TFP, Capital Deepening, and Gains from Trade

B. Ravikumar, Ana Maria Santacreu, and Michael Sposi

Abstract

Using a dynamic, multicountry model with capital accumulation, we compute the exact transition paths for 93 countries following a permanent, uniform, unanticipated trade liberalization and calculate the resulting welfare gains from trade. We find that while the dynamic gains are different across countries, consumption transition paths look similar except for scale. In addition, dynamic gains accrue gradually and are about 60 percent of steady-state gains for every country. Finally, the contribution of capital accumulation to dynamic gains is four times that of total factor productivity.

JEL codes: E22, F11, O11

Federal Reserve Bank of St. Louis *Review*, forthcoming.

1. INTRODUCTION

Recently, there have been a few papers computing gains from trade in dynamic models, e.g., Anderson, Larch, and Yotov (2020), Brooks and Pujolas (2018), Alvarez (2017), Ravikumar, Santacreu, and Sposi (2019), Mix (2020), and Alessandria, Choi, and Ruhl (2021). In this article, we decompose the dynamic gains from trade into gains from capital accumulation versus gains due to total factor productivity (TFP) changes.

Comparative advantage dictates that trade liberalization results in allocations that increase measured TFP. The increase in TFP, in turn, increases the rate of return to capital and hence the investment rate in a dynamic model. Under the assumption that the production of investment goods is more tradables intensive than the production of consumption goods, trade liberalization reduces the price of investment relative to the price of consumption; this also increases the investment rate. Thus, trade liberalization yields a higher stock of capital, higher output, and higher consumption.

Our trade environment is the multicountry model of Eaton and Kortum (2002), and our capital accumulation environment is a two-sector neoclassical growth model. We combine the two models, similar to Alvarez (2017) and Ravikumar, Santacreu, and Sposi (2019) (henceforth RSS), and study the interaction between international trade and capital accumulation. A continuum of tradable intermediate goods is used to produce investment goods, final consumption goods, and intermediate goods. A key assumption in our model is that the intensity of tradables is higher in the production of investment goods than in the production of consumption goods. Trade is balanced in each period. Each country is endowed with an initial stock of capital, and capital is accumulated in the same manner as in the neoclassical growth model.

We calibrate the steady state of the model to reproduce the observed bilateral trade flows across 93 countries. We then conduct a counterfactual exercise in which there is an unanticipated, uniform, and permanent reduction in trade frictions for all countries. We compute the exact levels of endogenous variables along the

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transition path from the calibrated steady state to the counterfactual steady state and calculate the welfare gains using a consumption-equivalent measure as in Lucas (1987).

We find that the consumption transition paths look similar across countries except for scale and comparing only steady states overstates the gains from trade; the dynamic gains accrue gradually and are about 60 percent of steady-state gains for every country. We also find that both the dynamic gains and the steady-state gains differ across countries: The dynamic gain for Belize is five times that of the United States, and the contribution of capital accumulation to dynamic gains is four times that of TFP.

In a closely related paper, Anderson, Larch, and Yotov (2020) also compute dynamic gains from trade. In their model, the transition path is a solution to a sequence of static problems since changes in trade frictions have no effect on the investment rate and the relative price of investment. In our model, the changes in trade frictions affect the transition dynamics of the investment rate and the relative price of investment.

The rest of the article proceeds as follows. Section 2 presents the model. Section 3 derives the welfare gains from trade. Section 4 describes the calibration, while Section 5 reports the quantitative results from the counterfactual exercise. Section 6 concludes.

2. MODEL

Our model is a simpler version of that in RSS: We do not have trade imbalances or adjustment costs to accumulating capital. There are I countries indexed by $i = 1, \ldots, I$, discrete time, running from $t = 1, \ldots, \infty$, and three sectors: consumption, investment, and intermediates, denoted by c, x, and m, respectively. There is a unit interval of varieties in the intermediates sector. Each variety within the sector is tradable and indexed by $v \in [0, 1]$. Neither consumption goods nor investment goods are tradable. The production of all goods is carried out by perfectly competitive firms.

As in Eaton and Kortum (2002), each country's efficiency in producing each intermediate variety is a realization of a random draw from a country-specific distribution. Trade in intermediate varieties is subject to iceberg costs. Each country purchases each intermediate variety from its lowest-cost supplier, and all of the varieties are aggregated into a *composite* intermediate good. The composite good is used as an input along with capital and labor to produce the consumption good, the investment good, and the intermediate varieties.

There is a representative household in country i endowed with a labor force of size L_i in each period, which it supplies inelastically, and an initial stock of capital, K_{i1}.

Composite good Within the intermediates sector, all of the varieties are combined with constant elasticity of substitution

$$M_{it} = \left[\int_0^1 q_{it}(v)^{1-1/\eta} \, dv \right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between any two varieties, $q_{it}(v)$ is the quantity of good v used by country i to construct the composite good at time t, and M_{it} is the quantity of the composite good available in country i to be used as an input.

Varieties The technologies for producing each variety are given by

$$Y_{mit}(v) = z_{mi}(v) \left(K_{mit}(v)^{\alpha} L_{mit}(v)^{1-\alpha} \right)^{\nu_m} M_{mit}(v)^{1-\nu_m}.$$

The parameter $\nu_m \in [0, 1]$ denotes the share of value added in total output and α denotes capital's share in value added. These parameters are constant across countries and over time. The term $M_{mit}(v)$ denotes the quantity of the composite good used by country i as an input to produce $Y_{mit}(v)$ units of variety v, while $K_{mit}(v)$ and $L_{mit}(v)$ denote the quantities of capital and labor used.

The term $z_{mi}(v)$ denotes country i's productivity for producing variety v. The productivity draw comes from independent Fréchet distributions with shape parameter θ and country-specific scale parameter T_{mi}, for $i = 1, 2, \ldots, I$. The cumulative distribution function for productivity draws in country i is $F_{mi}(z) = \exp(-T_{mi} z^{-\theta})$.

In country i the expected value of productivity is $\gamma^{-1} T_{mi}^{\frac{1}{\theta}}$, where $\gamma = \Gamma(1 + \frac{1}{\theta}(1-\eta))^{\frac{1}{1-\eta}}$ and $\Gamma(\cdot)$ is the gamma function and $T_{mi}^{\frac{1}{\theta}}$ is the fundamental productivity in country i. If $T_{mi} > T_{mj}$, then on average, country i is more efficient than country j at producing intermediate varieties. A smaller θ implies more room for specialization and hence more gains from trade.

Consumption and Investment goods The final consumption good and investment good are produced according to

$$Y_{cit} = A_{ci}\left(K_{cit}^{\alpha}L_{cit}^{1-\alpha}\right)^{\nu_c} M_{cit}^{1-\nu_c}$$
$$Y_{xit} = A_{xi}\left(K_{xit}^{\alpha}L_{xit}^{1-\alpha}\right)^{\nu_x} M_{xit}^{1-\nu_x}.$$

The parameters α, ν_c, and ν_x are constant across countries and over time. The term A_{ci} denotes country i's productivity in the consumption goods sector, and A_{xi} captures country i's productivity in the investment goods sector. The terms K_{cit}, L_{cit}, and M_{cit} denote the quantities of capital, labor, and the composite good used by country i to produce Y_{cit} units of consumption at time t. The terms K_{xit}, L_{xit}, and M_{xit} denote the quantities of capital, labor, and the composite good used by country i to produce Y_{xi} units of investment at time t.

Trade International trade is subject to frictions that take the iceberg form. Country i must purchase $d_{ij} \geq 1$ units of any intermediate variety from country j in order for one unit to arrive; $d_{ij}-1$ units melt away in transit. We normalize $d_{ii} = 1$ for all i.

Preferences The representative household's lifetime utility is given by

$$\sum_{t=1}^{\infty} \beta^{t-1} \frac{(C_{it}/L_i)^{1-1/\sigma}}{1-1/\sigma},$$

where C_{it}/L_i is consumption per capita in country i at time t, $\beta \in (0,1)$ denotes the discount factor, and σ denotes the intertemporal elasticity of substitution. Both parameters are constant across countries and over time.

Capital accumulation The representative household enters period t with K_{it} units of capital, which depreciates at the rate δ. Investment, X_{it}, adds to the stock of capital.

$$K_{it+1} = (1-\delta)K_{it} + X_{it}.$$

Budget constraint The representative household earns income by supplying capital and labor inelastically to domestic firms earning a rental rate r_{it} on capital and a wage rate w_{it} on labor. The household purchases consumption at the price P_{cit} and purchases investment at the price P_{xit}. The budget constraint is given by

$$P_{cit}C_{it} + P_{xit}X_{it} = r_{it}K_{it} + w_{it}L_i.$$

Equilibrium A competitive equilibrium satisfies the following conditions: (i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technology for accumulating capital; (ii) taking prices as given, firms maximize profits subject to the available technologies; (iii) intermediate varieties are purchased from their lowest-cost provider subject to the trade frictions; and (iv) all domestic markets clear, and trade is balanced in each period. At each point in time, we take world gross domestic product (GDP) as the numéraire: $\sum_i r_{it}K_{it} + w_{it}L_i = 1$ for all t. We describe each equilibrium condition in more detail in Appendix Appendix 1.

3. WELFARE GAINS

We measure changes in welfare using consumption-equivalent units as in Lucas (1987). In steady state, consumption is proportional to income, and the ratio of consumption to income, $1 - \frac{\alpha\delta}{\frac{1}{\beta}-(1-\delta)}$, is the same across countries. Real income per capita at time t is $y_{it} \equiv \frac{r_{it}K_{it}+w_{it}L_{it}}{P_{cit}L_{it}}$. Appendix Appendix 2 shows that

$$y_{it} \propto A_{ci}\left(\frac{T_{mi}}{\pi_{iit}}\right)^{\frac{1-\nu_c}{\theta\nu_m}}\left(\frac{K_{it}}{L_i}\right)^{\alpha}. \quad (1)$$

3.1 Steady-State Gains

We measure the steady-state gains in country i, λ_i^{ss}, according to

$$1+\frac{\lambda_i^{ss}}{100}=\frac{c_i^{\star\star}}{c_i^{\star}}=\frac{y_i^{\star\star}}{y_i^{\star}}, \tag{2}$$

where $c_i^{\star}$ and $y_i^{\star}$ are the per capita consumption and per capita income, respectively, in the initial steady state in country i and $c_i^{\star\star}$; and $y_i^{\star\star}$ are the per capita consumption and per capita income, respectively, in the counterfactual steady state in country i.

Contribution of capital Country i's steady-state per capita income is given by

$$y_i \propto \underbrace{A_{ci}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1-\nu_c}{\theta\nu_m}}}_{\text{TFP contribution}}\underbrace{A_{xi}^{\frac{\alpha}{1-\alpha}}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}}}_{\text{Capital contribution}}. \tag{3}$$

See Appendix Appendix 2 for the derivation.

Changes in steady-state per capita income are completely accounted for by changes in home trade share resulting from changes in trade costs. Equation (3) allows us to compute the contributions of TFP and capital in accounting for the steady-state gains. The log-change in steady-state welfare due to a log-change in the home trade share is

$$\frac{\partial\ln(y_i)}{\partial\ln(\pi_{ii})}=-\left(\underbrace{\frac{1-\nu_c}{\theta\nu_m}}_{\text{through TFP}}+\underbrace{\frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}}_{\text{through capital}}\right). \tag{4}$$

Two remarks are in order here. First, If the intensity of tradables is the same in the production of consumption goods and investment goods, i.e., $\nu_x=\nu_c$, then the contribution from capital would be just one-half of the contribution from TFP, assuming a typical capital share of $\alpha=\frac{1}{3}$. If investment goods are more tradables intensive, i.e., $1-\nu_x>1-\nu_c$, then the contribution of capital would be more than one-half. Second, the decomposition is constant across countries in our model since $(\theta,\alpha,\nu_c,\nu_m,\nu_x)$ are all constant across countries. This does not imply that the change in income is the same across countries, only that the *relative* contributions from TFP and capital are the same.

