The Great Recession, which was preceded by the Financial Crisis, resulted in higher unemployment and income inequality. We propose a simple model where firms producing varieties face labor-market frictions and credit constraints. In the model, tighter credit leads to lower output, a lower number of vacancies, and higher directed-search unemployment. If workers are more productive at higher levels of firm output, then a lower credit supply increases firm capital intensity, raises income inequality by increasing the rental of capital relative to the wage, and has an ambiguous effect on welfare. With an initially high share of labor costs in total production costs, tighter credit lowers welfare. This pattern reverses during an expansionary phase when there is higher credit availability. (JEL D43, E24, G21, J31, J64, L11)
times.” What are the forces that affect income and wealth inequality during “bad times” when incomes decline and unemployment rises? The Great Recession is a case in point. According to Perri (2013), income inequality increased during the Great Recession: Top-earner income declined by 4 percent, median household income declined by 9 percent, and bottom household income decreased by 20 percent (due to a large rise in long-term unemployed members). In addition, Haskel et al. (2012) show a few key developments since 2000: (i) U.S. corporate profits as a share of GDP have shown an upward trend, reaching a record 12.4 percent in 2010. (ii) Average income for all groups except the top 1 percent has declined continually. (iii) The top 1 percent of U.S. income earners, whose income depends in great part on capital earnings, has risen. Haskel et al. (2012) argue that these developments were in part a result of high unemployment and low labor earnings during the Great Recession.

The rise in income inequality and capital income as a share of GDP during the Great Recession raises several questions. Did the recent Financial Crisis, which generated a credit crunch, play any role in the rise of unemployment, reduction in aggregate output, and rise in capital earnings relative to wages? Under what conditions does a reduction in credit availability generate a recession and change factor prices? What are the welfare effects of a tighter credit supply?

The relationship between functional and personal income inequality is complex and depends on several factors. In addition to average labor and capital earnings, these factors include the role of government taxes and transfers, the distribution of physical and human capital, and the incidence of unemployment and saving rates across heterogeneous households. Analyzing formally the effects of credit availability on all these factors and their interactions is beyond the scope of the present article. Instead, we formally model the nexus among credit availability, unemployment, and the functional distribution of income captured by the rental of capital relative to the wage of labor (the rental-wage ratio). The analysis identifies conditions under which a reduction in credit availability raises unemployment, increases the rental-wage ratio, and reduces welfare. Interestingly, these conditions, which apply symmetrically to all firms and households, include non-homothetic increasing-returns production technology, the elasticity of substitution between labor and capital, and a sufficiently high share of labor in total production costs.

We model an economy populated by a mass of symmetric firms producing similar products (varieties) under monopolistic competition. Each product is produced under a non-homothetic, increasing-returns production technology using capital and labor. We abstract from dynamic considerations and propose a simple static framework. We assume that, in the beginning of a single period, firms borrow externally from a competitive banking sector to finance the process of building a factory (consisting of a firm’s capital stock). The factory is used as collateral in the borrowing process. After the factory is built, each firm posts job vacancies and completes the hiring process. Afterward, each firm pays its hired workers and produces its product using its factory as collateral. If the firm does not default, it pays back its loan and keeps its capital stock; if the firm does default, it does not pay back its loan and its capital stock goes to the bank. Labor-market frictions generate directed-search unemployment.

The main finding of the article is that tighter credit availability captured by a higher cost of funds or a higher probability of firm default reduces firm and aggregate output, raises the
rate of unemployment, and changes functional income inequality by increasing the rental-wage ratio. Intuitively, a tighter credit supply raises the rate at which firms borrow from banks and thus the marginal cost of production. Firms cut output and the number of job vacancies; as a result, aggregate output declines and unemployment rises.

Because capital income is the main source of very top incomes (Piketty and Saez, 2014, and Jones, 2015), the rise in unemployment and increase in the rental of capital worsen extreme inequality. In the absence of unemployment insurance, the rate of unemployment captures the proportion of workers with no income, whereas top income inequality measures the proportion of income earned by a fixed percentage of the highest income earners. One can construct an index of extreme inequality by simply adding the rate of unemployment to a measure of top income inequality.

In the presence of non-homothetic, increasing-returns production technology, changes in firm output affect the relative demand for labor for any given factor prices. This dependence is governed by the output elasticity of substitution, which captures the percentage change in the firm capital-labor ratio caused by a 1 percent change in firm output for any given wage of labor and rental of capital. Here, we focus on the case of capital-output substitutability by assuming that a reduction in firm output reduces labor efficiency and therefore makes hired workers more expensive relative to capital for any given factor prices. As a result, firms substitute capital for labor and raise their capital intensity. In other words, a credit-induced reduction in firm output reduces the demand for capital by less than the demand for labor and thus raises the capital-labor ratio at the firm level, the rental-wage ratio, and the share of capital in total production costs. In sum, non-homothetic production technology and capital-output substitutability will transmit the recessional effects of a lower credit supply and, thus, will reduce capital income less than wage income, making capital owners better off relative to workers.

