Are Options on Treasury Bond Futures Priced Efficiently?

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UNTIL recently, trading in commodity options has been viewed with a great deal of suspicion in the United States by both the general public and market regulators. The low margin required by option markets has led many people to believe that unsophisticated investors with limited resources were being encouraged to speculate and that commodity price movements could be manipulated by sophisticated speculators using a high degree of leverage.[1] Few people realized the useful role that speculators in futures and options markets play in assuming risk that others desire to avoid (thus providing hedging opportunities) and providing better estimates of future spot prices.[2]

The Commodity Futures Trading Commission (CFTC) is gradually lifting restrictions on option trading by allowing each commodity exchange to open trading in options on one of its futures contracts. The first phase of the CFTC pilot program introduced in 1982 saw eight commodity exchanges participate by offering options on several different futures contracts; these contracts covered three different stock market indices, two weights of gold, heating oil, sugar and U.S. Treasury bonds.[3] This article focuses on the pricing of options on Treasury bond futures.

The behavior of this particular option price series is interesting for at least two reasons. First, if the options market is efficient, no arbitrage opportunities will exist between any two option contracts.[4] Stated differently, an efficient options market is one in which the same market price will be observed for options with the same level of risk and rate of return. Because efficiency is one criterion that the CFTC is likely to consider when deciding the future of this market, it is important to assess whether the options market in U.S. Treasury bond futures contracts satisfies this criterion.

The second motivating interest of this study is the usefulness of Black's theoretical model in estimating the prices of American-type options on futures.[5] American options permit the holder to exercise the option at any time before the option contract expires. Most option pricing formulas, however, attempt to explain the prices of European options, which can be exercised only on the expiration date of the option contract.

Although the Black model is widely accepted as a theoretical representation of option price determination, some recent studies using stock options suggest that its predecessor, the Black-Scholes model, does not fit market data well.[6] Limited applications of the Black

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[1]A recent overview of problems associated with options trading in the early 1900s is provided in Wall (1983).

[2]One notable exception to this was Holbrook Working, who wrote extensively on the potentially useful role of speculators. The interested reader is referred to Working (1977).

[3]For more detail on the specifics of the CFTC pilot programs and a general background to options trading, see Wolf (1982); and Belongia (1983).

[4]Efficient markets are those that reflect all available information. Weak form market efficiency implies that all information contained in past price movements are fully reflected in current prices. Semistrong efficiency suggests that current prices reflect all publicly available information. Strong form efficiency means that prices reflect all information, both public and private. A considerable body of empirical work suggests that heavily traded capital markets are at least semistrong efficient. See Fama (1970).

[5]Black (1976).

[6]See, for example, Black and Scholes (1972); Gulteken, Rogalski and Tinic (1982); Finnerty (1978); Whaley (1982); and O'Brien and Kennedy (1982).

model to the pricing of London commodity options have produced contradictory results about market efficiency and the model's applicability.[7] In view of these results and the recent availability of options data from U.S. markets, it is of some interest to determine whether the Black model accurately describes the process by which prices on U.S. Treasury bond options are determined. From a different perspective, the research question is whether judgments about the observed behavior of option prices can be based on comparisons to prices predicted by this theoretical model.

This article first describes some basic principles of options contracts and their relationship to futures markets. The behavior of prices in the Treasury bond options market then is examined using a test proposed by Latane' and Rendleman.[8]

OPTIONS AND FUTURES IN THE CFTC PILOT PROGRAM

Options trading may be clarified somewhat by first comparing it with futures trading. A futures contract *obligates* the holder to buy (or sell) a specific volume of the underlying commodity at a specified price at some future date. An agreement to buy the commodity is a "long" futures position; a "short" position is an agreement to sell. If futures prices rise, holders of long positions realize a profit that is exactly offset by the losses of the holders of short positions that day, and vice-versa. Futures contracts are settled each day with debits or credits to the margin accounts of individuals holding a futures position. For example, if an individual bought a Treasury bond futures contract and, by the end of that day, Treasury bond futures "settled" at a higher price, he would realize a profit equal to the change in the value of the futures contract less transaction costs. He then would have the choice of liquidating the futures contract or holding it in hope of further price appreciation.