3.2 Transition and Dynamic Gains

Along the transition path, consumption might not be proportional to income. The dynamic gain in country i, λ_i^{dyn}, solves

$$\sum_{t=1}^{\infty}\beta^{t-1}\frac{\left(\left(1+\frac{\lambda_i^{dyn}}{100}\right)c_i^{\star}\right)^{1-1/\sigma}}{1-1/\sigma}=\sum_{t=1}^{\infty}\beta^{t-1}\frac{(\tilde{c}_{it})^{1-1/\sigma}}{1-1/\sigma}, \tag{5}$$

where $\tilde{c}_{it}$ is the per capita consumption at time t in the counterfactual. Note that in steady state, Equation (5) collapses to Equation (2).

Trade liberalization results in an immediate and permanent drop in the home trade shares and hence permanently higher-measured TFP on impact. The increase in TFP yields a higher rate of return to capital. While capital stock does not change on impact, the higher rate of return induces capital to increase gradually. The rate of accumulation depends on the investment rate, which is governed by the intertemporal Euler equation:

$$\frac{c_{it+1}}{c_{it}}=\beta^{\sigma}\left(1+\frac{r_{it+1}}{P_{xit+1}}-\delta\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma}, \tag{6}$$

where the relative price

$$\frac{P_{xit}}{P_{cit}} \propto \left(\frac{A_{ci}}{A_{xi}}\right)\left(\frac{T_{mi}}{\pi_{iit}}\right)^{\frac{\nu_x-\nu_c}{\theta\nu_m}}. \tag{7}$$

The lower home trade share implies a lower relative price of investment since $\nu_x < \nu_c$, so a larger share of income can be allocated to investment without sacrificing consumption.

The dynamics of capital in country i depend on the capital stocks in all other countries due to trade. That is, the prices faced in country i depend on the productivity and trade costs around the world together with market-clearing conditions. The solution is characterized by a system of I simultaneous, second-order, nonlinear difference equations. The optimality conditions for the firms combined with the relevant market-clearing conditions and trade balance pin down the prices as a function of the capital stocks in all countries.

Let $\mathbf{K}_t$ denote the vector of capital stocks across countries at time t. Combining the Euler equation with the budget constraint and the capital accumulation technology, the equilibrium law of motion for capital in every country must obey

$$\begin{aligned}
&\left(1+\frac{r_{it+1}(\mathbf{K}_{t+1})}{P_{xit+1}(\mathbf{K}_{t+1})}-\delta\right)\left(\frac{P_{xit+1}(\mathbf{K}_{t+1})}{P_{cit+1}(\mathbf{K}_{t+1})}\right)K_{it+1}\\
&+\left(\frac{w_{it+1}(\mathbf{K}_{t+1})}{P_{cit+1}(\mathbf{K}_{t+1})}\right)L_i-\left(\frac{P_{xit+1}(\mathbf{K}_{t+1})}{P_{cit+1}(\mathbf{K}_{t+1})}\right)K_{it+2}\\
&=\beta^{\sigma}\left(1+\frac{r_{it+1}(\mathbf{K}_{t+1})}{P_{xit+1}(\mathbf{K}_{t+1})}-\delta\right)^{\sigma}\left(\frac{P_{xit+1}(\mathbf{K}_{t+1})/P_{cit+1}(\mathbf{K}_{t+1})}{P_{xit}(\mathbf{K}_t)/P_{cit}(\mathbf{K}_t)}\right)^{\sigma}\\
&\times\left[\left(1+\frac{r_{it}(\mathbf{K}_t)}{P_{xit}(\mathbf{K}_t)}-\delta\right)\left(\frac{P_{xit}(\mathbf{K}_t)}{P_{cit}(\mathbf{K}_t)}\right)K_{it}+\left(\frac{w_{it}(\mathbf{K}_t)}{P_{cit}(\mathbf{K}_t)}\right)L_i-\left(\frac{P_{xit}(\mathbf{K}_{it})}{P_{cit}(\mathbf{K}_t)}\right)K_{it+1}\right].
\end{aligned} \tag{8}$$

This notation emphasizes the nonlinear dependence of prices in country i on the contemporaneous world distribution of capital stocks (i.e., $w_{it}(\mathbf{K}_t)$). The pricing equations depend on firms' first-order conditions and market-clearing conditions, which also involve underlying model parameters including the trade frictions. In other words, the solution to Equation (8) involves finding a fixed point in the space of capital sequences across countries. In turn, the equation reveals that a change in the trade friction for one country affects the dynamic path of all countries.

4. CALIBRATION

Our calibration strategy closely follows that in RSS. We calibrate the parameters of the model to match several observations in 2011, and we assume that the world is in steady state at this time. Table Appendix 4.1 provides the equilibrium conditions that describe the steady state. Our technique for computing the steady state is standard.

Our data cover 93 countries (containing 91 individual countries plus two regional country groups). Appendix Table Appendix 5.1 provides a list of the countries. This set of countries accounts for 90 percent of world GDP as measured by the Penn World Tables version 8.1 (hereafter PWT 8.1; Feenstra, Inklaar, and Timmer, 2015) and for 84 percent of world trade in manufactures as measured by the United Nations Comtrade Database. Appendix Appendix 3 provides the details of our data.

4.1 Common Parameters

Table 1 reports the values for the common parameters. Our elasticity of substitution parameter $\eta = 2$ plays no quantitative role in our results and satisfies the condition $1+\frac{1}{\theta}(1-\eta) > 0$. In line with the literature, we set the discount factor to $\beta = 0.96$, so that the steady-state real interest rate is about 4 percent, and the intertemporal elasticity of substitution to $\sigma = 0.67$.

We compute $\nu_m = 0.28$ by taking the cross-country average of the ratio of value added to the gross output of manufactures. We compute $\nu_x = 0.33$ by taking the cross-country average of the ratio of value added to the gross output of investment goods. Computing ν_c is slightly more involved since there is no clear industry classification for consumption goods. Instead, we infer this share by interpreting the national accounts through the lens of our model. We begin by noting that by combining firm optimization and market-clearing conditions for capital and labor, we get

$$r_iK_i = \frac{\alpha}{1-\alpha}w_iL_i. \tag{9}$$

Table 1
Common Parameters

θ	Trade elasticity (Simonovska and Waugh, 2014)	4
η	Elasticity of substitution between varieties	2
α	Capital's share in value added (Gollin, 2002)	0.33
β	Annual discount factor	0.96
δ	Annual depreciation rate for stock of capital	0.06
σ	Intertemporal elasticity of substitution	0.67
ν_c	Share of value added in final goods output	0.91
ν_x	Share of value added in investment goods output	0.33
ν_m	Share of value added in intermediate goods output	0.28

In steady state, the Euler equation and the capital accumulation technology imply

$$P_{xi}X_i = \frac{\delta\alpha}{\frac{1}{\beta} - (1-\delta)} \frac{w_i L_i}{1-\alpha} = \phi_x \frac{w_i L_i}{1-\alpha}.$$

We compute ϕ_x by taking the cross-country average of the share of gross fixed capital formation in nominal GDP. The household's budget constraint then implies that

$$P_{ci}C_i = \frac{w_i L_i}{1-\alpha} - P_{xi}X_i = (1-\phi_x)\frac{w_i L_i}{1-\alpha}.$$

Consumption in our model corresponds to the sum of private and public consumption, changes in inventories, and net exports. We use the trade balance condition together with the firm optimality and the market-clearing conditions for sectoral output to obtain

$$P_{mi}M_i = [(1-\nu_x)\phi_x + (1-\nu_c)(1-\phi_x)] \frac{w_i L_i}{1-\alpha} + (1-\nu_m)P_{mi}M_i, \tag{10}$$

where $P_{mi}M_i$ is the total absorption of manufactures in country i and $\frac{w_i L_i}{1-\alpha}$ is the nominal GDP. We use a standard method of moments estimator to back out ν_c from Equation (10).[1]

Given the value of ϕ_x and the relation $\phi_x = \frac{\delta\alpha}{\frac{1}{\beta}-(1-\delta)}$, the depreciation rate for capital is $\delta = 0.06$.

4.2 Country-Specific Parameters

We set the workforce, L_i, equal to the population in country i documented in PWT 8.1. The remaining parameters A_{ci}, T_{mi}, A_{xi}, and d_{ij}, for $(i,j) = 1,\ldots,I$, are not directly observable. We back these out by linking steady-state relationships of the model to observables.

The unobserved trade frictions between any two countries are related to the ratio of intermediate goods prices in the two countries and the trade shares by

$$\frac{\pi_{ij}}{\pi_{jj}} = \left(\frac{P_{mj}}{P_{mi}}\right)^{-\theta} d_{ij}^{-\theta}. \tag{11}$$

Appendix Appendix 3 describes how we construct the empirical counterparts to prices and trade shares. For observations in which $\pi_{ij} = 0$, we set $d_{ij} = 10^8$. We also set $d_{ij} = 1$ if the inferred value of trade cost is less than 1.

1. Using input-output data for 40 countries, we find that there is indeed variation in ν_c and ν_x. In every one of these countries, $\nu_x - \nu_c < 0$, with a range from –0.35 to –0.71. However, we assume that both ν_c and ν_x are constant across countries since (i) we do not have data on these shares for our sample of 93 countries and (ii) country-specific values for these parameters add noise to the channels that we explore. Allowing for these shares to differ across countries is straightforward with our solution algorithm.

Last, we use three structural relationships to pin down the productivity parameters A_{ci}, T_{mi}, and A_{xi}:

$$\frac{P_{ci}/P_{mi}}{P_{cU}/P_{mU}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}} / A_{ci}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}} / A_{cU}} \right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}} \right)^{\frac{\nu_c - \nu_m}{\nu_m}}, \tag{12}$$

$$\frac{P_{xi}/P_{mi}}{P_{xU}/P_{mU}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}} / A_{xi}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}} / A_{xU}} \right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}} \right)^{\frac{\nu_x - \nu_m}{\nu_m}}, \tag{13}$$

$$\frac{y_i}{y_U} = \left(\frac{A_{ci}}{A_{cU}}\right) \left(\frac{A_{xi}}{A_{xU}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}} \right)^{\frac{1-\nu_c+\frac{\alpha}{1-\alpha}(1-\nu_x)}{\nu_m}}. \tag{14}$$

Equations (12)–(14) are derived in Appendix Appendix 2. The three equations relate observables—the price of consumption relative to intermediates, the price of investment relative to intermediates, income per capita, and home trade shares—to the unknown productivity parameters. We set $A_{cU} = T_{mU} = A_{xU} = 1$ as a normalization, where the subscript U denotes the United States. For each country i, system (12)–(14) yields three nonlinear equations with three unknowns: A_{ci}, T_{mi}, and A_{xi}. Information about constructing the empirical counterparts to $P_{ci}, P_{mi}, P_{xi}, \pi_{ii}$, and y_i is in Appendix Appendix 3.

These equations are quite intuitive. The expression for income per capita provides a measure of aggregate productivity across all sectors: Higher income per capita is associated with higher productivity levels, on average. The expressions for relative prices boil down to two components. The first term reflects something akin to the Balassa-Samuelson effect: All else equal, a higher price of capital relative to intermediates suggests a low productivity in capital goods relative to intermediate goods. In our setup, the measured productivity for intermediates is endogenous, reflecting the degree of specialization as captured by the home trade share. The second term reflects the relative intensity of intermediate inputs. If measured productivity is high in intermediates, then the price of intermediates is relatively low and the sector that uses intermediates more intensively will have a lower relative price.

4.3 Model Fit

The correlation between the model and the data is 0.96 for the bilateral trade shares (see Figure 1), 0.97 for the absolute price of intermediates, 1.00 for income per capita, 0.96 for the price of consumption relative to intermediates, and 0.99 for the price of investment relative to intermediates. Our model does not perfectly replicate the data as there are more data points than parameters.

The correlation between the model and the data is 0.93 for the *absolute* price of consumption and 0.97 for the absolute price of investment. The correlation for the price of investment relative to consumption is 0.95. Recall that there are more data points than parameters in our calibration. Our model also has implications for the (untargeted) cross-country differences in capital and investment rates. Figure 2 shows that the model matches the data on capital-labor ratios; the correlation is 0.93.

