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What Do New-Keynesian Phillips Curves Imply for Price-Level Targeting?

Robert Dittmar and William T. Gavin

Would we be better off if the Federal Reserve had an inflation target or a price-level target? In a previous paper, Dittmar et al. (1999a) used a simple Phillips Curve model and evidence about the persistence in output gaps to show that a price-level-targeting regime would likely result in a better inflation-output variability tradeoff than an inflation-targeting regime. That was an extension of work by Svensson (1999). The Phillips Curve specification was consistent with one derived from a Lucas Island model with persistent supply shocks or a Fischer (1977) wage-contracting model. McCallum (1994) refers to this as a Neoclassical Phillips Curve because it is consistent with the Natural Rate Hypothesis (NRH)—monetary policy cannot keep output permanently above its natural rate because only unanticipated monetary policy affects real output.

Kiley (1998) argues that the Neoclassical specification is inconsistent with U.S. data because he believes there is historical evidence that anticipated monetary policy has had real effects. He attributes Svensson's (1999) favorable finding for price-level targeting to his choice of Phillips Curve specification. Kiley concludes that, compared to the case with inflation targeting, price-level targeting would have been found to result in a worse inflation-output variability tradeoff if Svensson had started with a New-Keynesian version of the Phillips Curve. Kiley derives the expectation for the mean of output in a New-Keynesian model, shows that the expectation depends on the lagged price level, and infers from this that trying to stabilize the price level would raise the variability of output. He does not derive the inflation-output variability tradeoff implied by the model nor does he experiment with alternative policy rules using his New-Keynesian specification.

In this paper, we extend the analysis of price-level targeting of Dittmar et al. (1999a) to a model including the New-Keynesian Phillips curve recommended by Kiley. We examine the inflation-output variability tradeoffs implied by optimal inflation and price-level rules. To be consistent with our earlier work and that of Svensson (1999), we assume that lagged output enters the aggregate supply function. The introduction of lags is consistent with both the theoretical model of Taylor (1980) who includes both leads and lags of unemployment in the Phillips Curve and the empirical work of Roberts (1995), who finds serial correlation in the error terms of his estimated Phillips Curves.

Our intuition is that price-level targeting should be preferable in a sticky-price world where prices are costly to adjust. If prices were perfectly flexible, alternative monetary policy rules would have almost no effect on real output. But in a world where it is costly to adjust prices, a policy that reduces price fluctuations would seem to be appropriate. Indeed, we find that the New-Keynesian Phillips Curve provides even stronger support for price-level targeting than did the model with the Neoclassical Phillips Curve.

In previous work with the Neoclassical Phillips Curve, we found that the choice between inflation targeting and price-level targeting depended on the amount of persistence in the output gap. That is, if the output gap was not too persistent, or if lagged output did not enter the aggregate supply function, then inflation targets were preferred to price-level targets. Empirical evidence, however, showed a very high level of persistence in the output gap, suggesting that price-level targets offer the policymaker a better menu of tradeoffs between output and inflation variability.

To preview the results in this article, we show that when we start with a New-Keynesian Phillips Curve, the amount of persistence in the output gap still affects the relative placement of the inflation-output variability tradeoff. Contrary to the Neoclassical case, however, even where the persistence of the output gap in the aggregate supply function is small or nonexistent, the price-level targeting regime still results in a more favorable tradeoff between output and inflation variability than does an inflation-targeting regime.

In the first section, we briefly describe the New-Keynesian model and compare it to the Neoclassical specification. In the second section, we construct the inflation-output variability curves implied by alternative parameterizations of the model. In the conclusion, we discuss the assumptions that are apparently needed to find that price-level targeting would destabilize output.

A NEW-KEYNESIAN PHILLIPS CURVE

We begin with the same infinite-horizon quadratic loss function used in our earlier work. The central bank with an inflation target minimizes

$$L^A = \sum_{t=0}^{\infty} \beta^t \left(\lambda y_t^2 + (\pi_t - \pi^*)^2 \right), \tag{1}$$

where the superscript A refers to the loss function of an inflation targeting central bank, β is the central bank's discount factor, y_t is the deviation of output from the target level, and $(\pi_t - \pi^*)$ is the deviation of inflation from the central bank's inflation target. The term, λ, gives the weight on output gap relative to the weight on inflation in the central bank's loss function.

