William A. Barnett and Ge Zhou

William A. Barnett is professor of economics at Washington University, St. Louis. Ge Zhou recently received a doctorate in economics from Washington University, St. Louis. *The authors benefitted from useful discussions with Athanasios Orphanides. Research on the project was partially supported by NSF grant SES 9223557.*

Commentary

WE ARE VERY PLEASED to be invited to comment on the new M2+ index, recently proposed in interesting papers by some Federal Reserve Board staff members (Collins and Edwards, 1994; and Orphanides, Reid and Small, 1994). The two papers presented at this conference by those Board staff members raise important and challenging questions that we believe should motivate much research in future years. In addition, we wish to commend those Board staff economists for their courage and integrity in pushing past barriers that have intimidated prior researchers and thereby precluded prior research on these difficult matters.

The basic issue is whether riskiness of the investment rate of return on an asset is a characteristic that rules out the possibility of an asset's contribution to the economy's liquidity. Oddly, that issue has largely precluded prior consideration of risky assets as components of central bank monetary aggregates. Yet clearly the position is groundless. While it is clear that risky assets are not good candidates for legal means of payment, monetary aggregates now contain many assets that are not legal means of payment. It has long been recognized that currency and demand deposits provide much, but by no means all, of the economy's monetary service flow.

No one has suggested that bond or stock mutual funds should be made legal means of payment. In addition, stock and bond mutual funds currently are bundled by companies into packages of funds that include money market funds within the bundle. Hence, it often is as easy as a telephone call to transfer funds from stock and bond funds into checkable money market funds. Although stock and bond funds certainly should not be made legal means of payment, it simply makes no sense to exclude bond and stock mutual funds from consideration as assets contributing monetary liquidity to the economy.

There is no necessary conflict between the existence of risky return and the contribution of liquidity to the economy. The two are not mutually exclusive.[1] Yet prior researchers have excluded assets having substantial principle risk from consideration as components of monetary aggregates. It indeed is odd that such an obviously groundless prejudice has precluded research by the entire economics profession on an important

[1] Formally, the correct method used to determine the clustering of components within an aggregation-theoretic monetary aggregate is testing for blockwise weak separability. An innovative new approach to testing for weak separability was recently proposed by Swofford and Whitney (1994). Although risky return complicates testing for weak separability, risk in no way precludes acceptance of that hypothesis. In fact, a successful test of weak separability with random rates of return is included in Barnett and Zhou (1994).

topic. The authors of the two Board staff papers are right. The authors have done a service to the profession by exploring the topic for the first time.

CHALLENGES PRESENTED TO ECONOMIC THEORY

Riskiness of the rate of return simply does not preclude the production of monetary services by an asset. Riskiness of the rate of return, however, certainly does make life more difficult for index number theorists and aggregation theorists. Most of the literature in those fields is produced under the assumption of perfect certainty or risk neutrality. Extensions of that literature to risk aversion were begun recently by Poterba and Rotemberg (1987), Barnett and Yue (1991), Barnett, Hinich and Yue (1989) and Barnett and Zhou (1994). We believe that the important issues raised by Collins and Edwards (1994) and Orphanides, Reid and Small (1994) at this conference should serve as motivation for further research on index number theory and aggregation theory under risk aversion. We do indeed welcome the increased motivation in that area provided by the work of those Board staff researchers.

But there is an even more fundamental problem. The existence of an investment rate of return, even a perfectly certain one, raises questions about how an asset should be incorporated into an aggregate. While the existence of such a rate of return does not prevent an asset from producing monetary services, the share of the asset's services that can be viewed as "monetary" is strongly affected. This comment is directed towards an investigation of that share.

HISTORICAL BACKGROUND

At one time, money was cash plus demand deposits. No controversy existed on that topic. But a sequence of technological changes and innovations occurred, and continued to occur, such that an increasingly large number of substitutes for money produced an increasingly large share of the economy's monetary services. The result was the Bach Commission, the Gurley and Shaw (1960) book, the Pesek and Saving (1967) book, and many other important contributions that influenced the growing movement towards the construction of increasingly broad monetary aggregates. The response of most central banks, however, has been to accept one aspect of that research while conveniently overlooking another closely related aspect.

