Arbitrage-Free Bond Pricing with Dynamic Macroeconomic Models

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The authors examine the relationship between changes in short-term interest rates induced by monetary policy and the yields on long-maturity default-free bonds. The volatility of the long end of the term structure and its relationship with monetary policy are puzzling from the perspective of simple structural macroeconomic models. The authors explore whether richer models of risk premiums, specifically stochastic volatility models combined with Epstein-Zin recursive utility, can account for such patterns. They study the properties of the yield curve when inflation is an exogenous process and compare this with the yield curve when inflation is endogenous and determined through an interest rate (Taylor) rule. When inflation is exogenous, it is difficult to match the shape of the historical average yield curve. Capturing its upward slope is especially difficult because the nominal pricing kernel with exogenous inflation does not exhibit any negative autocorrelation—a necessary condition for an upward-sloping yield curve, as shown in Backus and Zin. Endogenizing inflation provides a substantially better fit of the historical yield curve because the Taylor rule provides additional flexibility in introducing negative autocorrelation into the nominal pricing kernel. Additionally, endogenous inflation provides for a flatter term structure of yield volatilities, which better fits historical bond data. (JEL G0, G1, E4)

Federal Reserve Bank of St. Louis *Review*, July/August 2007, *89*(4), pp. 305-26.

The response of long-term interest rates to changes in short-term interest rates is a feature of the economy that often puzzles policymakers. For example, in remarks made on May 27, 1994, Alan Greenspan (1994, p. 5) expressed concern that long rates moved too much in response to an increase in short rates:

> In early February, we had thought long-term rates would move a little higher temporarily as we tightened…The sharp jump in [long] rates that occurred appeared to reflect the dramatic rise in market expectations of economic growth and associated concerns about possible inflation pressures.

Then in his February 16, 2005, testimony, Chairman Greenspan (2005) expressed a completely different concern about long rates:

> [L]ong-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points…Historically, though, even these distant forward rates have tended to rise in association with monetary policy tightening...For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.

Chairman Greenspan's comments reflect the fact that we do not yet have a satisfactory under-

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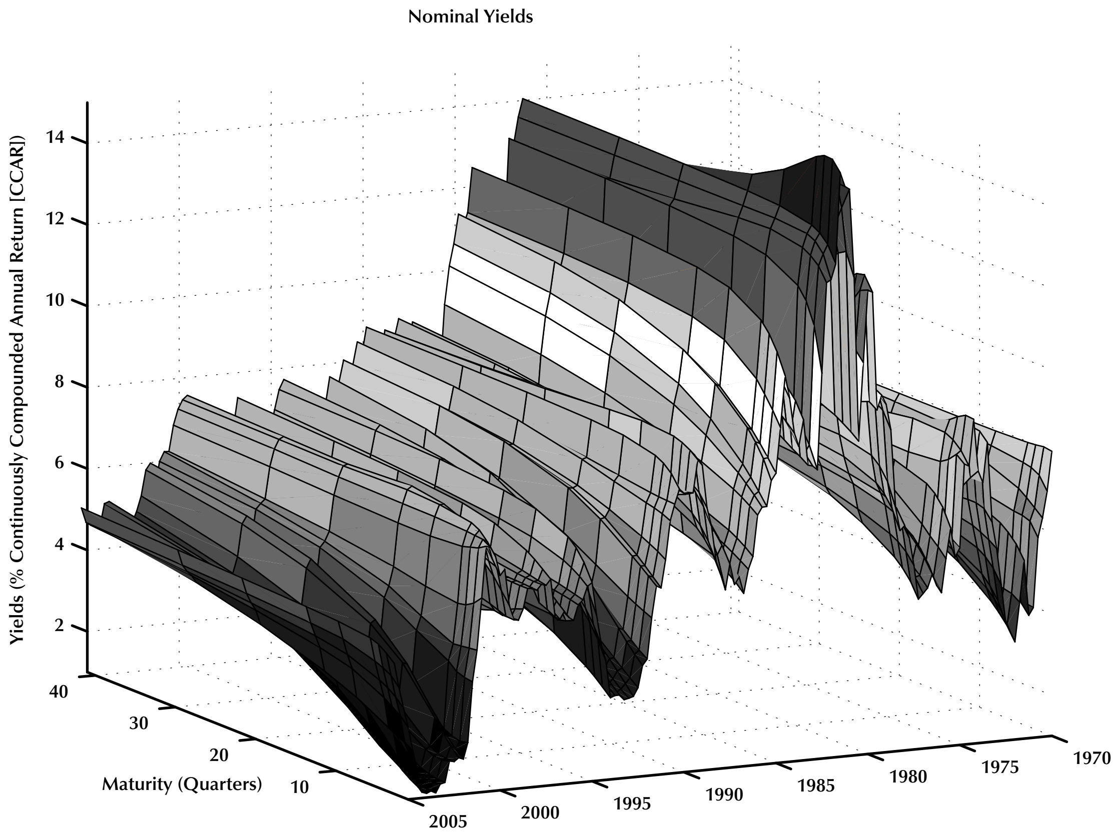
Figure 1

Time-Series Properties of the Yield Curve, 1970:Q1–2005:Q4



standing of how the yield curve is related to the structural features of the macroeconomy, such as investors' preferences, the fundamental sources of economic risk, and monetary policy.

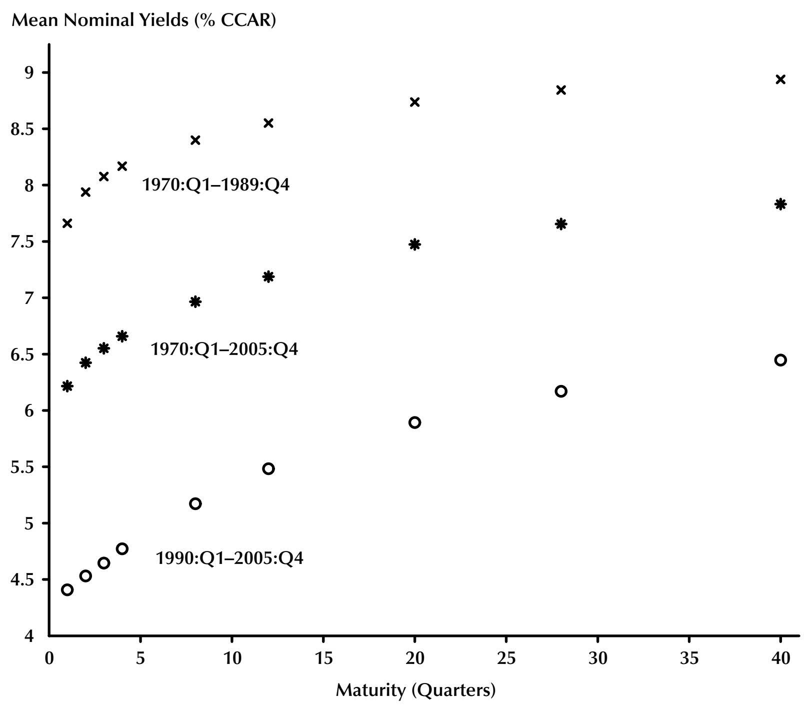
Figure 1 plots the nominal yield curve for a variety of maturities from 1 quarter—which we refer to as the short rate—up to 40 quarters for U.S. Treasuries, starting in the first quarter of 1970 and ending in the last quarter of 2005.[1] Figure 2 plots the average yield curve for the entire sample and for two subsamples. Figure 3 plots the standard deviation of yields against their maturities. Two basic patterns of yields are clear from these figures: (i) On average, the yield curve is upward sloping and (ii) there is substantial volatility in yields at all maturities. Chairman Greenspan's comments, therefore, must be framed by the fact that long yields are almost as volatile as short rates. The issue, however, is the relationship of the volatility at the long end to the volatility at the short end, and the correlation between changes in short-term interest rates and changes in long-term interest rates.

We can decompose forward interest rates into expectations of future short-term interest rates and interest rate risk premia. Because long-term interest rates are averages of forward rates, long-run interest rates depend on expectations of future short-term interest rates and interest rate risk premiums. A significant component of long rates is

[1] Yields up to 1991 are from McCulloch and Kwon (1993) then Datastream from 1991 to 2005.

Figure 2

Average Yield-Curve Behavior



the risk premium, and there is now a great deal of empirical evidence documenting that the risk premiums are time-varying and stochastic. Movements in long rates can therefore be attributed to movements in expectations of future nominal short rates, movements in risk premiums, or some combination of movements in both.

Moreover, if monetary policy is implemented using a short-term interest rate feedback rule—for example, a Taylor rule—then inflation rates must adjust so that the bond market clears. The resulting endogenous equilibrium inflation rate will then depend on the same risk factors that drive risk premiums in long rates. Monetary policy itself, therefore, could be a source of fluctuations in the yield curve in equilibrium.

