A Microeconomic System-Wide Approach to the Estimation of the Demand for Money

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A STABLE demand-for-money relationship is a necessary condition for the viability of a monetary policy based on the use of monetary aggregates as intermediate policy targets. In recent years, standard money-demand formulations have exhibited large shifts that remain largely unexplained today despite extensive research efforts devoted to determining the reasons for these shifts.

This paper presents an alternative to the standard single-equation method of estimating the demand for money. The alternative, called the microeconomic system-wide approach to demand analysis, differs in several fundamental ways from the usual money-demand specification. The purpose of this article is to show how the system-wide approach can be applied to estimating the demand for money. The results indicate that in-sample predictions made using this approach closely track the actual data over the 1969–85 period.

THE STANDARD MONEY DEMAND FORMULATION: A BRIEF REVIEW

Over the past three decades, most demand-for-money studies have employed similar specifications. Typically they use income (as a transaction variable) and one or more (typically two) interest rates (to capture the effect of the opportunity cost of holding money) as explanatory variables; the dependent variable is generally the stock of real M1 balances.

The wide acceptance of the standard money demand specification is understandable. It embodies a proposition, which, since Keynes' *General Theory*, has constituted a key tenet of the received wisdom on the demand for money: the desire to hold money balances is directly related to the need to conduct transactions and inversely related to the opportunity cost of holding money balances. In addition, it performed remarkably well in a statistical sense. The coefficients of the explanatory variables had "sensible" signs and magnitudes, and the estimated model fit the data very well.

The disquietude accompanying Goldfeld's (1976) discovery that his standard formulation of the demand-for-money function began in 1973 to systematically overpredict the real money balances underscores the importance that has been attached to the stability, and hence predictability, of the demand for money. It is not surprising that the reported shift in Goldfeld's specification, or what, after 1976, became generally known as the "case of the missing money," instigated a seemingly tireless search for a verifiable explanation of what happened.[1]

A review of the vast literature devoted to finding the reasons for the shift in money demand reveals that these studies are largely unsuccessful in accounting

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[1]A recent study suggests that the demand-for-money function has undergone shifts in the periods III/1962, IV/1973, IV/1979, and I/1980. See Mizrach and Santomero (1986).

for it. For example, Laidler has commented that:

> The first thing to be said . . . is whatever else they do, they do not rescue the demand for M, function from the suspicion of instability. . . . [T]he often unsatisfactory results . . . indicate that further work is required, rather than that the line of inquiry that they represent should be abandoned.[2]

The inconclusiveness of the evidence on what caused the standard money-demand specification to shift in 1973 can be viewed as an indication that examining alternative approaches to the demand-for-money formulation might be useful. The alternative offered here is derived from a microeconomic system-wide approach to demand analysis.

A NEW APPROACH TO MODELING MONEY DEMAND: THE MICROECONOMIC SYSTEM-WIDE APPROACH

The basic premise which underlies any micro theoretic approach to consumer demand analysis is that the consumer maximizes a neoclassical utility function subject to a budget constraint.[3] A model consistent with both the principles of microeconomic theory and aggregation theory yields specific behavioral implications which can be tested using available data aggregated over goods and consumers. (Some criticisms of including money in this approach appear on page 24.)

This study uses a neoclassical utility function defined over five expenditure categories, two of which are presumed to capture the "monetary services" in the U.S. economy. By restricting the analysis to five expenditure categories, this study assumes the existence of a macro utility function that is weakly separable in these categories.[4]

The solution to the consumer choice problem when the utility function is defined over five goods is a system of five demand equations. In each equation, the quantity demanded of a specific good is expressed as a function of the total amount available for spending on all five goods, and (in the general case) of their prices. Naturally, the exact specification of these demand equations will depend on the specific form of the utility function chosen. This study, however, uses a general demand system consistent with the maximization of an arbitrary neoclassical utility function. Thus, while the system is subject to all the restrictions that economic theory implies, the results are invariant to the functional form of the utility function being maximized. This choice avoids the loss of generality which may result if a particular functional form is specified and permits testing of hypotheses about the structure of the utility function itself.

The microeconomic system-wide approach to demand analysis deals with the allocation of total spending among the individual goods considered. Thus, for the specific set of goods chosen, the explanatory variables in the demand system are the amount available for spending and the prices of these goods. This approach provides a convenient means for acquiring detailed information about utility-based attributes of goods; this information is readily available by inspecting the signs (and, of course, the statistical and economic significance) of estimated income, own- and cross-price parameters.

The demand model used in this study, the absolute price version of the Rotterdam model, was chosen primarily because the theoretical restrictions are readily expressed in terms of the model's parameters. This makes it relatively easy to impose and to test the validity of these restrictions.[5] Another attractive feature of the Rotterdam model is that it can be used to provide predictions of the value (budget) shares of the goods included in the analysis. These predictions can be used to compute measures of informational inaccuracies useful in assessing the performance of the demand system as a whole and the individual demand equations as well.

The monetary variables used are the real flow of monetary services provided by various monetary assets, not the simple sum of the real stocks of monetary aggregates generally used in standard money-demand analysis. A measure of the monetary-service flows was obtained by evaluating the stocks of monetary assets at their corresponding user-cost prices (see the discussion of the data below). The user-cost price of each monetary asset is the difference between the interest

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[2]See Judd and Scadding (1982), p. 1014.

[3]A neoclassical utility function is one that is continuously twice differentiable and quasiconcave with positive marginal utility everywhere.

[4]A neoclassical utility function is weakly separable in a block of goods if and only if the marginal rate of substitution between any two goods inside the block is independent of consumption outside that block. While this separability assumption may seem overly restrictive, it is actually less restrictive than that maintained by studies in which money is considered to be the sole argument in the utility function. See, for example, Ewis and Fisher (1984).

