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Monetary Aggregates, Monetary Policy and Economic Activity

Almost a quarter century has passed since the publication of the (in)famous Andersen-Jordan (AJ) equation.¹ For a good portion of that time, Ted Balbach has been associated with the research department of the Federal Reserve Bank of St. Louis, and for a significant fraction of the period directed the research efforts of that department.² Throughout that period the Bank consistently advocated a monetarist approach to monetary analysis and monetary policy. It is appropriate at this point to look back and examine what lasting influence this perspective has contributed, both to analysis and to policymaking.

This study has three parts. The first is a re-examination of what monetarism and the St. Louis empirical representation thereof contributed. In particular, what controversies of the late 1960s and 1970s now can be considered settled? The second examines the empirical failures of the AJ equation in the 1980s and argues that these failures represent specification problems of the "Lucas variety" and not a rejection of the underlying theoretical framework. The implication of such a "Lucas effect" for prominent monetarist policy prescriptions is then analyzed. The third part examines the monetarist proposition that has remained most controversial in recent years, namely the short-run impact of changes in nominal money growth on real economic activity. In particular, the analysis attempts to address the question raised by Cagan—why do vector autoregressions (VARs) produce inferences about the impact of money on economic activity that contrasts dramatically with the conclusions of historical analyses?³

ST. LOUIS ON THE ROLE OF MONEY

Two aspects of the AJ equation seemed particularly controversial in the late 1960s. First, the analysis focused on the relationship between nominal measures of fiscal and monetary policy and nominal income. Second, the analysis focused on growth rates or first differences. Reduced to simplest terms, the analysis stated that the growth in velocity of narrow money, defined as the ratio of nominal GNP to a weighted moving average of M1, fluctuated around a positive deterministic trend and that some fraction of these fluctuations were correlated with fluctuations in the growth

¹See Andersen and Jordan (1968).
²A precursor of the AJ equation can be found in Brunner and Balbach (1959). Michael Belongia is responsible for bringing this well known article to my attention.
of nominal government spending. This contrasts sharply with macroeconometric models that were developed contemporaneously. The implicit reduced forms of the latter models specified relationships between the level of nominal money balances and the level of real output. The models also endogenized the price level or inflation rate, but the typical reduced forms implied little if any price level response over the time periods in the AJ specification.

The lightning rod in the AJ equation was the conclusion that a maintained change in nominal government spending, unaccompanied by changes in the nominal money stock did not produce a permanent change in nominal income (or velocity) and that changes in high employment nominal tax receipts produced no statistically significant changes in nominal income (or velocity). These implications, which dramatically refuted the fixed-price Keynesian model, did not go unchallenged. Numerous counter regressions were published which reported that the implied fiscal policy implications of the AJ equation were artifacts of measurement error and/or sample specific. The point that seems to get lost in the background of these challenges is the robustness of the long-run response of nominal income growth to monetary growth shocks: the conclusion that monetary shocks, in the absence of fiscal shocks, have only transitory impacts on velocity growth held its ground in the face of repeated “regression attacks”.

In retrospect it appears that in two significant respects the macroeconomics profession has largely surrendered and accepted the perspective of the AJ equation. First, velocity has been rehabilitated as a useful theoretical device across a broad range of macroeconomic thought. Monetarists have steadfastly maintained the usefulness of this concept. Two of Greg Mankiw’s (1991) “dubious Keynesian propositions” speak directly to the points raised in the AJ equation: Point No. 2—

“[The lessons of classical economics are not helpful in understanding how the world works]; and Point No. 4—[Fiscal policy is a powerful tool for economic stabilization and monetary policy is not very important].” Mankiw further asserts “for purposes of analyzing economic policy, a student would be better equipped with the quantity theory of money (together with the expectations augmented Phillips curve) than with the Keynesian cross.” Some new Keynesians may repudiate Mankiw, since this statement could be paraphrased that a student would be better equipped with the AJ equation (together with the St. Louis model) than with the Keynesian cross. Nevertheless, a statement such as this (original or paraphrase) was heresy 25 years ago, and it can only be said of the St. Louis view of monetary analysis and monetary policy “you’ve come a long way baby.”

Most of the attention that real-business cycle theorists give to money has focused on the relationship between money and real output in the short run. Proponents of this approach generally dismiss any causal effect from money to real output, arguing that correlations between changes in money and changes in real output reflect feedbacks from real output onto an endogenous money stock. This is not a denial of all significant parts of the St. Louis position. Plosser (1991), for example, argues that “money, without question, plays the dominant role in determining the rate of inflation.” Presumably money then also has important impacts on the path of nominal income, though real shocks are also important from this perspective. Real-business cycle specifications have recently expanded to include inflation and nominal variables. At least some of these expanded specifications incorporate a traditional demand-for-real-balances function, with point estimates of long-run income elasticities that are fairly close to unity. Thus these models do not reject the usefulness of velocity.

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*This interpretation of the AJ equation was not widely recognized at the time of publication, I suspect in part because the original specification was published in first differences rather than log differences and also because the specification was never was presented as a hypothesis about velocity. The original presentation was intended as a sequel to the Friedman-Meiselman debate. See Jordan (1986).

*See, for example, deLeeuw and Kalchbrenner (1969), Corrigan (1970) and Davis (1969).

*For example, Benjamin Friedman (1977) argued that the original Andersen-Jordan conclusion with respect to fiscal policy was sample specific. However, the permanent effect of money growth on velocity is robust to his changes in sample periods.

*See Anderson and Carlson (1970) for a discussion of the St. Louis model.

*That the St. Louis view is still contested in discussions of public policy is evidenced by the report of March 31, 1992, that 100 economists, including six Nobel Memorial Prize laureates, sent in an open letter to President Bush, Chairman Greenspan and members of Congress calling for additional government spending, lower interest rates and tax credits for business investment to stimulate economic growth (“Top Economists Urge Officials to Boost Federal Spending to Stimulate Growth”, Wall Street Journal, March 31, 1992, p. A2).

as a long-run concept relating money to nominal income.

The second aspect of the evolution of macroeconomic thought toward the AJ equation involves the modeling of shocks to velocity. The AJ equation was consistently estimated in differenced form, and thus the implicit assumption of the specification is that shocks to the level of velocity are permanent. At the time this analysis was constructed, the discussion of the role of permanent and transitory shocks that is so prominent in recent analyses was unforeseen. Nevertheless, there is vindication for the St. Louis modeling approach in the now conventional wisdom that many macroeconomic time series (including velocity) appear to be "difference stationary" and that there are serious problems of "spurious regressions" in estimations involving levels of such data series. 10

The conclusion from this discussion is that from current theoretical and econometric perspectives there are important ways in which the original St. Louis analyses "got things right." Nevertheless, the AJ equation has disappeared from contemporary discussions of monetary policy. 11 Why then the demise of the AJ equation?

THE DEMISE OF THE AJ EQUATION: ANALYSIS AND SOME IMPLICATIONS FOR MONETARIST POLICY PRESCRIPTIONS

The demise of the AJ equation is well illustrated in figure 1. Two different measures of velocity are plotted there. The first is the conventional ratio of nominal GNP to M1. The second is the ratio of nominal GNP to a geometric moving average of M1, where the weights in the moving average approximate the weights in the lag polynomial of the log of differences in money in the AJ equation. 12 It is clear that the velocity measure implicit in the AJ equation replicates the behavior of the traditional M1 velocity quite closely, both before and after 1980. Both measures have a strong positive deterministic trend that ends in the early 1980s. This trend was captured in the AJ equation by a significant positive intercept on the order of 2.5 percent to 3.0 percent per year. With the break in the trend in velocity in the 1980s, it is clear that the AJ equation falls apart.

In Rasche (1987) I showed that essentially all narrowly defined monetary aggregate velocities in the United States exhibit similar breaks in their deterministic trends in the early 1980s but that once these breaks are considered, the time series properties of the various velocities are not substantially different in the 1980s compared with the earlier period (see figure 2). 13 Thus to understand the demise of the AJ equation, it is crucial to understand the origins of the trend in velocity.

