Growth and the Kaldor Facts

Berthold Herrendorf, Richard Rogerson, and Ákos Valentinyi

We revisit the Kaldor growth facts for the United States and the United Kingdom during the post-war period. We find that while overall the original Kaldor facts continue to hold, deviations occurred along several dimensions: Instead of staying constant, the growth rates of real GDP per worker and of real capital per worker have slowed down in the United States and the United Kingdom since the 1970s, the capital-to-output ratio has increased in the United Kingdom, and the share of income paid to labor has decreased in the United States since 1990. We discuss how to calculate the Kaldor facts in multi-sector growth models and establish that a slowdown in GDP-per-worker growth naturally results from secular changes in relative prices. (JEL O41, O47)

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1 INTRODUCTION

The process of building economic models benefits from the existence of stylized facts that discipline the modeling choices. In the theory of economic growth, these stylized facts were first stated by Kaldor (1961) and are called the Kaldor growth facts (or sometimes for short the Kaldor facts or the growth facts). These facts are that the growth rates of real GDP per worker and real capital per worker exhibit no trend and that the gross return on capital, the capital-to-output ratio, and the GDP share of the payments to capital exhibit no trend growth. Although the field of economic growth has rapidly developed since Kaldor's article, it is fair to say that being consistent with the original Kaldor facts is still viewed as the minimum requirement for a credible model of economic growth.[1]

In this article, we have two primary goals. The first one is to empirically revisit the Kaldor facts more than half a century after Kaldor wrote his article. The second goal is to discuss how to address the Kaldor facts in multi-sector versions of the growth model.

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Given the long-standing and seemingly well-established position of the Kaldor facts in the literature, revisiting them may seem a frivolous task to undertake. There are three good reasons for nonetheless doing this. First, and somewhat ironically, the Kaldor (1961) paper that is cited for laying out the Kaldor facts does not actually contain any presentation of facts. Rather, Kaldor reported what he saw as the key patterns to be distilled after looking at various and disparate pieces of information that had been documented by other authors. Second, Kaldor described the experiences of the pre-1950 world. In contrast, many current macroeconomic studies focus on the post-1950 world, making it relevant to assess the state of the Kaldor facts for this period. Third, revisiting the Kaldor facts highlights a number of measurement issues that one needs to take a stand on. Although they are not usually discussed in detail, they can be subtle and are particularly relevant in multi-sector growth models.[2]

We find that overall the Kaldor facts continue to hold, in that constant trends provide a reasonable first-order description to most of the data. But there are sizeable short- and medium-term fluctuations around the trend. In particular, we find evidence of deviations from the Kaldor facts along several dimensions: Instead of staying constant, the growth rates of real GDP per worker and of real capital per worker have slowed down in the United States and the United Kingdom since the 1970s, the capital-to-output ratio has increased in the United Kingdom, and the share of income paid to labor has decreased in the United States since 1990.

Establishing the existence of a balanced growth path in the one-sector growth model and connecting its properties to the Kaldor facts is a standard exercise for first-year Ph.D. students. In recent years, the profession has gone beyond that and studied multi-sector versions of the growth models that capture the effects of secular changes in relative prices and the sectoral composition of the economy ("structural change"). It is therefore natural to ask (i) under what conditions a standard multi-sector model has a balanced growth path along which the updated Kaldor facts hold and (ii) what this multi-sector model has to say about the deviations from the original Kaldor facts. This is the second goal of our article.

One of our key points is that, with the one-sector growth model, it is immediate to go from model-based measures to empirical measures; this is not the case for multi-sector growth models. The reason for this is that the one-sector model has no changes in relative prices, implying that many statistics are effectively unit free. This is not the case for multi-sector growth models, which were constructed precisely to capture the effects of secular changes in relative prices. As a result, the issue of which units should be used to measure the statistics behind the Kaldor facts takes center stage in multi-sector models.

A basic principle of connecting models with data requires measuring objects in the data and the model in the same way. It turns out that this principle has important implications for connecting multi-sector models to the Kaldor facts, in particular with regard to quantities such as gross domestic product (GDP). We report and discuss a recent result by Duernecker, Herrendorf, and Valentinyi (2017a), who showed that if GDP per worker is measured in the model as it is in the data by chaining the Fisher quantity index, then the multi-sector model can naturally generate the first deviation from the original Kaldor facts: GDP-per-worker growth slows down over time, instead of remaining constant.

The rest of the article is organized as follows. Section 2 first presents the original Kaldor facts and then restates them in the context of modern growth theory. Section 3 revisits the Kaldor facts in the context of post-WWII data from the United States and the United Kingdom, paying special attention to how to define the relevant objects in the data. Section 4 introduces a simple two-sector growth model and revisits balanced growth and the Kaldor facts in this setting, paying particular attention to the conditions under which a growth slowdown arises. Section 5 concludes.