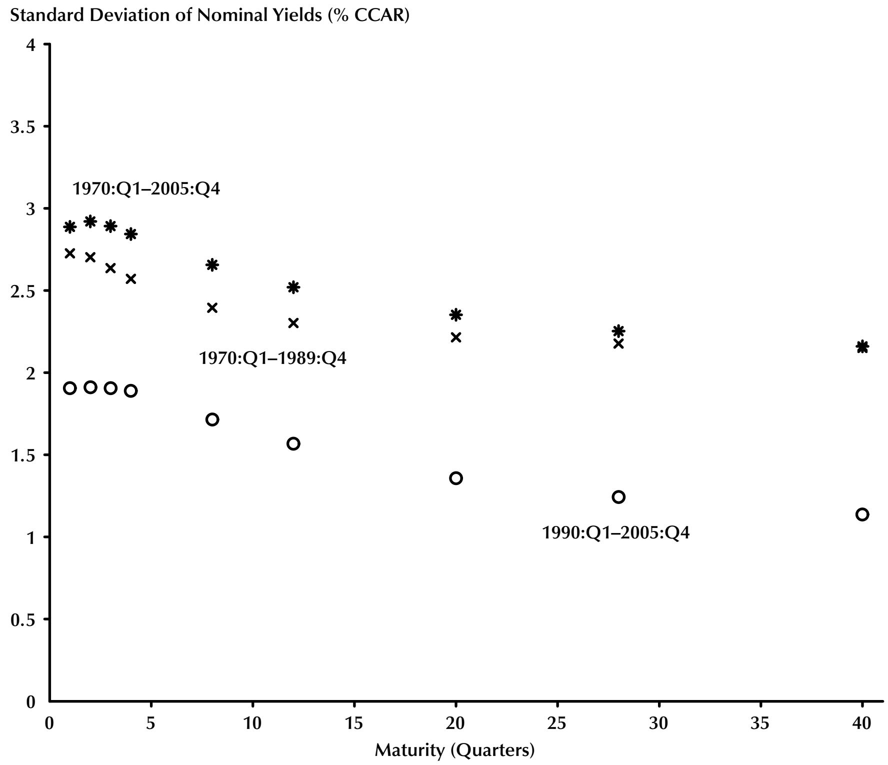
We explore such possibilities in a model of time-varying risk premiums generated by the recursive utility model of Epstein and Zin (1989) combined with stochastic volatility of endowment growth. We show how the model can be easily solved using now-standard affine term-structure methods. Affine term-structure models have the convenient property that yields are maturity-dependent linear functions of state variables. We examine some general properties of multi-period default-free bonds in our model, assuming first that inflation is an exogenous process and by allowing inflation to be endogenous and determined by an interest rate feedback rule. We show that the interest rate feedback rule—the form of monetary policy—can have significant impacts on properties of the term structure of interest rates.

THE DUFFIE-KAN AFFINE TERM-STRUCTURE MODEL

The Duffie and Kan (1996) class of affine term-structure models, translated into discrete time

Figure 3

Volatility of Yields of Various Maturities



by Backus, Foresi, and Telmer (2001), is based on a k-dimensional vector of state variables z that follows a "square root" model:

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + \Sigma(z_t)^{1/2}\varepsilon_{t+1},$$

where $\{\varepsilon_t\} \sim \text{NID}(0,1), \Sigma(z)$ is a diagonal matrix with a typical element given by $\sigma_i(z) = a_i + b_i'z$, where b_i has nonnegative elements, and Φ is stable with positive diagonal elements. The process for z requires that the volatility functions, $\sigma_i(z)$, be positive, which places additional restrictions on the parameters.

The asset-pricing implications of the model are given by the pricing kernel, m_{t+1}, a positive random variable that prices all financial assets. That is, if a security has a random payoff, h_{t+1}, at date $t+1$, then its date-t price is $E_t[m_{t+1}h_{t+1}]$. The pricing kernel in the affine model takes the form

$$-\log m_{t+1} = \delta + \gamma' z_t + \lambda' \Sigma(z_t)^{1/2}\varepsilon_{t+1},$$

where the $k \times 1$ vector γ is referred to as the "factor loadings" for the pricing kernel, the $k \times 1$ vector λ is referred to as the "price of risk" vector because it controls the size of the conditional correlation of the pricing kernel and the underlying sources of risk, and the $k \times k$ matrix $\Sigma(z_t)$ is the stochastic variance-covariance matrix of the unforecastable shock.

Let $b_t^{(n)}$ be the price at date t of a default-free pure-discount bond that pays 1 at date $t+n$, with $b_t^{(0)} = 1$. Multi-period default-free discount bond prices are built up using the arbitrage-free pricing restriction,

(1) $$b_t^{(n)} = E_t\left[m_{t+1} b_{t+1}^{(n-1)}\right].$$

Bond prices of all maturities are log-linear functions of the state:

$$-\log b_t^{(n)} = A^{(n)} + B^{(n)} z_t,$$

where $A^{(n)}$ is a scalar and $B^{(n)}$ is a $1 \times k$ row vector.

The intercept and slope parameters, which we often refer to as "yield-factor loadings," of these bond prices can be found recursively according to

(2)

$$A^{(n+1)} = A^{(n)} + \delta + B^{(n)}(I - \Phi)\theta - \frac{1}{2}\sum_{j=1}^{k}\left(\lambda_j + B_j^{(n)}\right)^2 a_j,$$

$$B^{(n+1)} = \left(\gamma' + B^{(n)}\Phi\right) - \frac{1}{2}\sum_{j=1}^{k}\left(\lambda_j + B_j^{(n)}\right)^2 b_j',$$

where $B_j^{(n)}$ is the jth element of the vector $B^{(n)}$. Because $b^{(0)} = 1$, we can start these recursions using $A^{(0)} = 0$ and $B_j^{(0)} = 0$, $j = 1,2,\ldots,k$.

Continuously compounded yields, $y_t^{(n)}$, are defined by $b_t^{(n)} = \exp(-ny_t^{(n)})$, which implies $y_t^{(n)} = -(\log b_t^{(n)})/n$. We refer to the short rate, i_t, as the one-period yield: $i_t \equiv y_t^{(1)}$.

This is an arbitrage-free model of bond pricing because it satisfies equation (1) for a given pricing kernel m_t. It is not yet a structural equilibrium model, because the mapping of the parameters of the pricing model to deeper structural parameters of investors' preferences and opportunities has not yet been specified. The equilibrium structural models we consider will all lie within this general class, hence, can be easily solved using these pricing equations.

A TWO-FACTOR MODEL WITH EPSTEIN-ZIN PREFERENCES

We begin our analysis of structural models of the yield curve by solving for equilibrium real yields in a representative-agent exchange economy. Following Backus and Zin (2006), we consider a representative agent who chooses consumption to maximize the recursive utility function given in Epstein and Zin (1989). Given a sequence of consumption, $\{c_t, c_{t+1}, c_{t+2}, \ldots\}$, where future consumptions can be random outcomes, the intertemporal utility function, U_t, is the solution to the recursive equation,

(3) $$U_t = \left[(1-\beta)c_t^{\rho} + \beta\mu_t(U_{t+1})^{\rho}\right]^{1/\rho},$$

where $0 < \beta < 1$ characterizes impatience (the marginal rate of time preference is $1-1/\beta$), $\rho \le 1$ measures the preference for intertemporal substitution (the elasticity of intertemporal substitution for deterministic consumption paths is $1/(1-\rho)$), and the certainty equivalent of random future utility is

(4) $$\mu_t(U_{t+1}) \equiv E_t\left[U_{t+1}^{\alpha}\right]^{\frac{1}{\alpha}},$$

where $\alpha \le 1$ measures static risk aversion (the coefficient of relative risk aversion for static gambles is $1 - \alpha$). The marginal rate of intertemporal substitution, m_{t+1}, is

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho}.$$

Time-additive expected utility corresponds to the parameter restriction $\rho = \alpha$.

In equilibrium, the representative agent consumes the stochastic endowment, e_t, so that, $\log(c_{t+1}/c_t) = \log(e_{t+1}/e_t) = x_{t+1}$, where x_{t+1} is the log of the ratio of endowments in $t+1$ relative to t. The log of the equilibrium marginal rate of substitution, referred to as the real pricing kernel, is therefore given by

(5) $$\begin{aligned}\log m_{t+1} &= \log\beta + (\rho-1)x_{t+1}\\ &\quad + (\alpha-\rho)\left[\log W_{t+1} - \log\mu_t(W_{t+1})\right],\end{aligned}$$

where W_t is the value of utility in equilibrium.

The first two terms in the marginal rate of substitution are standard expected utility terms: the pure time preference parameter, β, and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution. The third term in the pricing kernel is a new term coming from the Epstein-Zin preferences.

The endowment-growth process evolves stochastically according to

$$x_{t+1} = (1-\phi_x)\theta_x + \phi_x x_t + v_t^{1/2}\varepsilon_{t+1}^x,$$

where

$$v_{t+1} = (1-\phi_v)\theta_v + \phi_v v_t + \sigma_v \varepsilon_{t+1}^v$$

is the process for the conditional volatility of endowment growth. We will refer to v_t as stochastic volatility. The innovations ε_t^x and ε_t^v are distributed NID(0,1).

Note that the state vector in this model conforms with the setup of the Duffie-Kan model above. Define the state vector $z_t \equiv [x_t\ v_t]'$, which implies parameters for the Duffie-Kan model:

$$\theta = [\theta_x\ \theta_v]'$$

$$\Phi = \operatorname{diag}\{\phi_x, \phi_v\}$$

$$\Sigma(z_t) = \operatorname{diag}\{a_1 + b_1' z_t,\ a_2 + b_2' z_t\}$$

$$a_1 = 0,\ b_1 = [0\ 1]',\ a_2 = \sigma_v^2,\ b_2 = [0\ 0]'.$$

Following the analysis in Hansen, Heaton, and Li (2005), we will work with the logarithm of the value function scaled by the endowment:

$$W_t / e_t = \left[(1-\beta) + \beta\left(\mu_t(W_{t+1})/e_t\right)^{\rho}\right]^{1/\rho}$$

$$= \left[(1-\beta) + \beta\left(\mu_t\left(\frac{W_{t+1}}{e_{t+1}} \times \frac{e_{t+1}}{e_t}\right)^{\rho}\right)\right]^{1/\rho}, \tag{6}$$

where we have used the linear homogeneity of μ_t (see equation (4)). Take logarithms of (6) to obtain

$$w_t = \rho^{-1}\log\left[(1-\beta) + \beta\ \exp(\rho u_t)\right],$$

where $w_t \equiv \log(W_t/e_t)$ and $u_t \equiv \log(\mu_t(\exp(w_{t+1} + x_{t+1})))$. Consider a linear approximation of the right-hand side of this equation as a function of u_t around the point $\bar{m}$:

$$w_t \approx \rho^{-1}\log\left[(1-\beta) + \beta\exp(\rho\bar{m})\right] + \left[\frac{\beta exp(\rho\bar{m})}{1-\beta+\beta\exp(\rho\bar{m})}\right](u_t - \bar{m}) \equiv \bar{\kappa} + \kappa u_t,$$

where $\kappa < 1$. For the special case with $\rho = 0$, that is, a log time aggregator, the linear approximation is exact, implying $\bar{\kappa} = 1 - \beta$ and $\kappa = \beta$ (see Hansen, Heaton, and Li, 2005). Similarly, approximating around $\bar{m} = 0$, results in $\bar{\kappa} = 0$ and $\kappa = \beta$.

