Taylor-Type Rules and Total Factor Productivity

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This paper examines the impact of a persistent shock to the growth rate of total factor productivity in a New Keynesian model in which the central bank does not observe the shock. The authors then investigate the performance of alternative policy rules in such an incomplete information environment. While some rules perform better than others, the authors demonstrate that inflation is more stable after a persistent productivity shock when monetary policy targets the output growth rate (not the output gap) or the price-level path (not the inflation rate). Both the output growth and price-level path rules generate less volatility in output and inflation following a persistent productivity shock compared with the Taylor rule. (JEL E30, E42, E58)

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Gross domestic product (GDP) in the United States fell about 8.7 percent below its estimated long-run trend (i.e., potential GDP) during the last quarter of 2008 and the first quarter of 2009. Since that time, actual and potential GDP have grown at about the same rate but with actual GDP considerably lower than potential GDP. Figure 1 shows logged values of actual GDP and two estimates of potential GDP as calculated by the Congressional Budget Office (CBO). The higher level of potential GDP was estimated in 2007 and the lower level in 2011. The reduced 2011 estimate reflects the impact of sluggish GDP growth over the past few years. Uncertainty about how long actual GDP will remain below potential GDP and how much estimates of potential GDP will decline if actual GDP continues to grow slowly are just some of the problems in evaluating the current state of the economy. Obtaining reliable estimates of potential output is particularly important because potential GDP is a key benchmark used by the Federal Reserve to set its federal funds rate target. If the estimates of potential GDP are incorrect, the central bank could make a mistake when setting the federal funds rate target and trigger an unintended shift in inflation.

In recent U.S. history, two episodes occurred in which statistical agencies were initially unaware of a substantial shift in trend GDP growth. Orphanides et al. (2002) argue that rising

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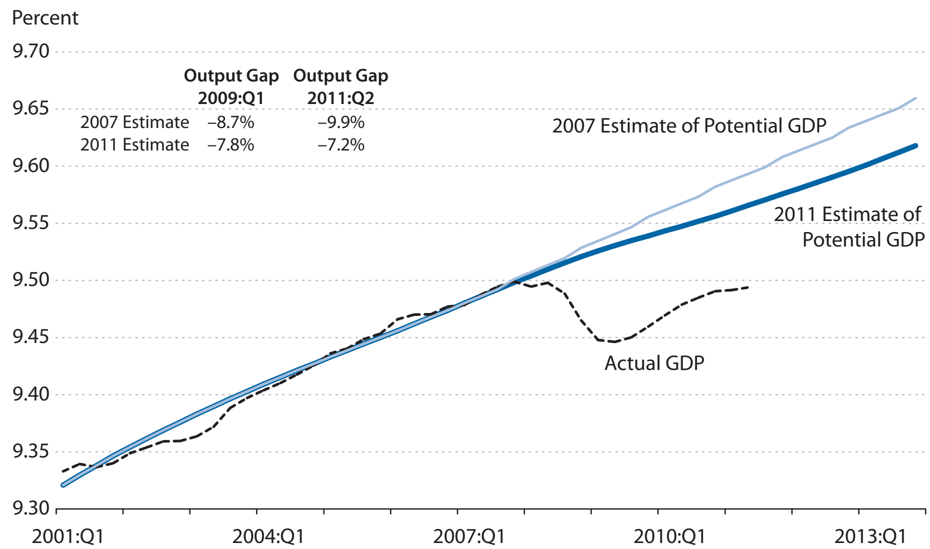
Figure 1

CBO Potential Real GDP and Actual Real GDP



NOTE: Data shown are logged values.

U.S. inflation from 1965 to 1980 was the result of real-time errors in the measurement of trend GDP.[1] They contend that an unexpected productivity slowdown reduced the actual growth rate of potential output below its expected trend, which inadvertently led policymakers to follow an inflationary policy. In the second case, the U.S. inflation rate averaged 3 percent per year during the 1990s, which was well below the 5- to 10-year-ahead forecasts of 5 percent made in 1989. Many economists now believe that the surprisingly low inflation of the 1990s was caused by an unexpected increase in productivity growth.

Taylor (1993) outlines a simple monetary policy rule that performs well in describing how the Federal Reserve conducted monetary policy between 1987 and 1993. The Taylor rule states that the nominal interest rate target responds to deviations of output from its potential and the inflation rate from its target. The fact that the Taylor rule has successfully accounted for monetary policy actions has led economists to examine how well the rule achieves the objectives of the central bank. Research finds that the Taylor rule, while not the optimal monetary policy rule, performs very well in a variety of macroeconomic models.[2] Such analysis, however, generally has omitted consideration of shifts in productivity growth trends.

This article shows how alternative monetary policy rules may prevent unintentional changes in inflation following a persistent productivity growth shock. Our results indicate that a persistent increase in the productivity growth rate causes inflation to fall when the central bank follows the Taylor rule but does not observe the productivity shock. The decline continues until policymakers recognize the shock and adjust to the level of productivity. We demonstrate that when the central bank targets the output growth rate or the price-level path instead of the level of output, inflation initially changes but eventually returns to its target with no further intervention by the central bank. Furthermore, the model predicts that inflation and the output gap vary much less when the output growth rate or the price-level path is the target of monetary policy.

The paper proceeds in the following manner. The next section provides an overview of the New Keynesian model. We then examine how the economy responds to a persistent productivity growth shock under various monetary policy rules. To assess the approximate welfare implications of alternative policy rules, we investigate the volatility of inflation and output over horizons ranging from 1 quarter to 5 years after a permanent productivity shock.

THE MODEL

Our model is a standard New Keynesian specification with Calvo (1983)-style price setting. A basic overview of the model is presented below. Those familiar with the standard New Keynesian model may wish to go directly to the discussion of calibration and parameter assignments in the next section.

Households

Households are infinitely lived agents who seek to maximize the discounted value of their expected lifetime utility from consumption, c_t, and leisure, l_t,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t, l_t\right)\right], \tag{1}$$

where E_0 is the expectation operator at time 0 and β is the discount factor. For simplicity, we assume that consumption and leisure are separable in the momentary utility function

$$u(c_t, l_t) = \ln\left(c_t\right) + \chi\frac{l_t^{1-\omega}}{1-\omega}, \tag{2}$$

where χ measures the relative weight of leisure in the household's utility function and ω determines the elasticity of the labor supply with respect to the real wage.

Households' utility maximization problem is subject to constraints on spending, time, and capital accumulation. Households begin each period with their initial real money balances, M_{t-1}/P_t, and income from the sale of bonds purchased in the previous period, $R_{t-1}B_{t-1}/P_t$, where M_t is nominal money balances, B_t is nominal bond holdings, P_t is the price level, and R_t is the gross nominal interest rate earned on bonds from period t to t+1. During the period, households receive resources from labor income, $w_t n_t$, capital rental income, $q_t k_t$, profits from ownership of firms, d_t, and a transfer payment from the monetary authority, T_t/P_t, where w_t is the real wage, n_t is labor, q_t is the capital rental rate, and k_t is the capital stock. The households then use those resources to fund their consumption, investment, i, and their end-of-period real money and bond holdings, M_t/P_t and B_t/P_t, respectively. Thus, households' budget constraint is expressed as

$$c_t + i_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = w_t n_t + q_t k_t + d_t + \frac{M_{t-1}}{P_t} + \frac{R_{t-1}B_{t-1}}{P_t} + \frac{T_t}{P_t}. \tag{3}$$

Households' time, which is normalized to unity, is divided among labor, leisure, and time spent in transaction-related activities, s_t:

(4) $$l_t + n_t + s_t = 1.$$

The time households spend in transaction-related activities—often called shopping time costs—will rise as the nominal value of consumption purchases rises. It will drop as households set aside more money to facilitate such transactions. This is denoted as follows:

(5) $$s_t = \zeta \left(\frac{P_t c_t}{M_{t-1}} \right)^{\gamma},$$

where $\zeta > 0$ is set to match the average velocity of money balances, defined as currency plus checkable deposits, and $\gamma > 0$ determines the interest elasticity of money. Our function for s_t depends on beginning-of-the-period money balances and does not include the monetary transfer. That assumption makes our specification more similar to a cash-in-advance model rather than a money-in-the-utility-function specification, which typically uses end-of-the-period money balances.

