New Economy—New Policy Rules?

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INTRODUCTION

The New Economy

United States economic performance during the latter portion of the 1990s far exceeded even optimistic forecasts. From 1996 through 2000, nonfarm business sector productivity grew by about 3.0 percent per year, on average. In the ten years previous to this period, from 1986 through 1995, it had increased at an average rate of only 1.4 percent per year. The late 1990s coincided with a spell of accelerated progress in computer technology and a widening adoption of the Internet by businesses and consumers. U.S. real output increased about 4.3 percent per year, on average, from 1996 through 2000, while, at the same time, inflation pressures remained rather subdued, with the personal consumption expenditures price index increasing at an average rate of only about 1.9 percent per year.

Economists in the United States have been cognizant of these changing trends. Many commentators have argued that technological change may be increasing American productivity, making it possible for the economy to grow at a faster rate without creating inflation. And, in fact, Federal Reserve officials have made many such arguments in recent years. Consider, for example, the May 6, 1999, Congressional testimony by Federal Reserve Chairman Alan Greenspan: "…the evidence appears to be mounting that, even if productivity does not continue to accelerate, the pickup already observed does seem to explain much of the extraordinary containment of inflation despite the ever-tightening labor markets of recent years." The next day the *Washington Post* reported: "Greenspan said the unexpected jump in productivity is the major reason that for the past three years so many forecasters, including those at the Fed, have underestimated economic growth while overestimating inflation."

This set of events is sometimes collectively called "the new economy," and we will use this meaning of the term for the purposes of this paper.

Optimal Monetary Policy Rules in the New Economy

The U.S. monetary policy debate has been importantly influenced by Taylor (1993), who argued that simple, nominal interest rate–based monetary policy rules might produce good stabilization performance.[1] Taylor's (1993) ideas were based on a given, constant inflation target for the monetary authorities and, especially important for this paper, a given, constant long-run level of productivity. Nearly all rules in this literature are then specified relative to these fundamental objects. In addition, Taylor's (1993) analysis was not of an optimal policy rule, but of an ad hoc rule that Taylor reasoned would perform well based on historical experience. Svensson (1997) showed how a version of the Taylor rule could be viewed as the *optimal* monetary policy rule in a simple dynamic macroeconomic model, again for a given inflation target and a given underlying level of productivity. In addition, the papers in the Taylor (1999) volume generally favor the idea that something close to optimal stabilization performance could be obtained by adhering to a Taylor rule, across a wide variety of macroeconomic models.

However, one of the key "new economy" events is the shift in productivity. It seems natural that a fully optimal monetary policy rule would take account of the changing nature of the supply side. Our main goal in this paper is to derive an optimal monetary policy rule in an environment with unobserved shifting productivity, so that the policy authorities must infer the underlying regime from observed data. We wish to accomplish this in the simplest possible framework, but one in which we are sure that a Taylor-type rule would be optimal were it not for the productivity changes. Accordingly, we adopt Svensson's (1997) model as a baseline, and we augment the model with two-state regime switching in the level of long-run productivity. We wish to understand how a Taylor-type rule would have to be altered to allow for the possibility that underlying productivity shifts may occur. Although

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[1] For some of the related recent research on monetary policy rules, see Taylor (1999), King and Plosser (1999), and Clarida, Gali, and, Gertler (1999).

we use this model to keep our exposition relatively simple, we also think that it is reasonably clear that the basic findings here would hold in far more elaborate models.

Main Results

Our main finding is that the optimal policy rule in an environment with unobserved productivity shifts involves important lagged terms in inflation and the output gap. The role of these lagged terms is to help the policy authority react with appropriate interest rate adjustments when unobserved shifts in underlying productivity occur. In certain special cases, our optimal policy rule collapses to Svensson's (1997) rule for the same model, which involves only contemporaneous data. These special cases occur when (i) the probability of remaining in each regime is exactly one-half, so that productivity regimes are not persistent and can be interpreted simply as noise, or (ii) when the levels of productivity in the two regimes approach one another, so that there is effectively no difference between the two regimes. Intuitively, we think our main finding is an important one that would extend to a wide variety of models: In the face of possible unobserved changes in regime, the policy authority must optimally consider recent trends in the data to infer whether the regime shift has occurred.

While our main results are analytical, we also consider a calibration of the model in order to fix ideas and provide illustrations of our findings. Adhering to a Taylor-type rule as derived by Svensson (1997) when there are, in fact, unobserved switches in productivity regimes implies significantly worse macroeconomic performance, relative to the optimal rule that we derive. Policymakers using a Svensson-Taylor rule would typically observe inflation that is persistently above or below target. This would appear to them to be due to unobserved special factors. But, with the optimal rule, inflation remains near target at all times and output fluctuates in response to the changes in productivity regimes and normal macroeconomic shocks. Thus the shift from low to high productivity in conjunction with a policy authority adhering to a Svensson-Taylor rule produces the events associated with the "new economy" as described in the opening paragraph: output persistently higher than expected, inflation persistently lower than expected, measured productivity higher, and policymakers arguing that a productivity shift has contained inflation. The reverse case, a regime shift from high to low productivity, generates some of the features of the stagflation of the 1970s.

