William A. Barnett and Ge Zhou[1]

Response to Brainard's Commentary

WE HAVE GREATLY BENEFITED from Brainard's stimulating comments on our paper. We agree with his suggestions for extensions to this research, and in fact expect to have extended versions available in the near future. For example, Barnett, Kirova and Pasupathy (1994) are including an extended model in their paper being prepared for the Federal Reserve Bank of Cleveland conference in September 1994. That model contains dynamic capital growth through Tobin's q, both for financial intermediaries producing monetary services as outputs and for manufacturing firms demanding monetary services as inputs. In addition, that model contains an endogenous dividend payout decision, produced by entering loans into technology as an output, along with the deposits which currently are the sole outputs. Introducing loans into the technology as an output eliminates the need for Barnett and Zhou's (1994) equation 2, which determines loans as a function of deposits under the assumption that all earnings are paid out as dividends.

Some readers may find Brainard's comments difficult to interpret, however, since they reflect unpublished background material developed during correspondence. The following few theorems are relevant to understanding the nature of the model's dynamics and the merits of extending the model further to exhibit deeper dynamics.

Equation 2 in our paper is imposed to require the firm to pay out all earnings as dividends, since introduction of an endogenous dividend payout decision is beyond the scope of our paper. Equation 1 is in the general form of the profit function used by Hancock (1985, equation 3.1) in her book and in her papers on financial intermediation under certainty. Equation 1 holds, regardless of how the dividend payout decision is made. Our equation 3 is acquired by imposing equation 2 on equation 1 through direct substitution. Hence, equation 3 is Hancock's variable profit function under the restriction that all earnings are paid out as dividends.

Under exactly that same payout restriction, Barnett (1987, equation 3.7) derived a different variable profit function for the same financial intermediary, and Barnett and Hahm (1994) recently have used Barnett's formulation of the variable profit function in estimating the technology of commercial banks. In correspondence, we found that Brainard had a very strong preference for use of Barnett's, rather than Hancock's, variable profit function under the payout restriction. The discussion about dynamics in Brainard's comment, including his discussion of the accounting for required reserves,

[1]Barnett's research on this paper was partially supported by NSF grant number SES 9223557. We have benefited from many discussions with Richard Anderson on this subject, and a lengthy and highly informative exchange of faxes with William Brainard.

explains his reasons for preferring Barnett's function to Hancock's.

Obviously Barnett would not dispute the merits of Barnett's variable profit function, and he is not at all displeased that Brainard so strongly prefers his variable profit function to Hancock's. Nevertheless, it may seem paradoxical that two different variable profit function formulas exist for the same firm under the same assumptions, and that we chose to use Hancock's rather than Barnett's formula in our paper in this volume. As we shall observe below, the distinction between the two variable profit functions is actually much more subtle than may appear to be the case, and the choice between them in our paper is little more than an econometric trick—barring empirical evidence to the contrary.

We begin by verifying that equation 1 in our paper is indeed exactly Hancock's variable profit function, with only the notation changed.

Theorem 1: The variable profit function defined by equation 1 in Barnett and Zhou (1994) is identical to Hancock's (1991, equation 3.1) variable profit function.[2]

Proof: Hancock's (1991, equation 3.1) variable profit function, using her notation, is

$$\pi_{Ht} = -B_t - \sum_{i=1}^{N_1+N_2} b_i[(1+h_{i,t-1})y_{i,t-1}\,P_{t-1} - y_{it}P_t],$$

where B_t = expenditure on variable factors; P_t = the general price level; y_{it} = a financial asset quantity if $i=1,\dots,N_1$, or a financial liability quantity if $i=N_1+1,\dots N_1+N_2$; h_{it} = the holding period yield on y_{it} if y_{it} is an asset, or the holding cost on y_{it} if y_{it} is a liability; and we define the indicator function b_i such that $b_i = 1$ if y_{it} is a liability and $b_i = -1$ if y_{it} is an asset.