[5]In the aggregated-over-consumers version of the model derived by Barnett (1979), the macro parameters are subject to the same restrictions as their micro counterparts. (See footnote 9.)

Can Money Be Included in a Microeconomic System-Wide Demand Model?

The applicability of the theoretical restrictions in the general case of demand for money and goods has been questioned. In fact, if, as in the Samuelsonian tradition, the utility-based analysis of the demand for money is handled by putting money and prices in the utility function, then, by the strong results produced by Samuelson and Sato (1984), the restrictions in question, *as they pertain to the demand for goods,* are simply unattainable. While potentially disquieting, the Samuelson-Sato results are not unqualifiedly binding. Indeed, these results are founded in the view, long espoused by Samuelson, that, in connection with the inclusion of money in the preference structures, money is wanted solely for the purposes of facilitating transactions. As Samuelson (1983), p. 117, states:

> In this connection, I have reference to none of the tenuous concepts of money, as a numeraire commodity, or as a composite commodity, but to money proper, the distinguishing features of which are its indirect usefulness, not for its own sake but for what it can buy, its acceptability, its not being "used up" by use, etc., etc.

This is the rationale behind Samuelson's inclusion of prices and money in the utility function specified to be homogeneous of degree zero in both money and prices. It is precisely this formulation to which the Samuelson-Sato results pertain.

One could argue that Samuelson's view of what money is wanted for is unduly restrictive. In fact, of the assets currently regarded as potential sources of monetary services in the U.S. economy (see table 1), only a few are "generally acceptable in exchange."[1] Furthermore, the supposition, based on Samuelson's view, that money cannot properly be treated like other commodities can also be questioned. Households consume the services provided by various expenditure categories ostensibly because of the utility they derive from these expenditure categories: in general, little, if any, effort is directed toward deciphering the nature of the utility involved in those cases. By the same token, it can be maintained that money is held because of the utility it provides, without having to speculate as to whether that utility derives from money's "general acceptability in exchange," the serenity its holders experience by holding it, or from any other knowable or even unknowable attribute.[2] Indeed, if money can be treated like other goods, then it can be included in the utility function in precisely the same manner as any other good. In that case, the Samuelson-Sato results would not apply, and one could thus impose or test for any of the restrictions implied by economic theory.

Interestingly, even within the Samuelson-Sato framework, the theoretical restrictions would not be unattainable if the utility function were weakly separable in the block of goods (see Samuelson and Sato (1984), pp. 592–95). Hence, it is legitimate to impose or test for any of the restrictions implied by theory if one assumes, or, even better, tests for and (where applicable) imposes blockwise weak separability in goods. The latter was done in this study, since weak separability in the block of goods could not be statistically rejected.[3]

[1]In this study, the Federal Reserve's definition of monetary assets was taken as given. The use of the Fed's definitions does not mean that the list of assets which appears in table 1 includes all assets that provide monetary services in the U.S. economy or, for that matter, that all assets included in the list provide such services.

[2]Of course, this amounts to suggesting that money is held for the "moneyness" of it. While tautological, this statement can be made operational by hypothesizing that, on the margin, the extent to which income is forgone when monetary assets are held is a measure of the moneyness that these assets possess. The gain that is realized by adopting this hypothesis is considerable; not only does it play a key role in the measurement of the flow of monetary services in terms of readily observable data, it also inherently captures the various degrees of moneyness provided by various monetary assets.

[3]The manner in which weak separability was tested for is discussed at length in Fayyad (1986), chapter 4. A preliminary draft of a paper on this subject is available on request.

rate paid on that asset and the maximum available holding-period yield.[6]

THE MODEL

For n goods, the (discrete-time) absolute price version of the Rotterdam model is given by

$$(1)\quad w_{it}^{*}\, Dx_{it} = \mu_i\, Dm_t^{*} + \sum_{j=1}^{n} \pi_{ij}\, Dp_{jt} + \varepsilon_{it} \qquad i=1, ..., n.^{7}$$

The x's and p's denote the quantities and prices of the various goods, respectively, and the subscript t indexes time. D is the log-change operator; thus $Dx_{it} = \Delta(\log x_{it})$. w_i denotes the expenditure (value) share of the ith good. $w_{it}^{*} = (w_{i,t-1} + w_{it})/2$ is that good's average value share over two successive time periods. Thus, the dependent variables in the model are the (average) share-weighted growth rates of the quantities of goods. The explanatory variables are the growth rates of real income (m_t^{*}) and prices.[8] The last term in the system of demand equation 1, ε_t, denotes the demand disturbance. The properties of this term are discussed in appendix A in conjunction with the estimation procedure employed in this study.

The parameters of the model are μ_i and π_{ij}. μ_i $(= p_i \frac{\partial x_i}{\partial m})$ is the *marginal* budget share of the ith good. π_{ij} $(= \frac{p_i p_j}{m} \frac{\partial x_i}{\partial p_j}$, $(j = 1, ..., n))$ is the ith good's price coefficient. Under the standard assumptions of economic theory, if the consumer maximizes a neoclassical utility function subject to a budget constraint, then the above parameters satisfy the following constraints:

$$(2)\quad \sum_i \mu_i = 1\,,$$

$$(3)\quad \sum_j \pi_{ij} = 0\,,$$

(4) $[\pi_{ij}]$ is symmetric and negative semidefinite.[9]

In the estimation procedure employed in this study, the constraints $\Sigma\mu_i = 1$, $\Sigma_j\pi_{ij} = 0$, and $[\pi_{ij}] = [\pi_{ji}]$ were imposed. The negative semidefiniteness of the $[\pi_{ij}]$ matrix was not imposed; however, it was checked for.

