Panel Discussion

The Role of Potential Output Growth in Monetary Policymaking in Brazil

Carlos Hamilton Araujo

Potential output is important in policymaking for a number of reasons:

- It is a key variable in most macroeconomic models because it enables construction of measures of the output gap. These measures are often used in the IS and Phillips curves and the Taylor rule, among others.
- It provides a measure of economic slack (i.e., its cyclical position).
- It helps to gauge future inflation pressures.
- It is important for estimating cyclically adjusted variables (e.g., structural fiscal deficit).

However, potential output is difficult to handle. As a latent variable, it is hard to measure in any circumstance, and frequent data revisions worsen the accuracy of any estimation. For example, in Brazil, the available time series has a short data span and the methodology for calculating gross domestic product (GDP) has changed frequently. Another potential problem is that geographic data might also be inadequate (e.g., the unemployment rate).

The Central Bank of Brazil uses a variety of statistical methods to measure potential output. The most common are statistical filters, including the Hodrick-Prescott filter, band-pass filters, Kalman filters, and Beveridge-Nelson decompositions. These methods are not based on economic theory or models, and each has its idiosyncrasies—sometimes with opposite identifying assumptions. As a general rule, the rationale is the same for all: to decompose the GDP time series into a permanent component and a transitory, cyclical component to measure the output gap. It is a shortcoming of these measures that they do not consider information other than GDP itself. They often behave like moving averages and, hence, perform poorly when the original GDP series faces large and sudden changes. In addition, the resulting filtered time series is frequently judged too volatile relative to the prior beliefs of senior policymakers.

We also use macroeconomic methods, including Cobb-Douglas production functions, structural vector autoregressions, dynamic stochastic general equilibrium models, and other macro models. To some extent, these are based on economic theory and may impose quite strong restrictions on the data. In addition, estimates are model dependent, which often leads to disagreement regarding the "true" model; furthermore, estimates are sensitive to model specification error. Given these restrictions, these models might be more difficult to estimate than with the previous statistical methods and may ignore key determinants of potential output.

Now, consider the simplest production function approach:

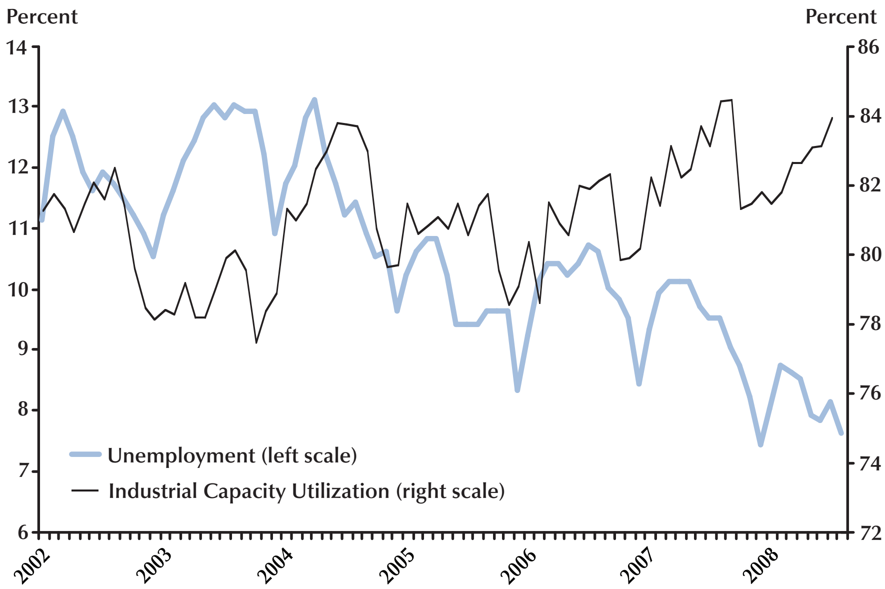
$$\bar{Y}_t = \bar{A}_t \bar{K}_t^{\alpha} \bar{L}_t^{1-\alpha}.$$

Carlos Hamilton Araujo is the head of the research department at the Central Bank of Brazil.

Federal Reserve Bank of St. Louis *Review*, July/August 2009, *91*(4), pp. 383-85.

Figure 1

Unemployment Rate and Capacity Utilization



This approach is based on widely accepted economic theory and explicitly specifies the sources of economic growth. An alternative specification that we find useful is the following:

$$Gap_t = \alpha\left[\log\left(cu_t\right) - \ln\left(NAIRCU_t\right)\right] + \left(1-\alpha\right)\left[\log\left(1-un_t\right) - \ln\left(1-NAIRU_t\right)\right],$$

where *NAIRCU* is the natural rate of capacity utilization and *NAIRU* is the natural unemployment rate. The unemployment rate and capacity utilization rate for Brazil are shown in Figure 1.

