A GROWING BODY of literature now argues that the public capital stock has significant, positive effects on private sector output, productivity and capital formation. Most of this literature suggests that a decline in the growth of the public capital stock since the early 1970s caused a “productivity slump” in the private sector lowering profitability and investment. Unless these trends are reversed, say the studies, the nation’s standard of living will be further threatened. This article explains this public capital hypothesis and evaluates the evidence supporting it.

THE PUBLIC CAPITAL HYPOTHESIS AND PRIVATE SECTOR PRODUCTIVITY

Public capital comprises federal, state and local government capital goods. It includes highways, streets and roads, mass transit and airport facilities, educational buildings, electric, gas and water supply facilities and distribution systems, wastewater treatment facilities, and administration, police, fire, justice and hospital facilities and equipment.

The public capital hypothesis is that the stock of public capital raises private sector output both directly and indirectly. The direct effect arises, according to the hypothesis, because public capital provides intermediate services to private sector firms, or the marginal product of public capital services in the private sector is positive. The indirect effect arises from an assumption that public and private capital are “complements” in production—that is, the partial derivative of the marginal product of private capital services with respect to the flow of public capital services is positive. Thus, a rise in public capital raises the marginal productivity of private

1 This argument will be referred to as the public capital hypothesis; it has been developed most fully by Rainer (1983), Aschauer (1989a), (1989b) and (1990) and Munnell (1990). Also see Deno (1988) and Eberts (1990). The hypothesis was suggested earlier by Schultze (1981), Arrow and Kurz (1970), Eisner (1980) and Ogura and Yohe (1977).

2 See The National Council on Public Works Improvement (1988), Malabre (1990) and Reich (1991) for analyses that attribute such consequences to the slowdown in public capital formation. A previous article, Tatom (1991), explains how several factors account for the decline in the rate of growth of the public capital stock. These factors suggest that a reversal of this past decline would not be economically justifiable, even if the public capital stock has the effects emphasized by the public capital hypothesis.

3 The notion of complementarity and substitutability used here has been called q-substitutability and q-complementarity. It refers to the effect of the quantity of one resource on the marginal product of another resource. The concept of p-substitutes or p-complements is more common; these terms refer to the effect on the demand for a resource of a rise in the price of another resource, holding other resource prices and output constant. See Sato and Koizumi (1973) for a discussion of this distinction, or its use in Tatom (1979b).
capital services so that, given the rental price of such services, a larger flow of private capital services and a larger stock of private assets producing them are demanded. The rise in the marginal product of capital increases private capital formation, further raising private sector output.

The indirect effect of a rise in public capital on private output, however, is not necessarily positive. In fact, this effect is negative if public and private capital are substitutes. Economic theory does not dictate whether private and public capital are complements or substitutes. The analysis below focuses on estimating the direct effect, the private sector's marginal product of public capital. If public capital does not enter the production function for private output, as is demonstrated below, the sign of the indirect effect must also be zero.

**The Productivity Decline and Public Capital Formation: A Look at the Record**

Advocates of the public capital hypothesis argue that a slowdown in public capital formation caused a "productivity slump" beginning in the early 1970s. Some perspective on this issue is provided by figure 1, which shows output per worker in the business sector and the real nonmilitary net stock of public capital (1982 prices) per business sector worker from 1947 to 1989. Public capital per worker is measured by

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*There is, however, a growing literature that suggests that government spending is a substitute for private sector spending. Recent attention to this view owes much to its elaboration by Aschauer (1985). Other research that develops the direct substitution channel for crowding out include Kormendi (1983), Kormendi and Meguire (1986) and (1990) and Tatom (1985). Critics of the Kormendi and Meguire view include Barth, Iden and Russek (1986), Feldstein and Elmendorf (1990), and Modigliani and Sterling (1986) and (1990).*
the capital stock at the end of the previous year divided by the average level of business sector employment during the year.\(^5\)

The growth of output per worker slowed from a 2.5 percent annual rate from 1948 to 1973 to nearly zero (−0.1 percent rate) from 1973 to 1982, before rebounding to a 1.8 percent rate from 1982 to 1989. Advocates of the public capital hypothesis emphasize only the post-1973 slowing. The growth of the public capital stock per worker also slowed abruptly in the early 1970s. Public capital per worker rose at a 3 percent rate from 1948 to 1971, then showed no growth from 1971 to 1982, much like the 1973-82 slowing in productivity. Since 1982, however, the growth in the stock of public capital per worker slowed further, falling at a 1.6 percent rate. This inconsistent shift in trends after 1982 does not contradict the fact that both variables grew more slowly after the early 1970s than they did before then.

