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The Inflation Tax and the Marginal Welfare Cost in a World of Currency and Deposits

HOW HIGH IS THE OPTIMAL rate of inflation? The answer depends on the range of benefits and costs associated with inflation that are considered by the monetary authority in choosing the inflation rate. For example, if one considers the effects of inflation on the distributions of income and wealth, its interactions with the tax code or the transition cost of changing the expected rate of inflation, or if one adopts the alternative perspectives of different economic agents, the benefits and costs can be relatively large and difficult to assess. This article abstracts from transitory and largely avoidable aspects of inflation, and focuses instead on the fundamental public finance aspects of the monetary authority's problem. In this case, the net benefits and costs are those associated with an inflation rate that is perfectly anticipated; the benefit of inflation that accrues to the monetary authority (typically the government) is the revenue from inflationary money creation. This benefit is analogous to the revenue arising from a specific tax on any other good or service.

Inflation imposes a tax on money holdings because it is the rate at which individuals lose the purchasing power of a dollar. To lower the total cost of holding money, individuals change their holdings and their use of money when inflation rises. Their efforts to do so, however, reduce their total services from real money balances, thereby lowering individuals' real income. This loss is the welfare cost of inflation. The optimal rate of inflation is found by comparing the marginal welfare cost of revenue from inflation with the marginal cost of alternative sources of revenue. An efficient system of tax collection minimizes the welfare cost of a given flow of tax revenue; this requires that the inflation rate must be chosen so that the marginal cost per dollar of revenue from inflation is the same as the marginal cost of alternative sources of revenue.

In the analysis below, these concepts are developed for models involving a money stock made up of currency only, competitively priced bank deposits only, and a mix of both. The differences in each case clarify the analysis as well as provide some insight into the implications of the analysis for the optimal inflation rate.

THE MARGINAL WELFARE COST OF REVENUE FROM MONEY CREATION: THE CURRENCY CASE

Almost two decades ago, it was shown that a simple formula provides a method of calculating the additional welfare cost of collecting a dollar of revenue from money creation. This measure is the ratio of the marginal welfare cost of inflation to the marginal revenue from a change in anticipated inflation.[1] To derive this formula, assume the only money is currency and that the demand for real money balances depends only on the nominal rate of interest, holding other influences constant:

$$(1)\ m = \varphi(i)$$

The welfare loss, W, is

$$(2)\ W = \int_0^i \varphi(x)dx - i\varphi(i),$$

and the marginal welfare loss from a rise in inflation, π, is reflected in the incremental loss from a rise in the nominal interest rate:

$$(3)\ \frac{dW}{di} = -i\varphi'(i).$$

Using the Phelps-Auernheimer (Phelps, 1973; Auernheimer, 1974) definition of the revenue, R, we have

$$(4)\ R = \varphi(i)i,$$

and the marginal revenue is

$$(5)\ \frac{dR}{di} = \varphi(i) + i\varphi'(i).$$

Since the elasticity of demand for real balances is

$$(6)\ N_i = -\frac{i\varphi'(i)}{\varphi(i)},$$

the marginal welfare cost per unit of revenue, the ratio of equations 3 and 5, is

$$(7)\ \frac{dW}{dR} = \frac{N_i}{1 - N_i}.$$

Equation 7 is a variant of the well-known Ramsey tax rule (tax more heavily goods in inelastic demand) and assumes, as does the Ramsey rule, cross effects absent within the taxed sector. The formula is useful in answering the question: What rate of inflation (money rate of interest) would equalize the marginal welfare cost per dollar revenue accruing to inflation tax with an index of such costs due to other distortionary taxes?

The analysis we are conducting is in the realm of balanced budget incidence. We raise the inflation tax on real balances until the marginal welfare cost per dollar of revenue is equal to an index of these per dollar distortions for other taxes. The increase in revenue is used by the government for exhaustive expenditures rather than rebated to consumers directly, or indirectly through reduced taxes.