2 THE ORIGINAL KALDOR FACTS

2.1 Kaldor's Statement of the Growth Facts

The seminal article by Kaldor (1961, pp. 178-79) states the following:

> As regards the process of economic change and development in capitalist societies, I suggest the following "stylized facts" as a starting point for the construction of theoretical models:
>
> 1. The continued growth in the aggregate volume of production and in the productivity of labour at a steady rate; no recorded tendency of a *falling* rate of growth of productivity.
> 2. A continued increase in the amount of capital per worker. ...
> 3. A steady rate of profit on capital, at least in the 'developed' capitalist societies. ...
> 4. Steady capital-to-output ratios over long periods; at least there are no clear long-term trends, either rising or falling, if differences in the degree of utilization of capital are allowed for. ...
> 5. A high correlation between the share of profits in income and the share of investment in output; a steady share of profits (and of wages) in societies and/or in periods in which the investment coefficient (the share of investment in output) is constant. ...
> 6. Finally, there are appreciable differences in the *rate* of growth of labour productivity and of total output in different societies, the range of variation (in the fast-growing economies) being of the order of 2-5 percent. These are associated with corresponding variations in the investment coefficient, and in the profit share, but the above proposition concerning the constancy of relative shares and of the capital-to-output ratio are applicable to countries with differing rates of growth.

The first five facts have become known as the Kaldor growth facts, or, for short, the Kaldor facts or the growth facts. The sixth fact usually receives less attention and is dropped by many authors.

2.2 The Kaldor Facts in the One-Sector Growth Model

The one-sector, closed-economy growth model is a benchmark model for aggregate analysis of economic growth because it generates the Kaldor growth facts in a rather robust and tractable fashion. In what follows, we briefly describe the one-sector model and explain how it generates the Kaldor growth facts.

There is a representative household of size N_t at time t, with preferences over streams of consumption $\{C_t\}$ described by

$$\sum_{t=0}^{\infty}\beta^t N_t \frac{\left(C_t/N_t\right)^{1-1/\sigma}-1}{1-1/\sigma},$$

where $\beta \in (0,1)$ is the discount factor and $\sigma > 0$ is the household's elasticity of substitution between consumption per member at different dates. The household does not value leisure, and the time available for work for each household member is normalized to 1. The total time available for work then equals N_t.

There is an aggregate production function of the Cobb-Douglas form

$$Y_t = A_t K_t^{\theta} L_t^{1-\theta},$$

where A_t captures exogenous technological progress, θ is the capital-share parameter, K_t is the capital stock at time t, and L_t is labor input at time t.[3]

The feasibility constraint and the capital-accumulation equation are

$$Y_t = C_t + X_t,$$

$$K_{t+1} = (1-\delta)K_t + X_t,$$

where X_t is investment and $\delta \in [0,1]$ is the depreciation rate.

The one-sector growth model is completed by assuming that the population and technological progress grow at constant rates η and γ, respectively:

$$N_{t+1} = (1+\eta)N_t,$$

$$A_{t+1} = (1+\gamma)A_t.$$

Since the growth facts involve prices, it is natural to focus on a competitive equilibrium of the one-sector growth model. Let p_t denote the price of output in period t in current dollars and w_t and r_t denote the prices of labor and capital, respectively, in period t in terms of units of output. As is standard in the literature, we focus on a balanced-growth-path equilibrium (or balanced growth path for short), along which all variables grow at constant rates including zero (that is, they may be constant). Such a balanced-growth-path equilibrium is of interest for two reasons. First, from a theoretical perspective, having a balanced-growth-path equilibrium anchors the asymptotic behavior of the model, which helps in analyzing and solving the model. Second, from an empirical perspective, establishing the existence of a balanced-growth-path equilibrium turns out to be tantamount to establishing consistency with the Kaldor facts. In what follows, we provide a brief summary of the arguments involved in doing this.

The relevant textbook result in our context is that the above economy possesses a balanced-growth-path equilibrium.[4] It is straightforward to show that the balanced-growth-path equilibrium generates the Kaldor growth facts. Specifically, along the balanced-growth-path equilibrium, w_t grows at the constant rate γ and r_t is constant. The values of aggregate output, consumption, investment, and the capital stock all grow at the same constant rate

$\gamma + \eta$.[5] The per-worker values of output, consumption, investment, and the capital stock grow at the same constant rate γ. Note that per-worker values equal per capita values in the one-sector growth model. Total labor input grows at the rate η, and labor input per worker is constant. It immediately follows that all of the Kaldor facts hold along this balanced-growth-path equilibrium:

(i) The growth rate of real GDP per worker is constant: $\%\Delta Y_t/N_t = \gamma$.

(ii) The growth rate of real capital per worker is constant: $\%\Delta K_t/N_t = \gamma$.[6]

(iii) The gross return on capital is constant: r_t constant.

(iv) The capital-to-output ratio is constant: K_t/Y_t constant.

(v) The share of capital income in GDP is constant: $r_t K_t/Y_t$ constant.

Note that (i) to (v) are not a minimal set of facts, because any two of (iii) to (v) imply the remaining third one.

The requirement that population and technology grow at constant rates is stringent and unlikely to hold in the data. However, this is a nonissue because the Kaldor facts do not state that certain growth rates are literally constant, but rather that trend growth rates do not vary. As a practical matter, the key property is that if the paths for population and technology exhibit fluctuations around unchanging trends, then the economy's equilibrium will basically fluctuate around the balanced growth path.

3 REVISITING THE KALDOR FACTS

In this section, we revisit the Kaldor facts. As noted in the introduction, there are three reasons for doing this. First, by today's standards the paper by Kaldor (1961) did not present any systematic evidence. Second, Kaldor was describing the pre-1950 world. And third, revisiting the Kaldor facts allows us to highlight a number of measurement issues that one needs to take a stand on. This becomes particularly relevant in multi-sector versions of the growth model.