The remainder of this paper is organized as follows. First, we describe the model we will employ. Then, we derive our optimal monetary policy rule when there are regime switches in productivity; in the next section we compare the economic performance of our simple macroeconomy under the Svensson-Taylor rule and under the optimal rule. Then we offer some closing comments and use two appendices to discuss mathematical details.

ENVIRONMENT

As in Svensson (1997), we assume that inflation and the output gap are linked by the following short-term Phillips curve relationship:

$$\pi_{t+1} = \pi_t + \alpha y_t - u_{t+1}\,, \tag{1}$$

where $\pi_t = p_t - p_{t-1}$ is the inflation rate from period $t-1$ to period t; p_t is the natural logarithm of the price level in period t; y_t is the natural logarithm of the output gap at t; and the parameter α measures the slope of the Phillips curve. We interpret u_t as a productivity (supply) shock, and we put more structure on it below. We normalize the natural level of output to zero, so that y_t is zero when output is at this steady-state or "trend" level. Following Svensson (1997, p. 1115), we assume that the output gap is serially correlated, decreasing in the short-term real interest rate and increasing in an exogenous shock to the gap:

$$y_{t+1} = \beta_1 y_t - \beta_2\left(i_t - \pi_t\right) + x_{t+1}, \tag{2}$$

where $\beta_2 > 0$, $0 < \beta_1 < 1$, i_t is a short-term nominal interest rate controlled by the monetary authority, and x_{t+1} is a stochastic disturbance term. As can be seen from equations (1) and (2), the real interest rate affects the output gap with a one-period lag, and hence affects inflation with a two-period lag, which is the control lag in the model. The shock to the output gap is serially correlated and assumed to be subject to i.i.d. noise ε_{t+1}, with mean zero and variance σ_ε^2 according to

$$x_{t+1} = \rho x_t + \varepsilon_{t+1}\,. \tag{3}$$

We want to think in terms of persistent productivity regimes in which switches are relatively rare events, corresponding to the U.S. productivity experience in the postwar era. To study persistent changes in productivity, we extend this system with a stochastic process for productivity, u_t. We use a two-state process defined by

$$u_{t+1} = a_h s_{t+1} - a_\ell = \begin{cases} a_h - a_\ell & \text{if } s_{t+1} = 1 \\ -a_\ell & \text{if } s_{t+1} = 0 \end{cases}, \tag{4}$$

where $a_h > a_\ell > 0$. Under this specification, as $a_h \to 0$, there is no difference between the regimes, and so we think of a_h as scaling the effect of the productivity differences in the two regimes. The unobserved state of the system s_t takes on a value of zero or one and follows a two-state Markov process.[2] There is an associated transition probability matrix given by

$$T = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}, \tag{5}$$

where

$$\begin{aligned} &\Pr[S_{t+1} = 1 | S_t = 1] = p, \\ &\Pr[S_{t+1} = 0 | S_t = 1] = 1-p, \\ &\Pr[S_{t+1} = 0 | S_t = 0] = q, \end{aligned} \tag{6}$$

and $\Pr[S_{t+1} = 1 | S_t = 0] = 1-q.$

Thus, the probability of remaining in the high (low) state conditional on being in the high (low) state in the previous period is $p(q)$, the probability of switching from the high to the low state is $1-q$, and the probability of switching from the low state to the high state is $1-p$. Because we wish to think of persistent regimes, we restrict our analysis to the case where both $p \geq 1/2$ and $q \geq 1/2$.

As suggested by Hamilton (1989), the stochastic process for equation (6) admits the following AR(1) representation:

$$s_{t+1} = (1-q) + \gamma s_t + \upsilon_{t+1}, \tag{7}$$

where $\gamma \equiv p + q - 1$ and υ_t is a discrete, white noise process with mean zero and variance σ_υ^2. From equation (2) it follows that the unconditional mean steady-state level of output, $\bar{y}$, associated with a zero steady-state real interest rate is zero. To be consistent, we impose that this level from equation (1) should also be zero. This implies

$$a_\ell = \bar{p} a_h, \tag{8}$$

where

$$\bar{p} \equiv \frac{(1-q)}{2-p-q}.$$

We give the details of this calculation in Appendix A.