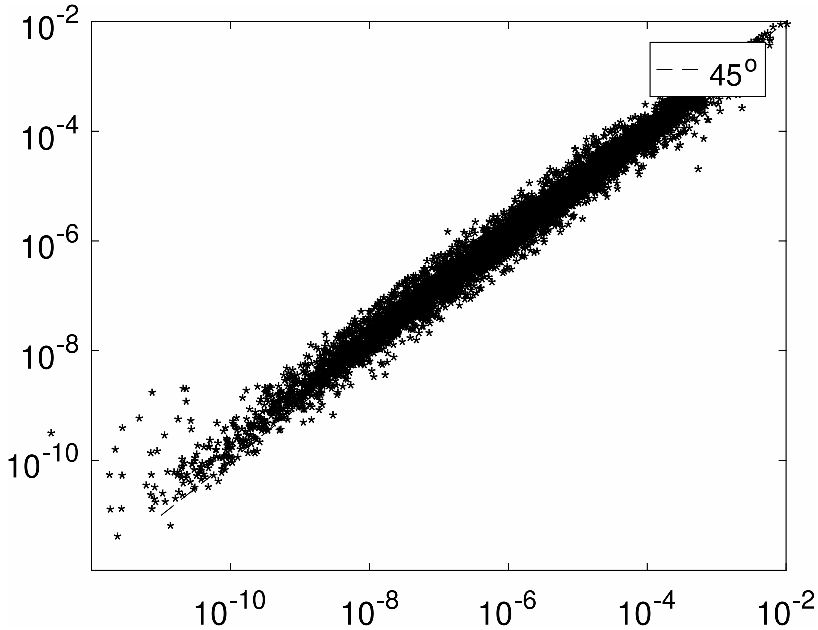
Our model is also broadly consistent with the real investment rate, $\frac{X}{yL}$. The nominal investment rate, $\frac{P_x X}{P_c y L}$, is the same across countries in the steady state and is equal to 19.5 percent; in the data, it is 23.3 percent and is uncorrelated with economic development. Since we assume that the world is in steady state in 2011, the investment rate is proportional to the capital-output ratio. Our model matches GDP by construction and also does well matching capital stocks, so our ability to replicate the investment rate is limited to the extent that the steady-state assumption is valid in the data.

5. TRADE LIBERALIZATION AND WELFARE GAINS

In our counterfactual trade liberalization, the world begins in the calibrated steady state. At the beginning of period $t = 1$, trade frictions fall uniformly in all countries such that the ratio of world trade to GDP increases from 50 percent in the calibrated steady state to 100 percent in the new steady state. This amounts to reducing $d_{ij} - 1$ by 55 percent for each country pair i, j. All other parameters are fixed at their calibrated values. The decline in trade frictions is unanticipated and permanent.

Appendix 4 describes our algorithm for computing the transition path in the counterfactual. Our solution method is gradient free and generalizes the algorithm of Alvarez and Lucas (2007) by iterating on a subset of

Figure 1
Model Fit for Bilateral Trade Share



NOTE: Horizontal axis, data; Vertical axis, model.

prices using excess demand equations. Our algorithm in this article is a special case of that in RSS. We deliver the entire transition path for 93 countries in less than four hours on a basic laptop computer. Our method is computationally less demanding than nonlinear solvers.

Specifically, we first reduce the infinite dimension of the problem down to a finite time model with $t = 1, \ldots, T$ periods. We make T sufficiently large to ensure convergence to a new steady state. This requires us to first solve for a terminal steady state to use as a boundary condition for the path of capital stocks. The other boundary condition is the set of capital stocks in the calibrated steady state; the transition path starts from this set. We guess the entire sequence of wages and rental rates in every country. Given the wages and rental rates, we recover all remaining prices and trade shares using optimality conditions for firms and then solve for the optimal sequence of consumption and investment in every country using the intertemporal Euler equation. Finally, we use deviations from domestic market-clearing and trade balance conditions to update the sequences of wages and rental rates. We continue the process until we reach a fixed point where all markets clear in all periods.

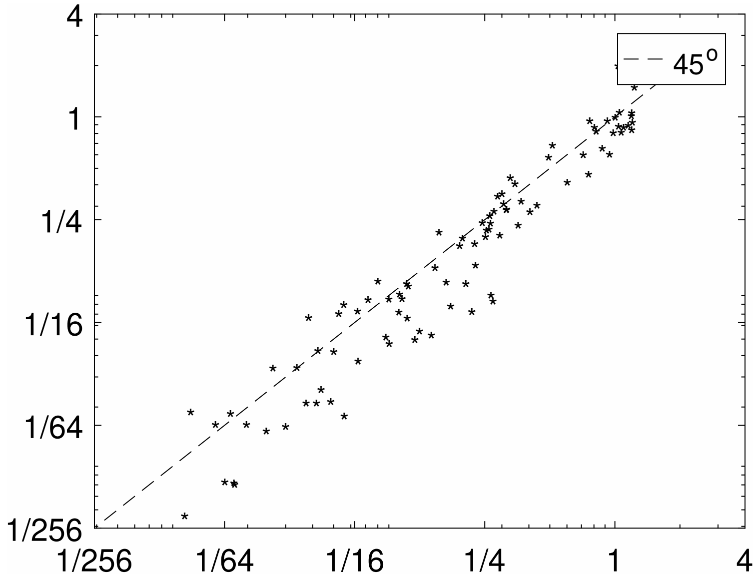
Steady-state gains We compute the steady-state gains from trade using Equation (2). The steady-state gains vary substantially across countries, ranging from 18 percent for the United States to 92 percent for Belize (Figure 3). The median gain (Greece) is 53 percent. (Recall that consumption is proportional to income in steady state, so the welfare gain can be measured by the change in per capita income.)

The change in capital accounts for 79 percent of the change in income per capita across steady states; the change in TFP accounts for the remaining 21 percent. Recall from Equation (3) that the relative contributions from TFP and capital are the same across countries since $(\theta, \alpha, \nu_c, \nu_m, \nu_x)$ are all constant across countries even though the change in income is different across countries.

Dynamic gains We compute the dynamic gains from trade using Equation (5). We calculate sums in (5) using the counterfactual transition path from $t = 1, \ldots, 150$ and setting the counterfactual consumption equal to the new steady-state level of consumption for $t = 151, \ldots, 400$.

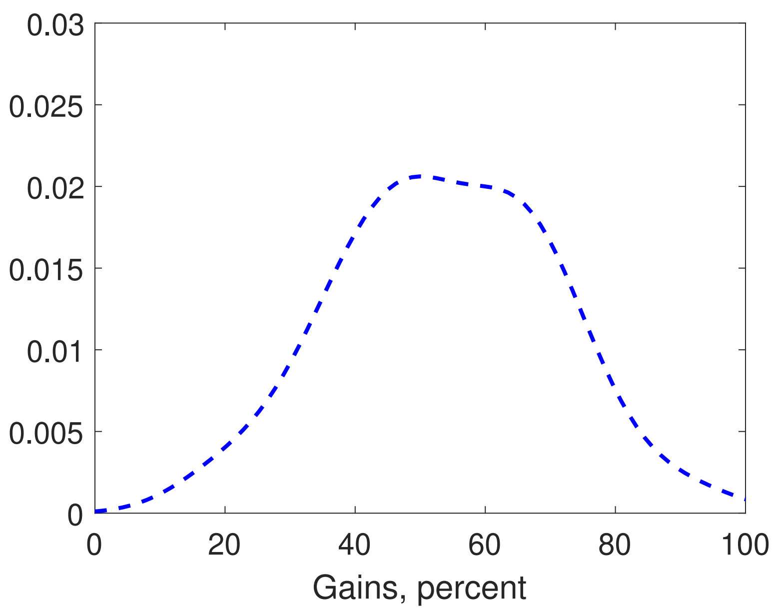
Dynamic gains also vary substantially across countries, ranging from 11 percent for the United States to 56 percent for Belize, with the median country (Greece) being 32 percent (see Figure 4a). Similar to the findings in the existing literature (Waugh and Ravikumar, 2016; Waugh, 2010), the gains are systematically smaller for large, developed countries and countries with smaller export frictions. Furthermore, the magnitude of our changes in welfare is similar to that in Desmet, Nagy, and Rossi-Hansberg (2015), who consider a

Figure 2
Model Fit for Capital-Labor Ratio



NOTE: Horizontal axis, data; Vertical axis, model.

Figure 3
Distribution of Steady-State Gains from Trade

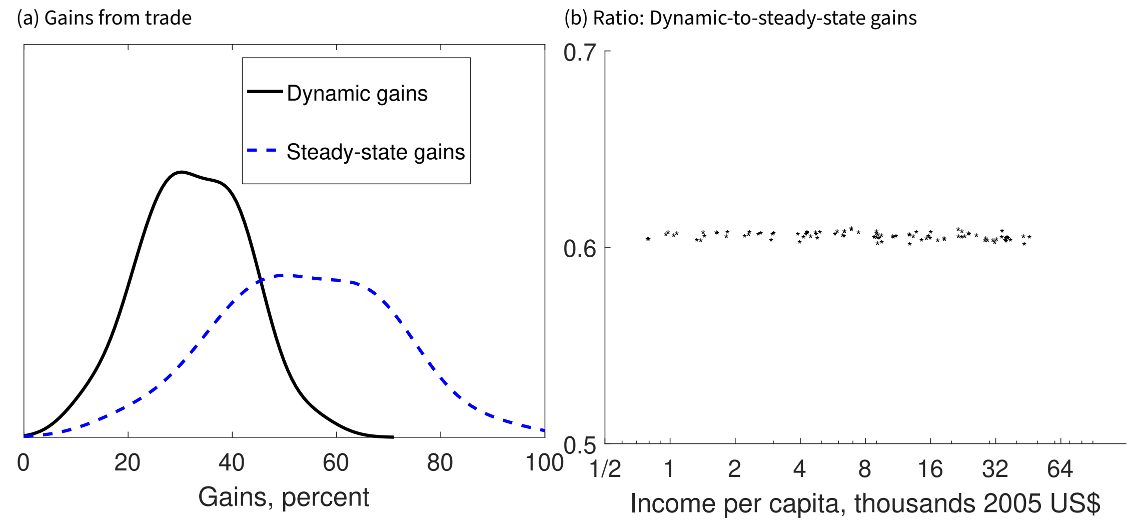


counterfactual increase of 40 percent in trade costs in a model of migration and trade and find that welfare decreases by around 34 percent.

Figure 4a shows that the dynamic gains are smaller than the steady-state gains. The average ratio of dynamic gains to steady-state gains is 60.2 percent and varies from a minimum of 60.1 percent to a maximum of 60.5 percent (see Figure 4b). This result is not specific to the magnitude of the trade liberalization: The ratio of dynamic to steady-state gains is about 60 percent in every counterfactual where trade frictions are uniformly reduced across countries.

The ratio of roughly 60 percent is a result of (i) the initial change in consumption and (ii) the rate at which consumption converges to the new steady state. If consumption jumped to its new steady-state level on impact,

Figure 4
Distribution of Gains from Trade



then the ratio would be close to 100 percent. If instead consumption declined significantly in the beginning and then converged to the new steady state slowly, then the ratio would be closer to 0 percent since there would be consumption losses in earlier periods, while higher levels of future consumption would be discounted.

Role of capital deepening Trade liberalization increases each country's output, making more consumption and investment feasible. First, the immediate increase in output is driven by an immediate increase in measured TFP; capital does not change on impact. Optimal allocation of the higher output to consumption and investment determines the dynamics and is governed by the relative price of investment and the return to capital, as revealed by the Euler Equation (6). Second, the changes in measured TFP are pinned down by the changes in home trade share; see Equation (1). Since the home trade share jumps on impact to the new steady-state level and remains constant thereafter, TFP also jumps on impact to the new steady state and remains constant thereafter; see Figure 5.

Trade liberalization reduces the relative price of investment. The reason is that trade liberalization decreases the price of traded intermediates, and intermediates are used more intensively in the production of investment goods than in consumption goods ($\nu_x < \nu_c$). Hence, the price of investment goods falls relative to that of consumption goods. The decline in the relative price of investment implies that investment can increase by a larger proportion than the increase in output without giving up consumption. This would result in a higher investment rate, i.e., a higher rate of capital accumulation.