The simple correlation coefficient for the logarithms of the two measures shown in the figure is 0.95 for the period 1947 to 1989. This strong positive relationship is a classic case of a spurious time-series relationship. In fact, changes in the public capital stock per worker are not statistically significantly related to changes in business sector output per worker. The correlation coefficient for changes in the logarithm of each measure is negative and equals only −0.03 for the period 1948 to 1989. The reversal in the size, significance and sign of the correlations among levels and first-differences illustrates the importance of the issues explored below in assessing the public capital hypothesis.

**THE BUSINESS SECTOR PRODUCTION FUNCTION AND PUBLIC CAPITAL FORMATION**

The most direct aggregate evidence on the positive effect of public sector capital formation can be obtained from production function estimates. The aggregate production function indicates the maximum output that can be produced with labor and capital given technology and other factors influencing production. The marginal product of each resource is assumed to be positive and inversely related to the quantity of the resource (diminishing returns).

**The Conventional Approach**

Ratner (1983) provides the first model that explicitly adds public capital to the production function to test whether the marginal product of public capital is positive. More recent analyses by Aschauer (1989a) and Munnell (1990) use a similar approach. Ratner assumes that the business sector production function (Q) can be represented by a Cobb-Douglas function:

\[
Q_t = Ah_t^Bk_t^Ck_g^D e^{rt+t'},
\]

where \(A\) = a scale parameter,

\(h_t = \) business sector hours,

\(k_t = \) the flow of services from \(K\), the constant-dollar net nonresidential stock of private capital at the end of the previous year,

\(k_g = \) the flow of services from \(KG\), the public capital stock at the end of the previous year,

\(r = \) a rate of disembodied technical change,

\(t = \) a time trend, and

\(\epsilon_t = \) a normally and independently distributed random disturbance term.

The same utilization rate, \(c\), is used for public capital as for private capital, so the flow of private capital services, \(k_c\), is \(cK\), and the flow of services from government capital, \(k_g\), equals \(cKG\). The utilization rate is measured by the

---

\(^5\) The net public and private capital stock data used in this article were provided by John Musgrave from data he prepares for the U.S. Department of Commerce, which is described in U.S. Department of Commerce (1987) and Musgrave (1988). The series are constructed by deducting depreciation from gross stock measures, which cumulate gross investment less discards (assets that are scrapped). The depreciation methods use straight-line depreciation for service lines equal to 85 percent of the U.S. Treasury Department's Bulletin F service lines. Constant cost measures (1982 prices) are used throughout this article.

\(^6\) The questionable assumption of an identical utilization rate for public and private capital is not required, however. The derivation of equation 2 requires only that the use of public capital be proportional to that of private capital. In this case, the constant term \(A\) in equation 2 will include this factor of proportionality.
The public capital hypothesis that KG affects Q as:

\[ I \text{Thus, the production function can be rewritten each resource raises scale, which means that a proportional rise in } Q \text{ by the same proportion; this assumption is expressed as } a + \beta + \delta = 1. \]

Thus, the production function can be rewritten as:

\[ \ln(Q/\kappa) = \ln A + a \ln(h/k) + d \ln(KG/k) + \varepsilon. \]

The public capital hypothesis that KG affects Q is tested by determining whether \( d \), the output elasticity of public capital, is positive.\(^7\) The output elasticity is the marginal product of public capital services divided by the average product of these services (Q/kg). The coefficient \( a \) is the output elasticity of labor which, in principle, should equal the share of labor cost in total cost.

Ratner estimated equation 2 using data for 1949 to 1973. Since then, the data have been revised numerous times, including changing the base period for computing constant-dollar output and capital stock data. When this equation is estimated for the original sample period 1949-73, using the latest available data, the estimate is (t-statistics in parentheses):

\[ \ln(Q/\kappa) = 1.410 + 0.548 \ln(h/k) \]
\[ + 0.277 \ln(KG/k) + 0.0128 t \]
\[ (11.52) (9.32) (2.81) \]
\[ R^2 = 0.93 \quad \text{S.E.} = 1.02% \quad \text{D.W.} = 1.65. \]

The statistically significant output elasticity of public capital is estimated to be 27.7 percent. This is much larger than Ratner’s earlier estimate of 5.8 percent.\(^8\) The rise in the output elasticity of the public capital stock arises from data revisions subsequent to Ratner’s study.\(^9\)

Aschauer (1989a) and Munnell estimate similar production functions for the business sector over the period 1949-85 and the non-farm business sector over the period 1949-87, respectively.\(^10\) They both find a positive and significant output elasticity for public capital; their estimates, however, are about 30 to 40 percent, somewhat larger than that in equation 3.\(^11\)

**Three Potential Shortcomings of Existing Estimates**

Estimates like that in equation 3 are suspect for three reasons. First, they ignore the significant influence of the relative price of energy on capital stock measure and its utilization rate used in this article are also used by Ratner and Aschauer. Both Munnell and Aschauer (1989a) include the capacity utilization rate in manufacturing as a separate variable to capture the influence of the business cycle on productivity. They provide no theoretical justification, nor do they indicate whether the capacity utilization rate is intended to capture any influences besides the varying use of the stock of business sector capital.