A considerable number and variety of explanations have been advanced for the change in velocity behavior observed in the 1980s, but most of these are not consistent with the patterns observed in the data. 14 Monetarism in general, and the AJ equation in particular, is based on the proposition that a stable long-run demand function for money exists; that is, the demand for real balances depends on relatively few variables, including real income, or wealth, and various rates of return on nonmoney assets.


11 Relatively few attempts to reestimate the St. Louis equation have occurred in recent years. Batten and Thornton (1993) extend the sample period through third quarter 1992. Belongia and Chalfant (1989) estimate regressions using M1A, M1 and Divisia variants of both those aggregates (including variables for relative energy prices and strike dummies, but excluding fiscal policy variables) over a first quarter 1970–third quarter 1987 sample. With the exception of Divisia M1A they find money growth elasticities that are significantly less than 1.0. They conclude that the AJ equation falls apart.


13 The weights are taken from Appendix Table 2 in Carlson (1982) as .40, .40 and .20 on lnM1, lnM1-1 and lnM1-2 respectively.

14 These conclusions are not altered by updated data. Over the sample period first quarter 1948–fourth quarter 1981 the mean change in velocity (St. Louis velocity) is 3.45 (3.46) percent per year, and the standard deviation is 4.74 (4.74) percent per year. The mean for the first quarter 1982–third quarter 1990 is –.72 (-.72) percent per year and the standard deviation is 6.04 (5.75) per cent per year. The mean change in the second sample is not significantly different from zero [p = .49 (.46)]. In Rasche (1990) I concluded that the velocities of the broadly defined monetary aggregates M2 and M3 showed little if any changes in trends at this time.

15 See Rasche (1987).
Figure 1
Velocity Measures First Quarter 1948 through Third Quarter 1990

Figure 2
Growth Rate of St. Louis Equation Velocity Measures First Quarter 1948 through Third Quarter 1990
The theory relates the level of real balances demanded to the level of specific variables. However, the AJ equation, proposed as a reduced form of a model containing such a money-demand specification, is estimated in difference form. Such statistical methodology is correct in that it properly adjusts for the apparent non-stationarities of the observed data series. Unfortunately, differencing data series maintains only short-run relationships among the various series and overlooks any long-run relationships that may exist simultaneously.

In the last decade, particularly in the past five years, innovations in econometric technique allow for the simultaneous treatment of nonstationary data and estimation of long-run relationships among the levels of variables. These techniques, namely cointegration analyses, maintain the spirit of the reduced form approach in differences of the data, but permit the analysis to incorporate the specification of long-run relationships among the levels of the variables, if such relationships exist. If identifying restrictions are satisfied, such a relationship can be interpreted as the long-run money demand function that is fundamental to the AJ analysis.16

Some studies have documented the existence of such a cointegrating relationship among real balances, real income and nominal interest rates. The implied long-run income elasticity of money demand in such estimated equations is not significantly different from unity; hence there is a long-run stationary relationship between the level of velocity and the level of nominal interest rates.

What then of the changes in the mean growth rate of velocity in the 1980s relative to the mean growth rate in previous decades? If a stable long-run money demand equation that relates the level of velocity to the level of nominal interest rates exists and if the deterministic trend (drift) in nominal interest rates changes, then the drift in velocity must change correspondingly to accommodate the stable money demand specification. Hence a reduced form in differences of velocity such as the AJ equation, given a stable money demand function, implies an unchanged constant only as long as there are no significant changes in interest rate trends. Since during the 1980s there is a complete break from the upward trend of nominal rates of the previous two decades, the break in velocity drift is completely consistent with stability of the money demand function.

Although the velocity break of the 1980s does not invalidate the theoretical propositions on which the AJ equation is based, it suggests that some rethinking of traditional monetarist policy prescriptions is in order. What forces are likely to generate breaks in interest rate trends? A plausible candidate, and the one of most concern for monetary policy prescriptions, is inflation expectations. Assume that there is an established initial regime in which expected inflation has a positive trend. Assume that the monetary authorities take successful actions to stabilize the inflation rate and that this regime change is reflected in the expectation of future inflation at some constant rate. The likely outcome of such a policy shift is that the drift in nominal interest rates will disappear as will the drift in velocity.19

This suggests that the time series properties of velocity and the constants in reduced form equations specified in differences are dependent on specific monetary policy regimes through expected inflation trends specific to the policy regimes. If true, this stands as one of the few clear-cut examples of a “Lucas effect” beyond the original Phillips curve example.20

One of the consequences of such a “Lucas effect” is that straightforward application of no-feedback monetary growth rules for narrowly defined monetary aggregates can lead to outcomes different from those predicted or desired.21 A monetary authority that desires to stabilize an inflation that has been drifting upward might be inclined to set a monetary growth objective equal to a projected growth rate for natural output plus a desired stabilized inflation rate, minus the historically observed drift in the

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19 See Lucas (1976).
20 In Milton Friedman’s defense it must be noted that he originally proposed a no-feedback rule in terms of a more broadly defined aggregate, old M2. An aggregate such as new M2, in a regime without interest rate ceilings, is unlikely to suffer from the problem discussed here. For some evidence on the stationarity of new M2 velocity over the post-Accord period, see Hallman, Porter and Small (1991).
velocity of a narrowly defined monetary aggregate. If the authority maintains this money growth rate after expected inflation has stabilized, under the above "Lucas effect" the drift in velocity will have disappeared and the actual steady rate of inflation will prove to be lower than the planned inflation rate. During the transition period to the steady inflation regime, the drift in velocity will be slowing and hence the growth of nominal income will drop below the planned inflation rate plus the projected growth rate of natural output. If the aim of the monetary authorities is to reduce, as well as to stabilize inflation and if actual and expected inflation adjust to the change in monetary policy slowly so that \( p > p^* \) while the drift in velocity is in transition, then real output growth will fall below \( q^* \) for some time during the transition period.\(^{22}\)

Meltzer [1987] and McCallum [1988] propose alternatives to a fixed money (base) growth rule that allow feedback from velocity to the planned growth in money (base). The rules are designed to account for permanent shocks to velocity, but not to respond to transitory velocity shocks. The rules set the growth rate of the monetary base equal to a desired growth of nominal income \((p^* + q^* \text{ in the above notation})\) less a moving average of the drift in base velocity.\(^{23}\) The rules establish base growth consistent with the planned stable inflation once stabilization is achieved, and the rules also adjust base growth to compensate for the declining velocity drift during the transition period to the stabilized inflation rate. Thus on the surface it appears that these feedback rules immunize monetary policy from the adverse consequences of the "Lucas effect" on velocity drift.

However, this conclusion depends critically on the credibility of the monetary authority. As long as private agents believe that the monetary authority is following the feedback rule consistently, inflation expectations should adjust either in anticipation of or with the observation over time of falling inflation. The feedback mechanism will adjust base growth as desired. Both the Meltzer and McCallum rules are deterministic. In practice, stochastic fluctuations around such deterministic rules will be observed which may make direct verification of the rule difficult. If the monetary authority lacks credibility, feedback rules such as these could prove unstable. Suppose the rule is implemented by the monetary authority and inflation and inflation expectations begin to stabilize. This lowers the drift in velocity, and the feedback rule calls for base growth to be adjusted upward (see figure 3). The McCallum rule, which ultimately restores nominal income to the specified path of nominal potential income, requires that base growth and nominal income growth overshoot equilibrium base growth during the transition period (see figures 3 and 4). If private agents do not understand the rule well, or if the increase in base and nominal income growth is interpreted by such agents as an abandonment of the rule, then inflation expectations could start adjusting upward. This would change the drift of velocity, and the rule would then call for reductions in base growth. It is not difficult to conceive of a situation where the monetary authority lacks credibility, in which the Meltzer-McCallum rules suffer from instrument instability (Holbrook [1972]) if the observed behavior of the monetary base affects inflation expectations, and through this the drift in base velocity.\(^{24}\)

The conclusions from these observations on the reduced form behavior of velocity is that constant growth rules applied to narrowly defined monetary aggregates are unlikely to be successful in stabilizing a nonzero inflation trend. The success of feedback rules that depend on observed velocity behavior can depend critically on the credibility of the monetary authority. In the

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\(^{22}\)Set \( m_t = p^* + q^* - v \), where \( m_t \) is the maintained growth rate of the nominal money stock, \( p^* \) is the planned steady inflation rate, \( q^* \) is the projected growth rate of natural output and \( v \) is the historically observed drift of velocity. Then during a transition period \((p_t + q_t) = (m_t + v_t) = (p^* + q^*) + (v_t - v)\). When the drift in velocity starts to react to the change in expected inflation, \((v_t - v) < 0 \) so \((p_t + q_t) < (p^* + q^*)\).

