Parsing Shocks: Real-Time Revisions to Gap and Growth Projections for Canada

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The output gap—the deviation of output from potential output—has played an important role in the conduct of monetary policy in Canada. This paper reviews the Bank of Canada's definition of potential output, as well as the use of the output gap in monetary policy. Using a real-time staff economic projection dataset from 1994 through 2005, a period during which the staff used the Quarterly Projection Model to construct economic projections, the authors investigate the relationship between shocks (data revisions or real-time projection errors) and revisions to projections of key macroeconomic variables. Of particular interest are the interactions between shocks to real gross domestic product (GDP) and inflation and revisions to the level of potential output, potential growth, the output gap, and real GDP growth. (JEL C53, E32, E37, E52, E58)

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Potential output is an important economic concept underlying the design of sustainable economic policies and decisionmaking in forward-looking environments. Stabilization policy is designed to minimize economic variation around potential output. Estimates of potential output may be used to obtain cyclically adjusted estimates of fiscal budget balances; projections of potential output may indicate trend demand for use in investment planning or trend tax revenues for use in fiscal planning; and potential output provides a measure of production capacity for assessing wage or inflation pressures.

Although potential output is an important economic concept, it is not observable. The Bank of Canada defines "potential output" as the sustainable level of goods and services that the economy can produce without adding to or subtracting from inflationary pressures. This definition is intrinsic to the methodology used by the Bank of Canada to construct historical estimates of potential output. In addition to using a production function to guide estimation of long-run trends influencing the supply side of the economy, the procedure incorporates information on the demand side that relates inflationary and disinflationary pressures to, respectively, situations where output exceeds and falls short of potential output.

Potential output and the "output gap," defined as the deviation of output from potential output, play central roles in monetary policy decisionmaking and communications at the Bank of Canada. Macklem (2002) describes the information and analysis presented to the Bank's Governing Council in the two to three weeks preceding a fixed announcement date.[1] As described in that document, the output gap—both its level and rate of change—is the central aggregate-demand

[1] In late 2000, the Bank of Canada adopted a system of eight pre-announced dates per year when it may adjust its policy rate—the target for the overnight rate of interest. The Bank retains the option of taking action between fixed dates in extraordinary circumstances.

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link between the policy actions and inflation responses.[2]

In addition to being central to policy deliberations, the output gap has played a critical role in Bank of Canada communications. The concept of the output gap is simple to explain and understand. It has been used effectively to simultaneously provide a concise and intuitive view of the current state of the economy and inflationary pressures. It also provides a point of reference in relation to current policy actions and helps align the Bank's current thoughts on the economy with those held by the public.

Use of the output gap as a key communications device with the public is clearly seen in *Monetary Policy Reports* (*MPRs*) and speeches by governors and deputy governors of the Bank. The Bank of Canada began publishing *MPRs* semiannually in May 1995 (with two additional *Monetary Policy Report Updates* per year starting in 2000), and the output gap has been prominent in the reports from the beginning.[3] Indeed, a Technical Box appears in the first *MPR* regarding the strategy used by the Bank to estimate potential output.[4] Not only is the Bank's estimate of the output gap referenced in the text of the *MPR* as a source of inflationary (or disinflationary) pressure in the economy, but the estimates of recent history of the output gap up to the current quarter are also charted.

Governors and deputy governors have extensively used the output gap to explain to the general public how the monetary policy framework works. Common elements across these speeches include discussions on how potential output is estimated, how it is used to construct the output gap, and how the output gap affects monetary policy decisions. These discussions are nontechnical to enhance understanding by noneconomists. For instance, when discussing the factors affecting potential output in a speech to the Standing Senate Committee on Banking, Trade and Commerce in 2001, Governor David Dodge stated:

> [T]he level of potential rises over time as more workers join the labour force; businesses increase their investments in new technology, machinery and equipment; policy measures are taken to make product and labour markets more flexible; and all of us become more efficient and productive in what we do.

One important challenge associated with the use of potential output and the output gap as tools for communication of monetary policy decisions is that they cannot be directly observed and must be estimated. Moreover, estimates are prone to revision as historical data are revised and new information becomes available. Consequently, the Bank has directly addressed uncertainty surrounding estimates of the output gap and the drivers behind revisions in policy communications. A discussion of the implications of uncertainty for the conduct of monetary policy appeared in the May 1999 *MPR* (Bank of Canada, 1999, p. 26):

> [P]oint estimates of the level of potential output and of the output gap should be viewed cautiously. This has particular significance when the output gap is believed to be narrow and when inflation expectations are adequately anchored. In this situation, to keep inflation in the target range, policy-makers may have more success by placing greater weight on the economy's inflation performance relative to expectations and less on the point estimate of the output gap. At about the same time, the Bank started providing standard error bands around recent estimates of the output gap.[5]

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[2] The important role of the output gap as a guide to monetary policymakers, over and above that of growth, was expressed by Governor Thiessen (1997):

> Some people apparently assume that it is the *speed* at which the economy is growing that determines whether inflationary pressures will increase or decrease. While the rate of the growth is not irrelevant, what really matters is the *level* of economic activity relative to the production capacity of the economy—in other words…the output gap in the economy. The size of the output gap, interacting with inflation expectations, is the principal force behind increased or decreased inflationary pressure.

[3] By contrast, incorporation of Governing Council projections has been more recent, with projections of core inflation first appearing in the April 2003 *MPR* and projections of gross domestic product (GDP) growth first appearing in the July 2005 *MPR*.

[4] The material in this box (May 1995) gives readers an idea of how the output gap is constructed without being overly technical. Publishing such statistics and the methods underlying their estimation has contributed importantly to monetary policy transparency in Canada.

[5] Standard error bands were provided around recent estimates from 1998 to 2007.

Revisions to historical estimates of potential output and the output gap also have been explicitly discussed in *MPRs*.[6] The discussions relate the revisions to recent developments in wage and price inflation and revised assessments of trends in labor input and labor productivity. Overall, transparency in the construction of the output gap, in understanding sources of revisions to past estimates of the output gap, and in uncertainty around the output gap has contributed to the effectiveness of the output gap as a key communications tool for enhancing understanding of the monetary policy process and of policy decisions in real time.

Implicit in the policy use of potential output and the output gap has been an effective strategy for managing volatility in estimates of the output gap. In particular, given the central role of potential output and the output gap in monetary policy, volatility in time series of the output gap or in revisions to estimates of the output gap can hinder the effectiveness of monetary policy communications, and therefore of monetary policy itself.

