Dennis W. Jansen

Dennis W. Jansen, an associate professor of economics at Texas A&M University, is a visiting scholar at the Federal Reserve Bank of St. Louis. *Scott Leitz provided research assistance.*

Does Inflation Uncertainty Affect Output Growth? Further Evidence

ECONOMISTS have long been interested in the effects of inflation on real economic variables. In the past two decades, this line of research has expanded greatly, spurred on by the relatively high inflation rates in the developed economies beginning in the 1970s and the coincident slowing in the rate of output growth. One traditional and widely accepted notion is that anticipated inflation has little or no effect on real variables, except for those effects arising from institutional features such as incompletely indexed tax codes and zero interest payments on currency and reserves.[1] It is also widely accepted that unanticipated inflation affects real variables, at least in the short run.

Many analysts also hold that uncertainty about future inflation rates affects real variables. Indeed, Marshall (1886) expressed concern about the negative effects of an uncertain future value of the English pound on output over 100 years ago. More recent arguments in this spirit are contained in Okun (1971) and Friedman (1977), who argue that uncertainty about future inflation is detrimental to real economic activity.

Furthermore, they suggest that uncertainty about future inflation is linked to the mean rate of inflation by the policy environment. Friedman, in particular, argues that nations might temporarily pursue a set of goals for real variables (for example, output, unemployment) that leads to a high inflation rate. The high inflation rate induces strong political pressure to reduce it, leading to stop-go policies and attendant uncertainty about future inflation. Thus, high inflation coexists with increased inflation uncertainty, as individuals become less certain about the political choice over future inflation paths.

Friedman postulates a negative effect of a highly volatile inflation rate on economic efficiency for two reasons. First, increased volatility in inflation makes long-term contracts more costly because the future value of dollar payments is more uncertain. Second, increased volatility in inflation reduces the ability of markets to convey information to market participants about relative price movements. By reducing economic efficiency, greater inflation uncertainty should at least temporarily increase

[1]Surveys reporting on this general consensus are Taylor (1981), Cukierman (1983) and Fischer (1981).

the rate of unemployment and reduce economic growth.[2]

Though these theoretical concerns about the effect of inflation uncertainty seem reasonable and persist in economic discussions, existing studies provide only mixed support for them. This paper studies the relationships between the mean and variance of the inflation rate and output growth for the United States in another attempt to identify the hypothesized negative relationship of inflation uncertainty on output growth. To put this study into perspective, the following section briefly reviews the findings of several previous studies, with particular attention to the relationship between the measure of inflation uncertainty employed in each study and evidence about the link between inflation uncertainty and real economic variables.

A REVIEW OF THE RECENT LITERATURE

Empirical studies of the effect of inflation uncertainty tend to follow one of three broad approaches. The first is that used by Okun (1971), who gathers data for 17 developed countries over 17 years and calculates the mean and variance of the inflation rate for each country. By plotting the mean inflation rate vs. the standard deviation of the inflation rate for these countries, he finds that these two variables are positively related. Logue and Sweeney (1981) use Okun's methodology and find that both the mean and variance of inflation are positively related to the variance of output growth.[3]

This approach has been criticized largely on two grounds. First, the sample variance of the inflation rate for a country over 15 or 20 years is unlikely to be the best measure of uncertainty about future inflation rates, because the sample variance of inflation confounds predictable and unpredictable changes in the inflation rate. For example, if the inflation rate moves in a perfectly predictable way, inflation uncertainty is zero, but the computed sample variance of inflation would be positive. A second criticism is that this approach requires a certain homogeneity across countries to make valid inferences about the variation of inflation and output growth across those countries. Gale (1981) gives reasons to doubt that this homogeneity exists, including noncomparability of indexes and different levels of development across countries. Indeed, Katsimbris (1985) strongly rejects the hypothesis of homogeneity across countries.

A second approach allows the mean and variance of inflation to change within a country through time. Katsimbris (1985) does this for 18 OECD countries. He constructs proxies for the time-varying mean and variance of inflation and output growth as eight-quarter, non-overlapping, moving averages. He finds few countries for which the mean and variance of inflation are related in a statistically significant way and even fewer for which the variance of inflation and the mean or variance of output growth are related. In particular, he finds no significant relationship between inflation uncertainty and output growth in the United States. Thornton (1988), in a recent study employing this methodology, obtains the same results.

Katsimbris' study of individual countries is but one example of a number of studies that use this second approach. Their main feature is the construction of proxies for inflation uncertainty. In addition to Katsimbris' eight-quarter, non-overlapping, moving averages, others estimate time series models for the inflation rate and the real variables and use the residuals to construct overlapping moving-average measures to proxy for the time-varying variance of inflation.

All of these studies lack a parametric model for the time-varying variance of inflation. For instance, Katsimbris' moving averages for the mean inflation rate does not necessarily capture the predictable elements of the inflation process. Therefore, his measure of the variance confounds the uncertainty of future inflation with predictable changes in inflation. In contrast, studies using proxies for inflation uncertainty constructed from the residuals of a model

---

[2]Recent theoretical work demonstrates that, under plausible conditions, increases in inflation uncertainty lead to reductions in output. Surveys of the theoretical rationales underlying relationships between inflation uncertainty and real variables are contained in Taylor (1981) and Cukierman (1983). These surveys also discuss some of the extant empirical literature on this topic.