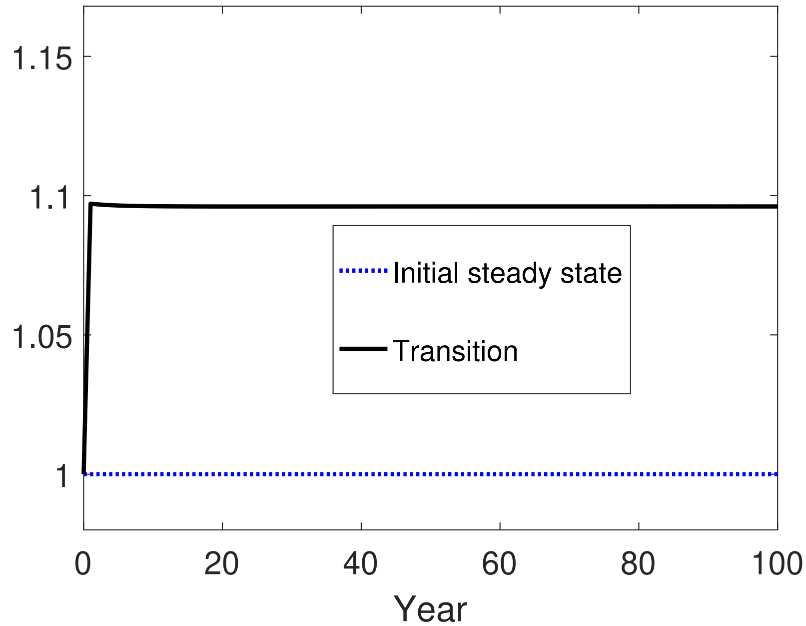
Figure 6 shows the transition paths for the relative price of investment and the return to capital for the country with the median gain. The transition paths for other countries are similar but differ in their magnitudes: Belize is at one extreme, and the U.S. is at the other. The relative price differences across countries are large in our calibrated steady state and in the data: The relative price in Belize is more than twice that in the U.S. Trade liberalization reduces these price differences: The relative price in Belize is only 10 percent higher than that in the U.S. after liberalization.

The return to capital on the transition path, $\left(1 + \frac{r_{it+1}}{P_{xit+1}} - \delta\right)$, is higher than the steady-state return, $\frac{1}{\beta}$. This is because, following the trade liberalization, measured TFP is higher. With the higher return, households invest more and the capital-labor ratio increases along the transition. This drives the return back to its initial steady-state level. As capital accumulates, output increases. Recall that the increase in output on impact is entirely due to TFP, whereas the increase in output after the initial period is driven entirely by capital accumulation. With higher output, both consumption and investment increase and settle to the new, higher steady-state levels.

Remarks First, one could imagine computing the welfare gain along the transition path by using Equation (4) to compute the change in the income due to the change in home trade share, period by period:

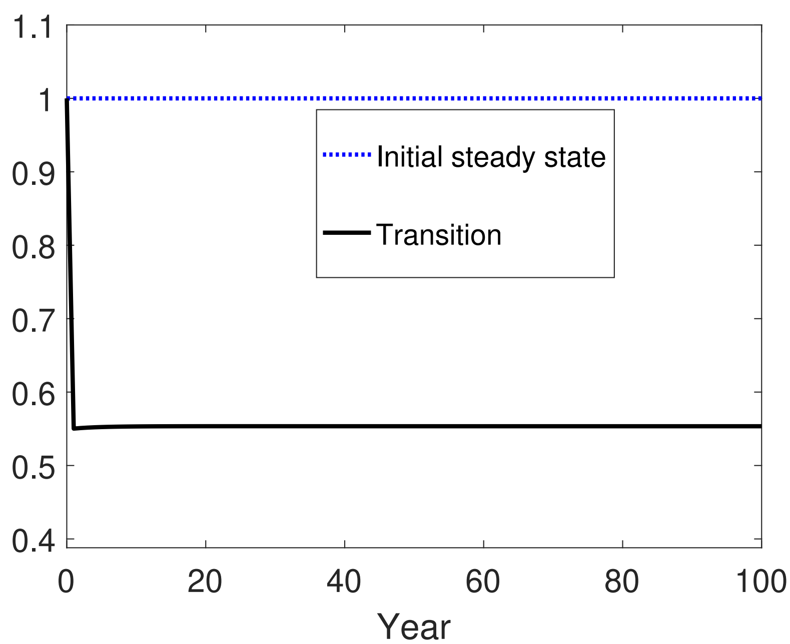
$$\Delta \ln(y_i) = -\left(\frac{1-\nu_c}{\theta\nu_m} + \frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}\right)\Delta \ln(\pi_{ii}).$$

Figure 5
Transition Path for TFP in the Median Country



NOTE: The country with the median dynamic gain is Greece. TFP is indexed to 1 in the initial steady state. Year 0 is the initial steady state, and year 1 is the period of liberalization.

Figure 6
Transition Paths for Relative Price in the Median Country

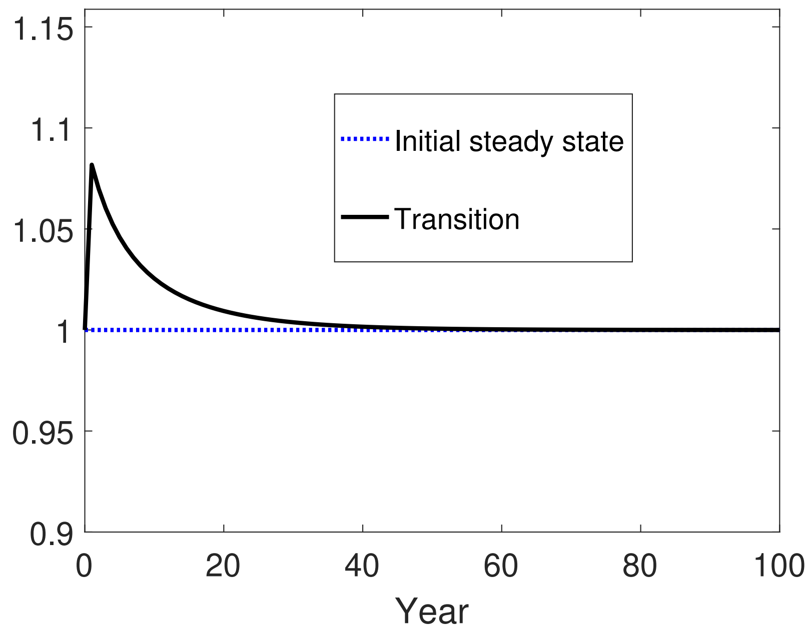


NOTE: The country with the median dynamic gain is Greece. Variables are indexed to 1 in the initial steady state. Year 0 is the initial steady state, and year 1 is the period of liberalization.

This is invalid for two reasons. First, along the transition, consumption is not proportional to income. For instance, consumption in the median-gain country grows by 4.7 percent between periods 2 and 3, while income grows by only 3.4 percent. Second, even if consumption was roughly proportional to income, changes

Figure 7

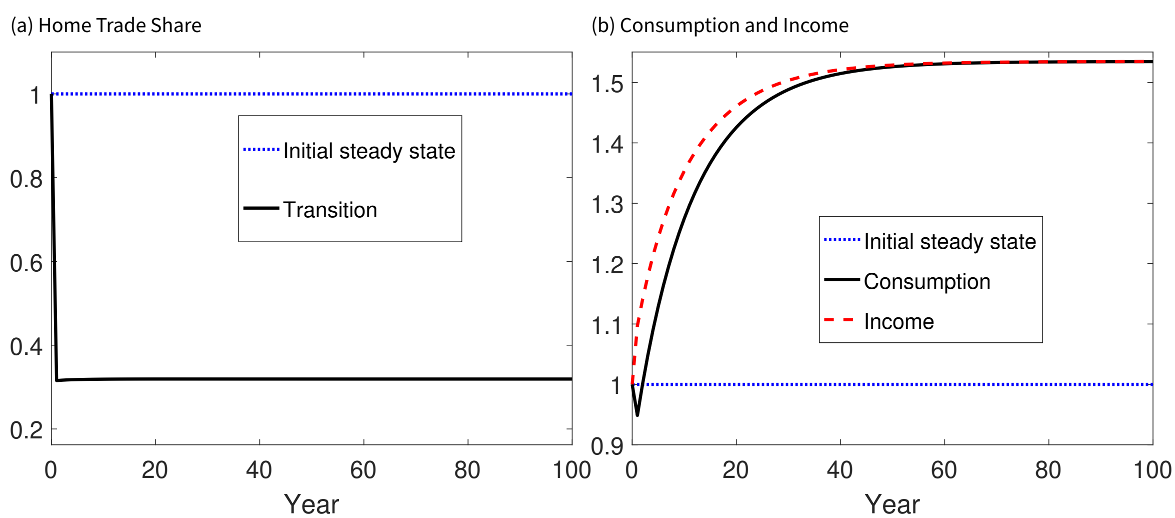
Transition Paths for Return to Capital in the Median Country



NOTE: The country with the median dynamic gain is Greece. Variables are indexed to 1 in the initial steady state. Year 0 is the initial steady state, and year 1 is the period of liberalization.

Figure 8

Transition for Home Trade Share, Consumption, and Income in the Median Country



NOTE: The country with the median dynamic gain is Greece. Variables are indexed to 1 in the initial steady state. Year 0 is the initial steady state, and year 1 is the period of liberalization.

in the home trade share between any two periods do not describe changes in income. Figure 8 plots the transition path for the home trade share, consumption, and income in our model for the median-gain country. The home trade share jumps on impact to the new steady-state level and remains constant thereafter (see Figure 8a). Income grows gradually to the new steady state; see Figure 8b.

Second, as noted earlier, the contribution of capital deepening to the gains from trade depends on the values of ν_x and ν_c. The contribution of capital deepening is just one-half of that of TFP when $\nu_x = \nu_c$, whereas the contribution is four times that of TFP in our calibration. In our calibration we interpret $P_{mi}M_i$ as the absorption of manufactures in country i and use (10) to determine ν_c. Alternatively, one could interpret $P_{mi}M_i$ as the *total* amount of absorption of intermediate goods in country i as RSS do. Under the alternative interpretation, $P_{mi}M_i$ is greater than the value of gross manufacturing output, which results in a smaller value of $\nu_c = 0.56$. The lower value of ν_c reduces the contribution of capital deepening to three-fourths of that of TFP.

Third, we assume balanced trade in our computation of dynamic gains: Households cannot borrow from or lend to other countries. The welfare gains from trade under this assumption could thus be an underestimate. RSS study a model with endogenous trade imbalances by introducing one-period bonds that can be traded freely across countries and show that each country's gain from trade depends on its net foreign asset position.

6. CONCLUSION

We build a multicountry trade model with capital accumulation where the relative price of investment and the investment rate are affected by trade frictions. We use this framework to study dynamic welfare gains after a trade liberalization and quantify the contribution of measured TFP and capital deepening to gains from trade. Our computational algorithm efficiently solves for the exact transitional dynamics for a system of second-order, nonlinear difference equations.

Our counterfactual trade liberalization suggests that the dynamic gains differ by a factor of five across countries. The dynamic gains are 60 percent of the steady-state gains, and almost 80 percent of the gains are due to capital deepening. Trade liberalization reduces the relative price of investment, allowing countries to invest more without forgoing consumption and therefore attain permanently higher capital-labor ratios. Trade liberalization also increases TFP, which increases the rate of return to investment, affecting the dynamic path of investment. As capital accumulates, consumption increases and the welfare gains accrue over time.

Almost all of the changes in measured TFP occur at the time of the liberalization. This immediate change in measured TFP suggests that one can compute the transition paths in our model by solving the transition paths for *closed* economies as follows. First, compute the home trade share in the counterfactual steady state for each country; this can be done without solving for the transition paths. Second, use the home trade shares to compute the measured TFP in each country. Third, endow each country with its "new" measured TFP, and compute its transition path assuming it is closed. Our conjecture is that the path computed in this manner would be almost identical to the one in our open economy model.

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APPENDIX 1. EQUILIBRIUM CONDITIONS

Household optimization The representative household chooses a path for consumption that satisfies the following Euler equation:

$$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(1+\frac{r_{it+1}}{P_{xit+1}}-\delta\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma}. \quad \text{(Appendix 1.1)}$$

Combining the household's budget constraint and the capital accumulation technology and rearranging, we get

$$C_{it} = \left(1+\frac{r_{it+1}}{P_{xit}+1}-\delta\right)\left(\frac{P_{xit}}{P_{cit}}\right)K_{it}+\left(\frac{w_{it}}{P_{cit}}\right)L_i-\left(\frac{P_{xit}}{P_{cit}}\right)K_{it+1}. \quad \text{(Appendix 1.2)}$$

Firm optimization Markets are perfectly competitive, so firms set prices equal to marginal costs. Denote the price of variety v, produced in country j and purchased by country i, as $p_{mij}(v)$. Then $p_{mij}(v) = p_{mjj}(v)d_{ij}$; in country j, $p_{mjj}(v)$ is also the marginal cost of producing variety v. Since country i purchases each variety from the country that can deliver it at the lowest price, the price in country i is $p_{mi}(v) = \min_{j=1,\ldots,I}[p_{mjj}(v)d_{mij}]$. The price of the composite good in country i at time t is then

$$P_{mit} = \gamma\left[\sum_{j=1}^{I}(u_{jt}d_{ij})^{-\theta}T_{mj}\right]^{-\frac{1}{\theta}}, \quad \text{(Appendix 1.3)}$$

where $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{jt}}{1-\nu_m}\right)^{1-\nu_m}$ is the unit cost for a bundle of inputs for intermediate goods producers in country n at time t.