Futures contracts normally call for delivery of a homogeneous, standardized product. The delivery of homogeneous, standardized Treasury bonds is complicated by the fact that Treasury bond prices respond to factors such as coupon rates and callability features that are specific to individual issues of Treasury bonds. Thus, the Treasury bond futures contract, as specified by the Chicago Board of Trade, calls for delivery of a hypothetical 8 percent coupon Treasury bond not callable for at least 15 years from the date of delivery. If no call provision is present, the bond must not mature for at least 15 years from date of delivery.[9] These bonds have a face value of $100,000 at maturity. A price of 70 implies a contract valued at $70,000.

An option contract gives its purchaser the right, *but not the obligation,* to buy or sell a specified volume of a commodity for a set price at some future time. Within the CFTC pilot program, this right to buy or sell applies only to specific futures contracts and not to the physical commodities underlying those contracts. For example, the purchaser of a call option on Treasury bond futures buys the right to purchase a specific Treasury bond futures contract for a specified price prior to some agreed-upon future date.

If, before that date, the market price of that Treasury bond futures contract rises above a specific level (the sum of the exercise price, the price of the call option and any commission costs), the purchaser will find it profitable to exercise the rights of the call option. By doing so, he buys the futures contract (that is, holds a long position in the Treasury bond futures market) and obtains an immediate profit equal to the difference between what he paid for the futures contract (the exercise price of the call option) and the current market price, less the transaction costs.

The purchaser of a put option, conversely, purchases the right to sell a particular futures contract at a set price. In this case, if the futures price falls below a particular level, the purchaser will find it profitable to exercise the rights of the put option and, by doing so, enter into a short position in the futures market. This will enable the individual to sell futures contracts for Treasury bonds at a price above the current market price.[10] In practice, owners of both call and put options often choose to realize profits by selling the option

[7]Studies of London options include Hoag (1982); and Figlewski and Fitzgerald (1982).

[8]Latane' and Rendleman (1976).

[9]The CBT publishes tables of conversion factors that translate all of the deliverable Treasury bonds into 15 year, 8 percent coupon bonds. The conversion factors for bonds with coupons less than 8 percent are less than 1, and the factors for bonds with coupons over 8 percent are greater than 1.

[10]By selling the futures contract, the individual agrees to deliver a specific amount of Treasury bonds at a specified price at the expiration of the contract. Again, the individual realizes an immediate profit equal to the difference between what he sold the futures contract for (the exercise price of the put option), and that trading day's futures settlement price, less transaction costs. He also is faced with the decision to liquidate or hold further.

instead of exercising its privileges and entering into a futures market position.

The Commodity Option Contract

The key elements of a commodity option contract are the strike (or exercise) price, the futures contract to which the option applies and the premium. The premium — the price of the option — is competitively determined, whereas other elements of the option are part of the contract itself. An "in the money" call option is one whose strike price (the price at which the option owner may exercise the rights of the option) is less than the current price of the futures contract that underlies the option; a call option is "out of the money" if its strike price is greater than the price of the futures contract. The reverse is true for put options. For example, if the current futures price is at 75, call options whose strike prices are less than 75 and put options with strike prices greater than 75 are in the money. Call options with strike prices greater than 75 and put options with strike prices less than 75 are out of the money.

WHAT SERVICES DO TREASURY BOND OPTIONS PROVIDE?

One useful role that option and futures contracts play is to transfer the risk associated with adverse price swings from hedgers to speculators. Consider, for example, the manager of a pension fund who expected interest rates to rise. He could hedge against the risk of capital loss in the price of his bond holdings by selling Treasury bond futures. If rates did rise, losses in his long position (bond holdings) would be at least partially offset by gains in his short position (futures contracts).

Because an option's price changes in response to the price of its underlying commodity or security, options also can be used to hedge against risk. In fact, at the heart of the Black and Black-Scholes models is the assumption that a totally risk-free hedge can be constructed using options and either futures (Black model) or securities (Black-Scholes model).

How To Interpret Option Prices

Table 1, a reproduction of one day's report on trading in Treasury bond options, indicates that on September 13, 1983, options could have been bought on futures contracts dated for delivery in December 1983, March 1984 and June 1984; no options had yet been written on the September 1984 futures contract. The data in the table's first column show the strike prices of available options, while columns 2–4 give the premiums associated with call options at those strike prices.