Another observation is in order. Although I have illustrated the use of these formulas by plugging in estimates of the marginal welfare cost, the main contribution of the paper lies in the provision of the formulas themselves. If these formulas pass muster, other empirical observations can be plugged in.

Assume the demand for real cash balances follows the Cagan semi-log form $M/P = A \exp(-bi)$. Friedman (1971) uses three alternative values of b, 5, 10 or 20. Laidler (1986) cites .15 as the typical interest elasticity of demand for M1. If we assume the real rate is 1.5 percent, which equals the money rate at zero inflation, the value of b is .15/.015 or 10 percent. To err on the side of charity to inflationary finance, we use a value of 5 for b. Tower (1971) cites 10 percent as the upper limit to the index of the marginal welfare cost per dollar revenue for other distorting taxes. This estimate is considerably lower than those of Ballard, Shoven and Whally (1985), which range from 17 to 56. We assume 10 percent as the marginal welfare cost per dollar of revenue for other distortionary taxes. Using the Cagan function, we have in a currency-only world

$$(8)\ \frac{dW}{dR} = \frac{ib}{1 - ib},$$

where $dW/dR = 5i/(1\text{-}5i) = .1$ and $i^* = .018$. With the real rate = 0.015, the "optimal" inflation rate is approximately zero. Given the parameter values we have assumed, a very modest tax on real balances equalizes the marginal welfare cost per dollar of revenue to an index of distortions due to other taxes.

THE MARGINAL WELFARE COST OF REVENUE: COMPETITIVELY PRICED DEPOSITS ONLY

A variant of the above formula holds for a large number of competitive banks subject to a sterile legal reserve requirement, f, (Marty and Chaloupka, 1988). An individual bank would be forced by competition to pay $(1\text{-}f)i$ on its

[1] The real rate of interest is held constant as the money rate of interest, i, varies in such an analysis (Marty, 1976).

deposits where i is the yield on its assets. The opportunity cost of holding deposits is then fi. In a deposit-only world, the welfare cost becomes

$$(9)\ W = \int_0^{if} \varphi(x)dx - if\varphi(if).$$

The marginal change to welfare due to inflation is

$$(10)\ \frac{dW}{di} = f\varphi(if) - [if\varphi'(if)f + \varphi(if)f] = -if\varphi'(if)f.$$

In this case, revenue is

$$(11)\ R = fi\varphi(if).$$

The marginal increment to revenue as the interest rate changes, dR/di, is the bracketed term in equation 10. Since the elasticity of demand for deposits is

$$(12)\ N_{if} = -\frac{fi\varphi'(if)}{\varphi(if)},$$

the marginal welfare cost per dollar increment to revenue is

$$(13)\ \frac{dW/di}{dR/di} = \frac{N_{if}}{1-N_{if}}.$$

The authorities in a bank-only world can set a money rate of interest equal to that in the currency-only world divided by the reciprocal of the reserve ratio, f. If the optimal money rate in the currency world were 10 percent, that rate can be set at 40 percent in a world of deposits (assuming the reserve ratio is 25 percent). Both the welfare loss and the tax revenue, however, are the same as in a currency-only world. Although the tax rate (the money rate of interest) is higher by the reciprocal of the reserve ratio, the tax base is reduced by the share of high powered money in the total money supply $f(M/P)$.

Assume initially that the demand for deposits has the same functional form as that for currency, that the marginal welfare costs per dollar of revenue for other distortionary taxes is the same as in the world of currency, and that the reserve ratio is 13 percent (realistic for the United States). Since $dW/dR = ifb/(1\text{-}ifb)$, we have $.1 = i(.13)5/[1 - i(.13)(5)]$ then $i^* = 13.8$ percent, which is equal to the money rate in a world of currency, 1.8 percent, divided by the reserve ratio, 13 percent.