[3]Logue and Sweeney acknowledge in their text that an alternative to their approach is to use a time series approach that relates inflation and its variability to the variability of production. They write, "Unfortunately, a neat measure of the next period's uncertainty that might be suitable for use in such a time series test is not available" (p. 499). It is a contention of this paper that the ARCH-M model provides the requisite time series test.

for the inflation process can claim rightly that they are attempting to measure only unpredictable movements in inflation; but these studies are prey to an internal inconsistency. In particular, such an approach estimates a model of inflation under the maintained hypothesis of homoskedasticity and then estimates a proxy for the time-varying (heteroskedastic) conditional variance from the residuals.

A third approach to measuring inflation uncertainty uses survey data from individual inflation forecasts. A good example is Mullineaux (1980), who uses the standard deviation of individual inflation forecasts about the mean value to measure inflation uncertainty. He finds that the sum of current and lagged values of this measure of inflation uncertainty is significantly and positively related to the unemployment rate and significantly and negatively related to the level of industrial production. A more recent study by Hafer (1986) confirms these results with an alternative survey of inflation expectations.

A crucial problem with this approach, however, is that the inflation uncertainty measure actually measures the dispersion of point estimates of the inflation rate across individuals, which does not necessarily capture the degree of uncertainty about future inflation rates. Within a specific theoretical framework, Cukierman (1983) has shown that these two measures are related. It is clear, however, that the individual point estimates reported in the surveys do not indicate the certainty with which individuals make their forecasts, so that measuring inflation uncertainty by the dispersion of these estimates of the inflation rate across forecasters can be misleading.[4] Consider, for example, what would happen if all individuals surveyed reported the same forecast. Even if none of the individuals were very certain of the forecast, that is, if inflation uncertainty were considerable, the constructed measure would be equal to zero.[5]

ESTIMATION RESULTS

This study investigates the effects of inflation uncertainty by looking at a time series of data for the United States, following the second approach discussed above. Unlike most previous studies, however, this investigation uses a statistical technique, the ARCH model, that parameterizes the mean and variance relationships under investigation. This permits straightforward estimation and hypothesis testing in an internally consistent framework. The measure of inflation uncertainty employed here is the time-varying conditional variance of the inflation equation. A more detailed description of the class of ARCH models is provided in the shaded insert on pages 46 and 47.

We model the inflation, real output growth system over the I/1959-II/1988 period using seasonally adjusted quarterly data on real GNP and the GNP deflator. The regression model for the conditional means of inflation and output growth is a vector autoregression.

Preliminary diagnostic tests were conducted to check for unit roots and time trends in the variables. These are reported in table 1. Neither inflation nor output growth exhibited a time trend. For output growth, the null hypothesis of a unit root was rejected. Tests for a unit root in the inflation process are inconclusive: the Dickey-Fuller test rejected the unit root hypothesis, but the augmented Dickey-Fuller test failed to do so. It is well known that tests for a unit root have low power when the alternative is a root close to but less than one. Moreover, the augmented Dickey-Fuller test is more powerful when the time series in question is not white noise after differencing, a situation that appears to hold for the GNP deflator.[6] Additional infor-

[4]One well-known survey, the ASA-NBER survey of professional forecasters, makes an attempt to gather data on confidence bands corresponding to forecasts. These data are relatively crude, however, and are seldom used by authors investigating the neutrality of inflation uncertainty. See, e.g., Hafer (1986).

[5]This is not to say that the information in the dispersion of inflation forecasts across individuals is not useful. Such information is not captured by the assumption implicit in this paper that agents forecast the inflation rate based on common information. Moreover, other approaches have been employed to look at related aspects of the relation between inflation uncertainty and real variables. Blejer and Leiderman (1980) look at relative price variability, measured as the dispersion of price changes in a set of industries about the average price change of the industry. They test to see if real output and unemployment are adversely affected by increases in relative price variability. Notice that inflation uncertainty is not directly an issue in Blejer and Leiderman's work since they examine only the variability of relative prices. They report that relative price variability had significant adverse effects on real variables for the United States.

[6]It is also known, however, that the augmented Dickey-Fuller test has lower power than the unaugmented test when the series is white noise after differencing.

The ARCH Class of Models

In a series of papers, Robert Engle and his collaborators have developed a class of models that allow for explicit parameterization of the variance process for time series models. These models are known by the acronym ARCH, for autoregressive conditional heteroskedasticity, and by variants on that acronym such as GARCH (generalized ARCH) and ARCH-M (ARCH in mean).[1] In these models, the variance of a regression is allowed to change over time and, in particular, to vary with past realizations of variables, including the regression disturbances.