The next section reviews the methodology used by the Bank of Canada to estimate potential output and the output gap in Canada. While the methodology was designed to be consistent with the economic structure of the model used by Bank of Canada staff to construct projections, the Quarterly Projection Model (QPM), the procedure is designed to also incorporate information outside the scope of the model, such as demographics and structural details related to the labor market.[7] Features designed to contain end-of-sample revisions to estimates in response to updates of underlying economic data and to the availability of additional observations are discussed. This paper examines the extent to which such concerns were addressed by the methodologies developed to estimate and project potential output and the output gap in real time.

We next describe a dataset on real-time revisions to economic data and projections that has been constructed from a historical database of real-time economic projections made by Bank of Canada staff. The properties of these real-time revisions are explored in the subsequent text section. While the main focus of the analysis is the parsing of economic shocks into revisions to projections of (i) the level of potential output, (ii) the output gap, (iii) real GDP growth, and (iv) potential growth, the response of projections of inflation and short-term interest rates to shocks is also examined.

POTENTIAL OUTPUT IN CANADA

This section describes the techniques used by Bank of Canada staff to estimate historical values and project future values of potential output in Canada. In real time, Bank staff make ongoing marginal changes to the estimation methodology. Consequently, the description in this section should be taken only as broadly indicative of the procedures followed and the inputs to the estimation exercise.

A unifying assumption underlying both historical estimates and projections of potential output is that aggregate production can be represented by a Cobb-Douglas production function:

$$Y = (TFP) N^{a} K^{(1-a)}, \qquad (1)$$

where Y is output, N is labor input, K is the aggregate capital stock, TFP is the level of total factor productivity, and a is the labor-output elasticity (or labor's share of income). This production function also was used in the now-discontinued model QPM to describe the supply side of the Canadian economy.

The next subsection describes the process by which historical estimates of potential output were estimated, while the following section focuses on assumptions underlying projections of potential output.

Historical Estimates of Potential Output

The methodology used to estimate potential output was heavily influenced by the requirements

6 See, for instance, Technical Box 3 in Bank of Canada (2000).

7 The QPM was used for economic projections between September 1993 and December 2005. Although there have been marginal changes in the procedure used to estimate the output gap over time, at the time of writing, the Bank continued to use basically the same methodology to generate its "conventional" estimate of the output gap in Canada.

of the monetary policy framework in which it was to be used. Thus, it was judged that the methodology should be consistent both with the QPM and the requirements associated with using the model to prepare economic projections. In this context, Butler (1996) notes that the following properties were judged to be of prime concern: consistency with the economic model (QPM); the ability to incorporate judgment in a flexible manner; the ability to both reduce and quantify uncertainty about the current level of potential output; and robustness to a variety of specifications of the trend component. In addition, given concerns about the feasibility and efficiency of estimates of potential output based solely on a model of the supply side of the economy, use of information from a variety of sources to better disentangle supply and demand shocks was deemed desirable.

With these guiding principles in mind, in the 1990s researchers at the Bank of Canada developed a new methodology to estimate potential output based on a multivariate filter that incorporates economic structure, as well as econometric techniques designed to isolate particular aspects of the data.[8] The main innovation was the development of a filter, known as the extended multivariate filter (EMVF), that solves a minimization problem similar to that underlying the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997), but the EMVF also incorporates information on economic structure and includes modifications to penalize large revisions and excess sensitivity to observations near the end of the sample. For a variable or vector of variables, x, the general filter estimates the trend(s), x^*, as follows:

$$x^* = \max_{\hat{x}} \left\{ \left\{ -\left(x-\hat{x}\right)' W_x \left(x-\hat{x}\right) - \lambda \hat{x}' D' D \hat{x} \right\} + \left\{ -\varepsilon' W_\varepsilon \varepsilon - \left(s-\hat{x}\right)' W_s \left(s-\hat{x}\right) \right\} + \left\{ -\hat{x}' P' W_g P \hat{x} - \left(x_{pr}^* - \hat{x}\right)' W_{pr} \left(x_{pr}^* - \hat{x}\right) \right\} \right\} \quad (2)$$

This filter nests the HP filter, which is clearly evident for univariate x, by setting W_ε, W_s, W_{pr}, and W_g to zero, leaving only

$$\left\{ -\left(x-\hat{x}\right)' W_x \left(x-\hat{x}\right) - \lambda \hat{x}' D' D \hat{x} \right\}.$$

Information on economic structure and judgment can be introduced through the two terms

$$\left\{ -\varepsilon' W_\varepsilon \varepsilon - \left(s-\hat{x}\right)' W_s \left(s-\hat{x}\right) \right\}.$$

The term $\varepsilon' W_\varepsilon \varepsilon$ is the main channel through which information on the demand side of the economy may be introduced to assist in better separating demand shocks and supply shocks. In general, ε represents residuals from key economic relationships that depend on $\hat{x}$. For instance, if the unobserved trend to be estimated is the non-accelerating inflation rate of unemployment (NAIRU), ε may contain residuals from a Phillips curve that relate inflation developments to deviations of the unemployment rate from the NAIRU. In this sense, residuals may be interpreted as deviations from a structural economic relationship, perhaps drawing on cyclical economic relationships in the QPM. With this term in the filter the estimate of the trend may be shifted to reduce such deviations from the embedded economic theory.

Additional external structural information on trends may be introduced through the term $(s-\hat{x})' W_s (s-\hat{x})$. In this expression, s generally represents an estimate of the trend based on information outside the general scope of the model. For instance, in the case of the trend participation rate, s may be based on external analysis including information on demographics and otherwise informed judgment.

Finally, the last two terms,

$$\left\{ -\hat{x}' P' W_g P \hat{x} - \left(x_{pr}^* - \hat{x}\right)' W_{pr} \left(x_{pr}^* - \hat{x}\right) \right\},$$

provide a means to limit revisions to trend estimates. In general, procedures such as the EMVF are subject to one-sided filtering asymmetries at the ends of the sample. Although the filter is a symmetric two-sided weighted moving average within the sample period, near the end (and begin-

[8] See the discussion in Laxton and Tetlow (1992), Butler (1996), and St-Amant and van Norden (1997).

ning) of the sample, filter weights become one-sided. Intuitively, weights that would have been assigned to future observations if they were available are redistributed across recent observations. As a consequence, trend estimates near the end of the sample place large weights on recent data and tend to be revised considerably as additional observations become available.[9] The term $\hat{x}'P'W_gP\hat{x}$ penalizes large end-of-sample changes in the trend estimates and reduces the importance of the last few observations for the end-of-sample estimate of the trend. The term $(x^*_{pr}-\hat{x})'W_{pr}(x^*_{pr}-\hat{x})$ penalizes revisions to trend estimates between two successive projection exercises attributable to any source. In the absence of such a penalty, trend estimates could be revised more than is judged desirable due to (i) revisions to historical data, (ii) the availability of data for an additional quarter, or (iii) changes to external information or judgment as summarized by s.[10]

In many ways, the methodology of the EMVF was at the leading edge of research contributions in this area. For instance, although the methodology tends to be applied to estimate the trend in a single trending variable at a time, the theory is sufficiently general to include joint estimation of multiple trends, including situations with common trend restrictions. Stock and Watson (1988) developed a common trends representation for a cointegrated system, and state-space models as outlined in Harvey (1989) could also accommodate common trend restrictions. However, within the context of filters such as the HP filter, the band-pass filter of Baxter and King (1999), or the exponential-smoothing filter used by King and Rebelo (1993), imposition of common trend restrictions was not explored elsewhere in the academic literature until Kozicki (1999).

