On the Use of Option Pricing Models to Analyze Deposit Insurance

The failure rate of banks and thrifts has exploded over the past decade, making reform of the deposit insurance system a topic of considerable interest to regulators, bankers, and economists. As illustrated by figure 1, which shows the total number of failed commercial banks (excluding thrifts) for each year since the chartering of the Federal Deposit Insurance Corporation (FDIC), the annual number of commercial bank failures in each of the last several years has exceeded its previous peak, attained during the Great Depression. The status of the thrift industry is even more grim, with losses to the Federal Savings and Loan Insurance Corporation (FSLIC) estimated at $160 billion or more. The primary consequence of these failures for public policy is the enormous losses, especially to the FSLIC, as depositors in these failed institutions are reimbursed.

This article considers a particular set of economic tools used to evaluate deposit insurance. Option pricing models are among the techniques available for analyzing the deposit insurance system. These models can be used to assign specific values to the claims of each of the interested parties involved in the deposit insurance system — the insurer, financial institutions, and depositors. Such valuations can then be used, for example, to estimate the net value of the government’s insurance fund or to determine a fair price that a bank should pay for its insurance. More generally, by comparing insurance valuations with different model parameters, one can investigate the system of incentives under a given regulatory scheme, such as the risk incentives for bank shareholders and depositors under the present system. Finally, comparisons of insurance values and incentives can be made across various proposed regulatory schemes. These applications are illustrated below with some examples.

The usefulness of option pricing models for evaluating deposit insurance is of special interest for two reasons. First, the consensus among the interested parties is that the present deposit insurance system has contributed to the current crisis. Second, in the context of this debate, a number of economists have used the

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1This article does not address the issue of why we should have deposit insurance. The rationale for the current system of bank regulation and for deposit insurance in particular is based on two related principles: protection of the depositor and the mitigation of contagious bank runs. See FDIC (1984) and U. S. Treasury (1985). Benston and Kaufman (1988) identify three reasons for bank regulation in addition to the two traditional rationales, namely disruption to communities from localized bank failures, moral hazard induced by deposit insurance and restrictions on competition.
modern theory of option pricing to explain the incentives and measure the costs both of the current system of deposit insurance and of some suggested alternatives. This paper presents the basic theory of option pricing, explains how it can be applied to deposit insurance, and analyzes some of the issues involved in its use.

A PRIMER ON OPTION PRICING

This paper presumes no knowledge of options or of the various economic models that have been used in the academic literature to assign values to options. Thus, it begins with a brief description of options and some of the major contributions to the theory of pricing options that have been made in the past two decades.

Contingent Claims, or Options

A call option is a legal contract that gives its owner the right to buy a specified asset at a fixed price on a specified date. Similarly, a put option gives its owner the right to sell a specified asset at a fixed price on a specified date. Option contracts are usually sold by one party to another. The person who owns an option contract is called the holder of the option. The person who sells an option contract — that is, the person who will be compelled to perform if the option holder invokes her right as specified in the contract — is called the writer of the option. The act of invoking the contract is called exercising the option. The fixed price identified by the option contract is called the striking price. The date at which the option can be exercised is called the expiration date of the option.

These legal contracts are probably best known by the stock options that are bought and sold by brokers in the trading pits of organized options exchanges in Chicago, New York and elsewhere. In addition to options on common stock, there are active markets for options on agricultural commodity futures, foreign currencies, stock index portfolios, and government securities, to name only a few. The definition of an option, however, does not limit the term to those contracts actively traded on the floors of organized financial exchanges. By definition, an option is any appropriately constructed legal contract between the writer and the holder, regardless of whether it is ever traded.

Expiration-date Values of Options

Consider now the value to the holder of an expiring put option, as illustrated in figure 2. The value of the underlying asset specified by the contract is given on the horizontal axis, while the value of the option itself is given on the vertical axis. The point K on the horizontal axis is the specified striking price for the asset. If the value of the underlying asset is above the striking price on the expiration date, then the put option will not be exercised; anyone who truly wanted to sell the asset would do so outright at the going price, rather than using the option and receiving the striking price. In this case, the option expires worthless, and the option holder experiences no gain or loss on the expiration date.

On the other hand, if the value of the asset is below the striking price, then the holder will exercise her option and receive the striking...
price for the asset. In this case, her net gain on the expiration date will be \((K - A_T)\), the difference between the striking price and the current price, since she can turn around and replace the asset immediately, if she wants to. Thus, the expiration value of the option and the decision about whether to exercise are contingent upon the value of the underlying asset at that time:

For this reason, options are also referred to as “contingent claims” on the underlying assets.

The corresponding net payoffs to the writer of the put option are given in figure 3. Notice that his payoffs are exactly the inverse of those for the option holder. Also note that the payoff at expiration to the writer of an option is never positive; at best it is zero. It is for this reason that options are sold to the holder, rather than being given away free of charge. The price initially paid for the option — the option price or option premium — could be incorporated into the figures by simply shifting the holder’s payoffs down and the writer’s payoffs up by the appropriate amount.

The payoffs at expiration to the holder and writer of a call option are given in figures 4 and 5, respectively. The corresponding analysis for call options is precisely analogous to the analysis just given for put options.