Next we define total factor usage in the intermediates sector by aggregating across the individual varieties.

$$K_{mit} = \int_0^1 K_{mit}(v)dv, \qquad L_{mit} = \int_0^1 L_{mit}(v)dv,$$
$$M_{mit} = \int_0^1 M_{mit}(v)dv, \qquad Y_{mit} = \int_0^1 Y_{mit}(v)dv.$$

The term $L_{mit}(v)$ denotes the labor used in the production of variety v at time t. If country i imports variety v at time t, then $L_{mit}(v) = 0$. Hence, L_{mit} is the total labor used in sector m in country i at time t. Similarly, K_{mit} is the total capital used, M_{mit} is the total intermediates used as an input, and Y_{mit} is the total output of intermediates.

Cost minimization by firms implies that, within each sector $b \in \{c, m, x\}$, factor expenses exhaust the value of output:

$$\begin{aligned} r_{it}K_{bit} &= \alpha\nu_b P_{bit}Y_{bit}, \\ w_{it}L_{bit} &= (1-\alpha)\nu_b P_{bit}Y_{bit}, \\ P_{mit}M_{bit} &= (1-\nu_b)P_{bit}Y_{bit}. \end{aligned}$$

That is, the fraction $\alpha\nu_b$ of the value of each sector's production compensates capital services, the fraction $(1-\alpha)\nu_b$ compensates labor services, and the fraction $1-\nu_b$ covers the cost of intermediate inputs; there are zero profits.

Trade flows The fraction of country i's expenditures allocated to intermediate varieties produced by country j is given by

$$\pi_{ijt} = \frac{(u_{mjt}d_{ijt})^{-\theta}T_{mj}}{\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}}, \quad \text{(Appendix 1.4)}$$

where u_{mjt} is the unit cost of intermediate varieties in country j.

Market-clearing conditions The domestic factor market-clearing conditions are

$$\sum_{b\in\{c,m,x\}} K_{bit} = K_{it}, \qquad \sum_{b\in\{c,m,x\}} L_{bit} = L_{i}, \qquad \sum_{b\in\{c,m,x\}} M_{bit} = M_{it}.$$

The first two conditions impose that the capital and labor markets clear in country i at each time t. The third condition requires that the use of the composite good equals its supply. Its use consists of demand by firms in each sector, and its supply consists of both domestically and foreign-produced varieties.

The next set of conditions require that goods markets clear:

$$C_{it} = Y_{cit}, \qquad X_{it} = Y_{xit}, \qquad \sum_{j=1}^{I} P_{mjt}\left(M_{cjt} + M_{mjt} + M_{xjt}\right)\pi_{jit} = P_{mit}Y_{mit}.$$

The first condition states that the quantity of (nontradable) consumption demanded by the representative household in country i must equal the quantity produced by country i. The second condition says the same for the investment good. The third condition imposes that the value of intermediates produced by country i must be absorbed globally. Recall that $P_{mjt}M_{bjt}$ is the value of intermediate inputs that country i uses in production in sector b. The term π_{jit} is the fraction of country j's intermediate good expenditures sourced from country i. Therefore, $P_{mjt}M_{bjt}\pi_{jit}$ denotes the value of trade flows from country i to j.

Finally, we impose an aggregate resource constraint in each country: Net exports equal zero. Equivalently, gross output equals gross absorption:

$$P_{mit}Y_{mit} = P_{mit}M_{it}.$$

APPENDIX 2. DERIVATIONS OF STRUCTURAL RELATIONSHIPS

This appendix shows the derivations of key structural relationships in the model. These relationships can also be derived from RSS as a special case without trade imbalances and adjustment costs. We refer to Table Appendix 4.2 for the derivations and omit time subscripts to simplify notation. We begin by deriving an expression for $\frac{w_i}{P_{mi}}$ that will be used repeatedly.

Combining conditions 17 and 19, we obtain

$$\pi_{ii} = \gamma^{-\theta}\left(\frac{u_{mi}^{-\theta}T_{mi}}{P_{mi}^{-\theta}}\right).$$

Use the fact that $u_{mi} = B_m r_i^{\alpha\nu_m} w_i^{(1-\alpha)\nu_m} P_{mi}^{1-\nu_m}$, where B_m is a collection of constants; then rearrange to obtain

$$P_{mi} = \left(\frac{T_{mi}}{\pi_{ii}}\right)^{-\frac{1}{\theta}}\left(\frac{r_i}{w_i}\right)^{\alpha\nu_m}\left(\frac{w_i}{P_{mi}}\right)^{\nu_m} P_{mi}$$

(Appendix 2.1)
$$\Rightarrow \frac{w_i}{P_{mi}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1}{\nu_m}}\left(\frac{w_i}{r_i}\right)^{\alpha}.$$

Note that this relationship holds in both the steady state and along the transition.

Relative prices We show how to derive the price of consumption relative to intermediates; the relative price of investment is analogous. Begin with condition 16 to obtain

$$P_{ci} = \left(\frac{B_c}{A_{ci}}\right)\left(\frac{r_i}{w_i}\right)^{\alpha\nu_c}\left(\frac{w_i}{P_{mi}}\right)^{\nu_c} P_{mi},$$

where B_c is a collection of constants. Substitute Equation (Appendix 2.1) into the previous expression, and rearrange to obtain

(Appendix 2.2)
$$\frac{P_{ci}}{P_{mi}} = \left(\frac{B_c}{A_{ci}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{\nu_c}{\nu_m}}.$$

Analogously,

$$\text{(Appendix 2.3)} \qquad \frac{P_{xi}}{P_{mi}} = \left(\frac{B_x}{A_{xi}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{\nu_x}{\nu_m}}.$$

Note that these relationships hold in both the steady state and along the transition.

Capital-labor ratio We derive a structural relationship for the capital-labor ratio in the steady state only and refer to conditions in Table Appendix 4.1. Conditions 1–6 together with conditions 10 and 11 imply that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_i}{r_i}\right).$$

Using condition 23, we know that

$$r_i = \left(\frac{1}{\beta} - (1-\delta)\right)P_{xi},$$

which, by substituting into the prior expression, implies that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{(1-\alpha)\left(\frac{1}{\beta} - (1-\delta)\right)}\right)\left(\frac{w_i}{P_{xi}}\right),$$

which leaves the problem of solving for $\frac{w_i}{P_{xi}}$. Equations (Appendix 2.1) and (Appendix 2.3) imply

$$\begin{aligned}\frac{w_i}{P_{xi}} &= \left(\frac{w_i}{P_{mi}}\right)\left(\frac{P_{mi}}{P_{xi}}\right) \\ &= \left(\frac{A_{xi}}{B_x}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{\nu_m}}\left(\frac{w_i}{r_i}\right)^{\alpha}.\end{aligned}$$

Substituting once more for $\frac{w_i}{r_i}$ in the previous expression yields

$$\left(\frac{w_i}{P_{xi}}\right)^{1-\alpha} = \left(\frac{1}{\beta} - (1-\delta)\right)^{-\alpha}\left(\frac{A_{xi}}{B_x}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{\nu_m}}.$$

Solve for the aggregate capital-labor ratio

$$\text{(Appendix 2.4)} \qquad \frac{K_i}{L_i} = \left(\frac{\frac{\alpha}{1-\alpha}}{\left(\frac{1}{\beta} - (1-\delta)\right)^{-\frac{1}{1-\alpha}}}\right)\left(\frac{A_{xi}}{B_x}\right)^{\frac{1}{1-\alpha}}\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{(1-\alpha)\nu_m}}.$$

Note that we invoked steady-state conditions, so this expression does not necessarily hold along the transition path.

Income per capita We define (real) income per capita in our model as

$$y_i = \frac{r_i K_i + w_i L_i}{L_i P_{ci}}.$$

We invoke conditions from Table Appendix 4.2 for the remainder of this derivation. Conditions 1–6, 10, and 11 imply that

$$\begin{aligned} r_i K_i + w_i L_i &= \frac{w_i L_i}{1-\alpha} \\ \Rightarrow y_i &= \left(\frac{1}{1-\alpha}\right)\left(\frac{w_i}{P_{ci}}\right).\end{aligned}$$

To solve for $\frac{w_i}{P_{ci}}$, we use condition 16:

$$P_{ci} = \frac{B_c}{A_{ci}}\left(\frac{r_i}{w_i}\right)^{\alpha\nu_c}\left(\frac{w_i}{P_{mi}}\right)^{\nu_c} P_{mi}$$
$$\Rightarrow \frac{P_{ci}}{w_i} = \frac{B_c}{A_{ci}}\left(\frac{r_i}{w_i}\right)^{\alpha\nu_c}\left(\frac{w_i}{P_{mi}}\right)^{\nu_c-1}.$$

Substituting Equation (Appendix 2.1) into the previous expression and exploiting the fact that $\frac{w_i}{r_i} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{K_i}{L_i}\right)$ yields

$$y_i = \left(\frac{1}{1-\alpha}\right)\left(\frac{w_i}{P_{ci}}\right)$$

(Appendix 2.5)
$$= \alpha^{-\alpha}(1-\alpha)^{\alpha-1}\left(\frac{A_{ci}}{B_c}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_c}{\nu_m}}\left(\frac{K_i}{L_i}\right)^{\alpha}.$$

Note that this expression holds both in the steady state and along the transition path.

The steady-state income per capita can be expressed more fundamentally by invoking Equation (Appendix 2.4) as

(Appendix 2.6)
$$y_i = \left(\frac{\left(\frac{1}{\beta}-(1-\delta)\right)^{-\frac{\alpha}{1-\alpha}}}{1-\alpha}\right)\left(\frac{A_{ci}}{B_c}\right)\left(\frac{A_{xi}}{B_x}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_c+\frac{\alpha}{1-\alpha}(1-\nu_x)}{\nu_m}}.$$

Income per capita as a function of capital-output ratio Define $Z \equiv A_{ci}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1-\nu_c}{\theta\nu_m}}$ and $k = \frac{K}{L}$. Then, we can write (Appendix 2.5) as

$$y_i \propto Z{k_i}^{\alpha}.$$

Dividing both sides by y^{α} and rearranging, we get

$$y_i \propto Z^{\frac{1}{1-\alpha}}\left(\frac{k_i}{y_i}\right)^{\frac{\alpha}{1-\alpha}}.$$

The first term is the direct effect of measured TFP on per capita income, and the second term is the effect of capital accumulation. Unlike the neoclassical growth model, the two effects are not orthogonal. To determine the capital-output ratio, note that (Appendix 2.4) implies

$$k_i \propto A_{xi}^{\frac{1}{1-\alpha}}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1-\nu_x}{(1-\alpha)\theta\nu_m}}.$$

Using the fact that $y_i \propto Z{k_i}^{\alpha}$, we get

$$\frac{k_i}{y_i} \propto \frac{1}{Z}A_{xi}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1-\nu_x}{\theta\nu_m}},$$

so

$$y_i \propto Z^{\frac{1}{1-\alpha}}\left(\frac{A_{xi}}{Z}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}}$$
$$\propto Z^{\frac{1}{1-\alpha}}\left(\frac{A_{xi}}{A_{ci}}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{\alpha(\nu_c-\nu_x)}{(1-\alpha)\theta\nu_m}}.$$

Recall that Z is a function of π_{ii}. Thus,

$$\frac{\partial \ln(y_i)}{\partial \ln(\pi_{ii})} = -\left(\underbrace{\frac{1-\nu_c}{(1-\alpha)\theta\nu_m}}_{\text{through TFP}} + \underbrace{\frac{\alpha(\nu_c-\nu_x)}{(1-\alpha)\theta\nu_m}}_{\text{through k/y}} \right).$$

For our parameters, the capital-output ratio effect is twice as large as the TFP effect.