Regardless of the adopted measure, potential output estimates are always uncertain. In this sense, the Bank relies on additional economic variables as a cross-check of economic activity; these variables include unemployment, capacity utilization, industrial production, retail sales, wage growth, and surveys of corporate confidence. Thus, various potential output measures are compared by computer simulations, focused on using Phillips curves to forecast inflation and on comparisons with predictions from Okun's law.

The relationship between output gap estimates and potential inflationary pressures is of utmost importance to the Monetary Policy Committee. Yet, in my view, indicators of inflation expectations are more important drivers of policy decisions than the output gap.

Potential output and capital growth are essential elements of capital deepening and output growth. Both have shown significant acceleration in recent years. Although explanations for the acceleration are uncertain, possible reasons include increased macroeconomic stability due to a new political environment (more favorable to planning); strong inflows of foreign capital in the form of foreign direct investment, bringing with it new technology; exchange rate appreciation, which sharply reduced the cost of imported capital goods; and the culmination of educational improvements, resulting in a higher-quality labor force.

Looking forward, we anticipate a slowing of economic growth. It is likely that slower GDP growth will adversely affect potential output through similar channels: a reduction in foreign direct investment inflows; less-accommodative credit conditions, both in the domestic and international markets; and exchange rate depreciation that will increase the cost of imported capital goods.

In a nutshell: Potential output is regarded as a key indicator for assessing the slack in the economy and gauging the buildup of inflationary pressures. Because it is not observable, potential output estimates are imprecise and worsened by short and volatile time series. At the Central Bank of Brazil, we use many purely statistical and structural methods to assess potential output. Evidence and experience favor structural methods. We seek to mitigate the related uncertainties by using several methods, as well as other excess demand indicators. In policymaking, the Bank places a larger weight on inflation expectations, in addition to its estimates of the output gap.

The Role of Potential Growth in Policymaking

Seppo Honkapohja

DEFINITIONS OF POTENTIAL OUTPUT AND POTENTIAL GROWTH

Currently, differing concepts of potential output and potential growth are used in both academic research and policy discussions. Traditionally, potential output and potential growth are measures of the average productive capacity of an economy and its change over time. Correspondingly, the output gap is the deviation of actual output from its potential value, that is, from average output. If potential output is viewed as (in some sense) average output, then potential output is naturally measured by fitting a statistical trend on the path of output over time. John Taylor's (1993) seminal paper on estimated interest rate rules used such a traditional measure for the output gap. Alternatively, potential growth might be measured by fitting trends to paths of factor supplies and using these in an estimated production function.

Nowadays, it is common to use a specific economic model to estimate potential output, its growth rate, and the output gap. Here is a simple example. Consider an economy with perfect competition and a Cobb-Douglas production function, $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. The log-linearization of this production function is

$$y_t = \alpha k_t + (1-\alpha) n_t + a_t, \tag{1}$$

where lower-case letters denote logarithms of output, capital, labor input, and total factor productivity (TFP). The log of TFP, a_t, evolves exogenously, while the actual values of y_t and n_t are determined as part of the competitive equilibrium. The difficulty is that TFP cannot be directly observed but must be obtained as a residual from equation (1), using an estimate or calibrated value for α.[1] With such an estimate, we obtain potential output, y_t^p, as

$$y_t^p = \bar{\alpha}_t k_t + (1-\bar{\alpha}_t) n_t + \bar{a}_t,$$

where $\bar{a}_t$ and $\bar{\alpha}_t$ are estimates of TFP and parameter α, respectively, for period t. Although model-based, this calculation often produces measures

[1] As is well known, there are also more sophisticated ways to estimate TFP. For example, see Chambers (1988, Chap. 6) for an introductory discussion.

Seppo Honkapohja is a member of the Board of the Bank of Finland and a research fellow at the Centre for Economic Policy Research.

Federal Reserve Bank of St. Louis *Review*, July/August 2009, *91*(4), pp. 385-89.

close to the traditional statistical measures. In this case, the output gap is $y_t - y_t^p$.

In the preceding formulation, $\bar{a}_t$ and $\bar{\alpha}_t$ may be ex post or real-time estimates of TFP and α, respectively. In practice, there are short- to medium-term policy concerns that require real-time measurement of the output gap. A possible policy objective can be to smooth fluctuations in aggregate output. Of course, in a competitive economy without distortions, there is no reason to offset the random fluctuations in y_t, as the equilibrium is Pareto efficient. If there are distortions, one might have some interest in measuring the real-time output gap, but this depends on the nature of the distortions and whether they vary cyclically. Naturally, measurement of potential output and potential growth is also important for setting growth policy; for example, the so-called Lisbon Agenda was devised to address the sluggish growth of most Western European Union countries. Such issues are long-run policy concerns and the background studies are based on, for example, growth accounting methodologies with ex post estimates for parameters. In such cases, there is no urgency to obtain real-time measurement for potential output.