### The Black-Scholes Option Valuation Model

Having described the value of an option at expiration leaves the question of its value prior to expiration unanswered. Instead of being a simple function of \(A_t\) and \(K\), the value of an option before maturity depends on several additional factors. Although a number of bounds had been placed on the value of an unexpired option by using relatively simple arbitrage arguments, an important advance in the valuation of unexpired options was made by Black and Scholes (1973). They obtained an exact equation for the value of a put option under an unrestrictive set of assumptions. Their result has since been elaborated and generalized by others.

In their model, the value of an unexpired option depends on five things:

<table>
<thead>
<tr>
<th>State</th>
<th>Action</th>
<th>Option value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_T &lt; K)</td>
<td>Exercise</td>
<td>(P = K - A_T)</td>
</tr>
<tr>
<td>(A_T \geq K)</td>
<td>No exercise</td>
<td>(P = 0)</td>
</tr>
</tbody>
</table>

4For an exposition of the arbitrage bounds on option prices, see Merton (1973), or Cox and Rubinstein, ch. 4.

5Almost all derivations of option pricing models, including that of Black and Scholes, are stated in terms of call rather than put options. As it happens, this distinction is largely irrelevant, because call option valuations are readily converted to put option valuations, and vice-versa, via an arbitrage relationship known as “put-call parity”. Put-call parity is an exact relationship for European options and an approximate one for American options (see Cox and Rubinstein, pp. 150-52); throughout this paper it is treated as exact. Put-call parity is first presented by Stoll (1969).

6One such generalization is found in the shaded insert. For a partial survey of option pricing models, see Cox and Rubinstein, ch. 7.
K = the striking price
A = the current asset price
T = the time remaining to expiration
σ = the volatility of the asset price
R = the risk-free interest rate.

Almost as notable is what the option's value does not depend on: any characteristic of the holder or the writer. Under their assumptions, Black and Scholes are able to include an option in a riskless portfolio. Such a portfolio must earn the risk-free interest rate, and they are able to use this result, along with an assumption about the probability distribution of the asset price, to identify an exact value, P, for a put option:

\[ P = (K e^{-RT}) N(X + \sigma \sqrt{T}) - A N(X), \]

where:
\[ X = \frac{1}{\sigma \sqrt{T}} [\ln(K e^{-RT}) - \ln(A)] - \frac{1}{2} \sigma \sqrt{T} \]
\[ N(*) = \text{the standard normal cumulative probability function} \]
\[ \ln(*) = \text{the natural logarithm function} \]
\[ e = \text{the base of the natural logarithm} \]

Although this formula may at first appear complicated, a rough intuition can be provided relatively painlessly. First, e^{-RT} is just the present value discount factor for T periods at interest rate R with continuous compounding, so that Ke^{-RT} is the present value of the striking price. Keeping in mind that N(X + \sigma \sqrt{T}) is a probability, the first term is the expected present value of the striking price at expiration, given that A < K. Similarly, the second term, A N(X), is the expected present value of the expiration-day asset price, again given that A < K. Thus, the value of the option is the expected present value of its value at expiration, given by condition 1 above.

Unfortunately, no easy, correct interpretation can be attached to the specific probabilities, N(X + \sigma \sqrt{T}) and N(X), in the two terms. These probabilities are closely related to the probability that A < K, but they are not quite the same, because the present value of the striking price is known with certainty, whereas the present value of the asset's price on the expiration day, A e^{-RT}, is not; the current asset price, A, appears instead.

\[ \text{Figure 4: Value of Call Option to Holder} \]

\[ \text{Expiration Value of Option} \]

\[ \text{Value of Underlying Asset at Expiration} \]

\[ \text{Figure 5: Value of Call Option to Writer} \]

\[ \text{Expiration Value of Option} \]

\[ \text{Value of Underlying Asset at Expiration} \]

\[ 7 \text{For example, one might suspect that the holder's attitudes toward risk or her beliefs about the asset price at expiration should influence the option's value to her. This is not the case, however. Also note that four of the five factors, at least theoretically, are well-defined and directly observable at the time of valuation. The exception is asset volatility, which must be estimated from observable factors; see Cox and Rubinstein, pp. 280-87, for an example of an estimation technique.} \]

\[ 8 \text{The corresponding expected present values for the case when A is greater than K are both zero, because then the expiring option is worthless and will not be exercised; hence, this possibility adds nothing to the current value of the option.} \]

Malliaris (1983) for a mathematically advanced approach or to Cox and Rubinstein, ch. 5, for a longer but less technical derivation.
In spite of its complexity, the option pricing equation is still a useful tool. In one sense, the formula can be treated as a black box in which the five parameters (K, A, T, σ and R) enter at one end, and the value of the put option, \( P \), comes out at the other; a computer spreadsheet or calculator can be programmed to perform the intervening calculations defined by the formula. For example, if the current asset value is \( A = \$985 \), the standard deviation of asset returns is \( σ = 0.3 \) percent, the striking price of the option is \( K = \$1000 \), the time to expiration is one year, and the riskless interest rate is \( R \) percent per year, then the Black-Scholes equation tells us that the put option is worth \$85.45. Figure 6 graphs the Black-Scholes value of a put option for a range of current asset values from zero to \$1500, where the values of the other four parameters are the ones just given.

**The Brownian Motion Assumption**

Not surprisingly, the distribution of asset prices is a crucial factor in determining the exact form of the option pricing equation. In their derivation, Black and Scholes assumed that the price of the underlying asset progressed randomly through time according to geometric Brownian motion. This is the assumption that leads to the specific normal probability functions in their pricing equation.

