I would like to start my discussion with some quotes from Ben Bernanke’s (2003) speech about the behavior of employment during the past recovery. In this speech Bernanke thoroughly reviews the main features of the data and several of the hypotheses advanced to understand the sluggishness of employment during the recovery.

I suspect that some of the recent expansion in productivity is instead the delayed result of firms’ heavy investment in high technology equipment in the later part of the 1990s. Only over time have managers learned how to organize their production and distribution so as to take full advantage of these technologies...

[Productivity growth] has also enabled firms to meet the demand for their output without hiring workers.

...Although other explanations for the jobless recovery...have played a role, in my view the productivity explanation is, quantitatively, probably the most important.

Bernanke’s interpretation, while not identical to the one in the paper by Koenders and Rogerson (2005), is very close: It emphasizes technology as the driving force, as opposed to aggregate demand fluctuations, and locates the seed of the sluggishness of the recovery in the strength of the previous expansion.

My discussion centers on a presentation of a general equilibrium version of the model by Koenders and Rogerson. Nevertheless, before offering it, I would like to briefly mention three other points.

First, as it is well known, the magnitude of the decline in employment during and after the past recession differs according to the data sets used to measure it. The establishment survey, which is the one used in the paper, shows a much larger decline than the household survey does. There are several differences in the design of the two surveys, which can, in principle, account for the difference in employment. The establishment survey measures employment from payroll information from a large number of firms, so it gives an estimate of the total employment for a large sector of the U.S. economy. The household survey measures the employment status from a given set of households, so it gives an estimate of the employment/population ratio. One explanation that has been advanced for the difference between the two estimates is that, if population decreases, it is possible that employment/population stays constant and that total employment decreases. Because population is measured accurately only every decade, a decline in population may explain part of the difference between the two estimates. Indeed, it has been proposed that immigration may have sharply declined during the recession and the beginning of the recovery, as a consequence of policies enacted after September 11, 2001. The paper by Koenders and Rogerson uses Hodrick-Prescott (HP) filtered data for employment, which, in principle, should remove slow-moving variations (such as the one in population) from employment. Nevertheless, it is well known that the HP filter is much less accurate at the end points, so one may be suspicious of its ability to remove a...
change in the trend that happened in the past three years of the sample. Additionally, the models we typically write are in terms of employment/population ratios, so it seems that, even though the household survey may be noisier, its results should be used or at least compared with the ones for the establishment survey used in the paper.

My second comment is that the model focuses on the (time-varying) cyclicality of firing. Nevertheless, recent studies by R. Shimer and R. Hall document that firing is less cyclical than hiring. While the paper recognizes this possibility, the model is still one in which the variations are mostly in firing.

My third point is related to the measurement of the hypothesis of the paper: The strength of the recovery after a recession depends on the length of the expansion that precedes it. There are only so many recessions in the United States after World War II. Thus, I think it may be useful to repeat the empirical exercise for the Organisation for Economic Co-operation and Development (OECD) countries. Of course, business cycles across these countries are correlated, but not perfectly so; thus, I think the international comparison will add valuable information to the hypothesis.

A GENERAL EQUILIBRIUM VERSION OF THE KOENDERS AND ROGERSON MODEL

Here I describe a general equilibrium (GE) version of the Koenders and Rogerson model (KR hereafter). This GE version endogenizes wages and interest rates. I first describe a “reduced form” model, which I hope makes the main ingredients of the theory easy to see. Then I show that the reduced form is the outcome of a GE version of the KR model.

Although the preferences and technology are similar, there are two main differences between the KR model and the model here. One difference is that I consider a planner’s problem instead of a market economy. The second difference is that I allow heterogeneity in the cost of reorganization, as explained in the next section. In the following description, I keep their notation as much as possible.

The way I converted their model into a GE problem is by postulating preferences of the form

\[
\sum_{t=0}^{\infty} \beta^t E_0 \left( \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{A}{\epsilon_{2t}} n_t \right),
\]

where \(C_t\) is aggregate consumption and \(n_t\) is employment. An increase in \(\epsilon_t\) makes the household less willing to work, and hence they will, everything else equal, end up producing less and consuming less. In this sense, \(\epsilon_t\) is a “demand shock.” As in KR, there is a constant number of firms in the economy. Let \(\mu_{2t}\) denote the beginning period fraction of non-reorganized firms and \(\mu'_{2t}\) the next period value of this variable. The more non-reorganized firms are in the economy, the fewer reorganized firms are. A non-reorganized firm incurs an extra fixed cost of production relative to reorganized firms. A non-reorganized firm can become reorganized by incurring a costly action that involves producing with a higher marginal cost. The technological possibilities for this economy are described by