Monetary policy is conducted by a central bank that controls a short-term nominal interest rate, i_t, and that has an exogenously given inflation target, π^*. The authorities aim to minimize deviations of inflation from this assigned target, on the one hand, and fluctuations of output around its trend level (which is normalized to zero, i.e., $\bar{y} = 0$), on the other. Consequently, the central bank will choose a sequence of current and future short-term nominal rates to meet the objective

$$\underset{\{i_t\}_{t=0}^{\infty}}{Min}\ E_t \sum \delta^t \left[\frac{1}{2}\left(\pi_t - \pi^*\right)^2 + \frac{\mu}{2}\left(y_t - \bar{y}\right)^2 \right]. \tag{9}$$

Here $0 \leq \mu < \infty$ represents the central bank's relative weight on output stabilization, while the parameter $\delta \in (0,1)$ denotes the discount factor. The expectation is conditional on the central bank's information set in period t. This information set contains the current output gap, y_t, the current inflation rate, π_t, its forecast of the shock to the output gap, x_{t+1}, its forecast of the productivity shock—which depends on the unobserved regime s_{t+1}—and the structure of the economy as described by equations (1) through (8).

IMPLEMENTING INFLATION TARGETING

To get some straightforward results, we interpret inflation targeting as implying *strict* inflation targeting, in the sense that inflation is the only argument in the loss function (9). This means that we set $\mu = 0$.[3]

Applying $(1-\gamma L)$, where L is the lag operator defined by $L^j x_t = x_{t-j}$, to equation (1) and taking account of equation (4),

(10)

$$(1-\gamma L)\Delta\pi_{t+1} = \alpha(1-\gamma L)y_t - a_h(1-\gamma L)s_{t+1} + (1-\gamma)a_\ell.$$

Substituting for $(1-\gamma L)s_{t+1}$ from equation (7), we may rewrite equation (10) as

$$\pi_{t+1} = (1+\gamma)\pi_t - \gamma\pi_{t-1} + \alpha y_t - \alpha\gamma y_{t-1} - a_h \upsilon_{t+1}, \tag{11}$$

where we have used the fact that

[2] We adopt the usual convention that, for discrete-valued variables, capital letters denote the random variable and small letters a particular realization. If both interpretations apply we use small letters.

[3] In the case where $\mu > 0$, the intuition and main findings change little while the mathematics becomes considerably more complex. To keep our main points clear we have simply decided to omit analysis of this case. We discuss the $\mu = 0$ assumption in more detail near the end of the paper.

$$a_\ell = \left(\frac{1-q}{1-\gamma}\right) a_h .$$

Hence, in the absence of control, the combination of the Phillips curve (equation (1)) and the AR(1) representation of the productivity state (equation (7)) gives rise to a second-order stochastic difference equation for inflation. An important limiting case of equation (11) is when $p = q = 0.5$; that is, in each period either regime is equally likely to occur, regardless of the current state. In this case the productivity shock becomes serially uncorrelated and the reduced form parameter $\gamma \to 0$. When this occurs, the Phillips curve approaches the standard Svensson (1997) first-order form.

We have noted that in the above model the control lag is two periods. The *current* inflation rate and the *current* output gap are predetermined variables that cannot be influenced by policy. Therefore, the one-period-ahead inflation forecast is also predetermined[4] (independently of the current level of interest rates). However, by changing the current nominal interest rate, the policymaker can affect the one-period-ahead output gap forecast and thereby the one-to-two-period inflation forecast. Thus, i_0 affects $E_0 y_1$, which in turn affects $E_0 \pi_2$; i_1 affects $E_1 y_2$, which in turn affects $E_1 \pi_3$, and so on. The proper intermediate target for policy is thus the one-to-two-period inflation forecast, and the instrument (control) of policy is the nominal interest rate at time t, or equivalently the one-period-ahead output gap forecast.

When $\mu \to 0$, the above problem becomes straightforward. The monetary authority then needs to set $E_t y_{t+1}$ so as to ensure that (today's forecast of) the two-period-ahead inflation rate is equal to the inflation target. Thus, we have $\pi^* = E_t \pi_{t+2}$.[5] Leading the inflation equation (1) by one period and taking expectations at time t yields

$$E_t \pi_{t+2} = E_t \pi_{t+1} + \alpha E_t y_{t+1} - E_t u_{t+2} . \tag{12}$$

Setting this expression equal to the inflation target and then rearranging gives the optimal value of the control:

$$E_t y_{t+1} = -\frac{1}{\alpha}\left(E_t \pi_{t+1} - \pi^*\right) + \frac{1}{\alpha} E_t u_{t+2}, \tag{13}$$

where the (predetermined) one-period-ahead inflation forecast is given by[6]

$$E_t \pi_{t+1} = \pi_t + \alpha y_t - E_t u_{t+1}. \tag{14}$$

If $p = q = 1/2$, we get $E_t u_{t+1} = E_t u_{t+2} = 0$ and the policy rule (13) is identical to the optimal rule for the Svensson (1997) model (for the case of strict inflation targeting). This rule says that, if the one-period-ahead inflation forecast exceeds the target, *ceteris paribus*, the one-to-two-period inflation forecast will exceed the target. To compensate, the policymaker then needs to contract next period's forecast of the level of the output gap in the economy.