\(^{23}\)McCallum’s rule provides an additional adjustment to base growth as nominal output is observed to deviate from nominal natural output.

\(^{24}\)See Holbrook (1972). Consider, for example, a feedback rule of the form: \( b_t = \theta(L)v_t + LX_t + \epsilon_t \), where \( b_t \) is the growth rate of base velocity, \( X_t \) is other factors to which the feedback rule responds, and \( \epsilon_t \) are random fluctuations generated by fluctuations of sources of monetary base outside the control of the monetary authorities and that cannot be perfectly forecasted. Let inflation expectations respond to observed base growth \( p_t^* = \delta(L)b_t \). Finally, let velocity growth respond to trends in inflation expectations: \( v_t = \omega(L)p_t^* \). Substituting the latter two equations into the first equation gives \( 1 - \theta(L)\omega(L)\delta(L) = 0 \). Invertibility of the polynomial \( 1 - \theta(L)\omega(L)\delta(L) \), and hence the absence of instrument instability depends upon the expectation formation mechanism, \( \delta(L) \).
Figure 3
McCallum Rule: Nominal Income Growth

Percent

0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0

1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97

DYF

Figure 4
McCallum Rule: Base and Base Velocity Growth

Percent

0.01
0.0075
0.005
0.0025
0

1 7 13 19 25 31 37 43 49 55 61 67 73 79 85 91 97

DBF

DVF

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absence of credibility, the adjustments to the growth of the aggregate required by the feedback rule can provoke adjustments to inflation expectations that introduce instrument instability into the feedback rule.

**CAN THE TRANSITORY RESPONSE OF REAL OUTPUT TO MAINTAINED CHANGES IN MONEY GROWTH BE INFERRED FROM REDUCED-FORM MODELS?**

**The Role of Identifying Restrictions**

The focus of much of the recent discussion of the role of money and monetary policy is not on the response of nominal income, but rather on the response of real output. Cagan (1989) summarizes a large body of recent empirical research and reaches the conclusion that "lately ... monetary research has turned again ... and new studies claim that money has little or no effect on output and other real variables." VARs figure prominently in recent research and are the source of much of the evidence from which the negative conclusions about the impact of nominal money changes on real output are drawn. Cagan faults the VAR approach as follows: "The VAR seems ... to be hopelessly unreliable and low in power to detect monetary effects of the kind that we are looking for and believe, from other kinds of evidence, to exist." I will argue here that Cagan's skepticism about the conclusions of VAR analysis is justified, but for reasons beyond those he enumerated.

The most important aspect of VAR analysis is the one most frequently slighted in drawing conclusions about policy shocks from such analyses. VARs are reduced forms of some unspecified economic model; as such they have common roots with the AJ equation. Reduced forms, in themselves, provide no information about the impact of nominal money shocks, or any other policy shocks of interest to economists. To provide such information, VARs must be supplemented with sufficient identifying restrictions, derived from some economic model, to uniquely extract information about the impact of monetary shocks on real output within the economic structure defined by the identifying restrictions.

Sims (1986) clearly explains the critical role of identifying restrictions in VAR analysis. Sims defines the economic model as follows:

\[
(1) \sum_{s=0}^{\infty} A(s) Y(t-s) = \sum_{s=0}^{\infty} B(s) e(t-s); \quad \text{Var}(e(t)) = \Omega
\]

and the corresponding VAR (reduced-form) model for \( Y \) as follows:

\[
(2) Y(t) = \sum_{s=1}^{\infty} C(s) Y(t-s) + \eta(t); \quad \text{Var}(\eta(t)) = \Sigma
\]

Sims notes the following:

The most straightforward example of identifying restrictions on \( A(0), B(0) \) and \( \Omega \) is the Wold causal chain. According to this idea, \( \Omega \) should be diagonal, \( B(0) = I \) and \( A(0) \) should be triangular and normalized to have ones down the main diagonal when the variables are ordered according to causal priority. Using the fact that with \( B(0) = I \), \( \Sigma = A(0) \Omega A(0)' \), the triangularity of \( A(0) \) implies that, once we have put the variables in proper order, we can recover \( A(0) \) and \( \Omega \) from \( \Sigma \) as \( \Sigma \)'s unique LDL decomposition. That is, it is known that there is a unique way to express a positive definite matrix \( \Sigma \) in the form \( LDL' \), where \( L \) is lower triangular with ones down its diagonal and \( D \) is diagonal. Applying the standard LDL algorithm to \( \Sigma \) gives us \( A(0) \) as \( L \) and \( \Omega \) as \( D \). This triangular orthogonalization has become a standard practice as part of the interpretation of econometric models (emphasis added) (p. 10).

Though this set of identifying restrictions has become so common in VAR analysis that only rarely is it acknowledged explicitly, it is neither unique nor uncontroversial. Criticisms of and arguments against both the appropriateness and necessity of the causal-chain (triangular) specification are longstanding.\(^\text{26}\) A simple example of the nonuniqueness of this approach is given by the three separate sets of identifying restrictions that Sims applies to his six-variable VAR. All of these identification schemes maintain the assumption that \( \Omega \) is diagonal, but they impose different exclusion restrictions on \( A(0) \), including restrictions that do not impose a triangular structure on \( A(0) \).

Recently, attention has turned to identification by restrictions on the steady-state coefficient

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\(^{26}\)See Basmann (1963) and Leamer (1985).
matrix, $A = \sum A(s)$, rather than by restrictions on $A(0)$. This latter approach seems more promising because there appears to be considerable agreement over a broad range of macroeconomic theories on identifying restrictions that apply to a steady-state macroeconomic model. In contrast, economic theory provides little if any information about identifying restrictions on the dynamic structure of macroeconomic specifications. In particular, during the past 10 years researchers have broadly debated the identification of a short-run money-demand function, to the extent that alleged short-run money demand functions are at best problematic and at worst fall into a class of "incredible" identifying restrictions.

If identification of a short-run money demand function is "incredible," then any "shocks" extracted from VARs under these restrictions will at best represent linear combinations of money-demand and money-supply shocks.

Under these conditions it is impossible to separate the impact effects of money on output from the reaction of money to output through whatever reaction function characterizes the behavior of the monetary authorities.

**The Importance of Specification and Identifying Assumptions**

The questions discussed previously are particularly important in the discussion of the effect of nominal money shocks on real output. To illustrate this, consider a four-variable VAR, that includes real output, inflation, nominal money ($M_1$) and a short-term nominal interest rate (Treasury bill rate). The general conclusion that has emerged from the study of such VARs is that "most of the dynamic interactions among the key variables can best be explained as arising from an economic structure in which monetary phenomena do not affect real variables. Thus ... monetary instability has not played an

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27See Bernanke (1986); Blanchard and Quah (1989); and King, Plosser, Stock and Watson (1991).
28See Hoffman and Rasche (1991c) for an illustration of how the restrictions on the KPSW (1991) common trends model are consistent with the identifying restrictions for the steady-state of a standard textbook macroeconomic model.
29These VARs are in the form of Sims (1980) and Litterman and Weiss (1985).