STICKY PRICES

Though some disagreements exist, it is now a common view that the perfectly competitive, flexible-price model is not relevant for short- to medium-run policymaking. The current workhorse for monetary policy analysis is the New Keynesian (NK) model, which differs from the perfect-competition model in two crucial respects: In it the economy is imperfectly competitive and displays nominal price and/or wage rigidity.[2]

We modify the model outlined above by introducing differentiated goods and imperfect competition. Log-linearized optimal consumption behavior as a log-deviation from the steady state is described by the Euler equation,

$$c_t = E_t c_{t+1} - \sigma^{-1}\left(i_t - E_t \pi_{t+1} - \rho\right),$$

where $\rho = -\log\beta$ and β is the subjective discount factor of the economy and σ is a utility function parameter. In equilibrium,

$$C_t = Y_t$$

and, therefore, we obtain the dynamic IS curve,

$$y_t = E_t y_{t+1} - \sigma^{-1}\left(i_t - E_t \pi_{t+1} - \rho\right). \tag{2}$$

Here lower-case variables denote log-deviations from the steady state. Equation (2) indicates that aggregate output in the economy depends positively on expectations of next-period output and negatively on the real rate of interest, where the latter is defined in terms of expected next-period inflation.

The dynamics of inflation are described by an aggregate supply curve, also called the NK Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda\left(mc_t - mc\right),$$

where inflation (as a deviation from the steady state) depends on expected inflation and the deviation of marginal cost from its steady-state value. Here λ is a function of several structural parameters. It can be shown that

$$mc_t = w_t - p_t - \frac{1}{1-\alpha}\left(a_t - \alpha y_t\right) - \log\left(1-\alpha\right).$$

It is possible to write $mc_t - mc$ in terms of a new measure of the output gap:

$$\tilde{y}_t = y_t - y_t^n, \text{ where}$$

$$y_t^n = \frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha} a_t - \frac{(1-\alpha)\left(\mu - \log(1-\alpha)\right)}{\sigma(1-\alpha)+\psi+\alpha}.$$

Here y_t^n is the natural level of output, that is, aggregate output at the flexible price (but monopolistically competitive) level.[3] Note that $y_t^n < y_t^{CE}$ because of imperfect competition. Note also that the natural level of output is different from potential output of the economy.

[2] There are several good expositions of the NK model. The formal details below are based on the excellent exposition of the NK model in Galí (2008).

[3] ω is a utility function parameter, whereas κ is the log of the steady-state markup.

Using the output gap, the inflation equation can be written

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t,$$

which implies that from a business cycle viewpoint the output gap, measured as just explained, is the relevant concept for monetary policy analysis. The dynamic IS curve can also be written in terms of the output gap as

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma^{-1}\left(i_t - E_t \pi_{t+1} - r_t^n\right), \text{ where}$$
$$r_t^n = \rho + \sigma \frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha} E_t\left(\Delta a_{t+1}\right).$$

Here r_t^n is the natural rate of interest.

To summarize, monetary policy analysis uses two different concepts of the output gap and both are used in monetary policy analysis. The traditional concept of potential output and the output gap are defined by the deviation from trend, whereas the recent model-based notion of the output gap is defined as the difference between actual output and the flexible price level of output. The two concepts are different and can behave in different ways, as vividly illustrated by Edge, Kiley, and Laforte (2008, Figure 1) and also studied by, for example, Andres, Lopez-Salido, and Nelson (2005) and Justiniano and Primiceri (2008).

NOISY DATA

The NK model outlined above suggests that, in theory, the output gap measure $\tilde{y}_t$, derived in the NK model (or analogously in dynamic stochastic general equilibrium models), is the appropriate measure of potential output for monetary policy analysis. It should be emphasized that this view holds only in theory for several reasons. First, any model-based output-gap measure is model dependent and thus capable of generating misleading recommendations. One should always examine the robustness of conclusions based on a specific model and the corresponding measure.

Second, how a measure will be used should be considered before deciding which model to use. The output gap measure based on the NK model is intended for analysis of inflation control and often does not measure well the economy's deviations from its long-term productive capacity.

Third, even if one opts for the measure based on the NK model, assumptions about the availability of output gap information are very strong in the standard analysis of monetary policy in dynamic stochastic general equilibrium models. Policies that perform well under the usual rational expectations (RE) assumption, for example, often do not perform well if the measurement of the output gap and other variables contain significant noise. Orphanides emphasizes this problem in a number of papers (e.g., see Orphanides, 2003). In particular, he states that naive optimal policies derived under RE often do poorly if there are noisy measurements of the true variables.[4]

In principle, optimal control policies that take into account the measurement problem can be calculated using Kalman filters; however, this approach can be sensitive to measurement problems caused by imperfect knowledge. Neither the "correct model" nor the data are, in practice, fully known to the policymaker (further discussed below). The use of well-performing simple rules offers another approach to the problem of noisy measurements. In some cases, though not optimal, simple rules work better than naive optimal rules. Such simple rules have the same functional form as naive optimal rules but respond to noisy real-time data appropriately when the policy coefficients are chosen optimally.

