Student Loans Under the Risk of Youth Unemployment

Alexander Monge-Naranjo

While most college graduates eventually find jobs that match their qualifications, the possibility of long spells of unemployment and/or underemployment—combined with ensuing difficulties in repaying student loans—may limit and even dissuade productive investments in human capital. The author explores the optimal design of student loans when young college graduates can be unemployed and reaches three main conclusions. First, the optimal student loan program must incorporate an unemployment compensation mechanism as a key element, even if unemployment probabilities are endogenous and subject to moral hazard. Second, despite the presence of moral hazard, a well-designed student loan program can deliver efficient levels of investments. Dispersion in consumption should be introduced so the labor market potential of any individual, regardless of the family's financial background, is not impaired as long as the individual is willing to put forth the effort, both during school and afterward, when seeking a job. Third, the amounts of unemployment benefits and the debt repayment schedule should be adjusted with the length of the unemployment spell. As unemployment persists, benefits should decline and repayments should increase to provide the right incentives for young college graduates to seek employment. (JEL D82, D86, I22, I26, I28, J65)

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Many college graduates may face spells of unemployment and/or underemployment before they find jobs that match their qualifications. These spells may be long, especially for some college majors, and can lead to serious financial difficulties, including obtaining credit and repaying student loans and other forms of debt. Aside from their direct welfare costs, the hardship and volatility during the early stages of labor market participation can impair—and even dissuade altogether—productive investments in human capital, especially for those from more modest family backgrounds.

In this article, I explore the optimal design of student loan programs in an environment in which younger individuals, fresh out of college, may face a substantial risk of unemploy-

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ment. In my model, the risk of unemployment is endogenous and subject to incentive problems. In particular, I assume that a problem of "moral hazard" (hidden action) distorts the implementation of credit contracts. More specifically, I examine an environment in which costly and unverifiable effort determines the probability of younger workers finding a job. The costs of unemployment for a young worker are in terms of zero (or very low) earnings and missing opportunities to gain experience that would enhance his or her labor earnings for subsequent periods. Moral hazard and other incentive problems have been studied extensively by economists in a wide array of areas ranging from banking and insurance to labor markets. Yet, only recently has the explicit consideration of incentive problems been introduced in the study of optimal student loan programs.[1] Despite the extensive literature on unemployment insurance since the 1990s (e.g., Wang and Williamson, 1996, and Hopenhayn and Nicolini, 1997), the integration of an unemployment insurance scheme within the repayment structure of student loans and the optimal design of such a scheme is an aspect that remains unexplored.

In this article, I first consider a simple three-period environment. In the first period, a young person decides on his or her level of schooling investment. In the second period, a hidden effort governs the probability of unemployment. In the third period, all workers find employment, but their earnings are affected by their schooling level and their previous employment. I contrast the resulting allocations from two contractual arrangements: the first-best (i.e., unrestricted efficient) allocations and the optimal student loan programs when effort is a hidden action (moral hazard). I then extend the simple environment by dividing the potential postcollege unemployment spell into multiple subperiods. I use this extension to examine the optimal design of unemployment insurance and compare the human capital investments resulting from a suboptimal scheme without unemployment insurance. In all these cases, I restrict the credit arrangement so the creditor expects to break even in expectation (i.e., in average over all possible future outcomes). Therefore, my conclusions can apply not only to government-run programs, but also, under similar enforcement conditions, to privately run student loan programs.

I derive three main conclusions. First, the optimal student loan program must incorporate, as a key element, a transfer mechanism should college graduates face post-schooling unemployment. This conclusion holds even if unemployment probabilities are endogenous and job searching might be subject to moral hazard. This simple and perhaps not surprising result is worth highlighting given the limited scope for insurance in existing student loans. An unemployment insurance mechanism not only alleviates the welfare cost of potentially catastrophic low consumption for the unemployed, but can also help to enhance human capital formation as individuals and their families would not need to self-insure by means of lower-return assets and reduced schooling.

Second, and related to the last point, despite the presence of moral hazard, a well-designed student loan program can deliver efficient levels of investments for at least a segment of the population. Here, dispersion in consumption should be introduced so the labor market potential of any individual, regardless of family financial background, is not impaired. This result is conditional on the individual's willingness to exert effort, which might be subject to wealth effects. However, once the effort and abilities of a person are factored in, the investments in schooling should be completely independent of one's family's wealth.

A third important result concerns the dynamics of the unemployment benefits and the repayment of debts. Once I consider a model with possible multiperiod unemployment and repeated search effort, the unemployment benefits should decline with the length of the unemployment spell. Moreover, the debt balance and its repayment should also be adjusted upward the longer a person stays unemployed. While these two features are well understood in the literature on unemployment insurance, they are not incorporated in actual student loan programs. I believe this is an interesting margin to explore: By enhancing the ability to provide both insurance and incentives. it also can enhance the formation of human capital, especially for those individuals with high ability but low family resources.

In the next section, I examine data for recent cohorts of U.S. college graduates and show that unemployment and underemployment are significant risks for them right after college. In Section 2, I describe the basic environment for analysis; in Sections 3 and 4 I characterize the allocations under the first-best and under optimal loan programs under moral hazard. Section 5 solves the optimal repayment in the multiperiod environment and discusses the allocations. Section 6 concludes. The appendix discusses additional aspects of the optimal student loan and compares it with other contractual environments.

1 POSTCOLLEGE UNEMPLOYMENT AND UNDEREMPLOYMENT

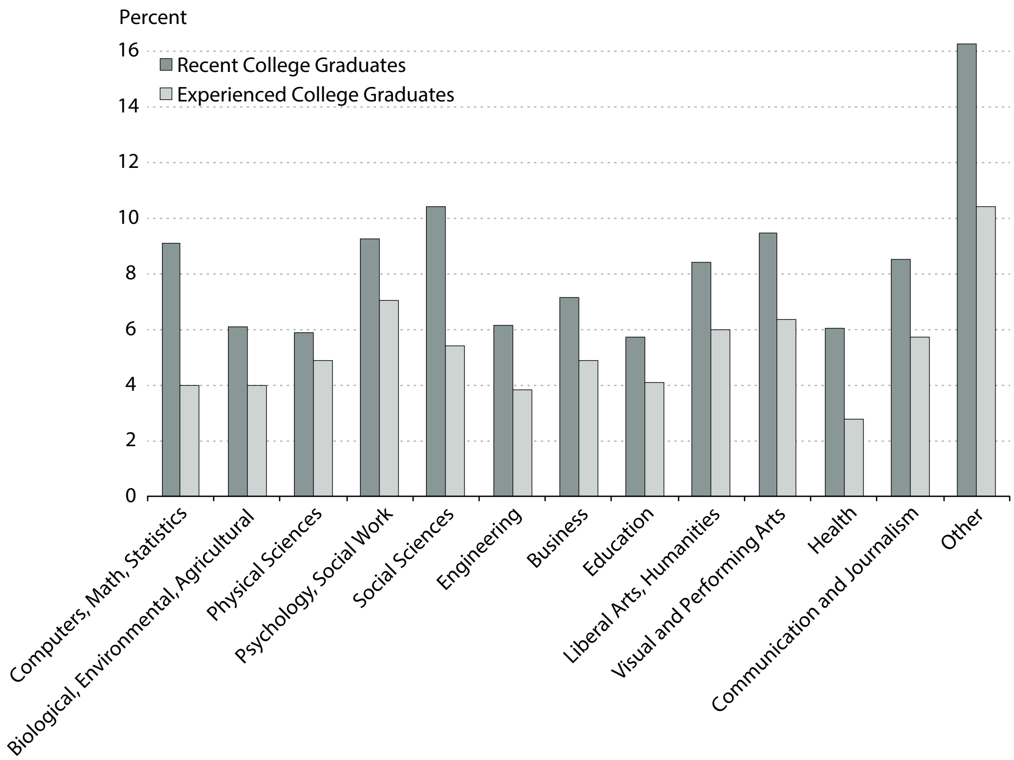
Recent work on college education choices has called attention to the rather high risk involved in investments in education. For instance, Chatterjee and Ionescu (2012) highlight the fact that a sizable fraction of college students fail to graduate. Furthermore, as emphasized by Lee, Lee, and Shin (2014), even successful graduates face a large and widening dispersion in labor market outcomes, possibly including the option of working in jobs and occupations that do not require their college training. Thus, even if a college education might greatly enhance the set of labor market opportunities, such an education is a risky investment that comes at the cost of tuition and forgone earnings; also, graduates' ex post returns may even render repayment of student loans difficult.

To be sure, some of these risks and volatilities are more prevalent at the beginning of a person's labor market experience. A college education does not fully preclude a younger, unexperienced worker from facing more difficulties in finding a job than an older, more mature, experienced, and better-connected worker. To illustrate this point, I use 2011 cross-sectional earnings and unemployment data from the American Community Survey (ACS) to report the unemployment and earnings of college graduates (Figures 1 and 2).[2] In both figures, the blue columns correspond to the average recent college graduate (between 22 and 26 years of age), while the red columns represent the average more experienced graduate (between 30 and 54 years of age). In both figures, graduates from more than 170 majors are grouped into 13 broader areas.

Figure 1 shows that the unemployment rates are uniformly higher for recent graduates than for more experienced ones. The differences in the rates are very pronounced for some

Figure 1

2011 Unemployment Rates for Recent and Experienced College Graduates by Major



SOURCE: American Community Survey.