\[
n_t = a(\mu_{2t}, \mu_{2t+1}) C_t + \mu_{2t} \phi,
\]

where \(n_t\) is the labor units required to produce \(C_t\) units of the aggregate consumption if the number of non-reorganized firms today is \(\mu_{2t}\), and its number next period is \(\mu_{2t+1}\). The parameter \(\phi > 0\) measures the fixed cost, in terms of labor, that each non-reorganized firm incurs. The function \(a\) gives the marginal cost, in terms of labor, of producing each unit of \(C_t\). It is assumed that \(a(\mu_{2t}, \mu_{2t+1})\) is decreasing in \(\mu_{2t+1}\) because reorganization is costly; lowering \(\mu_{2t+1}\) saves on the current costly action of reorganization. It is assumed that \(a(\mu_{2t}, \mu_{2t+1})\) is increasing in \(\mu_{2t}\), because non-reorganized firms are less productive. The next section derives these assumptions from a GE model that uses the same technology as the partial equilibrium KR model. Having described the preferences and technology I consider the following reduced form planning problem:
empirically, they are similar. In the GE version with \( \gamma = 1 \), the decision rule \( g \) is increasing in \( \varepsilon_2 \) if the distribution of \( \varepsilon_{2t+1} \) is stochastically higher when \( \varepsilon_{2t} \) is larger. In other words, in the case of persistent “demand shocks,” reorganization is countercyclical. Hence, the GE version of the model reproduces the key results sought by KR.

**DETAILED DESCRIPTION OF THE GE MODEL**

I first describe the technology and preferences of the GE model and then show that they imply the reduced form introduced here—the \( \alpha' \) function—and that the optimal decision rules have the properties previously described.

I introduce heterogeneity in the reorganization cost, \( \bar{\eta} \), as a way to smooth out the problem. In each period there is a continuum of goods indexed by \( i \in [0, \mu] \). These goods correspond to three types of firms. There are \( \mu_i \) firms of each type for \( i = 1,2,3 \). Type 1 goods are those of firms with low idiosyncratic demand shock \( \varepsilon^i \) and labor requirement \( \alpha^i \). Type 3 goods corresponds to reorganized large firms with high idiosyncratic demand shock \( \varepsilon^l \) and labor requirement \( \alpha^l \). Type 2 goods correspond to firms with high idiosyncratic demand shock \( \varepsilon^l \) and labor requirements that depend on whether they are reorganizing or not and on the idiosyncratic reorganization cost, \( \bar{\eta} \). These costs are drawn i.i.d. each period from a distribution with \( \mu_i \) firms of each type for \( i = 1,2,3 \).

The decision rule balances two effects: Employment increases with the “demand” shock, \( \varepsilon_{2t} \) (the household is more willing to work and consume), and also increases with \( \mu_{2t} \) because the reorganized firms have larger fixed costs.

I can now reproduce the main result in their paper, explained in section 4.1. Consider an initial value of \( \mu_{20} \) that is low, reflecting that a long expansion has occurred up to now. Consider an initial condition for \( \varepsilon_{20} \) low enough, so there will be reorganization and a subsequent increasing sequence of \( \varepsilon_{2t} \), which is meant to imitate the impulse response of a persistent shock. Then, \( \mu_{2t} \) will be increasing and hence the path of employment given by the optimal decision rule just described will be u-shaped, with its lower point after the lower point of \( \varepsilon_{2t} \)—hence, the sluggish recovery after a long expansion.

One point that differs between the GE and the partial equilibrium versions is the characterization of the conditions under which reorganization does occurs in relation to the persistence of the demand shock \( \varepsilon_2 \). In the GE version of the model with \( \gamma = 1 \), if \( \varepsilon_2 \) is i.i.d., then \( g \) is independent of \( \varepsilon_2 \). This differs from the partial equilibrium version. But for the case that is arguably more interesting
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\( n = \mu_1 (\alpha^s y_1) \)
\[ + \mu_2 \left[ a_2 \int \left[ \eta (1 + \eta) y_2(\eta) \right] dG(\eta) + a_1 (1 - G(\eta^*)) y_2(\eta) \right] \]
\[ + \mu_3 (a^l y_3). \]