Table 1
A Typical Summary of One Day's Trading in Options on Treasury Bond Futures

CHICAGO BOARD OF TRADE Treasury Bond Option Prices, 9/13/83, points and 64ths of 100 percent ($100,000)

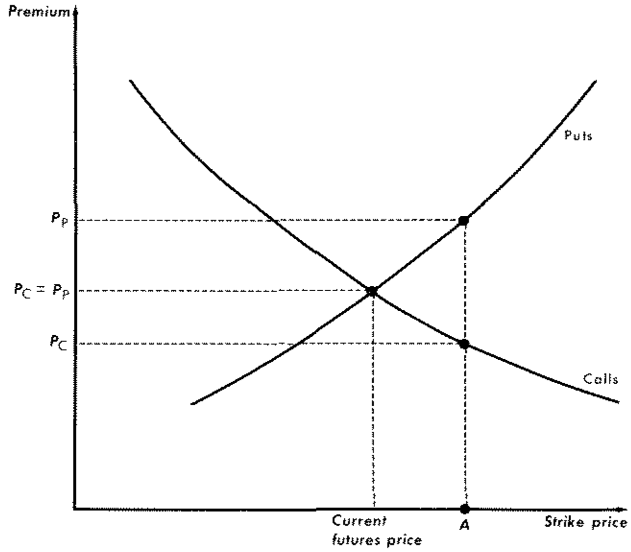
Strike	Calls — Settlement			Puts — Settlement		
Price	Dec	Mar	Jun	Dec	Mar	Jun
66	5–41	—	—	0–16	0–58	—
68	3–62	4–22	4–44	0–35	1–32	—
70	2–35	3–16	3–42	1–06	2–20	—
72	1–33	2–25	2–45	2–05	3–21	—
74	0–52	1–34	2–11	3–17	4–36	—
76	0–24	1–03	—	4–56	5–63	—
78	0–10	0–45	—	6–40	—	—
80	0–06	0–26	—	8–36	—	—
82	0–03	—	—	—	—	—

The data in the table show, for example, that call options on March 1984 Treasury bond futures had been written with strike prices between 68 and 80; the futures price on this date was 70-29/32. Therefore, the premium on a call option with a strike price of 68 is expected to be the highest premium since it offers the option purchaser the right to buy Treasury bond futures at a 2-29/32 discount to the current market price. The difference between this discount and the price of the call (4-22/64) represents the market's evaluation of the potential for future price appreciation of this contract.

The table also shows that call premiums fall as strike prices increase. Higher strike prices offer the option purchaser the right to buy Treasury bond futures at a price above the current market price. A buyer would purchase these options only if he expected futures prices to increase substantially above the option's strike price before the option's expiration date. This negative relationship between call option premiums and strike prices also is illustrated in figure 1.

The data in columns 5–7 of table 1 show the premiums on put options for the same strike prices listed

Figure 1
Relationship Between Premiums on Put and Call Options



P_P = premium on a put option with strike price = A.
P_C = premium on a call option with strike price = A.

in column 1. Because a put option gives the purchaser the right to sell Treasury bond futures, put option premiums tend to increase with strike prices; that is, the right to sell at a higher price has a greater value than the right to sell at a lower price. This relationship is depicted by the upward-sloping line in figure 1. In this and other respects, the properties of put options are the mirror image of properties associated with call options.

USING THE BLACK MODEL TO DERIVE CALL OPTION PREMIUMS

The Black model can be written as:[11]

(1) $P_c = e^{-rt} [F^*N(d_1) - X^*N(d_2)]$ (see insert).

The only two parameters of the model that are not directly observable are r, the risk-free nominal interest rate, and σ^2, the variance of expected future returns of the underlying futures contract. The risk-free nominal interest rate can be proxied, however, by the current market rate on Treasury bills with maturities near the expiration dates of the various futures contracts.[12] The determination of an appropriate value for σ^2, the expected variance of future returns, is the last piece of information needed to estimate the price of a particular option with the Black model.

The test of the Black-Scholes model suggested by Latane' and Rendleman provides an interesting approach to comparing theoretical and actual option prices. Their reasoning is that if the market is pricing options and risk efficiently, then, given r, the same estimate of σ^2 should apply to all options traded for a given futures contract on a particular day. For example, all options offered on October 26, 1982, for the December 1983 futures contract should yield the same implied expectation of future returns if the assumptions that underlie the Black model are true. This result holds because the same risk-free hedge can be constructed over this interval by constructing a portfolio using different options on the same futures contract, if markets are efficient.[13]

The Latane' and Rendleman test of the Black-Scholes model for data on stocks and stock options also can be used to test the applicability of the Black model for determining prices of options on futures contracts.[14] Their test involves the following steps. On a particular day, observe data on a variety of different options on futures contracts for the same commodity — for example, all of the data for options on U.S. Treasury bond futures shown in table 1. Insert these data, a value for r and a starting value of σ^2 into the Black model and solve for a final value of σ^2 that minimizes the differences between actual and estimated call option prices. If the Black model is a correct representation of commodity options pricing *and* if the market is pricing options efficiently, one would expect to find estimates of σ^2 that were nearly identical across all options traded that day for the same futures contract.[15] Con-

[11] Black (1976).