Brownian motion was first used to describe the random progress of a single molecule through a gas from a given starting point. It is a mathematical model of motion that identifies the way the particle can move. Three restrictions are implied by Brownian motion:

1) The path followed must be continuous;
2) All future movements are independent of all past movements;
3) The change in position between time \( s \) and time \( t \) is normally distributed with mean equal to zero and a standard deviation equal to \( σ\sqrt{t-s} \).

Note that standard deviation is directly proportional to the amount of time that has passed.

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\(^{10}\) We are here concerned with only a single dimension of motion, for example, the East-West coordinate of the molecule or the price of an asset.

\(^{11}\) Although this may be true of molecules, it need not be the case for asset prices, as is considered in the shaded insert on Merton’s jump-diffusion model.

\(^{12}\) This implies, for example, that the molecule cannot build up momentum or that prices do not have a predictable trend. It does not mean that the future location is independent of the past location.

\(^{13}\) By way of terminology, simple Brownian motion (also known as arithmetic Brownian motion) and geometric Brownian motion are examples of the Wiener process (also known as the Gauss-Wiener process), which, in turn, is a special case of the Itô process.
change in position itself, is normally distributed with mean zero and standard deviation \( \sigma \sqrt{t-s} \). This distributional assumption gives us the specific functional form which appears in the Black-Scholes equation. Thus, this assumption is important: a different distribution would generally yield a different pricing equation, as illustrated by Merton's (1976) jump-diffusion option pricing model, which is presented in the shaded insert on the opposite page.

**Risk and Hedging in Options**

The option pricing equation has the paradoxical property that, although risk (as measured by volatility in the asset price) is itself a factor in the option's value, the attitudes toward risk of the holder and the writer (and anyone else) are not. The option's value is a function of five variables, none of which depends on the characteristics of the individuals involved. Black and Scholes achieved this by showing that the option can be made part of a completely hedged (that is, riskless) portfolio. Any option writer who offered a risk discount when selling an option would find himself selling many option contracts to investors who, in turn, could hedge the risk completely and pocket the risk discount as an arbitrage profit. It is for this reason that the discount rate which appears in the pricing equation is the risk-free rate of interest, and attitudes toward risk are irrelevant to the value of the option.

To see how the hedged portfolio works, consider the value of a put option to the holder before expiration, depicted in figure 6, and the value of the underlying asset purchased for the amount \( A \), depicted in figure 7. The value of the net asset investment increases one for one as the price of the asset increases, and the value of the option decreases, although not in a constant proportion.

The key to the hedged portfolio is to buy put options and underlying assets in the appropriate ratio, so that, when the asset price increases, the increase in value of the net asset investment will be precisely offset by the decrease in value of the option position, and vice-versa. This implies a riskless total portfolio. Of course, the appropriate ratio (called the "hedge ratio" or "option delta") also changes as the asset price changes, because the value of a put option does not decrease as a constant proportion of the asset value (the put option's value is represented by a curved line). This implies that the holder of a completely hedged portfolio must continuously adjust the relative proportions of options to assets if the hedged portfolio is to remain riskless. Black and Scholes presume that at least some investors are large and sophisticated enough to do this.

Because the risk of an option can be completely diversified, the risk-free rate is the appropriate interest rate to use for discounting the option's uncertain payoff at expiration. Nevertheless, the risk (defined as price volatility) of the underlying asset is a factor in the option's value, because asset risk affects the expected value of the option's payoff. This is due to the limited liability nature of the option. Although increasing the volatility of the asset price increases both the chance of getting a very high expiration-day asset price and the chance of getting a very low expiration-day asset price, the bad (high price) outcomes all have a weight of zero in the put option valuation, while the good (low price) outcomes have a weight of \( (K-A) \). The volatility of the option's value also increases with that of the asset price, but the volatility of the option's value is irrelevant, because it can be completely hedged.

**DEPOSIT INSURANCE AND OPTIONS**

The analysis of deposit insurance is a natural, albeit not obvious, extension of option pricing models. The connection between the two comes through the limited liability property common to both options and common stock. This property implies an "expiration-day" payoff for deposit insurance that can be modeled as an ordinary put option. Similarly, other claims on a financial intermediary's assets can be modeled as options or combinations of options. The benefit is that, given such a model, option pricing theory allows us to assign values to each of the claims. These values are the key to option pricing's usefulness in this context, because they allow two sorts of comparisons to be made.

First, variations in the parameters of the option pricing equation can be considered. Such variations are of special interest, because, in the

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14 This connection was first made by Black and Scholes and first applied to deposit insurance by Merton (1977).

15 For example, in Black and Scholes' model, the five parameters: \( K, A, T, \sigma \) and \( R \) would be varied.
Merton's Jump-Diffusion Model

The Black and Scholes (1973) derivation of an option's value was based, in part, on a particular assumption about the random behavior of the price of the underlying asset. Their assumption of geometric Brownian motion as a description of the movements of asset prices is by no means the only possibility. In general, different assumptions about the statistical properties of asset price behavior produce different valuation equations for an option on that asset. Merton's jump-diffusion model is one of several alternative formulations that have been developed.¹

Just as arithmetic Brownian motion was not an apt model of asset price movements, empirical research suggests that geometric Brownian motion, at least for some assets, is similarly inappropriate. An alternative, proposed by Merton (1976), is a combination of geometric Brownian motion with random, discontinuous jumps in the asset price, such as might occur at the announcement of some news event.² This arrangement violates the first condition for Brownian motion and modifies the third condition again.³ When a jump occurs, the asset price is abruptly shifted by a random amount; the logarithm of this shift is normally distributed, analogously to geometric Brownian motion.