Schultze (1990) has criticized such estimates for implying implausibly large estimates of the rate of return to infrastructure. Aaron (1991) also questions the magnitude of the effect, the conceptual basis for such an effect and whether the estimate is spurious. A counterpart to the relatively high rate of return, at least in equation 3, is that the output elasticity of hours is only 54.8 percent. This is well below the theoretically expected value, which equals the share of labor cost in total cost of about 66.7 percent.

\[^{11}\text{Schultze (1990) has criticized such estimates for implying implausibly large estimates of the rate of return to infrastructure. Aaron (1991) also questions the magnitude of the effect, the conceptual basis for such an effect and whether the estimate is spurious. A counterpart to the relatively high rate of return, at least in equation 3, is that the output elasticity of hours is only 54.8 percent. This is well below the theoretically expected value, which equals the share of labor cost in total cost of about 66.7 percent. The output elasticity of private capital is about 17.5 percent, about half of the expected value.}\]
productivity found in similar studies (see the shaded insert on page 10). Second, they omit a significant time trend or reductions in the trend found in other studies. Third, they contain variables that are not stationary, raising the possibility of spurious estimates.\textsuperscript{12}

Consider the Cobb-Douglas production function including the flow of energy, $E$:

\begin{equation}
Q = Ah^\gamma k^\beta E^\epsilon \text{e}^{(\epsilon t + 1)},
\end{equation}

where $\gamma$ is the output elasticity of energy. The quantity of energy is assumed to satisfy the first-order condition for its employment, $E_t = \gamma Q_t / p^\prime_t$, where $p^\prime$ is the price of energy measured relative to the price of business sector output.\textsuperscript{13}

In addition, the production function is assumed to be characterized by constant returns to scale, $(\alpha + \beta + \gamma + \delta) = 1$.\textsuperscript{14} Substituting these two assumptions into equation 4 and taking logarithms of both sides yields:

\begin{equation}
(5) \ln(Q/k_t) = \ln(A_t^\prime + a_t^{\prime ln(h_t/k_t)} + \delta_t^{\prime ln(KG_t/K_t)}) + \gamma_t^{\prime ln(p_t^\prime)} + \tau_t^{\prime 1} + \epsilon_t^{\prime},
\end{equation}

where $a^{\prime} = a/(1 - \gamma)$, $\delta^{\prime} = \delta/(1 - \gamma)$, $\gamma^{\prime} = -\gamma/(1 - \gamma)$, $r^{\prime} = r/(1 - \gamma)$, $A^{\prime} = A^{(1 + r)/(1 - \gamma)}$, and $\epsilon_t^{\prime} = \epsilon_t^{\prime}/(1 - \gamma)$.

The omission of energy price effects on productivity after 1973 could result in attributing energy-related productivity losses to the decline in the growth of public capital.

The second potential shortcoming of existing tests using production functions is that they omit significant time trends or significant breaks in the time trend found in similar studies.\textsuperscript{15} Trends are intended to control for the influence of the pace of technical change: their omission could bias the coefficients and the standard errors for the included variables, especially those correlated with the omitted time trends.\textsuperscript{16}

\textsuperscript{12}Eberts (1990) raises the issue of whether there is "reverse causation" in estimates of the effect of public capital on private output; in other words, does a significant positive correlation indicate that public capital raises private output or does a rise in private output raise the demand for and quantity of public capital? He provides regional evidence suggesting that causality runs both ways.

\textsuperscript{13}The quantity of energy is assumed to be proportional to stock of energy-using capital and to its services. It is included because conventional measures of the flow of capital services, $k$ (or $K$), are not expected to reflect the differential effect of energy price changes on the economic value of the capital stock and its flow of services. Reduced energy usage is only one of several reasons why higher energy prices affect private sector output. The domestic and foreign capital stocks (for example, the pools of oil and gas, beds of coal and hydroelectric power sources) that produce the energy used in U.S. production are not included in the measured domestic nonresidential capital stock. Therefore, if the relative price of energy rises and producers respond by using less of this capital, the reduced flow of services from this capital will not be reflected in $k$. Moreover, the decline in the real value of the rest of the capital stock due to higher operating costs also is not reflected in $k$, since replacement cost rather than market prices are used to measure the value of existing assets in computing the constant dollar net stock. See Rasche and Tatom (1977a), (1977b) and (1981). Also see Helliwell, Sturm, Jarrett and Salou (1989) for an alternative approach.