Given the state variables and the log-linear structure of the model, we conjecture a solution for the log value function of the form

$$w_t = \bar{\omega} + \omega_x x_t + \omega_v v_t,$$

where $\bar{\omega}$, ω_x, and ω_v are constants to be determined. By substituting,

$$w_{t+1} + x_{t+1} = \bar{\omega} + (\omega_x + 1)x_{t+1} + \omega_v v_{t+1}.$$

Because x_{t+1} and v_{t+1} are jointly normally distributed, the properties of normal random variables can be used to solve for u_t:

$$\begin{aligned} u_t &\equiv \log\left(\mu_t\left(\exp(w_{t+1} + x_{t+1})\right)\right) \\ &= \log\left(E_t\left[\exp(w_{t+1} + x_{t+1})^{\alpha}\right]^{\frac{1}{\alpha}}\right) \\ &= E_t\left[w_{t+1} + x_{t+1}\right] + \frac{\alpha}{2}\operatorname{Var}_t\left[w_{t+1} + x_{t+1}\right] \\ &= \bar{\omega} + (\omega_x + 1)(1-\phi_x)\theta_x + \omega_v(1-\phi_v)\theta_v \\ &\quad + (\omega_x + 1)\phi_x x_t + \omega_v \phi_v v_t \\ &\quad + \frac{\alpha}{2}(\omega_x + 1)^2 v_t + \frac{\alpha}{2}\omega_v^2 \sigma_v^2. \end{aligned}$$

We can use the above expression to solve for the value-function parameters and verify its log-linear solution:

$$\begin{aligned} \omega_x &= \kappa(\omega_x + 1)\phi_x \\ \Rightarrow \omega_x &= \left(\frac{\kappa}{1-\kappa\phi_x}\right)\phi_x \\ \omega_v &= \kappa\left[\omega_v\phi_v + \frac{\alpha}{2}(\omega_x + 1)^2\right] \\ \Rightarrow \omega_v &= \left(\frac{\kappa}{1-\kappa\phi_v}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1-\kappa\phi_x}\right)^2\right] \\ \bar{\omega} &= \frac{\bar{\kappa}}{1-\kappa} + \frac{\kappa}{1-\kappa}\left[\begin{aligned}&(\omega_x + 1)(1-\phi_x)\theta_x \\ &+ \omega_v(1-\phi_v)\theta_v + \frac{\alpha}{2}\omega_v^2\sigma_v^2\end{aligned}\right]. \end{aligned}$$

The solution allows us to simplify the term $[\log W_{t+1} - \log \mu_t(W_{t+1})]$ in the real pricing kernel in equation (5):

$$
\begin{aligned}
&\log W_{t+1} - \log \mu_t(W_{t+1}) = w_{t+1} + x_{t+1} - \log \mu_t(w_{t+1} + x_{t+1}) \\
&= (\omega_x + 1)\left[x_{t+1} - E_t x_{t+1}\right] + \omega_v\left[v_{t+1} - E_t v_{t+1}\right] \\
&-\frac{\alpha}{2}(\omega_x + 1)^2 \operatorname{Var}_t\left[x_{t+1}\right] - \frac{\alpha}{2}\omega_v^2 \operatorname{Var}_t\left[v_{t+1}\right] \\
&= (\omega_x + 1) v_t^{1/2}\varepsilon_{t+1}^x + \omega_v \sigma_v \varepsilon_{t+1}^v - \frac{\alpha}{2}(\omega_x + 1)^2 v_t - \frac{\alpha}{2}\omega_v^2 \sigma_v^2.
\end{aligned}
$$

The real pricing kernel, therefore, is a member of the Duffie-Kan class with two factors and parameters:

$$
\begin{aligned}
&\delta = -\log(\beta) + (1-\rho)(1-\phi_x)\theta_x + \frac{\alpha}{2}(\alpha-\rho)\omega_v^2\sigma_v^2 \\
&\gamma = \left[\gamma_x \quad \gamma_v\right]' \\
&\quad = \left[(1-\rho)\phi_x \quad \frac{\alpha}{2}(\alpha-\rho)\left(\frac{1}{1-\kappa\phi_x}\right)^2\right]' \\
&\lambda = \left[\lambda_x \quad \lambda_v\right]' \\
&\quad = \begin{bmatrix} (1-\alpha) - (\alpha-\rho)\left(\dfrac{\kappa\phi_x}{1-\kappa\phi_x}\right) \\ -\left(\dfrac{\alpha}{2}\right)\left(\dfrac{\kappa(\alpha-\rho)}{1-\kappa\phi_v}\right)\left(\dfrac{1}{1-\kappa\phi_x}\right)^2 \end{bmatrix}.
\end{aligned}
\tag{7}
$$

We can now use the recursive formulas in equation (2) to solve for real discount bond prices and the real yield curve.

Note how the factor loadings and prices of risk depend on the deeper structural parameters and the greatly reduced dimensionality of the parameter space relative to the general affine model. Also, for the time-additive expected utility special case, $\alpha = \rho$, the volatility factor does not enter the conditional mean of the pricing kernel because $\gamma_v = 0$; and the price of risk for the volatility factor is zero because $\lambda_v = 0$. Finally, we can see from the expressions for bond prices that the two key preference parameters, ρ and α, provide freedom in controlling both the factor loadings and the prices of risk in the real pricing kernel.

NOMINAL BOND PRICING

To understand the price of nominal bonds, we need a nominal pricing kernel. If we assume that there is a frictionless conversion of money for goods in this economy, the nominal kernel is given by

$$
\log\left(m_{t+1}^{\$}\right) = \log\left(m_{t+1}\right) - p_{t+1}, \tag{8}
$$

where p_{t+1} is the log of the money price of goods at time $t+1$ relative to the money price of goods at time t, that is, the inflation rate between t and $t+1$. Clearly then, the source of inflation, its random properties, and its relationship to the real pricing kernel is of central interest for nominal bond pricing. We next consider two different specifications for equilibrium inflation.

Exogenous Inflation

If we expand the state space to include an exogenous inflation process, p_t, the state vector becomes $z_t = [x_t \ v_t \ p_t]'$. The stochastic process for exogenous inflation is

$$
p_{t+1} = \left(1-\phi_p\right)\theta_p + \phi_p p_t + \sigma_p \varepsilon_{t+1}^p,
$$

where ε_{t+1}^p is also normally distributed independently of the other two shocks. In this case, the parameters for the affine nominal pricing kernel are

$$
\begin{aligned}
\delta^{\$} &= \delta + \left(1-\phi_p\right)\theta_p \\
\gamma^{\$} &= \left[\gamma_x \ \ \gamma_v \ \ \phi_p\right]' \\
\lambda^{\$} &= \left[\lambda_x \ \ \lambda_v \ \ 1\right]'.
\end{aligned}
$$

In the exogenous inflation model, the price of inflation risk is always exactly 1 and does not change with the values of any of the other structural parameters in the model. In addition, the factor loadings and prices of risk for output growth and stochastic volatility are the same as in the real pricing kernel. We will refer to this nominal pricing kernel specification as the exogenous inflation economy.

Monetary Policy and Endogenous Inflation

We begin by assuming that monetary policy follows a simple nominal interest rate rule. We will abuse conventional terminology and often refer to the interest rate rule as a Taylor rule. Although there are a variety of ways to specify a Taylor rule—see Ang, Dong, and Piazzesi (2004)—we will consider a rule in which the short-term interest rate depends on contemporaneous output, inflation, and a policy shock:

$$i_t = \bar{\tau} + \tau_x x_t + \tau_p p_t + s_t, \tag{9}$$

where the monetary policy shock satisfies

$$s_t = \phi_s s_{t-1} + \sigma_s \varepsilon_t^s$$

and where $\varepsilon_t^s \sim \mathrm{NID}(0,1)$ is independent of the other two real shocks.

Because this nominal interest rate rule must also be consistent with equilibrium in the bond market, that is, it must be consistent with the nominal pricing kernel in equation (8) as well as equation (9), we can use these two equations to find the equilibrium process for inflation. Conjecture a log-linear solution for p_t,

$$p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t, \tag{10}$$

with $\bar{\pi}$, π_x, and π_s constants to be solved.

To solve for a rational expectations solution to the model, we substitute the guess for the inflation rate into both the Taylor rule and the nominal pricing kernel and solve for the parameters $\bar{\pi}$, π_x, π_v, and π_s that equate the short rate determined by the pricing kernel with the short rate determined by the Taylor rule.