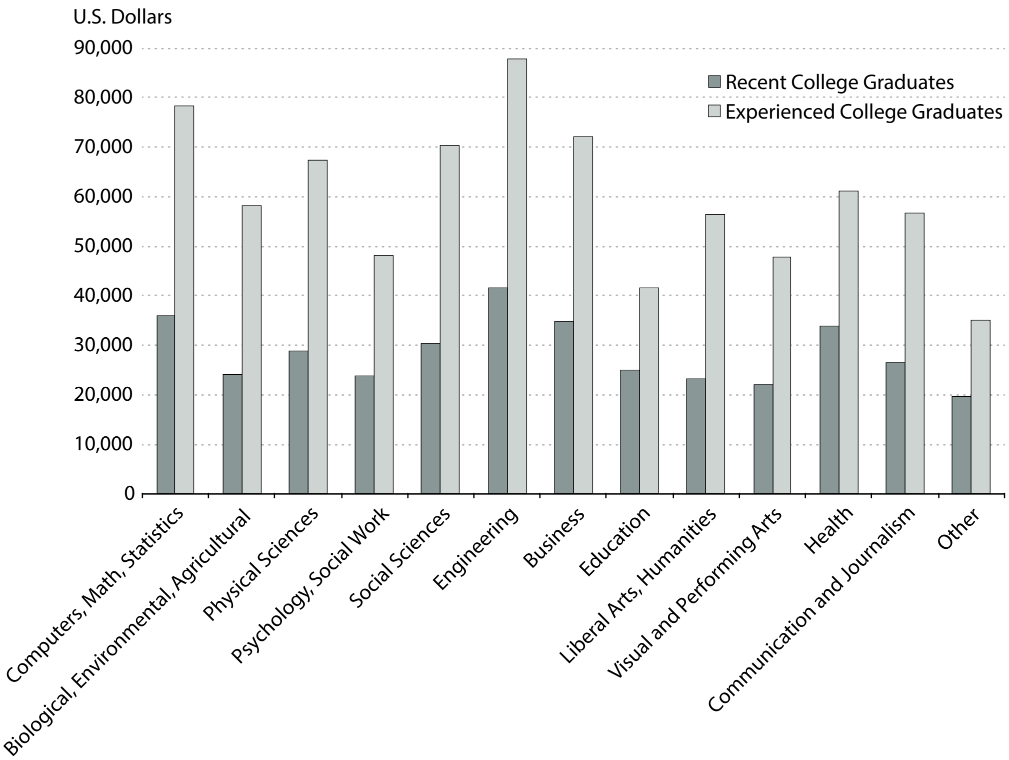
groups of majors: as much as 5 percentage points higher for fields such as computer science, math and statistics, social sciences, and others. The rates are much lower for other fields such as education, business, and, especially, physical sciences. However, the unemployment gaps are significant in all groups of majors, supporting the notion that graduates in all fields take time to find jobs. In the meantime, they experience higher rates of unemployment than their more established peers.

Figure 2 shows the 2011 labor earnings for the same groups of graduates who are employed by major. This figure also shows a very clear pattern: More recent graduates earn significantly less than more experienced graduates. In fact, for all but three majors (education, health, and other), recent graduates earn less than half that of their more experienced peers. Indeed, recent graduates earn as little as 41 percent as much as their older peers in biological sciences and liberal arts and humanities; in education that ratio is the highest among all majors, at 60 percent.

In sum, a simple look at the cross-sectional data from the ACS clearly indicates that within each field younger graduates (i) have more difficulty finding a job than more experienced ones and (ii) their earnings are lower when they are employed. But while the ACS makes it easy to compare different cohorts of college graduates, it does not follow them over time. The ACS

Figure 2

2011 Labor Earnings for Recent and Experienced College Graduates by Major



SOURCE: American Community Survey.

data cannot establish the transitions of college graduates from the early periods of their labor market experience to the more mature ones in terms of employment, earnings, and repayment of their student loans. To this end, I now consider the Baccalaureate and Beyond Longitudinal Survey 2008-12 (B&B:08/12) of college students who graduated in the 2007-08 academic year.[3] The survey collects the employment records of individuals in 2009 and in 2012, about one and four years after graduation, respectively. It follows just one cohort of graduates, so comparison across cohorts cannot be made with this dataset.

As in Lochner and Monge-Naranjo (2015a)—but for the B&B:93/03 survey—I aim to report unemployment and underemployment for a typical American college student. In what follows, I exclude noncitizens, the disabled, and individuals who received their baccalaureate degree at age 30 or older as their labor market experience involves a number of other issues. For the same reason, I also exclude those with more than 12 months of graduate work. Tables 1 and 2 document that unemployment and underemployment are very relevant risks for recent U.S. college graduates, especially in their first few years following graduation. Table 1 shows the average percentage of the months in which students remained unemployed since graduation (i.e., Number of months unemployed/Numbers of months since graduation × 100). The

Table 1

Mean Percent of Time Unemployed Since Graduation (July 2008)

College major	2009	2012
Business	10.0	6.1
Education	10.1	6.7
Engineering	6.9	4.2
Health professions	6.5	3.1
Public affairs	8.8	5.6
Biological sciences	9.3	6.0
Math/science/computer science	6.6	4.1
Social science	10.7	8.9
History	12.9	7.5
Humanities	12.0	9.1
Psychology	9.3	7.0
Other	10.9	7.1
All	9.8	6.6

SOURCE: Baccalaureate and Beyond, 93/03.

Table 2

Percent of Graduates with Primary Job Unrelated to College Education

College major	2009	2012
Business	19	14
Education	12	9
Engineering	9	11
Health professions	10	9
Public affairs	21	19
Biological sciences	20	23
Math/science/computer science	18	10
Social science	33	30
History	45	36
Humanities	43	29
Psychology	29	27
Other	29	23
All	24	19

SOURCE: Baccalaureate and Beyond, 93/03.

first column reports the percentage up until 2009 and the second column the average percentage three years later, in 2012. Each row reports the average by college major and for the whole sample.

The results show fairly high unemployment rates. On average, one of every 10 college graduates remains unemployed in the first year after graduation. Of course, 2009 is not a typical year since the United States was in the middle of the so-called Great Recession and the overall unemployment rate was high.[4] Moreover, the unemployment rate for this sample of college-educated individuals, on average, is much lower than for the rest of U.S. workers. Note also that there is significant dispersion. While health professionals, math/science, and computer science professionals all had unemployment rates lower than 7 percent, most others were closer to 10 percent. In the extreme, history majors found themselves unemployed 13 percent of the time—that is, almost one of every seven.

The other salient result is the rapid decline in this measure of unemployment three years later. The overall unemployment rate falls by one-third, from 9.83 percent to 6.55 percent. These employment gains occur across all majors, with remarkable gains in history, business, and education. With just four years of labor market experience, the unemployment rate for this young cohort compared favorably with the overall U.S. civilian unemployment rate: 8.2 percent in July 2012.

Table 2 reports the other form of labor market unemployment: the possibility of employment that does not use the person's main skills. From the B&B:08/12, I obtain the fraction of individuals reported as employed at the time of the survey but whose job or occupation is not directly related to the person's college education.

Table 3

Repayment Status of Borrowers Graduating in 1992-93

Status	1998	2003
Fully repaid (%)	26.9	63.9
Repaying (%)	65.1	27.8
Deferment/forbearance (%)	3.8	2.5
Default (%)	4.2	5.8

SOURCE: Lochner and Monge-Naranjo (2015a).

Table 2 also shows some remarkable results. For the first year after graduation, one of every four employed college graduates ends up working in a job unrelated to his or her education. For some majors such as social sciences, humanities, and especially history, the ratios are much higher. After four years, the ratios are lower but still high, around one of every five. With the exception of biological sciences and engineering, the ratios decline for all other majors; in some cases such as business, history, humanities, and especially math and computer science, the ratios decline substantially.

In sum, Tables 1 and 2 support the view hinted at by the ACS data that it not only may take time for a recent college graduate to find a job, but also may take an even longer time to find a job matching his or her acquired skills, abilities, and vocations.

The early postcollege stages are also associated with higher difficulties in repaying student loans. For a better perspective on the life cycle of payments for student loans, Lochner and Monge-Naranjo (2015a) examine the repayment patterns of an older cohort of borrowers: those in the B&B:93/03, the cohort of students who graduated in the 1992/93 academic year. Table 3 (from Lochner and Monge-Naranjo, 2015a) reports repayment status for borrowers as of 1998 and 2003—around 5 and 10 years after graduation. In both years, graduates repaying their loans plus those who had already fully repaid their loans account for 92 percent of the borrowers. Not surprisingly, the fraction of those with fully repaid loans is much higher 10 years after graduation.

More interestingly, the fraction of borrowers who applied for and received a deferment or a forbearance (postponement of repayment without default) was significantly higher in the early years after graduation. In 1998, this fraction accounted for 3.8 percent of borrowers. Five years later, in 2003, the percentage fell to 2.5 percent. These figures suggest that deferment and forbearance are important forms of non-repayment. The declining share of borrowers engaging in this form of non-repayment may be a reflection of lower volatility in the labor market of graduates as times passes, but it also may reflect the fact that fewer borrowers can quality for deferment and forbearance as they age. Indeed, the counterpart is that default rates rise from 4.2 percent to 5.8 percent between 1998 and 2003.[5]

Table 4

Repayment Status Transition Probabilities

	Repayment status in 2003		
Repayment status in 1998	Repaying/fully paid	Deferment/forbearance	Default
Repaying/fully paid (%)	93.9	2.0	4.0
Deferment/forbearance (%)	74.9	16.5	8.5
Default (%)	54.4	3.8	41.8

SOURCE: Lochner and Monge-Naranjo (2015a).