3.1 Revisiting the Kaldor Facts in the Data

Given that the one-sector growth model possesses a balanced-growth-path equilibrium with the properties noted above, we ask, How would one proceed to confirm these implied patterns in the data? An important, general principle to adopt when comparing model-based outcomes with the data is to apply the same measurement practices in both the model and data. Unfortunately, this principle is essentially vacuous in the one-sector growth model. The reason for this is that the one-sector model has just a single homogeneous good in each period, whereas in reality many goods and relative prices exhibit potentially large secular changes. Having multiple goods with changing relative prices gives rise to several questions. First, while measuring the growth rate of real output is straightforward in the model, how should we do it in the data? In the model, capital and output are the same thing and they have a relative price of unity, so K_t/Y_t is both a real ratio and a nominal ratio. But which of these ratios should we focus on in the data? In the model, the rate of return on capital is unambiguous, given that there is a single good, but in a world with many goods, one could express the return on capital in terms of different numeraires. Which one should we focus on?

Best-practice methods for many measurement issues have evolved over time. For example, not that long ago the standard practice for measuring real quantities was to use base-year prices, which were then updated at regular intervals. But the growth rates of real variables so constructed depended on the choice of the base year. Current best practice addresses this problem by computing real quantities using chained Fisher indexes that eliminate the dependence of growth rates on the base-year prices; see Whelan (2003) and Duernecker, Herrendorf, and Valentinyi (2017a) for discussion of calculating GDP with the chained Fisher index. We follow this best practice in reporting the facts for aggregate output and capital per worker.

Contemporaneous ratios may be measured as ratios of nominal or real variables. Using nominal values has the advantage that the resulting ratios are unit free, are unaffected by the choice of numeraire or base year, and are based on the same current prices that people face when they make their choices (Caselli and Freyer, 2007). In contrast, ratios of real variables tend to have undesirable properties and are often hard to interpret. For example, if the real variables are calculated in fixed prices of a base year, then their ratios depend on the choice of the base year. And if the real variables are calculated with the chained Fisher index, then it is not clear what they mean conceptually, because the chained Fisher index does not preserve additivity.[7]

We measure the return to capital as the nominal payments to capital divided by the nominal value of the capital stock. Doing this avoids the pitfalls around using the real variables described in the previous paragraph. It also avoids the issue of having to choose a numeraire in which to calculate rates of return over time.

Summarizing the above discussion, we calculate the Kaldor statistics as follows:

(i) Real output per worker: GDP calculated with the chained Fisher index divided by persons engaged[8]

(ii) Real capital per worker: capital calculated with the chained Fisher index divided by persons engaged

(iii) Return to capital: payments to capital in current dollars divided by the capital stock in current dollars

(iv) Capital-output ratio: capital in current dollars divided by GDP in current dollars

(v) Capital's share of income: payments to capital in current dollars divided by GDP in current dollars

Note that the statistics for (iii) to (v) are unit free because we calculated them with nominal, not real, variables.[9] While capital and output are naturally in the same units in the one-sector model and the capital-to-output ratio is unit free, this is not necessarily true in the data or in multi-sector growth models.

3.2 The Updated Kaldor Facts for the United States and United Kingdom

We are now ready to present the analogues to the Kaldor facts for both the United States and the United Kingdom over the post-WWII period. We think that it is of interest to extend the analysis to other countries as data permit, but note that it seems wise to separately consider countries that experienced prolonged transition periods, either because of significant destruction during a war or because the balanced growth path shifted after a large-scale change in the institutional environment. We think that both the United States and United Kingdom avoided prolonged transitions, and so they are good candidates for learning about the relevance of balanced growth as an empirical phenomenon. Other potential candidates that share this feature include Australia, Canada, and New Zealand. In contrast, the large economies of continental Europe (e.g., France, Germany, and Italy) and East Asia (e.g., Japan) seem less promising in this regard given their prolonged transitions with strong catch-up growth after WWII.

Figure 1 displays the Kaldor facts for the United States. Several comments are in order. First, the graphs support facts (i) to (iv): Although there are indeed sizeable short- and medium-term fluctuations around trend, constant trends provide a reasonable description of the data. Second, as is well known by now, the constancy of the labor share does not provide a good description of the U.S. economy in the period since 1990. Prior to 1990 there were significant medium-run departures, but the data are still well described as being without trend.

Figure 2 displays the Kaldor facts for the United Kingdom. Once again, there are substantial medium-term departures from each of the trend lines shown, but with the exception of the series for K_t/Y_t, the series seem consistent with the Kaldor facts.[10] The K_t/Y_t series first trended upward, then stabilized, and then has trended upwards again since 1990. The main reason for the upward trend is the large increase in the price of dwellings in the United Kingdom since 1990. We also note that the behavior of the labor share in the United Kingdom differed from what we found for the United States: In the United Kingdom, the labor share declined in the 1970s and 1980s but has since bounced back to its long-run average value of two-thirds.[11]