OTHER ASPECTS OF IMPERFECT KNOWLEDGE

Noisy data are just one aspect of knowledge imperfections that policymakers face, and although there are several others, I focus on learning effects—that is, that economic evolution in the short to medium run can be significantly influenced by learning effects from economic agents trying to improve their knowledge. The literature on learning and macroeconomics has been widely researched in recent years, and mone-

[4] In addition, data revisions are often significant and make it difficult to use model-based measures, so they will not be discussed further.

tary policy design has been shown to be affected by one's learning viewpoint.

The basic ideas in learning are that (i) agents and policymakers have imperfect knowledge, (ii) expectations are based on existing knowledge and updated over time using econometric techniques, and (iii) expectations feed into decisions by agents and hence to actual outcomes and future forecasts. Learning dynamics converge to an RE equilibrium, provided that the economy satisfies an expectational stability criterion. Good policy facilitates convergence of learning.

Basic learning models use fairly strong assumptions: (i) Functional forms of agents' forecasting models are correctly specified relative to the RE equilibrium, (ii) agents accurately observe relevant variables, and (iii) economic agents trust their forecasting model. Most of these assumptions have been weakened in the recent literature. Misspecification is certainly one concern because it can inhibit convergence to an RE equilibrium and create a restricted-perceptions equilibrium. However, the implications of this for policy design are not further discussed here.

Noisy measurements have been incorporated into some models of monetary policy that include learning, most notably by Orphanides and Williams (2007 and forthcoming). Basically, these models show that the ideas discussed above still hold. One can try to consider filtering and learning together, but this is likely to be formally demanding and has not been studied. Alternatively, one can use simple rules that work well. In particular, the recent papers by Orphanides and Williams (2007, forthcoming) suggest the use of rules that do not rely on data subject to significant noise.

A specific measurement problem is agents' private expectations. It has been shown that expectations-based optimal rules would work well for optimal monetary policy design. If there are significant errors in measuring private-sector expectations, one can try to develop proxies for them. This is, in fact, typically done, perhaps using survey data from either professional forecasters or consumer surveys. An alternative is model-based proxies from a variety of sources, including indexed and non-indexed bonds, swaps, or information from purely statistical forecasting models.

If agents do not trust their personal forecasting model, then they may wish to allow for uncertainty in their forecasting model and/or their behavioral attitudes. If one allows for unspecified model uncertainty in estimation, then robust estimation methods can be used. In fact, a "maximally robust" estimation leads to so-called constant-gain stochastic gradient algorithms, which have been studied for learning in Evans, Honkapohja, and Williams (forthcoming). Of course, literature on economic behavior in the presence of unstructured model uncertainty abounds (see Hansen and Sargent, 2007). In policy design, one can also incorporate aspects of robust policy with respect to learning by private agents. Usually, it is assumed that the policymaker does not know the learning rules of private agents,[5] but considers as policy constraints *E*-stability conditions for private-agent learning; that is, recursive least squares learning is assumed. One could make additional assumptions about learning or even identify stability conditions that are robust in some sense (see, e.g., Tetlow and von zur Muehlen, 2009).

REFERENCES

Andres, Javier; Lopez-Salido, J. David and Nelson, Edward. "Sticky-Price Models and the Natural Rate Hypothesis." *Journal of Monetary Economics*, July 2005, *52*(5), pp. 1025-53.

Chambers, Robert G. *Applied Production Analysis*. Cambridge: Cambridge University Press, 1988.

Edge, Rochelle M.; Kiley, Michael T. and Laforte, Jean-Philippe. "Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy." *Journal of Economic Dynamics and Control*, August 2008, *32*(8), pp. 2512-35.

Evans, George W.; Honkapohja, Seppo and Williams, Noah. "Generalized Stochastic Gradient Learning." *International Economic Review* (forthcoming).

[5] A few papers consider policy optimization with respect to learning rules of private agents.

Galí, Jordi. *Monetary Policy, Inflation, and the Business Cycle*. Princeton: Princeton University Press, 2008.

Hansen, Lars Peter and Sargent, Thomas J. *Robustness*. Princeton: Princeton University Press, 2007.

Justiniano, Alejandro and Primiceri, Giorgio E. "Potential and Natural Output." Unpublished manuscript, Northwestern University, June 2008; http://faculty.wcas.northwestern.edu/~gep575/JPgap8_gt.pdf.

Orphanides, Athanasios. "Monetary Policy Evaluation with Noisy Information." *Journal of Monetary Economics*, April 2003, *50*(3), pp. 605-31.

Orphanides, Athanasios and Williams, John C. "Robust Monetary Policy with Imperfect Knowledge." *Journal of Monetary Economics*, July 2007, *54*(5), pp. 1406-35.