THE DATA

The data consist of U.S. quarterly time series of expenditures on, and prices of, food, nondurables, services and two blocks of monetary assets for the period I/1969–I/1985. Together, the two blocks of monetary assets, M1 and ABM1, comprise the 27 assets that the Federal Reserve Board currently recognizes as potential sources of monetary services in the U.S. economy. M1 is the narrow monetary aggregate, consisting of currency and total checkable deposits. ABM1 consists of the non-M1 monetary assets shown in table 1.

Data on the first three commodity groups (food, nondurables and services) were obtained as follows: A time series on the price of each commodity group (p_{it}) was generated from available time series on current-dollar and (1972) constant-dollar consumption expenditures ($p_{it}\, q_{it}$ and $p_{i,72}\, q_{it}$, respectively) and the identity $(p_{it}/p_{i,72}) = (p_{it}\, q_{it}/p_{i,72}\, q_{it})$. Per-capita constant-dollar expenditures in each quarter were then obtained by dividing the aggregate constant-dollar expenditures by the corresponding mid-quarter population size (N_t). Thus, in terms of the variables which appear in the estimated system, $x_{it} = \frac{1}{N_t} p_{i,72}\, q_{it}$.

One can generate the data on the quantities and prices of M1 and ABM1 monetary services as follows:

(1) Convert the nominal balances of monetary assets into real balances by deflating the former by the "true cost-of-living index." In this study, this index was the geometric mean of the Consumer Price Index and the Commerce Department's implicit

[6]The maximum available holding-period yield is the highest yield of those available either on the monetary assets or on Baa-rated bonds.

[7]A detailed discussion of the model's derivation and applications can be found in Theil (1971, 1975, 1976, 1980), Barten (1969), and Barnett (1979, 1981).

[8]In this study, the terms "expenditure" and "income" are used interchangeably. When the latter is used, however, it means "full income," that is, income augmented by expenditure on the monetary assets that are included in this study. In the estimation procedure employed in this study, Dm_t^{*} is replaced by Dx_t, where $Dx_t = \sum_i w_{it}^{*}\, Dx_{it}$. See Theil (1971), pp. 331–32.

[9]A question may arise as to whether these restrictions are applicable, given that the data are aggregated over both goods and consumers. Insofar as goods are concerned, Hicks' composite commodity theorem can be used, assuming that each of the commodity groups is an elementary good. Resolving the more formidable issue of aggregation over consumers requires using the aggregation results produced by Barnett (1979). In his aggregated-over-consumers absolute price version of the Rotterdam model, Barnett treats the macrocoefficients, μ and π, as population versions of weighted average microcoefficients, with the weights proportional to corresponding incomes. He then shows that the macrocoefficients have the same properties as their micro counterparts, μ_i and π_{ij}.

Table 1
Potential Monetary Assets

Component	Asset Description
1	Currency and traveler's checks
2	Demand deposits held by households
3	Demand deposits held by business firms
4	Other checkable deposits less Super NOW accounts
5	Super NOW accounts at commercial banks
6	Super NOW accounts at thrifts
7	Overnight repurchase agreements
8	Overnight Eurodollars
9	Money market mutual fund shares
10	Money market demand deposit accounts at commercial banks
11	Money market demand deposit accounts at thrifts
12	Savings deposits less MMDAs at commercial banks
13	Savings deposits less MMDAs at savings and loans
14	Savings deposits less MMDAs at mutual savings banks
15	Savings deposits less MMDAs at credit unions
16	Small time deposits and retail repurchase agreements at commercial banks
17	Small time deposits and retail repurchase agreements at thrifts
18	Small time deposits at credit unions
19	Large time deposits at commercial banks
20	Large time deposits at thrifts
21	Institutional money market mutual funds
22	Term repurchase agreements at commercial banks and thrifts
23	Term Eurodollars
24	Savings bonds
25	Short-term Treasury securities
26	Banker's acceptances
27	Commercial paper

price deflator for personal consumption expenditures.

(2) Evaluate the real balances of each monetary asset in the base period at its real user-cost price to obtain the real expenditure on that asset during the base period.[10]

(3) Sum the expenditures thus obtained over the components of M1 and ABM1.

(4) Compute the Törnqvist-Theil Divisia quantity indexes for M1 and ABM1 for the entire sample period.

(5) Set the base-period expenditures obtained in (3) equal to the respective quantity indexes computed in (4) and interpolate to acquire complete series on the real expenditures on the monetary services provided by M1 and ABM1.

(6) Construct Törnqvist-Theil Divisia price indexes of the user-cost prices of the respective components of M1 and ABM1.

THE ESTIMATION RESULTS

The maximum likelihood estimates of the parameters of the absolute price version of the Rotterdam model and the associated income and price elasticities are reported in table 2.[11] Estimates of the income coefficients (μ_i) are all positive and statistically significant at usual significance levels, indicating that the five commodity groups included in this study are normal goods.

The income elasticity of demand for M1 shown in table 2 (0.53) is similar to those reported in other studies. Moreover, it corresponds closely to its theoretical value of 0.50 implied by the Baumol (1952)–Tobin (1956) inventory — theoretic model of the transactions demand for money.

The own- and cross-price coefficients (π_{ij}) are generally estimated with less precision than the income coefficients. Consistent with the standard assumptions of economic theory, estimates of the (Slutsky) own-price coefficients are negative, although not all are statistically significantly different from zero at usual significance levels.[12]

[10]The user-cost price of money was derived by Barnett (1978). See also Barnett (1986) and (1981), chapter 7.