APPENDIX 3. DATA

This section describes the sources of data and any adjustments we make to the data to map it to the model.

Production and Trade Mapping the trade dimension of our model to the data requires observations on both production and international trade flows. Our focus is on manufactured intermediate goods. We interpret manufacturing broadly as defined by the International Standard Industrial Classification (ISIC).

We obtain production data from multiple sources. First, we use value added and gross output data from the INDSTAT database, which are reported at the two-digit level using ISIC. The data for countries extend no further than 2010 and not even to 2010 for many countries. We use data on value added output in the UN National Accounts Main Aggregates Database (UNNAMAD, http://unstats.un.org/unsd/snaama/Introduction.asp), for 2011. For countries that report both value added and gross output in INDSTAT, we use the ratio in the year closest to 2011 and apply that to the value added from UNNAMAD to recover gross output. For countries with no data on gross output in INDSTAT for any years, we apply the average ratio of value added to gross output across all countries and apply that ratio to the value added figure in UNNAMAD for 2011. In our dataset, the ratio of value added to gross output does not vary significantly over time and is also not correlated with the level of development or country size.

For trade data, we use the UN Comtrade Database (http://comtrade.un.org). Trade is reported for goods using Standard International Trade Classification Revision 2 (SITC2) at the four-digit level. We use the correspondence tables created by Affendy, Sim Yee, and Satoru (2010) to map SITC2 to ISIC. We also omit any petroleum-related products.

Using the trade and production data, we construct bilateral trade shares for each country pair by following Bernard, Eaton, Jensen, and Kortum (2003) as follows:

$$\pi_{ij} = \frac{X_{ij}}{ABS_{bi}},$$

where i denotes the importer, j denotes the exporter, X_{ij} denotes manufacturing trade flows from j to i, and ABS_i denotes country i's absorption defined as gross output less net exports of manufactures.

National Accounts and Price

GDP and population We use data on output-side real GDP at current purchasing power parity (PPP; 2005 U.S. dollars) from PWT 8.1 using the variable `cgdpo`. We use the variable `pop` from PWT 8.1 to measure the population in each country. The ratio $\frac{\texttt{cgdpo}}{\texttt{pop}}$ corresponds to GDP per capita, y, in our model.

In our counterfactuals, we compare changes over time with past trade liberalization episodes using the national accounts from PWT 8.1: *rgdpna*, *rkna*, and *rtfpna*.

We take the price of household consumption and the price of capital formation (both relative to the price of output-side GDP in the Unites States in constant prices) from PWT 8.1 using variables `pl_c` and `pl_i`, respectively. These correspond to P_c and P_x in our model.

We construct the price of intermediate goods (manufactures) by combining disaggregate price data from the World Bank's 2011 International Comparison Program (ICP; http://siteresources.worldbank.org/ICPEXT/Resources/ICP_2011.html). The data have several categories that fall under what we classify as manufactures: "food and nonalcoholic beverages," "alcoholic beverages, tobacco, and narcotics," "clothing and foot wear," and "machinery and equipment." The ICP reports expenditure data for these categories in both nominal U.S. dollars and real U.S. dollars. The PPP price equals the ratio of nominal expenditures to real expenditures. We compute the PPP for manufactures as a whole of manufactures for each country as the sum of nominal expenditures across categories divided by the sum of real expenditures across categories.

There is one more step before we take these prices to the model. The data correspond to expenditures and thus include additional margins such as distribution. To adjust for this, we first construct a price for distribution services. We assume that the price of distribution services is proportional to the overall price of services in each country and use the same method as above to compute the price across the following categories: "housing, water, electricity, gas, and other fuels," "health," "transport," "communication," "recreation and culture," "education," "restaurants and hotels," and "construction."

Now that we have the price of services in hand, we strip it away from the price of goods computed above to arrive at a measure of the price of manufactures that better maps to our model. In particular, let P_d denote the price of distribution services and P_g denote the price of goods that includes the distribution margin. We assume that $P_g = P_d^{\psi} P_m^{1-\psi}$, where P_m is the price of manufactures. We set $\psi = 0.45$, a value commonly used in the literature.

APPENDIX 4. SOLUTION ALGORITHM

In this appendix, we describe the algorithm for computing (i) the steady state and (ii) the transition path. Before going further into the algorithms, we introduce some notation. We denote the steady-state objects using the $\star$ as a superscript; that is, $K_i^\star$ is the steady-state stock of capital in country i. We denote the vector of capital stocks across countries at time t as $\vec{K}_t = \{K_{it}\}_{i=1}^I$.

Computing the Steady State The steady state consists of 22 objects: $\vec{w}^\star$, $\vec{r}^\star$, $\vec{P}_c^\star$, $\vec{P}_m^\star$, $\vec{P}_x^\star$, $\vec{C}^\star$, $\vec{X}^\star$, $\vec{K}^\star$, $\vec{M}^\star$, $\vec{Y}_c^\star$, $\vec{Y}_m^\star$, $\vec{Y}_x^\star$, $\vec{K}_c^\star$, $\vec{K}_m^\star$, $\vec{K}_x^\star$, $\vec{L}_c^\star$, $\vec{L}_m^\star$, $\vec{L}_x^\star$, $\vec{M}_c^\star$, $\vec{M}_m^\star$, $\vec{M}_x^\star$, and $\vec{\vec{\pi}}^\star$ (we use the double-arrow notation on $\vec{\vec{\pi}}_t$ to indicate that this is an $I \times I$ matrix). Table Appendix 4.1 provides a list of 23 conditions that these objects must satisfy. One market-clearing equation is redundant (condition 12 in our algorithm).

We use the technique from Mutreja, Ravikumar, and Sposi (2014), which builds on Alvarez and Lucas (2007), to solve for the steady state. The idea is to guess a vector of wages and then recover all remaining prices and quantities using optimality conditions and market-clearing conditions, excluding the trade balance condition. We then use departures from the trade balance condition in each country to update our wage vector and iterate until we find a wage vector that satisfies the trade balance condition. The following steps outline our procedure in more detail:

(i) We guess a vector of wages $\vec{w} \in \Delta = \{w \in \mathbb{R}_+^I : \sum_{i=1}^I \frac{w_i L_i}{1-\alpha} = 1\}$; that is, with world GDP as the numéraire.

(ii) We compute prices $\vec{P}_c$, $\vec{P}_x$, $\vec{P}_m$, and $\vec{r}$ simultaneously using conditions 16, 17, 18, and 23 in Table Appendix 4.1. To complete this step, we compute the bilateral trade shares $\vec{\vec{\pi}}$ using condition 19.

(iii) We compute the aggregate capital stock as $K_i = \frac{\alpha}{1-\alpha}\frac{w_i L_i}{r_i}$, for all i, which derives easily from optimality conditions 1 and 4, 2 and 5, and 3 and 6, coupled with market-clearing conditions for capital and labor 10 and 11 in Table Appendix 4.1.

(iv) We use condition 22 to solve for steady-state investment $\vec{X}$. Then we use condition 21 to solve for steady-state consumption $\vec{C}$.

(v) We combine conditions 4 and 13 to solve for $\vec{L}_c$, combine conditions 5 and 14 to solve for $\vec{L}_x$, and use condition 11 to solve for $\vec{L}_m$. Next we combine conditions 1 and 4 to solve for $\vec{K}_c$, combine conditions 2 and 5 to solve for $\vec{K}_M$, and combine conditions 3 and 6 to solve for $\vec{K}_x$. Similarly, we combine conditions 4 and 7 to solve for $\vec{M}_c$, combine conditions 5 and 8 to solve for $\vec{M}_m$, and combine conditions 6 and 9 to solve for $\vec{M}_x$.

(vi) We compute $\vec{Y}_c$ using condition 13, compute $\vec{Y}_m$ using condition 14, and compute $\vec{Y}_x$ using condition 15.

(vii) We compute an excess demand equation as in Alvarez and Lucas (2007) defined as

$$Z_i(\vec{w}) = \frac{P_{mi}Y_{mi} - P_{mi}M_i}{w_i}$$

(the trade deficit relative to the wage). Condition 20 requires that $Z_i(\vec{w}) = 0$ for all i. If the excess demand is sufficiently close to 0, then we have a steady state. If not, we update the wage vector using the excess demand as follows:

$$\Lambda_i(\vec{w}) = w_i\left(1 + \psi \frac{Z_i(\vec{w})}{L_i}\right),$$

Table Appendix 4.1
Steady-State Conditions

1	$r_i^\star K_{ci}^\star = \alpha\nu_c P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
2	$r_i^\star K_{mi}^\star = \alpha\nu_m P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
3	$r_i^\star K_{xi}^\star = \alpha\nu_x P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
4	$w_i^\star L_{ci}^\star = (1-\alpha)\nu_c P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
5	$w_i^\star L_{mi}^\star = (1-\alpha)\nu_m P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
6	$w_i^\star L_{xi}^\star = (1-\alpha)\nu_x P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
7	$P_{mi}^\star M_{ci}^\star = (1-\nu_c) P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
8	$P_{mi}^\star M_{mi}^\star = (1-\nu_m) P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
9	$P_{mi}^\star M_{xi}^\star = (1-\nu_x) P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
10	$K_{ci}^\star + K_{mi}^\star + K_{xi}^\star = K_i^\star$	$\forall(i)$
11	$L_{ci}^\star + L_{mi}^\star + L_{xi}^\star = L_i$	$\forall(i)$
12	$M_{ci}^\star + M_{mi}^\star + M_{xi}^\star = M_i^\star$	$\forall(i)$
13	$C_i^\star = Y_{ci}^\star$	$\forall(i)$
14	$\sum_{j=1}^I P_{mj}^\star \left(M_{cj}^\star + M_{mj}^\star + M_{xj}^\star\right)\pi_{ji} = P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
15	$X_i^\star = Y_{xi}^\star$	$\forall(i)$
16	$P_{ci}^\star = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_i^\star}{\alpha\nu_c}\right)^{\alpha\nu_c}\left(\frac{w_i^\star}{(1-\alpha)\nu_c}\right)^{(1-\alpha)\nu_c}\left(\frac{P_{mi}^\star}{1-\nu_c}\right)^{1-\nu_c}$	$\forall(i)$
17	$P_{mi}^\star = \gamma\left[\sum_{j=1}^I (u_{mj}^\star d_{ij})^{-\theta} T_{mj}\right]^{-\frac{1}{\theta}}$	$\forall(i)$
18	$P_{xi}^\star = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_i^\star}{\alpha\nu_x}\right)^{\alpha\nu_x}\left(\frac{w_i^\star}{(1-\alpha)\nu_x}\right)^{(1-\alpha)\nu_x}\left(\frac{P_{mi}^\star}{1-\nu_x}\right)^{1-\nu_x}$	$\forall(i)$
19	$\pi_{ij}^\star = \frac{(u_{mj}^\star d_{ij})^{-\theta} T_{mj}}{\sum_{j=1}^I (u_{mj}^\star d_{ij})^{-\theta} T_{mj}}$	$\forall(i,j)$
20	$P_{mi}^\star Y_{mi}^\star = P_{mi}^\star M_i^\star$	$\forall(i)$
21	$P_{ci}^\star C_i^\star + P_{xi}^\star X_i^\star = r_i^\star K_i^\star + w_i^\star L_i^\star$	$\forall(i)$
22	$X_i^\star = \delta K_i^\star$	$\forall(i)$
23	$r_i^\star = \left(\frac{1}{\beta} - (1-\delta)\right) P_{xi}^\star$	$\forall(i)$

NOTE: $u_{mj}^\star = \left(\frac{r_j^\star}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_j^\star}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mj}^\star}{1-\nu_m}\right)^{1-\nu_m}$.

which is the updated wage vector, where ψ is chosen to be sufficiently small so that $\Lambda > 0$. Note that $\sum_{i=1}^{I}\frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = \sum_{i=1}^{I}\frac{w_iL_i}{1-\alpha} + \psi\sum_{i=1}^{I} w_iZ_i(\vec{w})$. As in Alvarez and Lucas (2007), it is easy to show that $\sum_{i=1}^{I} w_iZ_i(\vec{w}) = 0$, which implies that $\sum_{i=1}^{I}\frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = 1$, and hence $\Lambda : \Delta \to \Delta$. We return to step (ii) with our updated wage vector and repeat the steps. We iterate through this procedure until the excess demand is sufficiently close to 0. In our computations we find that our preferred convergence metric,

$$\max_{i=1}^{I}\{|Z_i(\vec{w})|\},$$

converges roughly monotonically toward 0.