[12] Because Treasury bills are backed by the U.S. government, the risk of default generally is considered to be zero.

[13] In the abstract to their 1973 article, Black and Scholes assert "(i)f options are correctly priced in the market, it should not be possible to make sure profits by creating portfolios of long and short positions in options and their underlying stocks." Their use of the term "correctly priced" markets is synonymous with what we are calling efficient markets. Black's model uses the underlying futures contracts in place of the underlying stocks.

[14] A strict test of market efficiency would compare the yield on a safe asset with the yield on a portfolio of hedged options and futures with continuously changing hedge ratios. Our reasoning is, however, that if the Black model does not predict option prices well, either the model is incorrectly specified or markets are inefficient. Therefore, in the absence of any *systematic* relationship between actual and implied option prices, conclusions about market efficiency on the basis of our "buy and hold" strategy are still valid.

[15] We are indebted to Fischer Black for emphasizing the implied differences among estimates of σ^2 for the same contract and observation dates.

Interpreting the Black Model

This model was designed to price options on the futures contract of an underlying asset. As such, it is both a refinement of and equivalent to the original Black-Scholes option pricing model, which theoretically prices options on the underlying asset itself.

According to the Black model,

$$(2)\quad P_c = e^{-rt}\,[F^*N(d_1) - X^*N(d_2)],$$

where
- P_c = the price of the call option
- F = the price of the underlying futures contract
- X = the contract's exercise price
- r = the risk-free rate of interest
- t = the number of time periods before the option expires (expressed as a fraction of a year)
- $N(d_1)$ and $N(d_2)$ = the cumulative normal density function evaluated at points d_1 and d_2.

$$d_1 = [\ln(\frac{F}{X}) + \frac{\sigma^2 t}{2}] / \sigma\sqrt{t}$$

$$d_2 = [\ln(\frac{F}{X}) - \frac{\sigma^2 t}{2}] / \sigma\sqrt{t}$$

and σ^2 represents the variance of expected future returns on the underlying futures contract over time. If all of the parameters of the capital asset pricing model hold and are constant, if σ^2 is constant, and if taxes and transaction costs are zero, these results can be derived by solving a differential equation for the change in the value of a hedged risk-free portfolio over time (given certain boundary conditions).

Although the model is difficult to interpret intuitively, certain general observations may be made. The use of the cumulative normal density function is a result of the assumption that returns on the futures contract follow a normal distribution. $N(d_1)$ represents the number of futures contracts an investor should sell per call option purchased in order to create a risk-free portfolio. For example, if $N(d_1)$ were estimated to be 0.5, it would imply that the investor should sell one futures contract for every two options he purchased. (Of course, since this ratio changes over time as market conditions change, an investor would have to adjust his portfolio continually if a risk-free hedge were to be maintained at all times.) As long as $[F^*N(d_1) - X^*N(d_2)] \geq 0$, then $P_c \geq 0$. This is always true given the relationships between $\ln(F/X)$, F, X, d_1 and d_2 (since σ and t are always positive). Thus, the price of a call option can never be negative.

If S equals the spot price of the security or commodity underlying a futures contract, and h equals the cost of holding this asset over time (e.g., interest and storage costs), then the substitution of Se^{ht} for F in the Black model yields the original Black-Scholes formula. In markets that fully reflect carrying costs, the models could be used interchangeably to value options; certain assumptions that underlie option pricing theory, however, imply that the Black model should be a more accurate representation of actual option prices than the Black-Scholes model.[1]

[1]These include institutional imperfections, relative market liquidities and the theoretical distributions of the underlying spot and future prices. See Asay (1982); and Samuelson (1965).

versely, if the different estimates of σ^2 are not very nearly identical, one can conclude either that the Black model does not estimate option prices accurately (given the use of Treasury bill rates as proxies for r) or that this market does not price options efficiently.[16]