A more convenient notation would be to use the symbol A to denote assets instead of the notation N_1 and L to denote liabilities instead of the notation N_2. Making that change in notation and using the indicator function b_i as defined above, we acquire:

$$\pi_{Ht} = -B_t - \sum_{i=1}^{L} [(1+h_{i,t-1})y_{i,t-1}P_{t-1} - y_{it}P_t]$$
$$+ \sum_{i=L+1}^{A+L} [(1+h_{i,t-1})y_{i,t-1}P_{t-1} - y_{it}P_t].$$

Further changing to the notation in Barnett and Zhou (1994), let $B_t = \sum_{j=1}^{J} w_{jt}z_{jt}$, where z_{jt}, $J=1,\dots,J$, are the nonfinancial variable factors, and w_{jt}, $j=1,\dots,J$, are their prices. We then can rewrite the variable profit function as:

$$\pi_{Ht} = -\sum_{j=1}^{J} w_{jt}z_{jt} - \sum_{i=1}^{L} [(1+h_{i,t-1})y_{i,t-1}P_{t-1} - y_{it}P_t]$$
$$+ \sum_{i=L+1}^{A+L} [(1+h_{i,t-1})y_{i,t-1}P_{t-1} - y_{it}P_t].$$

As in Hancock (1991), the assets consist of loan investments and excess reserves, which in our notation are Y_t and C_t respectively. Furthermore, let R_t be the single period holding yield on Y_t and let the yield on C_t be zero, as in Hancock (1991). The variable profit function now becomes:

$$\pi_{Ht} = -\sum_{j=1}^{J} w_{jt}z_{jt} - \sum_{i=1}^{L} [(1+h_{i,t-1})y_{i,t-1}P_{t-1} - y_{it}P_t]$$
$$+ C_{t-1}P_{t-1} - C_tP_t + (1+R_{t-1})Y_{t-1}P_{t-1} - Y_tP_t,$$

which is exactly equation 1 in Barnett and Zhou (1994). Q.E.D.

In the next theorem, we prove that the difference between the discounted present value of the profit flow produced from Hancock's formula (that is, Barnett and Zhou's 1994 equation 1) and the discounted present value of the profit flow produced by Barnett's (1987, equation 3.7) formula is a function only of initial conditions. The proof is produced under imposition of Barnett and Zhou's equation 2, which requires all earnings to be paid out as dividends.

Let y_{it} be deposits in account type i and let r_{it} be the single period holding yield on that account. Let K_{it} be the required reserve ratio on that account. Before proving the equivalency theorem, we define the two formulations of the variable profit function as follows.

Definition 1: Barnett's (1987, equation 3.7) variable profit function is

$$\pi_{Bt} = \sum_{i=1}^{L} \eta_{it}y_{it} - \sum_{j=1}^{J} w_{jt}z_t - \eta_{ct}c_t,$$

[2]Also, see equation (3) in Hancock (1985) for another statement of Hancock's formula.

where the user cost of account y_{it} is

$$\eta_{it} = P_t \frac{(1-k_{it})R_t - r_{it}}{1+R_t},$$

and the user cost of excess reserves c_t is

$$\eta_{ct} = P_t \frac{R_t}{1+R_t}.$$

Definition 2: Barnett and Zhou's (1994, equation 3) variable profit function is

$$\pi_{BZt} = \sum_{i=1}^{L} \{[(1+R_{t-1})(1-k_{i,t-1})-(1+h_{i,t-1})]y_{i,t-1}P_{t-1} + k_{it}y_{it}P_t\} - R_{t-1}C_{t-1}P_{t-1} - \sum_{j=1}^{J}(1+R_{t-1})w_{j,t-1}z_{j,t-1},$$

where $h_{is} = r_{is} + k_{is}R_s$.

Note that by Theorem 1, Definition 2 also defines Hancock's (1991, equation 3.1) variable profit function under the restriction that all earnings are paid out as dividends (Barnett and Zhou's (1994), equation 2). After multiplying Barnett's variable profit function (defined by Definition 1 above) through by $1+R_t$, it is easily seen that the profit function preferred by Brainard in his comments (as further clarified by our private correspondence) is Barnett's profit function. The equivalency theorem, producing a connection between Definitions 1 and 2, follows.