Orphanides, Athanasios and Williams, John C. "Monetary Policy Mistakes and the Evolution of Inflation Expectations," in Michael D. Bordo and Athanasios Orphanides, eds., *The Great Inflation*. Chicago: University of Chicago Press (forthcoming).

Taylor, John B. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, December 1993, *39*, pp. 195-214.

Tetlow, Robert and von zur Muehlen, Peter. "Robustifying Learnability." *Journal of Economic Dynamics and Control*, February 2009, *33*(2), pp. 292-316.

The Role of Potential Output in Policymaking*

James Bullard

Often, economists equate potential output with the trend in real gross domestic product (GDP) growth. My discussion is focused on "proper" detrending of aggregate data. I will emphasize the idea that theory is needed to satisfactorily detrend data—explicit theory that encompasses simultaneously both longer-run growth and shorter-run fluctuations. The point of view I wish to explore stresses that both growth *and* fluctuations must be included in the same theoretical construct if data are to be properly detrended. Common atheoretic statistical methods are not acceptable. When detrending data, an economist should detrend by the theoretical growth path so as to correctly distinguish output variance in the model due to growth from the variation in the model due to cyclical fluctuations.

The quest to fully integrate growth and cycle was Prescott's initial ambition; however, it is difficult to develop a model that can match the curvy, time-varying growth path often envisioned as describing an economy's long-run development. Instead, the Hodrick-Prescott (HP) filter was proposed to remove from the data a flexible time-varying trend (Hodrick and Prescott, 1980). My argument is that this procedure is unsatisfactory.

The idea in question is: How can we specify a model that will make the growth path look like the ones we see in the data? My suggestion is that,

* This discussion is based on the panel discussion, "The Role of Potential Output in Policymaking," available at http://research.stlouisfed.org/econ/bullard/FallPolicyConferenceBullard16oct2008.pdf and on Bullard and Duffy (2004).

James Bullard is president of the Federal Reserve Bank of St. Louis.

Federal Reserve Bank of St. Louis *Review*, July/August 2009, *91*(4), pp. 389-95.

as an initial approach, we use a mainstream core growth model augmented with occasional trend breaks and learning. Learning helps the model fit the data and has important implications for policy analysis. I will discuss some applications of this idea in Real Business Cycle (RBC) and New Keynesian (NK) models from Bullard and Duffy (2004) and Bullard and Eusepi (2005).

MAIN IDEAS

The equilibrium business cycle literature encompasses a wide class of models, including RBC, NK, and multisector growth models. Various frictions can be introduced in all of these approaches. Many analyses do not include any specific reference to growth, but all are based on the concept of a balanced growth path.

I will focus on a framework that is very close to the RBC model. This will provide a well-understood benchmark. However, I stress that these ideas have wide applicability in other models as well, and I will briefly discuss an NK application at the end.

Empirical studies, such as Perron (1989) and Hansen (2001), have suggested breaks in the trend growth of U.S. economic activity. One reasonable characterization of the data is to assume log-linear trends with occasional trend breaks but no discontinuous jumps in level—that is, a linear spline. This is the bottom line of Perron's econometric work. These breaks, however, suggest a degree of nonstationarity that is difficult to reconcile with available theoretical models. This is where adding learning can be helpful.

CURRENT PRACTICE

The standard approach in macroeconomics today is to analyze separately business cycles and long-term growth. The core of this analysis is the statistical trend-cycle decomposition. The standard method in the literature for trend-cycle decomposition is to use atheoretic, univariate statistical filters, that is, to conduct the decomposition series by series (see, for example, King and Rebelo, 1999). This method ignores an important dictum implied by the balanced growth path assumption: There are restrictions as to how the model's variables can grow through time and in turn, therefore, how one is allowed to detrend data. In an appalling lack of discipline, economists ignore this dictum and detrend data individually, series by series, which makes little sense in any growth theory. An acceptable theory specifies growth paths for the model's variables (i.e., consumption, investment, output); individual trends should not be taken out of the data. Still, the ad hoc practice dominates the literature.

Most of my criticisms are well known:

- Statistical filters do not remove the "trend" that the balanced growth path requires.
- Current practice does not respect the cointegration of the variables, that is, the multivariate trend that the model implies.
- Filtered trends imply changes in growth rates over time; agents would want to react to these changes and adjust their behavior. A model without growth does not allow for this change in behavior.
- The "business cycle facts" are not independent of the statistical filter employed. The econometrics literature normally—but not always—filters the data so as to achieve stationarity for estimation and inference without regard to the underlying theory's balanced growth assumption. Even recent sophisticated models (for examples, Smets and Wouters, 2007) do not address this issue.

HOW TO IMPROVE ON THIS?

The criticisms are correct in principle. They are quantitatively important. And, these issues cannot be resolved by using alternative statistical filters, because those filters are atheoretic. Instead, theory should be used to tell us what the growth path should look like; then, this theoretical trend can be used to detrend the data.