Another interesting aspect of the EMVF is that the methodology proposed approaches to reduce the importance of the one-sided filtering problem well before it was addressed elsewhere in the literature. Orphanides and van Norden (2002) drew attention to the result that many estimation methodologies yield large revisions to real-time end-of-sample estimates of the output gap. One potential approach to mitigating the one-sided filtering problem was proposed by Mise, Kim, and Newbold (2005).

As noted, an important characteristic of the EMVF is its ability to incorporate, within a filtering environment designed to extract fluctuations of targeted frequencies, information drawn from structural economic relationships, information from data sources external to the QPM, and judgment. The next few paragraphs provide details on the economic structure incorporated in the EMVF and the mechanism by which demographic information and structural features of the Canadian labor market could influence estimates of potential output.

Estimates of potential output are based on the Cobb-Douglas production function in equation (1). Recognizing that for the specification in equation (1), the marginal product of labor is $\partial Y/\partial N = aY/N$, the logarithm of output can be represented as

$$y = n + \mu - a, \tag{3}$$

where each term is expressed in logarithms and n is labor input, μ is the marginal product of labor, and $\alpha = \log(a)$ is the labor-output elasticity (also labor's share of income). The decision to use μ in constructions of historical estimates of potential output rather than data on the capital stock was motivated by concerns about the lack of timely (or quarterly) data and measurement problems. To construct log potential output, y^*, trends in log employment, n^*, the log marginal product of labor, μ^*, and the log labor share of income, α^*, are estimated separately and then summed.

One component of log potential output, trend log employment, n^*, is estimated using another decomposition:

$$n^* = Pop + p^* + \log\left(1-u^*\right), \tag{4}$$

[9] Orphanides and van Norden (2002) show that revisions associated with the availability of additional data tend to dominate those related to revisions to historical data.

[10] An alternative possibility that was not explored was to penalize revisions of deviations from the trend rather than just revisions to the trend, by replacing the last term with the following:

$$((x-\hat{x})-(x_{pr}-x^*_{pr}))'W_{pr}((x-\hat{x})-(x_{pr}-x^*_{pr})).$$

In the absence of data revisions $x_{pr} = x$, all revisions to deviations would be due to revisions to the trend and both alternatives would yield the same results.

where *Pop* is the logarithm of the working-age population, p is the logarithm of the participation rate, and u^* is the NAIRU.[11] As for aggregate output, trend employment is constructed as the sum of the estimated trends of each component. The trend participation rate, p^*, is estimated with the EMVF using an external estimate of the trend participation rate for s, setting W_ε, W_{pr}, and W_ε to zero. Around the time of Butler's (1996) writing, the smoothness parameter λ was set to a very high number ($\lambda = 16000$) to obtain a very smooth estimate of the trend participation rate. However, the value of this parameter has been adjusted considerably over time and more recently has been set to $\lambda = 1600$, a value typically used to exclude fluctuations of "typical" business cycle frequencies from trend estimates. The external estimate of the trend participation rate accounts for demographic developments, including, for instance, trends in the workforce participation rate of women and school employment rates.[12] The NAIRU is also estimated using the EMVF, with an external estimate of the trend unemployment rate based on the work of Côté and Hostland (1996) used for s, and residuals ε, obtained from a price-unemployment Phillips curve drawing on the work of Laxton, Rose, and Tetlow (1993). The external estimate of the trend unemployment rate incorporates information on structural features of Canadian labor markets, including the proportion of the labor force that is unionized and payroll taxes.

A second component of log potential output is the trend value of the log labor-output elasticity, a^*. This component is estimated as the smooth trend obtained by applying an HP filter with a large smoothing parameter ($\lambda = 10000$) to data on labor's share of income.

The third component of log potential output, the trend log marginal product of labor, μ^*, is also estimated by applying the EMVF. The real producer wage is used for s rather than an external estimate of the trend, and ε is the residual from an inflation/marginal product of labor relationship. The latter is motivated by the idea that the deviation of the marginal product of labor from its trend level can be interpreted as a factor utilization gap and, hence, provides an alternative index of excess demand pressures.

Projecting Potential Output

Projections of potential output are based on the Cobb-Douglas production function, equation (1), but are driven by consideration of supply-side features:

$$y^* = tfp^* + a^* n^* + \left(1 - a^*\right) k, \tag{5}$$

where lower-case letters indicate the logarithm of the respective capitalized notation and an asterisk denotes that a variable is set to its trend or equilibrium value.[13] Thus, projections of potential output are constructed with projections of tfp^*, a^*, n^*, and k.

The capital stock, k, is constructed from the cumulated projected investment flows given the actual capital stock at the start of the projection. The equilibrium labor-output elasticity, a^*, is set to a constant equal to the historical average labor share of income.

The typical assumption is that in the medium to long term, trend total factor productivity, tfp^*, will converge toward the level of productivity of the United States at the historical rate of convergence.[14] A short-run path for tfp^* links the historical estimate at the start of the projection to the medium-term path for tfp^*, with short-run behavior based on typical cyclical variation.

The equilibrium employment rate, n^*, is based on an analysis of population growth, labor force participation, and structural effects on the NAIRU (Bank of Canada, 1995). Analysis draws on information outside the scope of the QPM. For instance, labor force participation is related to demographic factors (Bank of Canada, 1996); population growth may be influenced by immi-

[11] Barnett (2007) provides recent estimates and projections of trend labor input using a cohort-based analysis that incorporates anticipated demographic changes. Barnett's analysis also accounts for trend movements in hours.

[12] See Technical Box 2 of Bank of Canada (1996).

[13] See the discussion in Butler (1996).

[14] Crawford (2002) discusses determinants of trends in labor productivity growth in Canada.

gration policy; and the NAIRU may be related to structural factors.[15] To a large extent, this series can be thought of as corresponding to an external structural estimate of s as used in the EMVF. Thus, projections are as if they are generated from an application of the EMVF with all weights other than W_s set to zero.