The motivation behind the development of the ARCH class of models derives from several empirical features of economic data. First, the restrictive assumption of homoskedasticity often is rejected by the data. The ARCH model permits a general form of heteroskedasticity that nests the homoskedastic model as a special case. In particular, the variance is allowed to depend on realizations of past variables including past disturbances. Second, consistent with observed data, the ARCH model allows for the clustering of forecast errors that is often observed in econometric models. Thus, the ARCH model permits the occurrence of a large forecasting error today to increase the probability of observing a large forecasting error tomorrow. Third, the ARCH model explicitly allows for the leptokurticity that economic data exhibits. Leptokurticity is the phenomenon that a distribution has "fat tails."[2] Finally, the more general ARCH-M models are especially useful for conducting hypothesis tests relating means and variances.

The basic structure of the ARCH model is fairly simple. The univariate ARCH model can be represented as follows:

(1) $y_t = x_t'b + \varepsilon_t$

(2) $\varepsilon_t | I_{t-1} \sim N(0,h_t)$

(3) $h_t = \alpha_0 + \Sigma_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \qquad \alpha_i > 0$ for all i.

Equation 1 represents a standard univariate equation with y_t as the dependent variable, x_t as the vector of predetermined variables which can include lags of the dependent variable, b as the vector of parameters to be estimated and ε_t as the stochastic disturbance term. Equation 2 describes the properties of ε_t *conditional on information known at time t-1,* represented as I_{t-1}. The disturbance ε_t is conditionally normal, with mean zero and variance h_t. Note the explicit dependence of h on time, as specified in equation 3, so that h_t is dependent on q lags of the squared realizations of ε_t. (The homoskedastic model is a special case of the ARCH model when the parameters $\alpha_j = 0$ for $j > 0$.)

Equation 3 allows the variance h_t to be a function of past realizations of the disturbances, whereby the analysis can capture explicitly the possibility of phenomenon such as the clustering in time of large forecast errors. Such a phenomenon would be implied by finding that large past values of ε_t lead to a higher variance, h_t, and hence to a greater likelihood of a further large value of ε_t in the future.

It is important to note that the unconditional distribution of ε_t is not normal. For instance, the unconditional distribution of ε_t can be leptokurtic. The conditional distribution of ε_t and hence y_t is assumed to be normal, however, and thus the joint density is merely the product of the conditional densities. The log likelihood function (aside from a constant term) is given by:

(4) $L_T(b,\alpha) = \Sigma_{t=1}^{T} l_t$, where

(5) $l_t(b,\alpha) = (-1/2)[\log(h_t) + \varepsilon_t^2 h_t^{-1}]$.

Estimation of the ARCH model proceeds by choosing parameters b, α, that give the max-

[1]These include, in addition to the ARCH model, the GARCH or Generalized ARCH model and the ARCH-M or GARCH-M for ARCH in Mean models. Relevant citations are to Engle (1983), Bollerslev (1986) and Engle, Lilien and Robins (1987).

[2]For instance, the t-distribution (with finite degrees of freedom) is leptokurtic, as the tails of the distribution contain more mass than the tails of a standard normal distribution.

imum value for $L_t(b,\alpha)$, given the sample of T observations. In other words, we search for parameters b and α that maximize the probability of having observed the sample. Estimation is carried out by a numerical optimization procedure. In the case of the ARCH model, estimation is simplified somewhat by the fact that the two sets of parameters α, b are asymptotically independent, thereby allowing for maximization of $L_T(b,\alpha)$ with respect to each set of parameters separately.

The parameters α are restricted to be positive. As mandated by theoretical considerations, these restrictions preclude large realizations of ε_t from driving the variance negative. For stability, we also require that the sum of the α's is less than one. This is a necessary condition for restraining the unconditional variance to be finite.

In actual applications, it is desirable to be able to test for ARCH before specifying and estimating a model with ARCH. This is especially true because estimation of a model with ARCH involves nonlinear methods. Engle (1982) provides a straightforward test, the ARCH test, based on the Lagrange multiplier principle. As such, it requires only estimates of the homoskedastic model. The null hypothesis is homoskedasticity. The test is conducted by squaring the residuals from the homoskedastic model and regressing the squared residuals on various lags of the squared residuals. The test statistic is the sample size times the R^2 from this auxiliary regression, distributed as chi-square with degrees of freedom equal to the number of lags of the squared residuals included in the auxiliary regression. Large values for the test statistic lead to rejection of the null hypothesis of homoskedasticity and motivate estimation of an ARCH specification.

An important generalization of the ARCH model that we will employ in this paper is the ARCH-M model, that allows for the variance term h_t to enter the regression equation for y_t. The ARCH-M model is given by

$$(1')\ y_t = x_t'b + h_t^{.5}d + \varepsilon_t$$

$$(2')\ \varepsilon_t | I_{t-1} \sim N(0,h_t)$$

$$(3')\ h_t = \alpha_0 + \Sigma_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2, \qquad \alpha_j > 0 \text{ for all j.}$$

In equation (1'), d, a parameter to be estimated, measures the effect of the conditional variance on y_t. The term $h_t^{.5}$ entering equation 1 permits the conditional variance of the disturbance ε_t to affect the conditional mean of y_t. The form of the likelihood function for this model is the same as that given in equations 4 and 5 above, though clearly the parameter estimates will differ between the two models.