Computing the Transition Path Computing the transition path in our model is faster relative to RSS since we do not have trade imbalances or adjustment costs. The equilibrium transition path consists of 22 objects: $\{\vec{w}_t\}_{t=1}^{\infty}$, $\{\vec{r}_t\}_{t=1}^{\infty}$, $\{\vec{P}_{ct}\}_{t=1}^{\infty}$, $\{\vec{P}_{mt}\}_{t=1}^{\infty}$, $\{\vec{P}_{xt}\}_{t=1}^{\infty}$, $\{\vec{C}_t\}_{t=1}^{\infty}$, $\{\vec{X}_t\}_{t=1}^{\infty}$, $\{\vec{K}_t\}_{t=1}^{\infty}$, $\{\vec{Q}_{mt}\}_{t=1}^{\infty}$, $\{\vec{Y}_{ct}\}_{t=1}^{\infty}$, $\{\vec{Y}_{mt}\}_{t=1}^{\infty}$, $\{\vec{Y}_{xt}\}_{t=1}^{\infty}$, $\{\vec{K}_{ct}\}_{t=1}^{\infty}$, $\{\vec{K}_{mt}\}_{t=1}^{\infty}$, $\{\vec{K}_{xt}\}_{t=1}^{\infty}$, $\{\vec{L}_{ct}\}_{t=1}^{\infty}$, $\{\vec{L}_{mt}\}_{t=1}^{\infty}$, $\{\vec{L}_{xt}\}_{t=1}^{\infty}$, $\{\vec{M}_{ct}\}_{t=1}^{\infty}$, $\{\vec{M}_{mt}\}_{t=1}^{\infty}$, $\{\vec{M}_{xt}\}_{t=1}^{\infty}$, and $\{\vec{\vec{\pi}}_t\}_{t=1}^{\infty}$ (we use the double-arrow notation on $\vec{\vec{\pi}}_t$ to indicate that this is an $I \times I$ matrix in each period t). Table Appendix 4.2 provides a list of equilibrium conditions that these objects must satisfy.

Table Appendix 4.2
Dynamic Equilibrium Conditions

1	$r_{it}K_{cit} = \alpha\nu_cP_{cit}Y_{cit}$	$\forall(i,t)$
2	$r_{it}K_{mit} = \alpha\nu_mP_{mit}Y_{mit}$	$\forall(i,t)$
3	$r_{it}K_{xit} = \alpha\nu_xP_{xit}Y_{xit}$	$\forall(i,t)$
4	$w_{it}L_{cit} = (1-\alpha)\nu_cP_{cit}Y_{cit}$	$\forall(i,t)$
5	$w_{it}L_{mit} = (1-\alpha)\nu_mP_{mit}Y_{mit}$	$\forall(i,t)$
6	$w_{it}L_{xit} = (1-\alpha)\nu_xP_{xit}Y_{xit}$	$\forall(i,t)$
7	$P_{mit}M_{cit} = (1-\nu_c)P_{cit}Y_{cit}$	$\forall(i,t)$
8	$P_{mit}M_{mit} = (1-\nu_m)P_{mit}Y_{mit}$	$\forall(i,t)$
9	$P_{mit}M_{xit} = (1-\nu_x)P_{xit}Y_{xit}$	$\forall(i,t)$
10	$K_{cit} + K_{mit} + K_{xit} = K_{it}$	$\forall(i,t)$
11	$L_{cit} + L_{mit} + L_{xit} = L_i$	$\forall(i,t)$
12	$M_{cit} + M_{mit} + M_{xit} = M_{it}$	$\forall(i,t)$
13	$C_{it} = Y_{cit}$	$\forall(i,t)$
14	$\sum_{j=1}^{I} P_{mjt}\left(M_{cjt} + M_{mjt} + M_{xjt}\right)\pi_{jit} = P_{mit}Y_{mit}$	$\forall(i,t)$
15	$X_{it} = Y_{xit}$	$\forall(i,t)$
16	$P_{cit} = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_{it}}{\alpha\nu_c}\right)^{\alpha\nu_c}\left(\frac{w_{it}}{(1-\alpha)\nu_c}\right)^{(1-\alpha)\nu_c}\left(\frac{P_{mit}}{1-\nu_c}\right)^{1-\nu_c}$	$\forall(i,t)$
17	$P_{mit} = \gamma\left[\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}\right]^{-\frac{1}{\theta}}$	$\forall(i,t)$
18	$P_{xit} = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_{it}}{\alpha\nu_x}\right)^{\alpha\nu_x}\left(\frac{w_{it}}{(1-\alpha)\nu_x}\right)^{(1-\alpha)\nu_x}\left(\frac{P_{mit}}{1-\nu_x}\right)^{1-\nu_x}$	$\forall(i,t)$
19	$\pi_{ijt} = \frac{(u_{mjt}d_{ij})^{-\theta}T_{mj}}{\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}}$	$\forall(i,j,t)$
20	$P_{mit}Y_{mit} = P_{mit}M_{it}$	$\forall(i,t)$
21	$P_{cit}C_{it} + P_{xit}X_{it} = r_{it}K_{it} + w_{it}L_i$	$\forall(i,t)$
22	$K_{it+1} = (1-\delta)K_{it} + X_{it}$	$\forall(i,t)$
23	$\left(\frac{C_{it+1}}{C_{it}}\right) = \beta^{\sigma}\left(1 + \frac{r_{it+1}}{P_{xit+1}} - \delta\right)^{\sigma}\left(\frac{P_{xt+1}/P_{ct+1}}{P_{xt}/P_{ct}}\right)^{\sigma}$	$\forall(i,t)$

NOTE: $u_{mjt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mjt}}{1-\nu_m}\right)^{1-\nu_m}$.

We reduce the infinite horizon problem to a finite time problem from $t = 1, \ldots, T$, with T sufficiently large to ensure that the endogenous variables settle to a steady state by T. As such, solving the transition first requires solving for the terminal steady state. Also, it requires taking the initial stock of capital as given (either by computing an initial steady state or by taking it from the data, for instance).

Our solution method mimics the idea of that for the steady state but is slightly modified to take into account the dynamic aspect as in Sposi (2012). Basically, we start with an initial guess for the entire *sequence* of wage and rental rate vectors. From these two objects, we can recover all prices and quantities, across countries and throughout time, using optimality conditions and market-clearing conditions, excluding the trade balance condition and the market-clearing condition for the stock of capital. We then use departures from these two conditions at each point in time and in each country to update the sequence of wages and rental rates. Then we iterate until we find wages and rental rates that satisfy the trade balance condition and the market-clearing condition for the stock of capital in each period. We describe our procedure in more detail below.

(i) We guess the entire path for wages $\{\vec{w}_t\}_{t=1}^T$ and rental rates $\{\vec{r}_t\}_{t=2}^T$ across countries such that $\sum_i \frac{w_{it}L_i}{1-\alpha} = 1\ (\forall t)$. In period 1, set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\vec{w}_1\vec{L}}{\vec{K}_1}\right)$ since the initial stock of capital is predetermined.

(ii) We compute prices $\{\vec{P}_{ct}\}_{t=1}^T$, $\{\vec{P}_{xt}\}_{t=1}^T$, and $\{\vec{P}_{mt}\}_{t=1}^T$ simultaneously using conditions 16, 17, and 18, in Table Appendix 4.2. To complete this step, we compute the bilateral trade shares $\{\vec{\vec{\pi}}_t\}_{t=1}^T$ using condition 19.

(iii) Computing the path for consumption and investment is slightly more involved and requires solving the household's intertemporal problem. We do this in three steps. First, we derive the lifetime budget constraint. Second, we derive the fraction of lifetime wealth allocated to consumption at each period t. And third, we recover the sequences for investment and the stock of capital.

Deriving the lifetime budget constraint For the representative household in country i, begin with the period budget constraint from condition 21, and combine it with the capital accumulation technology in condition 22 to get

$$K_{it+1} = \left(\frac{w_{it}}{P_{xit}}\right)L_i + \left(1 + \frac{r_{it}}{P_{xit}} - \delta\right)K_{it} - \left(\frac{P_{cit}}{P_{xit}}\right)C_{it}.$$

We iterate the period budget constraint forward through time and derive a lifetime budget constraint. At time $t = 1$, the stock of capital, $K_{i1} > 0$, is given. Next, compute the stock of capital at time $t = 2$:

$$K_{i2} = \left(\frac{w_{i1}}{P_{xi1}}\right)L_i + \left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right)K_{i1} - \left(\frac{P_{ci1}}{P_{xi1}}\right)C_{i1}.$$

Similarly, compute the stock of capital at time $t = 3$, but do it so that it is in terms the initial stock of capital.

$$\begin{aligned}
K_{i3} &= \left(\frac{w_{i2}}{P_{xi2}}\right)L_i + \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right)K_{i2} - \left(\frac{P_{ci2}}{P_{xi2}}\right)C_{i2}. \\
\Rightarrow K_{i3} &= \left(\frac{w_{i2}}{P_{xi2}}\right)L_{i2} + \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right)\left(\frac{w_{i1}}{P_{xi1}}\right)L_i \\
&\quad + \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right)\left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right)K_{i1} \\
&\quad - \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right)\left(\frac{P_{ci1}}{P_{xi1}}\right)C_{i1} - \left(\frac{P_{ci2}}{P_{xi2}}\right)C_{i2}.
\end{aligned}$$

Continue to period 4 in a similar way:

$$\begin{aligned}
K_{i4} &= \left(\frac{w_{i3}}{P_{xi3}}\right)L_i + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right)K_{i3} - \left(\frac{P_{ci3}}{P_{xi3}}\right)C_{i3} \\
\Rightarrow K_{i4} &= \left(\frac{w_{i3}}{P_{xi3}}\right)L_i + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right)\left(\frac{w_{i2}}{P_{xi2}}\right)L_i \\
&\quad + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right)\left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right)\left(\frac{w_{i1}}{P_{xi1}}\right)L_i \\
&\quad + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right)\left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right)\left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right)K_{i1} \\
&\quad - \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right)\left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right)\left(\frac{P_{ci1}}{P_{xi1}}\right)C_{i1} \\
&\quad - \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right)\left(\frac{P_{ci2}}{P_{xi2}}\right)C_{i2} - \left(\frac{P_{ci3}}{P_{xi3}}\right)C_{i3}.
\end{aligned}$$

Before we continue, it is useful to define $(1+R_{it}) \equiv \prod_{n=1}^{t}\left(1+\frac{r_{in}}{P_{xin}}-\delta\right)$. Then,

$$\begin{aligned}
\Rightarrow K_{i4} = & \frac{(1+R_{i3})\left(\frac{w_{i3}}{P_{xi3}}\right)L_i}{(1+R_{i3})} + \frac{(1+R_{i3})\left(\frac{w_{i2}}{P_{xi2}}\right)L_{i2}}{(1+R_{i2})} + \frac{(1+R_{i3})\left(\frac{w_{i1}}{P_{xi1}}\right)L_i}{(1+R_{i1})} \\
& + (1+R_{i3})K_{i1} \\
& - \frac{(1+R_{i3})\left(\frac{P_{ci3}}{P_{xi3}}\right)C_{i3}}{(1+R_{i3})} - \frac{(1+R_{i3})\left(\frac{P_{ci2}}{P_{xi2}}\right)C_{i2}}{(1+R_{i2})} - \frac{(1+R_{i3})\left(\frac{P_{ci1}}{P_{xi1}}\right)C_{i1}}{(1+R_{i1})} \\
\Rightarrow K_{i4} = & \sum_{n=1}^{3}\frac{(1+R_{i3})\left(\frac{w_{in}}{P_{xin}}\right)L_{in}}{(1+R_{in})} - \sum_{n=1}^{3}\frac{(1+R_{i3})\left(\frac{P_{cin}}{P_{xin}}\right)C_{in}}{(1+R_{in})} + (1+R_{i3})K_{i1}.
\end{aligned}$$

By induction, for any time t,

$$\begin{aligned}
K_{it+1} &= \sum_{n=1}^{t}\frac{(1+R_{it})\left(\frac{w_{in}}{P_{xin}}\right)L_i}{(1+R_{in})} - \sum_{n=1}^{t}\frac{(1+R_{it})\left(\frac{P_{cin}}{P_{xin}}\right)C_{in}}{(1+R_{in})} + (1+R_{it})K_{i1} \\
\Rightarrow K_{it+1} &= (1+R_{it})\left(\sum_{n=1}^{t}\frac{\left(\frac{w_{in}}{P_{xin}}\right)L_i}{(1+R_{in})} - \sum_{n=1}^{t}\frac{\left(\frac{P_{cin}}{P_{xin}}\right)C_{in}}{(1+R_{in})} + K_{i1}\right).
\end{aligned}$$

Finally, observe the previous expression as of $t = T$, and rearrange the terms to derive the lifetime budget constraint:

(Appendix 4.1)
$$\sum_{n=1}^{T}\frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = \underbrace{\sum_{n=1}^{T}\frac{w_{in}L_i}{P_{xin}(1+R_{in})} + K_{i1} - \frac{K_{iT+1}}{(1+R_{iT})}}_{W_i}.$$

In the lifetime budget constraint (Appendix 4.1), we use W_i to denote the net present value of lifetime wealth in country i. We take the capital stock at the end of time, K_{iT+1}, as given; in our case, it is the capital stock in the new steady state with T sufficiently large. Note that the terminal condition, $K_{iT+1} = K_i^\star$, automatically implies the transversality condition since $\lim_{T\to\infty}(1+R_{iT}) = \infty$ and $\lim_{T\to\infty} K_{iT+1} = K_i^\star$.