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<td>(\ln(M_{t-3}))</td>
<td>-0.4082</td>
<td>-1.78</td>
<td>15.3508</td>
<td>0.40</td>
<td>-0.1368</td>
<td>-0.87</td>
<td>17.7711</td>
<td>0.93</td>
</tr>
<tr>
<td>(\ln(M_{t-4}))</td>
<td>0.0857</td>
<td>0.84</td>
<td>-7.5296</td>
<td>-0.30</td>
<td>-0.0316</td>
<td>-0.35</td>
<td>-9.6530</td>
<td>-0.89</td>
</tr>
<tr>
<td>(RTB_{t-1})</td>
<td>-0.0004</td>
<td>-0.31</td>
<td>0.4763</td>
<td>2.51</td>
<td>-0.0056</td>
<td>7.19</td>
<td>0.0099</td>
<td>10.45</td>
</tr>
<tr>
<td>(RTB_{t-2})</td>
<td>0.0031</td>
<td>1.83</td>
<td>-0.3581</td>
<td>-1.27</td>
<td>0.0057</td>
<td>6.88</td>
<td>0.6275</td>
<td>4.47</td>
</tr>
<tr>
<td>(RTB_{t-3})</td>
<td>0.0034</td>
<td>1.84</td>
<td>0.1920</td>
<td>0.42</td>
<td>0.0005</td>
<td>0.37</td>
<td>0.6282</td>
<td>3.42</td>
</tr>
<tr>
<td>(RTB_{t-4})</td>
<td>-0.0018</td>
<td>-1.24</td>
<td>0.0692</td>
<td>0.25</td>
<td>0.0015</td>
<td>1.47</td>
<td>-0.1952</td>
<td>-1.63</td>
</tr>
<tr>
<td>(\text{CONSTANT})</td>
<td>-0.0961</td>
<td>-1.03</td>
<td>0.5120</td>
<td>0.03</td>
<td>-0.0696</td>
<td>-1.10</td>
<td>-9.3111</td>
<td>-1.22</td>
</tr>
<tr>
<td>(D67)</td>
<td>-0.0031</td>
<td>-0.74</td>
<td>1.4683</td>
<td>2.06</td>
<td>0.0019</td>
<td>0.65</td>
<td>-0.1573</td>
<td>-0.44</td>
</tr>
<tr>
<td>(D79)</td>
<td>0.0027</td>
<td>0.39</td>
<td>-1.4610</td>
<td>-1.25</td>
<td>-0.0024</td>
<td>-0.50</td>
<td>1.8046</td>
<td>3.10</td>
</tr>
<tr>
<td>(D82)</td>
<td>-0.0010</td>
<td>-0.14</td>
<td>-3.8349</td>
<td>-3.18</td>
<td>-0.0111</td>
<td>-2.24</td>
<td>-0.1683</td>
<td>-0.28</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.9900</td>
<td></td>
<td>0.6800</td>
<td></td>
<td>0.9900</td>
<td></td>
<td>0.9300</td>
<td></td>
</tr>
<tr>
<td>SEE</td>
<td>0.0090</td>
<td></td>
<td>1.4900</td>
<td></td>
<td>0.0062</td>
<td></td>
<td>0.7440</td>
<td></td>
</tr>
</tbody>
</table>

Important role in generating fluctuations.\(^{31}\)

Estimates of this four-variable VAR are shown in table 1 and table 2, for sample periods that begin in second quarter 1955 and end in fourth quarter 1981 and third quarter 1990, respectively. The starting point for both samples is chosen to avoid the pre-Accord data. The first sample ends before the apparent break in the trend of M1 velocity discussed previously. The second sample includes the 1980s. The VAR is supplemented with three dummy variables chosen to define roughly four inflation regimes with different trends.\(^{32}\)

The implications of these VARs for the response of real output to “money shocks” identified by the Wold causal chain structure with variables ordered as real output, inflation, money and interest rates are quite sensitive to the choice of the sample period (figure 5). Closer examination reveals that this is associated with dramatically different long-run responses of the nominal money stock to the “money shock” (figure 6). Both samples show the real output response to the “money shock” rises to a peak and then trails off. However, the nominal interest rate exhibits a transitory positive response to the “money shock” in both samples which is difficult to reconcile with the identification of the “money shock” as a monetary policy action (figure 7).\(^{33}\)

Two other variables of interest are implicit in the VAR menu: real money balances and velocity. The impulse response function for velocity to a “money shock” is shown in figure 8. The implicit velocity response is almost uniformly negative in both sample periods, and in the third quarter 1990 sample has the peculiar characteristic of having a response below -1.0 even after 40 periods.

The realization that the four-variable VAR

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\(^{31}\)See Litterman and Weiss (1985).

\(^{32}\)The dummy variables are as follows: \(D67 = 1.0\) for 67:4 and subsequent observations; \(D79 = 1.0\) for 79:3 and subsequent observations; and \(D82 = 1.0\) for first quarter 1982 and subsequent observations.

\(^{33}\)It is also difficult to reconcile the “interest rate shock” identified by the Wold causal chain specification with a monetary policy action because although the immediate impact of such a shock on interest rates is positive, the permanent effects of this shock on nominal rates, inflation, money and real output are all negative in both sample periods.
Figure 5
Real Output irf to Nominal Money Shock

Figure 6
Nominal Money irf to Nominal Money Shock
Figure 7
Inflation if to Nominal Money Shock

Figure 8
Velocity if to Money Growth Shock

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defines additional interesting economic measures as linear combinations of the menu entries raises the following question: Are the results invariant to the explicit choice of menu entries? Clearly if the degree of differencing of the variables in the VAR were the same, the OLS estimates would produce the same results regardless of the particular linear combinations explicitly chosen. However, the degree of differencing varies among the variables in the typical VAR study as log levels of real output and nominal money appear along with log differences of the price level (inflation). An alternative menu is to enter real balances along with either inflation or nominal money growth. The advantage of these choices is that the three variables that are traditionally included in money-demand specifications—real balances, real output and nominal interest rates—now explicitly appear in the VAR.4

In table 3, some results are reported from the estimation of a VAR with real output, inflation, real money balances and the Treasury bill rate. These results indicate the tests for stationary linear combinations (cointegrating vectors) among the four variables using the Johansen maximum likelihood estimator under the restriction that the log of real balances and the log of real output enter any such cointegrating vectors with equal and opposite signs.5 Both of the likelihood ratio tests—the trace test and the maximum eigenvalue test—typically reject the hypothesis of one or fewer cointegrating vectors at the 5 percent level, and in some samples at the 1 percent level. In every case the tests fail to reject the hypothesis that two or fewer cointegrating vectors exist. Thus we conclude that among these four variables there are two permanent and two transitory shocks.

To obtain a unique (to a scalar multiple) economic interpretation of the two cointegrating

---

Table 3
Johansen Maximum Likelihood Estimation of Four Variable VECM
ln(M/P), 400•ΔlnP, lnQ, Rtb
(Real Balances and Real Income Coefficients Constrained)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Trace Test</th>
<th>Max Test</th>
<th>Normalized Cointegrating Vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r = 0</td>
<td>r = 1</td>
<td>r = 2</td>
</tr>
<tr>
<td>II/1956—IV/1975</td>
<td>52.1</td>
<td>21.7</td>
<td>.22</td>
</tr>
<tr>
<td>II/1956—IV/1976</td>
<td>52.7</td>
<td>20.3</td>
<td>.24</td>
</tr>
<tr>
<td>II/1955—III/1979</td>
<td>54.7</td>
<td>20.5</td>
<td>.31</td>
</tr>
<tr>
<td>II/1955—IV/1981</td>
<td>67.1</td>
<td>17.2</td>
<td>.77</td>
</tr>
<tr>
<td>II/1955— III/1990</td>
<td>64.8</td>
<td>22.6</td>
<td>.46</td>
</tr>
</tbody>
</table>

Critical values from Osterwald-Lenum (1990)

<table>
<thead>
<tr>
<th>Percent</th>
<th>r = 0</th>
<th>r = 1</th>
<th>r = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>26.7</td>
<td>15.7</td>
<td>6.5</td>
</tr>
<tr>
<td>5</td>
<td>31.5</td>
<td>18.0</td>
<td>8.2</td>
</tr>
<tr>
<td>1</td>
<td>37.2</td>
<td>23.5</td>
<td>11.7</td>
</tr>
</tbody>
</table>

---

4 Such a VAR is an expanded version of the VAR used by Hoffman and Rasche [1992] to investigate long-run money demand.
5 See Johansen (1988 and 1991). This restriction was imposed because it was never rejected in the three variable menus investigated by Hoffman and Rasche (1992) and because in that study the unrestricted long-run income and interest elasticities were found to be quite imprecise and sensitive to the choice of the sample period.
vectors present among these four variables, identifying restrictions must be imposed on the estimated matrix of cointegration vectors. In this case the exclusion of one variable from each cointegrating vector is sufficient to achieve identification. The exclusion restrictions introduced here eliminate the inflation rate from one cointegrating vector and real balances from the other. The resulting identified cointegrating vectors, normalized for real balances and inflation respectively, are reported as \( \beta_e \) in table 3. The remaining unconstrained coefficients in these matrices are quite stable across sample periods. The estimated interest rate coefficient in the cointegrating vector with real balances is close to the estimate that Hoffman and Rasche obtained for the long-run interest semielasticity of money demand in the United States. The estimated interest rate coefficient in the cointegrating vector with the inflation rate ranges from -0.9 to -0.7 and is not significantly different from -1.0 consistent with a long-run Fisher effect, which implies a stationary real interest rate.