Table 4 (from Lochner and Monge-Naranjo, 2015a) shows the transition rates for different repayment states from 1998 to 2003. The rows in the table list the probabilities of (i) being in repayment (including those whose loans are fully repaid), (ii) receiving a deferment or forbearance, or (iii) being in default 10 years after graduation in 2003 conditional on each of these repayment states five years earlier (in 1998). Note that most (94 percent) of those in good standing five years after graduation are also in good standing 10 years after graduation. Also, most (75 percent) of those in deferment or forbearance in 1998 transition to good standing five years later. More than half (54 percent) of those in default in 1998 transition to good standing five years later. The general pattern indicates that repayment is more difficult early on, but many who face hardship repaying and even declare default eventually move to good standing. And once a college graduate is in good standing, there is a strong tendency for him or her to remain in that state.

A few words of caution are in order. These findings do not indicate that the risks are irrelevant because they are not necessarily permanent. Temporary and transition costs can be high for borrowers who may respond by underinvesting in their education. Moreover, the low persistence (and eventual reduction) in the fraction in deferment/forbearance may not be driven by younger workers finding a good job but instead because those mechanisms are designed only to temporarily help borrowers early on, and older borrowers cannot typically qualify for a deferment or forbearance. Supporting this view is the fact that the default rate is higher 10 years after graduation.

To summarize, this section reviews consistent evidence that new college graduates have a fairly higher incidence of unemployment and underemployment and lower earnings, relative to both contemporaneous older cohorts (from ACS data) and their own future (from B&B data). These findings are valid for all majors but to different degrees.[6] The section also provides evidence that new graduates seem to encounter more difficulties repaying their loans and that existing insurance devices such as deferments and forbearances are more widely used during the early postcollege years. These empirical patterns motivate the simple question of this article: What should be the optimal design of student loan programs given the risk of unemployment for recent graduates?

2 A SIMPLE MODEL OF UNEMPLOYMENT FOR RECENT GRADUATES

I analyze schooling investments in the presence of youth unemployment in a very stylized and tractable environment. In such an environment, I consider the best feasible arrangement in the presence of moral hazard and compare it with the best possible arrangement when incentive problems can be ruled out—that is, the so-called first-best allocations.

2.1 The Environment

Consider an individual whose life is divided into three periods or stages: early youth, $t = 0$; youth, $t = 1$; and maturity, $t = 2$. Schooling takes place in early youth, whereas labor market activity takes places during the youth and maturity periods. Preferences are standard. As of $t = 0$, each person's utility is driven by consumption in all periods $\{c_t\}_{t=0}^{2}$ and effort during youth e while forming human capital in school:

$$U = u(c_0) - v(e) + \beta E\left[u(c_1) + \beta u(c_2)\right],$$

where $0 < \beta < 1$ is a discount factor and $u(\cdot)$ is a standard increasing, twice continuously differentiable, strictly concave utility function for consumption streams c_t during $t = 0, 1, 2$. I assume that the utility has a constant relative risk aversion:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$. For most of the analysis, I assume $\sigma < 1$. For brevity, in some formulas I retain the use of $u(\cdot)$ and its derivative $u'(c)$. The expectation $E[\cdot]$ is over the possible employment outcomes.

To remain focused solely on youth unemployment, I consider an environment in which the only risk is whether a college graduate finds a job right after school (i.e., at $t = 1$) or has to wait until maturity ($t = 2$). For the stylized model, the exertion of effort e is relevant only for the first period at a disutility cost $v(\cdot)$, an increasing function. The probability of finding a job during the youth period—and thus avoiding unemployment altogether—is an increasing function $p(\cdot)$ of the effort e exerted by the person at the end of period $t = 0$.[7] Naturally, the function $p(\cdot)$ is bounded between (0,1); moreover, when effort e is treated as a continuous variable, $p(\cdot)$ is a strictly concave function.

The exertion of effort is a central aspect of my analysis. I consider the two leading assumptions in the literature of optimal contracts under moral hazard. In the first case, effort is a continuous variable $e \in [0,\infty)$ and the higher the effort, the higher the probability of finding a job right after school. In the second case, effort is binary, $e \in \{0,1\}$; that is, a person either works hard or not at all. If the former, the probability of finding a job is $0 < p_H < 1$. If the latter, the probability is $0 < p_L < p_H$. Here, the subscripts L and H stand for low effort and high effort, respectively.

In the first period, $t = 0$, the key investment is schooling. It is denoted by $h \geq 0$ and measured in consumption goods. Aside from h, the effort e must also be considered investments during early youth. Investments h enhance the person's ability a to generate output once employed during both postcollege periods, $t = 1$ and $t = 2$, while e is an investment in increasing the probability of realizing the young person's earnings potential. The initial ability a is given as of $t = 0$ and summarizes innate talents and acquired skills during the person's earlier years of childhood. Given ability a and investments h, the labor market income profile of the person is described as follows: For $t = 1$, the person can be employed or unemployed:

$$y_1 = \begin{cases} ah^{\alpha} & \text{if employed at } t=1 \\ b & \text{otherwise,} \end{cases}$$

and

$$y_2 = \begin{cases} Gah^{\alpha} & \text{if employed at } t=1 \\ ah^{\alpha} & \text{otherwise,} \end{cases}$$

for $t = 2$. In this setting, I assume that mature workers are always employed, so the only possible incidence of unemployment is for the young. Here $G \geq 1$ allows for the possibility of on-the-job training/learning by doing during period $t = 1$, thereby increasing the earnings for the second period. Also, $0 < \alpha < 1$, indicating decreasing returns to investment in education. Finally, $b \geq 0$ is a possibly nonzero unemployment minimum consumption level. I allow $b > 0$ only to ensure that the problem is well defined for complete financial autarky when $\sigma > 1$.

All individuals are initially endowed with positive resources, $W > 0$. These resources can be used at $t = 0$ to consume, invest in human capital h, or save for future consumption. This initial wealth can be seen as a family transfer, but for concreteness, here I assume that it cannot be made contingent on the employment regardless of whether the person finds a job at $t = 1$ or only at $t = 2$.

Finally, for simplicity, I assume that the relevant interest rate (i.e., the cost for lenders of providing credit) and the rate of return to savings are both equal to the discount rate. That is, the implicit interest rate is given by $r = \beta^{-1} - 1$.

3 THE FIRST-BEST

I now consider the extreme when markets are complete and allocations are the unrestricted optima. In the case of complete markets, any intertemporal and interstate (insurance) exchange can be performed. Unrestricted allocations indicate that any possible incentive problems can be directly handled and do not distort the intertemporal and insurance exchanges. Such allocations are useful as the efficient benchmark for the allocations from any other market arrangements and government policies.

In my environment, it suffices that the asset structure is one in which an individual's loan repayment is fully contingent on the labor market outcomes at $t = 1$. Let d_e and d_u denote the amount of resources that an individual at $t = 0$ commits to pay to lenders, which are assumed to be risk neutral and competitive. The amount d_e is the repayment due if a graduate finds a job right after school (hence the subscript e), an event that happens with probability $p(e)$. The amount d_u is the repayment the borrower must deliver (or receive) when $d_u < 0$ if he or she is unemployed after school, an event that happens with probability $1 - p(e)$. The borrower's effort is fully known (and controlled) by lenders, as are the probabilities for the two outcomes. As borrowers are risk neutral, they are willing to give the borrower $\beta p(e)d_e$ at $t = 0$ in exchange for a repayment d_e at $t = 1$ if the borrower finds a job right away. Similarly, a borrower would receive (or pay) an amount $\beta(1{-}p(e))d_u$ at $t = 0$ in exchange for d_u units at $t = 1$ if unemployed.

The sequential budget constraints for the individual are as follows: For $t = 0$, the first-period consumption c_0 and the investments in human capital h are financed by either initial wealth W or borrowing:

$$c_0 + h = W + \beta\left[p(e)d_e + (1 - p(e))d_u\right]. \tag{1}$$

For periods $t = 1$ and $t = 2$, the constraints for the consumption levels of young employed ($c_{e,1}$ and $c_{e,2}$) and young unemployed individuals ($c_{u,1}$ and $c_{u,2}$) are given by the present value constraints:

$$c_{e,1} + \beta c_{e,2} = ah^{\alpha} - d_e + \beta G a h^{\alpha} \tag{2}$$

and

$$c_{u,1} + \beta c_{u,2} = -d_u + \beta a h^{\alpha}. \tag{3}$$

Then, the optimization problem for the young person at $t = 0$ is choosing the investments in human capital h; the exertion of effort in successfully finding a job right after school e; the financial decisions d_e and d_u; the initial consumption c_0; and the contingent plan $c_{e,1}$, $c_{e,2}$, $c_{u,1}$, and $c_{u,2}$ to maximize

$$U(a,W) = \max_{\vec{c},d,h,e} u(c_0) - v(e) + \beta p(e)\left[u(c_{e,1}) + u(c_{e,2})\right] + \left(1 - p(e)p(e)\left[u(c_{u,1}) + u(c_{u,2})\right]\right),$$

subject to constraints(1), (2), and (3). Here $\vec{c}$ is the vector of all date-state consumption.

