Commentary

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I would like to thank Ben for presenting a careful and thought-provoking paper. It is really two papers. The first part is positive. Ben notes that simple macroeconometric models based on a Taylor rule omit any reference to money and asks whether or not there is any problem with this. The second part is normative. He asks whether or not it is dangerous to set interest rates in response to expected inflation. In both cases, his answer is "yes" in theory and "no" in practice.

I will begin with the first part. Ben's point can be seen by looking at the following simple "new Keynesian" model of the economy. It is made up of three equations:

(1) $$y_t = -b\left(R_t - \pi^e_{t+1}\right) + y^e_{t+1} + e_t$$

(2) $$\pi_t = \beta\pi^e_{t+1} + \alpha y_t + v_t$$

(3) $$m_t - p_t = \gamma y_t - \delta R_t.$$

The variables represent log deviations from a steady state. Superscript e denotes expected values. The first equation is an "expectational" IS curve. It comes from the Euler equation for consumption and therefore relates current output, y, to future output and the real interest rate. The second equation represents the pricing decisions of firms in the economy. Firms are assumed to fix prices over an interval of time. Hence inflation, π, is high if firms expect inflation in the future or if firms are responding to excess demand in the present. The third equation represents money demand. Real balances, $m-p$, depend positively on output and negatively on the nominal interest rate, R. The model is closed with an equation describing monetary policy and an initial condition for the lagged price level. If this policy rule is of the Taylor variety, that is, if the Fed sets the nominal interest rate in response to output and inflation such as

(4) $$R_t = \left(1+\mu_1\right)\pi^e_{t+1} + \mu_2 y_t,$$

then output and inflation are determined solely by equations (1), (2), and (4). Money and the price level play no direct role. The price level can be determined from equation (2) and the initial condition, p_{t-1}. Given the price level, the money supply can be determined by equation (3). But these are after thoughts; the main action is in equations (1), (2), and (4).

Is this a problem? My first reaction was to think "no." The model is well specified. Money works in the way that it is supposed to. It is just that with this particular policy rule, one can solve the model without money. After all, the same recursivity is present in the old Keynesian IS-LM model.

The old Keynesian IS curve and the new Keynesian IS curve, however, are really very different objects. The old Keynesian IS curve represents supply and demand for goods. The new Keynesian IS curve reflects intertemporal optimization. In order to see why money might or might not matter in this case, it is useful to review the derivation of equation (1). We begin with the consumption Euler equation:

$$U_{c,t} + \beta\left(1+R_t-\pi^e_{t+1}\right)U_{c,t+1}.$$

This equation reflects optimal consumption smoothing. It relates the marginal utility of consumption today, $U_{c,t}$, to the marginal utility of consumption tomorrow. If in addition we assume that utility is separable between consumption, on the one hand, and leisure and money, on the other:

$$U\left(C, M/P, L\right) = \frac{C^{1-\gamma}}{1-\gamma} + V\left(M/P, L\right);$$

and if we assume that there is no investment so that all output is consumed, $Y=C$, we arrive at the relation (1).

From this derivation, it is clear that (given the real interest rate and consumption tomorrow) money might matter for output in two ways: (i) money shifts the utility of consumption and (ii) money shifts the relationship between consumption and output.

Seen in this way, one can think up dozens of theories by which money should enter equation (1). It is interesting to note that Pigou's real balance effect is not one of them. The Pigou effect is a wealth effect. It effects c_t only through c_{t+1} and has no independent effect on the consumption Euler equation.

A partial list of amendments follows:

- Non-separable utility: Here changes in the money supply affect U_c directly.

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- Transactions costs: Here the money supply affects the efficiency with which dollars may be translated into consumption and hence the total cost of goods.
- Liquidity constraints: When the constraints bind, the Euler equation does not hold and changes in the money supply may relax the constraint.
- Cash-in-advance constraint: In Svensson (1985), for example, a relaxation of the constraint has a direct effect on consumption in many states of the world.
- Segmentation of the goods and asset markets: In many models of the liquidity effect, equation (1) does not hold for agents temporarily cut off from the asset market.
- Lending view: Here money affects investment through bank balance sheets and hence the gap between consumption and output.