Although numerous studies—including Butler (1996), Guay and St-Amant (1996), St-Amant and van Norden (1997), and Rennison (2003)—have compared the properties of alternative approaches of historical estimates of the output gap across alternative estimation approaches, no similar studies exist to examine properties of projections of potential output or the output gap. This is one area to which the current study hopes to contribute.

MEASURING SHOCKS AND REVISIONS TO PROJECTIONS

The empirical analysis is designed to assess the sensitivity of economic projections to new information. If economic projections were "raw" outputs from application of the QPM, then our analysis would be merely recovering information about the structure of the QPM, which is available elsewhere.[16] However, in general, economic projections are influenced by judgment to account for features of the economy outside the scope of the economic model. In addition, the QPM is primarily a business cycle model, designed to project deviations of economic variables from their respective trend levels. Consequently, while potential output and other trends are constructed to be consistent with the economic structure of the QPM, evolution of these trends is modeled outside the QPM.

[15] Poloz (1994) and Côté and Hostland (1996) discuss the effects of structural factors, such as demographics, unionization, and fiscal policies influencing unemployment insurance, the minimum wage, and payroll taxation, on the NAIRU. More information on demographic implications for labor force participation is provided by Ip (1998).

[16] A nontechnical description of the QPM is provided in Poloz, Rose, and Tetlow (1994). Detailed information on the QPM is provided in the trio of Bank of Canada Technical Reports by Black et al. (1994), Armstrong et al. (1995), and Coletti et al. (1996).

Real-Time QPM Projections and Data

The analysis uses real-time data from the Bank of Canada's staff economic projection database. Bank staff generate projections quarterly to inform the policy decisionmaking process. The projection data analyzed in this project were generated by the QPM. It is important to note that the projections in these data correspond to staff economic projections and may not be the same as projections implicitly underlying policy decisions, or, in later years, as published in the *MPR*, as such projections would correspond to the views of the Governing Council.

Analysis is limited to projection data for the period September 1993 through December 2005, the period during which the QPM was used by Bank staff producing projections. By limiting empirical analysis to data within this period, the likelihood of structural breaks in projections associated with large changes in the projection model is small. An additional advantage of this sample is that it falls entirely within the inflation-targeting regime in Canada, removing concerns about structural breaks associated with changes in policy regime.

The database includes a total of 50 vintages of data, one vintage for each quarterly projection exercise. As is standard in the real-time-data literature, the term "vintage" is used to refer to the dataset corresponding to the data from a specific projection. Vintages are described by the month and year when the projection was made. Projections were generated four times per year, once per quarter, in March, June, September, and December. For each vintage, the database contains the history of the conditioning data as available at the time of the projection, as well as the projections.

This database is used to construct measures of shocks and projection revisions. Both shocks and revisions are constructed as the difference between values of economic variables (either historical observations or the projection of a specific variable) for a given quarter as recorded in two successive vintages of data. The term "revision" is reserved to reflect a change in the projection of a variable, whereas the term "shock" is used to reflect the difference between a new or revised observation for a variable and its value (either an observation or a projection) as recorded in the previous vintage of data. For each economic variable, 2 sets of shocks series and 12 sets of revisions series are constructed.

The timing of the publication of data is critical to understanding the distinction between shocks and revisions. In general, data for a full quarter, t, are not published until the next quarter, $t + 1$. Thus, for instance, in the month when Bank of Canada staff were conducting a projection exercise, the values of variables recorded for the current quarter were "0-quarter-ahead" projections; values for the next quarter were "1-quarter-ahead" projections; and values for the prior quarter were published data. Letting x_t^v denote the value of variable x for quarter t as recorded in vintage v of the dataset, x_t^v denotes a $(t - v)$-quarter-ahead projection for $t \geq v$ and is treated as an observation of published data if $t < v$. The term "published" is somewhat of a misnomer and is more appropriate for data on inflation, real GDP, and interest rates, for instance, than for potential output, potential growth, or the output gap as the latter three concepts are not directly observed, nor are they measured or constructed by the statistical agency of Canada, Statistics Canada. As discussed earlier, values of these variables are estimated internally by Bank of Canada staff. Nevertheless, for notational convenience and to facilitate parsimonious exposition, language such as "observation," "data," and "published" is used synonymously in reference to all series according to the timing convention previously described.

The term "shock" is generally used to refer to marginal information from one vintage to the next provided by new observations on market interest rates, new or updated data produced by Statistics Canada, or new or updated historical estimates of potential output (and related series) constructed by the Bank of Canada. Two measures of shocks are examined:

$$shock1_t = x_t^{t+1} - x_t^t \quad (6)$$

is the difference between the published value of variable x for quarter t as available in quarter $t + 1$ (the first quarter it is published) and the 0-quarter-ahead (or contemporaneous) projection of variable x as made in t and recorded in vintage $v = t$. Thus, *shock1* is a projection error. The second measure of shocks captures the first quarterly update to the published data and is constructed as

$$shock2_t = x_{t-1}^{t+1} - x_{t-1}^t. \quad (7)$$

The term "revisions" is used to refer to changes in Bank of Canada staff projections of a variable between successive vintages. Twelve measures of revisions are examined with each corresponding to a different projection horizon,

$$revisionk_t = x_{t+k+1}^{t+1} - x_{t+k+1}^t, \quad (8)$$

where $k = 0,\ldots 11$.

The analysis in this article concentrates on shocks and revisions to nine variables as defined below:

- *EXCH*: the bilateral exchange rate between Canada and the United States, expressed as \$US per \$CDN;
- *GAP*: the output gap defined as the percent deviation of real GDP from potential real GDP (potential output);
- *GDP*: real GDP growth (an annualized quarterly growth rate);
- *GDPLEV*[17]: log-real GDP level, constructed as an index for a given quarter by taking *GDPLEV* for the prior quarter and adding (100/4) * log(1 + *GDP*) to it, with current vintage data for a given quarter early in the sample used to initiate the recursive construction;
- *POT*: potential output growth, calculated as the annualized one-period percent change in *POTLEV*;
- *POTLEV*: log potential output level, constructed as *GDPLEV* – *GAP*, an index;
- *INF*: CPI inflation (annualized quarterly growth rate);

[17] During the period of analysis, Statistics Canada rebased GDP several times. From 1994 to 1996, the base year used for real GDP calculations was 1986. The base year changed to 1992 from 1996 to July 2001. From July 2001 to May 2007, the base year used was 1997. However, as *GDPLEV* and *POTLEV* were constructed as indices, these rebasings would not affect the analysis of this study.

- *INFX*: core CPI inflation (annualized quarterly growth rate). The definition of core CPI has changed over our period of analysis. Before May 2001 the Bank of Canada used CPI excluding food, energy, and indirect taxes (CPIxFET) as the measure of core inflation. After May 2001 the Bank changed its official measure of core inflation to CPI excluding the eight most volatile components (CPIX), and;
- *R*90: a nominal 90-day short-term interest rate.