The law of motion of the number of firms of each type is as follows. Each period, measure 1 of firms of type 1 enter, fraction \( \lambda^t \) of the existing firms die, and fraction \( \pi^e \) become type 2 firms, so
\[ \mu^t_2 = \mu_1 (1 - \lambda^s - \pi^e) + 1. \]
Fraction \( \lambda^t \) of type 2 firms die, and fraction \( G(\eta^*) \) engage in restructuring, with \( \pi^e \) of those restructuring doing it successfully, so
\[ \mu^t_2 = (1 - \lambda^t - G(\eta^*)) \mu_2 + \pi^e \mu_1. \]
Fraction \( \lambda^t \) of type 3 firms die, so
\[ \mu^t_3 = (1 - \lambda^t - G(\eta^*)) \pi^e \mu_2. \]
Letting
\[ \bar{\mu} = \frac{1}{\lambda^s + \pi^e}, \]
one can verify that if
\[ \mu_{10} = \frac{1}{\lambda^s + \pi^e} \]
and
\[ \mu_{10} + \mu_{20} + \mu_{30} = \bar{\mu}, \]
then
\[ \mu_{tt} = \bar{\mu}, \]
\[ \mu_{1t} = \mu_{2t} + \mu_{3t} = \bar{\mu}, \]
for all \( t \geq 0 \). Thus, without loss of generality, the number of type 1 firms is constant \( (\equiv \bar{\mu}_1) \), as well as the total number of firms \( (\bar{\mu}) \), so that
\[ \left( 1 \right) \]
\[ \mu_3 = \bar{\mu} - \bar{\mu}_1 - \mu_2 \]
and the law of motion of firms of type 2 is
\[ \left( 2 \right) \]
\[ \mu^t_2 = (1 - \lambda^t - G(\eta^*)) \pi^e \mu_2 + \pi^e \bar{\mu}. \]
Consumption, \( C_t \), is a Dixit-Stiglitz aggregate of a continuum of goods:
\[ C_t = \left[ \int_0^\phi \alpha \left[ c_{it} \right] \frac{\phi^t}{\phi-1} \right] \frac{\phi}{\phi-1}, \]
where \( \alpha > 0 \) are the weights of the different goods. The weight of each type is as follows: Type 1 has weight \( \alpha_1 = \epsilon_1 \), and type 2 and 3 goods have weights \( \alpha_t - \epsilon_t \). This captures the higher demand for type 1 and 2 firms in the partial equilibrium model. Hence,
\[ \left( 4 \right) \]
\[ C_t = \left[ \int_0^\phi \alpha \left[ c_{it} \right] \frac{\phi^t}{\phi-1} \right] \frac{\phi}{\phi-1} \]
\[ + (\mu_2 \epsilon_2) \left( y_{tt} \right) \frac{\phi^t}{\phi-1} \]
\[ + (\mu_3 \epsilon_3) \left( y_{3t} \right) \frac{\phi^t}{\phi-1} \]
Now I can write the planning problem as
\[ V(\mu_2, \epsilon_2) \]
\[ = \max \left\{ (G)^{1-\gamma} n + \beta E \left[ V(\mu'_2, \epsilon'_2)|\epsilon_2 \right] \right\}, \]
where \( y \) is the vector of output across firms,
\[ y = \left( y_1, y_2(\eta) y_3, y_3 \right), \]
and where the maximization is subject to the definition of the aggregate consumption \( C(4) \), the total labor requirement \( n (1) \), the implied value for \( \mu_3 (2) \), and the law of motion for \( \mu_2 (3) \).

**ANALYSIS OF THE GE MODEL**

First I turn to the derivation of the reduced form planning problem, and then I analyze the optimal decision rules of the reduced form planning problem.
Let \( e(\mu_2, \epsilon'_2) \) be the value of cutoff point \( \eta^* \) so that the law of motion implies \( \mu'_2 \) given the current \( \mu_2 \). The function \( e \) solves
\[\mu'_2 = (1 - \lambda' - G(e(\mu'_2, \mu_2))\pi^c)\mu_2 + \pi^d.\]

For future reference, notice that
\[\frac{\partial e}{\partial \mu'_2} = \frac{(1 - \lambda' - G\pi^e)}{G\pi^e \mu_2} > 0,\]
\[\frac{\partial e}{\partial \mu'_2} = -\frac{1}{\pi^d G' \mu'_2} < 0.\]