With the real rate equal to 1.5 percent, the "optimal" rate of inflation is 12.3 percent.

THE MARGINAL WELFARE COST OF REVENUE FROM INFLATION: CURRENCY AND COMPETITIVELY PRICED DEPOSITS

We now show that the above analysis can be extended to a world of both currency and deposits. The demand function for each component is still referred to as φ, but they are potentially different and the different measure of cost, i or if, is used to indicate this. The counterpart measures are

$$(14)\ W = \left(\int_0^{i} \varphi(x)dx - i\varphi(i)\right) + \left(\int_0^{if} \varphi(x)dx - if\varphi(if)\right),$$

$$(15)\ \frac{dW}{di} = -i\varphi'(i) - if\varphi'(if)f,$$

$$(16)\ R = i\varphi(i) + if\varphi(if),$$

$$(17)\ \frac{dR}{di} = i\varphi'(i) + \varphi(i) + if\varphi'(if)f + \varphi(if)f$$

and

$$(18)\ \frac{dW}{dR} = \frac{-i\varphi'(i) - iff\varphi'(if)}{\varphi(i) + i\varphi'(i) + iff\varphi'(f) + f\varphi(if)}.$$

Since

$$(19)\ N_{if} = \frac{-i\varphi'(if)f}{\varphi(if)},$$

we obtain

$$(20)\ \frac{dW}{dR} = \frac{\frac{C}{M}N_i + \frac{D}{M}fN_{if}}{\frac{C}{M}(1-N_i) + \frac{D}{M}f(1-N_{if})},$$

where C is currency, $\varphi(i)$, and D is deposits, $\varphi(fi)$.

Once again, set the index of the marginal welfare costs per dollar increment to revenue for other distorting taxes equal to 0.1. Let the reserve ratio be 13 percent and ratio of currency to the money supply be 30 percent (the ratio of bank deposits to money is then 70 percent). These figures correspond broadly to ratios in place in the United States for the early 1990s. Again set the semi-log slope of the Cagan function equal to 5. Then we have $dW/dR = [(.3)(5i) + .7(.65i)(-13)]/\{(.3 - 1.5i) + .091[1 - (.13)(5i)]\} = 0.1$. Then $i^* = 2.28$ percent. It should be noted that, although the formula is a weighted average of currency and deposits, the currency weight dominates the solution. While demand deposits are 70 percent of the money supply, the tax base is only the ratio of reserves to the money supply —that is, 9 percent. Given this low reserve ratio, currency commands dominate weight.

The formula makes intuitive sense. If the revenue ratio equals 100 percent, so that demand deposits pay no interest and, assuming for simplicity, that the demand function (φ) for deposits is the same as that for currency, the formula reduces to

$$(21)\ \frac{dW}{dR}=\frac{\frac{C}{M}N_i+\frac{D}{M}N_i}{\frac{C}{M}(1-N_i)+\frac{D}{M}(1-N_i)}=\frac{N_i}{1-N_i}\ .$$

In effect, currency and deposits are of the same stuff.

On the other hand, if the reserve ratio is zero, deposits produce neither seignorage nor a welfare loss; we are, in effect, in a currency-only world because the monetary authority receives no revenue from deposits. In this case, the formula reduces to

$$(22)\ \frac{dW}{dR}=\frac{N_i}{1-N_i}\ .$$

This again makes intuitive sense since only currency is taxable.