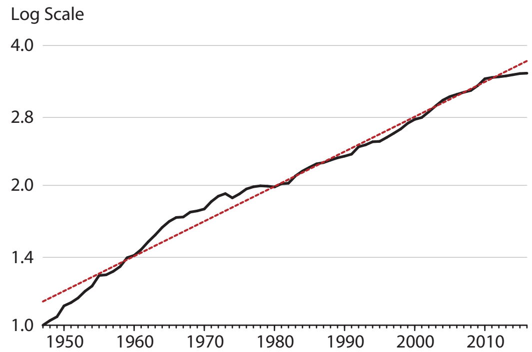
The main message that we take away from these pictures is that the Kaldor facts are indeed first-order features of the post-WWII evolution of both the U.S. and the U.K. economies. That notwithstanding, there are some departures worth noting as well as some interesting low-frequency fluctuations. For example, we do observe substantial differences in growth rates of GDP per worker and capital per worker across subperiods. To make this observation more precise, Table 1 shows the values of the GDP-per-worker growth rates for the postwar period 1947-2017 and four subperiods. The first three subperiods span 20 years each, starting in 1947 and lasting until 1967, 1987, and 2007. The last subperiod spans the 10 years 2007-17 that we treat separately because of the Great Recession. Clearly, the average growth rates were considerably stronger during 1947-67 than in the following three subperiods. This, of course, is a restatement of the well-documented fact that since the 1970s there has been a productivity growth slowdown. An additional observation is that in the last subperiod the average growth rates were particularly low, which is expected given that it includes the Great Recession.[12]

One of the hotly debated questions of the moment is whether the growth slowdown is a temporary or a permanent phenomenon. Fernald and Jones (2014) pointed out that engines

Figure 1

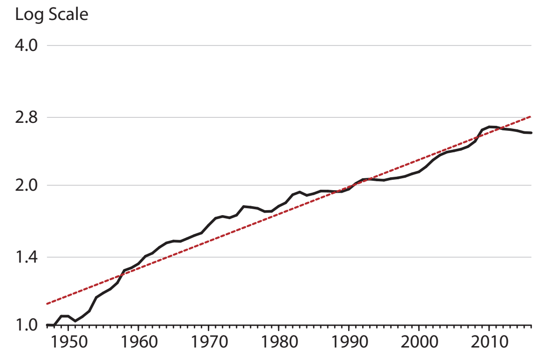
The Kaldor Facts for the Postwar United States

A. GDP Per Worker, 1947 = 1



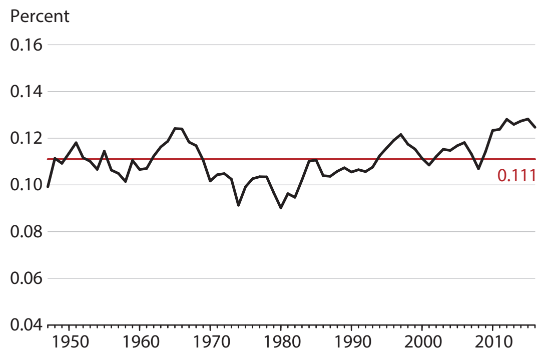
SOURCE: BEA NIPA.

B. Capital Stock Per Worker, 1947 = 1



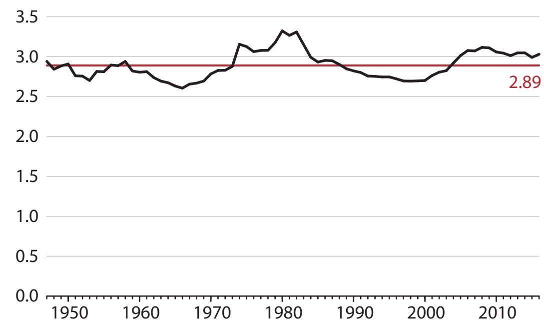
SOURCE: BEA NIPA Fixed Asset Tables.

C. Gross Return on Capital



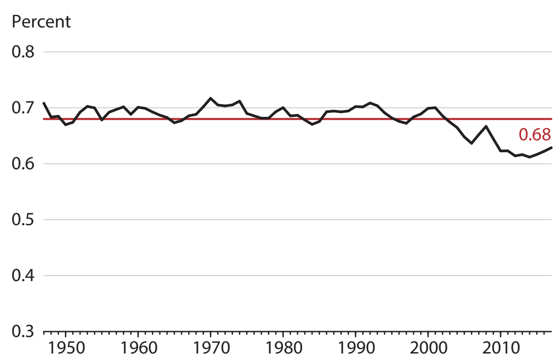
NOTE: The capital share of the corporate sector was used in the calculations.
SOURCE: BEA NIPA Fixed Asset Tables and authors' calculations.

D. Capital-to-GDP Ratio



SOURCE: BEA NIPA Fixed Asset Tables.

E. Labor Share

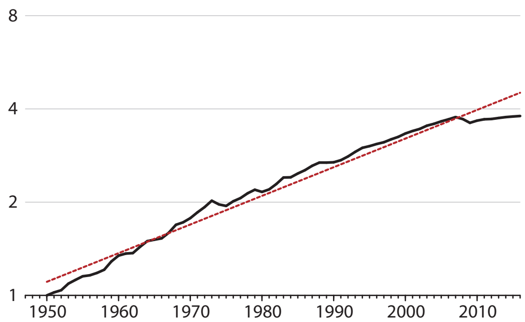


NOTE: The graph displays the labor share of the corporate sector.
SOURCE: BEA NIPA.