Define \(\bar{a}(\mu'_2, \bar{\eta}')\) as the minimum employment \(n\) needed to produce one unit of Dixit-Stiglitz \(C\), formally:
\[\bar{a}(\mu'_2, \bar{\eta}')C = \min_{y_1, y_2, y_3} \left\{ \bar{\mu}_1 a^e y_1 + \mu_2 a^l \left[ (1 + \bar{\eta})y_2 (\bar{\eta}) dG + \bar{y}_2 (1 - G(\bar{\eta}^)) \right] + \mu_2 \phi + \mu_3 a^r y_3 \right\}\]
subject to
\[C = \left[ \begin{array}{c}
\bar{\mu}_1 e^s y_1^\phi \\
\mu_2 e^l \left[ \int_0^1 (1 + \bar{\eta}) y_2 (\bar{\eta})^\phi dG + \bar{y}_2 \phi (1 - G(\bar{\eta}^)) \right] \\
+ \mu_3 e^r y_3^\phi
\end{array} \right] = \left( \begin{array}{c}
\mu_1 e^s y_1^\phi \\
\mu_2 e^l \left[ \int_0^1 (1 + \bar{\eta}) y_2 (\bar{\eta})^\phi dG + \bar{y}_2 \phi (1 - G(\bar{\eta}^)) \right] \\
+ \mu_3 e^r y_3^\phi
\end{array} \right].\]

Using the first-order conditions of this minimization problem into the objective function, it can be shown that
\[\bar{a}(\mu'_2, \bar{\eta}') = \left[ \begin{array}{c}
\bar{\mu}_1 e^s \left( \frac{a^l}{e^l} \right)^{1-\phi} \\
+ \mu_2 e^l \left[ \int_0^1 (1 + \bar{\eta}) y_2 \phi dG + \bar{y}_2 \phi (1 - G(\bar{\eta}^)) \right] \\
+ \left[ \bar{\mu} - \bar{\mu}_1 - \mu_2 \right] e^l \left( \frac{a^l}{e^l} \right)^{1-\phi}
\end{array} \right]^{1-\phi}.\]

For future reference notice that
\[\frac{\partial \bar{a}(\mu'_2, \bar{\eta}')}{\partial \mu_2} = \frac{-\phi \left( \int_0^1 (1 + \bar{\eta}) y_2 \phi dG + \bar{y}_2 \phi (1 - G(\bar{\eta}^)) \right) - 1}{\bar{\eta}' - \bar{\eta}'} > 0\]
if \(\bar{\eta}' > 0\) and that
\[\frac{\partial \bar{a}(\mu'_2, \bar{\eta}')}{\partial \bar{\eta}'} = -\frac{\alpha^l}{e^l} \left[ (1 + \bar{\eta})^{1-\phi} - 1 \right] G'(\bar{\eta}') > 0\]
if \(\bar{\eta}' > 0\) and \(G'(\bar{\eta}') > 0\).

Using the definitions of \(\bar{a}(\cdot)\) and \(e(\cdot)\), define the function \(a(\cdot)\), the marginal cost in terms of labor of producing one unit of the aggregate consumption, \(C\), given that the current and future states are \((\mu'_2, \mu_2)\). Then function \(a\) is given by
\[a(\mu'_2, \mu_2) = \bar{a}(\mu'_2, e(\mu'_2, \mu_2)).\]

It is immediate from the definition of \(a(\cdot)\) that the reduced form planning problem is equivalent to the planning problem.

Using the definitions of \(a, \bar{a}\), and \(e\),
\[\frac{\partial a(\mu'_2, \mu_2)}{\partial \mu_2} = \frac{\partial \bar{a}(\mu'_2, e(\mu'_2, \mu_2))}{\partial \mu_2} \frac{\partial e(\mu'_2, \mu_2)}{\partial \mu_2} > 0,\]
\[\frac{\partial a(\mu'_2, \mu_2)}{\partial \mu'_2} = \frac{\partial \bar{a}(\mu'_2, e(\mu'_2, \mu_2))}{\partial \mu'_2} \frac{\partial e(\mu'_2, \mu_2)}{\partial \mu'_2} < 0,\]
which is what we assume in the reduced form planning problem.