This new process has two important implications for the option pricing model. First, the option pricing equation is different from the Black and Scholes formula, since it must account for the new jumps. If we relabel the Black-Scholes value as \( P^\ast = P^\ast(K, A, T, \Theta, R) \), then we can write the Merton formula as a function of the Black-Scholes value:

\[
P^\ast = \sum_{i=0}^{\infty} \left[ \beta_i P^\ast(K, A, T, \Theta_i, R) \right] \cdot \left( 1 - \beta_i \right) \cdot (K \cdot e^{-\beta T} - A)
\]

where:

\[
\beta_i = e^{-\Theta_i} \cdot (gT)
\]

\[
\Theta_i = \sqrt{\sigma^2 + h^2 / t}
\]

\[
g = \text{the Poisson frequency of jumps}
\]

\[
h^2 = \text{the variance of the shift distribution.}
\]

Not surprisingly, the Merton equation is more complicated than the Black and Scholes equation. Nevertheless, it is still a function of variables that are at least hypothetically observable.⁴

Second, the presence of the jumps complicates the diversification problem. The simple hedging portfolio used for the Black-Scholes model will not do, because even continuous rebalancing cannot insure the portfolio's value at the jump points. If, however, the jumps are idiosyncratic (that is, firm-specific), then their risk can still be eliminated by holding a well-diversified portfolio that includes the assets of many firms in many industries. If the jump disturbances are idiosyncratic, then the risk-free interest rate is still the appropriate discount rate for the expected option payoff, and individual attitudes toward risk are not a factor in the option's value before expiration.

¹Some others are McCulloch's (1981, 1985) Pareto-stable process, and Cox and Ross's (1976) alternative jump processes and constant elasticity of variance (CEV) Ito process.
²Merton's derivation is only one of several alternatives to the Black and Scholes model. It was chosen to illustrate some of the issues involved in selecting an appropriate pricing model, not because it outperforms the others in some sense. See Rubinstein (1985) for a performance comparison of several models.
³The discontinuous jumps arrive according to a Poisson process, which conforms to the second condition.
⁴In practice, the statistical parameters \( \sigma, h \) and \( g \) would have to be estimated, either from previous observations or via some other technique. The equation given here is a special case of a more general formulation given by Merton (1976). His derivation is of a call option price, which has been re-arranged here using put-call parity.

context of deposit insurance, some of the parameters can be controlled or influenced by the parties to the option contract. Thus, each party has a clear interest in influencing the parameters to his own benefit and therefore to the detriment of the other. The risk-incentive problem presented below exemplifies this sort of application. Comparisons based on option pricing models indicate not only the direction, but also the magnitude, of such incentives.
Second, various deposit insurance structures can be compared. The structure of deposit insurance is defined here by the number and type of options pertaining to each of the interested parties. Changes in deposit insurance structure are different from the parameter changes within an insurance structure, considered in the preceding paragraph. Thus, for example, the FDIC could use option models to estimate the net increase or decrease in the present value of the insurance fund caused by a switch from one structure to another; or it could examine the change in risk incentives occasioned by the same switch. Three different structures, illustrating some of the issues involved, are presented in the following examples.16

100 Percent Deposit Insurance Coverage

To see how deposit insurance and options are related, consider the following simplified banking scenario. A single banker both owns and runs a bank, a single large depositor provides the entire liability portfolio of the bank, and a single insurer, the FDIC, insures deposits and will liquidate the bank in the event of insolvency.17 The liability portfolio consists of a single deposit due at year-end. Also at year-end, the FDIC examines the bank to determine the value of assets, which will, in turn, determine whether liquidation occurs. If the bank is economically insolvent, it is closed by the FDIC, which liquidates the assets at market values and pays off the depositor in full.18 If the bank is economically solvent, control remains with the banker, who can either renegotiate the deposit or liquidate the bank.

Now consider the payoffs to the three interested parties — banker, FDIC and depositor — when the year-end audit is performed. These payoffs are illustrated in figures 8-10. Each party’s year-end payoff is plotted as a function of the year-end value of the bank’s assets. Note that the sum of the payoffs to all of the parties (obtained by adding the graphs vertically) equals the value of the bank’s assets. These functions show how the bank’s assets will be distributed after the audit is performed. Also note the shape of the payoff functions for the banker and the FDIC; in effect, the banker’s portfolio consists of the bank’s assets, whose value is uncertain before the audit but known afterward, the bank’s deposits, whose value is known to be L, and a put option with striking price L written by the FDIC.19 The FDIC, on the other hand, has effectively written the put option on the assets of the bank and sold that option to the banker for the price of the deposit insurance premium.20 The depositor has issued the bank a risk-free loan, which pays off the amount L, including accrued interest.