The solution to this problem is very familiar. First, it is straightforward from their first-order conditions (FOCs) that $c_{e,1}$ and $c_{e,2}$ must be equal to each other; the same applies for $c_{u,1}$ and $c_{u,2}$. Then, from (2) $c_{e,2} = c_{e,1} = (ah^{\alpha}(1 + \beta G) - d_e)/(1 + \beta)$ and from (3) $c_{u,1} = c_{u,2} = (ah^{\alpha}(1 + \beta G) - d_e)/(1 + \beta)$. Second, from the FOCs of d_e and d_u, it is easy to show that early consumption c_0 must be equal to all consumptions—that is, $c_0 = c_{e,2} = c_{e,1} = c_{u,1} = c_{u,2} = c$ for some consumption level c to be determined by the individual's wealth W, ability a, and actions h and e. As expected, complete markets lead to perfect consumption smoothing over time and across states of the world, constraints (1), (2), and (3) reduce, after some trivial simplification, to the present value constraint:

$$c\left(1 + \beta + \beta^2\right) = W - h + \beta\left[\beta + p(e)\left(1 + \beta(G - 1)\right)\right]ah^{\alpha}.$$

Solving this expression for the consumption level c, the optimal schooling investment problem for a young person is simplified to

$$U(a,W)=\max_{e,h}\left(1+\beta+\beta^2\right)u\left(\frac{W-h+\beta\left[\beta+p(e)\left(1+\beta(G-1)\right)\right]ah^{\alpha}}{\left(1+\beta+\beta^2\right)}\right)-v(e).$$

Given any e, schooling investments should maximize $W-h+\beta[\beta+p(e)(1+\beta(G-1))]ah^{\alpha}$, the person's lifetime earnings net of schooling costs. The FOC for this implies an optimal investment function:

$$h^*(a;e)=\left\{a\alpha\beta\left[\beta+p(e)\left(1+\beta(G-1)\right)\right]\right\}^{\frac{1}{1-\alpha}}. \tag{4}$$

Some simple but very important implications are evident for schooling. First, conditional on effort e, investments are independent of the individual's wealth W. This simple result is the basis for a large empirical literature on credit constraints and education.[8] Second, investments are always increasing in the person's ability a, exerted effort e, the probability of finding a job right after school $p(\cdot)$, and the gains from experience G. Efficient investments are driven by expected returns as risks are efficiently insured by lenders.

With the efficient investment (4), the maximized lifetime present value of resources for the young person is $W+(1/\alpha-1)h^*(a;e)$. In the continuous case, the optimal exertion of effort would be given by the condition[9]

$$\left(\frac{W+\left(1/\alpha-1\right)h^*(a;e)}{\left(1+\beta+\beta^2\right)}\right)^{-\sigma}h^*(a;e)\times\frac{p'(e)}{p(e)}=\alpha\left[\frac{\beta}{p(e)\left[1+\beta(G-1)\right]}+1\right]v'(e),$$

where I have already used $u'(c)=c^{-\sigma}$.

A wealth effect implies that the optimal exertion of effort decreases with initial wealth W. With respect to ability, the implied relationship is more complex. If the wealth effect is weaker than the substitution effect—that is, if $\sigma<1$—then the relationship between ability and effort is always positive. However, depending on the level of wealth W, if $\sigma>1$, a negative relationship would hold.[10]

The case of binary effort can be substantially simpler. For any ability a, there is always a threshold level $\bar{W}(a)$ for which wealth levels W above the threshold imply zero effort, $e=0$, since consumption is so plentiful that the marginal gains of exerting effort are low compared with the utility cost of $e=1$. For wealth levels below $\bar{W}(a)$ the optimal exertion of effort is $e=1$. The attained utility is given by

$$U^*(a,W)=\begin{cases}\left(1+\beta+\beta^2\right)u\left(\frac{W+\left(1/\alpha-1\right)h^*(a;1)}{\left(1+\beta+\beta^2\right)}\right)-v(1) & \text{if } W\le\bar{W}(a),\\ \left(1+\beta+\beta^2\right)u\left(\frac{W+\left(1/\alpha-1\right)h^*(a;0)}{\left(1+\beta+\beta^2\right)}\right)-v(0) & \text{otherwise.}\end{cases}$$

That is, whether $\bar{W}(a)$ is increasing or decreasing is governed by whether or not $\sigma<1$.

3.1 Discussion: First-Best Allocations and Repayment

Except for the determination of effort, the first-best allocation is familiar to most readers. It is worthwhile, however, to reexamine its implications for consumption and investment, especially for borrowing and repayment of credit at $t = 0$, some of which is used to invest in college.

First, the optimal investment in schooling is determined entirely by the individual's ability a and effort e. While family wealth W may be correlated with a person's ability and may influence his or her exertion of effort, it does not have a direct impact on schooling per se.

Second, there is full insurance in consumption. The ability to (i) borrow and lend without restriction and (ii) differentiate the payments d_e and d_u, based on whether the student finds a job right after school, allows for perfect insurance. Perfect insurance is the equilibrium and optimal allocation since, in this environment, there is no reason to distort consumption over time and states of the world.

Implementing the first-best allocations may require large and stark transfers of resources between borrowers and lenders. First, a high-ability individual with low or no family wealth would need to borrow heavily at $t = 0$. In expectation, the lender would require a large net repayment, but this repayment would be very different depending on labor market outcomes. On the one hand, if the person fails to find a job, the lender might be required to make a transfer to the unemployed person. The optimal loan policy requires $d_u < 0$ (i.e., the lender making a payment to the student).

In a first-best world, therefore, student loan programs must also include an unemployment insurance mechanism. This type of unemployment insurance would differ greatly from conventional ones, since it is inherently associated with school investments, not previous employment. Also, note that existing mechanisms in U.S. student loan programs fall short of the insurance provisions in this environment. In practice, U.S. students can delay repayments using either deferments and/or forbearance for several different reasons. Moreover, students can opt for an income-contingent repayment program that fixes or limits the repayments to a fraction of their earnings. These provisions fall short of the prescriptions of the first-best allocations that might require compensation from the lender to the borrower, not just a limit or postponement of the payments from the borrower to the lender.

On the other hand, if the student finds a job right after school, then loan repayment must be expected over periods $t = 1$ and $t = 2$. This repayment is larger, in present value terms, than the original loan because it has to cover the expected cost of the unemployment insurance payments. Indeed, the high repayment of those loans can be a serious deterrent to student credit because of limited commitment and enforcement. This aspect has been extensively studied by itself (Lochner and Monge-Naranjo, 2011, 2012) and in relation to its interactions with other frictions (Lochner and Monge-Naranjo, forthcoming).

In the next two sections, I focus on moral hazard problems underlying the specific problem of finding a job right after school. Dealing with this problem explicitly provides guidance on how to optimally design repayment (and granting) of student loans.

4 OPTIMAL LOANS WITH MORAL HAZARD

A key assumption in the first-best allocation is the ability of the lenders to directly recommend and control the effort exerted by the borrowers. Since there is full insurance for the borrower, the optimal arrangement results in lenders being the only party really interested in the student finding a job right after school. Not surprisingly, a moral hazard problem arises if the student's effort cannot be perfectly monitored. In this section and in the next, I assume that effort cannot be observed or controlled by the lender in any way.

Under these circumstances, a contract that offers full insurance over youth unemployment is destined to fail unless it is based on the student exerting the minimum effort e to find a job. Under either continuous or discrete effort choice, $V_e = V_u$ unambiguously implies zero effort; $e = 0$. Clearly, to induce any positive effort, $e > 0$, it is necessary to break full insurance coverage by imposing $V_e > V_u$. Furthermore, the larger the difference between V_e and V_u, the larger would be the effort exerted by the student. This is called an incentive compatibility constraint (ICC) in any optimal arrangement with hidden action.

The optimal credit arrangement (and student loan contract) for a person with given ability a and family resources W maximizes the lifetime expected utility of the agent by choosing consumption levels for all dates and states of the world and investments in human capital and effort such that lenders break even (BE) in expectation and the ICC ensures that the borrower exerts the effort expected by the lender.

For brevity, in this section I consider only the discrete case. I set up the problem in terms of future utilities for the borrower, V_e and V_u, for students finding a job right after school or only at maturity. While this formulation requires additional explanation, it is very useful in Section 5, where I extend the model to a multiperiod setting.

First, note that if $e = 0$ is the effort sought by the lender, then the optimal contract is simply the first-best allocation described previously. Next, consider the optimal allocation if high effort, $e = 1$, is sought by the lender to maximize the chances of employment. Then, the optimal contract is given by the following loan program:

$$U^*(a,W;e=1) = \max_{c_0,h,V_e,V_u} u(c_0) - v(1) + \beta\left[p_H V_e + (1-p_H)V_u\right],$$

subject to the ICC that the expected gains from exerting effort more than compensate the cost

$$\beta(p_H - p_L)(V_e - V_u) > v(1) - v(0)$$

and subject to the BE constraint

$$-c_0 - h + W + \beta\left\{p_H\left[ah^\alpha(1+\beta G) - C(V_e)\right] + (1-p_H)\left[\beta ah^\alpha - C(V_u)\right]\right\} \geq 0,$$

where $C(\cdot)$ is the cost function that determines the cost for the lender to deliver a continuation utility level for the borrower. The function $C(\cdot)$ is strictly increasing and strictly convex as it is derived from the inverse of $u(\cdot)$. For constant relative risk aversion (CRRA) preferences, I can explicitly derive the form of this cost function as[11]

$$C(V)=(1+\beta)\left[\frac{(1-\sigma)V}{(1+\beta)}\right]^{\frac{1}{1-\sigma}}.$$

There are many alternative formulations of this principal-agent problem, all of which are equivalent. For example, in this case the lender takes possession of the labor earnings and commits to deliver continuation value V_e, letting $\mu \geq 0$ denote the Lagrange multiplier for the ICC and $\lambda > 0$ denote the multiplier for the BE constraint.[12]

After some straightforward simplifications, the FOCs that characterize the optimal allocations are

$$[c_0]: u'(c_0)=\lambda;$$
$$[V_e]: \lambda C'(V_e)=1+\mu\left(1-\frac{p_L}{p_H}\right);$$
$$[V_u]: \lambda C'(V_u)=1-\mu\left[1-\left(\frac{1-p_L}{1-p_H}\right)\right];$$
$$[h]: \beta\left[p_H(1+\beta G)+(1-p_H)\right]\alpha a h^{\alpha-1}=1.$$