Each period, households spend resources on investment in order to acquire capital. Some resources are exhausted during the process of converting investment into capital. These lost resources are referred to as "capital adjustment costs," AC_t. The capital accumulation equation then is

(6) $$i_t = k_{t+1} - (1-\delta) k_t + AC_t,$$

where δ is the depreciation rate and $AC_t = i_t - \varphi(i_t/k_t)k_t$. We assume that the average and marginal capital adjustment costs are zero around the steady state (i.e., $\varphi(i_t/k_t) = i/k$ and $\varphi'(i_t/k_t) = 1$). The capital adjustment costs are important in a model with sticky prices to prevent implausibly large movements in investment after most exogenous shocks to the economy.

Firms

Each firm produces a heterogeneous good in a monopolistically competitive market. The presence of monopoly power enables firms to optimally adjust their prices each period unless some friction exists to prevent it. The presence of a friction that prohibits all firms from optimally setting their prices every period is a common characteristic in most New Keynesian models.

Any model with heterogeneous firms requires a couple of assumptions to make it tractable. First, all firms have the same production function. Specifically, firm f produces its output, $y_{f,t}$, according to the following production function:

(7) $$y_{f,t} = k_{f,t}^{\alpha} \left(Z_t n_{f,t} \right)^{(1-\alpha)},$$

where $n_{f,t}$ is firm f's labor demand, $k_{f,t}$ is firm f's capital demand, Z_t is an economy-wide productivity factor, and $0 < \alpha < 1$. The productivity factor, Z_t, evolves as follows:

(8) $$\ln\left(Z_t / Z_{t-1}\right) = \rho_Z \ln\left(Z_{t-1}/Z_{t-2}\right) + \left(1-\rho_Z\right)\ln\left(\bar{g}\right) + \nu_t,$$

where $\bar{g}$ is the steady-state productivity growth rate, $0 \le \rho_Z < 1$, and $\upsilon \sim \mathrm{N}(0,\sigma^2)$.

The second assumption is that firms hire labor and rental capital in perfectly competitive factor markets, so that all firms pay the same wage and capital rental rate. The resulting first-order conditions from firm f's problem are

$$q_t = \alpha \psi_t \left(Z_t n_{f,t} / k_{f,t} \right)^{(1-\alpha)}, \tag{9}$$

$$w_t = (1-\alpha) \psi_t Z_t \left(k_{f,t} / Z_t n_{f,t} \right)^{\alpha}, \tag{10}$$

where ψ_t is interpreted as the real marginal cost of producing an additional unit of output. Since all firms have access to the same technology and pay the same price for capital and labor, the capital-to-labor ratio and the real marginal cost are identical for all firms. The capital and labor used by all firms is aggregated as follows:

$$k_t = \left[\int_0^1 k_{f,t} \, df \right], \text{ and } n_t = \left[\int_0^1 n_{f,t} df \right]. \tag{11}$$

Aggregate output, y_t, is a combination of the differentiated products using the Dixit-Stiglitz aggregator:

$$y_t = \left[\int_0^1 y_{f,t}^{(\epsilon-1)/\epsilon} \, df \right]^{\epsilon/(\epsilon-1)}, \tag{12}$$

where $-\epsilon$ is the price elasticity of demand for $y_{f,t}$. Cost minimization by households yields the following product demand equation for firm f's differentiated good:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon} y_t, \tag{13}$$

where $P_{f,t}$ is the price for $y_{f,t}$ and P_t is a nonlinear aggregate price index such that

$$P_t = \left[\int_0^1 P_{f,t}^{(1-\epsilon)} \, df \right]^{1/(1-\epsilon)}. \tag{14}$$

Price setting follows a Calvo (1983) model of price adjustment. Specifically, the probability that a firm can optimally reset its price in any given period is $(1-\eta)$, while the probability that that firm must charge the last period's price is η. Those firms with a price adjustment opportunity select the price that maximizes the present value of their current and expected future profits subject to the constraint in equation (13) that firms must satisfy all demand at their posted price. When the solution to this problem is linearized around its steady state, the equation for the New Keynesian Phillips curve is obtained[3]:

$$\hat{\pi}_t = \left[(1-\eta)(1-\beta\eta)/\eta \right] \hat{\psi}_t + \beta E_t \hat{\pi}_{t+1}, \tag{15}$$

where "^" represents the percent deviation of a variable from its steady state.

Scattered price adjustment enables individual firms to charge different prices, which leads to some firms producing less than their optimal allocation and others producing more. This dispersion in prices is especially prevalent after an exogenous shock hits the economy. If price stickiness is the only nominal friction in the model, the optimal monetary policy is to stabilize the price level with long-run inflation expectations equal to zero.[4] As the price level deviates further from its optimal level, output becomes more distorted and welfare is reduced. The welfare losses are directly related to the size of the gap between output in the sticky price model and the level of output that would occur with flexible prices. Thus, the goal of the policymaker is to reduce the size of that gap by aggressively targeting the price level so that output follows the path that it would in an economy with flexible prices.

The Monetary Authority

The monetary authority targets the nominal interest rate, R_t, as follows:

$$\hat{R}_t = \left(1+\theta_\pi\right)\hat{\pi}_t + \theta_y \hat{y}_t + \theta_g \hat{g}_t + \theta_P \hat{P}_t, \tag{16}$$

where $\theta_\pi \geq 0$, $\theta_y \geq 0$, $\theta_g \geq 0$, $\theta_P \geq 0$, and g_t is the growth rate of output. Equation (12) then resembles a Taylor (1993) rule in which θ_g and θ_P are set = 0. In our sticky price model, the optimal monetary policy rule, if it were implementable, prevents the inflation rate from deviating from its target by setting $\theta_\pi = \infty$. We initially analyze the effects of a persistent but temporary shock to the productivity growth rate on key economic variables under the optimal policy rule and then use those results to evaluate alternative monetary policies that are likely to be implementable.

Why is the optimal policy unrealistic? Essentially, under the optimal policy, the central bank promises to raise the interest rate by any amount necessary to prevent the inflation rate from deviating from the target rate. In theory, people expect the central bank to deliver inflation at the target rate; they make decisions, write contracts, and generally forecast inflation assuming inflation will be at the target rate. The central bank does not need to move the interest rate because people react to shocks in a manner that causes inflation to be equal to the target. In equilibrium there is almost no variability in either interest rates or inflation. This equilibrium outcome requires policy to be well defined and credible.

Models that are typically used in central banks to make forecasts and evaluate alternative policies generally assume that inflation expectations are mostly backward looking. That is, people do not have the opportunity to change their decisions in light of announced policy changes. Consequently, central bank simulations of switching to the optimal policy often find that doing so results in extreme variability for interest rates, inflation, and other economic time series. The bottom line is that central bank officials are often reluctant to commit to the optimal policy implied by the forward-looking model.

Fortunately, there are implementable policies that can approximate the optimal equilibrium. By "implementable," we mean that the central bank can make measured responses to incoming data in a way that will inform the public about the policy and help strengthen credibility if it is weak.[5] We demonstrate this result in our model using alternative policies defined by interest rate rules that react to output growth rather than the output gap and a price-level path rather than the period-by-period inflation rate.

CALIBRATING THE MODEL

Parameter values are specified based on a quarterly model. The households' discount factor, β, is set to 0.99, which is consistent with a steady-state annual real interest rate of about 4 percent.