The information content of contemporaneous, $k = 0$, projections will differ across variables projected, implying that for some variables *shock1* will be much smaller than for others. In particular, projections are made in the third month of each quarter. However, the initial release of the national accounts is at the end of the second month or early in the third month of the quarter. Data in these national accounts releases, such as GDP, extend only through the prior quarter. For example, for the national accounts release in late August 2008 (the second month of Q3), the latest GDP observations are for 2008:Q2. However, some statistics are available in a more timely manner. For example, interest rate data are available in real time. Thus, by the third month in a quarter, two months of interest rate data are already available. Likewise, for some variables *shock2* will be much smaller (and in some cases zero) than for others, because some published data series, such as GDP, are revised in quarters after the initial release, while others, such as interest rates, are not.

PROPERTIES OF PROJECTION REVISIONS

New information becomes available in the period between projection exercises. This information takes many forms, including new or revised data published by statistical agencies, new observations from financial markets, as well as anecdotal information from surveys or the press, among others. For interest rates, inflation, and real GDP growth, the information in *shock1* reflects projection errors, whereas the information in *shock2* reflects revised data. By contrast, shocks to potential output, the output gap, and potential growth generally are a function of shocks to data (including, but not limited to, interest rates, inflation, and real GDP growth), updated judgment on the part of Bank of Canada staff, and updates to external structural information on trends. Revisions may reflect some or all of the varying types of new information. New observations of some published data directly enter into model projections, but other information may inform judgment and also be incorporated.

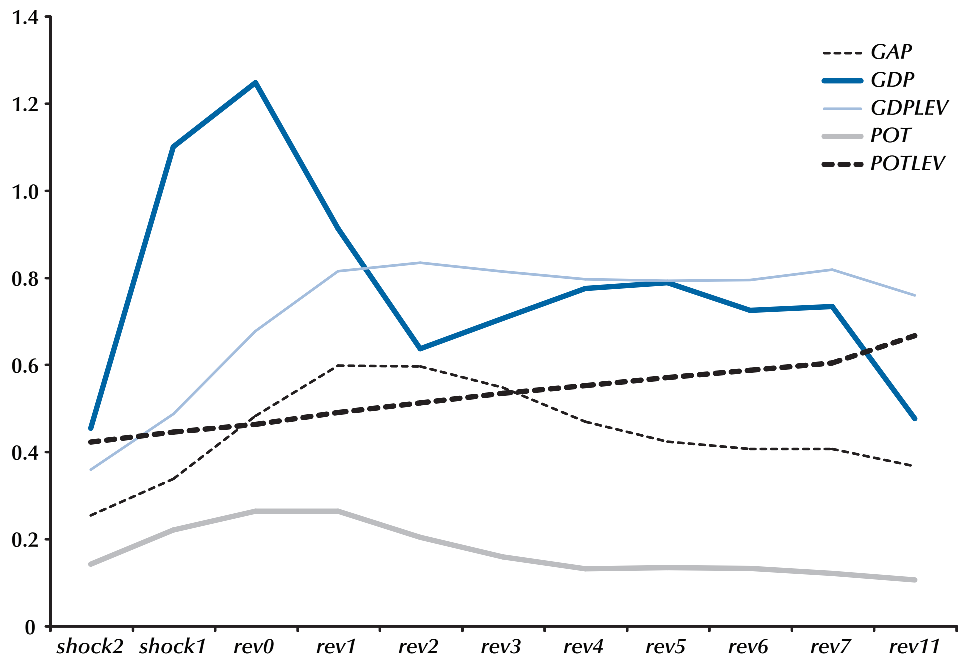
This section examines the properties of shocks and revisions. The analysis examines the relative size of revisions to projections of trends compared with revisions to projections of cyclical dynamics. Another issue of particular interest is the parsing of shocks to real GDP growth, interest rates, inflation, and exchange rates into permanent and transitory components that will, in turn, affect shocks and revisions of projections of potential output and the output gap.

Properties of Projection Revisions and Shocks

Figure 1 shows the standard deviations of shocks and revisions to *GAP*, *GDP*, *GDPLEV*, *POT*, and *POTLEV*. This figure shows that both shocks and revisions to potential output growth (*POT*) were small at all horizons. By contrast, projection errors (*shock1*) and near-term revisions to real GDP growth (*GDP*) tend to be considerably larger. Both results are consistent with what would generally be expected. By definition, potential is meant to capture low-frequency movements in output and is constructed to be smooth. Consequently, it would be surprising to see either volatile potential growth or frequent large revisions to potential growth. Real GDP growth, however, tends to be volatile. Thus, not surprisingly, revisions, particularly to current and one-quarter-ahead projections, can be sizable. Much of the volatility of both the underlying growth rate data and the revisions is likely related to the allocation and reallocation of inventory investment, imports, and exports across quarters. At longer horizons,

Figure 1

Standard Deviations



the standard deviation of *GAP* projection revisions remains quite large, and the standard deviation of revisions to projections of GDP growth are considerably larger than revisions to projections of potential growth. These observations suggest considerable persistence in business cycle propagation of economic shocks. Even at a 2- to 3-year horizon, real GDP growth does not consistently converge to potential output growth in projections.

Whereas shocks and revisions to potential growth are considerably smaller than revisions to GDP growth, the same is not true for the log levels of GDP (*GDPLEV*) and potential output (*POTLEV*). For these variables, the standard deviations of shocks are essentially the same. As expected, *GDPLEV* revisions tend to be larger than *POTLEV* revisions, but not by nearly as much as was the case for their growth rates (*GDP* and *POT*, respectively). In fact, at the longest horizon, $k = 11$, the magnitudes of revisions to the levels are, on average, essentially the same.