In particular, the researchers who first worked in that area were very clear on one simple, elementary fact: *Investment yield is not a monetary service.* There was a reason that all monetary economists once agreed that money included only cash and non-interest-bearing demand deposits. If investment yield were a monetary service, then coal mines would be money. Land would be money. The entire capital stock of the United States would be money.

This is not to deny that assets that produce an investment rate of return, whether risky or not, can produce monetary services. Interest yielding monetary assets, however, are joint products. Some of their services are monetary. Some are not. This fact seems to have escaped many of the world's central banks. To the degree that the economy equates marginal utilities per dollar across assets, the marginal utility of monetary services produced by an asset must decrease as its marginal non-monetary services increase—and investment return is very clearly not a monetary service.

To underscore our point, we bring up the famous diamonds-versus-water paradox. The total utility of water exceeds that of diamonds, even though the marginal utility of diamonds exceeds that of water. In fact, as one moves along a concave utility function, marginal utility varies inversely with total utility. Hence, the statements made above about marginal utilities should not be confused with the total or average monetary service flow produced by an asset. But it is the marginal utilities that are relevant to measuring the prices in index numbers, such as the Divisia, Fisher ideal, Paasche or Laspeyres quantity indexes. We hope that we also do not have to remind this audience that the prices (user costs) in such indexes are not the weights. We nevertheless find that this literature contains many misunderstandings of monetary index number theory, and most of those misunderstandings are produced by confusing prices with weights and marginal utilities with total or average utilities.

FERRARI SPORTS CARS

It has been asked at this conference whether stock funds or bond funds are "money," or are they not money. We would like to ask a different question. Are Ferrari sports cars transportation machines or recreational machines? We can imagine a Ferrari owner responding that a

Ferrari is strictly a transportation machine, and that the high price is produced by the Ferrari's superior performance on highways and on winding roads. Hence, the price of a Ferrari is the discounted present value solely of the transportation services. But I expect that most of the rest of us would view the price of a Ferrari as being the sum of the discounted present values of two different flows: transportation services and recreational services.

Ferraris are joint products, in terms of the services produced. Interest-bearing monetary assets similarly are joint products. Such assets produce both monetary and non-monetary services, whereby the interest yield unquestionably is in the latter category. Hence, the correct answer to the question asked by the Board's staff economists at this conference is that such assets, including stock and bond mutual funds, are partially money and partially not money.

WHAT TO DO NEXT

We see that we are presented with a paradox. More and more assets are contributing to the economy's monetary service flow. As made clear by the authors of the two Board staff papers, stock and bond funds now are among those assets. The investment yields of that growing collection of assets, however, are not monetary services. If we do not add such assets into the monetary aggregates, we overlook some of the economy's monetary service flow. If we do add those assets into the aggregates, we contaminate the aggregates with non-monetary services.

The answer should be obvious. We must untangle the two discounted present values: the discounted present value of the monetary service flow and the discounted present value of the investment yield. Indeed, it can be done.

THE THEORY

Barnett (1987) defined the economic stock of money to be the discounted present value of expenditure on the services of monetary assets. Barnett (1991) derived that discounted present value in the form that we display below. During period s let p_s^* = the true cost of living index, let M_{is} be nominal balances of monetary asset i, let r_{is} be the nominal expected holding period yield on monetary asset i, and define $m_{is} = M_{is}/p_s^*$ to be real balances of monetary asset i. The current period is defined to be period t so that $s \geq t$. Define the discount rate for period s to be

$$(1)\ \rho_s = \begin{cases} 1 & for\ s = t \\ \prod_{u=t}^{s-1} (1+R_u)\ for\ s > t. \end{cases}$$

By letting the planning horizon, T, go to infinity in the second term of Barnett (1978, eq. 2; 1980, eq. 3.3; 1981, eq. 7.3), we immediately acquire the following definition for the Economic Stock of Money, first derived as definition 1 and equation 2.2 in Barnett (1991):

Definition 1: Under risk neutrality, the economic stock of money during period t is

$$(2)\ V_t = \sum_{s=t}^{\infty} \sum_{i=1}^{n} \left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_{is})}{\rho_{s+1}} \right] m_{is}.$$

The concept of economic stock used to produce Definition 1 is the user-cost-evaluated expenditure on the services of the n monetary assets that are its components. It should be observed that the procedure used in Barnett (1978, eq. 2; 1980, eq. 3.3; 1981, eq. 7.3) to acquire that discounted present value for finite T was just back substitution and algebraic manipulation of the sequence of flow-of-funds identities. Hence, our conclusion is produced entirely from accounting identities.