The preference parameter, χ, is calibrated so that the steady-state labor supply, $\bar{n}$, works 30 percent of the available time. The other preference parameter, ω, is set to 7/9, which implies that the elasticity of labor supply with respect to the real wage is approximately equal to 3.[6] Parameters chosen for the shopping time function, s_t, are consistent with long-run studies of money demand. Specifically, the long-run elasticity of money demand with respect to consumption, γ, is set to 1, which implies that the interest rate elasticity of money demand equals –0.5.[7] The scale variable, ζ, is chosen to approximately match the income velocity of currency plus household checkable deposits. Furthermore, the steady-state shopping time cost, $\bar{s}$, is set to 1 percent of the time spent working.

The capital share of output, α, is set to 0.33 and the capital stock depreciates at 2 percent per quarter. The price elasticity of demand, ϵ, is set equal to 6, which is consistent with a 20 percent steady-state markup of price over marginal cost. The probability of price adjustment, $(1-\eta)$, is set to 0.25, which means that firms change prices on average once per year. Capital adjustment costs are calibrated so that the elasticity of the investment-to-capital ratio with respect to Tobin's q, $[(i/k)\varphi''(\cdot)/\varphi'(\cdot)]-1$, is equal to 5.

A persistent shock to the productivity growth rate will lead to a permanent change in the level of both productivity and output. Since the central bank in our model measures potential output as the original steady-state path for output, a productivity shock will cause policymakers to unknowingly respond to a flawed measure of the output gap.[8] This specification has some strong empirical support in the literature. Orphanides and van Norden (2002) and Orphanides (2003a,b) document that historically neither the Federal Reserve nor standard statistical methods have been able to detect large changes in potential output until well after they have occurred.

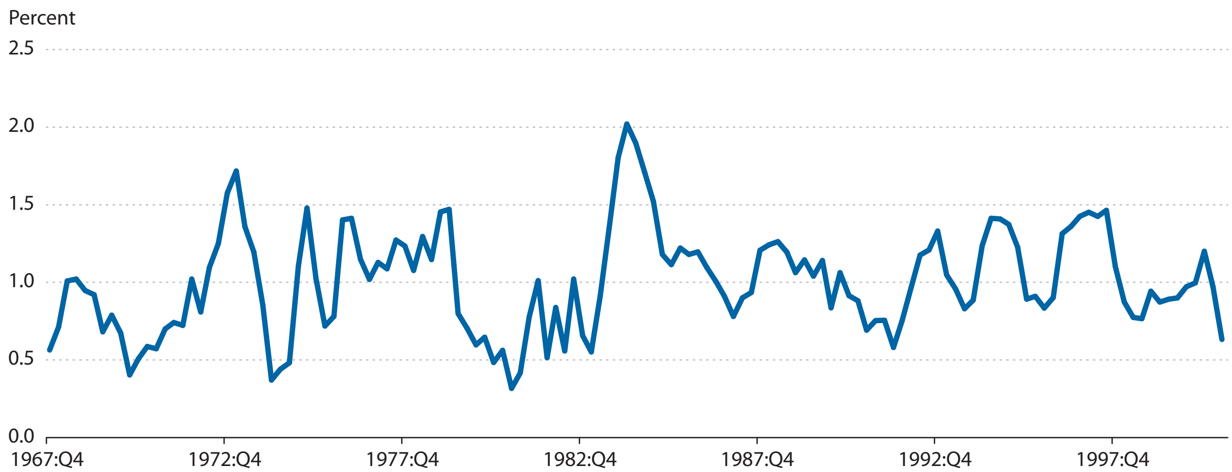
Identifying potential output changes in real time is complicated by frequent revisions to recent GDP data. Figure 2 plots the standard deviation (SD) of 2-year growth rates of real output from different vintages of the data.[9] For example, the growth rate for the 2-year period ending in 1984:Q1 has the largest standard error. It has been revised many times since the data were first computed in 1984:Q2. The SD of the 2-year growth rate ending in 1984:Q1 was 2.0 percent for all vintages of data published since 1984:Q2; that 2-year period had the largest number of revisions of any 2-year period in our sample. Overall, the average SD for the sample shown in Figure 2 is 1.0 percent.

The output gap is measured as the log difference between actual and potential output. Orphanides and van Norden (2002) show that revisions to actual output have a small effect on measured output gaps compared with the effect generated by revisions to potential output. Revisions to potential output for any particular quarter are so large because all statistical methods used to measure it rely on data both before and after the quarter in question. In real time, however, the policymaker has only past data available to measure potential output. As more data become available, the incoming information is used to refine estimates of the trend. For example, Figure 1 compares the 2011 estimate of potential GDP as measured by the CBO with its own 2007 estimate. Slow growth during the current recovery has led the CBO to lower its estimate of potential GDP. As a result, the estimate for 2011:Q2 potential GDP has fallen by about 2.7 percent since the beginning of the mortgage debt crisis.

Orphanides and van Norden (2002) report that the revisions of the Federal Reserve staff's estimates of the output gap for the 1980s and early 1990s have a root mean square error of 2.84

Figure 2

Standard Deviation of 2-Year GNP/GDP Growth Rates (Across Vintages)



SOURCE: Real-Time Data Research Center (www.philadelphiafed.org/research-and-data/real-time-center/).

percent, compared with an SD of 2.44 percent for earlier estimates available at the end of 1994. This means that revisions made in the second half of the 1990s to both the output data and the estimates of potential output have become larger and estimates of the size of the output gap have become smaller. That finding highlights a pattern in recent U.S. economic history: Depending on the particular statistical model used, the real-time estimate of the output gap can be reduced by half or more as new data arrive. If a model with a linear trend is used, Orphanides and van Norden (2002) show that the 11 percent negative output gap estimated for 1974-75 using real-time data nearly disappeared by 2000 as incoming information led to revised estimates. Revisions to the output gap have historically shown a high degree of positive correlation. Therefore, a downward revision to the output gap in the latest data release likely signals further downward revisions for future estimates of that output gap.

We calibrate the technology growth shock process using estimates by Kurmann and Otrok (2010) and Barsky and Sims (2011). The growth rate of technology follows a stochastic first-order autoregressive process around its nonstochastic steady state that is outlined in equation (8). We assume that the annual growth rate of productivity is 1.6 percent which, at a quarterly frequency, means that the steady-state gross growth rate of technology, $\bar{g}$, is 1.004. Following Barsky and Sims (2011), the first-order autocorrelation coefficient for the growth rate of productivity, ρ_Z, is set to 0.837.[10] With this calibration, a –0.1 percent shock lowers the level of technology by 0.6 percent in the long run and has a half-life of about one year.[11]

The equations describing the behavior of the households, firms, and monetary authority combine to form a nonlinear system describing the model's equilibrium. That system of equations is linearized around its deterministic steady state and then the model's rational expectations solution is obtained by standard solution methods (see Appendix). Our objective is to analyze the impact of a persistent but ultimately temporary shock to the growth rate of productivity.

PRODUCTIVITY GROWTH SHOCKS

This section evaluates the economic performance of monetary policy rules when a productivity growth shock shifts potential output but that shift is not immediately observed by the central bank. In all these rules, we assume that the policymaker measures the output gap as the deviation of the observed level of output from its original steady-state path. We examine the impact of a temporary increase in the productivity growth rate from 0.4 percent to 0.5 percent per quarter on capital stock growth, the inflation rate, real and nominal interest rates, real wage growth, real marginal costs, hours worked, and output for several monetary policy rules. Since the potential output shift is not immediately detected by policymakers, the steady-state output path in the policy rule remains unchanged.

Figure 3 presents the impulse responses for those variables when the monetary authority follows the optimal policy rule, the Taylor rule, and an inflation-only rule. In Figure 4, we repeat that experiment with an output growth rule suggested by Orphanides and Williams (2002) and Walsh (2003) and a price-level path rule recommended by many other authors.[12] The economy's response is limited to the first 5 years following the productivity shock because we suspect that after 5 years policymakers will begin to recognize the shift in potential GDP and make appropriate adjustments to its measure of the output gap. Furthermore, our model economy moves sufficiently far from its original steady state after 5 years to make approximation errors problematic.