Figure 2

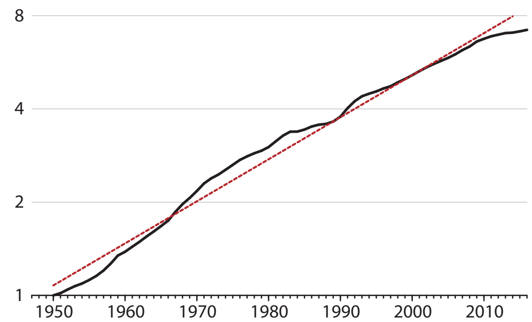
The Kaldor Facts for the Postwar United Kingdom

A. GDP Per Worker, 1950 = 1



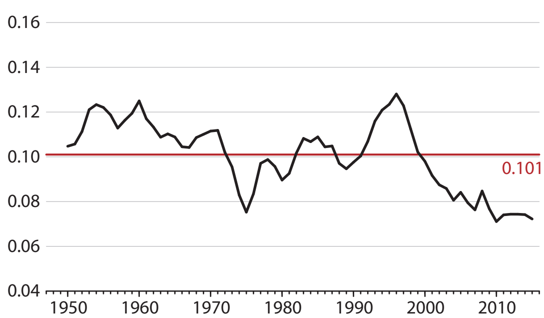
SOURCE: *A Millennium of Macroeconomic Data for the U.K.* Version 3.1.

B. Capital Stock Per Worker, 1950 = 1



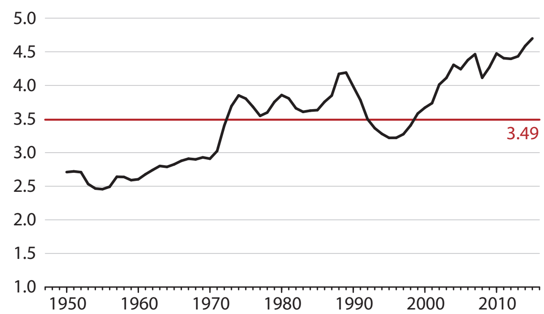
NOTE: Structures are excluded from the capital stock.
SOURCE: *A Millennium of Macroeconomic Data for the U.K.* Version 3.1.

C. Gross Return on Capital



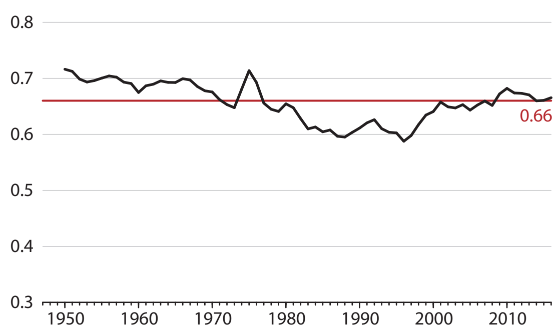
NOTE: Self-employed labor income has been imputed in the capital share calculation.
SOURCE: *A Millennium of Macroeconomic Data for the U.K.* Version 3.1 and authors' calculations.

D. Capital-to-GDP Ratio



SOURCE: *A Millennium of Macroeconomic Data for the U.K.* Version 3.1 and authors' calculations.

E. Labor Share



NOTE: Self-employed labor income has been imputed in the labor share calculation.
SOURCE: BEA NIPA.

Table 1

Annual Growth Rates Per Worker (percent)

	1947-2017	1947-67	1967-87	1987-2007	2007-17
U.S. GDP	1.8	2.7	1.4	1.7	1.1
U.K. GDP	2.0	2.8	2.5	1.8	0.3
U.S. capital	1.4	2.2	1.2	1.1	0.6

SOURCE: BEA NIPA and authors' calculations. *A Millennium of Macroeconomic Data for the U.K.* Version 3.1 and authors' calculations.

of economic growth such as education or research and development require the input of time that cannot be increased ad infinitum. This suggests that there is a natural limit to growth and that the slowdown might well be permanent. Gordon (2016) reached the same conclusion, arguing that we picked the "low-hanging fruit" (e.g., railroads, cars, and airplanes) during the "special century 1870-1970"and that more-recent innovations pale in comparison. Duernecker, Herrendorf, and Valentinyi (2017b) studied the extent to which structural change contributed to the growth slowdown.They found that although the effect of structural change on productivity was sizeable in the past, in the future it is likely to be considerable smaller.

In this article, our approach is somewhat more modest than those in the above papers. We just want to understand under what conditions the growth slowdown may occur in the growth model and which model features are in principle responsible for it. The material that follows answers these questions, drawing heavily on and Duernecker, Herrendorf, and Valentinyi (2017a).

4 THE KALDOR FACTS AND THE GROWTH SLOWDOWN

4.1 The Growth Slowdown in the One-Sector Growth Model

We previously argued that, subject to a few restrictions, one can find a balanced-growth-path equilibrium in the one-sector version of the growth model if the rate of technological change is constant. Along this balanced-growth-path equilibrium, GDP-per-worker growth is constant and a growth slowdown cannot occur by construction. However, a growth slowdown could still occur if a balanced growth path with high growth was unexpectedly replaced by a balanced growth path with low growth. Although the growth of GDP per worker is not constant overall, there would be two regimes with constant growth rates within each regime. The fact that one observes different growth rates for two subperiods would simply mean that the economy is shifting to the new balanced growth path.[13]

While this interpretation is a natural starting point to account for the growth slowdown, it relies on exogenous, unexpected, and permanent changes in the balanced growth path. Such changes are hard to detect and usually become identifiable only with hindsight long after they happened. It is therefore of interest to present an alternative interpretation of the growth slowdown, which relies on secular changes in relative prices in a two-sector version of the growth model.

4.2 A Two-Sector Growth Model

There are two main contexts in which economists have built multi-sector growth models to capture that changing relative prices are a quantitatively important empirical phenomenon. The first focuses on consumption versus investment, or more specifically, on nondurable consumption versus equipment; see Greenwood, Hercowitz, and Krusell (1997) for an early analysis of this in the context of a growth model. The second focuses on how the composition of GDP changes as the economy develops ("structural change"); see Baumol (1967) for an early analysis, which emphasized the implications of changing relative prices of goods and services, and Herrendorf, Rogerson, and Valentinyi (2014) for a recent review.