The presence of labor-market frictions and the absence of insurance markets for firm bankruptcy imply that the welfare effects of tighter credit are in general ambiguous because of the second-best nature of the initial equilibrium. When the labor share in total costs is high, tighter credit availability reduces aggregate welfare. Recessions are welfare-reducing in this case. Since changes in output work through labor efficiency in the present model, the effects of lower output are more profound when the share of labor in total costs is high. In this case, lower firm output has a greater effect on average firm output and firm productivity than when the labor share in total costs is low.

Our article is related to the vast literature on the determinants of income and wealth inequality. A strand of this literature focuses on forces shaping the rental of capital and the wage (Stiglitz, 1969; Piketty, 2014; Piketty and Saez, 2014; and Saez and Zucman, 2016). Another strand studies economic mechanisms that generate a Pareto distribution of income and/or wealth that captures top income inequality within the context of random growth models (Piketty and Saez, 2003; Benhabib, Bisin, Zhu, 2011; Jones, 2015; and Jones and Kim, 2015). Our article complements these seminal contributions by investigating how credit availability affects factor prices and unemployment in the presence of credit-market imperfections and labor-market frictions. These factors together with government policies constitute the primary forces governing personal income distribution.
Although implications of credit-market imperfections have been extensively investigated in many different contexts (such as growth, inequality, investment, and trade), the impact of credit constraints on unemployment has not received much attention. Acemoglu (2001) develops a model where unemployment rises because job creation stemming from technological progress is constrained by credit-market imperfections. Duygan-Bump, Levkov, and Montoriol-Garriga (2015) investigate the impact of the Great Recession on unemployment in the United States and find that sectors with high external-finance dependence are more likely to experience high unemployment.

This article is organized as follows. Section 2 describes the model and discusses its equilibrium properties. The effects of credit availability are investigated in Section 3, and Section 4 concludes.

**THE MODEL**

We consider an economy endowed with \( K \) units of capital and \( L \) workers. Individuals are identical and consume a set of differentiated products, each produced by capital and labor under increasing returns to scale. Each firm produces a distinct variety facing credit constraints and labor-market frictions. We assume that each firm lasts only one period.

In dynamic settings, firms borrow externally or use retained earnings to finance investments, augmenting their capital stock. The absence of insurance markets for firm bankruptcy necessitates the use of assets as collateral. The present model proposes a simple mechanism of credit constraints that mimics the standard dynamic framework. Specifically, we assume that, in the beginning of a single period, firms borrow externally from a competitive banking sector to finance the process of building a factory (consisting of a firm’s capital stock). The factory is used as collateral in the borrowing process. After the factory is built, each firm posts job vacancies and completes the hiring process. Afterward, each firm pays its hired workers and produces its product using its factory.

Each firm faces demand-based, product-specific uncertainty: With exogenous probability \( \delta \in (0, 1) \), there is no demand for its product, whereas with probability \( 1 - \delta \), the firm is able to sell its product and repay its loan. In the former case, the firm defaults and its factory (all capital stock) is confiscated by the bank.

Labor-market frictions generate equilibrium unemployment based on directed search. Finally, we assume that income transfers are used to equalize income among individuals. This assumption, which is used routinely in the literature on unemployment, allows us to abstract from unemployment-compensation issues.

**Consumers**

The economy is populated by a unit mass of identical households, each having a fixed supply of \( L \) workers. The preferences of the representative household are described by the following utility function over a continuum of goods indexed by \( j \):
where $\mathcal{J}$ is the set of varieties, $y_j$ is the quantity of consumed variety $j$, and $\epsilon > 1$ is the constant elasticity of substitution (CES) between any two goods.

The consumer maximizes her utility subject to the usual budget constraint. The demand for a typical variety is given by

$$y_j = Q \left( \frac{p_j}{P} \right)^{-\epsilon},$$

where $Q = U$ is the aggregate quantity, $p_j$ is the price of good $j$, and $P$ is the aggregate price given by

$$P = \left[ \int_{\mathcal{J}} p_j^{1-\epsilon} \, dj \right]^{\frac{1}{1-\epsilon}}.$$

Aggregate price $P$ is a dual price index associated with the aggregate quantity $Q$ such that $PQ = E$, where $E = \int p_j y_j \, dj$ is total consumer expenditures.

Substituting $Q = E/P$ into (2) and rearranging terms yields the following inverse market demand function:

$$p_j = Ay_j^{-\epsilon/\sigma}, \quad A \equiv E^{\frac{\sigma}{\sigma-1}} P^{\frac{\epsilon}{\sigma-1}},$$

which is more convenient to work with.