[11]The income elasticity of demand for the ith commodity group, μ_i, is given by $\mu_i = \frac{\mu_i}{w_i}$. This result can be verified by a simple manipulation of the definition $\mu_i = \frac{\partial x_i}{\partial m}$. On the other hand, the Hicks-Allen price elasticity of demand for the ith group, μ_{ij}, is given by $\mu_{ij} = \frac{\pi_{ij}}{w_i}$, which can also be verified by a simple manipulation of the definition $\pi_{ij} = \frac{p_i p_j}{m} \frac{\partial x_i}{\partial p_j}$.

[12]Negativity of the own-price coefficients, the diagonal elements in the $[\pi_{ij}]$ matrix, is a necessary, but not sufficient, condition for it to be negative semidefinite; a matrix is negative semidefinite if and only if all of its characteristic roots are nonpositive, and at least one root is zero. This property, which was not imposed in this study, was examined by computing the characteristic roots of the estimates of the $[\pi_{ij}]$ matrix in table 2. The computed characteristic roots are (0.0000, 0.0000, −0.0022, −0.0097, −0.1743); thus, the negativity condition is satisfied.

Table 2

Maximum Likelihood Estimates of the Absolute Price Version of the Rotterdam Model

Equation	μ_i	π_{ij}				
		Food	Nondurables	Services	ABM1	M1
Food	0.187083	−0.116241	0.058199	0.057593	0.000352	0.000097
	(6.682878)	(−7.380222)	(4.867328)	(3.167629)	(2.913345)	(1.779196)
Nondurables	0.291246	0.058199	−0.034326	−0.024572	0.000549	0.000150
	(9.685506)	(4.867328)	(−1.918331)	(−1.228305)	(2.129760)	(1.319010)
Services	0.467938	0.057593	−0.024572	−0.034144	0.000882	0.000242
	(10.304847)	(3.167629)	(−1.228305)	(−1.135707)	(1.596066)	(1.686502)
ABM1	0.042172	0.000352	0.000549	0.000882	−0.001399	−0.000384
	(2.090867)	(2.913345)	(2.129760)	(1.596066)	(−2.558969)	(−1.832747)
M1	0.011561	0.000097	0.000150	0.000242	−0.000384	−0.000105
	(3.043011)	(1.779196)	(1.319010)	(1.686502)	(−1.832747)	(−0.276950)

Note: t-ratios are in parentheses.

Average Income Elasticities

Food	Nondurables	Services	ABM1	M1
0.828044	1.390072	0.942341	1.027321	0.525813

Average Hicks-Allen Elasticities

	Food	Nondurables	Services	ABM1	M1
Food	−0.514493	0.257394	0.254911	0.001560	0.000428
Nondurables	0.277775	−0.163834	−0.117278	0.002619	0.000718
Services	0.115982	−0.049483	−0.068760	0.001775	0.000487
ABM1	0.008585	0.013365	0.021474	−0.034071	−0.009354
M1	0.004394	0.006841	0.010991	−0.017464	−0.004763

The Model's In-Sample Predictive Performance

As stated earlier, the Rotterdam model can be used to provide predictions of the value (budget) shares. The model's implied prediction of the share of the ith good at time t is given by

$$\hat{w}_{i,t+1} = w_{it} - e_{it},$$

where w_{it} is the actual value share of the ith good, and e_{it} is the residual of the ith demand equation at time t.

The in-sample predictive performance of the Rotterdam model can be evaluated in terms of its information-theory results; a general discussion of this method of assessing prediction accuracy is presented in appendix B. Computed measures of information (prediction) inaccuracies from the model are reported in table 3, along with information-inaccuracy measures for a naive (no-change) extrapolation of the value shares. The reported measures show substantial reductions in information inaccuracies when the model results are compared with the naive predictions.[13]

Further insight into the model's in-sample predictions may be gained by plotting the actual and predicted shares; this is done in charts 1–5. An inspection of these charts reveals that the model's in-sample predictions track the data extremely well; this is especially true for the M1 equation despite considerable variability in the actual shares of M1. These results suggest that the demand for M1, as derived in this

[13]In fact, in view of the greater variability of the shares of M1 and ABM1 relative to the shares of the other goods and services shown in charts 1–5, it is not surprising that predictions from the money equations "beat" the naive model by a larger margin than predictions from the other equations. In the presence of high period-to-period variation in the actual shares, the no-change naive model will always perform poorly.

Table 3

Average Information Inaccuracies[1]

	Rotterdam Model	Naive
System Results		
Uncorrected information inaccuracy	14.40	452.60
Information inaccuracy with d.f. correction	14.82	
Percent reduction from naive	96.73%	
Single Equation Results		
Food information inaccuracy	3.505	12.18
Percent reduction from naive	71.22%	
Nondurables information inaccuracy	4.141	11.95
Percent reduction from naive	65.35%	
Services information inaccuracy	5.943	34.54
Percent reduction from naive	82.79%	
ABM1 information inaccuracy	5.083	381.30
Percent reduction from naive	98.67%	
M1 information inaccuracy	0.646	51.30
Percent reduction from naive	98.74%	

[1]The information inaccuracies are to be multiplied by 10^{-4}.