If we now substitute equation 1 and $m_{is} = M_{is}/p_s^*$ into equation 2, we acquire the following result:

$$(3)\ V_t = \sum_{s=t}^{\infty} \sum_{i=1}^{n} \left[\frac{R_s - r_{is}}{\prod_{u=t}^{s}(1+R_u)} \right] M_{is}.$$

Unfortunately, this equation includes expected future values of interest rates and of monetary asset holdings. While it may be reasonable to assume that interest rates are stationary, no such easy simplification is available for the stochastic process of future monetary asset holdings. We believe that a useful way to proceed would be to use VAR forecasts of the monetary asset holdings and rates of return. We plan to produce results using that approach. But considering the time constraint that we faced with this conference, we had no choice but to make strongly simplifying assumptions. In particular, we make the assumption which causes equation 3 to collapse into the Rotemberg, Driscoll and Poterba (1994) CE index, first interpreted to be a stock index in Barnett (1991).

Definition 2: The CE index is

$$(4)\ V_t=\sum_{i=1}^{n}\frac{R_t-r_{it}}{R_t}M_{it}$$

We seek to find conditions under which equation 4 will equal equation 3. To that end, suppose that expectations are stationary in the sense that $r_{is} = r_{it}$ and $R_{is} = R_{it}$ for all $s \geq t$, and consider the static portfolio, $(M_{1s}, M_{2s}, ..., M_{ns}) = (M_{1t}, M_{2t}, ..., M_{nt})$, for all $s \geq t$. Equation 3 reduces to

$$(5)\ V_t=\sum_{i=1}^{n}\sum_{s=t}^{\infty}\left[\frac{R_t-r_{it}}{(1+R_t)^{s-t+1}}\right]M_{it}.$$

Observe, however, that

$$(6)\ \sum_{s=t}^{\infty}\frac{R_t-r_{it}}{(1+R_t)^{s-t+1}}=\frac{R_t-r_{it}}{R_t},$$

since the left side of equation 6 is a convergent geometric series (minus the first term in the series). Substituting equation 6 into equation 5, we acquire our result:

Theorem 1: Under stationary expectations, the CE index is equal to the Economic Money Stock.

Under the stationary expectations assumption, we easily can discount to present value the expected investment yield flows, $r_{is}M_{is} = r_{it}M_{it}$ for $s \geq t$ to get the following capitalized value:

$$(7)\ V_t^*=\sum_{s=t}^{\infty}\sum_{i=1}^{n}\left[\frac{r_{it}M_{it}}{(1+R_t)^{s-t+1}}\right].$$

Again, we have a convergent geometric series in the summation over s at any given i, so that we find

$$(8)\ V_t^*=\sum_{i=1}^{n}\frac{r_{it}}{R_t}M_{it}$$

Adding equation 8 to equation 4, we find that

$$(9)\ V_t+V_t^*=\sum_{i=1}^{n}M_{it}.$$

The conclusion is clear. The simple-sum monetary aggregates measure the stock of money only if the investment (interest) yield of the monetary components is treated as a monetary service. Yet it is difficult to think of any macroeconomic school of thought which has ever viewed the interest yield on monetary assets to be a monetary service. In fact, that possibility was considered carefully and rejected unequivocally in Pesek and Saving (1967).

In the discussion that follows, we shall use this result to decompose the simple-sum aggregates into their investment share and their monetary services share. In each case, the share is produced by discounting the flow to the present value, the interest yield in one and the service flow in the other. The decomposition then is into V_t and V_t^*, which partition

$$\sum_{i=1}^{n}M_{it}$$

into its two parts, in accordance with equation 9.