The ARCH-M model, by explicitly incorporating variance measures in the equation describing y_t, facilitates estimation and statistical inferences about the effects of variances on means.[3] For our purposes, the ARCH model allows the explicit parameterization and estimation of time-varying inflation uncertainty, defined as the conditional variance of the disturbance to an equation for the inflation rate. Further, with the ARCH-M generalization, we can estimate and test hypotheses about the effect of the time-varying inflation uncertainty on the conditional means of macroeconomic variables such as the inflation rate itself and the rate of growth of output.

[3]This model has been used by Engle, Lilien and Robins (1987) to estimate a model of the term structure in which the risk premium is modeled as time-varying and in which the risk premium affects the holding-period yield. The ARCH specification provides a way of estimating the time-varying risk premium, and the ARCH-M specification adds the ability to estimate the effect of the risk premium on the expected yield.

Table 1

Trend and Unit Root Tests
Sample: I/1960-II/1988

A. Unit root tests. Null hypothesis: Variable has a unit root.

Variable	Dickey-Fuller test t-ratio	Augmented Dickey-Fuller test t-ratio
log(p)	2.87	0.13
log(q)	−1.29	−1.06
log(m)	5.99	3.43
log(v)	−2.75 *	−2.10
$\triangle$log(p)	−4.16 ***	−2.18
$\triangle$log(q)	−8.09 ***	−5.11 ***
$\triangle$log(m)	−6.41 ***	−4.23 ***
$\triangle$log(v)	−7.52 ***	−4.39 ***

Approximate critical values for rejecting null hypothesis:

Significance	Critical value
10%	−2.58 *
5%	−2.89 **
1%	−3.17 ***

B. Tests for time trends. Null hypothesis: Variable has a unit root *and* no trend.

Variable	Dickey-Fuller Φ-statistic
log(p)	4.26
log(q)	2.98
log(m)	7.43
log(v)	1.41

Approximate critical values for rejecting null hypothesis:

Significance	Critical value
10%	5.47
5%	6.49
1%	8.73

mation on the hypothesis of a unit root in the inflation rate can be garnered from the empirical distributions of the Dickey-Fuller test statistic when the series has non-zero drift. These distributions have been tabulated by Schmidt (1988). For the inflation rate, the drift component would lead to a modification of the critical values tabulated by Dickey-Fuller, so that the 5 percent critical value is -2.11 and we reject the hypothesis of a unit root in the inflation series.[7]

The lag structure of the model was specified with the aid of the FPE (or Final Prediction Error) procedure.[8] Estimates of the model chosen under the assumption of homoskedasticity are provided in table 2. Diagnostic tests reported in table 3 indicate no statistically significant serial correlation and no significant evidence for a structural break in 1973, the approximate midpoint of the sample.[9] The ARCH test, also reported in table 3, rejects the null hypothesis of homoskedasticity for the inflation equation. There is little evidence for rejecting either a constant conditional variance of the disturbance to the output equation, or a constant covariance of disturbances to the output and inflation equations.

Given that the results of our specification tests indicated ARCH, at least for the inflation equation, we proceed to specify and estimate such a model. Since our concern is the effect of the variance of inflation on output growth, we allow the variance of inflation to enter the equations for inflation and output growth. As a further check of the specification, we also allow the variance of output growth to enter the inflation and output growth equations. That is, we specify an ARCH-M model. We can then directly estimate and test the hypotheses of interest.

Table 2
VAR Model of Output Growth and Inflation
Sample: I/1960-II/1988

$$DQ_t = \underset{(4.15)^{**}}{.00909} + \underset{(1.74)}{.164}\, DQ_{t-1} + \underset{(1.57)}{.144}\, DQ_{t-2} - \underset{(2.40)^{**}}{.310}\, DP_{t-1}$$

$$DP_t = \underset{(2.05)^{**}}{.00173} + \underset{(4.46)^{**}}{.413}\, DP_{t-1} + \underset{(2.21)^{**}}{.219}\, DP_{t-2} + \underset{(2.50)^{**}}{.232}\, DP_{t-3}$$

where the variance-covariance matrix of the disturbances is estimated to be

Var(eq) $= 1.86\ 10^{-5}$
Cov(ep,eq) $= -7.92\ 10^{-7}$
Var(ep) $= 8.73\ 10^{-5}$

and the log likelihood value is 830.1.