Ben focuses on transactions costs. He supposes that purchases absorb $\psi(C,M)$ in resources each period where ψ is increasing in C and decreasing in M. He does not find this channel important. To see why, consider the consumption Euler equation amended for transactions costs. Since reducing consumption by a unit reduces utility by $C^{-\gamma}$, but increases resources by $1 + \psi_c$, we replace U_c with $C^{-\gamma}/(1 + \psi_c)$. Log-linearizing and noting log $(1 + \psi_c) \sim \psi_c$:

$$y_t - \psi_{c,t} / \gamma = -b\left(R_t - \pi^e_{t+1}\right) + y^e_{t+1} - \psi^e_{ct+1} / \gamma + e_t .$$

Money matters if it shifts $\psi_{c,t} - \psi_{c,t+1}$. Since the transactions cost is small, one would not expect the change with respect to money of the first difference with respect to time of the derivative with respect to consumption of the transactions cost to be large. Although it could be.

Ben shows that for plausible parameterizations this effect is small. In a similar vein, Peter Ireland (2000) investigates the effect of non-separability between money and consumption in the utility function with similar results. Although it is just speculation, I would guess that this transactions effect of money is more important in cases in which the payments mechanism is under stress, such as during high inflations. During more stable times, the constraint models or the lending view make more plausible cases for the inclusion of money in the New Keynesian IS curve.

Now let's turn to the second topic of Ben's talk: the danger of using Taylor rules based on expected inflation. Consider again the simple new Keynesian model,

$$\begin{aligned} y_t &= -b\left(R_t - \pi^e_{t+1}\right) + y^e_{t+1} \\ \pi_t &= \beta \pi^e_{t+1} + \alpha y_t \\ R_t &= \left(1+\mu\right)\pi^e_{t+1}, \end{aligned}$$

where, for simplicity, we have removed the shocks and have specified that policy sets the nominal interest rate proportional to expected inflation. To see the potential problem, use the first and third equations to remove R and use the second equation to remove y_t and y^e_{t+1}; the result is

$$\pi_t - \left(1 + \beta - \alpha\mu b\right)\pi^e_{t+1} - \beta\pi^e_{t+2} = 0.$$

If μ is high enough or if μ is negative, then this equation has real non-zero solutions of the form $\pi_t = \lambda^t \pi_0$. Typically there are two of these λ, one greater than 1 in absolute value and one less than 1 in absolute value. Like the homogeneous solutions to a differential equation, these solutions may be tacked on to any particular solution that we find to the model.

While mathematically precise, this way of explaining the indeterminacy is not very enlightening. It does not explain why the indeterminacy arises and whether agents might actually fall into the trap of believing in one of these so-called sunspot or bubble solutions. To get a better handle on these issues I found it useful to rewrite the model in terms of the behavioral rules that agents follow. This requires taking a step back to the baseline model that these three equations represent. That model is a model of monopolistic competition in the spirit of Dixit and Stiglitz together with Calvo-style price rigidity.

The behavioral relations are:

$$\begin{aligned} y_{i,t} &= -b\left(R_t - \pi^e_{i,t+1}\right) + y^e_{i,t+1} \\ p^*_{i,t} &= p_t + \frac{\theta\alpha}{1-\theta}\bar{y}_{it} + \frac{\theta\beta}{1-\theta}\pi^e_{i,t+1} \\ R_t &= \left(1+\mu\right)\pi^e_{b,t+1} . \end{aligned}$$