The reduced form planning problem makes it easy to solve for aggregate consumption and labor, conditional on the current state \((\mu_2, \epsilon_2)\) and a choice of \(\mu'_2\). These conditional optimal policies are
\[C = \left( \frac{\epsilon_2}{Aa(\mu'_2, \mu_2)} \right)^{1/r},\]
\[n = a(\mu'_2, \mu'_2)^{1-\frac{1}{r}} \left( \frac{\epsilon_2}{A} \right)^{1/r} + \mu_2 \phi.\]
These choices for $C$ and $n$ can be replaced into the period utility of the reduced form planning problem so that the only choice is $\mu'_2$. In the special case of log preferences ($\gamma = 1$) this gives

$$V(\mu_2, \epsilon_2) = \max_{\mu'_2} \left\{ \log \frac{\epsilon_2}{A} - \log(a(\mu_2, \mu'_2)) - \frac{A}{\epsilon_2} \mu_2 \phi \right\} + \beta E \left[ V(\mu'_2, \epsilon'_2) | \epsilon_2 \right].$$

I turn to the analysis of the effect of the “demand shock,” $\epsilon_2$, into reorganization. In the log case, the period return function is additively separable in $\epsilon_2$ and $\mu'_2$. Thus if $\epsilon_2$ is i.i.d., then $g(\mu_2, \epsilon_2)$, the optimal decision for $\mu'_2$, is independent of $\epsilon_2$, and hence the amount of reorganization and $\mu'_2$ will be constant. This is to be compared with the partial equilibrium analysis wherein the i.i.d. case reorganization is countercyclical.

Assuming that $-\log(a(\mu_2, \mu'_2))$ is concave in $(\mu_2, \mu'_2)$, the first-order conditions and the envelope displayed below are sufficient for an interior solution:

$$\frac{1}{a(\mu_2, g(\mu_2, \epsilon_2))} \frac{\partial a}{\partial \mu_2}(\mu_2, g(\mu_2, \epsilon_2)) = \beta E \left[ \frac{\partial}{\partial \mu'_2} V(\mu_2, \epsilon_2, \epsilon'_2) | \epsilon_2 \right],$$

$$\frac{\partial}{\partial \mu_2} V(\mu_2, \epsilon_2) = -\frac{1}{a(\mu_2, g(\mu_2, \epsilon_2))} \frac{\partial a}{\partial \mu_2}(\mu_2, g(\mu_2, \epsilon_2)) - \frac{A}{\epsilon_2} \phi.$$

If the “demand shock,” $\epsilon_2$, is persistent in the sense that the distribution of $\epsilon'_2$ is stochastically higher if $\epsilon_2$ is larger, one can show that $\partial V/\partial \mu_2$ is increasing in $\epsilon_2$. This, in turn, implies that $g(\mu_2, \epsilon_2)$ is increasing in $\epsilon_2$. Thus, in the case where $\epsilon_2$ is persistent (provided $-\log a$ is concave), reorganization is countercyclical. I have solved this model numerically by making discrete the state space, so that the numerical solution does not depend on the assumption that $-\log a$ is concave. In my numerical examples, I have confirmed that $g$ is increasing in $\epsilon_2$ for $\gamma \leq 1$.

**REMARKS ON THE GE MODEL**

I have analyzed a planning problem instead of analyzing the decentralized equilibrium of the model. The partial equilibrium KR model is one of monopolistic competition. Given the Dixit-Stiglitz specification that I am using, it is easy to formulate the equilibrium problem corresponding to the economy whose optimal allocation I have analyzed. As is well known, the equilibrium allocation differs from the solution of the planning problem due to the effect of the mark-ups of firms on prices, which end up reducing real wages and output. One simple way to reconcile the decentralized economy and the planning problem is to consider a market economy with lump-sum taxes used to finance a subsidy to employment of $\phi/(\phi - 1)$, which undoes the effect of the mark-ups. Alternatively, I conjecture that the solution of a modified social problem gives the allocations of the decentralized equilibrium. The modification consists of changing the parameter $A$ in some of the expressions for the return function of the planning problem.

I believe that, given the question at hand, it is advantageous to consider a GE model. Nevertheless, the simple model I consider here is deficient as a model of business cycles. As the reduced form social planning problem makes clear, it is equivalent to a static model with productivity $1/a$, except that productivity is endogenous. For the case of log preferences, the model where $a$ is exogenous and random, employment, $n$, does not depend on productivity. To see this, notice that the conditional optimal decision rule is given by $n = A/\epsilon_2 + \phi \mu_2$, so it does not depend on productivity $1/a$ directly. Also, the static model has large variations in interest rates. Introducing capital, as in the standard neoclassical growth model, will solve most of these shortcomings.

**REFERENCES**

Koenders, Kathryn and Rogerson, Richard.