Theorem 2: *The discounted present value of the firm,* C_B, produced from Barnett's profit function flow, π_B, differs from the discounted present value of the firm, C_H, produced from Barnett and Zhou's (in other words, Hancock's with no retained earnings) profit function flow, π_{BZt}, by a function, $K(I)$, containing only initial conditions, I. In other words, there exists $K(I)$, depending only upon initial conditions, such that $C_H = C_B + K(I)$.

Proof: Define the discount factor δ_s such that

$$\delta_s = \begin{cases} 1 \text{ when } s=t \\ \prod_{a=t}^{s-1}(1+R_a) \text{ when } s \geq t+1, \end{cases}$$

where R_a, $a=t,\ldots,s-1$, are current and expected future values of the rate of return, R_t, defined above to be the single period holding yield on Y_t. The discounted capitalized value of the profit stream π_{BZt} at time t is

$$C_H = \sum_{s=t}^{\infty} \frac{1}{\delta_s} \pi_{BZs},$$

while the capitalized value of Barnett's profit stream π_B is

$$C_B = \sum_{s=t}^{\infty} \frac{1}{\delta_s} \pi_{Bs}.$$

Substituting the formulas for the profit streams into the two capitalized values and manipulating algebraically, we find

$$C_B = \sum_{s=t}^{\infty} \frac{1}{\delta_s} \pi_{Bs} = \sum_{s=t}^{\infty} \frac{1}{\delta_s} [\sum_{i=1}^{L} \eta_{is}y_{is} - \sum_{j=1}^{J} w_{js}z_{js} - \eta_{cs}c_s]$$

and

$$C_H = \sum_{s=t}^{\infty} \frac{1}{\delta_s} \pi_{BZs} = K + C_B,$$

where

$$K = \sum_{i=1}^{L} \{[(1+R_{t-1})(1-k_{i,t-1})-(1+h_{i,t-1})]y_{i,t-1}P_{t-1}\} - R_{t-1}C_{t-1}P_{t-1} - \sum_{j=1}^{J}(1+R_{t-1})w_{j,t-1}z_{j,t-1}.$$

Observe that K depends only upon initial conditions, since the intertemporal decision is made at time t over periods t, $t+1$, $t+2$,..... Q.E.D.

Theorem 2 proves that under the restriction that all earnings are paid out as dividends and except for a function of initial conditions, Barnett's variable profit function and Hancock's variable profit function are simply different ways of spreading the capitalized value of the firm over time. Any flow of funds or transaction that appears in one formula necessarily also appears in the other, but potentially with a time shift between them. Those time shifts are all properly discounted, however, as demonstrated by the fact that the two profit streams produce the same capitalized value up to $K(I)$. In his comment, Brainard observes correctly that the choice between the two profit flow formulas "has essentially no effect on the present value of the bank." Theorem 2 above makes that point clear.

The discussion that follows will extract from Theorem 2 its precise implications for the model estimated by Barnett and Zhou in this volume. Our discussion will compare the solutions of the two decisions defined below.

Decision 1: For some utility function, U, the firm determines its factor demands and output supplies by maximizing, $EU(C_B)$, which is the expected utility of the capitalized value C_B.

Decision 2: For some utility function, V, the firm determines its factor demands and output supplies by maximizing, $EV(C_H)$, which is the expected utility of the capitalized value C_H.

Observe that all terms in each capitalized value are inside the respective utility function, which is not assumed to be intertemporally separable in either case. The marginal utility of anything varied within either capitalized value depends upon everything else in that capitalized value. In short, neither utility function is intertemporally separable and the solution of either decision is deeply dynamic. In fact, each solution is intertemporally simultaneous with all time subscripts appearing in all Euler equations.

To determine whether there are any substantial differences between Decisions 1 and 2, we now define the following concept.

