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Does the Exchange Rate Regime Affect the Economy?

It seems to be a general rule that countries wish to peg their exchange rates but sometimes have floating rates thrust upon them. On three occasions during the twentieth century—the breakup of the international gold standard in the 1930s, the breakup of the Bretton Woods system in the 1970s and most recently the exodus of countries (notably Britain) from the exchange rate mechanism (ERM) of the European Economic Community (EEC)—external pressures led to the demise of fixed rate schemes and their replacement by some degree of exchange rate flexibility. In each case, the passing of the fixed rate scheme was mourned and within relatively short periods a new fixed rate plan was advanced to replace its fallen predecessor. In view of these failures, however, it is reasonable to ask: What makes pegged exchange rates so attractive?

Recently, in the context of the ERM, two arguments have been advanced. Exchange rate fixity is, as David Hume described in 1752, a way of importing another country’s monetary policy. In the case of the ERM, the deutsche mark served as the system’s anchor currency, and Germany’s low inflation rate was supposed to spread throughout the EEC. Moreover, the ERM’s member nations believed that the Bundesbank’s reputation would provide some credibility to the anti-inflation commitment of other central banks and therefore reduce the costs of lowering inflation throughout the EEC. A second motive for adopting fixed exchange rates has been the claim that they, and ultimately a single currency, are important to the EEC’s Single Market Programme. The logic is that the full benefits that could accrue from the free intra-European movement of goods, labor and capital will be realized only with a fixed exchange rate regime. A third argument, not emphasized recently but important on earlier occasions, is that economic performance—growth, inflation or any other important measure—is better under a fixed exchange rate system. This third argument differs from the second in that it identifies no specific causal chain from exchange rate regime to economic performance.

But does the exchange rate regime matter for economic performance? That is the question.
addressed in this paper. We examine empirically the relationship between the exchange rate regime and a number of key macroeconomic variables to see whether any systematic relationship exists between the behavior of these variables and the exchange rate regime. We have chosen to investigate this question for the United Kingdom because data over long periods are available for the variables we wish to examine and because the United Kingdom experienced a wide variety of exchange rate regimes over the period covered by these data.

TRADE AND THE EXCHANGE RATE REGIME

The claim that exchange rate flexibility hampers international trade in goods and in capital and thus depresses welfare and perhaps growth is based on the existence of uncertainty.

It is argued that removing the possibility of exchange rate change will remove an important nontariff barrier, because the possibility of exchange rate changes will deter some traders and investors altogether, whereas others will have to pay a substantial cost to fix the domestic value of their foreign currency receipts. Floating exchange rates, in other words, are believed to impose additional volatility, and hence costs, on international markets. If this is correct, a case for pegged exchange rates exists, and the case is particularly strong for any group of countries (such as the EEC) that wants to encourage mutual international trade and investment.

The proposition seems unexceptional, and for a number of years studies supported the proposition. For example, Cushman (1983) and de Graauwe and deBellefroid (1987), which are representative of the early literature, found that floating rates did impede trade. But as time passed, an increasing number of studies supported it.

By the early 1990s, not only had evidence shifted to support the notion that floating exchange rates do not impede trade, but Feldstein (1992) even went so far as to suggest that floating rates are more favorable to trade than are fixed rates. Attention is thus directed to other reasons for favoring pegged exchange rates.

There are two rather distinct types of effects of exchange rate fixing. The first arises because if a fixed exchange rate is in place, it is unlikely to stay fixed without policy actions. These can take several forms. Most common are foreign exchange intervention and short-term interest rate manipulation. Accurate figures on official intervention or the stock of foreign exchange reserves are not always available. Interest rate figures, however, are available, and several authors have found that unpredictable interest rate variability increased after exchange rates are pegged. These actions in turn make money growth more volatile, and this can have important consequences for the economy. It may create additional uncertainty about the future behavior of the price level and thus about real rates of return, which would affect investment. If future prices were uncertain, wage bargaining would be more complex because it would be harder to judge the future purchasing power of an agreed money wage. This uncertainty would also affect nominal variables. Risk-averse investors would be more reluctant to buy government bonds because they would be uncertain what the coupons would be worth and what the capital would be worth at maturity. This would raise nominal interest rates, the cost of debt service and thus the taxes necessary to service the debt. All these factors could have an adverse effect on long-term growth, depressing its trend.