The Neoclassical Phillips Curve used in our earlier paper is given by

$$y_t = \rho\, y_{t-1} + \alpha\,(\pi_t - {}_{t-1}\pi_t^e) + \varepsilon_t, \tag{2}$$

where ρ determines the persistence in the output gap, α determines the response of the output gap to unanticipated inflation, and ε_t is an independent and identically distributed technology shock with mean zero and variance σ^2_ε. We are making no distinction between the aggregate supply function and the Phillips Curve.[1]

Roberts (1995) shows that the sticky price models of Taylor (1980), Rotemberg (1982), and Calvo (1983) all imply the same Phillips Curve structure that has been called New Keynesian. Kiley (1998) uses the Calvo model to derive the following New-Keynesian Phillips Curve:

$$y_t = \rho\, y_{t-1} + \alpha\,(\pi_t - {}_t\pi_{t+1}^e) + \varepsilon_t. \tag{3}$$

This is deceptively similar to the Neoclassical version where the anticipated inflation that enters the function is the expectation for period $t+1$ rather than t. Kiley includes a discussion of the empirical support for this specification and a discussion of the research that has developed microeconomic foundations for this aggregate relationship. A comprehensive survey of the implications for monetary policy implied by New-Keynesian theories can be found in Clarida et al. (1999).

To solve this model, we must decide what to assume about how the central bank takes account of its effect on inflation expectations. Kydland and Prescott (1977) showed that the presence of forward-looking expectations in the central bank's Phillips Curve constraint causes a problem of time inconsistency if the bank tries to manipulate those expectations.

Intuitively, the problem of time inconsistency here results from the ability of a bank facing a New-Keynesian Phillips Curve to derive rewards today by creating expectations for tomorrow. When a new period arrives, the temptation is to confound expectations with new policy since the gains from the previously announced policy already have been taken. In equilibrium, the central bank cannot benefit from reneging on announced policies. If the bank reoptimizes each period, or only occasionally, then private agents will learn that the bank's announced future policy will not necessarily be implemented. When this occurs, the bank's ability to control expectations will be lost. Recent discussion of this issue can be found in Woodford (1999) and Clarida et al. (1999). To avoid this time inconsistency problem, we assume that the central bank takes private sector expectations as given. Under this assumption, the bank, recognizing that it may be unable to commit to policy announcements, forgoes any attempt to manipulate private expectations.

When the central bank regards expectations as given, the bank's optimization problem becomes a standard one, with a quadratic objective and linear constraints. Furthermore, first-order conditions take a standard form for all time periods, assuring policy rules are time consistent. Linear decision rules are assumed for the bank's optimal policy. Upon substituting the assumed linear rules into the first order conditions for the bank's optimization problem, we equate coefficients on the variables in the decision rules and derive the bank's policy function. We assume that the bank, in taking expectations as given, bases its time t decisions on current states, y_{t-1} and ε_t, in both regimes and p_{t-1} in the case of a price-level-targeting regime. Expectations then are assumed to be formed as a rational consequence of the bank's policy rule.

[1] Some would call our equation an aggregate supply function because the dependent variable is the output gap. If the equation were rearranged with inflation on the left-hand side, they would call it a Phillips Curve. King and Watson (1994) show that this distinction can be important when estimating the parameters from historical data, but it does not matter in our analytical work.

The derivation of policy rules under an inflation-targeting regime closely follows the derivation in the appendix of Dittmar et al. (1999a). The bank's constrained optimization problem is given as:

$$(4)\quad E_t\left\{\sum_{i=0}^{\infty}\beta^t\left[\left(\lambda\, y_{t+i}{}^2+(\pi_{t+i}-\pi^*)^2\right)-\mu_{t+i}\left(y_{t+i}-\rho y_{t-1+i}-\alpha\,(\pi_{t+i}-{}_{t+i}\pi^e_{t+1+i})-\varepsilon_{t+i}\right)\right]\right\},$$

with the μ_t's being a sequence of random multipliers and ${}_{t+i}\pi^e_{t+1+i}$ denotes the private sector's inflation expectations. First-order conditions for the bank take the form:

$$(5)\quad 2\lambda y_t-\mu_t+\beta\rho E_t\mu_{t+1}=0,$$

when taken with respect to the sequence of y_t's, and the form:

$$(6)\quad 2\left(\pi_t-\pi^*\right)+\alpha\,\mu_t=0,$$

when taken with respect to the sequence of π_t's. Eliminating the multipliers from these expressions gives the following Euler equation:

$$(7)\quad \lambda y_t+\frac{1}{\alpha}\left(\pi_t-\pi^*\right)-\frac{\beta\rho}{\alpha}E_t\left(\pi_{t+1}-\pi^*\right)=0.$$

Here the central bank wants to smooth inflation deviations from target with an adjustment for the current output gap. If there is no persistence in the output gap, then the desired inflation deviations today depend only on the output gap.

