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■ Measures of Money and the Quantity Theory

MANY ECONOMISTS BELIEVE THAT, over long periods of time, the quantity theory of money explains the relationship between money and inflation. In particular, many believe (generally speaking) that a permanent increase in the quantity of money will eventually produce an equiproportionate permanent increase in the general level of prices. Similarly, a constant rate of money growth will produce a constant rate of inflation. This belief is often summed up in the phrase "money is long-run neutral."

Unfortunately, it has been difficult for economists to investigate such claims satisfactorily. Part of the difficulty lies in defining what is meant by measurement at low frequencies, horizons long enough so that other economic adjustments have taken place. An additional problem has been one of designing investigations that do not rely critically on other details (sometimes called "structure") about how the economy works, details on which there is notoriously little consensus among economists.

In this paper, the basic proposition that money growth and inflation are closely related in the long run is examined from a nonstructural, low-

frequency point of view. The nonstructural aspect of the analysis is attained by using a technique that does not require a host of encumbering theoretical or econometric assumptions. The low-frequency aspect is achieved by using a certain filter that extracts a long-run signal from time series data. The filter was introduced to this literature by Lucas (1980). The purpose of the paper is to extend the analysis of Lucas, whose work is often cited as an illustration of the validity of the quantity theory, along two dimensions. The first is simply an extension of the quarterly data set up to the present.¹ The second is to check the robustness of the results across different measures of money, an issue not addressed in the original paper nor in subsequent comments on the paper by other authors.²

Authors commenting on Lucas (1980) tended to raise questions concerning the relationship of the graphically based, nonstochastic methodology to statistical techniques. Whiteman (1984) and McCallum (1984) in particular both suggested there were limits to the inferences that could be drawn using Lucas' empirical analysis. Recent developments in econometric theory due

¹Lucas (1980) used quarterly data on M1, the consumer price index and real GNP from 1953 to 1975.

²Lucas (1980, p. 1006) notes, "this question of which monetary aggregate one would theoretically expect to move in proportion to prices is much more open than has tradi-

tionally been recognized. In [this paper]...money means M1, but the arbitrariness of this measurement choice should be emphasized at the outset..." (italics in original).

to Fisher and Seater (1993) have suggested a framework that can be used to answer the questions raised by these authors, and also to put Lucas' original work into statistical perspective. This paper provides a summary of the Fisher and Seater framework as it pertains to the neutrality issues investigated here.

The data for the study consists of quarterly observations from the United States from 1960 through 1992. This data set includes, broadly speaking, a period of increasing inflation up to about 1980 and a period of disinflation thereafter. Thus, the data provide a useful natural experiment in that policymakers have evidently followed both relatively high and relatively low inflation policies during this era. This is useful because the methods used here would be uninformative if there were insufficient variation in policy. Two measures of inflation and 19 measures of money are used, the latter to check robustness of the results across different definitions of money. The measures of money used range from the very narrow to the very broad and include Divisia versions of some aggregates.

The results indicate, very broadly speaking, that quantity theory illustrations pan out in the sense that, by any combination of measures, higher money growth rates are associated with higher inflation rates at something like a one-for-one rate. When the measure of money is broad, such as M2, M3 or L, the illustrations can be striking, although when other measures of money are used, the results are weaker. In particular, the results of Lucas (1980), which were obtained using M1 as the measure of money, are less satisfactory when data from the 1980s are included.

A VERSION OF THE QUANTITY THEORY

The equation of exchange is defined as $MV \equiv PT$, where M is the quantity of money, P is the price level, T is a measure of the volume

of transactions and V is the transaction velocity of money, which is simply defined in terms of the other three variables. The transaction measure typically used is real output Y , so that $MV \equiv PY$. An assumption on the behavior of velocity is required in order to convert this tautology into a theory. The version of the quantity theory employed in this paper postulates that the growth rate of V is constant in the equation of exchange, and that output movements are uncorrelated with changes in the quantity of money. The constant velocity growth rate will be denoted by $\alpha > -1$; if $\alpha = 0$, the level of velocity is constant. Since the analysis is from a long-run perspective, these assumptions can be viewed as applying only over long horizons. Therefore, while it is true that velocity fluctuates over short time horizons, the nature of the analysis undertaken here makes a constant growth velocity assumption more attractive.³