In summary, the choice of exchange rate regime could affect the long-run behavior of the economy, influencing trends or cycles in important macroeconomic variables.

If the choice of exchange rate regime does not have these long-run consequences, then in terms of macroeconomic effects, all that the choice of exchange rate regime does is shift the distribution of short-run fluctuations from one market to another. This is the second type of effect noted above.

The question we examine is whether any as-

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†Haberler (1986) suggested the same thing some years earlier.

‡See Batchelor and Wood (1982), Wood (1983) and Belongia (1988). Wood and Belongia's research was conducted in the context of the ERM. In Wood (1983) there was an exception to this—Erie (South Ireland) after it joined the ERM. Unpredictable interest rate variability fell in that country, although it increased in every other ERM member country.
sociation exists between the exchange rate regime and the trend or cyclical behavior of some key macroeconomic variables—in other words, whether there is any evidence for the first type of effect. If no such association exists, then the only macroeconomic consequence of the choice of exchange rate regime is the change in the distribution of short-term volatility between the foreign exchange market and the short-term money markets. If, in contrast, such an association exists, then the choice of exchange rate regime may be a macroeconomic policy decision of considerable importance for national well-being.8

It is now appropriate to present the data we use for exploring this question. We then examine the properties of those data in light of the preceding discussion.

THE STOCHASTIC PROPERTIES OF U.K. MACROECONOMIC SERIES ACROSS EXCHANGE RATE REGIMES

In this section we consider the stochastic properties of five major U.K. macroeconomic series since the mid-nineteenth century. The exchange rate regimes since then have encompassed every possible type except the crawling peg. Until 1914, the United Kingdom was on the gold standard. That was suspended (that is, the United Kingdom left the standard but with the declared intention of returning) at the outbreak of World War I in 1914. After the war, the United Kingdom implemented a deliberate, discussed and announced policy of a return to the gold standard at the prewar parity. Monetary policy and foreign exchange intervention were used to this end, and the policy succeeded in 1925. The United Kingdom left the gold standard in 1931, however, and the exchange rate floated with varying degrees of intervention until the outbreak of World War II in 1939.9 The rate was then pegged to the U.S. dollar. After the war, the United Kingdom joined the Bretton Woods system. Several sterling devaluations occurred under Bretton Woods, but sterling did not finally float until 1972. Again, there were varying degrees of intervention under this regime of dirty floating, but the United Kingdom did not formally peg sterling until it joined the ERM in 1990 after shadowing the deutsche mark in 1988 and 1989. The United Kingdom subsequently left the ERM in 1992 to float once more. The series we examine across these various regimes are output, prices, money, and short- and long-term interest rates.

Our particular interest, and the focus of the empirical work that follows, is the trend and the cycle in output and prices primarily, but also in money and interest rates. We look to see how these variables have behaved over our close to a century-and-a-quarter of data, seeking changes in trend and changes in cyclical pattern. When these are identified, we examine whether any of these changes are associated with exchange rate regime changes and, if so, consider why this might be.

Output

Annual output in the United Kingdom (measured in logarithms) over the period 1855–1990 is shown in figure 1. Detailed econometric analyses of this series in Mills (1991) and Mills and Wood (1993) show that it can be represented as the sum of a segmented linear trend, with breaks at 1918 and 1921 and a stationary, autoregressive, cyclical component.10 Thus these results indicate that if output can be decomposed as \( y_t = \mu_t + n_t \), then the trend function \( \mu_t \) is

\[
(1) \quad \mu_t = \alpha + B t + \lambda_1 D_{1t} + \lambda_2 D_{2t}
\]