The Optimal Policy

We report the results for the optimal monetary policy as a benchmark for evaluating the alternative monetary policy rules. King and Wolman (1999), Woodford (2003), and Canzoneri, Cumby, and Diba (2005) show that the optimal monetary policy in a New Keynesian model eliminates the effect of distortions caused by nominal frictions. That rule is only approximately optimal because real distortions exist because of monopolistic competition in the goods sector and shopping time costs. Monetary policy, however, is unable to correct the monopolistic competition distortion, and the distortion due to the shopping time costs is usually small.[13] In our model, the only significant nominal friction is the Calvo price setting by firms, which can be eliminated by stabilizing the price level.

The solid lines in Figure 3 show the impulse responses of key economic variables to a productivity growth shock when the monetary authority follows the optimal policy rule ($\theta_\pi = \infty$, and all other θ_is equal zero). That shock causes a rise in households' permanent income, which in turn leads households to increase their consumption and leisure and decrease their labor supply.[14] Firms raise their demand for labor, which combined with the decline in labor supply, causes the real wage to rise. The decrease in hours worked almost fully offsets the rise in productivity, so output increases only slightly in the first quarter after the productivity shock. A surge in consumption accompanies a sharp decline in investment that initially lowers the capital stock before it starts to rise again. A temporary increase in productivity raises future capital rental rates, so the real interest rate jumps on impact. Under the optimal policy rule, the nominal interest rate mimics the real interest rate because inflation expectations remain unchanged. Finally, the optimal policy keeps the price markup and the real marginal cost constant at their steady-state levels.

Figure 3

Response to a Persistent 0.1 Percent Shock to TFP Growth with Inflation-Targeting Rules

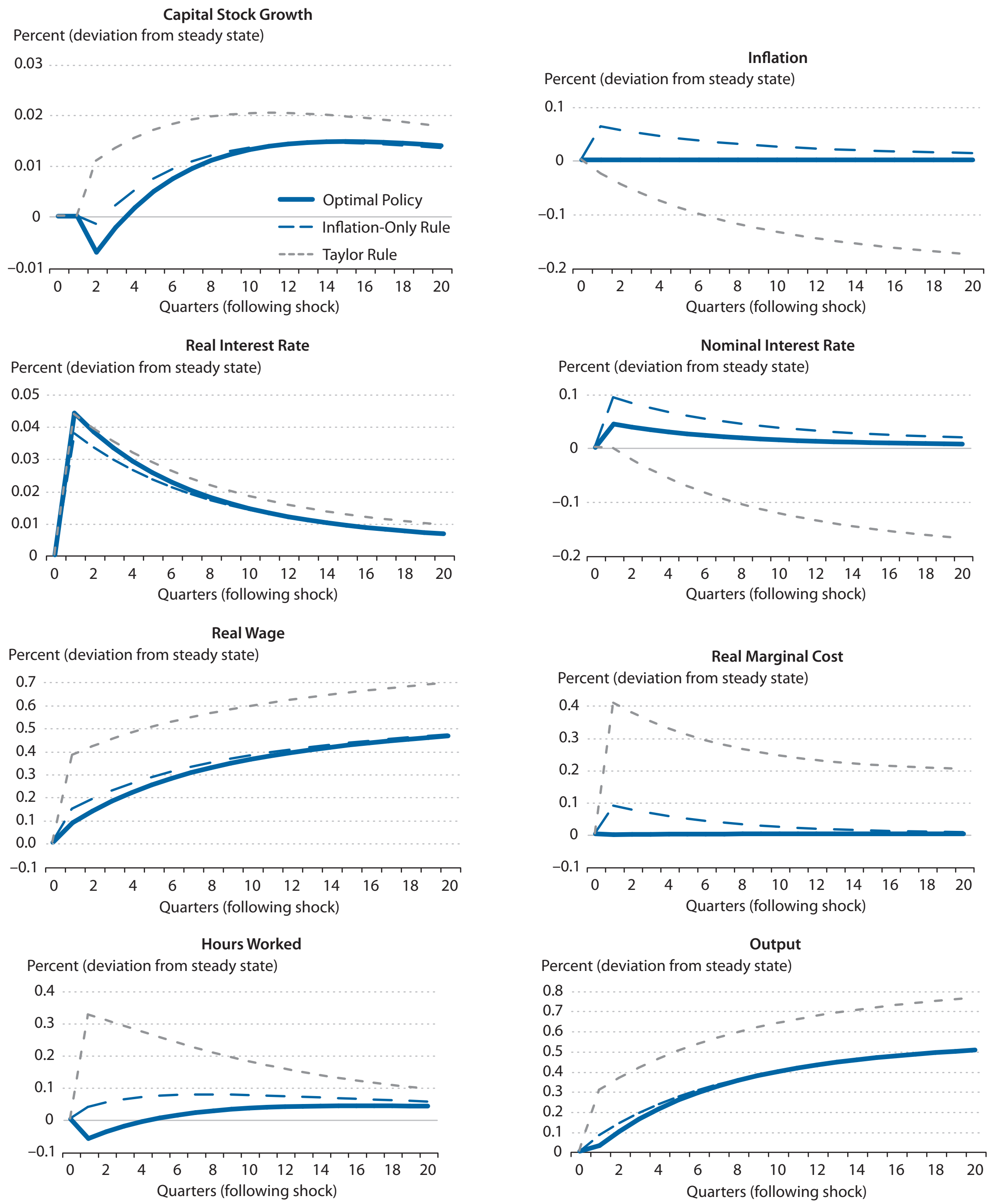
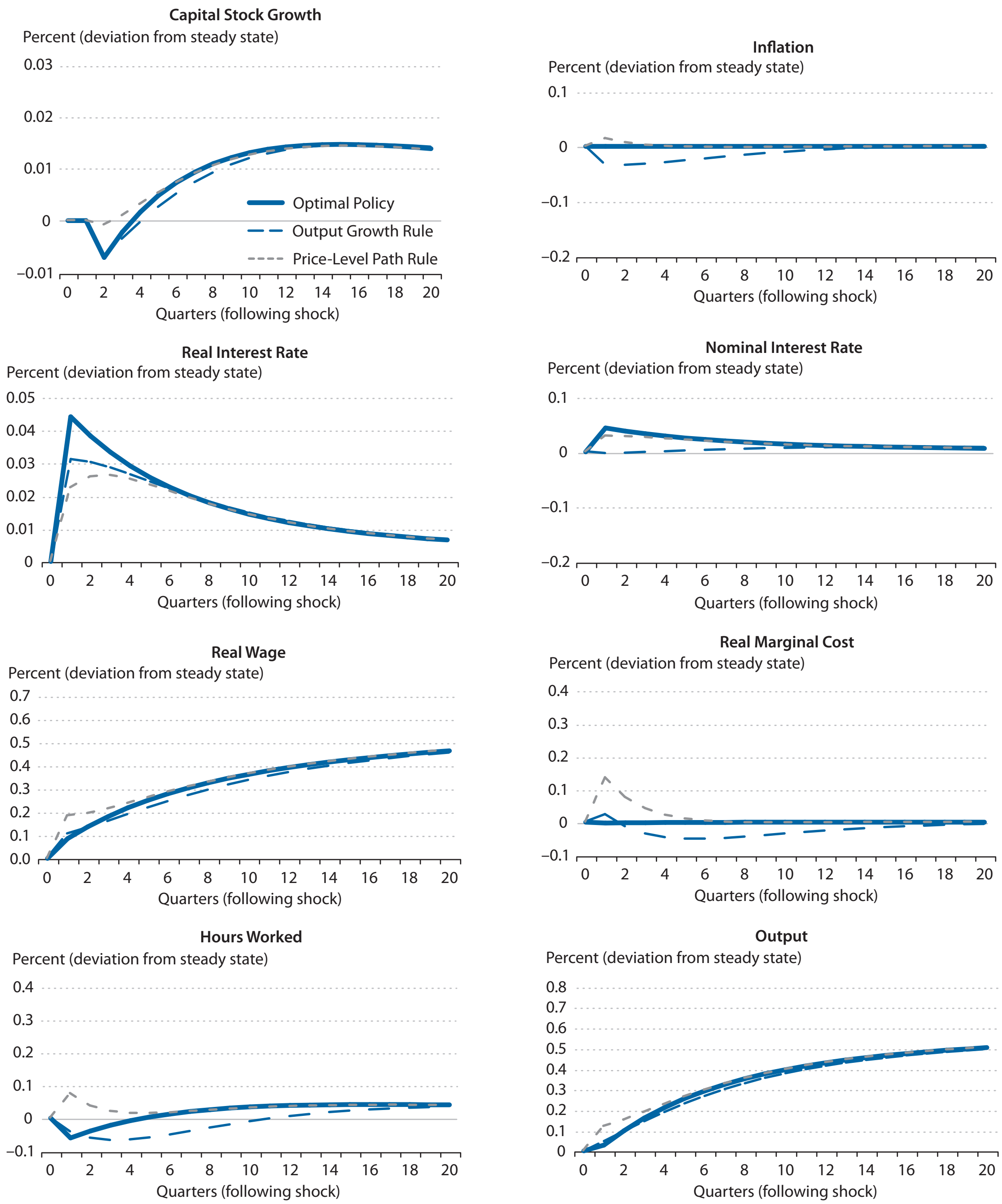