\textsuperscript{14}The use of the constant returns to scale constraint in estimating production functions is quite common because of the intuitive appeal of this property and, more importantly, because the high correlations between hours, the flow of private capital services, the time trend and, in this case, the flow of public capital services, are expected to make it difficult to interpret the coefficient estimates and to raise their standard error estimates without this constraint. When the constraint can be rejected, its imposition trades off some explanatory power in fitting the production func-

\textsuperscript{15}For example, Munnell (1990) includes no time trends and Aschauer (1989a) includes no shift in the trend.

\textsuperscript{16}A decline in the trend rate of technical change in 1967 is discussed in Rasche and Tatom (1977b) and several studies that discuss this trend-break are cited there. A break in the trend rate ($\tau$) in 1967 for the business sector was not significant in the data available at the time of Ratner's study, but it is significant in later data. See Tatom (1988), for example, for a discussion of this change in significance. Darby (1984) argues that a declining quadratic trend arises from a post-depression and post-WWII catch-up in the level of technology.
The effect of the first two shortcomings on an estimate of the production function like equation 3 can be seen by including the relative price of energy, as in equation 5, and by allowing for a quadratic trend component \( t^2 \). Nearly identical results arise from allowing for a one-time decline in the linear time trend from 1.5 percent per year before 1967 to a 1.0 percent rate afterward. For the period 1948 to 1989, estimate is:

\[
(6) \quad \ln(Q/k) = 1.595 + 0.614 \ln(h/k) \\
+ 0.132 \ln(KG/K) - 0.048 \ln p \\
+ 0.019 t - 0.0001 t^2 \\
\text{[15.29] [12.88] [2.77] [—6.41] [8.42] [—4.23]}
\]

\[
R^2 = 0.97 \quad \text{S.E. = 0.95%} \quad \text{D.W. = 1.49}
\]

This estimate indicates a statistically significant, positive effect of the public capital stock on output, but is less than half that given in equation 3 or the estimates obtained by Aschauer and Munnell. Both the relative price of energy and the slowing in the time trend are statistically significant. Updating the equation 3 estimate, but including the energy price and time trend slowing, does not alter the statistical significance of the public capital stock effect, however.

The third potential shortcoming in regression estimates like equations 3 or 6 is that they contain variables that are not stationary, and so are subject to a spurious regression bias. First-differencing typically renders the data stationary and removes the problem of justifying or explaining the existence of a deterministic trend or trends. The evidence concerning this potential difficulty and its implications is explained below.

**Are the Production Function Variables Stationary?**

Table 1 reports Dickey-Fuller tests for a unit root for the levels of the variables in equation 6—\( \ln(Q/k), \ln(h/k), \ln(KG/K) \), and \( \ln p \)—and for their first-differences. The relevant statistic for the unit root test is the \( t \)-statistic for the coefficient on the lagged level of the variable \( Z_{t-1} \) whose first-difference is used as the dependent variable; this coefficient is labeled \( b \) in the table. If this coefficient is significantly different from zero when the time trend is statistically insignificant and, therefore, omitted, then the variable \( Z \) is stationary. When the time trend is significantly different from zero, its coefficient, \( d \), is included in the reported test equation. In this case, if \( b \) is significant, the variable \( Z \) is said to be trend-stationary.

Only one lagged dependent variable is statistically significant in any of the tests for the levels of the data in the table; these significant instances are reported in the

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17The standard error of estimate using the time trend shift in 1967 instead of the quadratic trend is 0.99 percent; the estimated output elasticity of the public capital stock is 0.135 in this case. Equation 3 and the Aschauer and Munnell estimates are representative of estimates without a declining time trend and energy price effects. For example, when the quadratic trend term and energy price term are omitted from equation 6, the output elasticity of the public capital stock is 0.306 (\( t = 5.27 \)), about the same as in equation 3; when a significant first-order autocorrelation term is added, this output elasticity rises to 0.343 (\( t = 4.91 \)).

18Equation 6 contains the 1948 data point, as well, to include all available data. This does not affect the results, however. When the relative price of energy is added to the 1949-73 estimate in equation 3, its coefficient (—0.117) is not statistically significant (\( t = 1.35 \)). Its inclusion lowers the coefficient on \( \ln(KG/K) \) to 0.206, and it too becomes statistically insignificant (\( t = 1.88 \)) at a 95 percent confidence level using a one-tail test. When a first-order autocorrelation correction term is added to equation 6, its coefficient is not statistically significant.