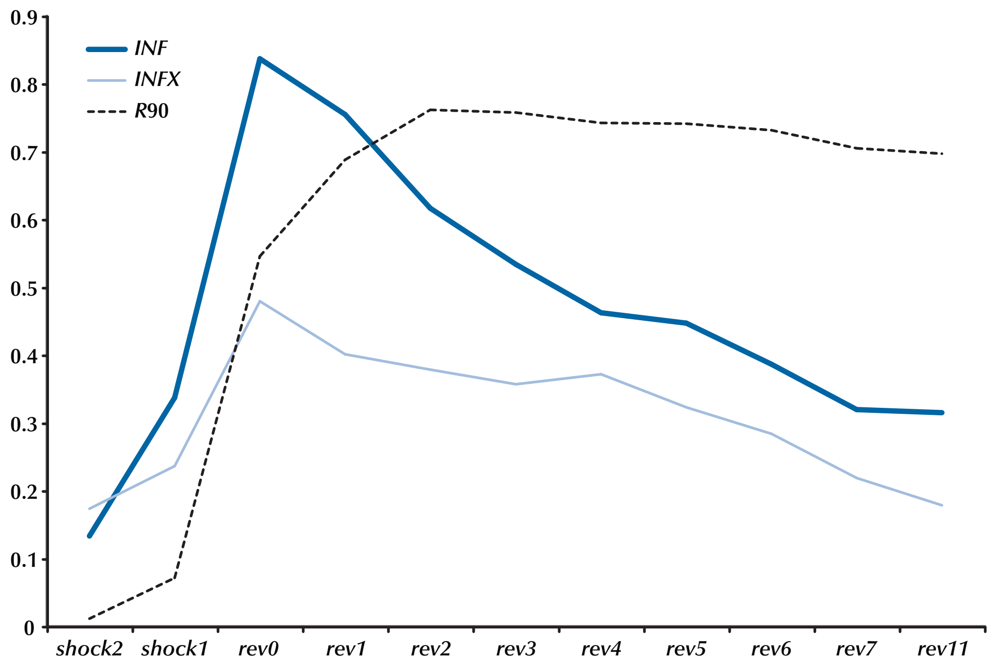
Figure 2 shows the standard deviations of shocks and revisions to *INF*, *INFX*, and *R*90. Shocks to all variables are quite small. As noted earlier, some monthly data are available for the contemporaneous quarter, likely explaining the larger differences in standard deviations of the projection error (*shock1*) relative to the first forecast revision (*rev0*).[18] Revisions to near-term projections tend to be larger than those to longer-horizon projections for inflation. This may reflect the effects of endogenous policy designed to achieve the 2 percent target at a roughly 2-year horizon.

Very different properties are evident for the short-term interest rate (*R*90). Shocks to interest rates are generally small, owing to the fact that interest rate data are available daily in real time (so that much of the current-quarter information

[18] For *INF* and *INFX*, annual updates to seasonal adjustments to the data are the main source of nonzero values of *shock2*. The change in definition of *INFX* in May 2001 also leads to a nonzero value of *shock2* for this variable.

Figure 2

Standard Deviations



is already available at the time of the contemporaneous-quarter projections) and are not very volatile. Standard deviations of *revision0* are similar to those of inflation. However, as the forecast horizon increases, standard deviations of revisions to interest rates rise somewhat before leveling off, and in contrast to the results for inflation, they do not noticeably decline for longer forecast horizons.

Table 1 provides information on the persistence of projection revisions across forecast horizons.[19] Persistence should vary considerably across different economic variables. In general, revisions to trend levels should be expected to be permanent, while revisions to cyclical variables should be expected to dissipate. Each column of Table 1 provides correlations of shocks and revisions with *revision0* for a single variable. When *revision0* of *GAP* is revised, so are revisions to *GAP* projections at other horizons, although the correlation diminishes as the projection horizon increases. Potential growth revisions are also positively correlated but display a somewhat different pattern, with much lower correlation at near-term horizons. GDP growth revisions show strong near-term momentum, but negative correlations suggest near-term revisions tend to be partially reversed further out.

Correlations across horizons of revisions to projections of the three level variables, *GAP*, *GDPLEV*, and *POTLEV*, clearly reveal the differing persistence properties of trends and cycles. When the level of potential output is revised, it tends to be revised by nearly equal amounts at all projection horizons. By contrast, as noted previously, when the contemporaneous-quarter projection of the output gap is revised, subsequent projections are revised in the same direction, but by diminishing amounts as the projection horizon increases. By construction, *GDPLEV* is the sum

[19] Note that an alternative definition of persistence would examine the persistence of revisions by horizon across time.

Table 1

Correlations of Revisions Across Projection Horizons

Revision	Gap	GDP growth	Potential growth	CPI inflation	Core inflation	Short-term interest rate	GDP level	Potential level	CPI level	Core CPI level	Exchange rate
Shock2	0.30	0.34	0.26	0.04	–0.11	0.16	0.70	0.98	0.64	0.97	–0.06
Shock1	0.79	0.50	0.56	0.53	0.38	0.08	0.91	0.99	0.80	0.98	0.35
Rev0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Rev1	0.94	0.71	0.36	0.09	0.19	0.76	0.97	0.99	0.90	0.99	0.92
Rev2	0.87	0.09	0.31	–0.10	0.32	0.55	0.93	0.98	0.82	0.96	0.87
Rev3	0.81	–0.17	0.29	–0.19	0.23	0.44	0.89	0.98	0.71	0.93	0.79
Rev4	0.69	–0.46	0.26	–0.12	0.22	0.42	0.84	0.97	0.62	0.90	0.70
Rev5	0.53	–0.44	0.25	–0.01	0.17	0.38	0.77	0.96	0.55	0.87	0.65
Rev6	0.28	–0.48	0.34	0.06	0.10	0.33	0.69	0.95	0.50	0.84	0.62
Rev7	0.01	–0.43	0.29	0.13	0.02	0.25	0.61	0.95	0.48	0.82	0.60
Rev11	–0.25	–0.04	0.06	0.45	–0.10	0.09	0.55	0.93	0.53	0.77	0.53

Table 2

Correlations among Projection Errors (*shock1*)

	Gap	GDP growth	Potential growth	Potential log level	CPI inflation	Core inflation	Short-term interest rate	Exchange rate
Gap	1.00	0.62	–0.08	–0.25	0.14	0.05	–0.01	–0.01
GDP growth	0.62	1.00	0.32	0.29	0.01	–0.01	0.07	–0.01
Potential growth	–0.08	0.32	1.00	0.47	0.09	–0.14	–0.13	0.11
Potential log level	–0.25	0.29	0.47	1.00	–0.15	–0.09	0.01	0.07
CPI inflation	0.14	0.01	0.09	–0.15	1.00	0.63	–0.09	–0.17
Core inflation	0.05	–0.01	–0.14	–0.09	0.63	1.00	–0.14	–0.31
Short-term interest rate	–0.01	0.07	–0.13	0.01	–0.09	–0.14	1.00	–0.02
Exchange rate	–0.01	–0.01	0.11	0.07	–0.17	–0.31	–0.02	1.00

Table 3

Correlations among Data Revisions (*shock2*)

	Gap	GDP growth	Potential growth	Potential log level
Gap	1.00	–0.07	–0.35	–0.53
GDP growth	–0.07	1.00	0.46	0.31
Potential growth	–0.35	0.46	1.00	0.59
Potential log level	–0.53	0.31	0.59	1.00

of *POTLEV* and *GAP*, so it should not be surprising that persistence properties are intermediate to the two components. On average, about half of the contemporaneous projection revision is permanent, whereas the other half shrinks with longer forecast horizons.