These simple conditions deliver a very sharp set of implications for the behavior of consumption over time and across states and for investments in human capital h. First, note that $C'(V)=\left[\frac{(1-\sigma)V}{(1+\beta)}\right]^{\frac{\sigma}{1-\sigma}}$. Second, in this environment the cheapest way to deliver the continuation utilities V_e and V_u is with different, but constant over time, consumption flows c_e and c_u; that is, $V_e=(1+\beta)\frac{c_e^{1-\sigma}}{1-\sigma}$ and $V_u=(1+\beta)\frac{c_u^{1-\sigma}}{1-\sigma}$. Then, $C'(V_e) = (c_e)^{\sigma} = 1/u'(c_e)$ and $C'(V_u) = (c_u)^{\sigma} = 1/u'(c_u)$. In this case, the result was directly obtained because there was a closed form for $C(\cdot)$. However, the result that $C'(V') = 1/u'(c')$ holds much more generally as a direct application of the envelope condition on the value function $C(\cdot)$. Then, the first three FOCs can be succinctly summarized as

$$u'(c_0)=\left[1+\mu\left(1-\frac{p_L}{p_H}\right)\right]u'(c_e), \tag{5}$$

$$u'(c_0)=\left[1-\mu\left(1-\frac{1-p_L}{1-p_H}\right)\right]u'(c_u). \tag{6}$$

First, note that if the ICC does bind (i.e., when $\mu > 0$), the term in brackets in the first expression is necessarily greater than 1; the opposite holds for the second expression. Obviously, this implies that $u'(c_e) < u'(c_0) < u'(c_u)$ and, because of strict concavity, these inequalities necessarily imply the following ordering for the consumption levels of initially employed c_e and initially unemployed c_u graduates, relative to their consumption while in school:

$$c_u < c_0 < c_e.$$

The crucial inequality is $c_u < c_e$, which is needed for the student to find it optimal to exert costly effort to find a job right after graduation. The inequalities with respect to c_0 are driven by my assumption that the implicit interest rate coincides with the borrower's discount factor. Of more interest, note that the wedges between consumption levels are determined by the ratio p_L/p_H, the likelihood ratio of finding a job without exerting any effort versus exerting it, and $(1 - p_L)/(1 - p_H)$, the likelihood ratio of not finding a job while not exerting effort versus not finding a job while exerting effort. When either (i) the ICC is not binding ($\mu = 0$) because effort is costless, $v(1) = v(0)$, or because the low effort is pursued by the credit arrangement or (ii) failing to find a job is uninformative about the effort exerted by the borrower, $p_L = p_H$, then consumption will be perfectly smooth. Otherwise, consumption will always be shifted in favor of those who find a job right after school as a mechanism to reward the exertion of effort.

Finally, despite the presence of moral hazard, there is a stark implication for the investment levels. Conditional on effort e, the investment in human capital h is always at the efficient level $h^*(a,e=1)=\left\{\beta\left[p_H(1+\beta G)+(1-p_H)\beta\right]\alpha a\right\}^{\frac{1}{1-\alpha}}$. That is, as long as the contract arrangement finds it optimal to request a high level of effort, the distortions on consumption not only can deliver the optimal investment, but also find it optimal to do so. The result, however, is conditional only on effort. In general, the sets of abilities a and family wealth levels W for which the individual exerts high effort are different in the first-best allocations than when moral hazard is a concern.

4.1 Implied Credit and Repayments

I have solved the optimal contract formulating the problem in what is called the *primal approach*—that is, looking at the allocations of consumption and human capital while imposing the ICC and the BE constraint for the lender. Underlying these constrained optimal allocations are transfers of resources—credit and repayment—between the borrower and the lender.

Let us first consider transfers when no effort is required (i.e., $e = 0$). As indicated previously, the ICC is not implemented, $\mu = 0$, and the perfect smoothing in consumption holds. The investments in schooling are

$$h_L^*(a)=\left\{\beta\left[p_L(1+\beta G)+(1-p_L)\beta\right]\alpha a\right\}^{\frac{1}{1-\alpha}},$$

where the subscript L stands for low effort, $e = 0$.

Initial wealth W implies a total lifetime of net resources $W + (1/\alpha - 1)h_L^*(a)$ and constant consumption levels $c_L^*(a,W) = [W + (1/\alpha - 1)h_L^*(a)]/(1 + \beta + \beta^2)$. With this information, at $t = 0$, an amount of borrowing $b_L(a,W)$

$$b_L(a,W)=\left(\frac{1/\alpha+\beta+\beta^2}{1+\beta+\beta^2}\right)h_L^*(a)-\left(\frac{\beta+\beta^2}{1+\beta+\beta^2}\right)W,$$

is needed to consume and invest $c_L^*(a,W) = h_L^*(a)$. Borrowing is always decreasing with own resources W. It is increasing with ability a not only because students with higher ability demand more current resources to invest, but also because higher future incomes are expected. Borrowing can be negative (savings) if the person is rich relative to his or her ability—that is, high W and/or low a.

The repayment of this loan is structured as follows: If the person is employed at $t = 1$, then he or she must make a repayment

$$p_{L,1}^e(a,W) = a\left[h_L^*(a)\right]^\alpha - \frac{\left[W + (1/\alpha - 1)h_L^*(a)\right]}{1+\beta+\beta^2}$$

and another one equal to

$$p_{L,2}^e(a,W) = Ga\left[h_L^*(a)\right]^\alpha - \frac{\left[W + (1/\alpha - 1)h_L^*(a)\right]}{1+\beta+\beta^2}$$

at $t = 2$. However, with competitive post-schooling borrowing and lending markets, the timing of the payments does not matter as long as the present value of the repayments as of $t = 1$ is equal to a debt balance equal to

$$D_L^e(a,W) = (1+\beta G)a\left[h_L^*(a)\right]^\alpha - \frac{1+\beta}{1+\beta+\beta^2}\left[W + (1/\alpha - 1)h_L^*(a)\right].$$

In contrast, if the person remains unemployed right after college, he or she would receive a transfer from the lender. The repayment, $p_{L,1}^u(a,W)$, would be negative:

$$p_{L,1}^u(a,W) = -\frac{\left[W + (1/\alpha - 1)h_L^*(a)\right]}{1+\beta+\beta^2};$$

as for $t = 2$, the payments would be

$$p_{L,2}^u(a,W) = a\left[h_L^*(a)\right]^\alpha - \frac{\left[W + (1/\alpha - 1)h_L^*(a)\right]}{1+\beta+\beta^2}.$$

Note that these payments incorporate not only an element of insurance, but also the returns to the person's own initial wealth W.

Now, consider the more interesting case in which high effort ($e = 1$) is induced. For brevity, I use the subscript H here and define

$$M \equiv \left[1 + \mu\left(1 - \frac{p_L}{p_H}\right)\right]^{\frac{1}{\sigma}} > 1$$

and

$$m \equiv \left[1 - \mu\left(1 - \frac{1-p_L}{1-p_H}\right)\right]^{\frac{1}{\sigma}} < 1.$$

From the FOCs (5) and (6), it can be verified that $c_e = Mc_0$ and $c_u = mc_0$, so what remains is the determination of $c_{0,H}(a,W)$, the initial consumption of someone who exerts high effort in school. Recall that (i) the first-best level of investment for $e = 1$ attains in this case and is equal to $h_H^*(a) = \left\{\beta\left[p_H(1+\beta G)+(1-p_H)\beta\right]\alpha a\right\}^{\frac{1}{1-\alpha}}$ and (ii) the net present value of lifetime resources at risk-neutral prices is $W + (1/\alpha - 1)h_H^*(a)$. Then, the budget constraint for the consumption profiles of the borrower is reduced to

$$c_0 + \beta\left[(1+\beta)p_H c_e + (1+\beta)(1-p_H)c_u\right] = W + (1/\alpha - 1)h_H^*(a)$$

or

$$c_0\left[1+\beta(1+\beta)\left(p_H M + (1-p_H)m\right)\right] = W + (1/\alpha - 1)h_H^*(a).$$

Thus, the terms m and M act as shifters in the relative probability weights for the consumption levels of individuals who find employment early and late, respectively. Since $m < 1 < M$, the shift increases the implicit probability weight for early employment, and since $c_e > c_0$ and $c_u < c_0$, the initial consumption c_0 is *lower* than in the first-best. In any event, the solutions for the consumption levels are

$$c_{0,H}(a,W) = \frac{1}{1+\beta(1+\beta)\left[m + p_H(M-m)\right]}\left[W + (1/\alpha - 1)h_H^*(a)\right],$$

$$c_{e,H}(a,W) = \frac{M}{1+\beta(1+\beta)\left[m + p_H(M-m)\right]}\left[W + (1/\alpha - 1)h_H^*(a)\right],$$

$$c_{u,H}(a,W) = \frac{m}{1+\beta(1+\beta)\left[m + p_H(M-m)\right]}\left[W + (1/\alpha - 1)h_H^*(a)\right].$$

Therefore, the borrowing and repayments for a person exerting effort $e = 1$ are as follows: At $t = 0$, the consumption $c_{0,H}(a,W)$ and investments $h_H^*(a)$ require borrowing

$$b_H(a,W) = \frac{\frac{1}{\alpha}h_H^*(a) + \beta(1+\beta)\left[m + p_H(M-m)\right]\left[h_H^*(a) - W\right]}{1+\beta(1+\beta)\left[m + p_H(M-m)\right]}.$$

Similar relationships with respect to a and W hold as in the case with low effort. However, note that the levels are different. First, high effort $e = 1$ implies higher investments, $h_H^*(a) > h_L^*(a)$. From here, for any W and a, borrowing with high effort is higher—potentially much higher—than borrowing with low effort. Second, a more subtle mechanism is at work: The weights for consumption with low effort are simply $1 + \beta + \beta^2$, but once the ICC is imposed to incentivize effort, it is $1 + \beta(1 + \beta)[m + p_H(M - m)]$. The difference between the two, of course, is determined by the magnitude of the multiplier μ.