Figure 4

Response to a Persistent 0.1 Percent Shock to TFP Growth with Output Growth and Price-Level Path Rules



In subsequent periods, the economy gradually returns to its steady-state growth path, but with higher levels for productivity, capital stock, output, the real wage, consumption, and investment. The higher levels for productivity further increase the demand for labor, which continues to push up the real wage and eventually encourages households to decrease leisure in favor of more work. The gradual increase in the capital stock over time exerts downward pressure on the capital rental rate until it returns to its original steady state. This response then is mimicked by both the real and nominal interest rates.

The Inflation-Only Rule

The next policy rule that we examine is one in which the monetary authority adjusts the nominal interest rate target in response to changes in the inflation rate but ignores the output gap:

(17) $$\hat{R}_t = \left(1+\theta_\pi\right)\hat{\pi}_t,$$

where $\theta_\pi > 0$ is a necessary condition for the model to have a stable and unique solution.[15] The blue dashed lines in Figure 3 depict the impulse responses to a 0.1 percent positive shock to the productivity growth rate when the monetary authority follows the inflation-only rule ($\theta_\pi = 0.5$). The key difference between the inflation-only rule and the optimal rule ($\theta_\pi = \infty$) is that a persistent productivity growth shock causes the inflation rate to rise under the inflation-only rule. Equation (17) can be rewritten so that inflation is a function of the nominal interest rate:

(18) $$\hat{\pi}_t = \frac{\hat{R}_t}{\left(1+\theta_\pi\right)}.$$

Since the productivity growth shock also raises the real interest rate, the nominal interest rate must increase. Equation (18) then indicates that the size of the inflation response is negatively related to the size of θ_π. Under the optimal policy rule, however, the value for θ_π is so large that inflation does not change after a productivity growth shock.

The inflation caused by the temporary productivity growth shock with the inflation-only rule also affects real variables, albeit only slightly. Firms, which can adjust their prices only infrequently, raise their prices more aggressively when given the opportunity because they expect inflation to increase. When only a fraction of firms can raise prices, the prices charged by different firms vary immediately following a productivity shock. This divergence generates a misallocation of labor and production that causes the economy to move away from potential output. Nevertheless, the real economy does not deviate too far from the optimal path because the productivity growth shock and the resulting inflation are temporary.

The Taylor Rule

The gray dashed lines in Figure 3 show the impulse responses for the Taylor (1993) rule in which the nominal interest rate target responds to both the inflation rate and the level of output:

(19) $$\hat{R}_t = \left(1+\theta_\pi\right)\hat{\pi}_t + \theta_y\hat{y}_t,$$

where $\theta_\pi = \theta_y = 0.5$. The impulse responses in Figure 3 demonstrate that setting $\theta_y > 0$ in the Taylor rule has a dramatic effect on both nominal and real variables. To understand the impact of θ_y, equation (19) is solved for the inflation rate:

$$\hat{\pi}_t = \frac{\hat{R}_t}{(1+\theta_\pi)} - \frac{\theta_y \hat{y}_t}{(1+\theta_\pi)}. \tag{20}$$

The increase in the productivity growth rate affects inflation by boosting both the real interest rate and the level of output. The inflation rate in equation (20), however, continues to fall as the deviation of output from the central bank's estimate of its potential continues to grow. If policymakers are slow to recognize a change in potential output, then the Taylor rule implies that a persistent increase in the productivity growth rate will generate an episode of surprisingly low inflation. This result is in contrast to the finding that inflation is unaffected by the optimal and inflation-only policy rules. The reason is simply that the monetary authority reacts to shifts in output under the Taylor rule, but it does not do so under the optimal and inflation-only policy rules.

Firms' pricing decisions are affected by the Taylor rule's endogenous response to the productivity shock. The expectation that inflation will decline leads firms, which adjust their prices infrequently, to select a lower price than if their prices could be adjusted every period. The lower prices lead to higher output demand, a smaller price markup, and a rise in the real marginal cost compared with the optimal and inflation-only policy rules. To raise production, firms further increase their demand for inputs, which not only raises the real wage and the rental rate of capital, but also increases the number of hours worked and investment in the capital stock. Furthermore, the higher capital rental rate puts more upward pressure on the real interest rate. The nominal interest rate initially rises with the real interest rate but then declines in subsequent periods as expected inflation falls.

An Output Growth Rule

Figure 4 examines the impact of a productivity growth shock on the optimal monetary policy rule and two alternative policy rules in which the policymakers respond to the output growth rate and the price-level path, respectively. Under the output growth rule ($\theta_\pi = 0.5$ and $\theta_g = 1$), the output growth rate replaces the output gap in the Taylor rule.[16] This specification is appealing because output growth converges back to the steady-state growth rate, whereas the perceived output gap grows until the monetary authority recognizes the change in potential output. Shifts in long-run productivity growth not only affect output growth but also exert a similar effect on the real interest rate. By including the output growth rate in the policy rule, the monetary authority can endogenously adjust its nominal interest rate target to unobserved changes in the real interest rate.

The output growth rule assumes that the monetary authority's nominal interest rate target responds to both the inflation rate and the output growth rate:

$$\hat{R}_t = (1+\theta_\pi)\hat{\pi}_t + \theta_g \hat{g}_t. \tag{21}$$

The blue dashed lines in Figure 4 display the impulse responses to the productivity shock under the output growth policy rule. In general, the economy's response to that shock under the output growth policy is very close to its response under the optimal policy. The link between the output growth rate and the real interest rate can be seen by substituting the Fisher equation, $\hat{R}_t = \hat{r}_t + \hat{\pi}^e_{t+1}$ into equation (21) and eliminating $\hat{R}_t$,

$$\hat{\pi}^e_{t+1} = \left(1+\theta_\pi\right)\hat{\pi}_t + \theta_g\,\hat{g}_t - \hat{r}_t.$$

The small differences between impulse responses for the two policy rules exist because the increase in the output growth rate exceeds the rise in the real interest rate over the first four years following the productivity shock. As a result, inflation continues to decline over that period. A falling inflation rate in an economy with sticky prices means the firms that cannot adjust their prices are charging higher prices than they would in the optimal policy environment. The sluggish downward adjustment in prices limits the increase in output, which dampens the rise in demand for factor inputs. That response leads to a reduction in hours worked and smaller increases in the real wage and the capital rental rate compared with the optimal policy. Lower capital demand also reduces the upward pressure on the real interest rate, which combined with a fall in inflation expectations, results in a lower nominal interest rate.