From the dynamics of x_{t+1}, v_{t+1}, and s_{t+1}, inflation, p_{t+1}, is given by

$$\begin{aligned}
p_{t+1} &= \bar{\pi} + \pi_x x_{t+1} + \pi_v v_{t+1} + \pi_s s_{t+1} \\
&= \bar{\pi} + \pi_x \left(1-\phi_x\right)\theta_x + \pi_v \left(1-\phi_v\right)\theta_v \\
&\quad + \pi_x \phi_x x_t + \pi_v \phi_v v_t + \pi_s \phi_s s_t \\
&\quad + \pi_x v_t^{1/2} \varepsilon_{t+1}^x + \pi_v \sigma_v \varepsilon_{t+1}^v + \pi_s \sigma_s \varepsilon_{t+1}^s.
\end{aligned}$$

Substituting into the nominal pricing kernel,

$$\begin{aligned}
-\log\left(m_{t+1}^{\$}\right) &= -\log\left(m_{t+1}\right) + p_{t+1} \\
&= \delta + \gamma_x x_t + \gamma_v v_t + \lambda_x v_t^{1/2} \varepsilon_{t+1}^x + \lambda_v \sigma_v \varepsilon_{t+1}^v + p_{t+1} \\
&= \delta + \bar{\pi} + \pi_x \left(1-\phi_x\right)\theta_x + \pi_v \left(1-\phi_v\right)\theta_v \\
&\quad + \left(\gamma_x + \pi_x \phi_x\right) x_t + \left(\gamma_v + \pi_v \phi_v\right) v_t + \pi_s \phi_s s_t \\
&\quad + \left(\lambda_x + \pi_x\right) v_t^{1/2} \varepsilon_{t+1}^x + \left(\lambda_v + \pi_v\right) \sigma_v \varepsilon_{t+1}^v + \pi_s \sigma_s \varepsilon_{t+1}^s.
\end{aligned}$$

From these dynamics, the nominal one-period interest rate, $i_t = -\log\left(E_t[m_{t+1}^{\$}]\right)$, is

$$\begin{aligned}
i_t &= \delta + \bar{\pi} + \pi_x \left(1-\phi_x\right)\theta_x + \pi_v \left(1-\phi_v\right)\theta_v \\
&\quad + \left(\gamma_x + \pi_x \phi_x\right) x_t + \left(\gamma_v + \pi_v \phi_v\right) v_t + \pi_s \phi_s s_t \\
&\quad - \frac{1}{2}\left(\lambda_x + \pi_x\right)^2 v_t - \frac{1}{2}\left(\lambda_v + \pi_v\right)^2 \sigma_v^2 - \frac{1}{2}\pi_s^2 \sigma_s^2.
\end{aligned}$$

Comparing this with the interest rate rule,

$$i_t = \bar{\tau} + \tau_x x + \tau_p \left(\bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t\right) + s_t,$$

gives the parameter restrictions consistent with equilibrium:

(11)

$$\begin{aligned}
\pi_x &= \frac{\gamma_x - \tau_x}{\tau_p - \phi_x} \\
\pi_v &= \frac{\gamma_v - \frac{1}{2}\left(\lambda_x + \pi_x\right)^2}{\tau_p - \phi_v} \\
\pi_s &= -\frac{1}{\tau_p - \phi_s} \\
\bar{\pi} &= \frac{1}{\tau_p - 1}\begin{pmatrix} \delta - \bar{\tau} + \pi_x \left(1-\phi_x\right)\theta_x + \pi_v \left(1-\phi_v\right)\theta_v \\ -\frac{1}{2}\left(\lambda_v + \pi_v\right)^2 \sigma_v^2 - \frac{1}{2}\pi_s^2 \sigma_s^2 \end{pmatrix}.
\end{aligned}$$

These expressions form a recursive system we use to solve for the equilibrium parameters of the inflation process. See Cochrane (2006) for a more detailed account of this rational expectations solution method.

It is clear from these expressions that the equilibrium inflation process will depend on the preference parameters of the household generally and attitudes toward risk specifically.

In a similar fashion, we can extend the analysis to any Taylor-type rule that is linear in the state variables, including (i) lagged short rates, (ii) other contemporaneous yields at any maturity, and (iii) forward-looking rules, such as those in Clarida, Galí, and Gertler (2000). Such extensions are possible because, in the affine framework, interest rates are all simply linear functions of the current state variables. See Ang, Dong, and Piazzesi (2004) and Gallmeyer, Hollifield, and Zin (2005) for some concrete examples.

A Monetary Policy–Consistent Pricing Kernel

Substituting the equilibrium inflation process from equations (10) and (11) into the nominal pricing kernel, we obtain an equilibrium three-factor affine term-structure model that is consistent with the nominal interest rate rule. The state space is

$$
\begin{aligned}
z_t &\equiv \begin{bmatrix} x_t & v_t & s_t \end{bmatrix}' \\
\Phi &= \operatorname{diag}\{\phi_x, \phi_v, \phi_s\} \\
\theta &= \begin{bmatrix} \theta_x & \theta_v & 0 \end{bmatrix}' \\
\Sigma(z_t) &= \operatorname{diag}\{a_1 + b_1' z_t,\ a_2 + b_2' z_t,\ a_3 + b_3' z_t\} \\
a_1 &= 0, b_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}' \\
a_2 &= \sigma_v^2, b_2 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}' \\
a_3 &= \sigma_s^2, b_3 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}',
\end{aligned}
$$

and the parameters of the pricing kernel are

$$
\begin{aligned}
\delta^{\$} &= \delta + \bar{\pi} + \pi_x (1 - \phi_x)\theta_x + \pi_v (1 - \phi_v)\theta_v \\
\gamma^{\$} &= \begin{bmatrix} \gamma_x + \phi_x \pi_x & \gamma_v + \phi_v \pi_v & \phi_s \pi_s \end{bmatrix}' \\
\lambda^{\$} &= \begin{bmatrix} \lambda_x + \pi_x & \lambda_v + \pi_v & \pi_s \end{bmatrix}'.
\end{aligned}
$$

We will often refer to this nominal pricing kernel specification as the endogenous inflation economy.

The Taylor rule parameters, through their determination of the equilibrium inflation process, affect both the factor loadings on the real factors as well as their prices of risk. Monetary policy through its effects on endogenous inflation, therefore, can result in risk premiums in the term structure that are significantly different from those in the exogenous inflation model. We explore such a possibility through numerical examples.

QUANTITATIVE EXERCISES

We calibrate the exogenous processes in our model to quarterly postwar U.S. data as follows:

1. Endowment growth: $\phi_x = 0.36$, $\theta_x = 0.006$, $\sigma_x = 0.0048(1 - \phi_x^2)^{1/2}$
2. Inflation: $\phi_p = 0.8471$, $\theta_p = 0.0093$, $\sigma_p = 0.0063(1 - \phi_p^2)^{1/2}$
3. Stochastic volatility: $\phi_v = 0.973$, $\theta_v = 0.0001825$, $\sigma_v = 0.9884 \times 10^{-5}$
4. Policy shock: $\phi_s = 0.922$, $\sigma_s = (0.023 \times 10^{-4})^{1/2}$

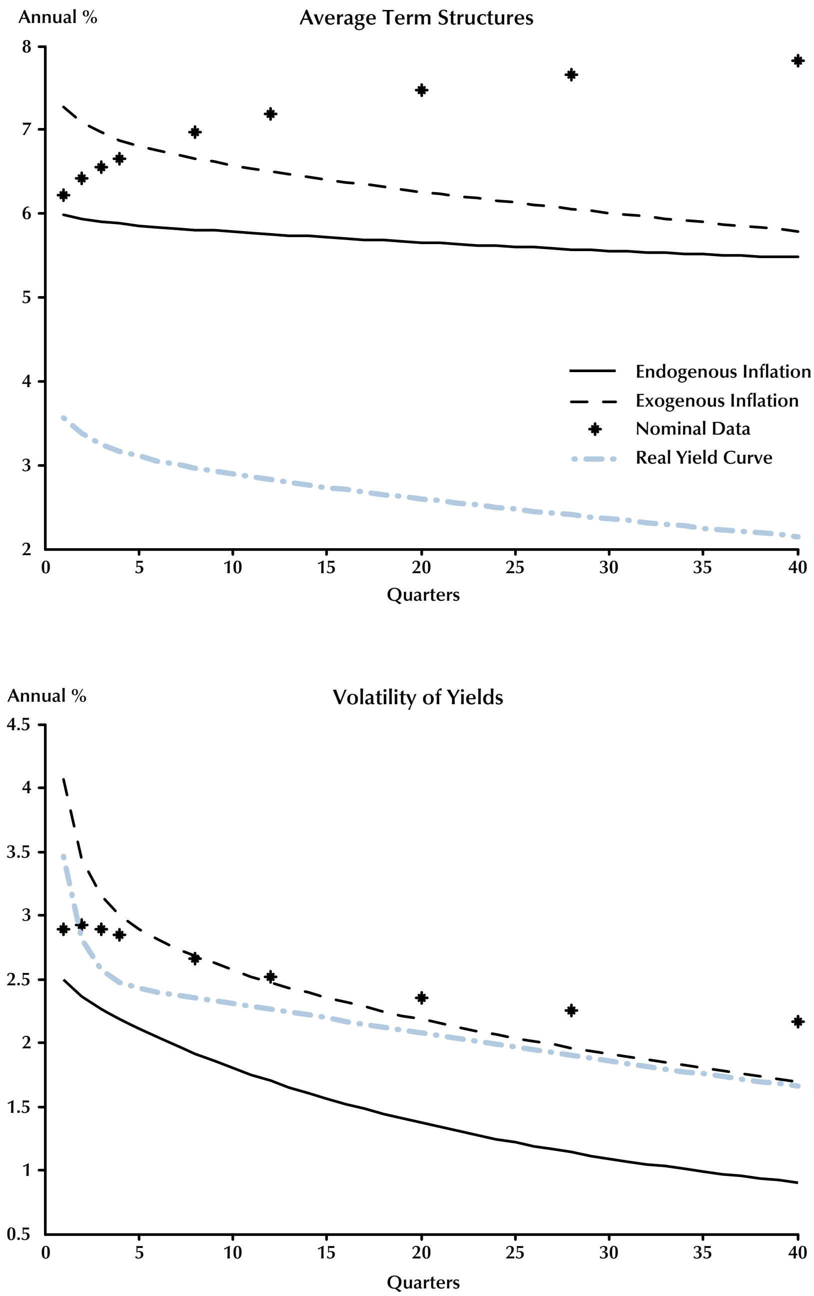
The endowment growth process is calibrated to quarterly per capita consumption of durable goods and services, and inflation is calibrated to the nondurables and services deflator, similarly to Piazzesi and Schneider (2007). The volatility process is taken from Bansal and Yaron (2004), who calibrate their model to monthly data. We adjust their parameters to deal with quarterly time-aggregation. We take the parameters for the policy shock from Ang, Dong, and Piazzesi (2004), who estimate a Taylor rule using an affine term-structure model with macroeconomic factors and an unobserved policy shock.