where \( D_{1t} = (t - T_1) \) if \( t > T_1 \), and zero otherwise. The identified breakpoints are at \( T_1 = 64 \) and \( T_2 = 67 \), which coincide with 1918 and 1921. The cyclical component, \( n_t \), on the other hand, is found to be adequately modeled as an AR(2)
process, leading to the fitted model (standard errors shown in parentheses),

\begin{equation}
Y_t = 3.474 + 0.0196t - 0.1137D_n + 0.1170D_{22} + n + a_t
\end{equation}

\begin{align*}
D_n & = 1.099n_{t-1} - 0.346n_{t-2} + a_t \\
D_{22} & = 0.0101
\end{align*}

This model has some simple properties. Trend growth is 1.96 percent per year until 1919 and 2.29 percent per year from 1922 on, with the level of trend output falling 28.3 percent in the intervening three years. The component \( n \) implies that output exhibits stationary cyclical fluctuations around the trend growth path, with cycles averaging 8.1 years. The residual standard error of the equation is 2.33 percent.

The trend component is shown superimposed on the output series in figure 1, and we thus conclude that, apart from the three years immediately after World War I, during which the series fell dramatically, the stochastic process generating output has remained remarkably stable. Output is a trend stationary process, irrespective of the exchange rate regime in force.

**Prices**

Figures 2 and 3 present plots of the (logarithmic) U.K. price level annually from 1870 to 1990 and monthly from January 1922 to May 1992, respectively, excluding the war years from 1940 to 1945. Unit root tests, calculated over various sample periods, provide little or no evidence against the hypothesis that prices are difference stationary, that is, \( I(1) \).

The post-1973 era may differ and is discussed later.

Two aspects of price behavior are worth fur-

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11 Details of these tests and similar tests for the other series investigated are reported in Mills and Wood (1993).
Figure 2
Annual U.K. Price Level (1870-1939)
Logarithms

Annual U.K. Price Level (1946-1990)
Logarithms
Figure 3
U.K. Price Level (1922-1939)

Logarithms

U.K. Price Level (1946-1992)

Logarithms
ther investigation. The first is the behavior of the price level before the United Kingdom abandoned the gold standard in 1931. Mills (1990) analyzes the long gold standard period from 1729 to 1931 and obtains an estimate of the largest autoregressive root of 0.93, identical to that obtained for the shorter sample beginning in 1870. The corresponding unit root test, though, rejects the unit root null hypothesis at the 5 percent significance level, and the process found to generate the price level (an autoregression of order two) yields cycles of around 50 years, close to the long swings thought to have characterized prices during this period.12

The second aspect concerns the post-1946 behavior of prices. Figures 2 and 3 show the series to have undergone slope changes around 1973 and 1983; possible explanations for these are discussed in the next paragraph and in the Interpretation and Conclusions section. Statistically, this behavior is typical of an I(2) process, and repeating the unit root tests for the (logarithmic) price changes, that is, for inflation, yields some evidence that postwar prices can be modeled as an I(2) process (evidence that inflation is nonstationary), particularly for the post-Bretton Woods era beginning in 1973.

The results are therefore suggestive of the U.K. price level undergoing two shifts in its generating process. The first might be associated with the abandonment of the gold standard, shifting the series from I(0) to an I(1) process. (From figure 3 it is in fact clear that prices did not start a secular increase until mid-1933, some two years after the move from the gold standard.)13 A stable price level is certainly in accordance with what would be expected under the gold standard (or, in principle, any commodity standard). There were fluctuations in the supply of gold, but in countries such as the United Kingdom, which had developed and stable banking systems, these fluctuations had only modest price level effects. The system was to some extent self-stabilizing. If prices were falling (the value of money rising) because the supply of gold was falling short of demand, there was an incentive to produce more gold. And if prices were rising (the value of money falling), then as the costs of gold production rose relative to what the monetary authorities would pay for gold, the incentive to produce gold would diminish.14 The second shift is around 1973 and could be associated with both the move to floating exchange rates and the first oil price shock.