A Price-Level Path Rule

Our last monetary policy rule considered is the price-level path rule. This policy is essentially a long-run inflation rate target as opposed to a period-by-period inflation rate target. The key difference between a price-level path target and an inflation target is the policy response when inflation rises above its target. In subsequent periods, a price-level path target automatically signals the monetary authority to set a short-run inflation objective below the average target to "undo" the previous inflation, whereas an inflation target ignores previous deviations and seeks to return the inflation rate to its target. Svensson (1999) shows that an economy with a discretionary price-level path target is equivalent to an economy with a commitment to an inflation target.[17] In other words, a monetary authority that is technically unable to commit to a strong period-by-period inflation target (i.e., $\theta_\pi = \infty$) can do so indirectly by adopting a long-run inflation target or, equivalently, a price-level path target.

Our price-level path rule assumes that the nominal interest rate target moves one for one with the inflation rate and also responds to deviations of the price level from its long-run price path:

$$\hat{R}_t = \hat{\pi}_t + \theta_p\ \hat{p}_t. \tag{22}$$

The gray dashed lines in Figure 4 show the impulse responses of key variables to a persistent productivity growth shock when policymakers implement a price-level path rule ($\theta_p = 1$).[18] As with the output growth rule, the price-level path target closely mimics the optimal policy response to a persistent productivity shock. The key difference is that the rise in the real interest rate initially generates a modest amount of inflation under the price-level path rule.

Targeting the price-level path, however, puts downward pressure on expected inflation. In a sticky price economy, price-adjusting firms limit their increase in prices due to an expected fall in inflation and instead temporarily increase their production. Higher output lifts the demand for factor inputs, which results in more labor hours and higher wages and capital rental rates. Given that the effects of the price-level path rule relative to the optimal policy are shorter in duration, most additional production is concentrated on investment. The extra investment keeps the capital stock from falling as it does under the optimal policy, which limits any production losses due to the capital adjustment costs. As a result, the real interest rate response is much smaller with the price-level path in the first year after the productivity shock than with the optimal policy.

Both the output growth and price-level path policy rules generate inflation responses that deviate from the steady-state rate. The inflation shift, combined with the sticky price assumption, generates price dispersion among firms, which causes output to deviate from its efficient level under the optimal policy. If the optimal policy is politically infeasible, then policymakers must recognize the trade-off between output and inflation variability when choosing between an output growth rule and a price-level path rule. We can see that result by examining the effect of the two policy rules on volatility of output and inflation induced by the productivity shock.

INFLATION AND OUTPUT VOLATILITY: A MEASURE OF WELFARE

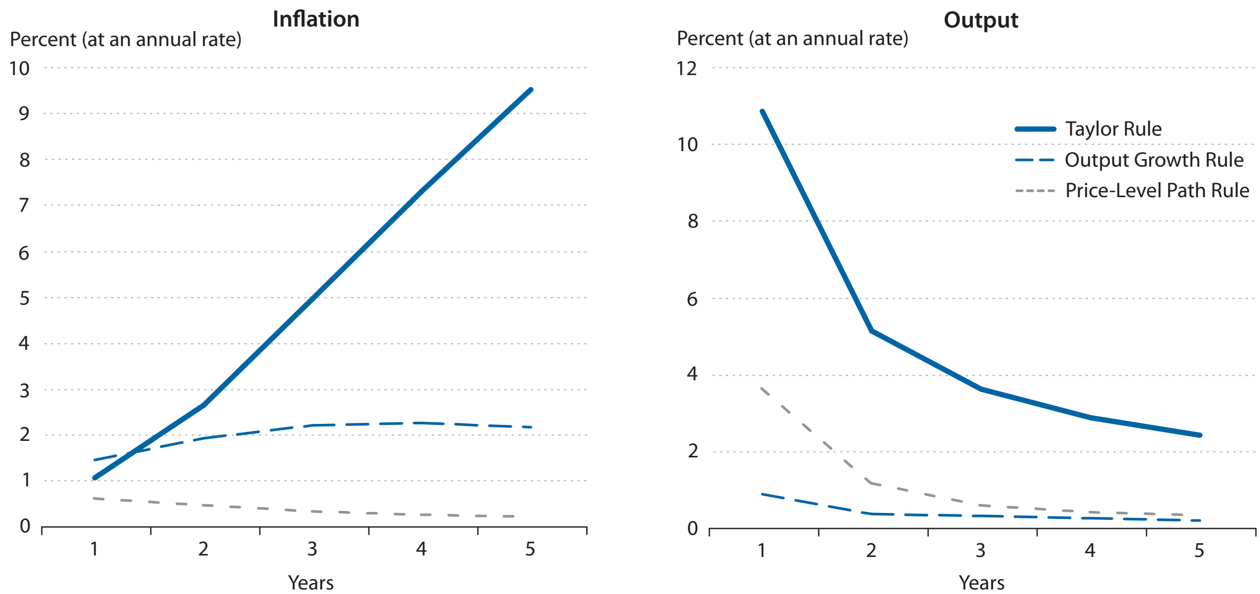
In New Keynesian models, monetary policy minimizes welfare losses by eliminating the output fluctuations caused by nominal frictions. The welfare loss in our model is proportional to the variance of the output gap (i.e., the deviation of output from its path in the absence of nominal rigidities).[19] Although welfare loss is properly measured using current-quarter output volatility, our New Keynesian model—like most other models—does not incorporate characteristics of the real economy that make the longer-term horizons relevant. For example, our model does not include long-term loans or long-term planning problems which, although difficult to model, are essential to the real economy. Given that central banks are concerned about the long-run consequences of their policy decisions, we examine the impact of persistent productivity growth shocks on the volatility of the output gap and inflation over 1- to 5-year horizons. Figure 5 compares the volatility of output and inflation under three rules: the Taylor rule, the output growth rule, and the price-level path rule.[20]

Our analysis focuses on fluctuations in output and inflation over forecast horizons as long as 5 years because that interval is a reasonable time for policymakers to recognize changes in potential output. We assume that the economy begins at its steady state and then simulate 5 years of persistent productivity growth shocks. Each simulation is repeated 1,000 times by drawing the shock from a normal distribution with mean zero and an SD equal to 0.1 percent per quarter. At each forecast horizon, we calculate the average deviation of the annual inflation rate and the output gap from their respective values under the optimal policy.

Figure 5 reports the impact of persistent productivity growth shocks on the SDs of output and inflation from the optimal policy over forecast horizons of 1, 2, 3, 4, and 5 years ahead. The left panel of Figure 5 displays the results for the inflation rate. When comparing the three rules, the Taylor rule generates the most variability in inflation at all forecast horizons except 1 year. Inflation volatility for the Taylor rule is very modest at the 1-year forecast horizon but accelerates as the forecast horizon increases. As for the other two monetary policy rules, inflation variability is considerably lower with the price-level path target than with the output growth target over all forecast horizons. In fact, inflation volatility continues to fall as the forecast horizon increases

Figure 5

Root Mean Square Deviations from the Optimal Path



with the price-level path rule, while it continues to rise mildly for the first three years with the output growth rule and remains elevated thereafter. Our results suggest that, on average, a price-level path rule minimizes inflation fluctuations after a persistent productivity growth shock.

The right panel of Figure 5 depicts the impact of persistent productivity growth shocks on the variability of the output gap over a forecast horizon ranging from 1 year to 5 years. Our findings show that the output growth policy rule produces the least output volatility at all forecast horizons, while the Taylor rule generates the most. Under each policy rule, output variability is highest at the 1-year forecast horizon and lowest at the 5-year horizon. Comparing the price-level path and output growth rules, the price-level path rule generates more output variability at the 1- and 2-year forecast horizons, but the volatility of output is nearly identical for both rules at forecast horizons of 3 years and longer. Overall, Figure 5 reveals that the price-level path rule is the most successful of the three rules at minimizing inflation fluctuations after a productivity shock, while the output growth rule is the best at minimizing the variability of output. Combining an output growth target with a price-level path target is a possible specification for a monetary policy rule that might further minimize the variability of both output and inflation. Determining the optimal coefficients for the output growth rate and the price level in a combined policy rule is beyond the scope of this paper, which considers only shocks to productivity growth.