"ANCIENT" HISTORY

There was a time—long, long ago—when money was currency and demand deposits, and demand deposits did not yield interest. In those days, we see that

$$V_t^* = 0, \text{ so that } V_t = \sum_{i=1}^{n} M_{it}.$$

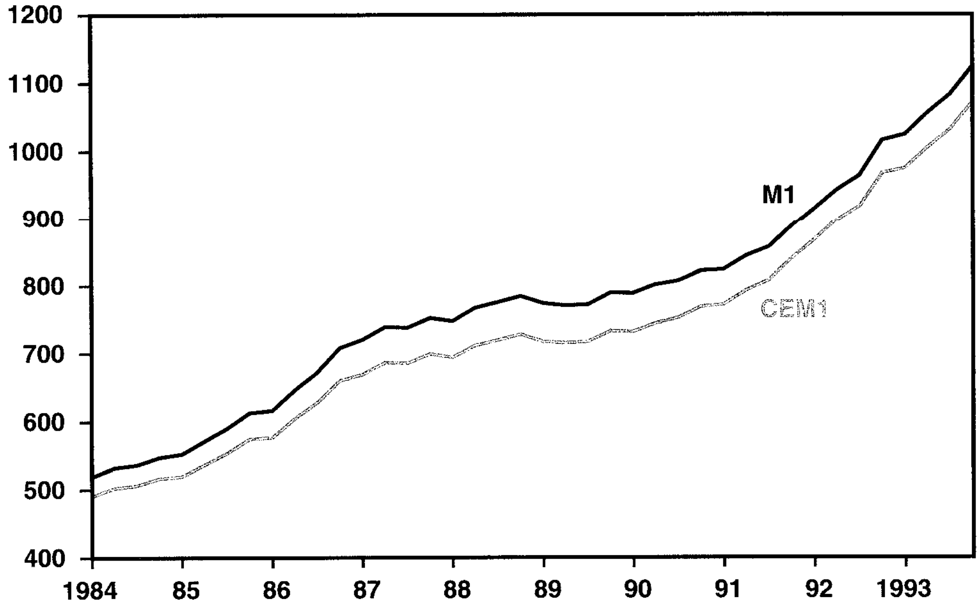
Those were the days when the simple-sum monetary aggregates were created, and we see that the people who created them knew what they were doing. But that simpler world is long gone. Many assets that contribute to the economy's monetary services also yield an investment rate of return.

THE DATA

We computed the decomposition into V_t and V_t^* of the official simple-sum M1 and M2 indexes along with the corresponding decomposition into V_t and V_t^* of the newly proposed simple-sum M2+ index. We also computed the decomposition into V_t and V_t^* of bond mutual funds and stock mutual funds as a means of further investigating the source of the difference in behavior of M2 versus M2+. The attached figures provide the results.

The decomposition depends upon the measurement of the benchmark rate of return R_t. Clearly, V_t increases as R_t increases, and V_t^* decreases as R_t increases. Hence, the monetary service share (versus the investment share) of the simple-sum aggregate "joint product" increases as R_t increases. The results can be biased in the direction favoring the inclusion of stock and bond funds by choosing an artificially high setting for R_t. For the purpose of biasing the

Figure 1
M1 Joint Product and Economic Capital Stock



M1= simple sum joint product
CEM1= economic capital stock part of the joint product

results in that direction intentionally, we chose the highest possible setting for R_t that could be connected in any way with the available data.