**Production Technology**

There is a continuum of firms, each producing a different variety. Firms use the same production technology, which exhibits internal (firm-level) economies of scale. We assume that firm output $y$ is produced by the following non-homothetic CES production function:

$$y = \left( \frac{\sigma l}{\sigma-1} + k \right)^{\frac{\sigma}{\sigma-1}},$$

where $l$ and $k$ are labor and capital employed by the typical firm, $\sigma$ is the constant capital-labor elasticity of substitution, and $\varphi$ is a function capturing labor efficiency (productivity), which will be specified shortly. Equation (5) states that output produced depends on capital and labor services, with the latter measured in efficiency units. The model can be solved for any $\sigma > 0$; however, the case of complementarity between labor and capital leads to comparative static properties that are not consistent with the main features of the Great Recession. Therefore, we hereafter assume that capital and labor are gross substitutes (i.e., $\sigma > 1$).

Turning to labor efficiency $\varphi$, we assume that it is a function of market size measured by each firm’s output, as in Dinopoulos et al. (2011). In other words, we assume that larger firms are more productive and exhibit higher labor productivity than smaller firms. This assump-
tion leads to a non-homothetic, increasing-returns-to-scale production technology that fits naturally with the monopolistic competition market structure. Specifically, we relate labor efficiency to scale economies by assuming that

\[ \varphi(y) = y^\theta, \]

where \( \theta \in (0,1] \), as in Panagariya (1981). Note that (5) becomes the standard constant-returns-to-scale CES production function when \( \theta = 0 \). When \( \theta > 0 \), the production technology exhibits “labor-saving” increasing returns: As firm output increases, labor becomes more efficient, leading to lower variable and average costs (as established below).

**Directed-Search Unemployment**

The labor market exhibits frictions leading to search unemployment. Labor-market frictions can be modeled in a variety of ways. In the present article, we focus on directed-search unemployment. Each firm hires workers by posting and maintaining \( v \) vacancies and offering a firm-specific wage \( w \). Each vacancy involves costs measured in units of capital (i.e., buildings and machines used in the hiring process).

Specifically, we assume that a fixed amount of capital \( \varphi \) is required for each job vacancy posted. We also assume that capital used in hiring \( \varphi v \) cannot be used in production. All workers search for jobs by directing their search to the firm offering the highest wage \( w \), with each worker applying to only one firm. A firm posting \( v \) vacancies faces a measure of \( n \) applicants and hires \( l < n \) workers. The measure of hired workers is given by the following firm-specific Cobb-Douglas matching function:

\[ l = mn^\gamma v^{1-\gamma}, \]

where \( m > 0 \) is an exogenous parameter representing matching efficiency and \( \gamma \in (0,1) \). The above matching function defines the following hiring rate, which equals the probability that a worker finds a job:

\[ \zeta = \frac{l}{n} = m(v/n)^{1-\gamma}. \]

In sum, each market segment is associated with a single firm and the firm-specific hiring rate \( \zeta \) increases with labor-market tightness (job vacancies per applicant \( v/n \)).

All workers are searching for jobs and only \( \zeta L \) are hired. It follows that the rate of unemployment is given by

\[ u = 1 - \zeta, \]

where the job-finding rate \( \zeta \) is given by (8).

**Cost Structure**

Let \( \bar{w} \) denote the expected wage (income) that each worker obtains at equilibrium. In the absence of unemployment, this wage would be identical to the standard wage of labor. Each
firm must offer a wage $w$ no less than $\bar{w}$, otherwise no worker would apply for jobs that the firm offers. The CES utility function implies that workers are risk-neutral and thus must be indifferent between a wage offered by a firm and $\bar{w}$. In other words, all firms must offer the same expected wage to workers; that is, $\zeta w = \bar{w}$. We also assume that each worker faces a zero reservation wage, and thus every worker always searches for a job. Without loss of generality, we choose the expected wage to be the numeraire by setting $\bar{w} = 1$. In other words, all variables are measured in units of hired labor.

Substituting (8) into $\zeta w = 1$ yields 

$$ v = m \frac{1}{1-\gamma} w^{1-\gamma} l^{\gamma}, $$

which together with (7) yield (10)

It then follows that the total cost of hiring $l$ workers is $r \phi m^{1-(1-\gamma)} w^{1-\gamma} l^{\gamma}$, where $r$ is the rental price of capital. In addition, firms pay each worker a wage $w$; as a result, the unit-labor cost (effective wage) is given by (11)

$$ \hat{w} = w + r \phi (1-\gamma) w^{1-\gamma} l^{\gamma}. $$

The total cost of using $l$ workers and $k$ units of capital is $C = \hat{w} l + rk$. Each firm chooses the wage it offers $w$, the measure of workers hired $l$, and the amount of capital $k$ to minimize the total cost $C$ subject to producing one unit of output; that is, $y = 1$. This minimization problem is recursive: In the first stage, the firm minimizes the cost per hired worker $\hat{w}$ with respect to the offered wage $w$ and then minimizes the total cost $C$ with respect to $l$ and $k$.

The first stage of the cost-minimization problem generates the following posted wage and the cost per hired worker (effective wage):

(12) 

$$ w = \gamma b_0 r^{1-\gamma} / m, \quad \hat{w} = b_0 r^{1-\gamma} / m, $$

where $b_0 = \phi^{1-\gamma} \left[ r^{\gamma} (1-\gamma)^{1-\gamma} \right]$. Thus, the wage posted by each firm and the minimum cost per worker hired increase with the rental of capital and the vacancy-capital requirement. In other words, labor-market frictions leading to costly job vacancies imply that the wage of labor and the rental of capital are positively related; that is, they are complements (as opposed to substitutes).