Figures 4 though 7 depict the average yield curves and yield volatilities for different preference parameters for the exogenous and endogenous inflation models. In the top panel of each figure, asterisks denote the empirical average nominal yield curve, a blue dashed-dotted line denotes the average real yield curve common across both inflation models, a dashed line denotes the average nominal yield curve in the exogenous inflation economy, and a solid line denotes the average nominal yield curve in the endogenous inflation economy. The bottom panel depicts yield volatilities for the same cases as the average yield curve in the top panel. (Asterisks in Figures 4 through 9 are the moments—means and standard deviations—of the data in Figure 1.)

Each figure is computed using a different set of preference parameters. We fix a level of the intertemporal elasticity parameter, ρ, for each panel and pick the remaining preference parameters—the risk aversion coefficient, α, and the rate of time preference, β—to minimize the distance between the average nominal yields and yield volatilities in the data and those implied by the exogenous inflation economy. We pick the Taylor rule parameters to minimize the distance between the average nominal yields and yield volatilities in the data and those implied by the endogenous inflation economy. Table 1 reports

Figure 4

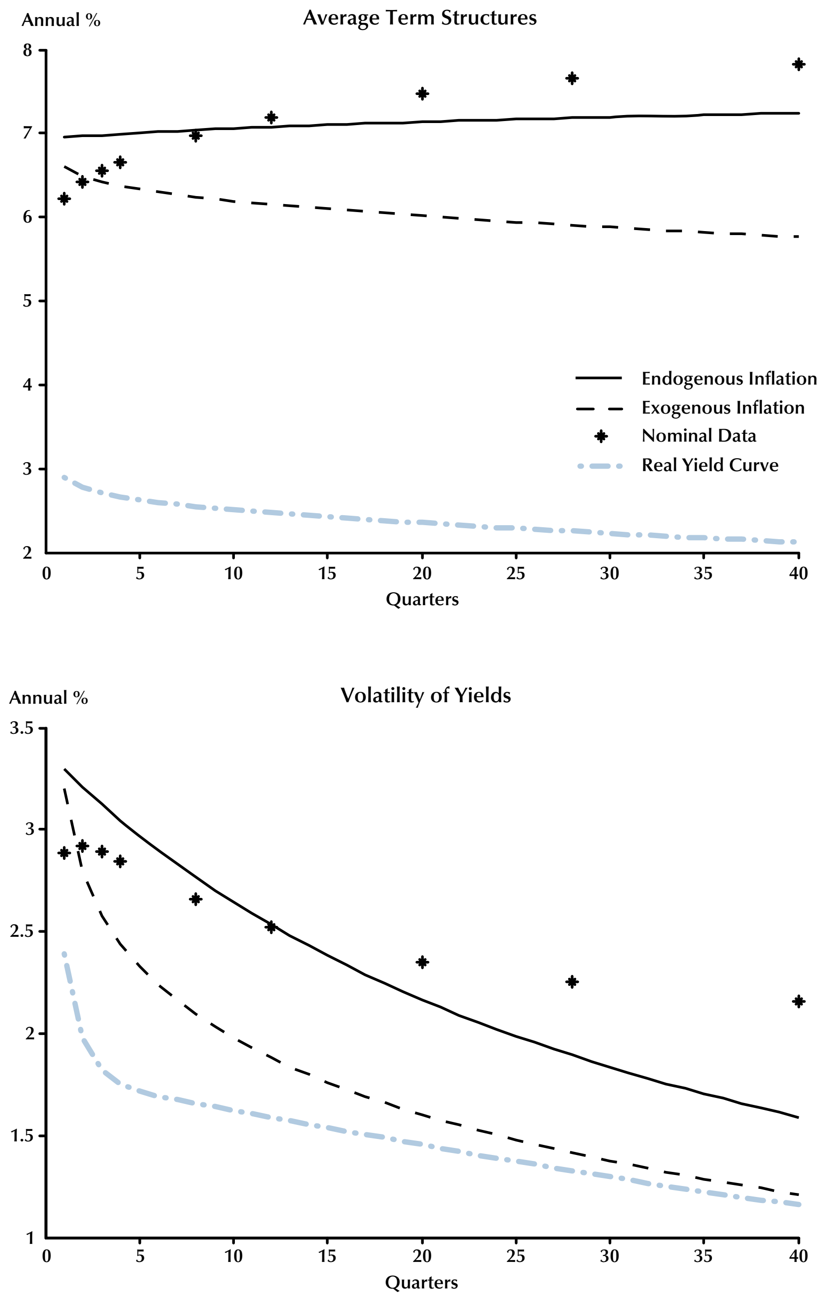
Average Yield Curve and Volatilities for the Epstein-Zin Model with Stochastic Volatility



NOTE: The parameters are $\rho = -0.5$, $\alpha = -4.835$, $\beta = 0.999$, $\bar{\tau} = 0.003$, $\tau_x = 1.2475$, and $\tau_p = 1.000$. The empirical moments for the full sample (1970:Q1–2005:Q4) are plotted with asterisks, properties of the real yield curve are plotted with a dashed-dotted blue line, properties of the yield curve in the exogenous inflation economy are plotted with a dashed black line, and properties of the yield curve in the economy with endogenous inflation are plotted with a solid black line.

Figure 5

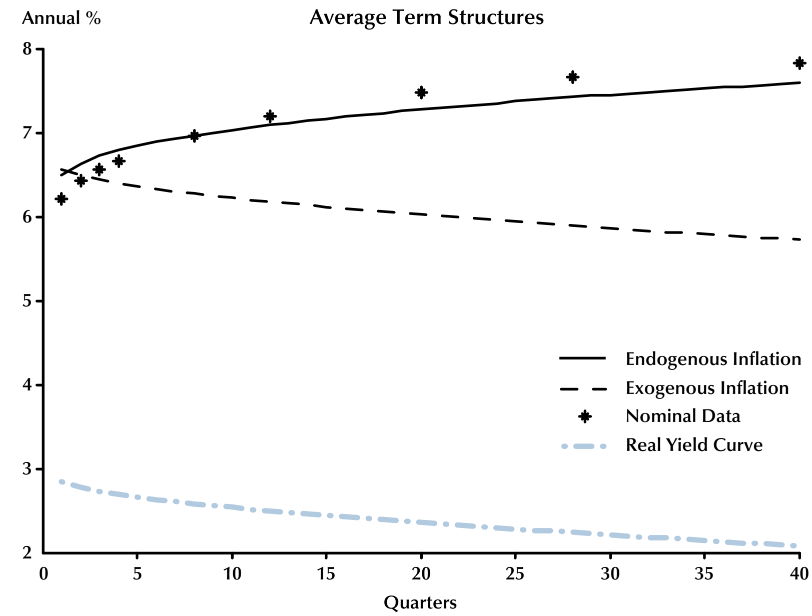
Average Yield Curve and Volatilities for the Epstein-Zin Model with Stochastic Volatility

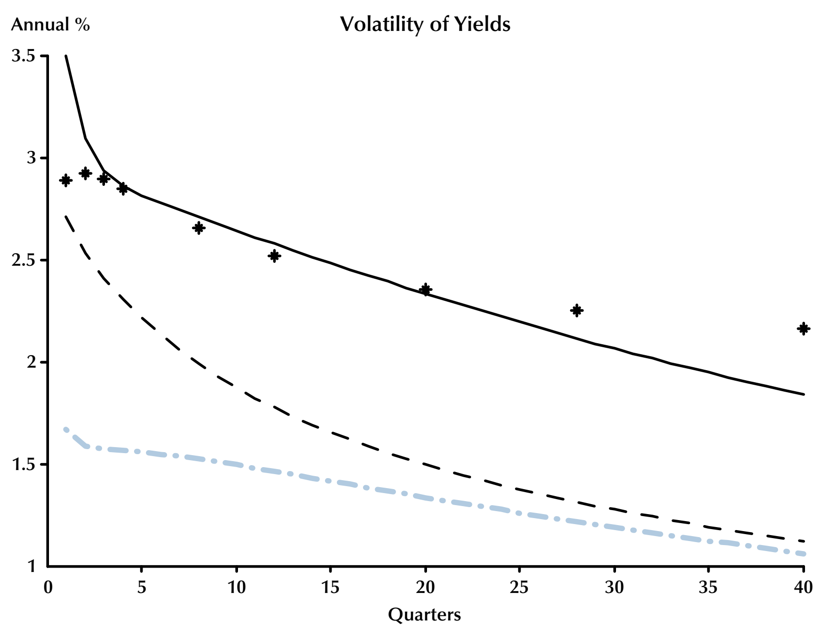


NOTE: The parameters are $\rho = 0.0$, $\alpha = -4.061$, $\beta = 0.998$, $\bar{\tau} = 0.003$, $\tau_x = 0.973$, and $\tau_p = 0.973$. The empirical moments for the full sample (1970:Q1–2005:Q4) are plotted with asterisks, properties of the real yield curve are plotted with a dashed-dotted blue line, properties of the yield curve in the exogenous inflation economy are plotted with a dashed black line, and properties of the yield curve in the economy with endogenous inflation are plotted with a solid black line.