19This bias is explained by Granger and Newbold (1974) and (1986). Some analysts refer to this potential bias as arising only when two random walk variables are used in a regression, because this was the example used by Granger and Newbold (1974). Granger and Newbold (1986) use other nonstationary variable combinations. Engel and Granger (1987) explain that a linear combination of stationary and nonstationary variables is nonstationary, unless the nonstationary variables are cointegrated. Thus, the error term in such an equation is potentially nonstationary, giving rise to a potentially spurious regression.

20The \( t \)-statistics for the \( b \) coefficients are estimates of Dickey-Fuller statistics called \( \tau (r) \), when the time trend is omitted (\( d = 0 \)), and \( \tau \), when the time trend is included. The critical values of \( \tau \) and \( \tau \), for this size sample are given in Fuller (1976) and equal about —2.95 and —3.53, respectively.
### Table 1

**Tests for Nonstationarity**

#### Test Equation for Levels of Variable (Z): \( \Delta Z_t = \alpha + bZ_{t-1} + dt + e_t \)

<table>
<thead>
<tr>
<th>Levels of Variable (Z)</th>
<th>( \alpha )</th>
<th>( b )</th>
<th>( d )</th>
<th>( e )</th>
<th>( R^2 )</th>
<th>S.E.</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln(Q/k) )</td>
<td>0.101</td>
<td>0.002</td>
<td>0.29</td>
<td>0.023</td>
<td>2.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.35)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln(h/k) )</td>
<td>0.324</td>
<td>0.003</td>
<td>0.12</td>
<td>0.034</td>
<td>2.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(1.33)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \ln(h/k) )</td>
<td>0.266</td>
<td></td>
<td>0.10</td>
<td>0.034</td>
<td>2.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.91)</td>
<td>(2.33)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln(h/k) )</td>
<td>0.185</td>
<td></td>
<td>0.22</td>
<td>0.031</td>
<td>1.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.15)</td>
<td>(3.50)</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \ln(p') )</td>
<td>0.252</td>
<td>-0.063</td>
<td>0.420</td>
<td>0.088</td>
<td>2.03</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td>(1.42)</td>
<td>(1.40)</td>
<td></td>
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</tr>
<tr>
<td>( \ln(p') )</td>
<td>0.401</td>
<td>0.002</td>
<td>0.436</td>
<td>0.17</td>
<td>0.087</td>
<td>2.06</td>
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<tr>
<td></td>
<td>(1.92)</td>
<td>(1.94)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \ln(KG/K) )</td>
<td>-0.005</td>
<td>0.602</td>
<td>0.68</td>
<td>0.007</td>
<td>1.91</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(1.46)</td>
<td>(5.2)</td>
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</tbody>
</table>

#### Test Equation for Differences (\( \Delta Z \)): \( \Delta Z_t = \alpha + b\Delta Z_{t-1} + dt + e_t \)

<table>
<thead>
<tr>
<th>First-difference (( \Delta Z ))</th>
<th>( \alpha )</th>
<th>( b )</th>
<th>( d )</th>
<th>( R^2 )</th>
<th>S.E.</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \ln(Q/k) )</td>
<td>0.006</td>
<td>0.352</td>
<td>0.68</td>
<td>0.025</td>
<td>2.03</td>
<td></td>
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<tr>
<td></td>
<td>(1.51)</td>
<td>(9.09)</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \Delta \ln(Q/k) )</td>
<td>0.013</td>
<td>1.364</td>
<td>0.0004</td>
<td>0.68</td>
<td>0.025</td>
<td>2.07</td>
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<tr>
<td></td>
<td>(1.64)</td>
<td>(1.03)</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>( \Delta \ln(h/k) )</td>
<td>0.031</td>
<td>0.141</td>
<td>0.57</td>
<td>0.036</td>
<td>1.57</td>
<td></td>
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<tr>
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<td>(4.38)</td>
<td>(7.21)</td>
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<tr>
<td>( \Delta \ln(h/k) )</td>
<td>0.062</td>
<td>1.266</td>
<td>0.64</td>
<td>0.032</td>
<td>1.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.99)</td>
<td>(6.41)</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>( \Delta \ln(p') )</td>
<td>0.004</td>
<td>0.612</td>
<td>0.29</td>
<td>0.089</td>
<td>1.98</td>
<td></td>
</tr>
<tr>
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<td>(0.26)</td>
<td>(4.09)</td>
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<tr>
<td>( \Delta \ln(p') )</td>
<td>0.0004</td>
<td>0.613</td>
<td>0.27</td>
<td>0.090</td>
<td>1.98</td>
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<td></td>
<td>(0.014)</td>
<td>(4.04)</td>
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<tr>
<td>( \Delta \ln(KG/K) )</td>
<td>-0.001</td>
<td>-0.323</td>
<td>0.23</td>
<td>0.008</td>
<td>1.53</td>
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<td>(0.09)</td>
<td>(3.53)</td>
<td></td>
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<tr>
<td>( \Delta \ln(KG/K) )</td>
<td>0.006</td>
<td>-0.425</td>
<td>-0.0004</td>
<td>0.41</td>
<td>0.007</td>
<td>1.84</td>
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<td></td>
<td>(2.35)</td>
<td>(5.00)</td>
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</tr>
</tbody>
</table>

* Significant at a 5 percent level.