The difficulty in interpreting results from this specification of the VAR is that nominal money or its growth rate does not appear explicitly among the variables in the VAR. An alternative specification is to replace the inflation rate with the growth rate of nominal money and allow the inflation rate to be determined implicitly by the identity relating nominal money growth and real balances to inflation. Some results from the estimation of this VAR are presented in table 4 using the same sample periods as in table 1 and table 2. These results are basically the same as those in table 3. The Johansen likelihood ratio tests again reject the hypotheses that one or fewer cointegrating vectors exist. When the identifying exclusion restrictions and normalization are applied to the two estimated cointegrating vectors (\( \beta_e \)), the interest semielasticity in the velocity vector is approximately 0.11 and the interest coefficient in the vector error with the money growth rate is between -0.8 and -0.9. The latter estimates are not significantly different from -1.0 on the basis of a Wald test.

---

**Table 4**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Trace Test</th>
<th>Max λ Test</th>
<th>Normalized Cointegrating Vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r=0</td>
<td>r&lt;1</td>
<td>r=0</td>
</tr>
<tr>
<td>II/1955—IV/1981</td>
<td>65.2</td>
<td>20.3</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>44.9</td>
<td>18.8</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>0.0</td>
<td>-1.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>-0.8867(22)</td>
<td>-0.8867(22)</td>
<td></td>
</tr>
</tbody>
</table>

Wald Test of Overidentifying Restriction \( \beta_{(2,1)} = -1.0 \chi^2(1) = 1.54 p = .21 \)

Wald Test of Overidentifying Restriction \( \beta_{(2,1)} = -1.0 \chi^2(1) = .27 p = .60 \)

Estimates of Restricted Cointegration Vectors

<table>
<thead>
<tr>
<th>Sample</th>
<th>1.0</th>
<th>0.0</th>
<th>-1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>II/1955—IV/1981</td>
<td>0.1217</td>
<td>0.0000</td>
<td>-1.0000</td>
</tr>
<tr>
<td>II/1955—IV/1990</td>
<td>0.1202</td>
<td>0.0000</td>
<td>-1.0000</td>
</tr>
</tbody>
</table>

---

Table 5  

<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>$\Delta \ln (M_{P1})$</th>
<th>1</th>
<th>$400 \Delta \ln (M_{P2})$</th>
<th>t</th>
<th>$\Delta \ln (Q_{1})$</th>
<th>t</th>
<th>$\Delta RBT_{2}$</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \ln (M_{P1})_{t-1}$</td>
<td>0.3418</td>
<td>2.07</td>
<td>14.87</td>
<td>0.78</td>
<td>0.1106</td>
<td>0.52</td>
<td>-41.32</td>
<td>-2.46</td>
</tr>
<tr>
<td>$\Delta \ln (M_{P1})_{t-2}$</td>
<td>0.3257</td>
<td>1.98</td>
<td>81.18</td>
<td>1.40</td>
<td>0.1793</td>
<td>0.84</td>
<td>-28.47</td>
<td>-1.68</td>
</tr>
<tr>
<td>$\Delta \ln (M_{P1})_{t-3}$</td>
<td>0.2738</td>
<td>1.62</td>
<td>81.06</td>
<td>1.37</td>
<td>0.1302</td>
<td>0.60</td>
<td>-6.98</td>
<td>-0.41</td>
</tr>
<tr>
<td>$\Delta \ln (M_{P2})_{t-1}$</td>
<td>0.0000</td>
<td>0.006</td>
<td>-0.75</td>
<td>-4.56</td>
<td>0.0004</td>
<td>-0.60</td>
<td>0.1166</td>
<td>2.44</td>
</tr>
<tr>
<td>$\Delta \ln (M_{P2})_{t-2}$</td>
<td>0.0004</td>
<td>-0.63</td>
<td>-0.79</td>
<td>-3.85</td>
<td>0.0001</td>
<td>-0.17</td>
<td>0.2007</td>
<td>3.35</td>
</tr>
<tr>
<td>$\Delta \ln (M_{P2})_{t-3}$</td>
<td>0.0011</td>
<td>-1.61</td>
<td>-0.99</td>
<td>-4.32</td>
<td>0.0007</td>
<td>-0.81</td>
<td>0.2795</td>
<td>4.18</td>
</tr>
<tr>
<td>$\Delta \ln (Q_{1})_{t-1}$</td>
<td>0.1312</td>
<td>0.18</td>
<td>-3.73</td>
<td>-0.14</td>
<td>0.0926</td>
<td>0.98</td>
<td>8.22</td>
<td>1.09</td>
</tr>
<tr>
<td>$\Delta \ln (Q_{1})_{t-2}$</td>
<td>0.0903</td>
<td>0.22</td>
<td>19.67</td>
<td>0.76</td>
<td>0.0598</td>
<td>0.63</td>
<td>22.31</td>
<td>2.97</td>
</tr>
<tr>
<td>$\Delta \ln (Q_{1})_{t-3}$</td>
<td>-0.0473</td>
<td>-0.64</td>
<td>-16.49</td>
<td>-0.64</td>
<td>0.1452</td>
<td>-1.54</td>
<td>0.2366</td>
<td>0.32</td>
</tr>
<tr>
<td>$\Delta RBT_{1}$</td>
<td>-0.0066</td>
<td>-7.55</td>
<td>-2.13</td>
<td>-6.93</td>
<td>0.0001</td>
<td>-0.11</td>
<td>0.0837</td>
<td>0.94</td>
</tr>
<tr>
<td>$\Delta RBT_{2}$</td>
<td>-0.0005</td>
<td>-0.49</td>
<td>-0.15</td>
<td>-0.43</td>
<td>0.0037</td>
<td>-2.91</td>
<td>-0.8973</td>
<td>-6.73</td>
</tr>
<tr>
<td>$\Delta RBT_{3}$</td>
<td>-0.0010</td>
<td>-0.87</td>
<td>-0.20</td>
<td>-0.50</td>
<td>0.0003</td>
<td>-0.19</td>
<td>0.1120</td>
<td>0.94</td>
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<tr>
<td>CONSTANT</td>
<td>-0.0118</td>
<td>-1.97</td>
<td>-8.99</td>
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<td>0.0014</td>
<td>-2.10</td>
<td>-0.3980</td>
<td>-0.99</td>
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<tr>
<td>D7</td>
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<td>0.26</td>
<td>1.06</td>
<td>1.91</td>
<td>0.0014</td>
<td>-0.67</td>
<td>-0.1429</td>
<td>-0.86</td>
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<tr>
<td>D70</td>
<td>0.0209</td>
<td>0.60</td>
<td>0.0116</td>
<td>0.01</td>
<td>0.0022</td>
<td>0.51</td>
<td>0.1394</td>
<td>3.25</td>
</tr>
<tr>
<td>D82</td>
<td>-0.0039</td>
<td>-1.04</td>
<td>-3.62</td>
<td>-2.77</td>
<td>0.0055</td>
<td>-1.14</td>
<td>-0.3777</td>
<td>-1.00</td>
</tr>
<tr>
<td>CIV1</td>
<td>-0.0121</td>
<td>-1.96</td>
<td>-0.03</td>
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<td>0.0258</td>
<td>-3.27</td>
<td>-0.5504</td>
<td>-0.88</td>
</tr>
<tr>
<td>CIV2</td>
<td>-0.0012</td>
<td>-1.81</td>
<td>-1.05</td>
<td>-4.62</td>
<td>0.0006</td>
<td>-0.93</td>
<td>0.1792</td>
<td>2.71</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.6100</td>
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<td>0.4100</td>
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<tr>
<td>SEE</td>
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<td>2.38</td>
<td>0.0087</td>
<td>0.6914</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: CIV1 and CIV2 are the two stationary linear combinations of the four dependent variables.