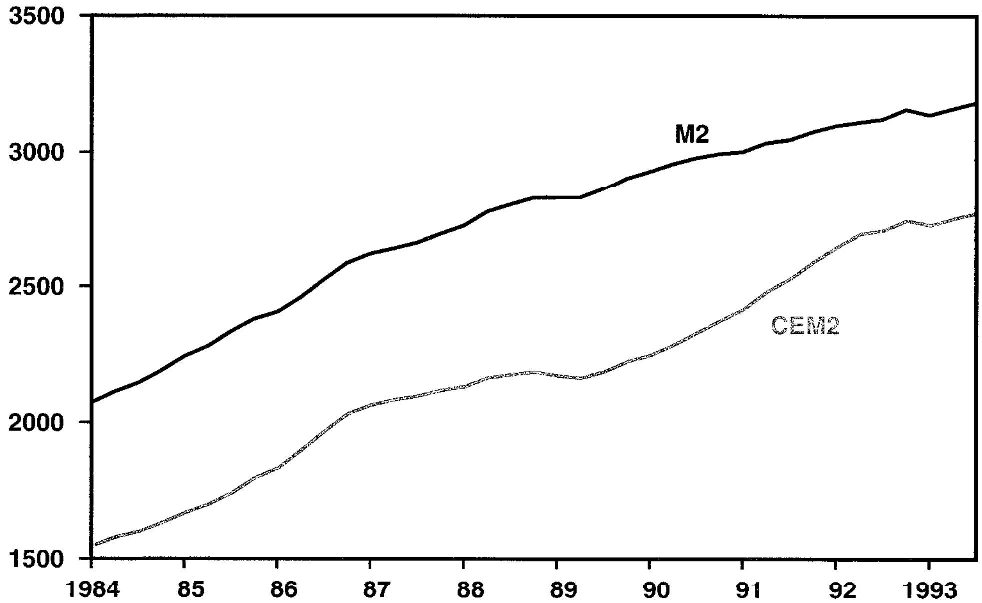
As shown by Rotemberg, Driscoll and Poterba (1994), V_t has a very volatile growth rate and, hence, they advocate smoothing the interest rates to produce smoother growth of the aggregates. This is not surprising, since V_t and V_t^* are stock aggregates which tend to have volatile growth rates. We use the same smoothing method advocated by Rotemberg, Driscoll and Poterba (1994). In particular, we replaced all of the interest rates in the index by 13-quarter centered moving averages. Since the moving averages are centered, they are not defined for the first six quarters or the last few six quarters. We used the method advocated by Rotemberg, Driscoll and Poterba and phased in the centered moving average from asymmetric averages computed during the first six and last six observations.

Once the smoothed interest data had been constructed, we searched over those series for the highest smoothed interest rate ever attained by any component asset during our sample. That ex post rate of return was 24.2 percent, which we selected to be the value of R_t for all t. In general, there is no reason for the benchmark rate to be a constant or to equal any ex post rate of return, since ex post rates of return tend to be much more volatile than ex ante expected rates of return. Our selection for the benchmark rate, however, produces the largest value that we could connect with the data, and we wanted to produce results that would be biased in favor of the Board staff members' proposal. In interpreting our results, the division of the simple sum into the components V_t and V_t^* should be understood to be biased very strongly towards V_t and away from V_t^*. Hence, the monetary services share in the joint product should be viewed as intentionally exaggerated.

THE RESULTS

Figure 1 contains the partition of simple-sum M1 into its investment share and its monetary services share. The solid line is the monetary service share produced from the computed value of V_t. The vertical gap between the solid line and the dotted line is the investment-motivated

Figure 2
M2 Joint Product and Economic Capital Stock



M2= simple sum joint product
CEM2= economic capital stock part of the joint product

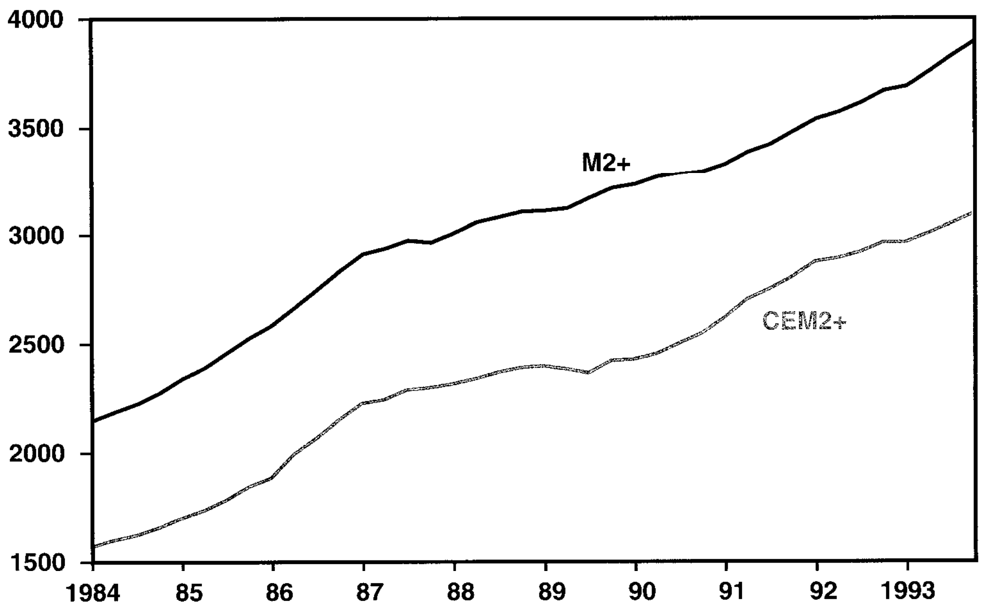
share, V_t^*, which could be interpreted as the "error-in-the-variable" embedded in the simple-sum index, M1. The height of the dotted line from the horizontal axis is the simple-sum index, equaling the sum of V_t and V_t^*. As is evident from Figure 1, the error-in-the-variable gap is relatively small and does not vary much over the sample. With a relatively constant vertical gap, the rate of growth of M1 is not greatly affected by the error-in-the-variable gap. For most statistical inferences and for policy, the growth rate of money is what matters. Hence, we see that the existence of the V_t^* error gap produces little difficulty for M1.