1 When \( e \) is not significantly different from zero at a 5 percent level of significance, it is constrained to zero and the period used begins one year earlier.

2 The lagged dependent variable is not significant at a 5 percent significance level for any of these variables, so it is omitted.
Alternative Hypotheses About the Productivity Decline

Several hypotheses have been put forth to explain the slowdown in productivity growth besides the one focusing on the slowdown in public capital formation originally formulated by Eisner (1980), Schultz (1981) and Ratner (1983). These other explanations include a slowing in the trend of resources moving from agriculture to the industrial sector; the return to "normality" from the temporarily rapid postwar growth of output and productivity associated with reversing the adverse effects of the Depression and World War II on the private sector; a slowdown in research and development spending; the costs of increased government regulation; a slowdown in the growth of the private capital stock per worker; and the rise in energy costs in 1973-74 and 1979-81.1

Rasche and Tatom (1977a) and (1981) explain how a rise in energy prices reduces the economic capacity of the typical firm and renders capital obsolete. In the short run, firms alter their optimal production techniques, reducing their use of energy and, in some cases, obsolete capital, substituting labor and other capital to economize on higher energy costs. In the long run, reductions in the productivity of labor and capital resources lead, in the case of capital, to a smaller desired capital stock and flow of its services.2

Except for an unexplained shift or a slowing in the trend time, only the energy price rise and associated slowing in the growth of the capital-labor ratio provides explanations that are consistent with the timing and magnitude of productivity movements since 1973. Tatom (1982), for example, provides evidence that the entire decline in productivity growth from late 1973 to 1981 resulted from the rise in energy prices and the associated reductions in the capital-labor ratio.3 Tests of the effects of public capital on private output have not controlled for these effects.

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2 Jorgenson (1988) provides evidence of these adjustments within individual industries, while Baily (1981) and Griliches (1988) have focused on energy price-induced obsolescence of capital and its effects on productivity. Also, see Tatom (1979a) and (1982).

3 The energy-price hypothesis is not universally accepted. For example, Berndt (1980), Dennison (1974), (1979) and (1985), Darby (1984) and Olson (1988) have been critical of its significance, arguing that the share of energy in costs is too small or, in Darby's case, that a quadratic trend fits the data without any energy price effect, so that the slowing in the 1970s was not a puzzle. With Darby's view, the productivity pick-up in the 1980s becomes a puzzle, however.

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No lagged value of the dependent variable is significant (or reported) in the bottom half of the table where the presence of a unit root for the first-differences is tested.

The evidence for the levels of the variables shown at the top of the table indicates that ln(Q/k) is trend-stationary. The level of ln(h/k) appears to be stationary when the insignificant lagged dependent variable is included to reduce the extent of autocorrelation indicated by the relatively high Durbin-Watson statistic (D.W.). Without this lagged dependent variable, the hypothesis that ln(h/k) has a unit root, or is not stationary, cannot be rejected. For this reason, ln(h/k) is considered to be nonstationary here. According to the tests, ln(k) and ln(KG/K) are also nonstationary. The latter has a significant trend term (d), but ln(KG/K) is not stationary when it is included. Based on the level results in the table, nonstationarity cannot be rejected for the four variables entering equation 6.

The bottom panel of the table conducts the same test for unit roots for first-differences of the variables. The test for each of the variables...
rejects a unit root, but two variables, \( \Delta \ln(h/k) \) and \( \Delta \ln(KG/K) \), are trend-stationary. The levels of \( \ln(Q/k) \) and \( \ln(p) \) are integrated of order 1, I(1), which means that these variables must be differenced once to achieve stationarity. The levels of \( \ln(h/k) \) and \( \ln(KG/K) \) are I(2), because they must be differenced twice to achieve stationarity. The presence of a significant trend in the first-differences suggests that there is a significant quadratic trend in the levels of the data.