In the second stage, each firm minimizes total cost $C$ subject to producing one unit of output $y = 1$. Because the adjusted wage $\hat{w}$ is independent of $l$ and $k$, we obtain the following unit-cost function:

(13) 

$$ c(\hat{w},r,y) = \left[ \frac{\hat{w}}{r^{\sigma}} \right]^{1-\sigma} + r^{1-\sigma} \left[ \frac{1}{r^{1-\sigma}} \right]^{1}. $$

It follows that the total cost of producing $y$ units of output is $C = c(\hat{w},r,y) y$. Observe that the unit-cost function declines with firm output $y$, indicating the presence of scale economies.

Let $a_l$ and $a_k$ represent the unit labor and capital requirements of the representative firm. Shephard’s lemma delivers the following expressions for these unit-factor requirements:
The production technology delivers two elasticities, which will play an important role in our subsequent analysis: the traditional wage elasticity of substitution $\sigma$ and the output elasticity of substitution $\lambda$.

Dividing $a_k$ by $a_l$ yields the firm-level capital-labor ratio:

$$a_k(\hat{w},r,y) = \frac{\partial c}{\partial \hat{w}} = \left(\frac{c}{\hat{w}}\right)^\sigma y^\lambda,$$

where $z = r/\hat{w}$ is the relative rental (price) of capital expressed in units of labor costs. We will refer to $z$ as the adjusted relative rental of capital. Substituting $\hat{w}$ from (12) into $z = r/\hat{w}$ yields

$$z = r/\hat{w} = m r^\gamma / b_0.$$

According to (16), an increase in $z$ is associated with a rise in $r$, leading to an increase in functional (as opposed to personal) income inequality. In addition, solving (16) for $r$ and using (12) yields

$$w = b_1 z^{(1-\gamma)/\gamma} / m^{1/\gamma},$$

where $b_1 = [\phi/(1-\gamma)]^{(1-\gamma)/\gamma}$. Thus, the wage offered by each firm increases with the adjusted relative rental of capital and decreases with efficiency parameter $m$.

Taking logs and differentiating (15) yields the following two elasticities of substitution:

$$\left|\frac{\partial \ln(a_k/a_l)}{\partial \ln z}\right| = \sigma, \quad \left|\frac{\partial \ln(a_k/a_l)}{\partial \ln y}\right| = \lambda.$$

The constant wage elasticity of substitution $\sigma > 1$ captures the percentage decline of the firm capital-labor ratio caused by a 1 percent increase in the adjusted relative rental of capital for any given amount of output. As capital becomes more expensive, firms substitute labor for capital, reducing their capital-labor ratio.

The constant output elasticity of substitution $\lambda = \theta(\sigma - 1) > 0$ stems from the non-homothetic production technology and captures the percentage change in the firm capital-labor ratio caused by a 1 percent change in output for any given adjusted relative rental of capital. We state that production technology (5) exhibits directed scale effects in the sense that an increase in firm output reduces the relative demand for capital (i.e., the capital-labor ratio) within each firm. Note that in the present model, the output elasticity of substitution $\lambda$ is positive and output has a similar effect as the adjusted relative rental of capital. The unit-cost function (13) indicates that average costs depend on the effective wage of labor $\hat{w}/y^\delta$. There-
fore, an increase in output reduces the unit labor cost without affecting the rental of capital. As a result, when labor becomes cheaper, the firm substitutes labor for capital, reducing its capital-labor ratio (for any given wage and rental rates). This is the case of output-capital substitutability, where an increase in output leads to a decline in the firm capital-labor ratio.

Another pivotal variable of our analysis is the capital share in total costs:

\[
s(z,y) = \frac{ra_y y + r\phi v}{cy}.
\]

Substituting \( v \) from (10) with \( l = a_t y \) into the above equation and using (12) and (13) yields

\[
s(z,y) = \frac{1 + (1 - \gamma)(zy^\theta)^{\sigma - 1}}{1 + (zy^\theta)^{\sigma - 1}},
\]

which indicates that \( s \) decreases with \( z \) and \( y \). Equation (20) further implies that \( s \in [1 - \gamma, 1] \) since \( zy^\theta \geq 0 \). In other words, the assumption that capital is needed for vacancy posting imposes a lower bound on the share of capital in total costs \( s \).