In the model I discuss, agents are allowed to react to trend changes. The ability to react to changes in trends alters agents' behavior—how much they save, how much they consume, and so forth. Of course, this is demanding territory. I am insisting that the theorist specify both the longer-term growth and short-run business cycle aspects of a model, and then explain the model's coherence to observed data. This is the research agenda I propose.

Core Ideas

The core idea is that modelers should use "model-consistent detrending," that is, the trends that are removed from the data are the same as the trends implied by the specified model. Presumably, changes in trend are infrequent and, perhaps with some lag, are recognized by agents who then react to them. This suggests a role for learning. In addition, the cointegration of the variables or the different trends in the various variables implied by the balanced growth path is respected.

FEATURES OF THE ENVIRONMENT

As an example, I will discuss briefly the most basic equilibrium business cycle model with exogenous stochastic growth, but replace rational expectations with learning as in Evans and Honkapohja (2001). This model perhaps is appropriate when there is an unanticipated, rare break in the trend (for example, a labor productivity slowdown or acceleration). I assume agents possess a tracking algorithm and are able to anticipate the characteristics of the new balanced growth path that will prevail after the productivity slowdown occurs. If there is no trend break for a sufficient period, then there is convergence to the rational expectations equilibrium associated with that balanced growth path. Learning helps around points where there is a structural break of some type by allowing the economy to converge to the new balanced growth path following the structural break. In order for this to work, of course, the model must be expectationally stable such that the model's implied stochastic process will remain near the growth path.

Environment

The environment studied by Bullard and Duffy (2004) is a standard equilibrium business cycle model such as the one studied by Cooley and Prescott (1995) or King and Rebelo (1999). A representative household maximizes utility defined over consumption and leisure. Physical capital is the only asset. Business cycles are driven by shocks to technology. Bullard and Duffy (2004) include explicit growth in the model. Growth in aggregate output is driven by exogenous improvements in technology over time and labor force growth. The growth rate is exogenous and constant, except for the rare trend breaks that are incorporated. The production technology is standard. Under these assumptions, aggregate output, consumption, investment, and capital will all grow at the same rate along a balanced growth path.

Structural Change

The idea of structural change in this setting is simply that either the growth rate of technology or of the labor force takes on a new value. In the model, changes of this type are unanticipated. This will dictate a new balanced growth path, and the agents learn this new balanced growth path.

In order to use the learning apparatus as in Evans and Honkapohja (2001), a linear approximation is needed. Using logarithmic deviations from steady state, one can define and rewrite the system appropriately, as Bullard and Duffy (2004) discuss extensively. One must be careful about this transformation because the steady-state values can be inferred from some types of linear approximations, but we really don't want to inform the agents that the steady state of the system has changed. We want the agents to be uncertain where the balanced growth path is and learn the path over time.

Recursive Learning

Bullard and Duffy (2004) study this system under a recursive learning assumption as in Evans and Honkapohja (2001). They assume agents have no specific knowledge of the economy in which they operate, but are endowed with a perceived law of motion (PLM) and are able to use this PLM—a vector autoregression—to learn the rational expectations equilibrium. The rational expectations equilibrium of the system is determinate under the given parameterizations of the model.

Should a trend break occur—say, a productivity slowdown or speedup—the change will be manifest in the coefficients associated with the rational expectations equilibrium of this system. The coefficients will change; agents will then update the coefficients in their corresponding regressions, eventually learning the correct coefficients. These will be the coefficients that correspond to the rational expectations equilibrium after the structural change has occurred.

Expectational Stability

For this to work properly the system must be expectationally stable. Agents form expectations that affect actual outcomes; these actual outcomes feed back into expectations. This process must converge so that, once a structural change occurs, we can expect the agents to locate the new balanced growth path. Expectational stability (*E*-stability) is determined by the stability of a corresponding matrix differential equation, as discussed extensively by Evans and Honkapohja (2001). A particular minimal state variable (MSV) solution is *E*-stable if the MSV fixed point of the differential equation is locally asymptotically stable at that point. Bullard and Duffy (2004) calculated *E*-stability conditions for this model and found that *E*-stability holds at baseline parameter values (including the various values of technology and labor force growth used).

WHAT THE MODEL DOES

The description above yields an entire system—one possible growth theory along with a business cycle theory laid on top of that. A simulation of the model will yield growing output and growing consumption, and so on, but at an uneven trend rate depending on when the trend shocks occur and how fast the learning guides the economy to the new balanced growth path following such a shock. The data produced by the model look closer to the raw data we obtain on the economy, and now we would like to somehow match up simulated data with actual data.

Of course, this model is too simple to match directly with the data, but it is also a well-known benchmark model so it is possible to assess how important structural change is when determining the nature of the business cycle as well as for the performance of the model relative to the data.