In what follows, we use a simple two-sector growth model with consumption and investment to study how a growth slowdown may endogenously arise. We can think of this two-sector model as representing the consumption-investment dynamics that result from a richer multi-sector model with several consumption goods and one investment good, in which structural change reallocates production from consumption goods with strong productivity growth to consumption services with weak productivity growth. For the sake of space, we present only the two-sector model, referring the reader to Duernecker, Herrendorf, and Valentinyi (2017a) for an analysis of the multi-sector model.

There is a representative household with preferences given by

$$\sum_{t=0}^{\infty} \beta^t \log C^t .$$

Note two simplifications compared with the one-sector model: The population size is constant and normalized to 1, and the period utility is log. Whereas the first simplification is merely for convenience, the second is crucial for deriving a balanced growth path of the multi-sector model; see Ngai and Pissarides (2007).

There are two sectors, which produce consumption and investment. The production functions are assumed to be Cobb-Douglas. They have the same capital intensity but potentially different total factor productivities (TFPs), which captures technological progress:

$$C_t = A_{ct} k_{ct}^{\theta} l_{ct}^{1-\theta},$$

$$X_t = A_{xt} k_{xt}^{\theta} l_{xt}^{1-\theta}.$$

TFP grows at rates γ_{ct} and γ_{xt}:

$$A_{ct+1} = (1+\gamma_{ct}) A_{ct},$$

$$A_{xt+1} = (1+\gamma_{xt}) A_{xt}.$$

The capital-accumulation equation is as usual:

$$K_{t+1} = (1-\delta)K_t + X_t.$$

4.3 Balanced Growth in the Two-Sector Model

Just as studies based on the one-sector growth model typically focus on properties of a balanced growth equilibrium, it is common for analyses of multi-sector versions of the growth model to look for and examine equilibrium paths that possess a similar constant growth property.

To proceed with the analysis of equilibrium, we normalize the price of investment in each period to be unity and let w_t, r_t, and p_{ct} denote the period-t prices for labor, capital, and consumption, respectively. We note that the price of consumption relative to investment is given by the inverse of the sector TFPs:

$$p_{ct} = \frac{A_{xt}}{A_{ct}}.$$

Hence, differential rates of TFP growth imply changing relative prices. Moreover, to replicate the secular increase in the relative price of consumption, TFP growth must be stronger in investment than in consumption. In what follows, we restrict γ_{xt} to be constant because that is required for the existence of a balanced growth path. We also let γ_{ct} change over time because that is required for matching the data feature that p_{ct} displays an increasing growth rate.

A natural first step in assessing the ability of the two-sector model to account for the Kaldor facts is to mimic what we did above and look for something analogous to the balanced-growth-path equilibrium in the one-sector model. One immediately realizes that it is no longer entirely clear how to extend the notion of a balanced-growth-path equilibrium to the two-sector model. For example, if one thought that a key feature of a balanced growth path in the one-sector model is that output grows at a constant rate, then one could not export this notion to the current setting without deciding how to measure output. As we discuss below, this choice can affect whether a balanced-growth-path equilibrium exists. Another feature of the balanced-growth-path equilibrium in the one-sector model is that the real rental rate of capital is constant, or equivalently, that the marginal product of capital is constant. But again, in an economy with multiple goods and multiple prices, it is not immediately obvious which prices should be used when obtaining these variables.

We describe next how the literature has typically looked for the analogue of the balanced-growth-path equilibrium found in the one-sector growth model. First, consistent with the assumption made in the one-sector model, it is assumed that technological progress of investment grows at a constant rate, but, importantly, it is not assumed that technological progress of consumption grows at a constant rate. The literature has typically looked for an equilibrium path in which the marginal value product of capital in the investment sector is constant when the investment good is the numeraire. In equilibrium, the marginal value product of capital is equalized across sectors, so the marginal value product of capital in all sectors will be constant when expressed in units of the investment good. This is a generalization of the condition that holds along a balanced-growth-path equilibrium in the one-sector model, since in that model both the physical marginal product and the marginal value product of capital are equal to each other and are constant.

It turns out that such an equilibrium path exists and has the following properties; see Duernecker, Herrendorf, and Valentinyi (2017a) for the proofs. If we define aggregate real output in current units of the numeraire investment as $Y_t = p_{ct}C_t + X_t$, then along this equilibrium path, Y_t will grow at the constant rate γ_x. The physical capital stock and investment will also grow at rate γ_x, and given that the investment good is the numeraire, this is also the growth rate of the physical capital stock when measured in units of the numeraire. Lastly, consumption expenditure $p_{ct}C_t$ in units of the numeraire will grow at rate γ_x as well. Because all production functions are Cobb Douglas with the same capital intensity, it trivially follows that in the competitive equilibrium the labor share of output will be constant.

We call this path an aggregate balanced growth path. The reason for the modifier "aggregate" is that whereas in the one-sector model all equilibrium variables grow at constant (though possibly different) rates, it is not necessarily the case here that the relative price of consumption and the real quantity of consumption grow at constant rates. In fact, as shown below, both of them must not grow at constant rates if the model is to generate the slowdown in the growth of GDP per worker.[14]

4.4 The Kaldor Facts and the Growth Slowdown in the Two-Sector Model

Our goal in this subsection is to highlight that the connection between the two-sector model and the data is not as simple or straightforward as is sometimes suggested. Given that along the aggregate-balanced-growth equilibrium path, Y_t and K_t grow at the same constant rate and that the rental rate of capital is constant, one might be tempted to conclude that the results from the one-sector model directly extend to the two-sector model and the two-sector model matches all of the Kaldor facts. However, as we shall see below, this conclusion is incorrect for the growth of aggregate quantities such as GDP.