**Money**

Figure 4 plots annual observations of the logarithms of M3 from 1871 to 1912 (the only aggregate apart from the monetary base available for this period), and figure 5 plots monthly observations of M3 from 1922 to 1989, excluding the war years. From a battery of unit root tests, we found that, for all sample periods investigated, the null hypothesis of a unit root cannot be rejected. Moreover, the series is indeed I(1) because we could not establish that further differencing was required for stationarity.

**Interest Rates**

Figures 6 through 8 plot monthly observations of short- and long-term interest rates from 1870 to 1992, excluding war years and related periods of interest rate restrictions.

From the results of unit root tests, we find that since the lifting of restrictions after World War II both short- and long-term interest rates have been I(1) processes, but their behavior before 1939 is rather different. Both are stationary between 1932 and 1939, but during the 1920s long-term rates are stationary (I(0)) and short-term rates are I(1), whereas before 1914 the orders of integration are reversed.15

**Trend and Cycle Decompositions**

Has the variability about trend of the series altered across regimes? This is an important question because of the widespread belief that floating exchange rates increase volatility in prices, interest rates, and economic activity and are in some general sense destabilizing. To answer this question, we need to decompose each series into trend and cycle components. There are many ways to do this, ranging from using a predetermined moving average to calculate trend to designing a signal extraction filter based on the stochastic process generating the data and a set of assumptions relating to the behavior of the unobserved components.

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13 See the discussion in Capie, Mills and Wood (1986a).
14 See Barro (1979) and Rockoff (1984) for a discussion of this.
15 Capie, Mills and Wood (1986b) provides an extended discussion of the behavior of these two interest rate series in relation to the Stock Conversion of 1932.
For output, equation (1) provides the appropriate decomposition. Table 1 thus reports the standard deviations of the cyclical component $n$, for a variety of sample periods. The sample periods shown were chosen by two quite distinct criteria—output trend change and exchange rate regime alteration. The 1922 break was used because after the 1919-22 discontinuity, output resumed a new trend. 1855-1913 were gold standard years, and 1925-31 were years during which the United Kingdom was either on or committed to returning to the gold standard. The period comprising 1855-1913 and 1922-31 is the same period omitting war and the postwar years of the break in output’s trend. The period 1922-31 has a stable output trend combined with commitment to gold; the period 1922-39 has a stable output trend with a change in exchange rate regime. The period 1932-90 is our whole sample period after gold. The years 1932-39 and 1946-90 are, of course, the same period excluding the World War II years. The period 1946-90 is simply postwar; 1946-72 is Bretton Woods; and 1973-90 is the period of various degrees of float. (Further subdivision of the series to examine the association with various exchange rate regimes more minutely, although appealing, is ruled out by many of these regimes having too few output observations for our statistical techniques.)

From all these statistics, one gets the impression that variability about trend has increased during the twentieth century. In particular, the abandonment of the gold standard in 1931 seems to have been accompanied by an increased variability of output about trend, even after the war years are excluded. In summary, the standard deviation almost doubled (from 2.87 percent to 5.49 percent) after 1931. But it should be noted that variability fell after the pound floated in 1972. From 1946 to 1972 the standard deviation was 4.45 percent; from 1973 to 1990 it was 3.64 percent.

For the other series, we have presented evidence of shifts in the stochastic processes generating them, so signal extraction techniques would be rather difficult to apply. We have chosen therefore to use a technique that has proved popular in recent years for re-examining the stylized facts of macroeconomic time series, namely the detrending filter proposed for use in economics by Hodrick and Prescott and used,
Figure 5
U.K. Money Supply: M3 (1922-1939)

Logarithms


Logarithms
Figure 6
U.K. Interest Rates (1870-1913)

Percent

Figure 7
U.K. Interest Rates (1922-1939)
for example, in Kydland and Prescott.\textsuperscript{16} This is an alternative to the method used earlier in the paper for separating a series into trend and cycle. It is described in the appendix, which also contains a summary of when this method is appropriate and when it may be misleading.