We now want to think of p and q as substantially greater than $1/2$, so that the model has persistent favorable or unfavorable supply side developments—regimes—which more closely approximate the postwar U.S. experience.

If the current-period forecast of next period's productivity state is favorable ($E_t u_{t+1} > 0$), this has two effects. In the first place it directly lowers the one-period-ahead inflation forecast (see equation (14)). This means that the central bank should allow next period's output gap to expand. The intuition is that, to prevent inflation from falling too far below the target, the demand side of the economy should move in tandem with the supply side. Thus, the sign of $E_t u_{t+1}$ in equation (13), through $E_t \pi_{t+1}$, is positive.

The second (or indirect) effect of a positive one-period-ahead productivity forecast is through its effect on the one-to-two-period productivity forecast. More specifically, any given productivity state is likely to persist into the future, so the expectation of a high state next period implies a similar outlook for the following period. In fact it can be shown that

$$E_t u_{t+2} = \gamma E_t u_{t+1} \tag{15}$$

(see Appendix C for details on the optimal predictor for productivity). In turn, if $E_t u_{t+2} > 0$, the one-to-two-period inflation forecast falls (see equation (12)), allowing the central bank to expand next period's level of the output gap. Thus, the sign of $E_t u_{t+2}$ in equation (13) is also positive. Substituting the right-hand sides of equation (14) for $E_t \pi_{t+1}$ and equation (17) for $E_t u_{t+2}$ in (15) gives

$$E_t y_{t+1} = -\frac{1}{\alpha}\left(\pi_t + \alpha y_t - \pi^*\right) + \frac{1}{\alpha}(1+\gamma) E_t u_{t+1}. \tag{16}$$

The first part of the expression for the optimal

[4] This can be seen by taking expectations at time t of equation (11). This yields $E_t \pi_{t+1} = (1 + \gamma)\pi_t - \gamma\pi_{t-1} + \alpha y_t - \alpha\gamma y_{t-1}$.

[5] For a derivation of this condition, see Appendix B.

[6] This can be seen by taking expectations at time t of equation (1).

control, $(-1/\alpha)(\pi_t + \alpha y_t - \pi^*)$, is identical to the Svensson (1997) derivation. This term can be interpreted as the demand component of the inflation process. The second component, $(1/\alpha)(1+\gamma)E_t u_{t+1}$, is new and contains the central bank's optimal reaction to its assessment of the (dis)inflationary consequences of the future *supply side* of the economy on the one-period-ahead and two-period-ahead inflation forecasts (the terms $(1/\alpha)\, E_t u_{t+1}$ and $(\gamma/\alpha)\, E_t u_{t+1}$, respectively).

In Appendix C we show that the central bank's optimal predictor for productivity is a function of the lagged output gap and the current acceleration of the inflation rate. That is,

$$E_t u_{t+1} = \alpha\gamma y_{t-1} - \gamma\Delta\pi_t, \tag{17}$$

where Δ is the backward difference operator. Thus, the central bank can use its observed values of y_{t-1} and $\Delta\pi_t$ to forecast next period's productivity level. Substituting (17) into (16), we obtain

$$E_t y_{t+1} = -\frac{1}{\alpha}\left(\pi_t + \alpha y_t - \pi^*\right) + \frac{1}{\alpha}(1+\gamma)\left(\alpha\gamma y_{t-1} - \gamma\Delta\pi_t\right). \tag{18}$$

The term $-\gamma\Delta\pi_t$ suggests that, if inflation accelerates, this is likely to be an indication of adverse developments on the supply side of the economy. Or, put differently, an accelerating inflation rate is a *leading indicator* of an adverse supply shock.

Similarly, the term $\alpha\gamma y_{t-1}$ suggests that if last period's output gap was negative—meaning that one period ago the economy was operating below its long-run potential—this is not an indication of lack of demand. Instead, it is indicative of the presence of an adverse *supply shock*. Under strict inflation targeting this means that the central bank demand should contract (its forecast of) the output gap. Otherwise, the policymaker risks further amplifying the inflation process. Similarly, if last period's output gap was positive, it indicates a positive supply shock, rather than excess demand. Now the central bank should allow the output gap to widen, since otherwise it risks creating disinflation.