With this in mind, the usefulness of an option pricing model to evaluate deposit insurance becomes more apparent. An option pricing model provides an estimate of the actuarial dollar value of deposit insurance, as well as a tool with which to analyze the economic incentives that deposit insurance creates. The depositor, for example, has a portfolio, D, that is worth, at the beginning of the year, simply the present value of the deposit liability discounted at the riskless rate, $L e^{-RT}$; if his year-end payoff, L, is $1000, and the riskless rate, R, is 8 percent, then the value of this portfolio at the beginning

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16Full coverage is considered first, because it is the simplest insurance structure possible, and because it approximates the current system of extensive coverage combined with the FDIC’s tendency to arrange purchase and assumption transactions, rather than deposit payouts, for failed banks.

17The assumptions of a single owner-manager for the bank and a lone depositor are clearly broad abstractions from reality. The owner-manager assumption allows us to ignore principal-agent incentives; see Barnea, Haugen and Senbet (1985). Similarly, the assumption of a single depositor effectively precludes the ability of depositors to withdraw their funds individually without forcing an immediate closure of the bank. Although these are both important issues, the purpose of the present analysis is to illustrate the general principles involved in the application of option models to deposit insurance, rather than to model a bank in its full complexity.

18The bank is defined as economically insolvent when the market value of its assets is less than the present value of its liability to the depositor. This terminology is meant to contrast with an illiquidity, or legal insolvency, which is brought on by an inability to meet maturing short-term liabilities with liquid assets. The legal profession has a separate terminology for these two concepts: the “balance sheet test” is used to determine economic insolvency, and the “equity test” is used to determine legal insolvency. See Symons and White (1984), pp. 603-16, for an exposition. Since there are no short-term liabilities in the simplified world here, legal insolvency is not germane.

19The bank’s deposits represent a “short” position, or borrowing, for the banker. The net payoff shown in figure 8 can be gotten by drawing the individual payoff graphs for the components of the portfolio and adding these together vertically as before. This portfolio is also equivalent, via put-call parity, to a simple call option on the assets of the bank.

20Compare figure 9 with figure 3. The insurance premium is considered a sunk cost at the time of the audit and hence is not included in the graphs.
of the year is $1000e^{-0.08} = $923.12. The FDIC, on the other hand, has written a put option with striking price $L = $1000; if, for example, the standard deviation of the bank's asset returns is $\sigma = 0.3$ percent, and the current value of the bank's assets is $985, then the value of the FDIC's portfolio is given by the Black-Scholes equation as $-P = -$85.45.

Of particular interest are the incentives created by deposit insurance. Under 100 percent insurance, the depositor does not care about the value of the bank's assets, since he receives his deposit back with interest, regardless of the bank's condition. The banker, however, receives the positive equity capital, if the bank is solvent; if the bank is insolvent, the loss is charged to the FDIC. This "heads I win, tails you lose" arrangement is certainly not peculiar to banks; it applies to any corporate entity with limited stockholder liability. In the absence of other incentives, the banker will make the corporation as risky as possible.

What is peculiar to banks under 100 percent flat-rate deposit insurance is the absence of such other incentives for the depositor and the banker to limit risk. Normally, creditors impose a risk premium on corporations, based on the riskiness of the firm's assets. By definition, flat-rate insurance implies that the FDIC charges no risk premium. Similarly, the depositors charge no risk premium, because they are fully insured. The result, in our simplified model, is that the banker has an unmitigated incentive to increase the riskiness of the bank's assets, while the FDIC has the incentive to reduce the bank's risk-taking. The risk incentive implied by extensive, flat-rate deposit insurance is the impetus for most of the current proposals for deposit insurance reform. In analyzing both the current system and proposed reforms, many authors have used option pricing models.

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21 Recall that the value of an option to the holder increases with the volatility of the underlying asset. For the Black and Scholes model, risk is defined as the standard deviation of the logarithm of the asset's value.

22 This is the function of bond rating services, such as Moody's and Standard and Poor's. See Barnea, Haugen, and Senbet, especially pp. 33-35, for an exposition of the risk-incentive problem.

23 For this reason, risk-taking is restricted by extensive regulation of commercial bank activities. In practice, the banker's incentive may also be mitigated by the potential loss of a valuable bank charter or by nonpecuniary factors, for example, the potential loss of a bank manager's professional reputation in the event of a failure. These factors are beyond the scope of the option model.

24 Reform proposals include: risk-based insurance premia, risk-based capital requirements, larger capital requirements, reduced insurance coverage, depositor co-insurance, subordinated debt requirements, increased supervision and more stringent asset regulation. See White (1989) for a survey of current proposals.

We can now extend the options model to other arrangements for deposit insurance, to evaluate their relative impacts. Two illustrative cases will be presented: a coverage ceiling clause and a deductible clause. A significant characteristic of both these cases is that they impose a portion of the bank’s asset risk on the depositor. The FDIC benefits directly from such provisions, because they shift some potential losses directly to the depositors. In addition, imposition of a possible loss on the depositor mitigates the risk-incentive problem that exists under 100 percent, flat-rate deposit insurance. In general, the depositor will monitor the bank more closely and will require a higher interest rate to compensate for the possibility of default.

Maximum Insurance Coverage Limit

Although 100 percent coverage is often treated as the status quo de facto of federal deposit insurance, coverage extends legally only to the first $100,000 per depositor per institution. A maximum coverage limit is a form of co-insurance, a technique used by insurers to reduce the moral hazard problem (the tendency of insurance to alter the behavior of the insured). Other basic forms of co-insurance are the deductible and fixed proportional sharing of losses.