** indicates significance at the 5 percent level

The bivariate ARCH-M model for inflation (dp) and real output growth (dq) that we estimate is given as:[10]

$$(1)\ dp_t = \beta_{10} + \beta_{11}dp_{t-1} + \beta_{12}dp_{t-2} + \beta_{13}dp_{t-3} + \beta_{14}H_{p,t} + \beta_{15}H_{q,t} + \varepsilon_{p,t}$$

$$(2)\ dq_t = \beta_{20} + \beta_{21}dq_{t-1} + \beta_{22}dq_{t-2} + \beta_{23}dp_{t-1}\ \ \beta_{24}H_{p,t} + \beta_{25}H_{q,t} + \varepsilon_{q,t}$$

where

$$(3)\ H_{p,t} = \alpha_{10} + \alpha_{11}\ [\ \Sigma_{i=1}^{4}(5\text{-}i)\varepsilon^2_{p,t-i}\ /10] + \alpha_{12}\ [\ \Sigma_{i=1}^{4}(5\text{-}i)\varepsilon^2_{q,t-i}\ /10]$$

[7]Further evidence may be obtained by looking at related series. Money and velocity are related to the inflation series and output growth in a known way. We present evidence in table 1 that M1 money growth and velocity growth (defined as the first difference of the log of nominal GNP minus the log of M1) do not contain a unit root. Since the growth rate of velocity is, by definition, output growth plus inflation minus money growth, the growth rate of velocity should exhibit the properties of the component series. As Engle and Granger (1987) write, "Because of the relative sizes of the variances, it is always true that the sum of an I(0) and an I(1) will be I(1)" (p. 253). Thus, velocity growth as a linear combination of inflation, money growth and output growth should be I(1), or integrated of order 1, if any of the component series are I(1). Since the evidence indicates that the growth of velocity does not contain a unit root, i.e., is I(0), this is indirect evidence that inflation is also I(0). The only exception would be if the variables money, output and inflation were cointegrated. Tests of cointegration failed to detect such a relationship. Thus, we find that the inflation series is highly persistent, but not nonstationary.

[8]This approach was first suggested by Akaike (1969). Hsiao (1981) presents a strategy for applying the technique in a multivariate setting.

[9]This year also approximately divides the sample into the fixed or managed exchange-rate period before 1973 and the relatively flexible exchange-rate period after 1973, as well as dividing the sample into the pre-1973 period of no oil price shocks and the post-1973 period marked by a number of oil price shocks, both positive and negative.

[10]A dummy variable for the price-control period, taking the value of 1 when the controls were in place during III/1971-I/1973, was found to be statistically insignificant.

Table 3
Diagnostic Tests on VAR
Sample: I/1960-II/1988

A. Test for serial correlation

	Output growth equation		Inflation equation	
Order of serial correlation	Test statistic	Marginal significance	Test statistic	Marginal significance
1	0.15	.70	1.29	.26
2	0.15	.93	1.61	.45
3	2.22	.53	2.18	.54
4	2.33	.68	2.18	.70

B. Test for ARCH
Single equation tests

	Output growth variance		Inflation variance		Covariance, output growth and inflation	
Order of ARCH	Test statistic	Marginal significance	Test statistic	Marginal significance	Test statistic	Marginal significance
1	0.05	.82	0.31	.58	0.02	.88
2	0.07	.96	0.33	.85	2.01	.37
3	1.45	.69	12.96	.00	2.05	.56
4	2.07	.72	13.24	.01	2.05	.73
6	2.21	.90	15.84	.02	3.22	.78
8	9.59	.30	17.05	.03	4.11	.85

C. Test for structural change
Subsamples: I/1960-IV/1973, I/1974-II/1988

Likelihood ratio test statistic: 4.1 ~ x^2 (8) Marginal significance .85

$$(4)\ H_{q,t} = \alpha_{20} + \alpha_{21}\ [\ \Sigma_{i=1}^{4}(5\text{-}i)\varepsilon^2_{p,t-i}\ /10]$$

$$+\ \alpha_{22}\ [\ \Sigma_{i=1}^{4}(5\text{-}i)\varepsilon^2_{p,t-i}\ /10]$$

and

$$h_t = \begin{vmatrix} H_{p,t} & H_{pq} \\ H_{pq} & H_{q,t} \end{vmatrix}$$

This specification of the variance process, with the conditional variance modeled as a declining lag structure in the squared residuals, has been employed extensively in applications of the ARCH model, but it is restrictive. For example, this specification allows just one free parameter to be estimated on the four lagged squared residuals and imposes a linearly declining lag structure. Therefore, other specifications of $H_{p,t}$ and $H_{q,t}$ were tried. One alternative specification had separate coefficients on each of the four lags of $\varepsilon^2_{p,t}$ and $\varepsilon^2_{p,t}$. This alternative did increase the estimated log likelihood, but only the coefficients on $\varepsilon^2_{p,t-3}$ and $\varepsilon^2_{q,t-3}$ were statistically significant. Further, a likelihood ratio test between the model with only $\varepsilon^2_{p,t-3}$ and $\varepsilon^2_{q,t-3}$ affecting the variance of inflation and output growth, respectively, and a model with all four lags of $\varepsilon^2_{p,t}$ and $\varepsilon^2_{q,t}$ in the respective variance equations, indicated no support for the additional lags. Also, lagged $H_{p,t}$ and $H_{q,t}$ were added to the variance specifications (yielding the generalization of the ARCH model called GARCH) and again the estimated log likelihood function did not increase significantly.