Chart 1

Actual vs. Predicted Value Shares of Food

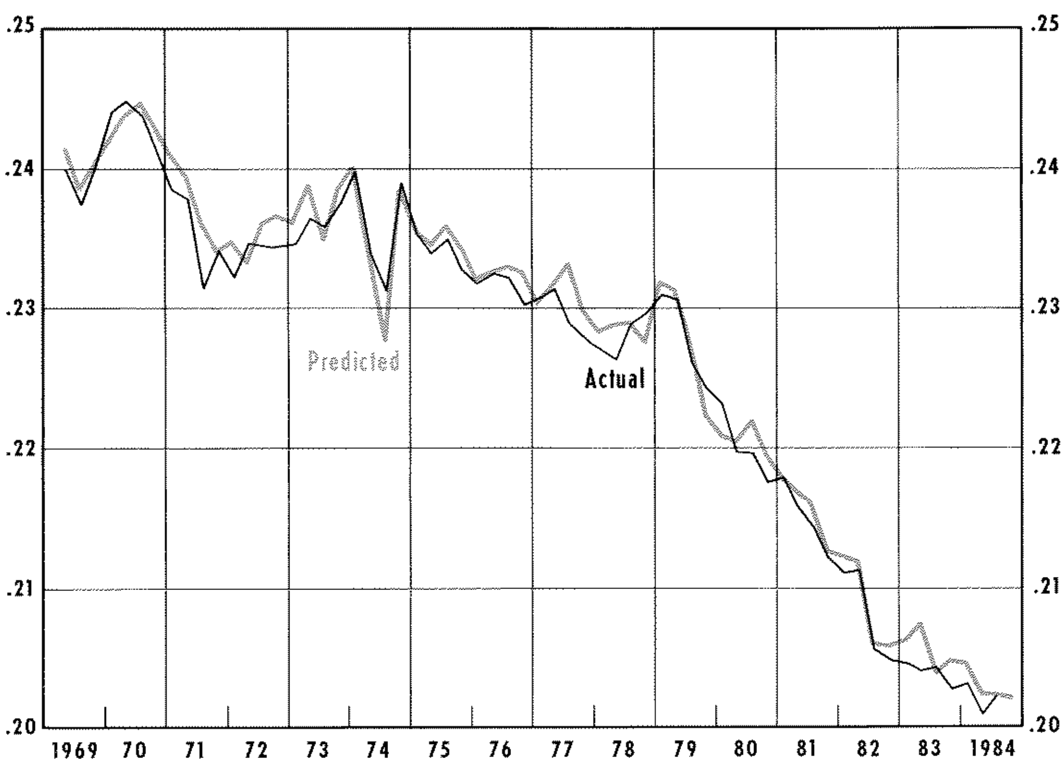


Chart 2

Actual vs. Predicted Value Shares of Nondurables

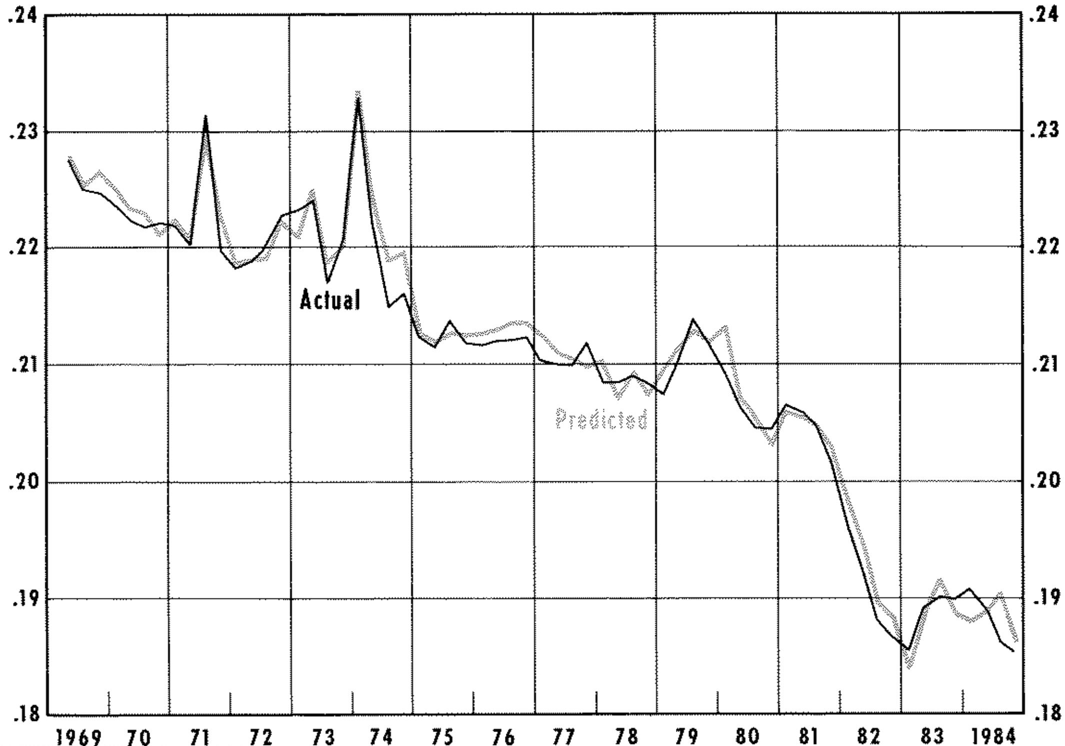


Chart 3