[16]This conclusion also depends on several other assumptions as well. Because the Black model is derived for application to European options that do not have early exercise privileges, a debate has developed in the literature concerning what value, if any, can be attributed to the early exercise privilege of American-type options. Based on the work of Robert Merton, who argued that early exercise of stock options had no value unless dividends were involved, one might conclude that this problem is irrelevant in a study of options on commodity futures because dividends are not involved. Moreover, in practice, American options are almost never exercised before expiration. The reason is that the option has two potential sources of value: its immediate exercise value (if any) and its potential for price appreciation in the future. Thus, an investor — in most cases — will be able to realize a greater profit by selling the option instead of exercising it. In efficient markets, if we exclude options on assets that pay dividends, American and European options should be priced similarly. See Merton (1973).

ESTIMATION AND RESULTS

Observations on Treasury bond options were taken at six dates between October 1982 and April 1983.[17] On each of these six dates, data were gathered for actively traded options with large open interest. In total, data were gathered on 53 call options with different strike prices or futures contracts. On these same dates, interest rates were observed for Treasury bills maturing near the delivery dates of the various futures contracts; these values were used to represent risk-free rates of return (r).[18]

These data and starting values for the unobservable variance of expected future returns (σ^2) were used to find values for d_1 and d_2, the two points at which the cumulative normal density must be evaluated. Equation 1 then was solved for an estimate of the call option price. By using different values of σ^2, the Black model was solved iteratively until a value of σ^2 was found that minimized the difference between actual and estimated option prices to within ± one cent. The values of σ^2 that produced the minimum differences for the 53 option contracts considered are reported in table 2.

The estimates of σ^2 in the fifth column of table 2, in general, suggest that estimates of the implied variation of future returns differ numerically across options written on the same futures contract on the same day. The spread between highest and lowest estimates of σ^2 range from 0.014 for options on September futures traded on February 23 to 0.110 for options on June futures traded on April 4. It is not clear, however, that it is possible to test whether these estimates of σ^2 are statistically different from one another. Unknown are the mean of expected returns, the number of traders determining the mean and variance of returns, and the shape of the distribution itself. Judgmentally, however, it would appear that these estimated differences are small. In half of the cases examined, the spread is 0.026 points or less. In economic terms, this result implies that, in one-half of the options examined, the range of estimates on expected variation of future returns was less than three basis points.

The last column of table 2 reports the *ex post* profit that could have been obtained — in the absence of transaction costs and taxes — if the individual option had been held until expiration. That is, the dollar figures listed show the change in the value of the option between the observation date and the last day it was traded. As the data indicate, options purchased on a particular day and held until expiration all tended to produce profits or losses, regardless of strike prices. In other words, no apparent *systematic* relationship between realized profits and certain characteristics of these options is revealed by the profit data in the table. The point with respect to judging market efficiency is that nothing in available market data indicate, *ex ante,* that these options would perform as they did. That is, none of the results in table 2 indicates a consistent *ex ante* signal for profit opportunities, a result consistent with an efficient market.

Testing the Model with Direct Estimates of σ^2

Another way to test the Black model might be to use historical price data to construct a proxy for the expected future variance of returns on the futures contract.[19] Given this estimate of σ^2 and using the Treasury bill rate to proxy the risk-free rate, we can obtain an implied value of a call option. If the Black model "predictions" represent the "efficient prices," an investor should buy those options that the model implies are underpriced and sell options that the model implies are overpriced. The results of this test are reported in table 3.

These results do not yield any consistent arbitragable profit opportunities. There is no apparent pattern either to the implied value of σ^2 or to the differences between the actual and implied call prices that, *ex ante,* would indicate profitable options. If an investor had bought *any* of the options in our sample on January 26, 1983, or any December 1982 call options on October 26, 1982, he would have earned a profit on the change in option prices. Likewise, anyone who bought March 1983 or June 1983 call options on November 23,

[17]The dates, which were not randomly chosen, are: October 26, November 23 and December 27, 1982; January 26, February 23 and April 4, 1983.

[18]The same risk-free hedge over different periods (using different futures contracts), may imply a different risk-free interest rate if the term structure of interest rates is not flat. That is, given a "normal" yield curve, the implied risk-free interest rate over a period of three months (the remaining duration of one option contract), should be less than the implied risk-free interest rate over a period of six months (the remaining duration of another option on a different futures contract), observed on the same day. Three-month and six-month Treasury bill rates were used to proxy the risk-free rate, depending on the remaining length of the option contract.