**Firm Behavior**

We are now ready to solve the firm value maximization problem. As said, firms face credit-market frictions and demand-based, product-specific uncertainty: With an exogenous probability \( 1 - \delta \), each firm is able to sell its product and repay its loan (the good state); and with probability \( \delta \), there is no demand for its product—leading to firm default (the bad state). The representative firm then chooses its output to maximize the expected firm value:

\[
E[\Pi_f] = (1 - \delta)\left[ py + scy - (1 + i)cy \right] = (1 - \delta)\left[ py - (1 + i - s)cy \right],
\]

where \( p \) is the price given by (4), \( s \) is the capital share given by (20), \( c \) is the unit cost of production given by (13), and \( i \) is the interest rate charged by banks (as determined below). According to (21), in the good state (which occurs with probability \( 1 - \delta \)) the firm’s value equals its revenue \( py \) plus the value of the factory \( scy = ra_y y + r\phi v \) (see equation (19)). The firm also repays its loan \( (1 + i)cy \). In the bad state (which occurs with probability \( \delta \)), the firm’s value is zero.

When maximizing expected firm value, each firm takes interest rate \( i \) as given. The first-order condition yields

\[
p\left(1 - \frac{1}{\varepsilon}\right) = (1 + i - s)y \frac{\partial c}{\partial y} - cy \frac{\partial s}{\partial y} + (1 + i - s)c.
\]

This condition states that each firm chooses its output such that the marginal revenue equals the credit-adjusted marginal cost. In addition, the assumption of free entry implies that the expected firm value must be zero (i.e., \( E[\Pi_f] = 0 \)) and thus the output price charged must be equal to the average credit-adjusted cost (that is, \( p = [1 + i - s]c \)). Substituting this expression for \( p \) into the above first-order condition yields
Differentiating $c$ from (13) and $s$ from (20) with respect to $y$ leads to

$$\frac{\partial c}{\partial y} = -c\theta(1-s)\gamma y,$$

$$\frac{\partial s}{\partial y} = \frac{\theta(s-1)(\gamma+s-1)(1-s)}{\gamma y},$$

where we use $\left(zy^\theta\right)^{\sigma-1}\sqrt[\gamma]{\left[1+(zy^\theta)^{\sigma-1}\right]} = (1-s)/\gamma$ from equation (20). Substituting these expressions into (22) and rearranging the terms yields

$$\frac{\gamma}{\theta\varepsilon(1-s)} = 1 - \frac{(\sigma-1)(\gamma+s-1)}{1+i-s},$$

where $s$ is the capital share in the total costs.

**Bank Behavior**

A competitive banking sector supplies credit to firms. The representative bank borrows funds at an exogenous interest rate $\rho$ and lends to firms at interest rate $i$. Recall that with probability $1 - \delta$ a firm repays its loan and the bank collects $(1+i)cy$, whereas with probability $\delta$ the firm defaults and the bank collects collateral $s cy$. The bank incurs a total cost of $(1+\rho)cy$ to obtain the necessary funds for each loan. Accordingly, the expected profit of a typical bank is given by

$$\mathbb{E}[\Pi_b] = (1-\delta)(1+i)cy + \delta scy - (1+\rho)cy.$$

Perfect competition among banks drives expected profits down to zero; that is, $\mathbb{E}[\Pi_b] = 0$. Using this property in equation (24), we obtain the following expression for the interest rate charged by banks:

$$i = \frac{\rho}{1-\delta} \frac{\delta(1-s)}{1-\delta}.$$

Equation (25) implies that $i > \rho$, because $\delta < 1$ and $s < 1$. The interest rate $i$ increases with the cost of funds $\rho$ and declines with the probability of survival $1-\delta$ and the share of costs used as collateral $s$. In other words, the use of collateral reduces the interest rate on firm loans.

**Factor Markets**

Let $M$ denote the ex ante measure of firms in equilibrium. The labor-market equilibrium requires that the demand for hired workers equals the measure of workers who find jobs (employed workers). The demand for workers equals $a_y y M$ and the supply of employed workers is given by $\zeta L$, where the probability of finding a job is equal to the inverse of the wage offer; that is, $\zeta = 1/w$. Thus, the labor-market equilibrium condition is
(26) \[ a_L yM = L/w, \]

where \( a_L \) is given by (14a) and the posted wage \( w \) is given by (17).

The demand for capital consists of capital devoted to output production and capital used in the hiring process. The former equals \( a_K yM \) and the latter equals \( \phi v M \). Using \( v \) from (10) with \( l = a_L y \), we obtain the following full-employment condition for capital:

(27) \[ a_K yM + \phi a_L yMm^{-\gamma(1-\gamma)}w^{-\gamma(1-\gamma)} = K, \]

where the unit-capital requirements \( a_L \) and \( a_K \) are given by equations (14a) and (14b), respectively, and \( K \) is the economy’s capital endowment. This completes the description of our model.