This result is rather striking, as is the result (evident in Figure 1) that the standard deviation of shocks to the level of GDP is about the same as the standard deviation of shocks to the level of potential GDP. Moreover, the standard deviations of revisions to projections of the level of potential output are only somewhat smaller than the standard deviations of revisions to projections of the output gap (and are smaller for only three near-term forecasting horizons). Overall, these results suggest almost the same amount of uncertainty is associated with the level of potential as with the gap. Of course, all else equal, revisions to the level of potential output do not have policy implications, whereas revisions to the output gap do.

In the case of inflation, revisions to core inflation projections tend to have some, albeit low, persistence, whereas those to overall inflation do not. This result is consistent with the observation that near-term revisions to overall inflation are generally driven by information on the volatile components excluded from the core measures. By contrast, revisions to exchange rate projections are very persistent. The persistence of revisions to projections of the short-term interest rate is roughly similar to the persistence of revisions to the gap, perhaps indicating a link between the two. This possibility is explored in the next subsection.

Correlations among projection errors (*shock1*) are presented in Table 2. A few interesting results emerge. First, correlations among projection errors to GDP growth, core inflation, *R*90, and the exchange rate are very low. Second, the correlation between projection errors to GDP growth and the output gap is quite high. This result likely signals

Table 4

Regression Results: Responses of Revisions to Shocks

Dependent variable (*revisionk*)	*k*	*GDP shock1*	*GDP shock2*	*INFX shock1*	*R90 shock1*	*EXCH shock1*	$\bar{R}^2$
GAP	0	0.30***	0.17	–0.22	–2.13***	0.12	0.65
	1	0.31***	0.33**	–0.22	–2.69***	0.16	0.57
	2	0.28***	0.38**	–0.20	–2.66***	–0.03	0.51
	3	0.24***	0.34**	–0.25	–2.29**	–0.11	0.46
	4	0.18***	0.29**	–0.23	–1.65*	–0.14	0.38
	5	0.13**	0.23	–0.24	–1.13	–0.12	0.27
	6	0.09	0.15	–0.19	–0.80	–0.15	0.14
	7	0.03	0.11	–0.15	–0.78	–0.04	0.05
	11	–0.02	0.08	0.11	–1.57**	0.32	0.16
GDP	0	0.47***	0.73**	–1.24*	–7.61***	0.48	0.49
	1	0.07	0.66**	–0.36	–1.79	–0.05	0.14
	2	–0.16*	0.13	–0.04	0.29	–0.71	0.10
	3	–0.18*	–0.14	–0.27	2.08	–0.32	0.16
	4	–0.24**	–0.21	0.12	3.10*	–0.11	0.21
	5	–0.17	–0.26	0.00	2.41	0.12	0.14
	6	–0.16	–0.29	0.26	1.83	–0.01	0.14
	7	–0.24**	–0.13	0.20	0.49	0.50	0.17
	11	–0.04	–0.14	0.02	–0.23	0.21	0.05
POT	0	0.02	0.00	–0.02	0.34	0.08	0.02
	1	0.02	–0.01	–0.37*	0.53	–0.24	0.14
	2	–0.02	–0.05	–0.12	0.19	0.08	0.06
	3	–0.01	–0.00	–0.05	0.50	0.02	0.06
	4	0.00	0.00	0.03	0.40	0.01	0.05
	5	0.01	0.03	0.04	0.26	0.07	0.05
	6	0.01	0.05	0.06	0.42	0.08	0.12
	7	0.02	0.03	0.04	0.42	0.07	0.13
	11	–0.01	0.10***	–0.03	0.34*	–0.07	0.28
POTLEV	0	0.08	0.26	–0.17	–0.38	0.15	0.15
	1	0.08	0.26	–0.26	–0.25	0.09	0.14
	2	0.08	0.25	–0.29	–0.20	0.11	0.13
	3	0.08	0.25	–0.30	–0.08	0.11	0.12
	4	0.08	0.25	–0.29	0.01	0.12	0.11
	5	0.08	0.26	–0.28	0.08	0.13	0.11
	6	0.08	0.27	–0.27	0.18	0.15	0.11
	7	0.08	0.27	–0.26	0.28	0.17	0.12
	11	0.08	0.36	–0.28	0.61	0.16	0.13

*Significant at 10 percent; **significant at 5 percent; ***significant at 1 percent.

Table 4, cont'd

Regression Results: Responses of Revisions to Shocks

Dependent variable (*revisionk*)	*k*	*GDP shock1*	*GDP shock2*	*INFX shock1*	*R90 shock1*	*EXCH shock1*	$\bar{R}^2$
POTLEV	0		0.35**	–0.17	–0.41	0.14	0.12
	1		0.35**	–0.26	–0.28	0.09	0.12
	2		0.33*	–0.29	–0.24	0.10	0.10
	3		0.33*	–0.31	–0.11	0.11	0.10
	4		0.33*	–0.30	–0.02	0.11	0.09
	5		0.34*	–0.29	0.04	0.13	0.09
	6		0.35*	–0.28	0.14	0.15	0.09
	7		0.36*	–0.27	0.24	0.16	0.10
	11		0.44**	–0.29	0.57	0.16	0.12
INF	0	–0.01	0.22	0.78	–0.68	0.73	0.07
	1	0.07	0.63**	–0.01	–0.51	–0.02	0.18
	2	0.14	–0.03	0.56	–1.75	0.25	0.15
	3	0.18**	0.03	0.32	–1.00	0.32	0.18
	4	0.10	0.16	0.57*	–1.35	0.23	0.23
	5	0.14**	0.04	0.57**	–1.01	0.42	0.24
	6	0.10**	0.07	0.61***	–1.00	0.29	0.28
	7	0.11***	0.07	0.50***	–0.99*	0.33	0.37
	11	0.07	–0.06	0.25	0.40	0.38	0.10
INFX	0	–0.06	0.03	0.71**	–0.89	–0.09	0.18
	1	0.09	0.19	0.04	–0.69	–0.13	0.15
	2	0.09*	0.14	–0.03	–1.65**	–0.23	0.21
	3	0.10**	0.22*	0.14	–1.09	–0.21	0.28
	4	0.04	0.34***	0.34	–1.05	–0.14	0.30
	5	0.08*	0.20*	0.33*	–0.86	–0.06	0.30
	6	0.07*	0.15	0.33*	–0.36	–0.04	0.28
	7	0.06**	0.13*	0.20	–0.23	0.01	0.29
	11	0.03	–0.01	0.05	0.43	0.30*	0.15
R90	0	0.01	0.21	–0.04	0.30	–0.09	0.04
	1	0.16*	0.51**	–0.06	–1.57	–0.11	0.25
	2	0.28***	0.61***	0.04	–0.41	0.32	0.41
	3	0.28***	0.60***	–0.00	–0.03	0.46	0.43
	4	0.25**	0.48**	–0.20	0.81	0.44	0.35
	5	0.22**	0.34	–0.25	1.03	0.68	0.26
	6	0.17	0.28	–0.18	1.05	1.01*	0.22
	7	0.15	0.19	–0.01	1.09	1.11*	0.19
	11	0.08	0.02	0.25	0.52	1.19*	0.10

*Significant at 10 percent; **significant at 5 percent; ***significant at 1 percent.

that for a given level of potential output, higher than expected GDP data would raise both GDP growth and the gap. Similarly, the correlation between projection errors to CPI inflation and core inflation is high, consistent with the fact that CPI inflation is an aggregate that contains core inflation, so that errors in core inflation would also show up in CPI inflation. Correlations among data revisions (Table 3) are of the same sign as those among projection errors, although the former are generally stronger.