[19]Historical values for σ^2 were determined by estimating the variance of the log of the ratio of successive days futures contract prices, up to the date at which a particular observation was taken; this variance, when multiplied by 365, approximates an annualized rate of return.

Table 2
Estimating Sigma, Given the Risk-Free Rate

Trading date	Futures contract delivery date	Strike price (thousands of dollars)	Futures price (thousands of dollars)	Sigma value	Estimated Ex post profit
10/26/82	12/82	70	75.25	0.255	$2140.63
10/26/82	12/82	72	75.25	0.233	2375.00
10/26/82	12/82	74	75.25	0.265	1921.88
10/26/82	12/82	76	75.25	0.239	1890.63
10/26/82	12/82	78	75.25	0.244	1531.25
10/26/82	12/82	80	75.25	0.245	1140.63
10/26/82	3/83	76	74.56	0.249	−2671.90
11/23/82	3/83	74	76.75	0.200	−2312.50
11/23/82	3/83	76	76.75	0.285	−3750.00
11/23/82	3/83	78	76.75	0.277	−2828.10
11/23/82	3/83	80	76.75	0.289	−2218.80
12/27/82	3/83	70	77.13	0.207	−1546.90
12/27/82	3/83	72	77.13	0.203	−1734.40
12/27/82	3/83	74	77.13	0.202	−2171.90
12/27/82	3/83	76	77.13	0.193	−2359.40
12/27/82	3/83	78	77.13	0.191	−1453.10
12/27/82	3/83	80	77.13	0.189	−765.63
12/27/82	6/83	68	76.41	0.201	−453.13
12/27/82	6/83	70	76.41	0.199	−1531.30
12/27/82	6/83	72	76.41	0.196	−1937.50
12/27/82	6/83	74	76.41	0.205	−2125.00
12/27/82	6/83	76	76.41	0.192	−1906.30
12/27/82	6/83	78	76.41	0.192	−1687.50
12/27/82	6/83	80	76.41	0.186	−1234.40
1/26/83	3/83	68	73.75	0.107	1921.88
1/26/83	3/83	70	73.75	0.074	1890.63
1/26/83	3/83	72	73.75	0.063	1671.88
1/26/83	3/83	74	73.75	0.062	875.00
1/26/83	6/83	68	73.03	0.130	3078.13
1/26/83	6/83	70	73.03	0.149	1781.25
1/26/83	6/83	72	73.03	0.149	1156.25
1/26/83	6/83	74	73.03	0.154	718.75
1/26/83	6/83	76	73.03	0.158	281.25
1/26/83	6/83	78	73.03	0.166	0.00
2/26/83	6/83	68	75.59	0.206	484.38
2/26/83	6/83	70	75.59	0.179	−343.75
2/26/83	6/83	72	75.59	0.154	−437.50
2/26/83	6/83	74	75.59	0.152	−359.38
2/26/83	6/83	76	75.59	0.146	−296.88
2/26/83	6/83	78	75.59	0.141	−171.88
2/26/83	9/83	76	74.97	0.150	−2296.90
2/26/83	9/83	78	74.97	0.144	−1468.80
2/26/83	9/83	80	74.97	0.158	−968.75
4/04/83	6/83	68	76.22	0.252	15.63
4/04/83	6/83	70	76.22	0.192	−687.50
4/04/83	6/83	72	76.22	0.158	−640.63
4/04/83	6/83	74	76.22	0.148	−234.38
4/04/83	6/83	76	76.22	0.142	46.88
4/04/83	9/83	70	75.72	0.138	−5078.10
4/04/83	9/83	72	75.72	0.131	−4328.10
4/04/83	9/83	74	75.72	0.126	−2953.10
4/04/83	9/83	76	75.72	0.134	−2031.30
4/04/83	9/83	78	75.72	0.131	−1218.80