Let us begin with the capital-to-output ratio. Our empirical measure was the nominal value of the capital stock relative to nominal GDP. In the one-sector model, this is equivalent to K_t/Y_t since both quantities have the same price. In our two-sector model with the price of investment normalized to unity, it turns out that the relevant object for the Kaldor facts is again K_t/Y_t, where K_t is the current capital stock measured in units of investment and Y_t is current output measured in units of investment. While in this regard the result from the one-sector model directly extends to the two-sector model, it only does so when investment is the numeraire. If instead one chose consumption as the numeraire, then one would have to include the price of capital relative to consumption, p_{Kt}, in the numerator of this expression, leading to $(p_{Kt}K_t)/Y_t$.

A similar result holds regarding the return on capital. Our empirical measure was the nominal payments to capital divided by the nominal value of the capital stock. In the one-sector model, this is simply the rental rate of capital, since the numerator is r_tK_t and the denominator is K_t. Choosing investment as the numeraire once again leads to a generalization from the one-sector model to the two-sector model in which the relevant object is the rental rate of capital. If, instead, we had chosen a different numeraire, then the relevant object from the model would have been $r_t K_t / p_{Kt} K_t = r_t / p_{Kt}$ instead of r_t. Moreover, note that if we had multiple capital goods, then the close parallel to the one-sector model would necessarily fail.

Lastly, we consider the model's implications for the behavior of the growth of GDP per worker along the aggregate balanced growth path. The model-based quantity Y_t was defined above as current output measured in units of the numeraire investment. We stress that, if the relative price of consumption to investment changes over time, then this model-based quantity will not correspond to the chained Fisher index of GDP per worker that is reported in the National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA); see and Duernecker, Herrendorf, and Valentinyi (2017a) for the details. This fact implies that constant growth of GDP per worker in the model will not in general correspond to constant growth of GDP per worker in the data. Differently from the one-sector model, the general principle of consistency put forth earlier has bite in the two-sector model: To connect the model with the data in a consistent fashion, one must use the same procedure for measuring GDP per worker in both the model and data.

There are two ways to satisfy the general principle of consistency: We could measure GDP per worker both in the model and the data with either the numeraire investment or the Fisher quantity index. Duernecker, Herrendorf, and Valentinyi (2017a) compare the two possibilities and come down strongly in favor of using the Fisher index. Their main argument is that in the current two-sector model, changes in GDP per worker measured with the Fisher index turn out to approximate to first-order approximate changes in a natural welfare measure. We follow their lead and focus on measuring GDP per worker in the model with the Fisher index as is done in the NIPA. In concrete terms, this means that we proceed in two steps: First, construct an aggregate balanced growth path with the numeraire investment, as outlined above; second, calculate GDP per worker by applying the Fisher quantity index to the model quantities generated from the aggregate balanced growth path. Having done this, Duernecker, Herrendorf, and Valentinyi (2017a) show the following key result:

If $\%\Delta p_{ct}$ is increasing, then the growth of GDP per worker measured with the Fisher index slows down along the aggregate balanced growth path constructed with the numeraire investment.

To provide intuition for this result, it is useful to ask under which conditions $\%\Delta p_{ct}$ is increasing. Since $\%\Delta A_{xt}$ is constant and in equilibrium $p_{ct} = A_{xt}/A_{ct}$, we have that $\%\Delta p_{ct}$ is increasing over time if and only if $\%\Delta A_{ct}$ is decreasing over time. Put differently, while TFP in investment grows at an unchanged rate, TFP in consumption must grow at a decreasing rate. If one uses the numeraire investment to measure GDP-per-worker growth, one nonetheless ends up with constant GDP-per-worker growth. The reason for this is that along the aggregate balanced growth path constructed above, the expenditure share of consumption, $p_{ct}C_t/Y_t$, is constant, and so the decreasing growth rate of C_t is exactly offset by the increasing growth rate of p_{ct}. This property will cease to hold when prices from periods other than the current one are used to evaluate the contribution of consumption expenditure to GDP. An example is the chained Fisher index, which combines current-period and last-period prices and thus picks up that $\%\Delta C_t$ slows down when $\%\Delta p_{ct}$ speeds up. For these reasons, the chained

Fisher index records a growth slowdown of GDP per worker; although measured in the numeraire investment, the growth of GDP per worker is constant.

In sum, applying the general principle of consistency to the current situation makes us change our conclusion dramatically. If we measure GDP growth consistently with the Fisher index, then it is no longer constant in the model but slows down as in the data. While this implies that the two-sector model is consistent with the dynamics in the data, it also implies that, measured with the Fisher index, an aggregate balanced growth path with constant GDP-per-worker growth no longer exists in the two-sector model.

Strucutural change is a natural microfoundation for the increase in $\%\Delta p_{ct}$; see Baumol (1967) for the original idea. Duernecker, Herrendorf, and Valentinyi (2017a) establish this by studying a model in which structural change within consumption reallocates production from goods, which have higher-than-average productivity growth, to services, which have lower-than-average productivity growth.[15] They show that structural change implies that $\%\Delta p_{ct}$ speeds up while $\%\Delta C_t$ slows down.