The theory's key proposition for the purposes of this paper can be found by now taking logarithms of both sides of the equation and differentiating with respect to time. This manipulation, combined with the velocity assumption, implies that

$$(1) \frac{1}{P} \frac{dP}{dt} = \alpha + \frac{1}{M} \frac{dM}{dt} - \frac{1}{Y} \frac{dY}{dt},$$

that is, the inflation rate is equal to the constant velocity growth rate plus the money growth rate less the growth rate of output. For convenience, denote $(1/x) (dx/dt)$ by Δx , so that

$$(2) \Delta P_t = \alpha + \Delta M_t - \Delta Y_t.$$

In the long run, then, according to this theory, a plot of inflation against money growth less output growth should produce data points that lie along a 45-degree line with intercept α . It is well known that such a proposition does not hold when the data are measured over short frequencies such as a quarter, but many economists believe that it does hold when the

³More complicated velocity assumptions are possible. One might suppose, for instance, that the trend in velocity sometimes changes or that it follows a quadratic. Generally, more creative velocity assumptions bring one closer to the tautological equation of exchange, and therefore may be of limited use. Still, it should be stressed that for any measure of P , M and Y there is a velocity assumption, sufficiently complicated, that will lead to a perfect illustration of the quantity theory by the methods used in this paper. The velocity assumption used here maintains comparability to Lucas (1980).

variables are viewed from a long-run perspective. To get at this notion, a filter is introduced in the next section which extracts a long-run signal from time series data.

LOW-FREQUENCY DATA ANALYSIS

A Two-Sided Filter

Lucas (1980) suggested the following filter for this problem:

$$(3) \chi_t(\beta) = \frac{(1 - \beta)}{(1 + \beta)} \sum_{k=-\infty}^{\infty} \beta^{|k|} \chi_{t+k},$$

where χ_t is the variable of interest, and β is a parameter restricted to be between 0 and 1. As β approaches zero, no filtering occurs, while as β approaches unity, the filtered $\chi_t(\beta)$ approach the sample mean of the original series. Higher values of β , but short of unity, imply greater smoothing of the time series. Lucas' original idea was to choose a value of β short of unity which would allow the filter to extract a long-run signal from the time series data, and then to compare filtered data on money and inflation to see if the long-run movements are along a 45-degree line, as suggested by the quantity theory. Lucas found that the value $\beta = .95$ worked well, and this value is employed throughout most of this paper.⁴ Of course, a value of $\beta = .95$ is close to 1, and, hence, the filtered data will be quite smooth relative to the unfiltered time series.⁵

The filter is two-sided and extends beyond the sample in both directions. A technique due to Cooley, Rosenberg and Wall (1977) can be used to assign beliefs via a diffuse prior on points outside the sample; the moving average can then be calculated as if the entire doubly in-

finite record existed. Lucas (1980) reports that filtered series using this technique are virtually identical to the filtered series calculated using zero values for points outside the sample, with the exception of the data points quite near the beginning and quite near the end of the sample data. Lucas discarded the first two years and last two years of the filtered data so as not to allow the zero values to have undue influence on the results. In this paper the same procedure is followed.⁶

The two-sided nature of the filter can be interpreted as incorporating within the data analysis the behavior of agents whose actions today depend on their expectations of the future. This point can be illustrated by envisioning a model economy with many individual agents. Suppose that such an economy is characterized by a growth rate of the money stock and an associated inflation rate which is equal to the money growth rate. The growth rate of the money stock generally has an invariant distribution with a fixed mean and constant variance; on occasion, however, the mean of the distribution changes according to decisions made by the policy authorities. Since agents need to know the inflation rate in order to make decisions, "structural" policy changes of this type play a role in influencing their behavior. Suppose finally that the agents have to learn the new inflation rate following a policy change. The learning implies a well-defined transitory dynamics following a policy change, and these transitory dynamics would tend to blur the period-by-period relationship between money growth and inflation in the model. The essential problem for the econometrician observing such an economy is to disentangle the actual long-run relationship from the surrounding noise introduced by the transitory learning dynamics. The filter used to analyze the data from this economy, then,

⁴To see the effects of other values of β , see the general equilibrium example in the next subsection.