Finally, using the fact that the one-period output gap forecast fulfills

$$E_t y_{t+1} = \beta_1 y_t - \beta_2 r_t + r^*, \tag{19}$$

where $r_t = i_t - \pi_t$ is the real ex post short-term interest rate and

$$r^* \equiv \frac{\rho x_t}{\beta_2},$$

the central bank's optimal monetary policy rule (interest rate reaction function) can be written as

$$r_t - r^* = \frac{1}{\beta_2\alpha}\left(\pi_t - \pi^*\right) + \frac{(1+\beta_1)}{\beta_2} y_t + \frac{\gamma(1+\gamma)}{\beta_2\alpha}\Delta\pi_t - \frac{\gamma(1+\gamma)}{\beta_2} y_{t-1}. \tag{20}$$

The first two terms in the rule, involving $(\pi_t - \pi^*)$ and y_t, are identical to those derived by Svensson (1997). These terms can be viewed as the demand components of the inflation process. The third and fourth terms, involving $\Delta\pi_t$ and y_{t-1}, are new and are leading indicators of future supply shocks.

An important limiting case of equation (20) is when $p = q = 1/2$. Then $\gamma \to 0$ and the supply-side terms drop out, so that the policy rule collapses to

$$r_t - r^* = \frac{1}{\beta_2\alpha}\left(\pi_t - \pi^*\right) + \frac{(1+\beta_1)}{\beta_2} y_t, \tag{21}$$

which—as in Svensson (1997, p. 1119)—is essentially a version of the simple policy rule popularized by Taylor (1993).[7] Another special case, less obvious from equation (20), is when $a_h \to 0$; here there is still regime switching, but the two regimes approach the same productivity levels and so the switching does not have any effect.

We now turn to a calibrated case to illustrate some of the differences between these rules.

COMPARING THE RULES

Calibration

Table 1 summarizes the parameter values used in our calibrated economy. We use standard, illustrative values for α, β_1, and β_2. The shock to the output gap is quite persistent, with $\rho = 0.9$. We chose the shock ε from a uniform distribution with minimum value $-1/2$ and maximum value $1/2$. The value of a_h scales the size of the effects of a productivity regime switch on the deviation of inflation from the policymakers' target value. Our choice of $a_h = 1$ limits this effect to 1 percentage point, but we could scale it up or down by choosing other values. Finally, we want to consider systems with very persistent regimes, and so we set $p = q = 0.975$, meaning that the chance of switching out of a given regime is only 0.025 in any period.

[7] Taylor rules are often written in terms of nominal interest rates, but given the definition of r_t the rules in equations (20) and (21) can easily be interpreted in these terms.

Table 1

Parameter Configuration

Parameter	Controls	Value
α	Response of inflation to the output gap	0.5
β_1	Output persistence	0.7
β_2	Response of the output gap to the real interest rate	1
ρ	Serial correlation in the shock to the output gap	0.9
σ_ε^2	Variance of the shock to the output gap	0.084
a_h	Productivity scale factor	1
p	Probability of high productivity, given high productivity	0.975
q	Probability of low productivity, given low productivity	0.975
π^*	Policymaker's inflation target	2.5

NOTE: We illustrate our analytical findings using this calibration.

Table 2

Asymptotic Loss

Case	Svensson-Taylor rule	Optimal rule
Baseline...	0.996	0.138
...with $p=q=1/2$	0.521	0.521
...with $a_h \to 0$	0.021	0.021

NOTE: In the baseline case, there are quantitatively important, persistent regimes. The optimal rule performs significantly better in this case. If the regimes are not persistent (second line) or not very different (third line), then the two rules perform equally well.

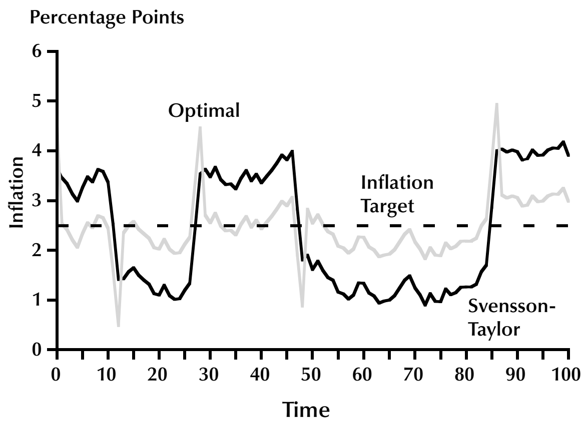
Optimality

We begin by demonstrating the superiority of the optimal rule given by equation (20) in the calibrated economy. Of course, in our derivation we assumed $\mu = 0$, meaning that the monetary authorities in the model economy direct policy solely toward keeping inflation near target because their objective function only involves inflation deviations. This was termed "strict inflation targeting" by Svensson (1997). Accordingly, we consider the asymptotic ($t \to \infty$) mean-squared inflation deviation from target for both the optimal rule given by equation (20) and for the Svensson-Taylor rule given by equation (21). We calculate the asymptotic mean-squared inflation deviation through a simulation using equations (1) through (3), and either (20) or (21), for a large enough number of periods that the mean-squared deviation no longer changes. Table 2 summarizes the results.