Definition 3: Two decision problems are observationally equivalent if the solution functions (factor demand and output supply) functions produced by solving one problem are identical to the solution functions produced by solving the other at any fixed setting of the initial conditions.

The following theorem and corollary are now easily proved.

Theorem 3: For any given fixed value of the initial conditions function, $K(I)$, and any given utility function, U, there exists a utility function, V, such that $V(C_H)=U(C_B)$ for all possible settings of the firm's decision variables (the controls).

Proof: For given $K(I)$ and U, define V such that $V(x+K(I))=U(x)$ for all nonnegative values of the scalar x. Now let $x=C_B$, and let $C_H=x+K(I)$. By substitution, the result is immediate. Q.E.D.

Corollary 1 to Theorem 3: Decisions 1 and 2 are observationally equivalent.

Proof: The corollary follows immediately from Definition 3 and Theorem 3. Q.E.D.

The implications of the above results at this point are the following. To justify the introduction of risk aversion into the decision of the firm, we implicitly assume the existence of incomplete markets.[3] How to model the decisions of firms with incomplete contingent claims markets is controversial. One approach that has been proposed is to apply principle agent theory in a form that produces incentive compatibility, when the decision is delegated by the owners to a professional manager. The source of the risk averse, concave utility function is the utility function of the principle agent.

Having introduced expected utility maximization into the firm's decision in that controversial manner, we then see from the above corollary that it makes no difference whether we use Hancock's variable profit function or Barnett's in producing the Euler equations to be estimated. The Euler equations are identical and the decision is deeply dynamic, with all time subscripts appearing in each Euler equation. The choice between the two profit formulas is a choice between two different methods of spreading the same capitalized value over time. But since it is the capitalized value itself that enters as the sole argument of the utility function, the method of spreading over time is irrelevant. Corollary 1 is the result.

The problem at this point is that estimating a system of simultaneous Euler equations is beyond the state of art. We need a means of decreasing the depth of the model's dynamics. An obvious method would be to impose a separability restriction on the utility of capitalized value. We could use complete separability, blockwise separability, weak separability, or strong separability. Separability restrictions are testable structural restrictions, and behavior is not invariant to such structural restrictions.[4] In addition, nothing in principle agent theory helps us to choose between such restrictions, which in fact all may be wrong. The utility function may indeed be nonseparable, and the decision may be unavoidably deeply dynamic. Furthermore, we are aware of no empirical results that would help us to choose between the many simplifying separability restrictions, and the few results in that area in Barnett (1981) indicate that separability restrictions are strong restrictions that often are rejected in empirical tests.

[3]If contingent claims markets are complete, then the owner will instruct the manager to maximize profits conditionally upon the prices in contingent claims markets. Those prices contain the information about the risk aversion of the owner and, hence, the managers will be instructed to behave in a risk-neutral manner relative to those prices. See Duffie (1991) and Magill and Shafer (1991).

[4]This issue does not exist in the perfect-certainty or risk-neutral case, since in those cases there is no utility function to be structurally separable. The invariance theorem, then, is the end of the story.

Under such circumstances, applied researchers regularly choose simplifying assumptions on the basis of their usefulness in estimation. One possibility is intertemporal strong separability in Hancock's profit stream. Another possibility is intertemporal strong separability in Barnett's profit stream. More formally, those two possibilities are Assumptions 1 and 2 below, respectively.

Assumption 1: The utility function, V, is intertemporally strongly separable in $\{C_{H,t};t=1,...,\infty\}$.

Assumption 2: The utility function, U, is intertemporally strongly separable in $\{C_{B,t};t=1,...,\infty\}$.

There are many other such possibilities produced by grouping together terms in the capitalized value in different manners. Behavior is not invariant to choices between those possible separability restrictions. In terms of the degree of simplification of the Euler equations, complete intertemporal separability in Barnett's profit stream (Assumption 2), as assumed by Barnett and Hahm (1994), produces the most extreme simplification. The decision becomes completely static. Complete intertemporal separability in Hancock's profit stream (Assumption 1) produces a more modest decrease in the depth of the dynamics: The solution becomes recursive, with two time subscripts appearing in the Euler equations.