Equilibrium Analysis

This section analyzes the properties of the equilibrium. The first step is to determine the equilibrium value of the capital share in total costs. Substituting the interest rate \( i \) from (25) into (23) leads to the following elasticity condition:

(28) \[ \frac{\gamma}{\theta e(1-s)} = 1 - \frac{(1-\delta)(\sigma-1)(\gamma + s - 1)}{1 + \rho - s}. \]

An economically meaningful solution is obtained under the parameter restriction \( \theta e > 1 \), and thus we hereafter assume that \( \theta e > 1 \). Parameter \( \epsilon > 1 \) represents the CES between varieties, and parameter \( \theta > 0 \) indicates the constant elasticity of labor efficiency with respect to firm output. The left-hand side (LHS) of elasticity condition (28) is a convex, increasing function of \( s \) starting at \( 1/\epsilon \) (where \( s = 1 - \gamma \)) and approaching infinity (as \( s \to 1 \)). The right-hand side (RHS) of equation (28) is a concave, decreasing function of \( s \) starting at 1 and reaching the value of \( 1 - (1 - \delta)(\sigma - 1)\gamma / \rho \) at \( s = 1 \). It then follows that the two curves defined by the LHS and RHS of (28) intersect only once. The unique intersection determines the general-equilibrium value of \( s \).

**Lemma 1.** There exists a unique value of the capital share in total costs \( s \in (1 - \gamma, 1) \) that satisfies elasticity condition (28).

Once the equilibrium value of the capital share in total costs is determined, one can solve for other endogenous variables as well. Dividing (27) by (26) and substituting \( a_K / a_L \) from (15) into the resulting expression and using (17) yields

\[ \left( z y^\theta \right)^{1-\sigma} + 1 - \gamma = b_1 \left( z/m \right)^{1/\gamma} K/L. \]

Note that equation (20) implies \( \left( z y^\sigma \right)^{1-\sigma} = \gamma / (1 - s) - 1 \), and substituting this into the above equation yields the following equilibrium value for the adjusted relative rental of capital:

(29) \[ z \equiv \frac{r}{w} = m \left( \frac{1 - \gamma}{\phi} \right)^{1-\gamma} \left( \frac{s}{1 - s} \right)^{\gamma} \left( \frac{\gamma L}{K} \right)^{\gamma}. \]
This condition indicates that a higher relative supply of capital (captured by a higher $K/L$) puts downward pressure on the relative rental of capital and a higher relative demand for capital (captured by a higher $s$) exerts upward pressure on the adjusted relative price of capital $z$.

Observe that the offered wage $w$ is an increasing function of the adjusted relative rental of capital $z$, as given by (17). In addition, determination of $z$ leads to the determination of the rental of capital $r$ and the hiring cost per worker $\hat{w}$ based on (16). Output per firm $y$ depends on the share of capital in total costs as well. Solving (20) for $zy^\theta$ and substituting $z$ from (29) yields

$$y = m \frac{1}{\phi} \left[ \frac{\phi}{1-\gamma} \right]^{1-\gamma} \left[ \frac{(1-s)K}{s\gamma L} \right]^{\gamma} \left[ \frac{1-s}{s+\gamma-1} \right]^{1/(\sigma-1)},$$

which indicates that, as the share of capital in total costs rises, output per firm must fall (since $\sigma > 1$). We can also determine the total output $Y = (1-\delta)My$, where $M$ is the number of goods produced in equilibrium. Equation (26) implies that $Y = (1-\delta)My = (1-\delta)L/(a_tw)$. Using equations (13), (14a), (17), and (20) yields

$$a_t w = \frac{s(s+\gamma-1)^{1/(\sigma-1)} L}{s^{1/(\sigma-1)} K}.$$ 

It then follows that the total output $Y$ is given by

$$Y = \frac{(1-\delta)\gamma^{1/(\sigma-1)}K}{s(s+\gamma-1)^{1/(\sigma-1)}}.$$

The total output is independent of the total labor supply $L$. It increases with aggregate capital stock $K$ and decreases with the share of capital $s$.

The rate of unemployment depends on the adjusted relative rental of capital and is calculated as follows. As said, all workers $L$ are searching for jobs, but only $\zeta L$ obtain jobs, where $\zeta = 1/w$ is the job-finding rate and $w$ is the wage offer given by (17). As a result, the rate of unemployment is

$$u = 1 - \zeta = 1 - \left[ \frac{(1-\gamma)\phi}{\phi z} \right]^{(1-\gamma)/\gamma}.$$

This equation states that unemployment and income inequality measured by the adjusted relative rental of capital are positively related to each other. One may interpret unemployment as the lower bound of extreme inequality—the percentage of individuals with zero income. Furthermore, one can interpret the adjusted relative rental of capital $z$ as another source of extreme inequality: An increase in $z$ raises the rental-wage ratio $r/w$, benefiting capital owners.\textsuperscript{11}

Since capital income is the main source of top income inequality (Piketty and Saez, 2014, and Jones, 2015), our model implies that an increase in the adjusted relative rental of capital raises extreme income inequality.