⁵For a detailed discussion of the filter, see Lucas (1980).

⁶The filter in the text employs the factor $(1 - \beta)/(1 + \beta)$. This factor is the inverse of the sum of the doubly infinite set of weights

$$\sum_{k=-\infty}^{\infty} \beta^{|k|},$$

and it serves to preserve the mean of the doubly infinite data set. Since we have assumed zeros for the points outside the actual sample, one might be tempted to preserve the mean of the actual finite sample with the factor

$$\left[\sum_{k=1-t}^{T-t} \beta^{|k|} \right]^{-1},$$

where T is the sample size and t is the point in the sample for which computation is being done. The results in this paper are qualitatively unchanged if an alternative filter of this type is employed. This confirms Lucas' (1980) claim that the results are not very sensitive to the way in which the points outside the sample are treated.

should be one that reliably distinguishes between "signal" induced by the structural policy changes that occur and the transitory noise. The filter employed here does extract signal from noise based on the variance of the noise term, and indeed this is the principle reason Lucas (1980, 1987) chose to use the filter. This motivation for the filter is illustrated in more detail in a general equilibrium example in the next section.

Before turning to the example, it is perhaps worth emphasizing that in an economy with a constant mean money growth rate and a constant mean inflation rate over the whole sample, an examination of the data such as the one carried out in this paper will yield no information. One cannot discern the effects of changes in money growth rates on inflation if there have been neither changes in money growth rates nor changes in inflation rates, by which I mean shifts in the entire distribution of these rates. In this sense, the structural policy changes are crucial to the successful verification of the quantity theoretic relationship; if no structural changes occur, the filtered data will simply be tightly clustered about the mean. Fortunately, the United States since 1960 has been characterized both by a period of accelerating inflation and a period of disinflation. It would appear, then, that the historical record contains enough variation in policy to be informative according to the methods employed here.

An Example in General Equilibrium

Some of these ideas can be made more concrete by illustrating the principles in a simple dynamic general equilibrium model with structural policy shifts. The model economy endures forever and consists of overlapping generations of identical two-period lived agents. The agents maximize utility $U = \ln c_i(t) + \ln c_i(t+1)$, where $c_i(t)$ is consumption, subscripts denoting birthdates and parentheses denoting real time. Each agent receives an endowment of the consumption good in each period of life, which we denote by $\{w_i(t), w_i(t+1)\}$. The endowments are the same for all agents regardless of birthdate. Agents can hold unbacked paper currency provided by the government; the government endures forever and provides currency at gross

rate θ . Currency holdings have a gross rate of return $P(t)/P(t+1)$, where $P(t)$ is the price of the consumption good at time t . The nominal amount of currency in circulation at time t is denoted by $H(t)$. The population size is constant, and the identical agents of each generation will be represented by a single agent.

If we solve the problem of the individual agent, we can write the equations describing equilibrium in this economy as

$$(4) H(t)/P(t) = [w_i(t) - w_i(t+1)\gamma(t)] / 2$$

$$(5) H(t) = \theta H(t-1)$$

$$(6) F[P(t+1)] \equiv \gamma(t)P(t),$$

where $\gamma(t)$ is the expected gross inflation rate at time t and $F[P(t+1)]$ is the time t forecast of the price at time $t+1$. The model can be closed with an assumption about how agents form expectations of the future price level. The learning assumption employed here is that agents use a first-order autoregression on prices using information available through time $t-1$:

$$(7) \gamma(t) = \left[\sum_{s=1}^{t-1} P(s-1)^2 \right]^{-1} \left[\sum_{s=1}^{t-1} P(s-1) P(s) \right].$$

These assumptions determine a dynamic system in $\gamma(t)$. For cases where $w_i(t) > w_i(t+1)$ and the pace of currency creation is relatively slow, this model has a locally stable steady state in which the gross rate of inflation is equal to the gross rate of currency creation.⁷ Local stability means that if the model is initialized at the steady state and then subjected to a small, one-time unanticipated change in the policy parameter θ , the dynamic path will eventually converge back to the steady state at $\gamma = \theta$. Thus, in the long run, the quantity theory holds in this model in the sense that the rate of inflation is equal to the rate of currency creation in the steady state.