Bars represent averages across agents. As before, the first equation follows from consumers' Euler equations. Each consumer sets his consumption, $y_{i,t}$, equal to his expected future consumption less b times his expected real interest rate. The second equation represents the optimal pricing choices of firms that set prices in period t, $p^*_{i,t}$. These prices are increasing in the current price level, p_t, current output, $\bar{y}_{i,t}$, and the firm's expectations of future inflation, $\pi^e_{i,t+1}$. Firms care about future inflation because they do not get the opportunity to change their price each period. The parameter θ represents the probability that a firm must keep its price unchanged. Note that the expectation of relevance is the firm's expectation of others' price changes. To get from this equation to equation (2), we note that a fraction, 1–θ, of firms change their prices each period so that

$$p_t = \theta\, p_{t-1} + (1-\theta)\bar{p}^*_{i,t} \;.$$

The last equation describes the rule of the monetary authority. They may choose their own expectation of inflation if they wish.

There are several strategic complementarities present in this model. First, there is an intratemporal pricing complementarity: price increases by some firms lead others to desire price increases. This is not a source of multiple equilibria in the present context. A 1 percent increase in prices by all firms that are adjusting prices, leads to a (1–θ) percent increase in the price level and therefore a (1–θ) percent increase in the desired price of each firm.

There is also an intertemporal pricing complementarity: Expectations of future inflation lead firms to raise prices today. As noted above, the firms raising their prices tomorrow are different than the ones acting today. This raises an important point. The multiple solutions to equation (5) all arise because one group of agents believes that another group of agents will do something in the next period, and not because individuals believe that they themselves will act in a certain way tomorrow.

The central bank's interest rate rule has an impact on this intertemporal complementarity. How this works depends on how the central bank forms its expectations and how the economy reacts to its announcements. One possibility is that the central bank responds directly to the private sector's expectations of inflation, $\pi^e_{b,t+1} = \bar{\pi}^e_{i,t+1}$. In this case we get equation (5) after imposing rational expectations and symmetry across agents.

There are, however, other possibilities. It is possible that the central bank uses its own model to generate its expectations and announces these expectations to the market along with its interest rate at the beginning of each period. If agents heed these announcements, they may coordinate agents on a single equilibrium of the central bank's choosing, and thereby eliminate solutions to equation (5) that are undesirable from the bank's perspective.

It is also possible that agents ignore the central bank's announcements and go with their own forecasts. Suppose, for example, that the central bank chooses $R_t = (1 + \mu)\pi^e_{b,t+1}$ where $\pi^e_{b,t+1}$ is its own internal forecast, but that private agents choose $\pi^e_{i,t+1} \neq \pi^e_{b,t+1}$. Solving for the expectations of private agents yields:

$$\bar{\pi}_{i,t} - (1+\beta+\alpha b)\bar{\pi}^e_{i,t+1} - \beta\,\bar{\pi}^e_{i,t+2} = \alpha b R_t \,.$$

The homogeneous part of this difference equation has non-zero real roots like equation (5). In fact it is the same as (5), but with $\mu = -1$. In this case, by sticking stubbornly to its inflation forecast while private agents choose expectations that are different, the central bank finds itself insufficiently responsive to bubbles in inflation that may develop. Note in this case, we are not imposing that the central bank has rational expectations, only that the private agents do.

Are these problems serious? The values of λ that arise when μ is large are negative. This implies that the inflation oscillates along the bubble path. It is hard to imagine the conditions under which these solutions would arise. It requires one group of agents to raise prices in anticipation that the next group would cut prices, and the next group would raise them again. A negative value of μ, on the other hand, leads to positive values for λ. Here the explosive root has a certain intuitive appeal, whereas the stable root, although mathematically just as valid, appears a bit far-fetched. One can imagine agents believing that things are getting out of hand and believing that others believe likewise. It is less likely that agents believe that future actions will cause the inflation rate to converge geometrically to the no-bubble solution. In any case, the easy way to avoid such situations is either to be credible or to set interest rates sufficiently responsive to agents' own expectations of inflation.

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