The applicability of the maximum coverage limit considered here is complicated by the FDIC’s current closure protocol. Bank closures by the FDIC can take one of two forms: purchase and assumption or deposit payout. Under a purchase and assumption closure, healthy assets and deposits are transferred to another healthy bank, with the FDIC absorbing the problem assets and any net loss. This sort of transaction is best modelled by the 100 percent coverage considered above.

Under a deposit payout closure, the FDIC itself takes all of the bank’s assets and liabilities into receivership. It then sells the assets and pays the depositors up to the maximum coverage limit plus any excess of asset sales over insurance claims, distributed on a pro rata basis. As a result, this method is best modelled by the deductible considered below. The upshot is that the payoffs under the FDIC’s maximum coverage limit do not conform to the familiar (from, say, automobile or health insurance) maximum coverage arrangement illustrated here.

Consider a maximum coverage limit of M dollars for the depositor (where M < L), illustrated in figures 11—13. Under this arrangement, the depositor receives the full deposit amount, L, in the event of any insolvency or shortfall, up to the amount, M, of the coverage limit. Thus, the depositor's portfolio contains the deposit amount, L, and he has written a put option on the bank’s assets with striking price (L − M). This put option is the result of the maximum coverage limit. The FDIC holds the put option with striking price (L − M), but has written a second put option on the bank’s assets with striking price L. As before, this put option (with striking price L) is held by the banker, who also owns the assets and owes the amount L to the depositor.

Because of the put option written by the depositor and held by the FDIC, the depositor now shares in the risk of the bank’s assets. His deposit is now worth less, and he will discount the promised payoff more steeply. Extending the example given above for the case of 100 percent coverage, the depositor’s portfolio, which contained only the riskless deposit, worth $932.12 when discounted, is now augmented by the put option written with a striking price of (L − M). If, for example, the coverage limit is set at M = $100, so that the striking price is $900, then with a = 0.3 percent, R = 8 percent and A = $985 as before, the Black-Scholes value of this put option to the depositor is \( P = -\$47.96 \), and the total value of his portfolio is \( D = \)

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26 The original limit was $2500 under the Banking Act of 1933; see FDIC (1984), pp. 44, 69. The impact of the coverage ceiling is limited by the availability of brokered deposits and by the tendency of the insurer to arrange purchase and assumption solutions to bank failures.

27 There are other possibilities. For example, deposit insurance in the United Kingdom involves fixed proportional sharing combined with a coverage ceiling (see Llewellyn (1986), p. 20), and a temporary deposit insurance program in the United States was to have sharing in staggered proportions [see FDIC (1984), p. 44]. Analyses of co-insurance tend to focus on proportional sharing arrangements. See Boyd and Polnick (1989), and Benston and Kaufman (1998), ch. 3.

28 Defining a “healthy” asset is a difficult chore. The task is generally accomplished by individual evaluation of assets, rather than the application of a generic rule.
Given deposit insurance with a $100 coverage limit, $884.16 is the amount deposited at the beginning of the year in exchange for a promised year-end payoff of $1000.

In general, we can use an option pricing model to solve algebraically for a measure of the risk premium that the depositor would charge under this risk-sharing arrangement. On the one hand, we can think of the deposit as a riskless deposit combined with a put option. On the other hand, we can think of it as a single risky promise of repayment from the banker, to be discounted at some risk-adjusted interest rate, \( r \), so that the value of the deposit is:

\[
D = L e^{-rT}.
\]

We can equate these two interpretations thus:

\[
D = L e^{-rT} - P(L - M, A, T, \sigma, R) = L e^{-rT},
\]

where \( P(\cdot) \) is the value of the put option before expiration, as defined, for example, by the Black-Scholes model, and \( r \) is the risk-adjusted discount rate implied by the presence of the coverage limit. Given the other variables, we can rearrange this to find the risk premium:

\[
2 \quad (r - R) = -\frac{1}{T} \ln \left[ \frac{e^{-rT} - \frac{1}{L} P(\cdot) \right] - R.
\]

Applying this to the numerical example, the stated risk-adjusted interest yield on the deposit is:

\[
2 \quad r = -\frac{\ln\left(e^{-0.0811} - \frac{47.96}{1000.00}\right)}{100} = -\ln(0.87515) = 13.3%,
\]

implying a risk premium, \( r - R \), of 5.3 percent.

In practical terms, such an estimate of the magnitude of the risk premium implied by a given coverage ceiling might be useful in calibrating the degree of market discipline in a reform of the insurance system. If bank risk-taking is to be curbed by limiting deposit insurance, forcing riskier banks to pay higher risk premia as some have suggested, then the insurance limitations must be such that the risk premium implied by equation 2 is large enough to make bankers alter their behavior.\(^{29}\)

The magnitude of the risk premium might also serve as a readily observable vital sign, registering the financial health of the bank’s assets, and aiding the regulator in scheduling audits. This presumes that depositors have some advantage over regulators in assessing the bank’s risk between audits.\(^{30}\) Such applications, how-
ever, are subject to some limitations which are illustrated by the next example.