If the policy parameter changed often enough, the transitory learning dynamics might cause money growth and inflation to appear to be unrelated period-to-period even though the quantity theory holds in the long run in this model. To consider a situation like this, view the agents as sophisticated enough to look forward via the first-order autoregression to make their savings decision, but not so sophisticated that they attempt to anticipate the next move of the policy

⁷See Bullard (1994).

authorities. In particular, ascribe to agents the belief that today's value of the policy parameter will persist into the next period (which is the only period that matters from the perspective of the young agents). Given this assumption, suppose that the actual law of motion for the money growth rate is given by

$$(8) \theta(t) = \theta(t-1) + \varepsilon(t) \text{ if } \theta(t-1) \in [\theta_l, \theta_u],$$

$$\theta(t) \sim U[\theta_l, \theta_u] \text{ otherwise,}$$

where $\varepsilon(t)$ is a mean zero noise term with variance σ_ε^2 , θ_l and θ_u represent lower and upper bounds, respectively, on the money growth rate, and $U[\cdot]$ represents a uniform distribution. If the variance of $\varepsilon(t)$ is chosen to be small relative to $\theta_u - \theta_l$, the policy parameter changes slowly within the bounds but can move sharply on occasions when the bounds are violated.

Because the system is locally stable near the monetary steady state, if policy was constant in the sense that $\sigma_\varepsilon^2 = 0$ and $\theta(0) \in [\theta_l, \theta_u]$, the system would converge to the steady state from an initial condition $\gamma(0)$ in the neighborhood of $\theta(0)$ and remain there for all time. Data plotted from such an experiment, with money growth on the horizontal axis and inflation on the vertical axis, would have virtually all of the observations on a 45-degree line at a single point. To obtain an illustration of the quantity theory—a movement along the 45-degree line—a policy change is required. If there were a single, unanticipated policy change at time τ such that $\theta(0) \neq \theta(\tau) \in [\theta_l, \theta_u]$, the system would first converge to the steady state at $\theta(0)$ and then, after some transitory dynamics following the policy change, converge to the steady state at $\theta(\tau)$.

The law of motion for the gross rate of money growth used here represents a more complicated situation, where policy changes occur every period, with most changes being small and some changes being large. By construction, the model obeys the quantity theoretic proposition that the rate of money creation is reflected in the rate of inflation in the long run. But because the policy parameter is constantly changing, the short-run (period-by-period) data might not provide evidence of such a relation-

ship. By simulating the model and using Lucas' filter, evidence of the long-run relationship can be recovered.

This principle can be shown through a simulation of the model with endowments for all agents set as $\{w_t(t), w_t(t+1)\} = \{2, 1\}$. The distribution of ε_t was set as triangular with mean zero and bounds $-.1$ and $.1$; this implies a variance of $.00167$.⁸ The system was initialized at the monetary steady state with $\gamma(0) = \theta(0) = 1.2$, and the upper and lower bounds on the money growth rate were set as $\theta_l = 1.1$ and $\theta_u = 1.3$, that is, between 10 percent and 30 percent per period. The simulation was run for 500 periods. The results are reported in Figures 1 through 4. Consistent with the earlier discussion of the distortion in the points near the beginning and end of the sample, the first and last 20 observations were omitted, leaving 460 in the charts. Figure 1 reports the raw, unfiltered data. There appears to be little or no evidence of a relationship between money growth and inflation, even though such a relationship exists by construction in this model. Figures 2, 3 and 4 show the same plot based on the filtered data, with the filtering parameter β set to $.5$, $.8$ and $.95$, respectively. In Figure 4, the filtered data lie virtually exactly on the 45-degree line and, thus, the long-run relationship between money growth and inflation that exists in the model is recovered using Lucas' (1980) procedure.