Figure 2 contains the analogous plot and decomposition for the official simple-sum M2 aggregate. Observe that the error-in-the-variable gap is large (and would be much larger for a more realistic choice of R_t). In addition, that gap is variable and trends downward, especially recently. Hence, the existence of the error gap not only effects the short-run growth rate dynamics of the aggregate, but also biases downward the long-term growth rate. Inferences and policy are not invariant to the existence of this gap.

Figure 3 contains the decomposition for the M2+ aggregate that has been proposed at this conference by Collins and Edwards (1994) and Orphanides, Reid and Small (1994). Observe that while the error gap is even larger than for M2, the size of the gap is less variable and no longer trends downward. Hence, the growth rate of the error-shifted dotted line approximately tracks the growth rate of the "correct" solid line.

To see why M2+ stabilizes the size of the gap and thereby improves on the aggregate's growth rate performance, see Figures 4 and 5, which display the decomposition of the stock mutual funds data and the bond mutual funds data into their economic capital stock share and their error-in-the-variable shift. Observe that in each of those two cases, the size of the gap grew rapidly during the past two years. This growing error gap offsets the declining error gap in M2, when the stock and bond fund data are added into M2.

Figure 3
M2+ Joint Product and Economic Capital Stock



M2+= simple sum joint product
CEM2+= economic capital stock part of the joint product

In short, we conclude that the authors of the two papers presented at this conference are correct in concluding that the growth rate behavior of M2 is improved by incorporating the stock and bond mutual fund data into M2. Indeed, it does appear that substitution from M2 components into stock and bond mutual funds has become important.

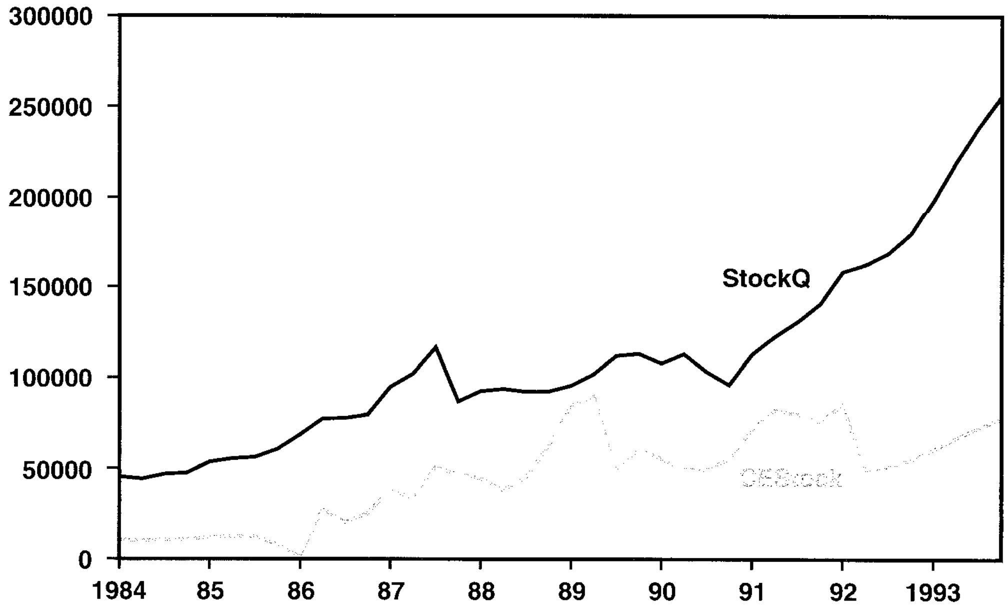
WHERE IS ALL OF THIS GOING?