Tables 2 through 5 report statistics assessing the variability of the trend and cycle components of the price level, money supply and short- and long-interest rates, and figures 9 through 12 present graphs for these components. Although these tables report results from the examination of monthly data, the breakpoints are at year ends except for 1992, whose data end with June.

This choice of breakpoints reflects two considerations. The first relates to when an exchange rate regime changed. Does change for our purposes relate to when the change was formally announced or to when it became expected and affected behavior? The latter is the more significant, but it is not clear \textit{a priori} when it would be. Nor as it turns out does detailed examination of the data case by case give clear-cut answers.\textsuperscript{17} Accordingly, the simple expedient of using calendar years as breakpoints was adopted, on the grounds that using other dates close to these would not change the results.

For the interwar years, the trend of the price level was relatively flat, with a slow decline until 1933 and an upward drift thereafter. The cyclical component, in contrast, is relatively volatile, no doubt, in view of the unchanged behavior of money, reflecting the changes in exchange rate policy in the United Kingdom, as well as the disturbed external environment. Not only did the interwar years include the Great Depression in the United States, with the associated severely depressing effects on the prices of commodities, but in continental Europe there were inflations—hyperinflations in some cases—civil war and revolutions. Meanwhile Britain’s exchange rate regime was changing rapidly. Between 1919 and 1925 there was a


\textsuperscript{17}See Mills and Wood (1993). For a subset covering the years 1870–1939, see Capie and Wood (forthcoming).
commitment to return to gold at the prewar parity, and the exchange rate rose steadily toward that. Gold was abandoned in 1931, and the exchange rate thereafter floated with various degrees of intervention until the outbreak of war in 1939.

After 1946, the trend is smooth and monotonically steady, and the cyclical component is less volatile than before. Trend money is rather similar to trend prices. Its variability is stable throughout the sample period, supporting the suggestion that external factors were important in interwar price volatility.

Pre-1914 trend interest rates fluctuate around 3 percent, although the far greater stability of long-term rates is reflected in the almost constant components of this series relative to short-term rates. Volatility is indeed fairly stable until 1972, after which both trend and cycle components became considerably more variable.

**INTERPRETATION AND CONCLUSIONS**

When discussing the preceding findings, it is convenient to consider the trend and cyclical behavior of each series together. We start with output. As noted previously, the trend growth...
of output changed from 1.96 percent per year to 2.29 percent per year between 1919 and 1922. Speculating on what produced that welcome change is outside the scope of this paper. What we would note is the stability of the post-1922 trend in the face of a wide variety of monetary experiences and exchange rate regimes, a finding clearly consistent with the long-run neutrality of money.

In contrast to that long-run neutrality, the cyclical behavior was affected. The variability of output rose substantially with the abandonment of the gold standard. The significance of this is discussed later.

Turning now to prices, what do we find? The first notable feature is the essentially flat trend, with long swings around it, under the gold standard. More dramatic and equally revealing about the nature of the monetary regime is the post-1946 period. The trend of prices was positive after 1946, accelerated sharply around 1973 and slowed around 1983. The United Kingdom went to a floating exchange rate in 1972, but at around the same time there was also the first oil price shock and the Heath-Barber monetary expansion. That the acceleration of prices was the result of these factors rather than the new exchange rate system is suggested by the slowing of prices around 1983, when the United Kingdom was still under a floating rate regime but had a government strongly committed to reducing inflation by introducing money supply targets and a commitment to budget balance over the cycle. The cyclical component of prices became much smoother and was unaffected by the exchange rate regime; its variability was unchanged from 1946 to 1992 and identical over subperiods and the period as a whole.

And finally, interest rates. The striking contrast is between the behavior in the pre-World War II period, when long-term rates were stable and short-term rates were volatile, an observation usually interpreted as reflecting expectations of long-run price level stability and behavior in the post-1972 period, when inflation first accelerated and then slowed, and both interest rate series displayed markedly increased variability.