CONCLUSION

This article analyzes the effect of a persistent productivity growth shock when the central bank does not immediately detect that such a shock has hit the economy. A productivity growth shock affects the economy by causing the growth path of potential output to change. If the central bank does not recognize this change, a monetary policy rule that targets potential output will produce unintended movements in inflation following the shock. Such a result is important because empirical evidence suggests that changes in the trend growth rate of potential output are usually not identified by the central bank or statistical agencies until well after the shift has occurred.

We show that a productivity growth shock has distorting effects when the central bank uses a Taylor rule but does not observe the shock. Specifically, the positive productivity growth shock raises both real and potential output. Given that the central bank does not notice the shift in potential output, its measure of the output gap in the Taylor rule rises. The perceived increase in a positive output gap causes the central bank to overly tighten monetary policy, which results in falling inflation.

Our results suggest that the Taylor rule can be improved in two ways. First, we find that central banks should target the long-run average inflation rate (a price-level path) as opposed to the period-by-period inflation rate. A commitment to a price-level path target stabilizes inflation over the long term and prevents drifting of the price level from its long-run trend. In practice, the central bank can anchor the price level because monetary policy is the primary determinant of prices in the long run.

Second, our findings indicate that central banks should target the growth rate of output rather than the level of the output gap. The rationale for the modification is that the growth rate of output is known, while the size of the output gap—or more specifically, potential output—is unobservable in real time and subject to substantial shifts over the short to medium term. In practice, potential output at any point in time is measured as a function of the real GDP data observed before and after that particular time. Most of the variation in potential output is attributable to movements in output. An output growth rate rule is a practical alternative to an output gap rule because the output growth rate remains fairly stable after a persistent productivity shock that is not observed immediately by the central bank. Lastly, the output gap is not the best policy instrument to target because potential output is determined by factors beyond the control of the central bank.

We have treated the output gap as the relevant measure of the state of the real economy. Our theoretical results and the empirical evidence about trends in potential output are also applicable to the unemployment rate since there is approximately a one-to-one relationship between the output gap and the unemployment gap (i.e., the unemployment rate minus the natural rate of unemployment). Nevertheless, it is just as difficult to measure the natural rate of unemployment as it is to measure potential output. An analogous policy that is not subject to large measurement errors is targeting the change in the unemployment gap rather than its level.[21]

Finally, our results about a price-level path rule depend critically on the assumption that people are rational and consider central bank behavior when forming expectations about inflation and nominal interest rates. This assumption does not mean that people have perfect knowledge about how the economy works or perfect foresight about what central bankers will do. It simply means that households and firms will gather and use information about how the central bank conducts monetary policy when making their own decisions.

NOTES

1 See also Orphanides (2003a,b). Edge, Laubach, and Williams (2007) investigate a model in which agents learn about shifts in long-run productivity growth.

2 See, for example, the papers collected in Taylor (1999b) and on the Monetary Policy Rule Home Page website (www.stanford.edu/~johntayl/PolRulLink.htm).

3 See the appendix to Gavin, Keen, and Pakko (2005) for a detailed description of the firm's pricing problem.

4 We are ignoring two distortions. The first is due to the monopolistically competitive firms that produce less than they would in a perfectly competitive world. The second is the loss associated with the shopping time constraint. At a zero inflation rate, the return on money is less than the return on bonds and people will hold lower real money balances and spend more time shopping than they would if the nominal interest rate were zero.

5 Note that none of the policies that stabilize inflation may perform well if the public is as irrational and backward looking as is typically assumed in forecasting models.

6 The elasticity of labor supply with respect to the real wage equals $(1 - \bar{n} - \bar{s})/(\bar{n}\omega)$.

7 The interest rate elasticity of money demand is approximately equal to $-1/(1+\gamma)$.

8 In New Keynesian theory, the concept of the efficient level of output is used to measure the output gap. The efficient level of output is that which would occur in the absence of sticky prices. Neither the Fed nor the statistical agencies attempt to measure the efficient level of output. There is the possibility that the distortion from sticky prices is actually quite small and that actual and efficient levels of output are similar.

9 Figure 2 excludes data after 2000:Q4 because those data are subject to future comprehensive revisions. The revision process is discussed in the National Income and Product Accounts (NIPA) Handbook (www.bea.gov/methodologies/index.htm#national_meth).

10 The value of 0.837 is not explicitly stated in their article but was verified in a private communication with Eric Sims.

11 In our model, this shock is less than one-quarter of the size necessary to account for the decline in potential GDP observed since 2007:Q4.

12 For a recent survey of the literature, see Gaspar, Smets, and Vestin (2010).

13 The optimal policy for the shopping time feature is to saturate the economy with money balances and drive the nominal interest rate to zero. We disregard issues surrounding operating a monetary policy with a zero nominal interest rate because the traditional solution methods used in this article are not easily adaptable to such a model. Wolman (2005), Coibion, Gorodnichenko, and Weiland (2010), and Gavin and Keen (2011) show that economies in which the central bank adopts some version of a price-level path target are not likely to hit the zero lower bound.

14 Basu, Fernald, and Kimball (2006) and Francis and Ramey (2005) provide empirical evidence that hours worked declines after a positive technology shock.

15 This condition, sometimes referred to as the "Taylor principle" (Taylor, 1999a), states that a percentage-point change in the nominal interest rate target must exceed the corresponding change in the inflation rate.

16 Several authors, including Orphanides and Williams (2002) and Walsh (2003), have recommended replacing the output gap with the output growth rate.

17 Gaspar, Smets, and Vestin (2007) survey the literature on price-level path rules, and Gorodnichenko and Shapiro (2007) show that including a price-level path target in the policy rule generally improves the performance of the economy in the presence of temporary shifts in productivity growth.

18 Our calibration of θ_p is based roughly on the relationship between Hodrick-Prescott-filtered data on the price level and the nominal interest rate. Specifically, volatility of the percent deviation of the consumer price index from its long-run trend is similar to that of the federal funds rate over the past two decades.

19 This definition of the output gap is suggested by Neiss and Nelson (2003).

20 The long-term volatility of the output gap and inflation is considered because many papers measure welfare loss as a weighted average of the fluctuations in the output gap and inflation.

21 See Orphanides and Williams (2002) for analysis of unobserved shifts in the unemployment gap.

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APPENDIX

A.1 Nonlinear Equations

(A.1.1)
$$\frac{\lambda_t}{P_t} = \beta R_t E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right]$$

(A.1.2)
$$w_t \lambda_t = \chi\left(l_t\right)^{-\omega}$$

(A.1.3)
$$\frac{1}{c_t} = \lambda_t + \gamma \frac{s_t}{c_t}\chi\left(l_t\right)^{-\omega}$$

(A.1.4)
$$\gamma w_t s_t = \left(1 - \frac{1}{R_t}\right) m_t$$

(A.1.5)
$$\lambda_t = \tau_t \varphi'\left(\frac{i_t}{k_t}\right)$$

(A.1.6)
$$\tau_t = \beta E_t\left[\tau_{t+1}\left(\left(1-\delta\right) + \varphi\left(\frac{i_{t+1}}{k_{t+1}}\right) - \varphi'\left(\frac{i_{t+1}}{k_{t+1}}\right)\left(\frac{i_{t+1}}{k_{t+1}}\right) + \varphi'\left(\frac{i_{t+1}}{k_{t+1}}\right) q_{t+1}\right)\right]$$