One aspect of this approach is that the model provides a global theory of the whole picture of the data. The components of the data have to add up to total output. This is because in the model it adds up and one is using that fact to detrend across all of the different variables. When considering the U.S. data, then, one has to think about the pieces that are not part of the model and how those might match up to objects inside the model. Bullard and Duffy (2004) discuss this extensively.

Breaks Along the Balanced Growth Path

The slowdown in measured productivity growth in the U.S. economy beginning sometime in the late 1960s or early 1970s is well known, and econometric evidence on this question is reviewed in Hansen (2001). Perron (1989) associated the 1973 slowdown with the oil price shock. The analysis by Bai, Lumsdaine, and Stock (1998) suggests the trend break most likely occurred in 1969:Q1.

The Bullard and Duffy (2004) model says that the nature of the balanced growth path—the trend—is dictated by increases in productivity units $X(t)$ and increases in the labor input $N(t)$. To find break dates, instead of relying on econometric evidence alone, Bullard and Duffy (2004) designed an algorithm that uses a simulated method of moments search process (genetic algorithm)[1] to choose break dates for the growth factors and the growth rates of these factors, based on the principle that the trend in measured productivity and hours from the model should match the trend in measured productivity and hours from the data. Table 1 reports their findings. The algorithm suggests one trend break date in the early 1960s for the labor input and two break dates for productivity: one in the early 1970s and one in the 1990s.

[1] See Appendix B in Bullard and Duffy (2004).

Table 1

Optimal Trend Breaks

	$N(t)$	$X(t)$
Initial annual growth rate (percent)	1.20	2.47
Break date	1961:Q2	1973:Q3
Mid-sample annual growth rate (percent)	1.91	1.21
Break date	—	1993:Q3
Ending annual growth rate (percent)	1.91	1.86

Table 2

Business Cycle Statistics, Model-Consistent Detrending

	Volatility		Relative volatility		Contemporaneous correlations	
	Data	Model	Data	Model	Data	Model
Output	3.25	3.50	1.00	1.00	1.00	1.00
Consumption	3.40	2.16	1.05	0.62	0.60	0.75
Investment	14.80	8.86	4.57	2.53	0.65	0.92
Hours	2.62	1.54	0.81	0.44	0.65	0.80
Productivity	2.52	2.44	0.77	0.70	0.61	0.92

According to Table 1, productivity grows rapidly early in the sample, then slowly from the ’70s to the ’90s and then somewhat faster after 1993. After each one of those breaks the agents in the model are somewhat surprised, but their tracking algorithm allows them to find the new balanced growth path that is implied by the new growth rates.

This model includes both a trend and a cycle. Looking at the simulated data from the model, what would a trend be? A trend is the economy’s path if only low-frequency shocks occur. Bullard and Duffy (2004) turn off the noise on the business cycle shock and just trace out the evolution of the economy if only the low-frequency breaks in technology and labor force growth occur. Importantly, the multivariate trend defined this way is then the same one that is removed from the actual data. In this sense, the model and the data are treated symmetrically: The growth theory that is used to design the model is dictating the trends that are removed from the actual data.

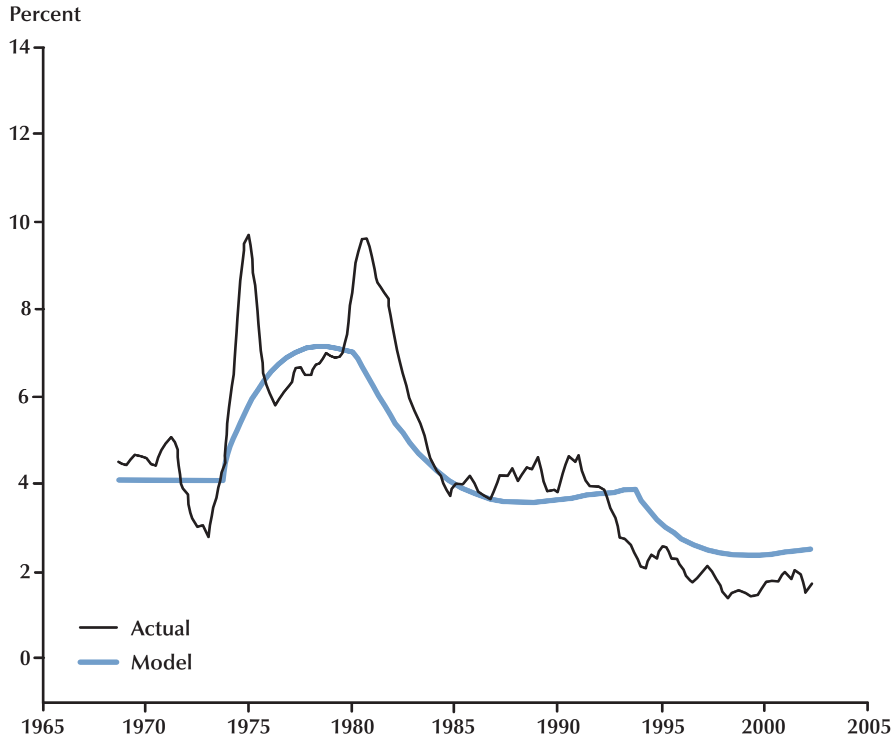
Business Cycle Statistics

The reaction of the economy to changes in the balanced growth path will depend in part on what business cycle shocks occur in tandem with the growth rate changes. Bullard and Duffy (2004) average over a large number of economies to calculate business cycle statistics for artificial economies. They collect 217 quarters of data for each economy, with trends breaking as described above. They detrend the actual data using the same (multivariate) trend that is used for the model data.