How do these findings as a whole bear on the hypothesis that the exchange rate regime is not a source of volatility? They support it. Of the variety of exchange rate regimes after 1913 (we turn to the gold standard in a moment), none seemed to increase the volatility of any series examined to any significant extent. The policy changes necessary to hold rates pegged may have appeared in foreign exchange reserves, a series that we did not examine because reliable data were not available. The policy changes did appear in movements that had higher frequencies than the trends and cycles we isolate. Interest rate cyclical variability did increase with the move to floating exchange rates in 1972, but there are numerous other factors to explain this. Shocks to the price of oil disturbed financial markets very substantially in this period. Two other shocks were superimposed on the oil price shocks. There was a commitment to reduce inflation—particularly after 1979. What this meant in terms of the operation of monetary policy was unknown, so the commitment increased uncertainty for a time. And further, monetary targets were adopted. These affected how the authorities used short-term interest rates; and as commitment to monetary targets

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**Table 5**

Component Variability of Long-Term Interest Rates

<table>
<thead>
<tr>
<th></th>
<th>$\bar{x}$</th>
<th>$s_x$</th>
<th>$s_T$</th>
<th>$s_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970.01-1913.12</td>
<td>2.94</td>
<td>0.29</td>
<td>0.22</td>
<td>0.04</td>
</tr>
<tr>
<td>1922.01-1931.12</td>
<td>4.45</td>
<td>0.13</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>1932.01-1939.12</td>
<td>3.31</td>
<td>0.35</td>
<td>0.31</td>
<td>0.14</td>
</tr>
<tr>
<td>1944.01-1972.12</td>
<td>6.35</td>
<td>1.71</td>
<td>1.67</td>
<td>0.24</td>
</tr>
<tr>
<td>1973.01-1992.04</td>
<td>11.32</td>
<td>1.91</td>
<td>1.74</td>
<td>0.61</td>
</tr>
</tbody>
</table>

$x$: sample mean  
$s_x$: sample standard deviation  
$s_T$: sample standard deviation of trend component  
$s_C$: sample standard deviation of cycle component

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19The role of the exchange rate regime in the 1970s episode is also discussed in Williamson and Wood (1976). The conclusion that the exchange rate regime was not at fault was also, by different means, argued there.


Figure 9
Price Level Trend and Cycle (1922-1992)

Figure 10
Money Supply: M3 Trend and Cycle (1922-1986)
Figure 11
Short Interest Rate Trend and Cycle (1870-1992)

Figure 12
Long Interest Rate Trend and Cycle (1870-1992)
became increasingly credible, the relationship between movements in short- and long-term rates changed.\textsuperscript{22}

It cannot but be observed that there was greater stability of output, interest rates and prices under the gold standard than under any subsequent exchange rate regime. But, of course, the gold standard was more than an exchange rate regime. It was a system, a set of rules, for the conduct of monetary policy. As Bordo (1993) wrote, “The gold standard rule can be viewed as a form of contingent rule or a rule with escape clauses. The monetary authority maintains the standard—that is, keeps the price of the currency in terms of gold fixed—except in the event of a well-understood emergency, such as a major war or a financial crisis. In wartime it may suspend gold convertibility and issue paper money to finance its expenditures, and it can sell debt issues in terms of the nominal value of its currency on the understanding that debt will eventually be paid off in gold. The rule is contingent in the sense that the public understands that the suspension will last only for the duration of the wartime emergency plus some period of adjustment. It assumes that afterward the government will follow the deflationary policies necessary to resume payments at the original parity.” It may be consistent with this interpretation of the gold standard that with the floating exchange rate of the 1970s, output variability fell, but not to where it had been under the gold standard. The argument would be that monetary policy was now clearly focused on internal objectives and not subject to the vicissitudes of a multitude of shocks from the outside world.

All in all, then, it appears clear that the exchange rate regime in the United Kingdom has not been a source of volatility for the main macroeconomic variables. For that reason we need not consider why exchange rate regimes might affect real economic performance—in the United Kingdom they did not. The case for a fixed rate regime in the United Kingdom apparently must depend only on its traditional source of support—the desire to import price level performance.