With respect to repayments, a person who finds employment at $t = 1$ and signed a contract that implements high effort must repay

$$p^e_{H,1}(a,W) = a\left[h^*_H(a)\right]^\alpha - c_{e,H}(a,W)$$

at $t = 1$ and

$$p^e_{H,2}(a,W) = Ga\left[h^*_H(a)\right]^\alpha - c_{e,H}(a,W)$$

at $t = 2$. As before, with competitive post-school borrowing and lending markets, the timing of the payments is immaterial as long as the $t = 1$ present value of payments from $t = 1$ and $t = 2$ adds up to

$$D^e_H(a,W) = (1+\beta G)a\left[h^*_H(a)\right]^\alpha - (1+\beta)c_{e,H}(a,W).$$

On the contrary, if the borrower remains unemployed after school, then the optimal contract would prescribe a *negative repayment* for $t = 1$:

$$p^u_{H,1}(a,W) = -c_{u,H}(a,W).$$

Unambiguously, the lender would have to transfer additional resources to pay for the consumption of the unemployed borrower. For the second period, the payments must be

$$p^u_{H,2}(a,W) = a\left[h^*_H(a)\right]^\alpha - c_{u,H}(a,W),$$

which might be positive. In any event, as before, with postcollege borrowing and lending, the key is that the debt balance is set to

$$D^u_H(a,W) = \beta a\left[h^*_H(a)\right]^\alpha - (1+\beta)c_{u,H}(a,W),$$

which might be positive or negative.

The optimal contract has clear prescriptions for the level of credit granted and the investments made by different individuals according to their initial wealth and ability. According to these characteristics, a level of effort would be implemented and repayments would be set contingent on whether the individual finds a job right after college. Such repayments necessarily entail elements of insurance, and unless the initial wealth W is very high, the lender must end up making a net transfer for the unemployed. While successful individuals—that is, those who are employed right after school—are required to make larger repayments, the repayments are limited by the need to reward effort in the first place.

I wish to comment on simple yet stark implications of my analysis. First, by the proper structure of consumption distortions, the optimal contract attains the first-best investment in human capital. Once ability and effort are controlled for, the investment of individuals should be independent of family wealth and background. Second, the structure of repayments is very different from the existing mechanisms of deferments, forbearances, and the emerging income-contingent repayment loans that limit the repayment to a fraction of the income accrued and, therefore, forgive debts of the unemployed. The optimal contract can indeed go beyond this by prescribing an unemployment insurance transfer.

4.2 Discussion: Incentives Versus Insurance

Hidden action problems while studying in college or later, when searching for a job, lead to a form of the proverbial trade-off between insurance and incentives. On the one hand, consider the case in which lenders, including the government, ignore the incentive problems. Aiming for the best competitive contract in their perceived environment, lenders would offer the first-best allocations. If, however, *none* of the borrowers, not a single one—regardless of their wealth W or ability a—would exert effort, the unemployment rate of this cohort of students would be $1 - p_L$, possibly much higher than the lenders' expectations. This higher unemployment rate would also be the culprit behind financial losses for lenders, as they receive lower repayments from the fewer borrowers who found employment and make higher transfers to those who remained unemployed. Over time, the lack of self-sustainability implies that private lending would simply disappear and government lending programs would have consistent deficits that should be financed with taxes.

On the other hand, credit arrangements that ignore the desire for insurance would be suboptimal, not only because of the lack of consumption insurance, but also because of suboptimal investments in human capital. For instance, consider a credit arrangement that requires the same repayment regardless of whether the person finds work. Trivially, unless there is a post-schooling credit market, the only arrangement is the autarkic one discussed previously since unemployed youth are unable to repay anything in this environment. With post-schooling credit markets, the set of student loans with constant repayments across youth unemployment or employment is significantly larger. But in any such case, the consumption across states of the world would be disrupted, especially for those with high ability and low family wealth, who might have invested and borrowed larger amounts. In response, the borrowing and investment in human capital of those individuals could be severely limited. See the appendix for further discussion.

Interestingly, in this environment under the optimal contract, the proper balance between incentives and insurance enables efficient investments in human capital. This efficiency result is conditional on effort and, thus, the distortions are on consumption and/or the exertion of effort, but, given the latter, not on the schooling investment level. For this efficiency result, the optimal contract arrangement may require additional transfers from the lender to the borrower in case of unemployment, and if so, high payments in case of employment. However, the efficient repayment must always reward the consumption levels of the successfully employed relative to those who are unemployed after school.

5 MULTIPERIOD UNEMPLOYMENT SPELLS

The stylized three-period model analyzed earlier (see Section 2.1) assumes that all college-educated workers eventually find employment. Unemployment may be avoided, but if it happens, it is only at the beginning of the labor market experience of college-educated workers. Within this model, I derived some basic implications for the optimal design of student loans and their repayment. In particular, I found that the efficient investment levels can be delivered as long as the repayment of the loans is designed with the proper balance between incentives and insurance. I also highlighted the fact that the optimal student loan program should incorporate an unemployment insurance mechanism as an integral component.

In this section, I explore an important aspect of such an unemployment insurance mechanism. To do so, I need to incorporate into the model the fact that finding a job may require repeated search efforts. The unemployment insurance mechanism should provide the right incentives and not dissuade new graduates from seeking employment. To explore this important dimension, in this section I change the specification for the post-schooling lifetime. Specifically, I now consider an environment in which the labor market participation is open-ended—that is, I consider a simple infinite horizon model for the job search after school. This version of the model is a simplification of the already simple and well-known Hopenhayn-Nicolini (1997) model of unemployment insurance. The model is highly stylized; in Section 6 I discuss a number of life cycle issues that would be desirable to explore.

Consider the following environment. As before, knowing a young person's ability a and initial wealth W, the person must decide how much to invest in human capital h and how much effort e to exert. Ability and schooling investments determine the income $y = ah^{\alpha}$ of a worker *once he or she is employed*. Similarly, a level of effort e during school determines the student's probability of finding a job right after school. Otherwise, if the student finishes school with no job, he or she can also search for employment after school. For concreteness, I restrict attention to the discrete case, $e \in \{0,1\}$; that is, where the student exerts effort either fully or not at all.[13] For simplicity, as in Hopenhayn-Nicolini (1997), I assume that once a worker has found a job, regardless of whether it is right after school, the worker will remain employed forever. Then, if unemployment happens, it is in the initial postcollege periods. The length of this initial unemployment spell is endogenous.

The time interval length for schooling differs from the length of each period in the labor market. College can be considered a period of four years; however, for modeling the job search a more meaningful breakdown of time is biweekly or monthly time intervals. To this end, the discount factor between the flow utility and the continuation utility for $t = 0$ is β, as before.[14] For all subsequent periods, the discount factor between the current flow and continuation utilities is $\delta \in (\beta,1)$. Likewise, the probability of getting a job while still in school is p_H if $e = 1$ or p_L if $e = 0$. After school, if the graduate is unemployed, the probabilities of finding a job are q_H or q_L, depending on the effort. Finally, the costs of exerting effort are given by the increasing functions $v(e)$ and $s(e)$ for the periods $t = 0$ and $t \geq 1$, respectively. For simplicity, I normalize the search costs so that $s(0) = 0$. In any event, the expected lifetime utility of a person as of $t = 0$ is given by

$$U = u(c_0) - v(e_0) + \beta E\left[\sum_{t=1}^{\infty}\delta^{t-1}\left[u(c_t) - s(e_t)\right]\right],$$

where $E[\cdot]$ is the expectation over consumption and search efforts and $u(\cdot)$ is the CRRA utility function used throughout the article.

Relative to the stylized model of previous sections, the enhanced search dynamics in this environment do not add anything interesting unless there is an incentive problem. All individuals would be fully insured in their consumption levels regardless of the length of unemployment spells. Conditional on search effort levels, which would depend on ability a and family wealth W, the investments in human capital h would maximize expected lifetime earnings.

Much more interesting is the behavior of consumption, effort, and investments when effort is a hidden action. I now explore the optimal contract with moral hazard, in which effort, if desirable, must be induced by the proper ICC. My treatment of the problem in the previous section can be conveniently adapted to the current environment: Consider the utility optimization of a student with ability a and wealth W, for whom the lenders aim to induce high effort. The optimization problem is very similar to the previous case:

$$U^*(a,W;e=1)=\max_{c_0,h,V_e,V_u} u(c_0)-v(1)+\beta\left[p_H V_e+(1-p_H)V_u\right],$$

subject to exactly the same ICC as before:

$$\beta(p_H-p_L)(V_e-V_u)>v(1)-v(0),$$

and to a different BE constraint:

$$-c_0-h+W+\beta\left\{p_H\left[\frac{ah^\alpha}{1-\delta}-\frac{\left[(1-\sigma)V_e\right]^{\frac{1}{1-\sigma}}}{1-\delta}\right]+(1-p_H)\left[-C\left(V_u,ah^\alpha\right)\right]\right\}\geq 0.$$

The first term inside the square brackets is the next payoff for the lenders if the student is employed right away. Here $\frac{ah^\alpha}{1-\delta}$ is the present value of the resources generated by the individual, while the term $\frac{\left[(1-\sigma)V_e\right]^{\frac{1}{1-\sigma}}}{1-\delta}$ is the present value of the cost of providing a (constant) consumption flow that delivers a lifetime utility equal to V_e. The second bracketed term, $-C(V_u,ah^\alpha)$, is the cost of delivering an expected continuation utility V_u for an unemployed person whose earnings potential, once employed, is equal to ah^α every period.