Solving for the path of consumption Next we compute the consumption expenditures in each period. The Euler equation (condition 23) implies the following relationship between consumption in any two periods t and n:

$$\begin{aligned}
C_{in} &= \beta^{\sigma(n-t)}\left(\frac{(1+R_{in})}{(1+R_{it})}\right)^{\sigma}\left(\frac{P_{xin}}{P_{xit}}\right)^{\sigma}\left(\frac{P_{cit}}{P_{cin}}\right)^{\sigma} C_{it} \\
\Rightarrow \frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} &= \beta^{\sigma(n-t)}\left(\frac{P_{xin}(1+R_{in})}{P_{xit}(1+R_{it})}\right)^{\sigma-1}\left(\frac{P_{cin}}{P_{cit}}\right)^{1-\sigma}\left(\frac{P_{cit}C_{it}}{P_{xit}(1+R_{it})}\right).
\end{aligned}$$

Since Equation (Appendix 4.1) implies that $\sum_{n=1}^{T}\frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = W_i$, we can rearrange the previous expression to obtain

(Appendix 4.2)
$$\frac{P_{cit}C_{it}}{P_{xit}(1+R_{it})} = \underbrace{\left(\frac{\beta^{\sigma t}P_{xit}^{\sigma-1}(1+R_{it})^{\sigma-1}P_{cit}^{1-\sigma}}{\sum_{n=1}^{T}\beta^{\sigma n}P_{xin}^{\sigma-1}(1+R_{in})^{\sigma-1}P_{cin}^{1-\sigma}}\right)}_{\xi_{it}} W_i.$$

That is, each period, the household spends a share ξ_{it} of lifetime wealth on consumption, with $\sum_{t=1}^{T}\xi_{it} = 1$ for all i. Note that ξ_{it} depends only on prices.

Computing investment and the sequence of capital stocks Given the paths of consumption, we solve for investment $\{\vec{X}_t\}_{t=1}^{T}$ using the period budget constraint in condition 21. The catch here is that there is no restriction that household investment be nonnegative up to this point. Looking ahead, negative investment cannot satisfy market-clearing conditions together with firm optimality conditions. As such, we restrict our attention to the transition paths for which investment is always positive, which we find is the case for the equilibrium outcomes in our article. However, off the equilibrium path, if during the course of the iterations the value of X_{it} is negative, then we set it equal to a small positive number.

The last part of this step is to use condition 22 to compute the path for the stock of capital. $\{\vec{K}_t\}_{t=2}^{T+1}$. Note that $\vec{K}_1$ is taken as given and $\vec{K}_{T+1}$ is equal to the (computed) terminal steady-state value.

(iv) We combine conditions 4 and 13 to solve for $\{\vec{L}_{ct}\}_{t=1}^{T}$, combine conditions 5 and 14 to solve for $\{\vec{L}_{xt}\}_{t=1}^{T}$, and use condition 11 to solve for $\{\vec{L}_{mt}\}_{t=1}^{T}$. Next we combine conditions 1 and 4 to solve for $\{\vec{K}_{ct}\}_{t=1}^{T}$, combine conditions 2 and 5 to solve for $\{\vec{K}_{mt}\}_{t=1}^{T}$, and combine conditions 3 and 6 to solve for $\{\vec{K}_{xt}\}_{t=1}^{T}$. Similarly, we combine conditions 4 and 7 to solve for $\{\vec{M}_{ct}\}_{t=1}^{T}$, combine conditions 5 and 8 to solve for $\{\vec{M}_{mt}\}_{t=1}^{T}$, and combine conditions 6 and 9 to solve for $\{\vec{M}_{xt}\}_{t=1}^{T}$.

(v) We compute $\{\vec{Y}_{ct}\}_{t=1}^{T}$ using condition 13, compute $\{\vec{Y}_{mt}\}_{t=1}^{T}$ using condition 14, and compute $\{\vec{Y}_{xt}\}_{t=1}^{T}$ using condition 15.

(vi) Until this point, we have imposed all equilibrium conditions except for two: the trade balance condition 20 and the capital market-clearing condition 10.

Trade balance condition We compute an excess demand equation as in Alvarez and Lucas (2007) defined as

$$Z_{it}^{w}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)=\frac{P_{mit}Y_{mit}-P_{mit}M_{it}}{w_{it}}$$

(the trade deficit relative to the wage). Condition 20 requires that $Z_{it}^{w}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)=0$ for all i. If this is different from zero in some country at some point in time, we update the wages as follows:

$$\Lambda_{it}^{w}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)=w_{it}\left(1+\psi\frac{Z_{it}^{w}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)}{L_i}\right)$$

is the updated wages, where ψ is chosen to be sufficiently small so that $\Lambda^{w}>0$.

Market-clearing condition for the stock of capital We compute an excess demand equation

$$Z_{it}^{r}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)=\frac{w_{it}L_i}{1-\alpha}-\frac{r_{it}K_{it}}{\alpha}.$$

Using conditions 1–6, we have imposed that within each sector, $\frac{r_{it}K_{bit}}{\alpha}=\frac{w_{it}L_{bit}}{1-\alpha}$. We have also imposed condition 11 that the labor market clears. Hence, the market for capital is in excess demand (i.e., $K_{cit}+K_{mit}+K_{xit}>K_{it}$) in country i at time t if and only if $\left(\frac{w_{it}L_i}{1-\alpha}\right)>\left(\frac{r_{it}K_{it}}{\alpha}\right)$ (it is in excess supply if and only if the inequality is <). If this condition does not hold with equality in some country at some point in time, then we update the rental rates as follows. Let

$$\Lambda_{it}^{r}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)=\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{L_i}{K_{it}}\right)\Lambda_{it}^{w}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)$$

be the updated rental rates (taking into account the updated wages).

We return to step (ii) with our updated wages and rental rates and repeat the steps. We iterate through this procedure until the excess demand is sufficiently close to 0. In our computations we find that our preferred convergence metric,

$$\max_{t=1}^{T}\left\{\max_{i=1}^{I}\left\{|Z_{it}^{w}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)|+|Z_{it}^{r}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)|\right\}\right\},$$

converges roughly monotonically toward 0.

Along the equilibrium transition, $\sum_i w_{it}L_i + r_{it}K_{it} = 1$ $(\forall t)$; that is, we have chosen world GDP as the numéraire at each point in time.

The fact that $\vec{K}_{T+1} = \vec{K}^\star$ at each iteration is a huge benefit of our algorithm compared to a shooting algorithm or algorithms that rely on using the Euler equation for updating. Such algorithms inherit the instability (saddle-path) properties of the Euler equation and generate highly volatile terminal stocks of capital with respect to the initial guess. Instead, we impose the Euler equation and the terminal condition for $\vec{K}_{T+1} = \vec{K}^\star$ at each iteration and use excess demand equations for our updating rules, just as in the computation of static models (e.g., Alvarez and Lucas, 2007). Another advantage of using excess demand iteration is that we do not need to compute gradients to choose step directions or step size, as in the case of nonlinear solvers. This saves computational time, particularly as the number of countries or the number of time periods is increased.

APPENDIX 5. LIST OF COUNTRIES AND THEIR GAINS FROM TRADE

Table Appendix 5.1

Gains from Trade (%) Following Uniform Reduction in Frictions by 55 Percent

Country	Dyn	SS
Armenia	47.2	77.8
Australia	20.8	34.5
Austria	26.9	44.6
Bahamas	40.8	67.2
Bangladesh	31.1	51.5
Barbados	52.7	86.6
Belarus	27.3	45.3
Belize	55.7	91.6
Benin	43.7	72.0
Bhutan	40.4	66.5
Brazil	14.6	24.3
Bulgaria	37.3	61.6
Burundi	25.3	42.0
Côte d’Ivoire	40.2	66.3
Cabo Verde	42.7	70.4
Cambodia	36.4	60.1
Cameroon	37.4	61.8
Canada	19.0	31.5
Central African Rep.	25.8	42.8
Chile	30.2	49.9
China, Hong Kong, Macao	11.5	19.1
Colombia	27.8	45.9
Costa Rica	40.7	67.0
Cyprus	40.0	66.0
Czech Rep.	28.9	47.8
Denmark	29.6	48.9
Dominican Rep.	23.7	39.2
Ecuador	39.0	64.4
Egypt	33.9	56.1
Ethiopia	35.7	59.1
Fiji	41.0	67.6
Finland	30.6	50.6
France	21.4	35.6
Georgia	46.6	76.8
Germany	20.7	34.4
Greece	32.3	53.5
Guatemala	27.8	46.0
Honduras	37.7	62.1
Hungary	21.6	35.9
Iceland	47.5	78.1
India	17.0	28.3
Indonesia	26.2	43.4
Iran	21.0	34.9
Ireland	29.6	49.0
Israel	37.7	62.2
Italy	21.9	36.3
Jamaica	41.8	68.9
Japan	13.1	21.8
Jordan	38.8	64.0

(Continued)

Table Appendix 5.1 – Continued

Country	Dyn	SS
Kyrgyzstan	40.4	66.7
Lesotho	30.6	50.6
Madagascar	36.9	60.9
Malawi	41.2	68.0
Maldives	46.2	76.1
Mauritius	44.8	73.8
Mexico	12.5	20.8
Morocco	38.7	63.9
Mozambique	43.1	71.0
Nepal	29.5	48.8
New Zealand	33.1	54.7
Pakistan	21.5	35.6
Paraguay	40.9	67.3
Peru	27.9	46.2
Philippines	29.5	48.8
Poland	26.4	43.8
Portugal	30.7	50.7
Rep. of Korea	24.2	40.0
Rep. of Moldova	40.5	66.8
Romania	32.8	54.2
Russian Federation	23.9	39.5
Rwanda	22.5	37.2
Saint Vincent and the Grenadines	56.4	92.7
Sao Tome and Principe	39.9	65.8
Senegal	46.6	76.8
South Africa	29.6	49.0
Southeast Europe	32.9	54.4
Spain	21.9	36.3
Sri Lanka	32.4	53.6
Sweden	28.1	46.5
Switzerland	28.7	47.5
TFYR of Macedonia	38.9	64.2
Thailand	32.4	53.6
Tunisia	40.7	67.1
Turkey	26.8	44.3
USA	11.0	18.3
Uganda	22.5	37.4
Ukraine	30.7	50.9
United Kingdom	23.8	39.5
United Rep. of Tanzania	44.1	72.7
Uruguay	36.9	60.9
Venezuela	29.2	48.4
Vietnam	36.5	60.2
Yemen	42.0	69.2

NOTE: "Dyn" refers to dynamic gains and "SS" refers to steady-state gains. The group "Southeast Europe" is an aggregate of Albania, Bosnia and Herzegovina, Croatia, Montenegro, and Serbia.