Table 4
ARCH-M Model of Output Growth and Inflation Sample: I/1960-II/1988

$$DQ_t = \underset{(.54)}{-.0109} + \underset{(2.33)}{.172}\, DQ_{t-1} + \underset{(2.33)}{.149}\, DQ_{t-2} - \underset{(3.55)}{.410}\, DP_{t-1} + \underset{(.93)}{2.53}\, Var(eq)^{.5} - \underset{(.13)}{.46}\, Var(ep)^{.5}$$

$$DP_t = \underset{(.38)}{-.0027} + \underset{(5.17)}{.384}\, DP_{t-1} + \underset{(2.79)}{.205}\, DP_{t-2} + \underset{(3.63)}{.245}\, DP_{t-3} + \underset{(.20)}{.188}\, Var(eq)^{.5} + \underset{(.48)}{.696}\, Var(ep)^{.5}$$

where the variance-covariance matrix of the disturbances is estimated to be:

$$Var(eq_t) = 6.99 \quad 10^{-5} + \underset{(1.85)}{.131} \sum_{i=1}^{4} [(5-i)eq^2_{t-i}/10] + \underset{(.72)}{.203} \sum_{i=1}^{4} [(5-i)ep^2_{t-i}/10]$$

$$CoV(ep_t, eq_t) = 3.39 \quad 10^{-7}$$

$$Var(ep_t) = 1.40 \quad 10^{-5} + \underset{(2.67)}{.244} \sum_{i=1}^{4} [(5\text{-}i)\, ep^2_{t-i}/10] + \underset{(.00)}{.0000} \sum_{i=1}^{4} [(5-i)eq^2_{t-i}/10]$$

and the log likelihood is 835.9.

Likelihood ratio test against homoskedastic VAR: $11.6 \sim X^2(8)$ (Marginal significance .17)

Estimates of the model in equations 1-4 are reported in table 4.[11] The coefficients on the conditional variance terms entering the output growth and inflation equations are insignificant at the 5 percent level. In addition, the lags of the output growth residuals have an insignificant coefficient in the inflation variance equation. Moreover, the lags of the inflation residuals have an insignificant coefficient in the output variance equation. Finally, a likelihood ratio test of the model reported in table 4 against the homoskedastic model reported in table 2 indicates that the null hypothesis, that the homoskedastic model is a valid restriction to the ARCH-M model, cannot be rejected at any reasonable significance levels. These results indicate that inflation uncertainty, measured as the conditional variance of inflation from an ARCH specification, does not have a significant effect on output growth.

To determine the sensitivity of the results to the model specification, we modified the model to include only the third lag of the squared inflation residual in the inflation variance equation and only the third lag of the squared output growth residual in the output variance equation. This specification was chosen from a preliminary model including separate coefficients on each of the four lags of the squared residuals in each variance equation. Estimates are reported in table 5. The estimated log likelihood function of this specification is nearly equivalent numerically (and certainly not statistically distinguishable) from the more general model. A likelihood ratio test against the homoskedastic VAR model leads to rejection at the 5 percent significance level of the null hypothesis that the homoskedastic VAR restrictions are correct relative to the ARCH-M alternative.[12]

[11]To estimate the ARCH-M model, indeed all the ARCH estimates reported in this paper, the ARCH parameters α_{11}, α_{12}, α_{21} and α_{22} were restricted to be non-negative. The shaded insert discusses the rationale for this restriction.

[12]One caveat to the interpretation of the likelihood ratio tests reported here, indeed to most of the statistical inference drawn in this paper, is that considerable pretesting was done in specifying both the VAR and ARCH models. This greatly complicates the inference problem. A good introduction to this issue is provided in Judge, et al (1988).

Table 5

ARCH-M Model of Output Growth and Inflation
Sample: I/1960-II/1988

$$DQ_t = -\ .0186 + .136\ DQ_{t-1} + .125\ DQ_{t-2} - .384\ DP_{t-1}$$
$$\qquad (0.89) \quad (2.00) \qquad (2.01) \qquad (4.29)$$

$$+\ 2.98\ [\mathrm{Var}(eq_t)]^{.5} + .474\ [\mathrm{Var}(ep_t)]^{.5}$$
$$(1.22) \qquad\qquad (0.46)$$

$$DP_t = .0047 + .345\ DP_{t-1} + .248\ DP_{t-2} + .296\ DP_{t-3}$$
$$(0.89) \quad (5.73) \qquad (4.01) \qquad (4.61)$$

$$-\ .582\ [\mathrm{Var}(eq_t)]^{.5} + .493\ [\mathrm{Var}(ep_t)]^{.5}$$
$$(0.98) \qquad\qquad (1.23)$$

where the variance-covariance matrix of the disturbances is estimated to be

$$\mathrm{Var}(eq_t) = 7.45\ 10^{-5} + .100\ eq_{t-3}^2$$
$$(2.82)$$

$$\mathrm{Cov}(eq_t, ep_t) = 1.16\ 10^{-6}$$

$$\mathrm{Var}(ep_t) = 1.26\ 10^{-5} + .301\ ep_{t-3}^2$$
$$(6.31)$$

and the log likelihood value is 840.9.