SOME ECONOMETRIC ISSUES

Empirical testing of the money growth-inflation relationship has been successfully undertaken by Vogel (1974), Dwyer and Hafer (1988), Duck (1993) and others using cross-country data. The general conclusion of these studies is that countries which experience high rates of inflation also have high rates of money growth, where inflation rates and money growth rates are typically averaged over many years. Unfortunately, as mentioned in the introduction, similar tests on time series data for a single country have been difficult to carry out.⁹ One element of the problem has been obtaining a suitable approach to defining the "long run" and detecting long-run relationships; an approach to this problem is the one used in this paper.

⁸The triangular distribution can be found by setting $\varepsilon_t = x_1 - x_2$, where $x_1, x_2 \sim U[0, 1]$.

⁹This fact motivated Lucas (1980).

Figure 1
A Theoretical Example

Figure 2
A Theoretical Example

Figure 3
A Theoretical Example

Figure 4
A Theoretical Example

One time series technique for testing neutrality has been developed recently by Fisher and Seater (1993).¹⁰ These authors show how non-structural tests of neutrality propositions depend importantly on the order integration of the variables being tested. The Fisher and Seater (1993) methodology can be used to provide a statistically based rationalization for the technique used by Lucas (1980, 1987) and in this paper, and also to clarify some questions raised by authors commenting on Lucas (1980).

Fisher and Seater examine tests of neutrality and superneutrality in a nonstructural two variable system. The first variable can be thought of as m , the natural logarithm of the nominal money stock, and the second variable can be thought of as p , the natural logarithm of the aggregate price level. Let $\langle x \rangle$ denote the order of integration of x , so that if x is integrated of order one, then $\langle x \rangle = 1$. Let the lag operator be denoted by L , and let $\Delta \equiv (1 - L)$. It follows that the growth rate of a variable can be denoted by Δx , and that $\langle \Delta x \rangle = \langle x \rangle - 1$. Fisher and Seater study a two-equation system given by

$$(9) a(L)\Delta^{\langle m \rangle} m_t = b(L)\Delta^{\langle p \rangle} p_t + u_t$$

$$(10) d(L)\Delta^{\langle p \rangle} p_t = c(L)\Delta^{\langle m \rangle} m_t + w_t,$$

where $a_0 = d_0 = 1$, and the vector $[u_t, w_t]'$ is independently and identically distributed with mean zero and covariance Σ . Constants and trends are suppressed, and variables stationary about a deterministic trend are treated as integrated of order zero. Fisher and Seater work with this model in some generality, considering cases of superneutrality as well as neutrality, and also considering cases where the variable opposite m could be either of real or nominal magnitude. To focus the discussion here, we will concentrate on the case in which the two variables are m and p and the only question is one of neutrality.¹¹

Fisher and Seater define neutrality in terms of a long-run derivative of p with respect to a permanent change in m . Their definition is that if

$$(11) \lim_{k \rightarrow \infty} \frac{\partial m_{t+k}}{\partial u_t} \neq 0,$$

then

$$(12) LRD_{p,m} \equiv \lim_{k \rightarrow \infty} \frac{\partial p_{t+k} / \partial u_t}{\partial m_{t+k} / \partial u_t}.$$

In the case where

$$(13) \lim_{k \rightarrow \infty} \frac{\partial m_{t+k}}{\partial u_t} = 0,$$

Fisher and Seater simply leave the long-run derivative undefined. In this case, there is no permanent movement in m and a neutrality proposition cannot be tested. Otherwise, Fisher and Seater interpret the long-run derivative as representing the ultimate long-run effect of a disturbance u on p relative to the effect of the disturbance on m itself. Fisher and Seater (1993, p. 404) show that

$$(14) \lim_{k \rightarrow \infty} \partial m_{t+k} / \partial u_t = \theta(1),$$

where $\theta(L) \equiv (1 - L)^{1 - \langle m \rangle} \alpha(L)$

and that

$$(15) \lim_{k \rightarrow \infty} \partial p_{t+k} / \partial u_t = \Gamma(1),$$

where $\Gamma(L) \equiv (1 - L)^{1 - \langle p \rangle} \gamma(L)$.