Figure 6

Average Yield Curve and Volatilities for the Epstein-Zin Model with Stochastic Volatility

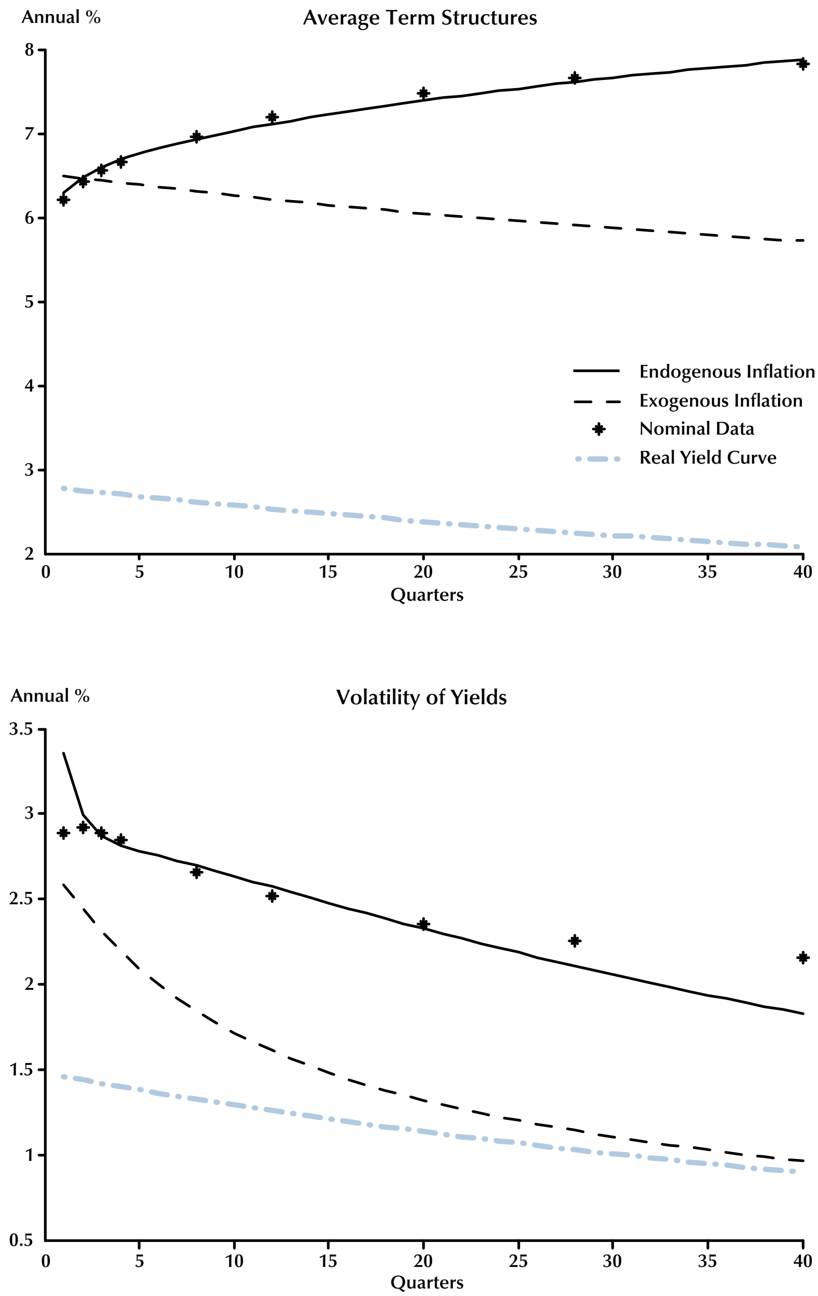




NOTE: The parameters are $\rho = 0.5$, $\alpha = -4.911$, $\beta = 0.994$, $\bar{\tau} = -0.015$, $\tau_x = 3.064$, and $\tau_p = 2.006$. The empirical moments for the full sample (1970:Q1–2005:Q4) are plotted with asterisks, properties of the real yield curve are plotted with a dashed-dotted blue line, properties of the yield curve in the exogenous inflation economy are plotted with a dashed black line, and properties of the yield curve in the economy with endogenous inflation are plotted with a solid black line.

Figure 7

Average Yield Curve and Volatilities for the Epstein-Zin Model with Stochastic Volatility



NOTE: The parameters are $\rho = 1.0$, $\alpha = -6.079$, $\beta = 0.990$, $\overline{\tau} = -0.004$, $\tau_x = 1.534$, and $\tau_p = 1.607$. The empirical moments for the full sample (1970:Q1–2005:Q4) are plotted with asterisks, properties of the real yield curve are plotted with a dashed-dotted blue line, properties of the yield curve in the exogenous inflation economy are plotted with a dashed black line, and properties of the yield curve in the economy with endogenous inflation are plotted with a solid black line.

Table 1

Factor Loadings and Prices of Risk

	Constant	Factor loadings (γs) x_t	v_t	p_t	s_t	Prices of risk (λs) ε_{t+1}^{x}	ε_{t+1}^{v}	ε_{t+1}^{p}	ε_{t+1}^{s}
A. ρ = –0.5, α = –4.835, β = 0.999, $\bar{\tau}$ = 0.003, τ_x = 1.2475, τ_p = 1.000									
Real kernel	0.01	0.54	25.45	—	—	8.25	–902.98	—	—
Exogenous inflation	0.01	0.54	25.45	0.85	—	8.25	–902.98	1.00	—
Endogenous inflation	0.02	0.14	21.63	—	–1.44	7.15	–906.90	—	–1.56
B. ρ = 0.0, α = –4.061, β = 0.998, $\bar{\tau}$ = 0.003, τ_x = 0.973, τ_p = 0.973									
Real kernel	0.01	0.36	20.07	—	—	7.34	–677.11	—	—
Exogenous inflation	0.01	0.36	20.07	0.85	—	7.34	–677.11	1.00	—
Endogenous inflation	0.02	0.00	33.56	—	–1.51	6.34	–663.24	—	–1.63
C. ρ = 0.5, α = –4.911, β = 0.994, $\bar{\tau}$ = –0.015, τ_x = 3.064, τ_p = 2.006									
Real kernel	0.01	0.18	32.23	—	—	8.93	–972.61	—	—
Exogenous inflation	0.01	0.18	32.23	0.85	—	8.93	–972.61	1.00	—
Endogenous inflation	0.02	–0.45	38.34	—	–0.56	7.18	–966.33	—	–0.61
D. ρ = 1.0, α = –6.079, β = 0.990, $\bar{\tau}$ = 0.004, τ_x = 1.534, τ_p = 1.607									
Real kernel	0.02	0.00	51.82	—	—	10.99	–1,398.00	—	—
Exogenous inflation	0.02	0.00	51.82	0.85	—	10.99	–1,398.00	1.00	—
Endogenous inflation	0.03	–0.44	58.30	—	–0.74	9.76	–1,391.30	—	–0.80

NOTE: The table reports the affine term-structure parameters for the real term structure, the nominal term structure in the exogenous inflation economy, and the nominal term structure in the endogenous inflation economy. The parameters in each panel are computed using a different set of preference parameters. We fix a level of the intertemporal elasticity parameter, ρ, and choose the remaining preference parameters—the risk aversion coefficient, α, and the rate-of-time preference, β, to minimize the distance between the average nominal yields and yield volatilities in the data and those implied by the exogenous inflation economy. We pick the Taylor rule parameters to minimize the distance between the average nominal yields and yield volatilities in the data and the those implied by the endogenous inflation economy.

the factor loadings and the prices of risk for each economy corresponding to the figures. Table 2 reports the coefficients on the equilibrium inflation rate and properties of the equilibrium inflation rate in the endogenous inflation economy.

Figure 4 reports the results with $\rho = -0.5$; here the representative agent has a low intertemporal elasticity of substitution. The remaining preference parameters are $\alpha = -4.835$ and $\beta = 0.999$. With this choice of parameters, the average real term structure is slightly downward sloping.

Backus and Zin (1994) show that a necessary condition for the average yield curve to be upward sloping is negative autocorrelation in the pricing kernel.[2] Consider an affine model with independent factors $z_t^1, z_t^2, \ldots, z_t^k$ with an innovation ε_t^j on the jth factor, a factor loading γ_j on the jth factor, and a price of risk λ_j on the jth factor. In such a model, the jth factor contributes

$$\gamma_j^2 \text{Autocov}\left(z_t^j\right) + \gamma_j \lambda_j \text{Cov}\left(z_t^j, \varepsilon_t^j\right) \tag{12}$$

to the autocovariance of the pricing kernel.

In our calibration, the exogenous factors in

[2] Piazzesi and Schneider (2007) argue that an upward-sloping nominal yield curve can be generated if inflation is bad news for consumption growth. Such a structure leads to negative autocorrelation in the nominal pricing kernel.

Table 2

Properties of p_t in the Endogenous Inflation Economy

	$\bar{\pi}$	π_x	π_v	π_s	$E(p_t)$	$\sigma(p_t)$	AR(1)
A. $\rho = -0.5$, $\alpha = -4.835$, $\beta = 0.999$, $\bar{\tau} = 0.003$, $\tau_x = 1.2475$, $\tau_p = 1.000$							
	0.01	–1.11	–3.92	–1.56	0.01	0.02	0.37
B. $\rho = 0.0$, $\alpha = -4.061$, $\beta = 0.998$, $\bar{\tau} = 0.003$, $\tau_x = 0.973$, $\tau_p = 0.973$							
	0.01	–1.00	13.87	–1.63	0.01	0.02	0.44
C. $\rho = 0.5$, $\alpha = -4.911$, $\beta = 0.994$, $\bar{\tau} = -0.015$, $\tau_x = 3.064$, $\tau_p = 2.006$							
	0.02	–1.75	6.28	–0.61	0.01	0.03	0.37
D. $\rho = 1.0$, $\alpha = -6.079$, $\beta = 0.990$, $\bar{\tau} = 0.004$, $\tau_x = 1.534$, $\tau_p = 1.607$							
	0.01	–1.23	6.66	–0.80	0.01	0.02	0.37

NOTE: The first four columns are coefficients of the inflation rate: the equilibrium inflation rate coefficients on a constant, output, stochastic volatility, and the monetary policy shock, respectively. The last two columns are properties of inflation: the unconditional standard deviation and the first-order autocorrelation of inflation, respectively.

the real economy—output growth and stochastic volatility—all have positive autocovariances and the factor innovations have positive covariances to the factor levels. This implies that $\gamma_j^2\text{Autocov}(z_t^j)$ and $\text{Cov}(z_t^j, \varepsilon_t^j)$ are both positive. For a factor to contribute negatively to the autocorrelation of the pricing kernel, the factor loading γ_j and the price of risk λ_j must have opposite signs. Additionally, the price of risk λ_j must be large enough relative to the factor loading γ_j to counteract the positive autocovariance term $\gamma_j^2\text{Autocov}(z_t^j)$.