The vector error correction model (VECM) in table 4 can be reestimated with the overidentifying restriction $\beta_{CIV} = 1.0$ imposed. The constrained estimates of $\beta_{CIV}$ are obtained using the two-step estimator in Rothenberg and the asymptotic covariance matrix for $\beta_{CIV}$ derived by Johansen.\textsuperscript{40} The restricted estimates of $\beta_{CIV}$ are given at the bottom of table 4. These estimates are used to construct two linear combinations of the levels of the four different variables to obtain estimates of the remaining parameters of the restricted VECM. The estimated coefficients of the restricted VECM are shown in table 5 for the II/1955–III/1990 sample.\textsuperscript{41}

The interesting question that these results raise is: Can the two permanent shocks among these four variables be associated with individual variables? Or in the terminology of King, Plosser, Stock and Watson (KPSW) (1991): Can we derive a structural model from the reduced-form model with steady-state characteristics suggested by economic theory? The interesting hypotheses to test are as follows:

- One permanent shock corresponds to a real-output (productivity) shock as suggested by real-business cycle theories; and
- The second permanent shock corresponds to a money growth–inflation–nominal interest rate shock consistent with a broad spectrum of macroeconomic theories.

The common-trends modeling approach of KPSW identifies the permanent components of each time series by restricting them to be random walks. A common-trends model exists if

\textsuperscript{40}Rothenberg (1973) proves that his two-step estimator is a restricted maximum likelihood estimator when the unrestricted estimator is asymptotically normal and converges at rate $T^{-1}$. Johansen (1991) shows that his estimator of $\beta_{CIV}$ is asymptotically normal, but converges at rate $T^{-1}$. The maximum likelihood properties of the restricted estimator have not been established for this case.

\textsuperscript{41}The dummy variables are not important for the estimation of the cointegrating vector (CIV) involving real balances. The separation of the shift in the constant of the VECM into components representing shifts in the deterministic trend and shifts in the mean of this cointegrating vector indicates that the mean of the CIV is little changed in the 80s compared with the previous 25 years. (See Yoshida and Rasche [1990].) The dummy variables are important for the estimation of the second (real interest rate) cointegrating vector. They suggest a large increase in the mean real interest rate during fourth quarter 1979–fourth quarter 1981 followed by a substantial, though not fully offsetting reduction in the mean real rate after 1981. This is consistent with the work of Clarida and Friedman (1984), Huizinga and Mishkin (1986) and Roley (1986) all of whom found shifts in the relationship of nominal rates and inflation in 1979 and 1982.

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the permanent components of each time series are equal to linear combinations of the orthogonal permanent shocks that are suggested by economic theory. In the case under consideration here, the existence of the hypothesized common-trends model requires that the permanent components of real output and money growth are equal to the two permanent shocks and hence are orthogonal. These correlations are 0.047 and -0.065 for the samples ending in fourth quarter 1981 and third quarter 1990, respectively. The extent that the permanent components of real output and money growth violate the necessary conditions for the existence of a common-trends model can be judged by the size of the off-diagonal element of the $\Pi$ matrix as defined by KPSW. In the sample ending fourth quarter 1981 the estimated restricted VECM implies that $\Pi_{21} = 0.107$ and in the sample ending third quarter 1990 the estimated restricted VECM implies that $\Pi_{21} = -0.007$ under the identifying restrictions that the permanent components are random walks. Because the absolute values of these estimates are both close to zero, we conclude that the data are consistent with a common-trends representation with independent, permanent real-output and permanent money-growth shocks.

KPSW (1991) show how impulse response functions are constructed for permanent shocks in such a common-trends model. Graphs of these impulse response functions are shown in figures 5–18. The long-run properties of these impulse response functions are completely determined by the cointegrating vectors and the near orthogonality of the permanent components of real output and money growth. The long-run responses of velocity, inflation, money growth and nominal interest rates (figures 14, 16, 17 and 18) to a permanent shock to real output are all identically equal to zero. This follows from the orthogonalization of the common trends when real output is ordered before money growth. The long-run responses of real output to a permanent money-growth shock are not identically zero (figure 10), reflecting the small correlations between the permanent components of real output and money growth. The long-run responses of inflation and nominal interest rates to a permanent money-growth shock (figures 11 and 13) are identically equal to 1.0 as determined by the values of the estimated coefficients in the cointegrating vectors. In the long run, the level of velocity is increased slightly by the permanent increase in money growth in response to the permanently higher value of nominal rates (figure 8). The long-run responses are consistent with the steady-state properties of most macroeconomic models, but this is not "news" once the elements of the cointegrating vectors have been estimated.

Additional interesting information can be found in these figures. Estimates for both samples suggest that the transitory responses to either permanent shock die out after two to three years. These implied lags in the adjustment to the steady state seem quite short relative to much of the conventional wisdom, though the length of the transitory reaction of velocity to a permanent money-growth shock is surprisingly similar to that in the AJ equation.

The reactions to a real-output shock are not exactly those implied by a pure real-business cycle model because output effects from this type of shock build only gradually (figure 15), during which period there are highly serially correlated negative impacts on the inflation rate (figure 16). The real output response here is quite similar to the output response to a "balanced-growth" shock obtained by KPSW in their six-variable restricted VAR model (figure 6). There is a transitory money-growth response (figure 17) associated with the output shock, but because the money measure here, M1, includes inside money, this response is consistent with the picture drawn by some real-business cycle theorists.

At first glance, it appears that the variance decomposition of real output in this model is consistent with the conclusion that "monetary instability has not played an important role in generating fluctuations." The variance decomposition of real output from the fourth quarter 1981 sample indicates that the permanent "money-growth" shock accounts for about 23 percent of the variance of real output at all forecast horizons. In contrast, the permanent "real-output" shock accounts for only 7 percent of the variance of real output at a one-period hori-

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Figure 11
Inflation irf to Money Growth Shock

Figure 12
Money Growth irf to Money Growth Shock
Figure 13
T-bill Rate irf to Money Growth Shock

Figure 14
Velocity irf to Real Output Shock
Figure 15
Real Output irf to Real Output Shock

Figure 16
Inflation irf to Real Output Shock

FEDERAL RESERVE BANK OF ST. LOUIS
Figure 17
Money Growth if to Real Output Shock

Figure 18
T-bill Rate if to Real Output Shock
zon but increases to 66 percent of the variance at a 12-period horizon. When the sample is extended through third quarter 1990, the permanent “money-growth” shock accounts for only 7 percent of the forecast variance at a one-quarter horizon, and this declines steadily to one percent of the forecast variance at a 12-quarter horizon. In this sample the permanent “real output” shock accounts for 31 percent of the forecast variance of real output at a one-quarter horizon, 66 percent at a 12-period horizon. When the sample is extended through third quarter 1990 (fourth quarter 1981). On the contrary, the opposite shock becomes a maintained change in the growth of nominal money. However this is not the case initially. For the first two to three years, the money growth response to the permanent “money-growth” shock contains a large transitory component and the net effect is frequently of the opposite sign to the permanent effect. This response pattern certainly does not conform to the traditional monetarist policy experiment. In the latter case, the policy intervention involves a shift from one maintained growth rate of money (or the monetary base) to a different maintained monetary growth rate. Under these conditions the traditional monetarist hypothesis is that the initial impact of the policy intervention will largely affect real output, but that over time this effect will disappear as the inflation rate approaches its new steady-state rate.47