The numbers in Table 2 are not the standard ones for this type of exercise. In fact, they are quite different from the ones that are typically reported for this model, both for the data and for the model relative to the data. This shows that the issues of the underlying growth theory and its implications for the trends we expect to observe are key issues in assessing theories. One simple message from Table 2 is we obtain almost twice as much volatil-

Figure 1

U.S. Core PCE Inflation



ity in this model as there would be in the standard business cycle in this economy. This is so even though the technology shock is calibrated in the standard way.

New Keynesian Application

A similar approach can be used in the NK model. This was done by Bullard and Eusepi (2005). In the NK model (with capital), a monetary authority plays an important role in the economy's equilibrium. In Bullard and Eusepi (2005), the monetary authority follows a Taylor-type policy rule. The trend breaks and the underlying growth theory are the same as in Bullard and Duffy (2004). Now, however, one can ask how the policymaker responds using the Taylor rule given a productivity slowdown that must be learned. The policymaker initially misperceives how big the output gap is and this is making policy set the interest rate too low, pushing the inflation rate up. How large is this effect? According to Bullard and Eusepi (2005), the effect is about 300 basis points on the inflation rate for a productivity slowdown of the magnitude experienced in the 1970s (Figure 1). So, this does not explain all of the inflation in the 1970s but it helps explain a big part of it.

CONCLUSION

The approach outlined above provides some microfoundations for the largely atheoretical practices that are currently used in the literature. Structural change is not a small matter, and structural breaks likely account for a large fraction of the observed variability of output. One way to think of structural change is as a series of piecewise balanced growth paths. Learning is a glue that can hold together these piecewise paths.

I think this is an interesting approach and I would like to encourage more research that goes in this direction. It doesn't have to be a simple RBC-type model; one could instead use a more elaborate model that incorporates more empirically realistic ideas about what is driving growth and what is driving the business cycle. The approach I have outlined forces the researcher to lay out a growth theory, which is a tough and rather intensive task, but also leads to a more satisfactory detrending method and a model that is congruent with the macroeconomic data in a broad way.

REFERENCES

Bai, Jushan; Lumsdaine, Robin L. and Stock, James H. "Testing for and Dating Common Breaks in Multivariate Time Series." *Review of Economic Studies*, July 1998, *65*(3), pp. 395-432.

Bullard, James and Duffy, John. "Learning and Structural Change in Macroeconomic Data." Working Paper No. 2004-016A, Federal Reserve Bank of St. Louis, August 2004; http://research.stlouisfed.org/wp/2004/2004-016.pdf.

Bullard, James and Eusepi, Stefano. "Did the Great Inflation Occur Despite Policymaker Commitment to a Taylor Rule?" *Review of Economic Dynamics*, April 2005, *8*(2), pp. 324-59.

Cooley, Thomas F. and Prescott, Edward C. "Economic Growth and Business Cycles," in T.F. Cooley, ed., *Frontiers of Business Cycle Research*. Princeton, NJ: Princeton University Press, 1995, pp. 1-38.

Evans, George W. and Honkapohja, Seppo. *Learning and Expectations in Macroeconomics*. Princeton, NJ: Princeton University Press, 2001.

Hansen, Bruce E. "The New Econometrics of Structural Change: Dating Breaks in U.S. Labour Productivity." *Journal of Economic Perspectives*, Fall 2001, *15*(4), pp. 117-28.

Hodrick, Robert J. and Prescott, Edward C. "Postwar U.S. Business Cycles: An Empirical Investigation." Discussion Paper 451, Carnegie-Mellon University, May 1980.

King, Robert G. and Rebelo, Sergio T. "Resuscitating Real Business Cycles," in J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*. Volume 1C. Chap. 14. Amseterdam: Elsevier, 1999, pp. 927-1007.

Orphanides, Athanasios and Williams, John C. "The Decline of Activist Stabilization Policy: Natural Rate Misperceptions, Learning and Expectations." *Journal of Economic Dynamics and Control*, November 2005, *29*(11), pp. 1927-50.

Perron, Pierre. "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Econometrica*, November 1989, *57*(6), pp. 1361-401.

Smets, Frank and Wouters, Rafael. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." CEPR Discussion Paper No. 6112, Centre for Economic Policy Research, February 2007; www.cepr.org/pubs/dps/DP6112.asp.