The cost function $C(V_u,ah^\alpha)$ is given by the following Bellman equation:

$$C\left(V_u,ah^\alpha\right)=\min\left\{C\left(V_u,ah^\alpha;e=0\right),C\left(V_u,ah^\alpha;e=1\right)\right\};$$

that is, the lowest cost between loan programs with low and high job search efforts. These programs are defined as follows: If the program does not require effort, the minimization is

$$C\left(V,ah^\alpha;e=0\right)=\min_{c,V_e',V_u'}\left\{c+\delta\left[q_L\left(\frac{ah^\alpha}{1-\delta}-\frac{\left[(1-\sigma)V_e'\right]^{\frac{1}{1-\sigma}}}{1-\delta}\right)+(1-q_L)C\left(V_u',ah^\alpha\right)\right]\right\},$$

subject to the promise-keeping constraint

$$V=u(c)+\delta\left[q_L V_e'+(1-q_L)V_u'\right].$$

If the program requires effort, the minimization is

$$C\left(V, ah^{\alpha}; e=1\right) = \min_{c, V_e', V_u'} \left\{ c + \delta \left[q_H \left(\frac{ah^{\alpha}}{1-\delta} - \frac{\left[(1-\sigma)V_e'\right]^{\frac{1}{1-\sigma}}}{1-\delta} \right) + (1-q_H) C\left(V_u', ah^{\alpha}\right) \right] \right\},$$

subject to the promise-keeping constraint

$$V = u(c) - s(1) + \delta\left[q_H V_e' + (1-q_H) V_u'\right],$$

and the ICC:

$$\delta(q_H - q_L)(V_e' - V_u') \geq s(1).$$

A number of interesting implications arise from this program. First, the low-effort level $e = 0$ would be optimal for very high and very low levels of V. On the one hand, for very high levels of utility V, the cost of compensating the agent for exerting effort might be too high in terms of consumption goods, given the decreasing marginal utility of consumption. That is, inducing effort from the very rich might be too costly. On the other hand, for very low levels of utility, if the coefficient of relative risk aversion $\sigma < 1$, then a low level of utility V may leave little room to generate the needed gap between V_e' and V_u' to generate effort. That is, it is very difficult to induce the very poor to exert the costly effort $s(1)$ since for them, there might be little to gain.

In either of these cases, from the FOCs of V_e' and V_u' and the envelope condition on $C(V, ah^{\alpha}; e = 0)$, I can show that

$$V_e' = V_u' = V;$$

that is, the person's continuation utilities would be the same as the initial one, regardless of whether he or she finds a job. In both extremes of utility entitlement levels, as long as a young person remains unemployed, he or she would receive a constant unemployment compensation (UE) equal to

$$UE(V) = u^{-1}\left[(1-\delta)V\right].$$

Upon obtaining a job, the person must repay a constant payment (p) to the lender equal to

$$p = ah^{\alpha} - u^{-1}\left[(1-\delta)V\right].$$

More interesting is the case when high effort $e = 1$ is implemented. In this case, the ICC requires that $V_e' > V_u'$. Indeed, the ICC will hold with equality because the borrower is risk averse and the lender is risk neutral, so $V_e' = V_u' + \frac{s(1)}{\delta(q_H - q_L)}$. Using the envelope condition on $C(V, ah^{\alpha}); e = 1)$, it can be shown that

$$V_u' < V.$$

That is, if the person fails to find a job in this period, his or her continuation utility will be lower next period. The optimal program prescribes a decreasing sequence of utilities $\{V_t\}$ for as long as the person remains unemployed.

Here I emphasize that both continuation utilities for those employed and those unemployed, V_e' and V_u', respectively, decline over time. Therefore, unemployment compensation declines with each period the person fails to find a job until, possibly, hitting a threshold after which no effort is induced. But the fact that V_e' also declines with the unemployment spell indicates that the amount to be repaid forever after finding a job has to be increasing; that is,

$$p_t = ah^{\alpha} - u^{-1}\left[(1-\delta)V_t\right]$$

is increasing as V_t declines. As shown by Hopenhayn and Nicolini (1997), most of the insurance and efficiency gains arise not only from a declining path in the unemployment compensation transfers, but also from an earnings tax that grows with the length of time a worker needs to finds a job.

In sum, once I incorporate the possibility of multiperiod search with hidden action, the optimal arrangement extends the results of the previous section. Although the optimal arrangement still implies unemployment insurance right after school, over the unemployment spell there is a clear downward trend in unemployment transfers and an upward trend in the flow of payments once a job is found. The first result is well known and the second is highlighted by Hopenhayn and Nicolini (1997) as a crucial aspect in improving the provision of insurance and incentives.

Adapting some aspects of the optimal declining unemployment compensation/increasing debt repayments may lead to substantial gains in the efficiency of human capital investments and welfare generated from student loan programs. Unfortunately, to my knowledge, the use of variations in the balance of the debt as a function of the length of the initial unemployment (or in richer models, in any subsequent spell) is not a feature of any current student loan programs.

CONCLUSION

Unemployment is a major risk for college investment decisions. Even if a college graduate eventually finds a job that matches his or her qualifications—thereby enabling the repayment of loans—the possibility of long spells of unemployment, underemployment, and difficulty repaying student loans may limit and even dissuade productive investments in human capital.

In this article, I explored the optimal design of student loans in the face of a higher incidence of unemployment for the earlier periods of a person's labor market experience. Using a highly stylized model, I derived three main conclusions. First, the optimal student loan program must incorporate an unemployment compensation mechanism as a key element, even if unemployment probabilities are endogenous and subject to moral hazard. Second, even under moral hazard, a well-designed student loan program can deliver efficient levels of investments. Distortions in consumption may remain, but the labor market potential of any individual, regardless of family background, should not be impaired as long as he or she is willing to put forth the effort. Third, the provision of unemployment benefits and debt balances must be set as functions of the unemployment spell to provide the right incentives for youth to seek employment.

These conclusions are valid for all forms of college education investment. However, the brief data exploration in this article uncovered substantial differences across college majors in terms of unemployment, underemployment, and levels and growth of earnings after graduation. In principle, the schooling costs of the different majors can also vary widely, partly because the effective time for graduation in different majors can vary but also because the cost of equipment and the salaries of professors and other instructors can vary widely. A natural extension of the analysis presented here would quantitatively examine how these differences should translate into the credit provision and repayment schedules for students opting for various majors. A subsequent step would be to examine the implications for income distribution and social mobility of reforming the current student loan system for schemes that efficiently cope with the different incentive problems involved in financing a country's higher education. Pursuing this line, of course, would require accounting for the general equilibrium implications of such a policy change, including the labor market tightness and job-finding rates for graduates in all the different fields. ■

APPENDIX: A WORLD WITH NO STUDENT LOANS

As additional benchmarks, in this appendix I explore the allocations when student loans are not available. I first consider the case in which there are no financial markets at all. Then, I consider the case in which financial markets are available for the post-schooling stage for those active in labor markets.

Case 1: Complete Autarky

Schooling investment and effort levels are the choices to be made by a young person in the first period, $t = 0$. Since no financial markets are available, the first-period consumption is simply $c_0 = W - h$; that is, initial resources minus investments h. After school, the inability to transfer resources across periods implies that consumption would always be equal to labor earnings. Then, the optimal schooling choices solve the problem

$$\max_{e,h} u(W-h)-v(e)+\beta\left\{p(e)\left[u\left(ah^{\alpha}\right)+\beta u\left(Gah^{\alpha}\right)\right]+\left(1-p(e)\right)\left[u(b)+\beta u\left(ah^{\alpha}\right)\right]\right\}.$$

Given an effort level e and the CRRA utility function, the optimal investment $h(a,W;e)$ is uniquely pinned down by the FOC[15]:

$$(W-h)^{-\sigma}=\beta\alpha a^{1-\sigma}\left[p(e)\left(1+\beta G^{1-\sigma}\right)+\left(1-p(e)\right)\beta\right]h^{\alpha(1-\sigma)-1}.$$

Given the lack of credit markets, schooling investments are limited by the individual's own wealth W; it is straightforward to show that schooling investments h are always increasing in W. Similarly, schooling investments are always increasing in the exertion of effort e, as it increases the probability of youth employment, with the potential added benefit of the labor market experience gains G.

From the point of view of efficiency, perhaps the most relevant relationship is the one between an individual's ability a and investments in schooling h. In this environment, this relationship is entirely driven by the opposing *wealth effects* (a smarter individual has more future earnings for the same investment) and *substitution effects* (a smarter individual gains more future income for each additional investment). The relative strength of these effects is governed by $1/\sigma$, the intertemporal elasticity of substitution (IES). If $\sigma < 1$, the IES is large and a positive relationship between ability and investment arises. The more problematic case, in terms of efficiency and also relative to empirical evidence, is when $\sigma > 1$, which is the most common condition for quantitative work. In such a case, the IES is low and a negative relationship between a and h is implied by the model. Here, the smarter a person is—that is, the higher a—the higher would be his or her future consumption for any investment level. In addition, his or her optimal investment would be lower to enhance consumption at time $t = 0$, as it is entirely limited by W. It is worth noting the relationship between schooling investment h and the gains from labor market experience G. These results are explored further in Lochner and Monge-Naranjo (2011).

In the case of $\sigma = 1$ (i.e., log preferences), the wealth and substitution effects cancel each other, leading to a simple formula:

$$h = \frac{\beta\alpha\left(p(e)+\beta\right)}{1+\beta\alpha\left(p(e)+\beta\right)}W;$$

that is, investment should always be a constant fraction of the individual's wealth. That fraction depends on (i) the discount factor, (ii) the elasticity of income relative to investment α, and (ii) the individual's exerted effort in school and the early labor market e, but not on his or her ability a or experience gains G.