Finally, we calculate aggregate welfare. To this end, note that utility function (1) can be written as
and substituting $y$ and $Y$ from equations (30) and (31) into the above equation and ignoring the constant yields the following indirect utility function:

$$V = \left(1 - \delta\right)\left(\frac{1}{\theta^{s-1}}\right)^{\frac{e}{s-1}} = \left(Y\frac{1}{\theta^{s-1}}\right)^{\frac{e}{s-1}},$$

where $\lambda = \theta(\sigma - 1)$. Note that $V \to \infty$ as $s \to 1 - \gamma$ or $s \to 1$. In addition, taking the derivative of $V$ with respect to $s$ yields

$$\frac{1}{V} \frac{dV}{ds} = -\frac{\theta \epsilon - \gamma - \frac{\theta \epsilon - 1}{\lambda s (s + \gamma - 1) (1 - s)}}{\lambda (s + \gamma - 1) (1 - s)}.$$
Intuition behind these results is as follows. The zero profit condition (25) requires that a rise in the cost of credit (rise in $\rho$) or an increase in the probability of default (rise in $\delta$) must be covered by raising the interest rate $i$ at which banks lend to firms. In response to higher interest payments, each firm reduces output produced. The assumption that the output elasticity of substitution is positive (i.e., $\lambda > 0$) implies that a reduction in firm output reduces firm demand for labor more than firm demand for capital, leading to an increase in firm capital intensity. At the aggregate level, this raises the relative demand for capital and, for a given factor abundance, raises the relative rental of capital $z$ as well as the absolute rental of capital $r$ (see equation (16)).

The rise in the relative demand for capital increases the capital share in production costs $s$. This leads to a decline in aggregate output according to equation (31). In other words, a rise in costs raises the labor cost per unit of output $a_Lw$ and requires a reduction in aggregate output $Y = (1 - \delta)My$ according to the labor-market equilibrium condition (26). The latter also implies that tougher credit conditions raise the mass of firms $M$. However, the reduction in output per firm dominates the increase in the mass of firms, leading to a reduction in aggregate output.

The rise in the adjusted relative rental of capital $z$ induces each firm to offer a higher wage $w$, as indicated by equation (17), leading to a reduction in the hiring rate $\zeta = 1/w$ and an increase in the rate of unemployment $u = 1 - \zeta$. Another way to reveal the mechanism relating lower credit availability to the rate of unemployment is as follows. Equation (10) indicates that each firm faces a trade off between maintaining vacancies $v$ and the wage posted $w$ when considering whether to hire a worker. A rise in $r$ makes posting vacancies more expensive because of the associated capital costs. This tilts the balance in favor of a higher $w$ (see equation (12)). As firms post higher wages, they reduce the number of vacancies $v$ and keep the expected wage $\zeta w$ constant at unity. As vacancies fall, $\zeta$ must fall, as indicated by (8), which means that the rate of unemployment $u = 1 - \zeta$ must rise.

Next, we analyze how an increase in the cost of external funds $\rho$ and/or default rate $\delta$ affects welfare. In general, the presence of labor-market frictions, monopolistic competition, and collateral requirements generates a second-best market environment. This means that changes in credit conditions affect welfare through (i) changes in consumed varieties, which is captured by the elasticity of substitution in consumption $\varepsilon$, and (ii) changes in income stemming from distorted factor prices, which is captured by the output elasticity of substitution $\lambda$. Specifically, the impact of tougher credit conditions on welfare depends on $\varepsilon \lambda$. Assuming that the initial capital share $s$ is not high and thus the unit labor cost is low and aggregate output is high, an increase in $\rho$ or $\delta$ decreases aggregate welfare.

**Proposition 2.** Tighter credit availability captured by a higher cost of funds $\rho$ or higher probability of firm default $\delta$ reduces aggregate welfare if the capital share in total costs is sufficiently small.

Another way to see how tighter credit conditions affect welfare is as follows. Recall from the previous section that the direct utility function can be written as

$$U = \left( Y^{\frac{1}{\varepsilon}} \right) ^{\frac{\varepsilon}{\varepsilon - 1}}.$$
As discussed earlier, tighter credit conditions decrease both firm output $y$ and aggregate output $Y$ and, as a result, tighter credit has an ambiguous effect on welfare. When the capital share in total costs is not too high, the reduction in the aggregate output $Y$ dominates the increase in $y^{-1/\varepsilon}$ (caused by a reduction in $y$), leading to lower aggregate welfare. Intuitively, firm output affects welfare mainly through labor efficiency by augmenting effective labor units and reducing average production costs. When the share of capital in total costs is not too large (and thus the share of labor is high), output has a more profound effect on firm productivity measured by average production costs. In this case, a reduction in firm output has a higher impact on welfare, leading welfare to decline as firms substitute capital for labor and increase their capital intensity. In contrast, when the share of capital in total costs is high, firm productivity is not affected as much by changes in output and welfare increases with lower output (and higher firm capital intensity).

**CONCLUSION**

Some of the greatest challenges facing the U.S. economy are sluggish wage growth and widening income and wealth inequality. In this article, we present a simple model showing that credit-market frictions can be a catalyst to widening extreme inequality by affecting capital income, wages, and the rate of unemployment. In our model, monopolistically competitive firms face credit-market imperfections when financing production costs. Credit-market frictions stem from the inability of firms to buy default insurance. In addition, firms use a non-homothetic increasing-returns production technology, where workers are more productive at higher firm output. This technology translates changes in firm output into changes in factor prices. Finally, labor markets exhibit frictions stemming from job search and matching. Thus, our model presents a unified general-equilibrium framework to address income inequality and unemployment in the presence of credit-market imperfections.