Likelihood ratio test against homoskedastic VAR: 21.6 $\sim X^2(6)$ (Marginal significance .001)

Likelihood ratio test against ARCH VAR: 4.0 $\sim X^2(4)$ (Marginal significance .21)

The estimated parameter values and the asymptotically valid t-statistics reported in table 5 provide further information about the hypotheses of interest. Table 5 shows that the variance of inflation had a positive but statistically insignificant effect on the rate of growth of output and a positive but statistically insignificant effect on the rate of inflation. These results provide no support for the hypotheses under investigation. We also find that the variance of output has an insignificant positive effect on the rate of growth of output and an insignificant negative effect on the rate of inflation.

Table 5 also reports estimates of the variance process. The third lag of squared realizations of the stochastic error in the inflation equation has a statistically significant effect on the conditional variance of the inflation error. In contrast, the lagged squared realization of the stochastic error in the output growth equation has a statistically insignificant effect on the conditional variance of output growth.

Table 5 provides no support for the hypotheses that inflation uncertainty, measured as the conditional variance of inflation forecast errors, has a negative effect on output growth. Indeed, of the six coefficients estimated for the ARCH-M model that were not estimated for the homoskedastic VAR model, five were statistically insignificant, including all of those measuring the effect of the conditional variance of inflation on the inflation rate and the rate of output growth. This observation leads to the suspicion that it is only the ARCH process itself that is important in the rejection of the VAR restrictions by the likelihood ratio test, a suspicion confirmed by estimation of an ARCH variant of the model in table 6. The ARCH model includes the conditional variance specification as in table 5, but does not allow the conditional variance to affect the conditional mean of the inflation process or the rate of output growth. Estimates of this model are reported in table 6.

In table 6 we see that the likelihood value is almost as high as that reported in table 5. A likelihood ratio test does not reject the null hypothesis that the ARCH model is a valid restriction to the ARCH-M model. Moreover, a likelihood ratio test of the null hypothesis of the

Table 6
ARCH Model for Output Growth and Inflation

$$DQ_t = \underset{(6.55)}{.00947} + \underset{(2.42)}{.157}\, DQ_{t-1} + \underset{(2.09)}{.129}\, DQ_{t-2} - \underset{(3.97)}{.353}\, DP_{t-1}$$

$$DP_t = \underset{(2.67)}{.00147} + \underset{(5.84)}{.352}\, DP_{t-1} + \underset{(4.26)}{.262}\, DP_{t-2} + \underset{(4.24)}{.268}\, DP_{t-3}$$

where the variance-covariance matrix of the disturbances is estimated to be

$$Var(eq_t) = 7.43\ 10^{-5} + \underset{(2.32)}{.125}\, eq^2_{t-1}$$

$$Cov(eq_t, ep_t) = 1.10\ 10^{-6}$$

$$Var(ep_t) = 1.30\ 10^{-5} + \underset{(5.76)}{.285}\, ep^2_{t-1}$$

and the log likelihood value is 838.9.

Likelihood ratio test against homoskedastic VAR: 17.6 ~ $X^2(2)$ (Marginal significance .0001)

homoskedastic VAR model against the ARCH alternative leads to a strong rejection of the null. It seems that the inflation-output growth process has ARCH disturbances, but that the changing conditional variance does not feed back to the inflation rate or the rate of output growth.

FURTHER PROBLEMS AND PROSPECTS

The evidence presented here lends no support to the hypothesis that uncertainty about the future inflation rate leads to a reduction in the rate of output growth. Further, this evidence, in accord with that provided by both Katsimbris and Thornton using an alternative methodology, casts doubt on the existence and relevance of the hypothesized positive relation between the rate of inflation and the uncertainty about future inflation.

One possible explanation for this lack of support is that the inflation rate was largely predictable over our sample. Indeed, it is difficult to detect much of an ARCH effect in the inflation data over this span, especially when the inflation forecasting equation is supplemented with other exogenous variables, most notably relative energy prices. Several recent studies, including Engle (1983), Holland (1984), Cosimano and Jansen (1988), and Rich, Kanago and Raymond (1988), all report either difficulty in detecting ARCH in the inflation equation or estimates of the ARCH conditional variance that are very flat over this period. This study identifies an ARCH inflation process, but the process may not have been sufficiently variable to generate precise measures of the effect of the conditional variance of inflation on output growth.

Because this study is limited to investigating the first two moments of the bivariate inflation rate-output growth rate process, it abstracts from some potentially important issues, one of which is the importance of relative energy prices after the 1973 oil price shock. Of perhaps more importance is the neglect of a measure of the mean and variance of the policy stance of the monetary authority. Uncertainty about the future inflation rate can arise from several sources, including uncertainty about future government policy or future values of exogenous variables impinging on the inflation rate. A measure of government policy, perhaps by some monetary aggregate, might be useful to supplement results from the bivariate system reported here.

REFERENCES

Akaike, Hizotogu. "Fitting Autoregressive Models for Prediction," *Annals of the Institute of Statistical Mathematics* (January 1969), pp. 243-47.

Blejer, Mario I., and Leonardo Leiderman. "On the Real Effects of Inflation and Relative-Price Variability: Some Empirical Evidence," *Review of Economics and Statistics* (November 1980), pp. 539-44.