For baseline parameters, Table 2 indicates that the optimal rule clearly dominates the Svensson-Taylor rule, as expected, with an asymptotic mean-squared inflation deviation of only 0.138, versus 0.996 for the Svensson-Taylor rule. The Svensson-Taylor rule does not take account of the changing nature of the supply side of the economy, and thus policymakers using it would end up with a suboptimally high inflation variance. As we have emphasized, in two special cases the Svensson-Taylor rule and the optimal rule perform equally well. One of these occurs when the two productivity regimes are not persistent, so that $p = q = 1/2$, and other parameters are left as in the baseline case. In this situation, regime switches occur as often as non-switches, which merely adds to the noise in the system and leaves the "leading indicator" feature of the optimal rule impotent. The asymptotic loss is then equal for the two rules at 0.521, as shown in the second line of Table 2. The other special case is when the two regimes are not very far apart, which is the case when $a_h \to 0$ in our model, and all other parameters are again at baseline values (including p and q). Here, regime switches occur, but they are not quantitatively important because the productivity levels in the two regimes are not sufficiently different. The asymptotic loss is 0.021 for both rules, as shown in the third line of Table 2. This is much smaller than in the other cases because the lack of important regime switches reduces the overall variance in the economy dramatically.

Figure 1

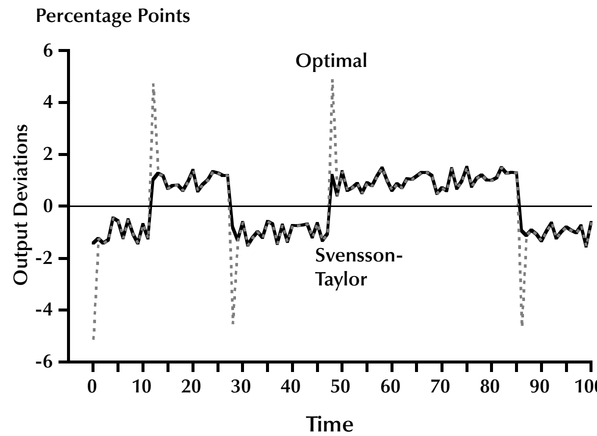
Inflation Stabilization Under Strict Inflation-Targeting Policies



NOTE: Inflation performance in model economies.

Figure 2

Output Performance Under Strict Inflation-Targeting Policies



We now turn to a particular realization of the model economy to illustrate some of the features of the optimal policy rule.

An Example

In Figure 1 we display the last 100 of 5,000 observations on inflation for simulated systems for both the optimal rule and the Svensson-Taylor rule. Both systems are calculated based on the same realized sequence of shocks. We use 100 observations to keep the Figure relatively clear. Figure 2 illustrates the implications for the output gap. Since our policymakers in these systems (under both policy rules) are strict inflation targeters ($\mu = 0$), they are of course only concerned with the inflation deviations as pictured in Figure 1.

In the Figures, regime shifts are realized in periods 0, 12, 28, 48, and 86. We think this provides enough switches to understand the main effects of the two rules. The primary feature of the optimal rule is that it tends to bring the inflation rate much closer to target following a productivity regime switch. The Svensson-Taylor rule, which leaves the policymakers without a response to the shifting productivity of the economy, does not bring inflation back toward its target; instead, regime shifts are associated with persistent movements in the level of inflation. In fact, inflation remains persistently above or persistently below target depending on the regime. Figure 1 clearly shows why the mean-squared deviation of inflation from target is higher for the Svensson-Taylor rule as compared with the optimal rule, as the systems are allowed to continue for a large amount of time.

It is interesting to see how the optimal rule fares in a period following an unfavorable supply disturbance, such as the regime switch realized in period 28 in the two Figures. As inflation starts to accelerate, the optimal rule fairly quickly infers the persistent change in the inflation environment and gets inflation back to target. This is in fact achieved by amplifying the structural economic slowdown, as shown in Figure 2. This is the correct policy response because a negative output gap in this case does not merely indicate lack of demand, but rather is indicative of the presence of an adverse supply shock. Thus, the optimal rule calls for contracting demand so as to avoid amplifying the inflationary effects of the low-productivity state. By way of contrast, the Svensson-Taylor rule fails to bring inflation down at all (even though the only goal here is to control inflation). In fact, inflation does not increase in response to the regime shift as much as under the optimal rule, but it stays persistently above target until the next regime shift is realized in period 48. Thus, a monetary policy response that is driven purely by demand factors amplifies the inflation problems associated with adverse supply shocks. We think that this "stagflation" example is reminiscent of the monetary policy responses of several

Organization for Economic Cooperation and Development (OECD) countries in the 1970s.