Output growth has a lower autocorrelation coefficient than stochastic volatility in our calibration, but because output growth has a much higher unconditional volatility, it has a much higher autocovariance than stochastic volatility. In the real economy, the factor loading γ_x on the level of output growth is equal to $(1-\rho)\phi_x$, which is nonnegative for all $\rho \le 1$. Also, the price of risk for output growth, λ_x, is positive at the parameter values used in Figure 4 because a sufficient condition for it to be positive is $\alpha \le 0$ and $|\rho| \le |\alpha|$. From (12), output growth contributes positively to the autocovariance of the pricing kernel.

From the real pricing kernel parameters given in (7), the price of risk for volatility is related to the factor loading on the level of volatility by

$$\lambda_v = -\frac{\beta}{1-\beta\phi_v}\gamma_v .$$

Because $1-\beta\phi_v > 0$, the volatility price of risk, λ_v, and the volatility factor loading, γ_v, have opposite signs, implying that the volatility factor can contribute a negative autocovariance to the pricing kernel. But output growth has the strongest effect on the autocovariance of the pricing kernel, leading to positive autocovariance in the pricing kernel. As a consequence, the average real yield curve is downward sloping. The numerical values for the real pricing kernel's factor loadings and prices of risk from Figure 4 are reported in panel A of Table 1.

In the exogenous inflation economy, shocks to inflation are uncorrelated to output growth and stochastic volatility—the factor loadings and prices of risk on output growth and stochastic volatility in the nominal pricing kernel are the same as in the real pricing kernel. Average nominal yields in the exogenous inflation economy are equal to the real yields plus expected inflation and inflation volatility with an adjustment for properties of the inflation process. The inflation shocks are positively autocorrelated, with a factor loading and a price of risk that are both positive. The average nominal yield curve has approximately the same shape as the real yield curve—it is downward sloping.

In the endogenous inflation economy, inflation is a linear combination of output growth, stochastic volatility, and the monetary policy shock. As shown in panel A of Table 2, endogenous inflation's loading on output, π_x, is negative. This implies that the nominal pricing kernel's output-growth factor loading and price of risk are lower than in the exogenous inflation economy. As a consequence, output growth contributes much less to the autocovariance of the pricing kernel with endogenous inflation. The factor loading and price of risk for stochastic volatility are also lower in the endogenous inflation economy. The policy shocks are positively autocorrelated, but the factor loading and the price of risk for the policy shock are of opposite sign. The average nominal yield curve in the endogenous inflation economy is therefore flatter than both the real yield curve and the nominal yield curve with exogenous inflation.

Turning to the volatilities in the bottom panel of Figure 4, the exogenous inflation economy exhibits more volatility in short rates and less volatility in long rates than found in the data. This is a fairly standard finding for term-structure models with stationary dynamics (see Backus and Zin, 1994). The volatility of long rates is mainly driven by the loading on the factor with the largest innovation variance and that factor's autocorrelation. The closer that autocorrelation is to zero, the faster that yield volatility decreases as bond maturity increases. In our calibration, output growth has the largest innovation variance and a fast rate of mean reversion, equal to 0.36. Yield volatility drops quite quickly as bond maturity increases. In general, the lower the loading on output growth, the slower that yield volatility drops as bond maturity increases. Because endogenous inflation is negatively related to output growth, the factor loading on output growth is lower. Yield volatility drops at a slower rate (relative to maturity) in the endogenous inflation economy than in the exogenous inflation economy.

Figure 5, panel B of Table 1, and panel B of Table 2 report yield-curve properties with a higher intertemporal elasticity of substitution ($\rho = 0$) or a log time aggregator. Piazzesi and Schneider (2007) study a model with the same preferences, but without stochastic volatility. The factor loading on output growth in the real economy is higher than in the economy with $\rho = -0.5$ reported in Figure 4 (compare panel A with panel B of Table 1). The average real yield curve and the average nominal yield curve with exogenous inflation are less downward sloping when $\rho = 0$ than when $\rho = -0.5$. Similarly, increasing ρ further to 0.5 (see Figure 6) or 1.0 (see Figure 7) leads to a real yield curve that is less downward-sloping. Because increasing ρ decreases the factor loading on output growth, it also decreases the volatility of real yields: See the bottom panels in Figures 4 through 7.

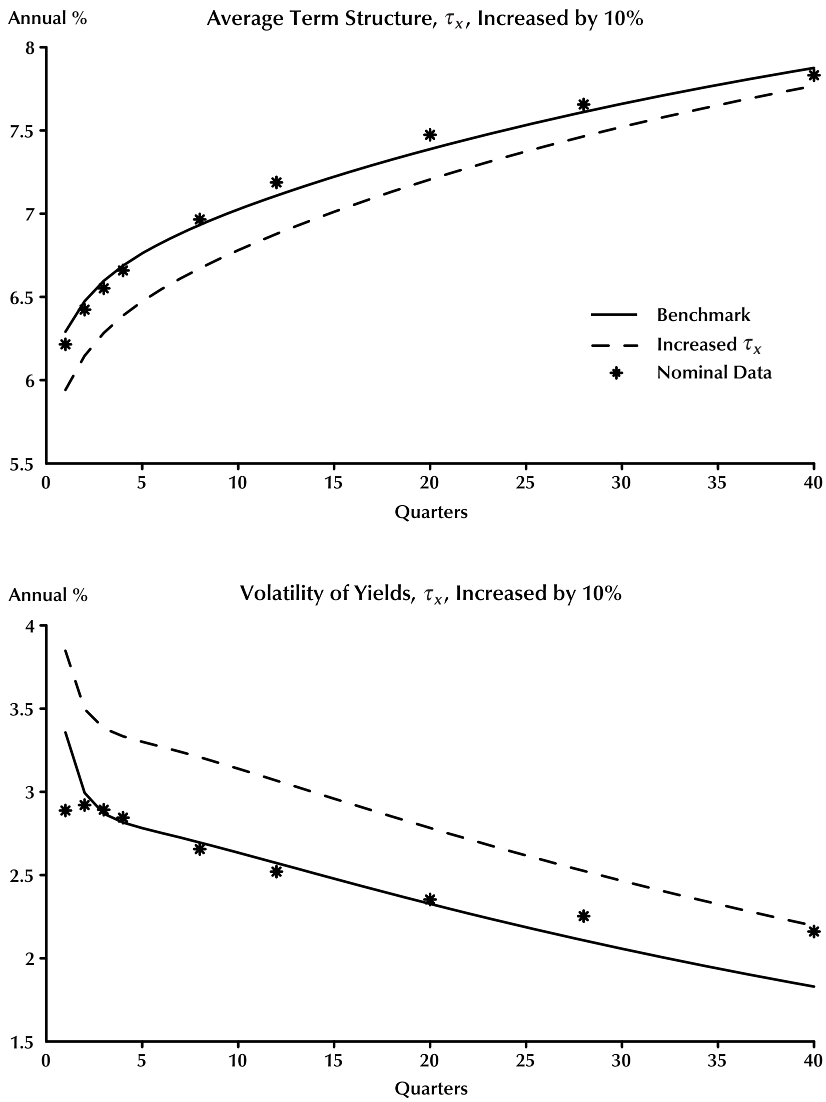
As ρ increases, the representative agent's intertemporal elasticity of substitution increases, implying less demand for smoothing consumption over time. Increasing ρ decreases the representative agent's demand for long-term bonds for the purpose of intertemporal consumption smoothing and leads to lower equilibrium prices and higher yields for real long-term bonds. The average real yield curve therefore is less downward-sloping as ρ increases. Increasing ρ also reduces the sensitivity of long-term real yields to output growth, leading to less volatile long-term yields: See the bottom panels in Figures 4 through 7.

Nominal yields in the economies with exogenous inflation are approximately equal to the real yields plus a maturity-independent constant. But in the economies with endogenous inflation, inflation and output growth have a negative covariance, leading to a decrease in the factor loading on output growth: See panels C and D of Tables 1 and 2. For $\rho \geq 0.5$ (see Figures 6 and 7), the average nominal yield curve is upward sloping and the shape of the volatility term structure decays similarly to that observed in the data.

The final three columns of Table 2 report unconditional moments of inflation in the economy with endogenous inflation. There are a few notable features. First, the unconditional moments