Consistent with what we observed during the Great Recession (which was caused by the tightening of credit-market conditions as a result of the Financial Crisis), our model predicts that a tighter credit supply widens the gap between wage earners and capital owners, raises the rate of unemployment, and reduces firm-level and aggregate output. When workers are less productive at lower firm output, effective labor costs increase as output declines. With a positive output elasticity of substitution, this leads to substitution toward more capital-intensive techniques and a rise in the share of capital in total costs. The increased relative demand for capital raises the rental of capital and thereby widens the gap between labor and capital earnings. Furthermore, as firms substitute labor with capital, the economy experiences a higher rate of unemployment. When the share of labor in total costs is sufficiently high, a tighter credit supply reduces welfare as well. However, as the economy recovers, this process is reversed. As credit conditions improve and output rises, the gap between capital and labor earnings narrows, as seems to be the case in the more recent data.

The proposed framework can be generalized along several dimensions. By adding an explicit distribution of capital among individuals, the model can offer insights on how credit availability affects within-group (as opposed to between-group) inequality, especially for top
income earners. The assumption of symmetric firms can be replaced with heterogeneous firms as in Melitz (2003) or Dinopoulos and Unel (2017), where heterogeneous firm profits can constitute another source of personal income inequality. Finally, recent studies (for instance, Foley and Manova, 2015; Dinopoulos, Kalivitis, and Katsimi, 2017; and Unel, 2018) have shown that credit constraints can substantially affect firm productivity, price elasticities, and trade patterns. Consequently, relaxing the closed-economy assumption is an important extension.

NOTES

1 Piketty and Saez (2003) and Kopczuk (2015) among many others have documented the long-run evolution of top incomes and wealth in the United States. Haskel et al. (2012) show that, since 2000, average incomes of all groups in the U.S. income distribution have declined except the top 1 percent. Jones and Kim (2015) present evidence of increasing income inequality in several advanced and developing countries.

2 Using household survey data for six advanced countries, including the United States, for a 20-year period, Maestri and Roventini (2012) argue that inequality in hours of work and market income is in general countercyclical, implying that recessions are associated with greater income inequality. In addition, they find that the impact of business cycles on disposable income depends on country-specific tax and transfer systems.

3 During the Great Recession, low-income households experienced a higher incidence of unemployment (Perri 2013, and Hoynes, Miller, and Shaller, 2012). In addition, government taxes and transfers reduced the impact of the Financial Crisis on disposable-income inequality. Moreover, the Financial Crisis had an asymmetric effect on asset prices: Stock prices rose after 2009, whereas the housing market remained stagnant at least until 2011, contributing to higher wealth (and capital income) inequality during the Great Recession. Finally, households with low incomes and wealth increased their saving rates more than households with high incomes and wealth (Krueger, Mitman, and Perri, 2016).

4 Variants of the above CES production function have been widely used in the literature on wage inequality. See, for example, Krussell et al. (2000); Acemoglu (2002); and Unel (2010) among many others.

5 This assumption is consistent with several recent studies, which argue that higher capital intensity and a reduced labor share can only be reconciled by \( \sigma > 1 \) (Elsby, Hobijn, and Sahin, 2013; Karabarbounis and Neiman, 2014; and Piketty and Zucman, 2014). Piketty and Zucman (2014), for example, argue that this elasticity must be between 1.3 and 1.6 to be consistent with observed income-inequality patterns in the United States and Europe. It should be stated, though, that there is a large literature that estimates the elasticity of substitution between capital and labor from a production or cost function, and estimates are usually less than 1 (Lawrence, 2015). However, these studies suffer from problems related to omitted-variable bias and reverse causality.

6 Although in practice worker hiring processes require labor, our assumption captures in an extreme fashion that labor hiring is a capital-intensive activity.

7 Directed-search unemployment is consistent with evidence provided by Hall and Krueger (2012). They surveyed a sample of U.S. firms and found that about one-third of workers had a take-it-or-leave-it wage offer. They also found that bargaining is more common among educated workers and about one-third of workers had bargained over pay before they were hired.

8 In our model, symmetric firms and their relation with workers last only one period. Consequently, there is no possibility of on-the-job search and renegotiation of wages, as more general wage-posting models assume (Mortensen, 2003). Without on-the-job search and dynamic interaction between firms and workers, there will be no wage dispersion.

9 Feenstra, Li, and Yu (2011) assume that credit is supplied by a single bank behaving as a monopolist, which addresses incomplete-information considerations. In contrast, we assume that credit is provided under complete information.

10 Note that \( \theta = 1 \) is sufficient for \( \theta \varepsilon > 1 \), since \( \varepsilon > 1 \). In other words, we analyze the case where firms face sufficiently large increasing returns to scale.
Equations (16) and (17) imply that \( r/w = z/\gamma > 0. \)

Solving \( dV/ds = 0 \) yields \( s_m \) as a highly complicated function of model parameters, and thus simulation analysis is more informative about its value.

REFERENCES