Trend versus Cycle: Projection Revisions in Response to Shocks

An important element of projection exercises is parsing shocks into permanent components that influence trends but do not have inflationary consequences, and transitory components that affect cyclical dynamics and generally affect inflationary pressures. The QPM was the primary tool used to map the implications of transitory structural shocks into economic projections. While judgment may have also entered into projections, particularly for understanding near-term economic variation, at medium to longer horizons, endogenously generated model dynamics would play a more dominant role. As noted earlier, the properties of the QPM are well documented. However, the implications of shocks for trend projections are less well understood.

In this section, the responses of projections of several main economic variables to shocks to *GDP*, *INFX*, *R*90, and *EXCH* are analyzed. To a certain extent, shocks to these variables might be considered exogenous, as they directly reveal new information from financial markets (in the case of interest rates and exchange rates) or as published by Statistics Canada. Revisions to potential output (and variables constructed using potential output) might be thought of as responses to this new information.[20] To assess the importance of these sources of new information, regressions of the following format were estimated:

$$\begin{aligned} &revisionk_t = c + \beta_{G1} GDPshock1_t \\ (9)\quad &+\beta_{G2} GDPshock2_t + \beta_I INFXshock1_t \\ &+\beta_R R90shock1_t + \beta_E EXCHshock1_t. \end{aligned}$$

Only one shock variable was included for inflation, the short-term interest rate, and the exchange rate, as these variables are essentially unrevised. Results are presented in Table 4.

The most important variable in terms of influencing projection revisions is *GDP*. Shocks to *GDP* tend to lead to revisions of the same sign to projections of the output gap, inflation, core inflation, the short-term interest rate, the level of potential output, and near-term projections of real GDP growth; and to revisions of the opposite sign to longer-term projections of real GDP growth. By contrast, there is no evidence that potential growth is responsive to these shocks. In terms of parsing *GDP* shocks, a fraction of these shocks (about 1/3) are mapped into permanent shocks that lead to parallel shifts of the level of potential without influencing the growth rate. The remainder of the *GDP* shocks are assessed as cyclical (transitory), with some persistence, and lead to revisions to gap projections at horizons out to five quarters, with the largest revisions being to *revision1* and *revision2* (about 2/3 of *GDP* shocks are mapped into *GAP* revisions for $k = (1,2)$). For positive shocks, near-term growth is revised upward and the output gap becomes larger. The additional inflationary pressures lead to tighter monetary policy, which is consistent with more rapid reductions in the size of the gap and therefore downward revisions to GDP growth, both of which are consistent with a closing of the gap after two years.

There are two noteworthy aspects to this parsing of *GDP* shocks into potential output and the output gap. First, parsing explicitly recognizes that not all shocks are transitory demand shocks. In the EMVF filter, the HP terms imply that estimates of potential output are informed by historical output data. Thus, shocks may lead to revised estimates of potential output for the last few observations of the historical data. The empirical results

[20] In examining the empirical results in the table, it is important to keep in mind that some shocks have smaller standard deviations than others. In particular, because interest rates tend to move gradually and two of three months of interest rate data are available for the contemporaneous quarter during the projection exercise, shocks to interest rates are generally of smaller magnitude. This feature may explain the somewhat larger coefficients on interest rate shocks in the tables.

suggest that this revision is linked into a new projection of potential output by shifting the previously projected level of potential output up or down in an essentially parallel fashion so that the shock has permanent effects.

Second, parallel revisions to the level of potential are consistent with smaller revisions to the output gap and potential growth, variables that play more prominent roles in communication. For communications purposes, it is preferable to focus on the main underlying signal of the state of the economy that indicates the extent of inflationary pressures. Large or frequent revisions to the recent history of the output gap or to projections of economic activity, particularly when reversed, would be undesirable. The historical mapping of a fraction of shocks into parallel shifts of potential output reduces the size of real-time revisions to the output gap and to projections of potential growth. In combination with communications about data revisions and uncertainty surrounding measures of potential output and the output gap, this may have provided a practical approach to dealing with real-time challenges of noisy and revised data.[21]

Finally, the pattern of revisions to projections of the output gap and *R*90 in response to GDP growth *shock1* may explain why there are only small effects of shocks on inflation. In particular, in general equilibrium, monetary policy responds (gradually according to the empirical results) to the revisions in the output gap projections. But with lags in the response of inflation to aggregate demand pressures, policy is "ahead of the curve" and attenuates inflationary implications. A similar outcome may occur with shocks to the exchange rate. In particular, projections of *R*90 at longer horizons respond positively to *EXCH* shocks (which are quite persistent, as evident in Table 1), possibly indicating slow pass-through of exchange rate movements to inflation, and therefore a delayed policy response to such shocks.

[21] Such a strategy is not unlike the strategy of using a measure of core inflation to indicate "underlying inflation" when a few components of the total CPI are subject to large transitory shocks.

CONCLUSION

The output gap plays a central role in monetary policy decisions and communications at the Bank of Canada. The methodology used to estimate and project potential output was designed to be consistent with the structure of the Bank's projection model (the QPM), allow estimates to be (flexibly) influenced by judgment and external structural estimates of trends, and incorporate information from a variety of sources to better disentangle supply and demand shocks. In practice, information sources that are external to the QPM, such as demographics or structural details of the Canadian labor market, are important drivers of the trend labor input component of potential output.

Analysis of revisions to real-time Bank of Canada staff economic projections reveals several interesting results. First, the similar size of typical revisions to projections of log potential output and the output gap suggest as much uncertainty about the trend as about the cycle. Second, real GDP shocks provided information about both the trend and the cycle. These shocks were parsed into permanent components that led to parallel shifts in projections of potential output and transitory components that led to persistent near-term revisions of the output gap that, with endogenous policy, dissipated over the projection horizon.

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