A first-difference version of equation 5 involves only stationary and trend-stationary variables. The first-difference of the time trend term, \( r_t \), in equation 5 is the constant, \( r \), which is the constant term in the first-difference equation. If the time trend consists of broken linear segments, then the average of the coefficients on these linear trends also is captured in the constant term. If there is a deterministic quadratic trend, first-differencing results in a linear trend remaining in the first-difference expression. Since two of the variables in equation 5 are only trend-stationary, however, a time trend must be included in the first-difference regression to maintain the desired stationarity. This is consistent with the presence of a deterministic quadratic trend in the production function, so that differencing does not avoid the consideration of deterministic trends in this case. Estimating a first-difference equation avoids both the problems arising from nonstationarity and the difficulties of selecting ad hoc breaks in the time trend in equation 5.\(^{22}\)

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NEW ESTIMATES OF THE EFFECT OF PUBLIC CAPITAL ON PRIVATE SECTOR OUTPUT

The variables used in equation 6 do not appear to be stationary, so the statistical significance of the public capital effect found there is potentially spurious. This problem is avoided by estimating the production function parameters in a first-difference specification with a time trend.\(^{23}\)

The first-differenced (\( \Delta \)) estimate of the production function for the period 1949 to 1989 is:

\[
\begin{align*}
\Delta \ln(Q/k) &= 0.025 + 0.042 \Delta \ln(KG/K) \\
&+ 0.737 \Delta \ln(h/k) \\
&- 0.058 \Delta \ln(p) - 0.0005 t \\
&= 0.85 SE. = 1.05% D.W. = 2.25
\end{align*}
\]

The coefficient on public capital, while positive, is much smaller than in estimates based on the levels of the variables, like that in equation 3 or even in equation 6.\(^{24}\) More importantly, however,
this coefficient is not statistically significant.\textsuperscript{25} The coefficient on energy prices is significantly negative, and that on hours per unit of capital remains significantly positive and rises to a value that is closer to its theoretically expected level.\textsuperscript{24} Thus, equation 7 indicates that the public capital stock has no significant influence on business sector output, given the capital-labor ratio and the relative price of energy.\textsuperscript{27}

The statistical insignificance of the public capital stock arises from first-differencing the data; it does not arise from the inclusion of energy prices in the estimation of equation 7 or from allowing for a trend.\textsuperscript{28} The omission of the significant energy price term does not produce a significant public capital stock effect either. When it is omitted in equation 7, the coefficient on the public capital stock variable, \( \Delta \ln(KG/K) \), rises to 0.108; however, it remains statistically insignificant (\( t = 0.77 \)).\textsuperscript{29}

The inferences from equation 7 are not subject to the spurious regression problem. Since tests of the variables in equation 6 generally fail to reject nonstationarity, the results in equation 7 offer the strongest evidence on the factors influencing, or not influencing, business sector output. This estimate rejects the public capital hypothesis.

\textsuperscript{25}If the insignificance of \( \ln(KG/K) \) arose from over-differencing an appropriate test equation, then the problem could be corrected by estimating equation 7 with a significant MA1 error process; the MA1 coefficient should be equal to minus one in this case. When equation 7 is estimated with an MA1 error process, the MA1 parameter is only \( -0.399 \); it is not statistically significant (\( t = -1.95 \)). More importantly, it is significantly less than one (\( t = 3.34 \)). The coefficient on \( \ln(KG/K) \) is reduced (0.021) and it remains statistically insignificant (\( t = 0.22 \)) when this term is included.

\textsuperscript{27}This theoretical value is derived in equation 5; it is conditional on the share of labor in total cost and the coefficient on the relative price of energy. For values of these parameters of 0.657 and -0.058, respectively, this theoretical value is 0.706.

\textsuperscript{29}Rubin (1990) regresses the growth rate of multifactor productivity in manufacturing and 11 two-digit SIC code industries on a constant, the growth rate of the Federal Reserve Board's measure of the industry's capacity utilization rate, and the growth rate of core infrastructure. Core infrastructure includes highways, streets, sewers and water systems in her analysis. The period she uses is generally from 1956 to 1986. She finds that there is no statistically significant effect for core infrastructure in any industry except petroleum refining, where a significant positive relationship is observed. Her result is consistent with the view suggested above, that the decline in public capital growth is, in part, a proxy for the pattern of increased energy prices.

In papers prepared after this research was completed, Hulten and Schwab (1991) and Jorgenson (1991) note the fragility of estimates of the marginal product of public capital. Hulten and Schwab provide evidence of this fragility, but in their first-difference estimates, the private sector input coefficients are fragile as well.