Actual vs. Predicted Value Shares of Services

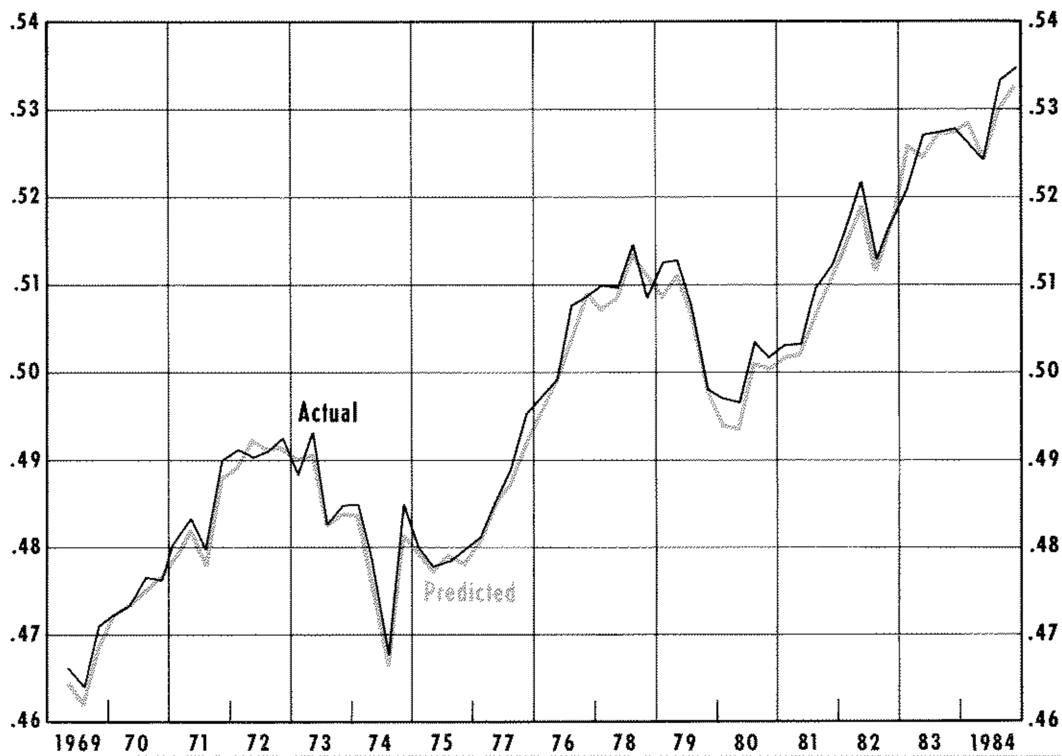


Chart 4

Actual vs. Predicted Value Shares of ABM1

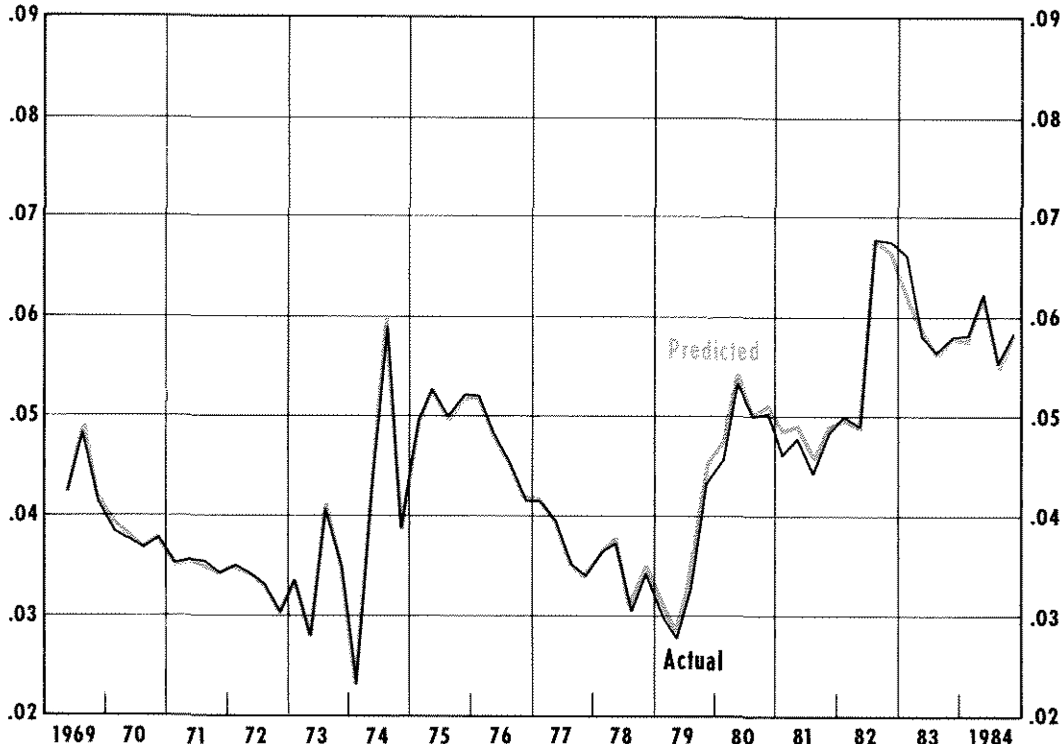
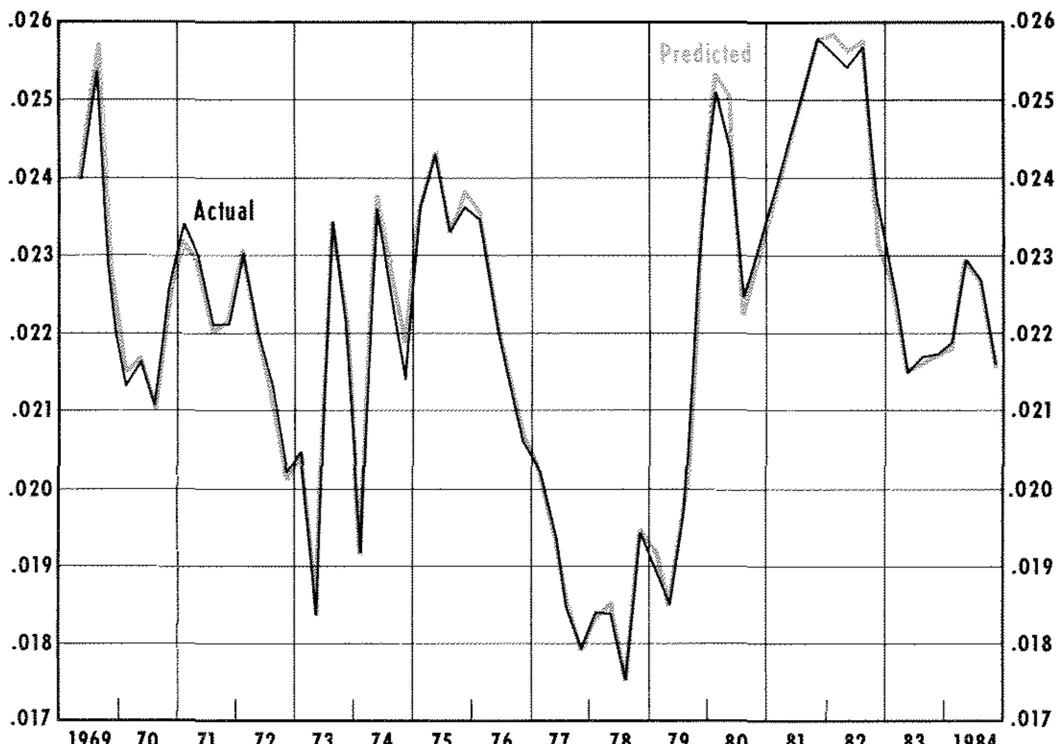