We end this section by pointing to three related papers. Ngai and Pissarides (2007) mentioned that structural change can lead to a slowdown of GDP growth when GDP is calculated with constant relative prices. However, they nonetheless framed their analysis in terms of constant GDP growth measured in a current numeraire. Moro (2015) provided a model in which structural change reduces GDP growth measured with the Fisher index, but his analysis focused on the role of differences in the sectoral intermediate-input shares. Leon-Ledesma and Moro (2017) also asked to what extent structural change may lead to violations of the Kaldor growth facts. Simulating a model with multiple consumption goods, they found that structural change leads to a growth slowdown of GDP per worker measured with the Fisher index. However, they did not provide analytical characterizations of the conditions under which a growth slowdown occurs in the standard two-sector model studied here.

5 CONCLUSION

We have revisited the Kaldor growth facts for the United States and the United Kingdom during the postwar period. We have presented evidence of deviations from the original Kaldor facts along several dimensions: Instead of staying constant, the growth rates of real GDP per worker and real capital per worker have slowed down in the United States and the United Kingdom since the 1970s, the capital-to-output ratio has increased in the United Kingdom, and the share of income paid to labor has decreased in the United States since 1990. We have discussed how to calculate the Kaldor facts in multi-sector growth models and have established that the first deviation naturally results from secular changes in relative prices once model GDP is measured by a chained Fisher index in the same way as it is in the NIPA.

We have remained quiet about the other deviations of the evidence from the original Kaldor facts. With regard to the growth slowdown of capital per worker, we did not wish to disaggregate capital into its components—equipment, structures, and intellectual property products. We conjecture that if we did this, then similar arguments as those put forth regarding the GDP growth slowdown would explain the capital growth slowdown. In particular, a sizeable part of the capital stock—structures—has experienced an accelerating relative price. Following the logic underlying the growth slowdown of GDP per worker outlined above, this acceleration is likely to reduce the model growth rate of capital per worker if it is measured with the chained Fisher index as it is in the NIPA. Be that as it may, the fact that dwellings have experienced a strong increase in their relative price explains why in the United Kingdom the capital-to-output ratio has shown a pronounced upward trend. An interesting question is why the relative price of dwellings has increased so much in the United Kingdom, but not in the United States. We conjecture that stringent regulation of building permits plays a central role in the United Kingdom.

With regard to the decline of the labor share, the standard multi-sector growth model has nothing to say because by construction the labor share is constant if all production functions are Cobb-Douglas with equal labor-share parameters. Without feeling overly apologetic for this, we refer the reader to the growing literature about the decline of the labor share. For example, Elsby, Hobijn, and Şahin (2013) discuss various reasons for why the labor share decreased. Other contributions to this literature include Karabarbounis and Neiman (2014), Bridgman (2018), Glover and Short (2018), and Koh, Santaeulália-Llopis, and Zheng (forthcoming). ■

NOTES

1 See Jones and Romer (2010) and Jones (2016) for summaries of a broader set of growth facts.

2 An exception is Kongsamut, Rebelo, and Xie (2001), who go into some detail to update the Kaldor facts.

3 While imposing the Cobb-Douglas functional form is convenient, it is not necessary for constructing a balanced growth path in the one-sector model. All arguments would go through for a general production function $Y_t = F(K_t, A_t L_t)$ as long as it had constant returns to scale and technological change was labor augmenting.

4 See, for example, Barro and Sala-i-Martin (2003). Conditional on there being constant growth rates for population and technology, the three core assumptions that guarantee the existence of such a balanced-growth-path equilibrium are that the utility function is of the CRRA (constant relative risk aversion) variety, the production function has constant returns to scale, and technological change augments only labor.

5 There is also a second-order term, $\gamma\eta$, which we ignore for simplicity.

6 When Kaldor wrote his study, capital data included only equipment and structures. The Bureau of Economic Analysis (BEA) now includes also intellectual property products in their investment and capital series. All statistics we report include intellectual property products in investment and capital. None of the conclusions would change if we left intellectual property products out.

7 To be specific, if each of the real components of a total is calculated with the chained Fisher index, then they do not add up to the real value of the total except in the reference year (Whelan, 2002).

8 Persons engaged are full-time equivalent workers plus the self-employed.

9 This differs from Leon-Ledesma and Moro (2017), who do not impose that ratios are unit free when they study how they change in the face of structural transformation from goods to services.

10 Note that the series for capital per worker excludes structures, because we do not have their real values for the United Kingdom.

11 We are not the first to discover that the labor share behaved differently in the United States and the United Kingdom. For previous evidence, see, for example, Annex B of the International Labour Organisation and Organisation for Economic Co-operation and Development (2015).

[12] Note that for the United Kingdom, Table 1 does not report the growth rates of capital per worker. The reason for this is that the U.K. data that we have for the real capital stocks exclude structures.

[13] Andolfatto and MacDonald (1998) elaborated on this idea. They modeled technological progress as resulting from irregular Schumpeterian innovations of different sizes. They showed that a slowdown in the growth of GDP per worker may occur in such an environment.

[14] The concept of aggregate balanced growth was introduced by Ngai and Pissarides (2007). It has since been used widely, including by Duernecker, Herrendorf, and Valentinyi (2017a) and Herrendorf, Rogerson, and Valentinyi (2018).

[15] For a generalization with structural change in both consumption and investment, see Herrendorf, Rogerson, and Valentinyi (2018).

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