Figure 8

The Effects of Increasing τ_x



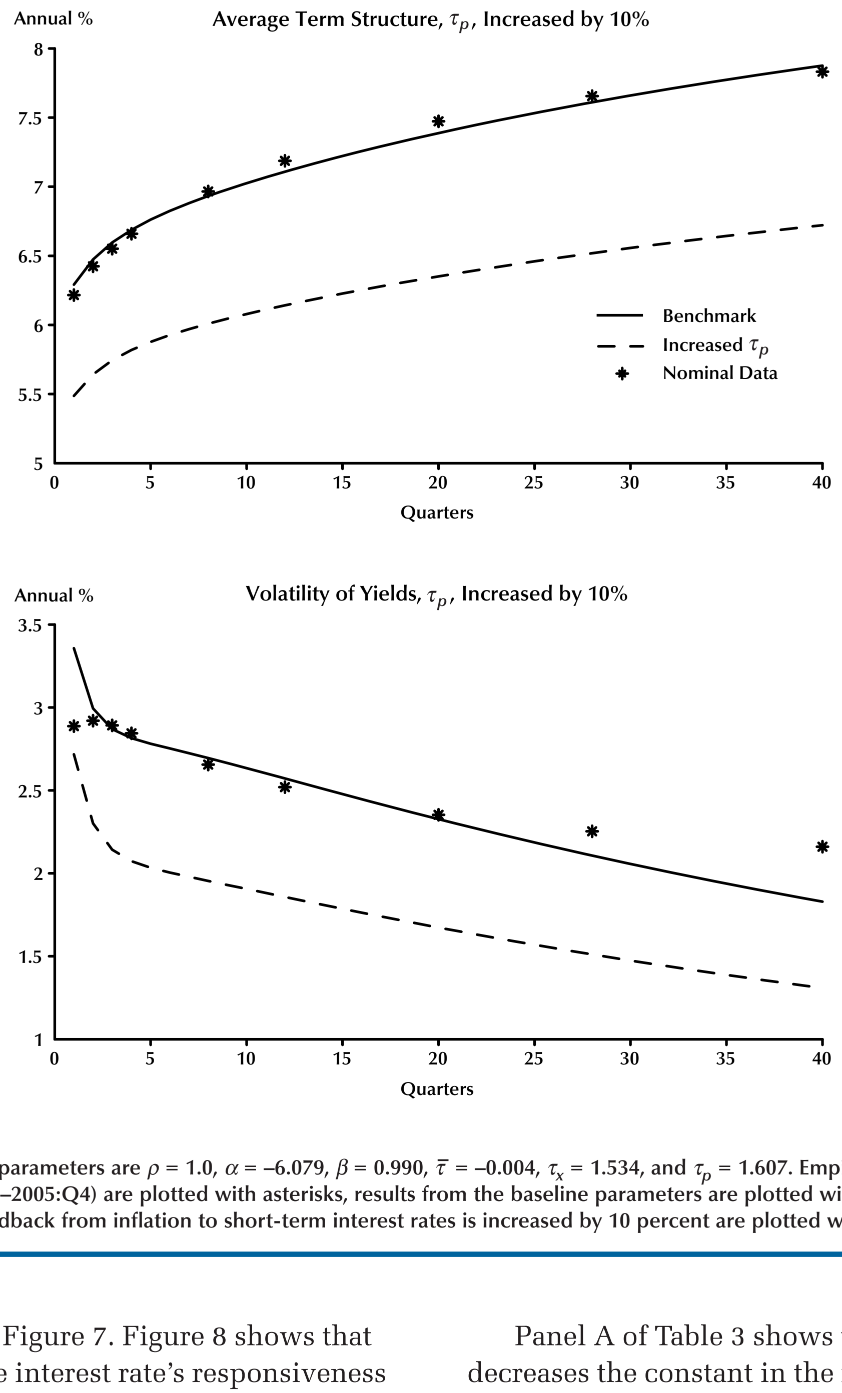
NOTE: The baseline parameters are ρ = 1.0, α = –6.079, β = 0.990, $\bar{\tau}$ = –0.004, τ_x = 1.534, and τ_p = 1.607. Empirical moments for the full sample (1970:Q1–2005:Q4) are plotted with asterisks, results from the baseline parameters are plotted with a solid black line, and results when the feedback from output growth to short-term interest rates is increased by 10 percent are plotted with a dashed black line.

are not particularly sensitive to the intertemporal elasticity of substitution. Second, the unconditional variance of inflation in the calibrated economy is an order of magnitude higher than that in the data: 0.0033 in empirical data and about 0.02 in these economies. Finally, inflation is much more autocorrelated in the data—the AR(1) coefficient is 0.85 in the data and about 0.4 in the model economies.

Figure 8, Figure 9, and Table 3 show results from changing the Taylor rule parameters. We keep the remaining parameters fixed at the values

Figure 9

The Effects of Increasing τ_p



NOTE: The baseline parameters are ρ = 1.0, α = –6.079, β = 0.990, $\bar{\tau}$ = –0.004, τ_x = 1.534, and τ_p = 1.607. Empirical moments for the full sample (1970:Q1–2005:Q4) are plotted with asterisks, results from the baseline parameters are plotted with a solid black line, and results when the feedback from inflation to short-term interest rates is increased by 10 percent are plotted with a dashed black line.

used to generate Figure 7. Figure 8 shows that increasing τ_x, the interest rate's responsiveness to output growth shocks, leads to a reduction in average nominal yields and a steepening in the average yield curve (top panel), as well as an increase in yield volatility (bottom panel).

Panel A of Table 3 shows that increasing τ_x decreases the constant in the nominal pricing kernel, decreases the factor loading on output growth, decreases the price of risk for output growth, and also increases the factor loading on stochastic volatility. The loading on output growth

Table 3

Comparative Statics for the Taylor Rule Parameters

		Nominal pricing kernel									
		Factor loadings (γs)			Prices of risk (λs)			Equilibrium inflation loadings			
	Constant	x_t	v_t	s_t	ε^x_{t+1}	ε^v_{t+1}	ε^s_{t+1}	$\bar{\pi}$	π_x	π_v	π_s
A. τ_x increased by 10%, from 1.53 to 1.69											
Baseline	0.03	–0.44	58.30	–0.74	9.76	–1,391.30	–0.80	0.01	–1.23	6.66	–0.80
Increased τ_x	0.02	–0.49	60.13	–0.74	9.63	–1,389.40	–0.80	0.01	–1.35	8.55	–0.80
B. τ_p increased by 10%, from 1.61 to 1.77											
Baseline	0.03	–0.44	58.30	–0.74	9.76	–1,391.30	–0.80	0.01	–1.23	6.66	–0.80
Increased τ_p	0.02	–0.39	55.30	–0.66	9.90	–1,394.40	–0.71	0.01	–1.09	3.58	–0.71

NOTE: The table reports the effect of changing the Taylor rule parameter τ_x or τ_p on the affine term-structure parameters as well as properties of p_t in the endogenous inflation economy. The equilibrium inflation rate coefficients on output, stochastic volatility, and the monetary policy shock are reported. The baseline parameters are $\rho = 1.0$, $\alpha = -6.08$, $\beta = 0.990$, $\bar{\tau} = -0.004$, $\tau_x = 1.53$, and $\tau_p = 1.61$.

in the pricing kernel drops because the sensitivity of the inflation rate to output growth drops; in turn, the sensitivity of inflation to stochastic volatility increases by a large amount—from 6.66 to 8.55.

Figure 9 shows that increasing τ_p, the interest rate responsiveness to inflation, leads to a reduction in average nominal yields and a flattening in the average yield curve (top panel) and a decrease in yield volatility (bottom panel).

Panel B of Table 3 shows that increasing τ_p decreases the constant in the nominal pricing kernel, increases the factor loading on output growth, increases the price of risk for output growth, decreases the factor loading on stochastic volatility, and also drops the factor loading on the monetary policy shock. The constant in the pricing kernel drops because the constant in the inflation rate drops, the factor loading on output growth increases because the sensitivity of the inflation rate to output growth increases; in turn, the sensitivity of inflation to stochastic volatility decreases by a large amount—from 6.66 to 3.58.

Overall, the experiments reported in Figure 8 and Figure 9 show that properties of the term structure depend on the form of the monetary authorities' interest rate feedback rule. In particular, the factor loading on stochastic volatility is quite sensitive to the interest rate rule. In this economy, because stochastic volatility is driving time-variation in interest rate risk premiums, monetary policy can have large impacts on interest rate risk premiums.

RELATED RESEARCH

The model we develop is similar to a version of Bansal and Yaron's (2004), which includes stochastic volatility; however, our simple autoregressive state-variable process does not capture their richer ARMA specification. Our work is also related to Piazzesi and Schneider (2007), who emphasize that, for a structural model to generate an upward-sloping nominal yield curve, it requires joint assumptions on preferences and the distribution of fundamentals. Our work highlights how an upward-sloping yield curve can also be generated through the monetary authority's interest rate feedback rule.

Our paper adds to a large and growing literature combining structural macroeconomic models that include Taylor rules with arbitrage-free term-structure models. Ang and Piazzesi (2003), following work by Piazzesi (2005), have shown that a factor model of the term structure that imposes arbitrage-free conditions can provide a better empirical model of the term structure than a model based on unobserved factors or latent variables alone. Estrella and Mishkin (1997), Evans and Marshall (1998 and 2001), Hördahl, Tristani, and Vestin (2004), Bekaert, Cho, and Moreno (2005), and Ravenna and Seppala (2006) also provide evidence of the benefits of building arbitrage-free term-structure models with macroeconomic fundamentals. Rudebusch and Wu (2004) and Ang, Dong, and Piazzesi (2004) investigate the empirical consequences of imposing a Taylor rule on the performance of arbitrage-free term-structure models.

For an alternative linkage between short- and long-maturity bond yields, see Vayanos and Vila (2006), who show how the shape of the term structure is determined in the presence of risk-averse arbitrageurs, investor clienteles for specific bond maturities, and an exogenous short rate that could be driven by the central bank's monetary policy.

CONCLUSIONS

We demonstrate that an endogenous monetary policy that involves an interest rate feedback rule can contribute to the riskiness of multi-period bonds by creating an endogenous inflation process that exhibits significant covariance risk with the pricing kernel. We explore this through a recursive utility model with stochastic volatility that generates sizable average risk premiums. Our results point to a number of additional questions. First, the Taylor rule that we work with is arbitrary, so how would the predictions of the model change with alternative specification of the rule? In particular, how would adding monetary nonneutralities along the lines of a New Keynesian Phillips curve as in Clarida, Galí, and Gertler (2000) and Gallmeyer, Hollifield, and Zin (2005) alter the monetary policy–consistent pricing kernel? Second, what Taylor rule would implement an optimal monetary policy in this context? Because preferences have changed relative to the models in the literature, this is a nontrivial theoretical question.

In addition, the simple calibration exercise in this paper is not a very good substitute for a more serious econometric exercise. Further research will explore the trade-offs between shock specifications, preference parameters, and monetary policy rules for empirical yield-curve models that more closely match historical evidence.

Finally, it would be instructive to compare and contrast the recursive utility model with stochastic volatility with other preference specifications that are capable of generating realistic risk premiums. The leading candidate on this dimension is the external habits models of Campbell and Cochrane (1999). We are currently pursuing an extension of the external habits model in Gallmeyer, Hollifield, and Zin (2005) to include an endogenous, Taylor rule–driven inflation process.

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