They thus conclude that the value of the long-run derivative, when it is defined, depends on $\langle m \rangle - \langle p \rangle$ through the formula

$$(16) LRD_{p,m} = \frac{(1 - L)^{\langle m \rangle - \langle p \rangle} \gamma(L) \big|_{L=1}}{\alpha(1)}.$$

Fisher and Seater then define long-run monetary neutrality as $LRD_{p,m} = 1$. They categorize the possibilities into several cases. In the first case, $\langle m \rangle < 1$ and the long-run derivative is not defined. Long-run neutrality cannot be addressed because there are no permanent changes in the money stock. In the second case, $\langle m \rangle \geq \langle p \rangle + 1 \geq 1$ and long-run neutrality fails immediately because (in the simplest case) there are permanent shocks to the money supply but no permanent shocks to the price level. A third case has $\langle m \rangle = \langle p \rangle \geq 1$, and here $LRD_{p,m} = 1$ if neutrality holds. Therefore, tests of long-run neutrality can be devised since both m and p possess permanent changes. Fisher and Seater also argue that tests can be devised in a fourth case where $\langle m \rangle = \langle p \rangle - 1 \geq 1$.

¹⁰For applications of the techniques Fisher and Seater (1993) describe, see King and Watson (1992) and Bullard and Keating (1993). Most of the material in the remainder of this section can be found in greater detail and generality in Fisher and Seater (1993).

¹¹In Lucas (1980), the relationship between money growth and interest rates is also examined. The question of superneutrality would be important in this context, but this issue is not dealt with in this paper.

Lucas' (1980) graphical technique can be viewed as equivalent to estimating a regression coefficient, and if money is assumed to be long-run exogenous, this coefficient can be identified with the long-run derivative. In particular, Fisher and Seater argue that if $\langle m \rangle = \langle p \rangle = 1$, one can interpret the slope coefficient in a regression of filtered Δp on filtered Δm as an estimate of $LRD_{p,m}$.¹² In this paper, tests of integration are not pursued, but there is ample evidence that $\langle m \rangle \geq 1$, and that $\langle p \rangle \geq 1$.¹³ Since such tests have low power, economists cannot say with precision what the order of integration of these variables is, but it seems reasonable to proceed for the purposes of the present paper on the assertion that one of the above conditions holds. Later in the paper, values of the regression coefficients of filtered Δp on filtered Δm are reported as estimates of $LRD_{p,m}$.

As mentioned in the introduction, two papers offering critiques of Lucas (1980) can be understood relatively easily in terms of the Fisher and Seater (1993) paradigm. McCallum's (1984) "second example" suggested that $LRD_{p,m}$ was not necessarily equal to unity even when long-run neutrality held. But in the example, $\langle m \rangle = 0$ so that the long-run derivative is not defined. Both Lucas (1980) and Fisher and Seater (1993) emphasized that permanent shocks to money were necessary to test neutrality propositions.

Whiteman (1984) critiqued Lucas (1980) from the point of view of a structural model that could display a Mundell-Tobin effect. In such a model, a permanent increase in the rate of money growth would permanently lower the real interest rate. Because of this, nominal interest rates would not rise one-for-one with increases in money growth, and superneutrality would be violated. This is an important consideration for Lucas' second set of scatterplots which are not replicated in this paper. The Mundell-Tobin effect does not bear on long-run

neutrality, however, and Whiteman confirmed this by showing that when $\langle m \rangle = \langle p \rangle \geq 1$, the long-run derivative would equal unity in his model regardless of the Mundell-Tobin effect. Whiteman's critique of Lucas (1980), although valid, does not impinge on the first part of Lucas' analysis or on the analysis here, both of which focus on long-run neutrality.

RESULTS

In this section, the filter is applied to all three series as described above, giving the maintained relationship as $\Delta P_t(\beta) = \alpha + \Delta M_t(\beta) - \Delta Y_t(\beta)$. If the filtered inflation data is plotted against the difference between filtered money growth and filtered output growth, the form of the quantity theory used here predicts that the data will lie on a 45-degree line with intercept α .¹⁴ The output measure employed is real gross domestic product.¹⁵ Two inflation measures are used: the consumer price index and the gross domestic product deflator. Along with 19 measures of money, this yields 38 illustrations of the quantity theory. The measures of money range from the very narrow to the very broad. These series are all available over the entire sample period of 1960-92. These years keep all measures on equal footing: although some measures could be taken further into the past, any comparisons among monetary aggregates would then be blurred.¹⁶

The results can be summarized in a number of ways. Lucas' (1980) method simply involves a graphical interpretation in which the data is plotted and examined to see if it appears to lie plausibly on a 45-degree line. A few selected plots of this type are shown in Figures 5 through 8. One of the main results of this paper is that, broadly speaking, these plots provide illustrations of the quantity theory in that higher inflation is associated with higher money growth regardless of the particular measure of money used.¹⁷ In this sense, the results are

¹²Fisher and Seater (1993) also argue that the LRD interpretation holds if $\langle m \rangle = \langle p \rangle = 2$ and Δm and Δp are co-integrated.