Variants of the above formulas can be derived. Consider, for example, a world in which an effective prohibition on the payment of interest on deposits is in effect. Then

$$(23)\ W=\int_0^i\varphi(x)dx-i\varphi(i)+\int_0^i\varphi(x)dx-i\varphi(i)$$

and

$$(24)\ R=i\varphi(i)+if\varphi(i).$$

It follows that

$$(25)\ \frac{dW}{dR}=\frac{\frac{C}{M}N_i+\frac{D}{M}N_i}{\frac{C}{M}(1-N_i)+\frac{D}{M}f(1-N_i)}\ .$$

This is similar to the formula in equation 20, where deposits pay interest, but without the reserve ratio, f, in the second term of the numerator. If the reserve ratio equals zero and there is no interest paid on deposits, bank deposits yield no government revenue, but a welfare loss accrues to both currency and deposits. If the reserve ratio equals 100 percent, the interest prohibition on deposits is unnecessary, but both currency and deposits incur a welfare loss and both provide seignorage. Once again, the formulas make intuitive sense.

CONCLUSIONS AND COMMENTARY

The above analysis has imposed the zero-profit condition that the return on interest-bearing assets is paid out in interest on deposits. This condition ignores the bank's intermediation function, which has a necessary supply price. If the marginal costs of intermediation are constant, the interest paid on deposits is reduced by a given proportion. Since the tax base (reserves) is independent of intermediation costs, but deposits pay less interest, it follows that we have underestimated somewhat the marginal welfare costs and have erred on the side of overestimating the optimal rate of inflation.

Although for purposes of exposition, the analysis has in the main assumed that the demand schedule for deposits is the same as that for currency, all the formulas hold if the demand schedule for deposits differs from that for currency. All one needs to do is change the form of the function and plug in the relevant interest elasticities. The formulas are general and can be applied to economies with different indexes of marginal distortions and varying interest elasticities.

A potential problem in using these formulas to predict dW/dR is that the ratio of currency to deposits may change with the rate of inflation. As an empirical matter, the currency-deposit ratio has remained remarkably stable in the United States since the period of financial deregulation in the late '80s, when deposits began paying explicit interest. Moreover, a theoretical argument that the currency-deposit ratio is independent of the money rate of interest has been made by Dwyer and Saving (1986). As we have seen the opportunity cost of holding currency is the rate of interest, i, and the opportunity cost of holding deposits is a fraction of the interest rate, if. Assuming the indifference curve between currency and deposits are homothetic, and that the ratio of these opportunity costs is the appropriate measure (by analogy with price theory) determining the currency deposit ratio, this ratio is independent of the money rate of interest.[2]

Although these formulas have been used to assess dW/dR at hypothetical inflation rates, which requires predicting the currency deposit ratio, the formulas also can be used to calculate the ex post measure dW/dR at a prevailing money rate. All that is required is to observe the prevailing currency-deposit ratio.

Finally, some caveats are in order. The analysis deals with alternative positions of steady-state inflation. It does not handle the welfare costs of variable inflation—costs which may well be more significant than those associated with steady-state inflation. Moreover, our analysis has treated real balances as part of an optimal tax menu; this usual assumption is not without its critics (Lucas, 1986).

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[2] This reasoning, however, is not fully compelling. The ratio of the price of sowbellies to that of caviar has the dimensionality of sowbellies to caviar and is independent of proportionate changes which leave the relative price ratio unchanged. The ratio of the opportunity cost of currency to that of deposits is a dimensionless number and taking the ratio of the opportunity costs (by analogy with commodities) implies that one's choice of currency and deposits is independent of the difference in their opportunity costs.

Tatom (1979) makes an early attempt to determine the marginal welfare costs per dollar of revenue in a world of both currency and deposits. He takes the ratio of the opportunity costs combined with homothetic indifference curves as a compelling reason to treat the currency-deposits ratio as independent of the inflation rate. More importantly, Tatom does not build up his welfare costs from an explicit consideration of the integral for currency and deposits separately, but conflates the two using a single integral running from zero to the money rate of interest. In fact, the integral for competitively priced deposits should run from zero to *if.* Interestingly enough, Tatom's analysis, although not general, applies to a world in which an effective prohibition on the payment of interest on deposits exists, and in which the form of the demand schedule is the same for currency and deposits. This is a special case of my analysis, and I am indebted to John Tatom for this reference and discussion.