Chart 5

Actual vs. Predicted Value Shares of M1



study, was more stable than the demand for other goods, services and financial assets over the sample period.

CONCLUDING REMARKS

This study has discussed an approach to the estimation of the demand for money that relies on a methodology markedly different from that employed in the conventional money demand analysis. The approach is explicitly derived from the principles of microeconomic theory and emphasizes the importance of interaction among goods. The modeling process is not influenced by a search for "goodness of fit"; instead the emphasis is placed on the model's consistency with explicit utility-maximizing conditions.

The empirical results produced in this study show that it is possible to specify a model of money demand that closely tracks the actual behavior of the flow of M1's monetary services despite its considerable variability over this period. Thus, there seems to be nothing mysterious about that variability; it can be explained in terms of changes in relevant economic variables. These results indicate that money demand has been considerably more stable over the past two decades than standard money demand analysis has suggested.

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(See appendixes A and B on following pages)

Appendix A

In order to estimate the functional form of the demand equations, a stochastic version of that form should be specified and the disturbance terms interpreted. The system of demand equations can be written as follows,

$$(1)\quad X_{it} = \mu_i X_t + \sum_{j=1}^{5} \pi_{ij} P_{jt} + \varepsilon_{it}, \qquad i=1, ..., 5,\; t=1, ..., T,$$

where X_{it} ($= w^*_{it} Dx_{it}$) is a T-dimensional vector of observations on the left-hand-side variables of the ith commodity group, P_{it}($=Dp_{it}$) is a T-dimensional vector of the log-change in the price of the ith commodity group, μ_i is the marginal budget share of the ith commodity group, $[\pi_{ij}]$ is a 5 x 5 matrix of the price coefficients, and X_t ($= \sum_{i=t}^{5} w^*_{it} Dx_{it}$) is a T-dimensional vector of the (budget-share) weighted sum of the log-change in expenditures on the five commodity groups.

The last term in equation 1, ε_{it}, is the disturbance term of the ith demand equation. The disturbance terms, $[\varepsilon_{it}]$, are assumed to capture the random effects of all variables other than income and all prices. The disturbance terms are further assumed to be normally distributed with mean zero and a variance-covariance matrix $\Sigma \otimes I_T$, such that

$$(2)\quad E(\varepsilon_{is}, \varepsilon_{jt}) = \sigma_{ij} \qquad \text{for } s=t,$$

$$(3)\quad \text{and } E(\varepsilon_{is}, \varepsilon_{jt}) = 0 \qquad \text{for } s \neq t,$$

where $\otimes$ is the Kronecker product, I_T is a T x T identity matrix, and σ_{ij} is the i, jth element of the 5 x 5 matrix, Σ.

Another property of the demand-disturbance terms is that their sum vanishes with unit probability (see Barten (1969), p. 16 and Theil (1971), p. 333). A potentially troublesome implication of this property is that

$$\sum_j \sigma_{ij} = E(\varepsilon_{it}(\varepsilon_{1t} + ... + \varepsilon_{st})) = 0.$$

Thus, the covariance matrix, Σ, is singular, and as such, cannot have a rank that is larger than $n-1$. In what follows, it is assumed that the rank of Σ is exactly $n-1$. In order to circumvent the complications posed by this singularity problem, one equation of the system (1) is deleted. The legitimacy of this procedure can be verified easily by summing over i any four of the five equations of the system and using the properties of that system in order to recover the deleted equation. In fact, a major advantage of the estimation method used in this study, full information maximum likelihood (FIML), is that the parameter estimates it produces are invariant to the equation deleted (see Barten (1969), pp. 25–27).

Formulation of the Likelihood Function

For notational convenience, the system of demand equations (1) may be written as follows,

$$(4)\quad y_t = g(x_t,\Theta) + \varepsilon_t,$$

where y_t are the vectors of the left-hand-side variables of (1), x_t are the vectors of the right-hand-side variables, ε_t are the vectors of the demand-disturbance terms, and Θ is the vector of the parameters μ_i and π_{ij}. Since the additive disturbance vectors $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{nt})$, $t=1, ..., T$, are assumed to be independently normally distributed with mean 0 and variance-covariance matrix Σ, it follows that the vectors y_t must also be independently normally distributed with mean $g(x_t, \Theta)$ and variance-covariance matrix Σ. In arriving at the vector-valued function g, it is assumed for notational convenience that prior restriction on the parameters has already been eliminated by substitution.