Barnett and Zhou (1994) selected and imposed the latter restriction, since the resulting recursive form of the solution assists in GMM estimation. Brainard (1994), in his commentary, argues forcefully for intertemporal strong separability in Barnett's profit stream. We have no reason to dispute his strong prior on this subject. His views are reasonable, and obviously Barnett (1987) and Barnett and Hahm (1994) must have had somewhat similar priors in mind when they published their work. Nevertheless, it is also possible that the opposite extreme may be true. The utility function may be completely nonseparable, so that both Assumptions 1 and 2, along with all other possible separability restrictions, may be wrong.[5] The Euler equations would thereby be intertemporally simultaneous, so that we cannot readily estimate the model with current methods because of the depth of the dynamics. Even worse, it may be the case that the use of a risk averse principle agent as a means of introducing risk aversion into the decision of the firm may be a defective approach. That question at present is unresolved in economic theory.[6]

Under these circumstances, we feel justified in choosing our separability restriction based upon the resulting estimation convenience. Producing interesting dynamics with long-run economic growth was not an objective of Barnett and Zhou (1994), which was an exploration in aggregation theory for firms under uncertainty. We agree with Brainard that far more interesting dynamics would be produced by introducing a law of motion for capital, which indeed will be included in Barnett, Kirova and Pasupathy (1994).

We wish to acknowledge that the above clarifying proofs resulted from our correspondence with Brainard, and we are indebted to him for motivating this exploration of the connection between Hancock's and Barnett's formulations. Many of his other suggestions will be used in future extensions of our research such as the estimation of the model with learning-by-doing technological change. Although we have not yet estimated that model, the Euler equations for that extended model are provided in Barnett and Zhou (1994) and the dynamics in that model are indeed dynamic in an interesting manner.

REFERENCES

Barnett, William A., Milia Kirova and Munich Pasupathy. "Estimating Policy Invariant Technology Parameters in the Financial Sector, When Risk and Growth Matter," Paper to be presented at the Federal Reserve Bank of Cleveland, September 1994 macroeconomics conference on Liquidity, Monetary Policy, and Financial Intermediation.

________, and Ge Zhou. "Financial Firm's Production and Supply-Side Monetary Aggregation Under Dynamic Uncertainty," this *Review* (March/April 1994).

________, and Jeong Ho Hahm. "Financial-Firm Production of Monetary Services: A Generalized Symmetric Barnett Variable-Profit-Function Approach," *Journal of Business and Economic Statistics* (January 1994).

________. "The Microeconomic Theory of Monetary Aggregation," in William A. Barnett and Kenneth J. Singleton, eds., *New Approaches to Monetary Economics.* Cambridge University Press, 1987, pp. 115–68.

________. *Consumer Demand and Labor Supply.* North-Holland, 1981.

[5] In fact, the assumption of intertemporal separability of preferences has become controversial in the real business cycle literature. See, for example, Kydland and Prescott (1982).

[6] See for example Magill and Shafer (1991).

Brainard, William C. "Commentary," this *Review* (March/April 1994).

Duffie, Darrell. "The Theory of Value in Security Markets," in Werner Hildenbrand and Hugo Sonnenschein, eds., *Handbook of Mathematical Economics,* volume IV. North-Holland, 1991.

Hancock, Diana. *A Theory of Production for the Financial Firm.* Kluwer Academic Publishers, 1991.

________. "The Financial Firm: Production with Monetary and Non Monetary Goods," *Journal of Political Economy* (October 1985) pp. 859–80.

Kydland, Finn E., and Edward C. Prescott. "Time to Build and Aggregate Fluctuations," *Econometrica* (November 1982), pp. 1345–70.

Magill, Michael, and Wayne Shafer. "Incomplete Markets," in Werner Hildenbrand and Hugo Sonnenschein, eds., *Handbook of Mathematical Economics,* volume IV. North-Holland, 1991.