¹³See, for instance, King and Watson (1992).

¹⁴Plots of this type differ somewhat from those found in Lucas (1980, 1987) in that the real output growth rate is also filtered; in the previous work, the output growth rate was set equal to the average output growth rate over the sample period.

¹⁵Replacing actual output with potential output produces qualitatively unchanged results. Here, actual output is used to maintain comparability with Lucas (1980, 1987).

¹⁶The measures of money used are adjusted reserves, total reserves, nonborrowed reserves, currency, adjusted monetary base (St. Louis), adjusted monetary base (Board of Governors), Divisia M1A, M1A, Divisia M1, M1, Divisia M2, M2, non-M1 components of M2, Divisia M3, M3, the non-M2 components of M3, Divisia L, L, and the non-M3 components of L. Barnett, Fisher and Serletis (1992) provide a survey of the construction and use of Divisia aggregates, a topic beyond the scope of this paper.

¹⁷Plots using the CPI as the measure of inflation are qualitatively similar.

Figure 5
Monetary Base (Board Series)
with GDP Deflator

Figure 6
M1 with GDP Deflator

Figure 7
M2 with GDP Deflator

Figure 8
L with GDP Deflator

consistent with those provided by Lucas even when the data from the last 17 years are included, years that are known for being rocky from the point of view of reliable empirical relationships involving monetary aggregates. The results are particularly striking if the measure of money is broad, such as M2 (Figure 7), M3 or L (Figure 8). Narrower measures, such as the monetary base (Figure 5) or M1 (Figure 6), tend not to provide as convincing an illustration.¹⁸

The results can be summarized more quantitatively by computing the mean-square error (MSE) from the 45-degree line that passes through the grand mean of the filtered data. This amounts to measuring the distance of the filtered data from a fitted regression line where the slope is forced to unity. Table 1 summarizes the results using all measures of money and inflation based on an MSE criterion. In the table, the results are presented in order from the lowest MSE to the highest when the measure of inflation is the deflator, but the results are also presented for the case where the CPI is the inflation measure. The MSE is the lowest when the measure of money is broad, with aggregates like M2, M3 and L and their Divisia counterparts provide the best performance.

The data in Figures 5 through 8 can be viewed as representing the coherence between long-run movements in inflation and long-run movements in money growth. That is, when the pace of monetary expansion is increasing, the quantity theory suggests that the rate of inflation should be increasing as well, again, in the special long-run sense used in this paper. Thus, regardless of the relationship to a 45-degree line that passes through the mean of the data, one would like to know if the data is moving in the "right direction"—along a line with slope one—most of the time. It may be, for instance, that the relationship between some measure of money and inflation is subjected to an occasional shift during the sample period. The filtered data in such a case might normally plot along a 45-degree line except for brief interludes corresponding to the occasional shifts. Thus, it may be useful to consider a coherence measure that does not require the data to stay on the same 45-degree line at all times in order to do well.

Table 1
Mean-Square Error Criterion

Measure	MSE	
	Deflator	CPI
M2	10.1	24.9
Divisia L	19.8	37.5
L	19.8	22.8
Divisia M3	27.3	53.9
Divisia M2	27.6	50.0
M3	36.1	52.7
Currency	90.2	68.8
Adjusted monetary base STL	96.0	81.4
Non-M1 components of M2	97.2	136.4
M1A	105.5	130.7
Divisia M1A	112.3	130.8
Adjusted monetary base BOG	119.2	98.6
Adjusted reserves	126.9	131.9
Divisia M1	135.8	117.5
M1	162.9	141.7
Non-M3 components of L	255.9	194.4
Total reserves	383.3	362.0
Nonborrowed reserves	462.7	437.8
Non-M2 components of M3	4233.3	4475.9