It is, of course, important to consider whether these results generalize to other economies.

There is virtually no feature of the U.K. economy to indicate that they should not.\textsuperscript{23} The composition of output is not unusual; the U.K. economy has always been fairly open. It was a dominant economy internationally for only a modest part of our period, and it has not gone through hyperinflation or recessions as severe as those in some other economies, so such problems cannot have biased our results. Though we would not claim that our findings are more than those of a case study, we would suggest that they are findings we would not be surprised to see roughly repeated in studies of other countries.

\textbf{REFERENCES}


\textsuperscript{22}Initially, rises in short-term rates produced rises in long-term rates. But as markets became convinced that the authorities were serious, short- and long-term rates started to move much more independently of each other.

\textsuperscript{23}This is also suggested by the similar structure of models used to explain and predict the economies of a wide range of countries.
Appendix

The Hodrick-Prescott Filter

The filter proposed by Hodrick and Prescott (1980) has a long tradition as a method of fitting a smooth curve through a set of points, versions of it being used as an actuarial graduation formula. Given the traditional decomposition $y_t = \mu_t + \eta_t$, the trend series $\mu_t$ is obtained as the solution to the problem of minimizing

$$\sum_{t=1}^{T} (y_t - \mu_t)^2 + \lambda \sum_{t=1}^{T} (\mu_t - \mu_{t-1} - (\mu_{t-1} - \mu_t))^2$$

with respect to $\mu_1, \mu_2, ..., \mu_T$. The first order condition for this minimization problem is

$$y_t = \mu_t - 4\mu_{t-1} + (6 + \lambda^{-1})\mu_{t-2} - 4\mu_{t-3} + \mu_{t-4}$$

Using the lag operator $B$, defined such that $B\mu_t = \mu_{t-1}$, this can be written as

$$Y_t = \lambda(B^{-3} - 4B^{-1} + (6 + \lambda^{-1})B^{-2})\mu_t$$

so that if an infinite series of $y$ values were available, $\mu_t$ would be given by the two-sided moving average

$$\mu_t = \sum_{j=-\infty}^{\infty} \alpha_j y_{t+j}, \quad \alpha_j = \frac{1}{2^j}$$
where the weights can be calculated from

\[(5) \ a(B) = [\lambda (1-B^2)(1-B^{-1})^2 + 1]^{-1}.\]

King and Rebelo (1989) provide expressions for the \(a_i\), which do not take a simple form. Fortunately, Hodrick and Prescott (1980) provide an algorithm that removes the need to calculate the moving average weights and so allows the trend to be computed when only a finite number of y observations are available. This algorithm was employed to compute the decompositions used here, noting that the cyclical component can be obtained by residual as \(n_i = y_i - \mu_i\). Typically, following Hodrick and Prescott, \(\lambda\) is set at 100 if annual data are used or 1,600 if quarterly or monthly data are used.¹

Harvey and Jaeger (1991), for example, show that the filter \(a(B)y_i\) can be interpreted as being the optimal estimate of \(\mu_i\) when \(y_i\) is generated by the structural model

\[(6) \quad y_i = \mu_i + n_i, \quad \mu_i = \mu_{i-1} + \Psi_{i-1}, \quad \Psi_i = \Psi_{i-1} + \zeta_i, \quad n_i \sim NID(0,\sigma_n^2), \quad \zeta_i \sim NID(0,\sigma_\zeta^2), \quad \lambda = \sigma_n^2/\sigma_\zeta^2.

An observed series may not be generated even approximately by such a model, and even if it is, the ratio of the two innovation variances may be very different from the assumed value of \(\lambda\). Harvey and Jaeger argue that the Hodrick-Prescott filter may create spurious cycles, distort the estimates of the components or both. King and Rebelo argue in similar vein, although they focus on the calculation of sample moments of the estimated trend and cycle components. Given these strictures, we emphasize that our use of the filter is purely for exploratory purposes outside the confines of any explicit model.

¹See Hodrick and Prescott (1980).