Bollerslev, Tim. "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics* (April 1986), pp. 307-27.

Cosimano, Thomas F., and Dennis W. Jansen. "Estimates of the Variance of U.S. Inflation Based upon the ARCH Model: Comment," *Journal of Money, Credit, and Banking* (August 1988, Part 1), pp. 409-21.

Coulson, N. Edward, and Russell P. Robins. "Aggregate Economic Activity and the Variance of Inflation: Another Look," *Economic Letters* (January 1985), pp. 71-75.

Cukierman, Alex. "Relative Price Variability and Inflation: A Survey and Further Results," *Carnegie-Rochester Series on Public Policy* (Autumn 1983), pp. 103-58.

Cukierman, Alex, and Paul Wachtel. "Differential Inflationary Expectations and the Variability of the Rate of Inflation: Theory and Evidence," *American Economic Review* (September 1979), pp. 595-609.

Dickey, David A., and Wayne A. Fuller. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica* (July 1981), pp. 1057-72.

________. "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association* (June 1979), pp. 427-31.

Engle, Robert F. "Estimates of the Variance of U.S. Inflation Based upon the ARCH Model," *Journal of Money, Credit, and Banking* (August 1983), pp. 286-301.

________. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica* (July 1982), pp. 987-1008.

Engle, Robert F., and C. W. J. Granger. "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica* (March 1987), pp. 251-76.

Engle, Robert F., David M. Lilien, and Russell P. Robins. "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model," *Econometrica* (March 1987), pp. 391-407.

Evans, Paul. "Price-Level Instability and Output in the U.S.," *Economic Inquiry* (April 1983), pp. 172-87.

Fischer, Stanley. "Towards an Understanding of the Costs of Inflation: II,"Carnegie-Rochester Conference Series on Public Policy (Autumn 1981), pp. 5-42.

Friedman, Milton. "Nobel Lecture: Inflation and Unemployment," *Journal of Political Economy* (June 1977), pp. 451-72.

Froyen, Richard T., and Roger N. Waud. "An Examination of Aggregate Price Uncertainty in Four Countries and Some Implications for Real Output," *International Economic Review* (June 1987), pp. 353-72.

Gale, William A. "Temporal Variability of United States Consumer Price Index," *Journal of Money, Credit, and Banking* (August 1981), pp 273-97.

Godfrey, L. G. "Testing Against General Autoregressive and Moving Average Error Models when the Regressors Include Lagged Dependent Variables," *Econometrica* (November 1978), pp. 1293-1301.

Hafer, R. W. "Inflation Uncertainty and a Test of the Friedman Hypothesis," *Journal of Macroeconomics* (Summer 1986), pp. 365-72.

Holland, A. Steven. "Does Higher Inflation Lead to More Uncertain Inflation?" this *Review* (February 1984), pp. 15-26.

Hsiao, Cheng. "Autoregressive Modelling and Money-Income Causality Detection," *Journal of Monetary Economics* (January 1981), pp 85-106.

Judge, George G., R. Carter Hill, William E. Griffiths, Helmut Lutkepohl, and Tsoung-Chao Lee. *Introduction to the Theory and Practice of Econometrics* (John Wiley and Sons, 1988).

Katsimbris, George M. "The Relationship between the Inflation Rate, Its Variability, and Output Growth Variability: Disaggregated International Evidence," *Journal of Money, Credit, and Banking* (May 1985), pp. 179-88.

Logue, Dennis E., and Richard J. Sweeney. "Inflation and Real Growth: Some Empirical Results," *Journal of Money, Credit, and Banking* (November 1981), pp. 497-501.

Marshall, Alfred. "Answers to Questions on the Subject of Currency and Prices Circulated by the Royal Commission on the Depression of Trade and Industry," *Official Papers of Alfred Marshall* (London: McMillan, 1926).

Mullineaux, Donald J. "Unemployment, Industrial Production, and Inflation Uncertainty in the United States," *Review of Economics and Statistics* (May 1980), pp. 163-69.

Okun, Arthur M. "The Mirage of Steady Inflation," *Brookings Papers on Economic Activity* (2: 1971), pp. 485-98.

Pagan, A. R., A. D. Hall, and P. K. Trivedi. "Assessing the Variability of Inflation," *Review of Economic Studies* (October 1983), pp. 585-96.

Rich, Robert W., Bryce Kanago, and Jennie Raymone. "New Evidence on the Variance of U.S. Inflation Based Upon the ARCH Model," Vanderbilt University Working Paper No. 88-W06, October 1988.

Schmidt, Peter. "Dickey-Fuller Tests with Drift," unpublished manuscript, Michigan State University, June 1988.

Taylor, John B. "On the Relation between the Variability of Inflation and the Average Inflation Rate," *Carnegie-Rochester Series on Public Policy* (Autumn 1981), pp. 57-85.

Thornton, John. "Inflation and Output Growth: Some Time Series Evidence, A Note," *American Economist* (Fall 1988), pp 55-58.