There is an underlying dynamic to this trend in monetary theory. Stabilizing the size of the error gap requires continually incorporating more assets into the monetary aggregates. The size of the gap keeps growing. The share of the monetary aggregate representing discounted monetary services continues to decrease, and the monetary aggregates look increasingly like pure investment capital rather than money. Even if stabilizing the size of the gap offsets long-run errors in growth rate paths, the short-run dynamics of the aggregates are likely to become increasingly disjoint from monetary services growth.

In this paper we use the CE index, equation 4, to permit easy decomposition of the simple-sum aggregate "joint product" into its monetary service and investment shares. Using the formula in equation 3 with forecasted variables, perhaps by a VAR, would be better. But generating data that depends upon forecasts is unpleasant for data-producing governmental agencies. Smoothing interest rates to decrease the volatility of the resulting aggregate is also unpleasant for governmental agencies. For this conference, decomposition of the stocks in that manner was revealing. But as a means to produce data for a central bank, there is a better way. It is the Divisia monetary aggregates long advocated by Barnett (1980). See Barnett, Fisher and Serletis (1992) and Belongia and Chalfant (1989) for an overview and some of the relevant empirical results.

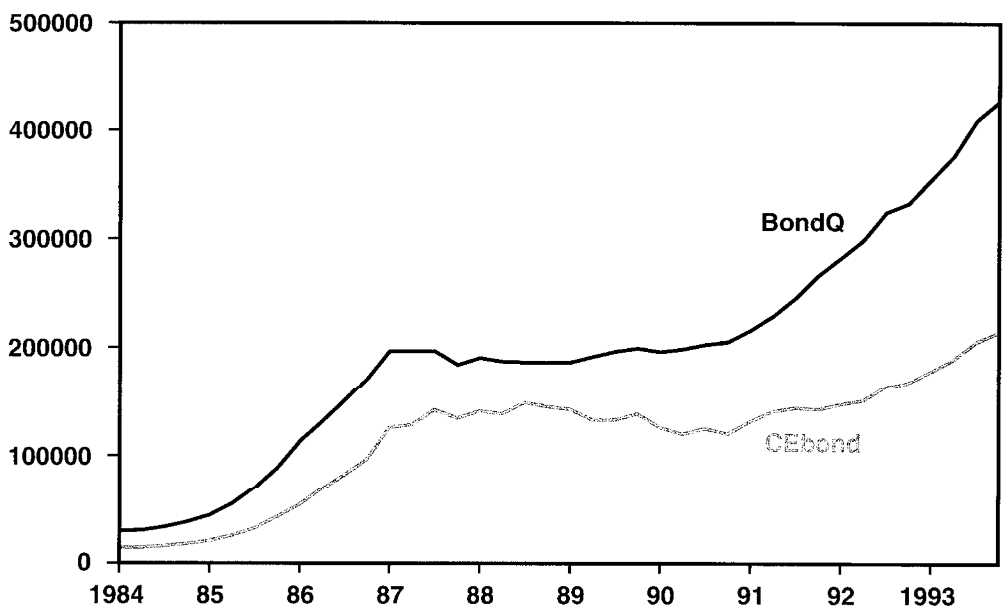
The Divisia monetary aggregates directly measure monetary service flows, not the discounted stock levels. The Divisia monetary aggregates do not require smoothing of interest rates to smooth the index's growth rate, and the Divisia monetary

Figure 4
Common Stock Mutual Funds Joint Product and Economic Capital Stock



StockQ= simple sum joint product
CEstock= economic capital stock part of the joint product

Figure 5
Bond Mutual Funds Joint Product and Economic Capital Stock



BondQ= simple sum joint product
CEbond= economic capital stock part of the joint product

aggregates contain no variables that need forecasting.[2] In addition, Barnett (1991) proved that if we could do the forecasting needed to compute the monetary capital stock (equation 3), the result would be identical to that produced by discounting to present value the future stochastic process of the Divisia monetary aggregate.