**Deductible**

Another form of co-insurance is a deductible. The case of a deductible on insurance coverage introduces a twist to the problem. Now the depositor’s portfolio effectively consists of two put options, one written and one held, in addition to the promised repayment of the deposit with interest. This case is of special interest, because it applies to a deposit payout closure as considered above and because it can also be applied to subordinated debt, which is the object of a recent debate on sources of market discipline of bank risk-taking. In both cases, the payoffs to one of the bank’s creditors can be modeled as a pair of put options with different striking prices.31

In this example, the depositor is promised the return of his deposit amount, with accrued interest, for a total of L dollars. Because of the deductible provision, however, this promised repayment is not certain; in the event of the bank’s insolvency, the depositor will be the first to share in the shortfall. For year-end asset levels below L, the shortfall is deducted from the depositor’s payoff until the deductible amount, U, is exhausted. Any shortfall beyond that is absorbed by the FDIC. Thus, the depositor effectively holds the deposit amount L and has written a put option with striking price L, that is held by the banker; in addition, he holds a put option with striking price (L – U), which is written by the FDIC. These payoffs are illustrated in figures 14-16.

The deductible provides a cushion for the FDIC, which, in the preceding examples, had written a put option with striking price L rather than (L – U). The year-end payoffs for the banker are the same as before. The real difference applies to the depositor’s incentives and the resulting impact on the price he charges the banker for the deposit. Although he always prefers a higher asset value, as before, his attitude toward the riskiness of the bank’s assets is now ambiguous, because he is long one put option and short another, with two different striking prices. Volatility in the bank’s asset returns increases the value of the long position and decreases the value of the short position.

As Black and Cox (1976) point out, the net impact of these countervailing forces will depend on the current asset value relative to the striking prices. Specifically, there is an inflection

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31The relevant creditors in each case are the depositor and the subordinated debt-holder, respectively. For an analysis of option models in the case of subordinated debt, see Black and Cox (1976) and Gorton and Santomero (1988, 1989). For an analysis of subordinated debt and bank regulation, see Gilbert (1990). The approach here is at odds with that of Ronn and Verma (1986), who calculate the value for a single put on total debt and then scale that value down by the proportion of insured to total liabilities.
point equal to the discounted geometric mean of the two striking prices.\(^{32}\) For an asset value above the inflection point, which includes all cases in which the bank is solvent (i.e., \(A > Le^{-RT}\)), the effect of the short position outweighs that of the long position, and the depositor will prefer less risk. Conversely, when the current market value of assets falls below the inflection point, the long position outweighs the short, and the depositor would prefer a riskier asset portfolio, given the low asset value. Thus, a decrease in the "risk premium" charged by the depositor no longer necessarily implies that the bank's assets are less risky; for example, such a decrease could instead be the result of an increase in the current asset value and an increase in the volatility of those assets.

Under such circumstances, it is a reasonable taxonomic question whether the interest rate markup over the riskless rate should be called a risk premium at all. The current value of the depositor's claim and the implicit risk premium can be calculated as before:

\[
D = Le^{-RT} - P(L, T, A, \sigma, R) + P(L - U, T, A, \sigma, R)
\]

\[
= Le^{-RT} \rightarrow (r - R) = -\frac{1}{T} \ln \left\{ e^{-RT} - \frac{1}{L} \left[ P(L, \ast) - P(L - U, \ast) \right] \right\} - R,
\]

but the risk premium so defined is a measure of the expected difference between cash promised, \(L\), and cash ultimately received. It is a poor measure of the volatility of the returns on the bank's assets, because the expected difference between cash promised and cash received depends on several factors and is no longer a simple direct relation of the volatility of assets.

Figure 17 graphs the value of the depositor's claim for a range of asset levels and volatilities.\(^{33}\) In interpreting this graph, note the connection between it and figure 16. In particular, as the volatility, \(\sigma\), goes to zero in figure 16, the value of the deposit resembles more and more the staggered year-end payoff function of figure 16. In fact, if the year-end payoff of figure 16 is scaled down by the risk-free present value discount factor, \(e^{-RT}\), it becomes identical to the extreme case of figure 17 where \(\sigma = 0\). Figure 17 also illustrates graphically the Black and Cox argument that for some (higher) asset levels, depositors will charge a risk premium, while for other (lower) levels, they will offer a risk discount.

It is also clear from the picture, however, that the asset level has a much more significant effect on the value of the claim than does the volatility.\(^{34}\) All of this suggests that bankers, depositors and policymakers should give considerable care to an appropriate definition of risk in this context, and that similar care should be given to designing a practical measure of that risk. Risk defined as volatility in bank asset returns and measured by the risk premium charged on equity, subordinated debt or uninsured deposits may not be apt for the tasks to which it has been applied.

**SOME CAVEATS**

The preceding analysis has illustrated some uses of option pricing models in evaluating deposit insurance. There are some limitations, however, on the use of options models in this context. Most of these limitations derive from the assumptions that form the basis for the option pricing equation, and the extent to which these assumptions are valid for the case at hand.