From the perspective of the "new economy," we can also consider the policy response to favorable supply shocks, such as those realized in periods 12 and 48 when the economy switches to a high-productivity state. Under the optimal policy the productivity shock drives inflation below target, but only temporarily. A few periods later inflation is back on target. The Svensson-Taylor rule, however, interprets the substantial increase in the output gap in these periods as evidence of excess demand. The central bank then responds by contracting aggregate demand. This in turn amplifies the downturn in inflation. As a result of systematically misreading the data, inflation falls below the target. Worse, it stays systematically below the target until the next regime switch.

As we have stressed, our exposition has been kept relatively simple by limiting the analysis to the strict inflation-targeting case ($\mu = 0$). The case when $\mu > 0$ is obviously an interesting extension in a quantitative sense, but we think our main points are better made in this simpler, strict inflation-targeting environment. If there are going to be unobserved shifts in productivity in the economy, then the optimal stabilization policy is naturally going to take these shifts into account. To accomplish this, an optimal policy rule will consider past data in addition to contemporaneous data in an effort to identify whether or not a regime shift has occurred. A policy rule that takes account of these factors is clearly going to perform better than one that does not. An optimal policy rule in the case with $\mu > 0$ will still have all of these features, except that it will mitigate output fluctuations to some extent at the expense of exacerbating fluctuations in inflation, as policymakers will in that case be attempting to optimally trade off these two types of fluctuations.

CONCLUDING REMARKS

In this paper we have investigated the implications of regime switching in productivity for optimal monetary policy rules. Our economy is simple and delivers a version of the Taylor rule as the optimal stabilization policy when there are no regime shifts in productivity. Thus, our analysis is able to isolate the additional components of an optimal policy rule in the face of persistent, unobserved productivity improvements or declines. We find that the optimal monetary policy rule in the regime-switching environment incorporates information about the changing nature of the supply side by considering lagged terms on inflation deviations and the output gap. We show that the optimal rule significantly outperforms a rule that ignores these terms in a quantitative simulation, provided the two regimes are persistent and sufficiently different. These conditions seem to characterize the postwar U.S. experience, as many analyses date a persistent productivity slowdown as beginning in the early 1970s followed by a "new economy" appearing in the 1990s.

We think our main findings are intuitively appealing and likely to carry over into more complicated environments, but this of course remains an open question, which we leave to future research.

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Appendix A

STEADY-STATE EQUILIBRIUM

The innovation sequence $\{V_t\}$ in equation (7) satisfies

$$\begin{aligned} &\Pr[V_{t+1} = (1-p) | S_t = 1] = p, \\ &\Pr[V_{t+1} = -p | S_t = 1] = 1-p, \\ &\Pr[V_{t+1} = -(1-q) | S_t = 0] = q, \\ &\Pr[V_{t+1} = q | S_t = 0] = 1-q, \end{aligned} \tag{22}$$

with $E_t V_{t+1} = 0$ and $\sigma_v^2 = E(V_t^2) = p(1-p)\bar{p} + q(1-q)(1-\bar{p})$, where we have used that[8]

$$\bar{p} \equiv \frac{(1-q)}{(1-p+1-q)} .$$

From equation (22) we see that $E_0 V_t = 0$ for all $t > 0$. Using this fact, and iterating equation (7) into the future, we can write

$$E_0 S_t = \gamma^t E_0 S_0 + \frac{(1-q)(1-\gamma^t)}{1-\gamma}, \tag{23}$$

where E_0 denotes the expectation conditional on information available at date zero (which need not include observation of S_0). Observing that $E_0 S_t$ can be interpreted as the probability that $S_t = 1$ given information at time zero (denoted $P_0[S_t = 1]$), equation (23) can be rewritten

$$P_0[S_t = 1] = \bar{p} + \gamma^t (\bar{p}_0 - \bar{p}), \tag{24}$$

where $p_0 \equiv P_0[S_0 = 1]$. From equation (24) we can see that for large t the economy will be in the high productivity state (state 1) with probability $\bar{p}$ in which case u would be $a_h - a_\ell$. Similarly, the economy will be in the low productivity state (state 0) with probability $1-\bar{p}$, in which case u would be $-a_\ell$. Hence, the *expected* long-run level of u (denoted as $\bar{u}$) is

$$\bar{u} = \bar{p} a_h - a_\ell . \tag{25}$$

From equation (2) it follows that the (unconditional mean) steady-state level of output ($\bar{y}$) associated with a zero steady-state real interest rate is zero. To be consistent, we impose that this level from equation (1) should also be zero. Taking account of (25) this implies

$$a_\ell = \bar{p} a_h, \tag{26}$$

which is equation (8) in the main text.