REFERENCES

Barnett, William A. "Reply to Julio J. Rotemberg," in Michael T. Belongia, ed., *Monetary Policy on the 75th Anniversary of the Federal Reserve System: Proceedings of the Fourteenth Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis*. Kluwer Academic Publishers, 1991, pp. 232-43.

______. "The Microeconomic Theory of Monetary Aggregation," in William A. Barnett and Kenneth J. Singleton, eds., *New Approaches to Monetary Economics*, Proceedings of the Second International Symposium in Economic Theory and Econometrics. Cambridge University Press, 1987, pp. 115-68.

______. "The Optimal Level of Monetary Aggregation," *Journal of Money, Credit and Banking* (February 1982), pp. 687-710.

______. *Consumer Demand and Labor Supply: Goods, Monetary Assets, and Time*. North-Holland, 1981.

______. "Economic Monetary Aggregates: An Application of Index Number and Aggregate Theory," *Journal of Econometrics* (summer 1980), pp. 11-48.

______. "The User Cost of Money," *Economics Letters*, vol. 1 (1978), pp. 145-9.

______, and Ge Zhou (1993), "Financial Firm's Production and Supply-Side Monetary Aggregation Under Dynamic Uncertainty," this *Review* (March/April 1994), pp. 133-65.

______, Douglas Fisher, and Apostolos Serletis. "Consumer Theory and the Demand for Money." *Journal of Economic Literature* (December 1992), pp. 2086-119.

______, and Piyu Yue. "Exact Monetary Aggregation Under Risk," Washington University (Department of Economics), St. Louis, *Working Paper #163* (December 1991).

______, Melvin Hinich, and Piyu Yue. "Monitoring Monetary Aggregates Under Risk Aversion," in Michael T. Belongia, ed., *Monetary Policy on the 75th Anniversary of the Federal Reserve System: Proceedings of the Fourteenth Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis*. Kluwer Academic Publishers, 1989, pp. 189-222.

Belongia, Michael T., and James Chalfant. "The Changing Empirical Definition of Money: Some Estimates From a Model of the Demand for Money Substitutes," *Journal of Political Economy* (April 1989), pp. 387-97.

Collins, Sean, and Cheryl L. Edwards. "An Alternative Monetary Aggregate: M2 Plus Household Holdings of Bond and Equity Mutual Funds," this *Review* (November/December 1994), pp. 7-29.

Gurley, John, and Edward S. Shaw. *Money in a Theory of Finance*. Brookings Institution, 1960.

Orphanides, Athanasios, Brian Reid, and David H. Small. "The Empirical Properties of a Monetary Aggregate That Adds Bond and Stock Funds to M2," this *Review* (November/December 1994), pp. 31-51.

Pesek, B.P., and T.R. Saving. *Money, Wealth and Economic Theory*. Macmillan, 1967.

Poterba, James M., and Julio J. Rotemberg. "Money in the Utility Function: an Empirical Implementation," in William Barnett and Kenneth Singleton, eds., *New Approaches to Monetary Economics*, Proceedings of the Second International Symposium in Economic Theory and Econometrics. Cambridge University Press, 1987, pp. 219-40.

Rotemberg, Julio J., John C. Driscoll, and James M. Poterba. "Money, Output, and Prices: Evidence From a New Monetary Aggregate," *Journal of Business and Economic Statistics* (forthcoming).

Swofford, James L., and Gerald A. Whitney. "A Revealed Preference Test For Weakly Separable Utility Maximization With Incomplete Adjustment," *Journal of Econometrics* (January/February 1994), pp. 235-49.

[2] It is necessary to measure the benchmark rate, *R*, to construct the Divisia monetary aggregates, and we advocate the use of the upper envelope of the yield-curve-adjusted, holding-period yields on all of the components in the broadest aggregate. Obviously, we do not advocate the use of the extreme, constant setting of 24.4 percent, chosen for an illustrative purpose in this paper. However, it should be observed that the behavior of the Divisia monetary aggregate is much more robust to variations in the method of measuring the benchmark rate than is the CE index, which is very sensitive to that rate's selection. The reason is that the benchmark rate appears symmetrically in both the numerator and denominator of the share weights of the Divisia index, which in turn is a growth-rate index. Since the CE index is a level index, variations in any interest rate, including the benchmark rate, produce jumps in the level of the unsmoothed index. Jumps in levels produce spikes in growth rates.