers 16 and over.
- \( ER \) (EXRUS) = log of weighted-average exchange rate.