Given the observed data on $y = (y_1, ..., y_T)$ and $x = (x_1, ..., x_T)$, the log-likelihood function on Θ and Σ is given by

$$(5)\quad L(\Theta, \Sigma; y, x) = -(T(n-1)/2) \log 2\pi \; |\Sigma| \; -1/2 \sum_{t=1}^{T} [(y_t - g(x_t, \Theta))' \; \Sigma^{-1} \; (y_t - g(x, \Sigma))].$$

This function is to be maximized with respect to the elements of the parameter vector Θ and the elements of the variance-covariance matrix, Σ. For computational convenience, however, and since the asymptotic distribution of Σ is not at all needed, a stepwise-optimization procedure is used. This procedure involves first maximizing the log-likelihood function (5) with respect to the elements of Σ, for a given value of Θ, to obtain an expression for Σ in terms of the elements of Θ. Thus, for $\Theta = \Theta^*$, the value of Σ that maximizes (5) is given by

$$(6)\quad \Sigma^*(\Theta^*; y, x) = \frac{1}{T} \sum_{t=1}^{T} (y_t - g(x_t, \Theta))(y_t - g(x_t, \Theta))'.$$

Substitution of Σ^* into (5) yields

$$(7)\quad L(\Theta; y, x) = -(T(n-1)/2) \log 2\pi |\Sigma(\Theta; y, x)| - \frac{1}{2}(T(n-1)),$$

which is the concentrated likelihood function.

The second step in the optimization procedure becomes immediately clear when one recognizes that maximizing the log-likelihood function (5) is equivalent to minimizing the determinant of Σ in (7). The latter is accomplished by searching the feasible parameter space for the value of Θ at which $|\Sigma|$ is minimized. The values of the elements of Θ thus obtained, $\hat{\Theta}$, are the maximum likelihood estimates of the system (1). The asymptotic covariance matrix of $\hat{\Theta}$ is obtained by inverting the matrix $-[\frac{\partial^2 L}{\partial\Theta\partial\Theta'}]$, which is numerically evaluated at $\Theta = \hat{\Theta}$. Naturally, the elements of that matrix pertain only to the estimated parameters. The asymptotic covariance matrix of the entire vector of (estimated as well as computed) parameter estimates is derived in Fayyad (1986).

Appendix B

Available results from information theory can be used to develop measures by which the performance of each of the estimated equations as well as that of the system as a whole can be gauged.[1] Consider an infinitesimal change in the budget share of the ith commodity ($w_i = p_i x_i/m$):

$$dw_i = \frac{x_i}{m}dp_i + \frac{p_i}{m}dx_i - \frac{p_i x_i}{m^2}dm,$$

from which it follows that

(1) $dw_i = w_i\, d\log p_i + w_i\, d\log x_i - w_i\, d\log m.$

The finite-change analog of equation 1 is given by

(2) $\Delta w_{it} = w^*_{it}\, Dp_{it} + w^*_{it}\, Dx_{it} - w^*_{it}\, Dm_t.$

Since $Dm_t = Dx_t + Dp_t$, it follows that equation 2 can be rewritten as

(3) $\Delta w_{it} = w^*_{it}\, Dx_{it} + w^*_{it}\,(Dp_{it} - Dp_t) - w^*_{it}\, Dx_t.$

Observe that the first term on the right-hand-side of equation 3, to be interpreted as the quantity component of the change in the budget share of the ith good, is the dependent variable of the ith demand equation of the estimates system. Thus, given the log-changes in real income and relative prices, the Rotterdam model can be used to provide conditional forecasts of $w^*_{it}\, Dx_{it}$ and, through equation 3 of $\Delta\, w_{it}$. Since the prediction of $w^*_{it}\, Dx_{it}$ is equal to the right-hand-side of the ith demand equation with the disturbance term deleted, it follows that

$$\hat{w}_{i,t+1} = w_{it} - e_{it},$$

where $\hat{w}_{i,t+1}$ is the implied prediction of w_{it}, and e_{it} is the residual of the ith demand equation in period t.

In view of the fact that the budget shares are positive and add up to unity, they may be viewed as probabilities. A measure of the model fit can be acquired by determining the expected gain in information from the actual shares, which can be viewed as posterior probabilities, when the implied predictions (or the fitted values) of these shares are viewed as prior probabilities. That measure is given by

(4) $$I_t = \sum_{i=1}^{n} w_{it} \log \frac{w_{it}}{\hat{w}_{it}},$$

where I_t is the information inaccuracy of the predictions provided by the system of demand equations. It is to be noted that not only is this measure of information inaccuracy additive over goods, as is indicated by the expression in (4), but it is also additive over time. Thus, it is possible to construct an average index of information inaccuracy, $\bar{I}$, over the period from t_1 to t_2 by using the following formula

(5) $$\bar{I} = \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} I_t.$$

Observe that the information-inaccuracy measures presented above pertain to the predictions which are provided by the system of demand equations as a whole. It is possible to acquire a single-equation measure of information inaccuracy by using the formula

(6) $$I_{it} = w_{it} \log \frac{w_{it}}{\hat{w}_{it}} + (1 - w_{it}) \log \frac{1 - w_{it}}{1 - \hat{w}_{it}},$$

where $1 - w_{it}$ is the combined budget share of all commodities other than the ith. As before, the average (over time) index of a single-equation information inaccuracy can be obtained by using formula 5.

In order to provide comparability of the information inaccuracy across models, a correction (adjustment) factor should be applied to these measures (see Theil (1971), pp. 651–52, and Barnett (1981), p. 150). Adjustment was achieved in this study by multiplying the information-inaccuracy measure in each case by a factor of ML/(ML-K), where M is the number of jointly estimated equations, L is the number of time periods (quarters), and K is the number of unrestricted parameters. Clearly, this procedure is closely akin to the degrees-of-freedom adjustment of the correlation coefficient.

[1]See Theil (1967), pp. 1–48; (1971), pp. 646–50, and Barnett (1981), pp. 149–54.