\textsuperscript{29}Without the trend term in equation 7, the coefficient on the public capital stock variable is 0.147, about the same as in equation 6, but it is not statistically significant (\( t = 1.07 \)). The trend term is necessary to ensure that the error term in equation 7 is stationary. A regression of the first-difference of the residuals from equation 7 on the lagged level of the residual, with no constant, yields a coefficient on the lagged residual equal to \( -1.152 \) (\( t = -7.32 \)). Even without the trend term in equation 7, the t-statistic on the resulting lagged residual is \( -5.86 \). Engel and Granger (1987) indicate that the critical value for these t-statistics is ~3.37. Thus, the residuals are stationary in either case.

\textsuperscript{29}Production function estimates are subject to simultaneous equation bias, but this has no effect here. Virtually the same results are obtained using a two-stage least-squares estimation procedure. The instruments for the right-hand-side variables include the first-difference of the logarithms of real wages, the AAA bond yield, and the relative price of private capital goods, as well as lagged dependent and independent variables.

\textbf{An Alternative Approach: Are Private Sector Output and Public Capital Cointegrated?}

According to the evidence in table 1, the variables in equations 6 are not stationary; two of them are \( I(1) \) and two are \( I(2) \). Engel and Granger (1987), Johansen (1988) and Johansen and Juselius (1989) develop procedures for examining whether \( I(1) \) variables have long-run relationships or are cointegrated. These methods cannot be used here because two of the variables in equations 6 are \( I(2) \). Neither procedure addresses the problem of how to incorporate a linear time trend, trend shift or quadratic trend in a cointegration test.

Stock and Watson (1988) have developed a method for testing cointegration among higher-order integrated variables, including variables that are integrated of different orders. They explain that one approach to the problem of non-stationarity is to include significant lags and leads of first-differences of the dependent and independent variables as right-hand-side variables in tests of functional relationships. They argue that this practice avoids the spurious regression problem for nonstationary variables pointed out by Granger and Newbold (1974) and that it indicates the presence of long-run (or
cointegrating) relationships between variables as the coefficients on the levels of the variables.

This approach was taken in estimating the level of the production function in equation 5.\textsuperscript{30} Up to two leads and lags of first-differences of each variable in equation 5 were examined. The equation estimate containing only significant leads or lags, estimated over the period 1950-88 is:

\[
(8) \ln(Q/k) = 0.489 + 0.105 \ln(h/k) \\
(10.76) \quad (9.86) \\
- 0.046 \ln p_t - 0.075 \ln(KG/K) \\
(-2.95) \quad (-1.47) \\
+ 0.762 \Delta \ln(KG_{t+1}/K_{t+1}) \\
(2.85) \\
+ 0.064 \Delta \ln p_{t+1} - 0.065 \Delta \ln p_{t+1} \\
(2.11) \quad (-2.40) \\
R^2 = 0.93 \quad \text{S.E.} = 1.36\% \quad \text{D.W.} = 1.69
\]

In this estimate, the coefficient on the nonmilitary net stock of public capital per unit of private capital (-0.075) has the wrong sign and is not statistically significant.\textsuperscript{31} Like the result reported above, the nonmilitary public capital stock has no statistically significant relationship with business sector output. The levels of business sector output or productivity are uncorrelated with the level of the nonmilitary public capital stock. The statistically significant t-statistics on the coefficients for the levels of \(\ln(h/k)\) and \(\ln p_t\) in equation 8 suggest that only the variables (\(\ln Q/k, \ln h/k, \ln p_t\)) are cointegrated.

CONCLUSION

An increasing number of people are advocating increased government capital spending to raise private sector output, productivity and private capital formation. The evidence presented here, based on the post-World War II experience, suggests that a rise in public capital spending would have no statistically significant effect on these measures.

Earlier claims of a positive and significant effect of public capital on private sector output have arisen from spurious estimates. In fact, most of these earlier estimates have ignored a trend or broken trends in productivity, as well as the statistically significant influence of energy price changes. Simply accounting for these two factors reduces the conventional estimates of the elasticity of private output with respect to public capital of about 30 to 40 percent, to about 13 percent. More importantly, however, both the earlier estimates and those reported here that find a statistically significant public capital effect use equation estimates that contain nonstationary variables. Thus, these estimates are likely to be spurious.

When all of these problems are addressed using a first-difference estimate of the production function, the public capital stock effect on private sector output is not statistically different from zero. Appropriately estimated, the hypothesis that public capital has a positive marginal private sector product cannot be supported.

The same result is found using a method that allows testing a long-run relationship among nonstationary variables.

REFERENCES


\textsuperscript{30}The quadratic trend is omitted because it is not an integrated stochastic process; thus, production cannot be cointegrated with it. See Stock and Watson (1988), p. 168, for example.

\textsuperscript{31}A first-order autocorrelation term is not statistically significant and its inclusion does not alter the statistical insignificance of \(\ln(KG/K)\).