The only identifying characteristic of a monetary shock in this analysis is the steady-state restriction that the impulse response of money growth to such a shock is one. However, this restriction does not define a unique monetary shock, but rather a whole class of such shocks. This is clear from the impulse responses of money growth to the two “transitory shocks” that are plotted in figures 19 and 20. By construction, in both samples the steady-state response of money growth (and all other variables defined by the VAR) is zero. Thus it is possible to define the class of monetary shocks equal to the permanent “monetary shock” plus any weighted sum of the two transitory shocks and satisfy the identifying restriction for a monetary shock. Within this class of monetary shocks it is impossible to determine the short-run impact of monetary policy on real output. For example, consider defining the response of real output as the sum of the responses to the permanent “money-growth” shock and the two transitory shocks. Such a composite shock has the identical steady-state response as the permanent “money-growth” shock plus the two transitory shocks. Such a composite shock has the identical steady-state response as the permanent “money-growth” shock and so satisfies the identifying restrictions for a permanent monetary intervention imposed by our model. Yet on a one-quarter forecasting horizon such a composite shock accounts for 69 (93) percent of the variance in real output for the sample period ending third quarter 1990 (fourth quarter 1981). On a 12-quarter horizon the fraction of the forecast

46See Appendix B.
Figure 19
Money Growth irf to Transitory Output Shock

Figure 20
Money Growth irf to Second Transitory Shock
variance in real output attributable to such a composite shock decreases to 13 (34) percent for the sample period ending in third quarter 1990 (fourth quarter 1981).

The fraction of the variance of deviations of real balances from equilibrium real balances attributable to this composite shock is 73 (76) percent at a one-quarter horizon and 75 (66) percent at a 12-quarter horizon for the sample period ending third quarter 1990 (fourth quarter 1981). The fraction of the variance of deviations of the real interest rate attributable to this composite shock is 99 (99) percent at a one-quarter horizon and 80 (73) percent at a 12-quarter horizon for the sample period ending third quarter 1990 (fourth quarter 1981).

In contrast, the monetary intervention of traditional monetarist analysis is not contained in the general class of monetary shocks defined as the permanent “monetary shock” plus a weighted sum of the transitory shocks. Consider a regression of the following form:

\[(\text{IMPMP}_t - 1.0) = \beta_1 \text{IMPMT1}_t + \beta_2 \text{IMPMT2}_t + \varepsilon_t\]

where IMPMP is the impulse response of money growth to the permanent “money shock” and IMPMT1 and IMPMT2 are the impulse responses of money growth to the transitory shocks. The traditional monetarist policy experiment is defined in the class of identified monetary shocks if there are \(\beta_s\) that produce an estimated impulse response pattern that replicates the deviations of the impulse response function to the permanent “money shock” from unity. This result does not hold for either sample period. For the sample ending fourth quarter 1981,

\[
(\text{IMPMP}_t - 1.0) = -11.39(\text{IMPMT1}_t) - 6.19(\text{IMPMT2}_t) + \varepsilon_t
\]

\[(-9.28)\]

\[R^2 = 0.81\]

\[\text{SEE} = 1.15\]

while for the sample period ending third quarter 1990,

\[
(\text{IMPMP}_t - 1.0) = -9.06(\text{IMPMT1}_t) - 0.94(\text{IMPMT2}_t) + \varepsilon_t
\]

\[(-4.31)\]

\[R^2 = 0.23\]

\[\text{SEE} = 2.47\]

The weighted-sum impulse response functions for money growth are shown in figures 21 and 22 for the two sample periods. Large transitory deviations from unity remain in both cases.

The lack of identification of the short-run real output response in the absence of a specification of the monetary rule, or monetary policy reaction function, that prevails during the sample period can be shown easily using a simple macroeconomic model that satisfies all of the steady-state identifying restrictions imposed on the VECM. Consider the following:

1. \(\ln P_t = \gamma \ln Q_t + \varepsilon_{pt}\)
2. \(\ln MR_t = \ln Q_t - \beta_t + \varepsilon_{mt}\)
3. \(\text{i}_t = r_t + \ln P_{t+1} - \ln P_t\)
4. \(\ln Q_t = k + \ln A - \alpha r_t + \varepsilon_{rt}\)

where equation (1) is an expectations-augmented Phillips curve (Lucas supply function) that relates deviations of real output \((Q_t)\) from natural output \((Q)\) to inflation expectation errors \((\ln P_t - \ln P_{t-1})\). Equation (2) is a money-demand function that relates real money balances \((\ln M_t - \ln P_t)\) to real output and nominal interest rates \((i)\) with a unitary income elasticity of money demand. Equation (3) defines nominal interest rates as the sum of the real rate \((r_t)\) and the expected future rate of inflation \((\ln P_{t+1} - \ln P_t)\). Equation (4) defines the demand for real output in terms of the real interest rate and autonomous planned expenditures \((A_i)\). This model is closed by two additional specifications. First, we assume that expectations are generated by adaptive expectations of inflation:46

\[\ln P_t = \gamma \ln P_{t-1} + \lambda (\ln P_t - \ln P_{t-1})\]

Second, a stochastic monetary rule (policy reaction function) is specified as follows:

\[\Delta \ln M_t = \mu + \phi \Delta i_t + \phi_2 (\Delta \ln M_{t-1} - \mu) + \varepsilon_{mt}\]

This rule allows for contemporaneous interest rate smoothing \((\phi_2 > 0)\) and for offsetting of past deviations from the steady-state money growth

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46Adaptive expectations in the inflation rate are chosen as an algebraically convenient way of generating a model that potentially has transitory real output responses to permanent nominal money growth shocks and has the steady-state characteristics of the estimated VECM. This is only for illustration of the identification problem. In particular, the type of inflation expectation shift discussed in the section beginning on p. 3 as the root cause of the shift in velocity drift is not consistent with an adaptive expectations mechanism.
Table 6

Adjoint Matrix (A*)

<table>
<thead>
<tr>
<th>Expression</th>
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<th>Expression</th>
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<tbody>
<tr>
<td>(-1 + \Phi_2 B)[(1 - (1 - \lambda)B + \lambda(1 - B)] + (\alpha B)]</td>
<td>(-1 + \Phi_2 B)[(1 - B)]²</td>
<td>([\Phi_2 B + \Phi_1] (1 - B)^2)</td>
<td>(\alpha (1 - B))</td>
</tr>
<tr>
<td>(-1 + \Phi_2 B)((\alpha + \beta)(1 - (1 - \lambda)B) + \alpha B)]</td>
<td>(-1 + \Phi_2 B)[(1 - (1 - \lambda)B] + (\Phi_1 [1 - \alpha \lambda - B][1 - B])</td>
<td>([-1 + \Phi_2 B] [\lambda (1 - (1 - \lambda)B] - \Phi_1 [1 - B]^2)</td>
<td>((\alpha + \beta)(1 - B) - \alpha \lambda B)</td>
</tr>
<tr>
<td>([1 + \Phi_2 B][1 + \alpha \lambda (1 - B)] + \lambda B]</td>
<td>([1 + \Phi_2 B][1 - \alpha \lambda - B][1 - B])</td>
<td>([1 + \Phi_2 B][\gamma (1 - (1 - \lambda)B) + (1 - B)^2])</td>
<td>(-[1 - \alpha \lambda - B])</td>
</tr>
<tr>
<td>(\Phi_1[1 + \alpha \lambda (1 - B) + \lambda B][1 - B])</td>
<td>(\Phi_1 [1 - \alpha \lambda - B][1 - B]^2)</td>
<td>(\Phi_1[\gamma (1 - (1 - \lambda)B) + (1 - B)^2][1 - B])</td>
<td>(\alpha \lambda (1 - B))</td>
</tr>
</tbody>
</table>

\[
det = [1 + \Phi_2 B][\alpha \gamma (1 - (1 - \lambda)B) + (\alpha + \beta)(1 - B)^2 - \alpha \lambda B][1 - B] + \Phi_1[1 - \alpha \lambda - B][1 - B]
\]
path ($\phi > 0$). Thus with appropriate parameter values this specification can accommodate a range of central bank behavior from nominal-interest rate smoothing to a stochastic no-feedback money growth regime. This model can be reduced to a four-variable VAR in $\ln Q_r$, $\ln MR_r$, $i_r$, and $\Delta \ln M_r$, driven by the exogenous variables $\ln Q_r$, $\ln A$, and the shocks $\varepsilon_r$. With some tedious algebra, the moving average representation of the model can be expressed as follows:

$$
\begin{bmatrix}
\ln Q_r \\
\ln MR_r \\
i_r \\
\Delta \ln M_r
\end{bmatrix}
= \left[ \frac{1.0}{\text{det}} \right] A^* 
\begin{bmatrix}
y \ln Q_r'' + \varepsilon_r \\
\varepsilon_r \\
k + \ln A + \varepsilon_{st} \\
\mu(1+\phi_2') + \varepsilon_{st}
\end{bmatrix},
$$

where the polynomial matrix $A^*$ and the polynomial $\text{det}$ are given in table 6. In the deterministic steady state, the impulse response functions are independent of the parameters of the monetary rule ($\phi_1$, $\phi_2$) and real output responds only to changes in $\ln Q_n$, (1,0). Similarly in the deterministic steady-state $\Delta \ln M_r$ responds only to $\mu$ (1,0). However, the transitory responses of real output to money-growth shocks are not zero. In particular, the greater is the interest-rate smoothing ($\phi_1$), the smaller are the transitory responses of real output to monetary shocks. Thus estimation of VARs in this type of model will produce different impulse response functions based on different behaviors of the monetary authorities, and it is not possible to infer from those impulse response functions the short-run impact of a change in money growth under a no-feedback rule, without prior knowledge of the form and parameter values of the sample period monetary rule(s).

A recent analysis by Strongin (1991) is an attempt at defining a monetary policy disturbance. His identifying restriction is that monetary policy shocks have exactly offsetting impacts on nonborrowed reserves and borrowed reserves and hence have no effect on total reserves. In contrast he assumes that “reserve-demand” shocks in principle affect all three aggregates. Much of Strongin's discussion of historical Federal Reserve operating procedures focuses on the likely distribution of reserve-demand shocks (his $\phi$ parameter) between nonborrowed and borrowed reserves. The size of this parameter is not relevant to his identification problem, though it is important for estimation if the parameter value differs across subsamples. The identifying restriction allows him to construct a measure of monetary policy shocks but does not address the structure of the monetary rule or policy reaction function. Strongin implicitly assumes that there is no contemporaneous feedback from interest rates onto his monetary-policy shock because both total and nonborrowed reserves precede the fed funds rate in his Wold causal chain. Thus his identifying restriction does not address all of the problems raised here.

Unfortunately, inference about monetary regimes (policy reaction functions) using regression techniques has proved illusory. Khoury (1990) reviews 42 attempts at the estimation of reaction functions for the Fed over various sample periods. He concludes that “the results were in disarray” and “the specification search showed that very few variables were robust in a reaction function ... consistent with the lack of robustness in the literature.” The additional attempts at developing reaction functions that are included in Mayer do not overturn this conclusion. Thus it appears appropriate to conclude that at present we lack adequate information to make inferences from time series analyses on the vexing question of the short-run impact of nominal shocks on real output.

**CONCLUSIONS**

Significant elements of the St. Louis research agenda are now widely accepted, at least in U.S. academic circles and to some extent within the Federal Reserve System. Nevertheless, issues of short-run impacts of monetary policy remain unresolved. Among these are the following two critical topics: 1) changes in the drift of velocity and the extent to which such changes are generated by changes in inflation expectations and 2) the short-run impacts of nominal money shocks on real output.

The first of these questions is critical to the design of monetary rules and/or operating procedures that will retain credibility during the...
transition to an alternative inflation regime. The second question has long been debated and appears to be re-emerging as a focus of time series analysis. The analysis presented here suggests that the information necessary to pursue this agenda successfully is not yet available. One critical precondition to such analysis is a reasonable specification of the monetary regime(s) during the sample period. In this respect, Cagan’s (1989) appeal for more “historical” research warrants careful consideration.

A potential application of such a historical analysis is a test of Strongin’s (1991) identifying restriction for monetary policy shocks. His restriction provides an estimated time series for borrowed reserves targets. If the identifying restriction is valid, time series estimates of the monetary policy shocks should correlate well with the data extracted from these historical records.

REFERENCES


51See Brunner and Meltzer (1964) and Rasche (1987).


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**Appendix A**

**Technical Description of the Assumptions in the Simulation of the McCallum Rule**

The initial regime (periods 1-19) in figures 3 and 4 before the implementation of the McCallum rule are base velocity growth at \( t_0 = .0075 \) per period. The monetary base is assumed to grow at a rate that increases at \( t_0 = .0001563 \) per period. Thus nominal income growth is increasing at a rate of \( t_0 = .0001563 \) per period. The rate of increase in nominal income growth is assumed to reflect the trend in inflation, which in turn is assumed to reflect the trend in nominal interest rates. The trend in nominal interest rates and the trend in velocity must satisfy the restriction \( t_t - e_t = 0 \) where \( e_t \) is the interest semielasticity of velocity if base velocity and interest rates are cointegrated. \( e_t \) is assumed to be 48, using the estimated semielasticity of M1
velocity from Hoffman and Rasche [1992]. Nominal income and nominal potential income are assumed equal throughout this period.

The regime switch to the McCallum rule is announced and implemented in period 20. The desired growth rate of nominal income in the new regime is .0075 per period. It is assumed that the announcement of the new policy results in an immediate elimination of base velocity growth.

It should be noted that the path of nominal income growth once the McCallum regime is implemented is independent of the assumptions about growth in the prior regime. Nominal income growth in the McCallum regime is totally determined by the assumed growth of velocity starting 16 periods prior to the implementation of the rule, and the reaction of velocity growth to the institution of the new regime. The particular initial conditions for base growth used here are chosen strictly for consistency with the assumed initial growth rate of base velocity.

Appendix B
Confidence Intervals for Impulse Response Functions

Estimates of the precision of the impulse response functions from the sample ending in 90:3 were constructed from a Monte Carlo integration. The estimated coefficients and covariance matrix of residuals from a VAR augmented by two error correction variables were shocked using the algorithm described in Doan [1990], example 10.1. The elements of the cointegrating vectors were held constant at their estimated values, since Johansen [1991], Theorem 5.5, proves that the estimated asymptotic covariance matrix of $\hat{\Pi} = \hat{\sigma} \hat{\beta}'$ depends only on the estimated asymptotic covariances of $\hat{\sigma}$ and the estimated $\hat{\beta}$ and not on the estimated asymptotic covariance of $\hat{\beta}$. 1000 replications on the parameter values were constructed and the parameters of the KPSW common trends model were recomputed for each replication. The mean value of KPSW's critical $\Pi_{11}$ parameter across all replications is -.0057 with a standard deviation of .0371. These parameters were used to derive impulse response functions. The means of various impulse responses across the 1000 replications are plotted in Figures A1–A10, together with confidence bands of $\pm 1.96^*$ (standard deviations of the impulse responses across replications). The graphs suggest that the short-run responses of real output and velocity with respect to both permanent shocks are measured with considerable precision, in particular that the real output response to a permanent "money growth" shock is initially significantly positive and that the real output response of a permanent "real output" shock is significantly less than 1.0 for about 10 quarters. In contrast, the measurement of the short-run responses of inflation, money growth, and interest rates to both permanent shocks is highly imprecise.

Appendix C
Sources of Data

All data series were extracted from Citibase. The primary sources are as follows:

Treasury Bill Rate: Three month secondary market rate. Federal Reserve Bulletin, Table I.35, line 15.


Appendix Figure 1
Velocity irf to Money Growth Shock

Appendix Figure 2
Real Output irf to Money Growth Shock
Appendix Figure 3
Inflation irf to Money Growth Shock

Appendix Figure 4
Money Growth irf to Money Growth Shock
Appendix Figure 5

T-bill Rate irf to Money Growth Shock

Appendix Figure 6

Velocity irf to Real Output Shock
Appendix Figure 9
Money Growth irf to Real Output Shock

Appendix Figure 10
T-Bill Rate irf to Real Output Shock