One way to measure coherence of this type is to proceed as follows. First, construct a line between each pair of adjacent filtered data points. Second, measure the angles in radians between the constructed lines and a 45-degree line. Finally, square each radian measure and sum across all data points to obtain a measure of coherence. This coherence measure has a maximum value which occurs when each constructed line is exactly perpendicular to the 45-degree line. The results according to a coherence criterion are presented in Table 2, and rankings are again computed using the deflator as the measure of inflation. The broad simple-sum measures M2, M3 and L again do well, but currency, base measures and the non-M1 components of M2 also fare well. The results concerning the monetary base (and to some extent currency, which is a large portion of the base) can be inferred from Figure 5. The base certainly moves in the right direction much of the time, as the coherence criterion requires, even though the plotted

¹⁸In the charts, the grand mean is the mean of all the plotted pairs of filtered money and filtered inflation.

Table 2
Coherence Criterion

Measure	Squared radians	
	Deflator	CPI
L	12.3	8.1
Currency	20.5	15.5
M3	23.2	17.5
Non-M1 components of M2	24.7	36.9
Adjusted monetary base BOG	26.0	20.5
M2	26.6	23.7
Adjusted monetary base STL	27.5	22.2
Divisia M3	30.1	44.6
Non-M3 components of L	30.4	25.5
Divisia M2	31.5	44.1
Divisia L	32.5	46.0
Divisia M1	35.1	29.4
M1	37.4	31.6
Adjusted reserves	63.2	57.7
M1A	75.5	82.3
Divisia M1A	76.8	82.4
Total reserves	80.6	75.5
Nonborrowed reserves	88.6	87.6
Non-M2 components of M3	90.9	92.8

data is rarely on the 45-degree line that passes through the grand mean.

Finally, the results can be summarized according to the estimate of the long-run derivative as defined by Fisher and Seater (1993), that is, by the slope of an ordinary least-squares line fitted to the filtered data. This time, both the slope and intercept are estimated, instead of forcing the slope to unity as in the MSE criterion. The main concern is whether the estimated slope is close to 1. As a simple metric, the squared difference between the estimated slope and unity is used as the measure of how close the estimated long-run derivative is to 1. In Table 3, the results are shown ranked according to this metric when the measure of inflation is the deflator. The table shows the estimated slope coefficient, instead of the squared difference between this coefficient and 1. Again, the broad aggregates and their Divisia counterparts tend to rank in the top half. In this case, Divisia M1A and adjusted reserves also perform well.

SUMMARY

The results presented in this paper are generally supportive of a quantity theoretic proposition that has been difficult for economists to investigate satisfactorily using time series data from a single country. The proposition is that money is long-run neutral. By using a certain filter suggested by Lucas (1980), a long-run signal can be extracted from time series data, and filtered data on money growth and inflation can be examined to see if it conforms to quantity theoretic predictions. When broad measures of money are used, such as M2, M3 and L, striking illustrations of the quantity theory are obtained. These results can be verified using either Lucas' original graphical procedure or by using alternative goodness-of-fit criteria. The results have some statistical basis in the sense that they can be described within the framework for testing neutrality and superneutrality propositions recently worked out by Fisher and Seater (1993).

Table 3
Long-Run Derivative

Measure	LRD	
	Deflator	CPI
M2	0.99	1.08
Divisia M1A	1.08	1.27
Adjusted reserves	0.83	1.02
L	0.80	0.88
M3	0.78	0.84
Divisia M3	1.24	1.32
Divisia L	1.25	1.37
Adjusted monetary base STL	0.75	0.90
Divisia M2	1.28	1.39
Currency	0.71	0.85
Non-M1 components of M2	0.68	0.70
Adjusted monetary base BOG	0.66	0.80
Divisia M1	0.64	0.78
M1A	1.38	1.58
M1	0.57	0.71
Non-M3 components of L	0.45	0.54
Total reserves	0.22	0.32
Nonborrowed reserves	0.16	0.25
Non-M2 components of M3	0.01	-0.01

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