(A.1.7)
$$l_t + n_t + s_t = 1$$

(A.1.8)
$$k_{t+1} - k_t = \varphi\left(\frac{i_t}{k_t}\right)k_t - \delta k_t$$

(A.1.9)
$$s_t = \zeta\left(\frac{c_t}{m_t}\right)^{\gamma}$$

(A.1.10)
$$c_t + i_t = y_t$$

(A.1.11)
$$y_t = \left(k_t\right)^{\alpha}\left(Z_t n_t\right)^{(1-\alpha)}$$

(A.1.12)
$$q_t = \alpha \psi_t \left(Z_t n_t\right)^{(1-\alpha)}\left(k_t\right)^{(\alpha-1)}$$

(A.1.13)
$$w_t = \left(1-\alpha\right)\psi_t\left(Z_t\right)^{(1-\alpha)}\left(k_t\right)^{\alpha}\left(nt\right)^{-\alpha}$$

(A.1.14)
$$P_t = \left[\eta\left(P_t^*\right)^{(1-\varepsilon)} + \left(1-\eta\right)\left(P_{t-1}\right)^{(1-\varepsilon)}\right]^{1/(1-\varepsilon)}$$

(A.1.15) $$P_t^* = \left(\frac{\varepsilon}{\varepsilon - 1}\right) E_t \left[\frac{\sum_{i=0}^{\infty} \beta^i \left(1-\eta\right)^i \lambda_{t+1} P_{t+i}^{\varepsilon} \psi_{t+i} y_{t+i}}{\sum_{i=0}^{\infty} \beta^i \left(1-\eta\right)^i \lambda_{t+i} P_{t+i}^{\varepsilon-1} y_{t+i}} \right]$$

(A.1.16) $$R_t = \pi_t^{\left(1+\theta_\pi\right)} y_t^{\theta_y} g_t^{\theta_g} P_t^{\theta p}$$

(A.1.17) $$g_t = \frac{y_t}{y_{t-1}}$$

(A.1.18) $$\pi_t = \frac{P_t}{P_{t-1}}$$

(A.1.19) $$\frac{Z_t}{Z_{t-1}} = \left(\frac{Z_{t-1}}{Z_{t-2}}\right)^{\rho_z} \left(\bar{g}\right)^{1-\rho_z} e^{\nu_t}$$

A.2 Steady-State Equations

(A.2.1) $$\beta \bar{R} = \overline{g\pi}$$

(A.2.2) $$\bar{w}\bar{\lambda} = \chi\left(\bar{l}\right)^{-\omega}$$

(A.2.3) $$\frac{1}{\bar{c}} = \bar{\lambda} + \gamma \frac{\bar{s}}{\bar{c}} \chi\left(\bar{l}\right)^{-\omega}$$

(A.2.4) $$\bar{\lambda} = \bar{\tau}\varphi'\left(\cdot\right)$$

(A.2.5) $$\bar{q} = \frac{\bar{g}}{\bar{\beta}} - 1 + \delta$$

(A.2.6) $$\bar{w}\ \bar{s}\ \gamma = \left(1 - \frac{1}{\bar{R}}\right)\bar{m}$$

(A.2.7) $$\bar{l} + \bar{n} + \bar{s} = 1$$

(A.2.8) $$\bar{i} = \left(\bar{g} - 1 + \delta\right)\bar{k}$$

(A.2.9) $$\bar{s} = \zeta\left(\frac{\bar{c}}{\bar{m}}\right)^{\gamma}$$

(A.2.10) $$\bar{c} + \bar{i} = \bar{y}$$

(A.2.11) $$\bar{y} = \left(\bar{k}\right)^{\alpha} \left(\bar{Z}\bar{n}\right)^{(1-\alpha)}$$

(A.2.12) $$\bar{q} = \alpha\bar{\psi}\left(\bar{Z}\bar{n}\right)^{(1-\alpha)} \left(\bar{k}\right)^{(\alpha-1)}$$

(A.2.13) $$\bar{w} = (1-\alpha)\bar{\psi}\left(\bar{Z}\right)^{(1-\alpha)} \left(\bar{k}\right)^{\alpha} \left(\bar{n}\right)^{-\alpha}$$

(A.2.14) $$\bar{\psi} = \frac{(\varepsilon - 1)}{\varepsilon}$$

(A.2.15) $$\bar{P} = \bar{P}^{*} = 1$$

A.3 Linearized Equations

(A.3.1) $$\hat{\lambda}_t - \hat{R}_t = E_t\left[\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\right]$$

(A.3.2) $$\hat{w}_t + \hat{\lambda}_t = -\omega\hat{l}_t$$

(A.3.3) $$\left[\left(\frac{1}{\bar{c}}\right) - \gamma\left(\frac{\bar{s}}{\bar{c}}\right)\chi\left(\bar{l}\right)^{-\omega}\right]\hat{c}_t + \bar{\lambda}\hat{\lambda}_t + \left[\gamma\left(\frac{\bar{s}}{\bar{c}}\right)\chi\left(\bar{l}\right)^{-\omega}\right]\left(\hat{s}_t - \omega\hat{l}_t\right) = 0$$

(A.3.4) $$\hat{w}_t + \hat{s}_t - \left(\frac{1}{\bar{R}-1}\right)\hat{R}_t = \hat{m}_t$$

(A.3.5) $$\left(\frac{\left(\frac{\bar{i}}{\bar{k}}\right)\varphi''(\cdot)}{\varphi'(\cdot)}\right)\left(\hat{i}_t - \hat{k}_t\right) + \hat{\tau}_t = \hat{\lambda}_t$$

(A.3.6) $$\hat{\tau}_t = E_t\left[\left[\left(\frac{\beta}{\bar{g}}\right)\left(\frac{\bar{i}}{\bar{k}}\right)\varphi''(\cdot)\left(\bar{q} - \left(\frac{\bar{i}}{\bar{k}}\right)\right)\right]\left(\hat{i}_{t+1} - \hat{k}_{t+1}\right) + \left(\frac{\beta\varphi'(\cdot)\bar{q}}{\bar{g}}\right)\bar{q}_{t+1} + \hat{\tau}_{t+1}\right]$$

(A.3.7) $$\bar{l}\ \hat{l}_t + \bar{n}\ \hat{n}_t + \bar{s}\ \hat{s}_t = 0$$

(A.3.8) $$\left(\frac{\bar{i}}{\bar{k}}\right)\varphi'(\cdot)\hat{l}_t + \left[1 - \delta + \varphi(\cdot) - \left(\frac{\bar{i}}{\bar{k}}\right)\varphi'(\cdot)\right]\bar{k}_t = \bar{g}\bar{k}_{t+1}$$

(A.3.9) $$\hat{c}_t - \hat{m}_t = \left(\frac{1}{\gamma}\right)\hat{s}_t$$

(A.3.10) $$\bar{c}\hat{c}_t + \bar{i}\hat{i}_t = \bar{y}\hat{y}_t$$

(A.3.11) $$\alpha\hat{k}_t + (1-\alpha)\left(\hat{Z}_t + \hat{n}_t\right) = \hat{y}_t$$

(A.3.12) $$\hat{\psi}_t + (1-\alpha)\left(\hat{Z}_t + \hat{n}_t - \hat{k}_t\right) = \hat{q}_t$$

(A.3.13) $$\hat{\psi}_t + (1-\alpha)\hat{Z}_t + \alpha\left(\hat{k}_t - \hat{n}_t\right) = \hat{w}_t$$

(A.3.14) $$\hat{\pi}_t = \left(\frac{(1-\eta)(1-\beta\eta)}{\eta}\right)\hat{\psi}_t + \beta E_t\left[\hat{\pi}_{t+1}\right]$$

(A.3.15) $$\bar{R}_t = \left(1+\theta_\pi\right)\hat{\pi}_t + \theta_y\hat{y}_t + \theta_g\hat{g}_t + \theta_p\hat{p}_t$$

(A.3.16) $$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$$

(A.3.17) $$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1}$$

(A.3.18) $$\hat{Z}_t = \left(1+\rho_z\right)\hat{Z}_{t-1} - \rho_z\hat{Z}_{t-2} + \nu_t$$

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