Perhaps the most basic problem is the question, who truly holds the option.\(^{35}\) Until now, it has been presumed, based on the end-of-period payoffs, that deposit insurance represents a put option written by the FDIC and held by the banker. In fact, however, the FDIC decides whether a bank is insolvent, and, more importantly, whether to close a bank that is already insolvent (or one that is not quite insolvent).\(^{36}\) In the face of a large-scale bank failure or run,
short-term political considerations may overwhelm any prior prescriptions on closure policy. The FSLIC's actions in the thrift crisis indicate that this is not idle speculation; numerous thrifts were left open long after their insolvency had been discovered. Conversely, as Benston and Kaufman (1988) suggest, the FDIC could close solvent banks, if they have come close enough to insolvency. If the insurer were to follow scrupulously a well-defined rule one way or the other, there would be little at issue, since the striking price and year-end payoffs could then be easily adjusted within the context of the current model. As matters stand, however, bankers and depositors effectively face a random striking price, because the insurer decides if the option will be exercised. As a practical matter, it is difficult to envision how such a well-defined closure rule might be implemented.37

A related issue is the measurement of bank asset values. The option pricing model presented above presumes that the current asset price can be readily observed. For stock options, this is an uncontroversial assumption, because stock prices can be observed on the floor of the exchange or in the over-the-counter market. For an option on the assets of the bank, however, the relevant price is not readily observable. Indeed, one of the primary functions of bank credit analysis is to assign values to assets for which there is no active market. Similarly,

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37A rule is well-defined if it leaves no doubt about the circumstances which imply closure, with no room for FDIC discretion. Note that defining a closure rule, in general, is not sufficient for our purposes; the closure rule must be defined so that the values of the resultant claims conform to the values given by the option pricing equation.
the insurer must invest significant effort, in the form of an audit, to determine the year-end asset value. The inherent inaccessibility of bank asset values has two implications for option models. First, it is no longer possible for an option holder to construct the appropriately hedged portfolio described in the Black and Scholes derivation, because the hedge ratio depends on the value of the underlying asset. This casts doubt upon the appropriateness of the riskless rate in discounting the expected end-of-period payoffs. Second, the current asset value is important, because it partly determines the probabilities for the various end-of-period payoffs. Ignorance of the current asset value adds another layer of uncertainty, and this additional uncertainty significantly affects the value of the option.

A closely related issue is the measurement of asset risk. The option pricing models presented here use the variance of the asset’s returns as a measure of risk. Producing an accurate assessment of the variance is problematic, even for stock options, because the volatility that matters is the variance of the process over the future life of the option. For bank assets, the measurement problem is compounded, because even past values are generally unavailable. Pyle (1983) and Flannery (1989b) consider some of the implications of this problem in using the option models to price deposit insurance.

Just as asset values and the volatility of returns are not observable directly, there is the more general moot question of which stochastic returns-generating process should be incorporated in the option pricing model. As we’ve seen above, the difference in the assumed returns process between Black and Scholes’s model and Merton’s model resulted in a substantially different pricing equation. Although the choice of an appropriate returns process for modeling a bank’s assets is beyond the scope of this paper, it is sufficient to note that this choice is a salient factor in the option’s value, because it determines the probability of each of the possible year-end payoffs.

The empirical evidence to date testifies to the sensitivity of the results to the specification employed. Marcus and Shaked (1984) use the basic Black-Scholes model, adjusted for dividends, and find that federal deposit insurance is currently substantially overpriced relative to the “actuarially fair” estimates provided by their option model. They note, however, that McCulloch’s (1981, 1983) estimates of insurance values derived from the Paretian-stable distribution greatly exceed their own. McCulloch (1987b) uses a more complicated model which includes the degree of regulatory control wielded by the insurer. He finds that deposit insurance may be either overpriced or underpriced, depending on the level of regulation assumed. McCulloch’s (1985) study assumes non-normal, Paretian-stable asset returns and non-stationary random interest rates. He finds that insurance values are highly sensitive to the level and volatility of interest rates. Ronn and Verma (1986), however, using a variant of Merton’s (1977) model, conclude that neither random interest rates nor non-stationary equity returns significantly affect the insurance valuations. In brief, the empirical evidence suggests that a wide range of insurance valuations can be reached by varying the returns process employed in the model.

Finally, it has been assumed in the preceding examples that there is a single deposit that does not mature until the end of the year. In fact, of course, banks maintain many deposit accounts with a wide range of maturities, starting with the instant maturity of demand deposits. This is significant, because it gives many depositors another type of insurance. A depositor who can withdraw his funds from a failing bank sooner than the FDIC can close it has 100 percent in -
result is that the simple application of an option pricing model does not provide an accurate evaluation of such deposits, because it ignores certain relevant strategies.

CONCLUSIONS

The usefulness of option models in the study of deposit insurance results from two important characteristics. First, these models distill the host of economic factors involved down to a handful of relevant parameters whose interaction is well-defined by the option-pricing equation. Second, they are able to evaluate deposit insurance claims under a wide variety of insurance structures. Although only three such structures were elaborated here, they can be generalized to other applications. Thus, option models provide a unified context for analyzing incentives within an insurance structure, as well as for comparing alternative insurance schemes.

Unfortunately, option pricing models, like most economic models, are an imperfect tool when directly applied to the complexities of the real world. Beyond certain fundamental qualitative results, there are theoretical and empirical reasons to believe that the insurance values given by any particular option pricing model will be incorrect, highly sensitive to changes in their specification, or both. As a result, the absolute dollar magnitudes provided by option models of the value of deposit insurance are suspect. The contradictory empirical evidence on fair pricing is indicative of this problem.

In defense of these models, however, there is no reason to believe that option models are any worse in this regard than any alternative economic model. Indeed, there is some reason to believe that, although the absolute magnitude of the valuations provided by option models may be unstable, the rankings they provide for a sample of banks are not. Similarly, inaccuracies in determining the scale of insurance values do not deny the ability of option models to identify the direction of incentives or the impact of marginal changes in the structure of deposit insurance. Therefore, used judiciously, option pricing models can be an effective analytical tool in the study of deposit insurance.

REFERENCES