8 For more details see Hamilton (1989, pp. 360-63).

Appendix B

DERIVATION OF THE FIRST-ORDER CONDITION

The problem is to choose $\{i_t\}_{t=0}^{\infty}$ to minimize

(27) $$E_t \sum_{t=0}^{\infty} \delta^t \left[\frac{1}{2} \left(\pi_t - \pi^* \right)^2 \right]$$

subject to

(28) $$\pi_{t+1} = \pi_t + \alpha y_t - u_{t+1}$$

and

(29) $$y_{t+1} = \beta_1 y_t - \beta_2 \left(i_t - \pi_t \right) + x_{t+1}.$$

This problem can be reformulated by choosing $\{c_t\}_{t=0}^{\infty}$ to minimize

(30) $$E_t \sum_{t=0}^{\infty} \delta^t \left[\frac{1}{2} \left(E_t \pi_{t+1} - \pi^* \right)^2 \right]$$

subject to

(31) $$z_{t+1} = z_t + c_t + \alpha x_{t+1} - (1+\gamma) a_h \upsilon_{t+1},$$

where $c_t = \alpha E_t y_{t+1} - E_t u_{t+2}$ is a new control variable and $z_t = E_t \pi_{t+1}$ is a new state variable.[9] We solve this problem using the method of Lagrange multipliers. We denote the Lagrange multiplier by λ and we write the Lagrangian as

(32)
$$\mathfrak{L} = E_t \sum_{t=0}^{\infty} \left\{ \delta^t \left[-\frac{1}{2} \left(z_t - \pi^* \right)^2 \right] - \delta^{t+1} \lambda_{t+1} \left[z_{t+1} - z_t - c_t - \alpha x_{t+1} + (1+\gamma) a_h \upsilon_{t+1} \right] \right\}.$$

The central bank's first-order conditions then take the form

(33) $$\frac{\partial \mathfrak{L}}{\partial c_t} = \delta^2 E_t \lambda_{t+1} = 0$$

and

(34) $$\frac{\partial \mathfrak{L}}{\partial z_t} = -\left(z_t - \pi^* \right) - \lambda_t + \delta E_t \lambda_{t+1} = 0.$$

Equation (33) implies $E_t \lambda_{t+1} = 0$. Using this in equation (34) yields

(35) $$\lambda_t = -\left(z_t - \pi^* \right).$$

Leading this expression one period and taking expectations implies that

(36) $$E_t \lambda_{t+1} = -\left(E_t z_{t+1} - \pi^* \right).$$

Since $E_t \lambda_{t+1} = 0$ and since $z_{t+1} = E_{t+1} \pi_{t+2}$, we conclude that the first-order condition for strict inflation targeting is given by

(37) $$E_t \pi_{t+2} = \pi^*,$$

which is the expression used in the text.

[9] This constraint is derived using the fact that π_{t+1}, y_{t+1}, and $E_{t+1} u_{t+2}$ can be written as $E_t \pi_{t+1} + (\pi_{t+1} - E_t \pi_{t+1})$, $E_t y_{t+1} + (y_{t+1} - E_t y_{t+1})$, and $E_t u_{t+2} + (E_{t+1} u_{t+2} - E_t u_{t+2})$, respectively.

Appendix C

DERIVATION OF THE OPTIMAL PREDICTOR FOR PRODUCTIVITY

Taking expectations at time t of equation (11) we obtain

(38) $$E_t \pi_{t+1} = \pi_t + \alpha y_t + \gamma \Delta \pi_t - \alpha \gamma y_{t-1}.$$

However, from equation (1) it follows that

(39) $$E_t \pi_{t+1} = \pi_t + \alpha y_t - E_t u_{t+1}.$$

Hence, consistency requires that $-E_t u_{t+1} = \gamma \Delta \pi_t - \alpha \gamma y_{t-1}$ or

(40) $$E_t u_{t+1} = \alpha \gamma y_{t-1} - \gamma \Delta \pi_t.$$

Along similar lines we can derive that $E_t u_{t+2} = \alpha \gamma y_t - \gamma E_t \Delta \pi_{t+1}$. Using equation (40), we find that

(41) $$E_t u_{t+2} = \gamma E_t u_{t+1} = \gamma \left(\alpha \gamma y_{t-1} - \gamma \Delta \pi_t \right).$$