Last but not least, the optimal exertion of effort is determined by the difference in the value of the career of a young person who is employed right after school and the value of the career of a young person who is unemployed during his or her youth and works only during maturity. Let V_e and V_u denote, respectively, these post-schooling labor market career values:

$$V_e = u\left(ah^{\alpha}\right) + \beta u\left(Gah^{\alpha}\right)$$

and

$$V_u = u(b) + \beta u\left(ah^{\alpha}\right).$$

Then, the optimal exertion of effort is as follows. If effort is a continuous variable, $e \in [0,\infty)$, the optimal is given by

$$\nu'(e) = \beta p'(e)\left[V_e - V_u\right]. \tag{7}$$

That is, the higher the *absolute gap* between the two career outcomes, the more effort the individual would exert in seeking employment right after school. In the alternative case where effort is a discreet choice—that is, $e \in \{0,1\}$—for brevity, let $p_H = p(1)$ and $p_L = p(0)$. Obviously, $0 \leq p_L < p_H < 1$ and the optimal exertion of effort would be given by

$$e = \begin{cases} 1 & \text{if } \beta\left[(p_H - p_L)(V_e - V_u)\right] \geq \nu(1) - \nu(0), \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

That is, a young person would be interested in assuming the higher cost of effort, $\nu(1) - \nu(0)$, only if the returns of that effort are more than compensated by the expected career gains, $\beta[(p_H - p_L)(V_e - V_u)]$. Here β captures the fact that (i) the gains occur in the future, (ii) $p_H - p_L$ is the increased probability of finding a job if effort is exerted, and (iii) $V_e - V_u$ is the net gain of being employed.

Case 2: Post-Schooling Borrowing and Lending

Credit cards, auto loans, mortgages, and many other forms of credit besides student loans might be available once a person has started participating in labor markets. While these forms of credit might not be available for individuals to finance their education, their presence may alter the human capital decisions, especially in the face of early unemployment.

To incorporate some of the main implications of these forms of credit into my simple model, consider the case in which students cannot borrow or save in $t = 0$ but between periods $t = 1$ and $t = 0$ they can fully smooth consumption by borrowing or lending. Given my assumption that $\beta(1 + r) = 1$, after realizing their employment status in $t = 1$, their consumption will be fully smoothed whether employed or unemployed between periods $t = 1$ and $t = 2$. If they are employed, then the present value of consumption will be $ah^{\alpha}(1 + \beta G)$, and consumption at $t = 1$ and $t = 2$ will be equal to $c_1 = c_2 = ah^{\alpha}\frac{(1+\beta G)}{1+\beta}$. The value of a career with early employment is then

$$V_e = \Theta_e \frac{\left[ah^{\alpha}\right]^{1-\sigma}}{1-\sigma},$$

where $\Theta_e \equiv (1 + \beta)^{\sigma}[1 + \beta G]^{1-\sigma}$.

Likewise, an unemployed young person would borrow to consume at $t = 1$ and repay at $t = 2$. The present value of resources βah^{α} will be equally consumed in both periods, leading to $c_1 = c_2 = \frac{ah^{\alpha}}{1+\beta}$,[16] and a value, V_u, as of $t = 1$ equal to

$$V_u = \Theta_u \frac{\left[ah^{\alpha}\right]^{1-\sigma}}{1-\sigma},$$

where $\Theta_u = (1 + \beta)^{\sigma}$.

With those values in place, I can succinctly define the problem for choosing effort e and schooling investments h as

$$\max_{e,h} \frac{[W-h]^{1-\sigma}}{1-\sigma} - v(e) + \beta\left[p(e)\Theta_e + \left(1-p(e)\right)\Theta_u\right]\frac{\left[ah^{\alpha}\right]^{1-\sigma}}{1-\sigma}.$$

For a given choice of effort e in finding a job while young, the optimal investment in schooling h is given by

$$\left[W-h\right]^{-\sigma} = \beta\alpha a^{1-\sigma}\left[p(e)\Theta_e + \left(1-p(e)\right)\Theta_u\right]h^{\alpha(1-\sigma)-1}.$$

Note that while the levels might differ, the implied relationship between investments h, and the individual's ability a and wealth W are both in the same direction as in the previous environment with no markets whatsoever.

Similarly, the optimal exertion of effort is given by the condition

$$v'(e) = \beta p'(e)[\Theta_e - \Theta_u]\frac{\left[ah^{\alpha}\right]^{1-\sigma}}{1-\sigma}$$

for the continuous effort case. For the discrete effort case, the condition is exactly the same for autarky. However, in this case I can write it more explicitly as

$$e = \begin{cases} 1 & \text{if } \beta\left[\left(p_H - p_L\right)\left(\Theta_e - \Theta_u\right)\right]\frac{\left[ah^{\alpha}\right]^{1-\sigma}}{1-\sigma} \geq v(1) - v(0), \\ 0 & \text{otherwise.} \end{cases}$$

I wish to highlight that although the patterns are similar, the allocations can differ substantially with or without credit markets after school. The ability to borrow from future income allows the person to partially insure against low consumption outcomes in case he or she does not find a job when young (and misses the experience gains for $t = 2$). But this insurance is partial at best. Moreover, the inability to borrow at time $t = 0$ can severely limit the investments of young persons, especially those with high ability but very little material support from their families.

NOTES

1 See Lochner and Monge-Naranjo (2015b) and references therein.

2 ACS data are available at https://www.census.gov/programs-surveys/acs/data.html.

3 The B&B:08/12 is available at https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2014641rev.

4 The U.S. civilian unemployment rate during the period ranged from 5.8 percent in July 2008, before the Great Recession, to 9.5 percent in July 2009.

5 Lochner and Monge-Naranjo (2015a) use repayment measures based on individual loan records from the National Student Loan Data System accessed in both 1998 and 2003. The loan status represents the most recent available status date at the time. See Lochner and Monge-Naranjo (2015a) for further details.

6 The data in this section show substantial differences across majors in terms of unemployment and underemployment levels and earnings growth. Schooling costs for the different majors can also vary widely as college professors and even fixed-term instructors command very different wages depending on their field. Examining these differences and whether they should be incorporated in the form of differences in the student loan programs offered to prospective students deserves ample attention—enough to warrant a separate work—and is beyond the scope of this article.

7 It is equivalent to assume that the effort must be exerted at the beginning of $t = 1$. Therefore, in the simple model I do not distinguish between whether the individual's effort is to succeed in school and find a job based on the recommendation of the school or to search early for jobs and interviews. That distinction is present in the multiperiod model later in the article.

8 See the discussion in Lochner and Monge-Naranjo (2012).

9 The FOC is

$$u'\left(\frac{W+(1/\alpha-1)h^*(a;e)}{(1+\beta+\beta^2)}\right)\left(\frac{1-\alpha}{\alpha}\right)\frac{\partial h^*(a;e)}{\partial e}=v'(e). \tag{4a}$$

However, notice that $\frac{\partial h^*(a;e)}{\partial e}=\frac{(1+\beta(G-1))}{1-\alpha}\beta\alpha a\left[h^*(a;e)\right]^{\alpha}\times p'(e)$. From the FOC on h, $[\beta + p(e)(1+ \beta(G - 1))]\beta\alpha a h^{\alpha} = h$. Substituting the latter expression into the former, I obtain

$$\frac{\partial h^*(a;e)}{\partial e}=\frac{1}{1-\alpha}\left(\frac{1+\beta(G-1)}{\beta+p(e)(1+\beta(G-1))}\right)h^*(a;e)\times p'(e).$$

Substituting this last equation into the FOC (4a) yields the expression in the text.

10 The FOC is necessary only because multiple crossings can happen unless additional restrictions on $p(\cdot)$ and $v(\cdot)$ are imposed. However, around an equilibrium, the left-hand side of this equation must be decreasing and the right-hand side increasing, otherwise the second-order conditions would be violated.

11 First, note that given the concavity of the utility function $u(\cdot)$, the cheapest form of delivering a utility level V is $(1 + \beta)c$ for a constant flow $c > 0$, such that $V = (1 + \beta)u(c)$. The level c is given by $c=u^{-1}\left[\frac{V}{1+\beta}\right]$. Then, the cost for the lender of providing V is as given in the text.

If $\sigma < 1$, the domain for the function $C(\cdot)$ is $[0,\infty)$; if $\sigma > 1$, its domain is $(-\infty, 0)$.

[12] The Lagrangian is standard:

$$L = u(c_0) - v(1) + \beta[p_H V_e + (1 - p_H)V_u]$$
$$+ \lambda[-c_0 - h + W + \beta\{p_H[ah^\alpha(1 + \beta G) - C(V_e)] + (1 - p_H)[\beta ah^\alpha - C(V_u)]\}]$$
$$+ \mu[\beta(p_H - p_L)(V_e - V_u) - (v(1) - v(0))],$$

and it is straightforward to show that the FOCs are sufficient since $u(\cdot)$ is strictly concave, $C(\cdot)$ is strictly concave, and the feasible set for $\{c_0,h,V_e,V_u\}$ is convex.

[13] The continuous effort case is very similar. Indeed, the Hopenhayn-Nicolini (1997) model is a continuous effort model.

[14] For an explicit model of a sequential investment in college education, see Garriga and Keightley (2007).

[15] The left-hand side of this equation is obviously increasing in h, while the right-hand side is strictly decreasing since the term within square brackets is positive and the exponent $\alpha(1 - \sigma) - 1$ on h is always negative.

[16] It is assumed that $b = 0$ for simplicity. The assumption of employment during $t = 2$ and borrowing available for $t = 1$ implies that the problem is always well defined.

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