Table 3
Implied Call Prices, Using Historical Sigma

Trading date	Futures contract delivery date	Strike price (thousands of dollars)	Futures price (thousands of dollars)	Treasury bill rate	Ex post profit
10/26/82	12/82	70	75.25	0.0796	$2140.63
10/26/82	12/82	72	75.25	0.0796	2375.00
10/26/82	12/82	74	75.25	0.0796	1921.88
10/26/82	12/82	76	75.25	0.0796	1890.63
10/26/82	12/82	78	75.25	0.0796	1531.25
10/26/82	12/82	80	75.25	0.0796	1140.63
10/26/82	3/83	76	74.56	0.0847	−2671.90
11/23/82	3/83	74	76.75	0.0795	−2312.50
11/23/82	3/83	76	76.75	0.0795	−3750.00
11/23/82	3/83	78	76.75	0.0795	−2828.10
11/23/82	3/83	80	76.75	0.0795	−2218.80
12/27/82	3/83	70	77.13	0.0791	−1546.90
12/27/82	3/83	72	77.13	0.0791	−1734.40
12/27/82	3/83	74	77.13	0.0791	−2171.90
12/27/82	3/83	76	77.13	0.0791	−2359.40
12/27/82	3/83	78	77.13	0.0791	−1453.10
12/27/82	3/83	80	77.13	0.0791	−765.63
12/27/82	6/83	68	76.41	0.0810	−453.13
12/27/82	6/83	70	76.41	0.0810	−1531.30
12/27/82	6/83	72	76.41	0.0810	−1937.50
12/27/82	6/83	74	76.41	0.0810	−2125.00
12/27/82	6/83	76	76.41	0.0810	−1906.30
12/27/82	6/83	78	76.41	0.0810	−1687.50
12/27/82	6/83	80	76.41	0.0810	−1234.40
1/26/83	3/83	68	73.75	0.0808	1921.88
1/26/83	3/83	70	73.75	0.0808	1890.63
1/26/83	3/83	72	73.75	0.0808	1671.88
1/26/83	3/83	74	73.75	0.0808	875.00
1/26/83	6/83	68	73.03	0.0795	3078.13
1/26/83	6/83	70	73.03	0.0795	1781.25
1/26/83	6/83	72	73.03	0.0795	1156.25
1/26/83	6/83	74	73.03	0.0795	718.75
1/26/83	6/83	76	73.03	0.0795	281.25
1/26/83	6/83	78	73.03	0.0795	0.00
2/26/83	6/83	68	75.59	0.0796	484.38
2/26/83	6/83	70	75.59	0.0796	−343.75
2/26/83	6/83	72	75.59	0.0796	−437.50
2/26/83	6/83	74	75.59	0.0796	−359.38
2/26/83	6/83	76	75.59	0.0796	−296.88
2/26/83	6/83	78	75.59	0.0796	−171.88
2/26/83	9/83	76	74.97	0.0797	−2296.90
2/26/83	9/83	78	74.97	0.0797	−1468.80
2/26/83	9/83	80	74.97	0.0797	−968.75
4/04/83	6/83	68	76.22	0.0864	15.63
4/04/83	6/83	70	76.22	0.0864	−687.50
4/04/83	6/83	72	76.22	0.0864	−640.63
4/04/83	6/83	74	76.22	0.0864	−234.38
4/04/83	6/83	76	76.22	0.0864	46.88
4/04/83	9/83	70	75.72	0.0871	−5078.10
4/04/83	9/83	72	75.72	0.0871	−4328.10
4/04/83	9/83	74	75.72	0.0871	−2953.10
4/04/83	9/83	76	75.72	0.0871	−2031.30
4/04/83	9/83	78	75.72	0.0871	−1218.80

1982, or December 27, 1982, or any September 1983 call options on February 23, 1983, or April 4, 1983, would have incurred losses. Some options that the model implied were underpriced eventually rose in price; others, however, declined further. Similarly, higher variance of expected returns is associated with both profitable and non-profitable options; relatively lower estimates of σ^2 yielded the same mixed results.

Additional evidence of market efficiency is shown by the absence of any consistent relationship between strike price and profit or loss. Profits are sometimes negatively associated with strike prices (for example, June 1983 options on January 26, 1983), while on other occasions losses are negatively associated with strike prices (September 1983 options on April 4, 1983). Thus, generally no predictable *ex ante* pattern between strike prices and profits can be identified.

CONCLUSIONS

The trading of options on commodity futures has been permitted only recently in the United States. Because the success and future of the CFTC's pilot program in options trading will depend, in part, on judgments about pricing efficiency, it is of interest to compare actual prices with those of a model whose fundamental assumption is that option pricing *is* efficient. In those instances where the Black model estimates of option prices differed from observed market values, we were unable to find consistent arbitragable profit opportunities. Thus, we were unable to reject the assumption that Treasury bond option prices are "efficient" in the fundamental economic sense.

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