We now seek a linear decision rule for inflation of the form:

$$(8)\quad \pi_t=A_1+A_2 y_{t-1}+A_3\varepsilon_t.$$

Expectations of the private sector are assumed to be rational, so at time t we have:

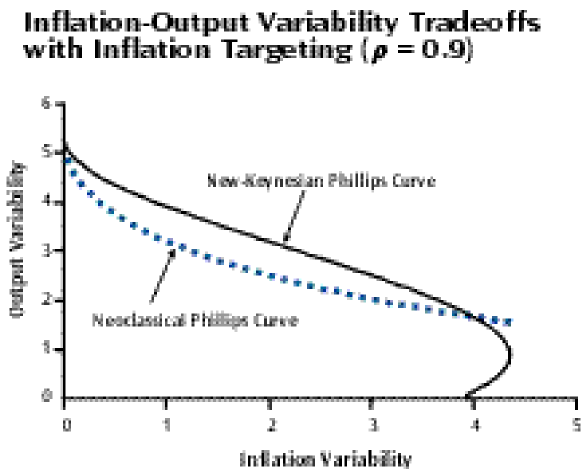
$$(9)\quad \pi^e_{t+1}=A_1+A_2 y_t.$$

Substituting these expressions into the Phillips Curve equation and solving the resulting equation for y_t yields a decision rule for y_t directly of the form:

$$(10)\quad y_t=\left(\frac{\rho+\alpha A_2}{1+\alpha A_2}\right)y_{t-1}+\left(\frac{1+\alpha A_3}{1+\alpha A_2}\right)\varepsilon_t.$$

Decision rules are invariant so we can determine π_{t+1} by iterating on the rule for π_t to yield the following expression:

Figure 1



$$(11)\quad \pi_{t+1}=A_1+A_2 y_t+A_3\varepsilon_{t+1}$$

$$=A_1+A_2\left(\frac{\rho+\alpha A_2}{1+\alpha A_2}\right)y_{t-1}+A_2\left(\frac{1+\alpha A_3}{1+\alpha A_2}\right)\varepsilon_t+A_3\varepsilon_{t+1}.$$

Taking time t expectations then yields a linear expression for $E_t\pi_{t+1}$. If we now substitute the expressions for y_t, π_t, and π_{t+1} into the first-order condition and equate constant terms and coefficients on y_{t-1} and ε_t, we obtain three equations that can be solved for the unknown A_1, A_2, and A_3.

When the central bank takes inflation expectations as given, the first-order conditions for the inflation targeting case are of the same form for both the New Keynesian and Neoclassical specifications of the Phillips Curve. The reason is simply that the difference in the specifications is in the way expectations enter. When different expressions for the Phillips Curve constraint and expected inflation for period $t+1$ (equations 10 and 11 for the New-Keynesian case) are substituted back into the first-order conditions, however, we get different monetary policy rules. In the New-Keynesian case, agents' inflation expectations at time t are for inflation at time $t+1$ and involve y_t; whereas in the Neoclassical case, inflation expectations at time t are for inflation at time t and involve y_{t-1}.

Figure 1 shows the inflation-output variability tradeoffs for the Neoclassical and New-Keynesian cases when the central bank has an inflation target. We graphically display the inflation/output variability tradeoffs in the two specifications by first expressing the output gap variance and the inflation variance as functions of the preference parameter, λ, while holding the parameters of the Phillips Curve constant. For a given λ, the bank's decision rules can be used to calculate an unconditional variance for both inflation and the output gap (a single point in Figure 1). Varying the bank's preferences by varying λ will determine the location of the curve representing the tradeoff between σ_π^2 and σ_y^2.

The parameterizations used here are the same as the ones used by Dittmar et al. (1999a): $\alpha = 0.5$, $\beta = 0.99$, and $\rho = 0.9$. These assumptions imply a Phillips Curve slope of 0.2.[2] We assume that the interest rate is 4 percent at an annual rate, so the quarterly discount factor is approximately 0.99. The variance of the output shock is normalized to one in the figures. As Figure 1 shows, the tradeoff is similar across the two model specifications, except for extreme cases where the central bank puts little weight on the deviation of inflation from target. In the Neoclassical case, the variance of inflation rises monotonically with λ, the relative weight the central bank puts on the output gap in its loss function. In the New-Keynesian case, with this parameterization, the curve bends back; that is, inflation variability stops rising and begins to decline when the central bank has a very strong preference for output stability (after λ goes above three). Inflation variability begins to rise again at very high values of λ (this second reversal is not discernable in Figure 1). We do not have any intuition about why this curve is oddly shaped when the central bank puts high weight on reducing variability of the output gap. Note, however, Cecchetti, McConnell, and Quiros (1999) estimated λ to be less than 0.33 (by our definition of λ) for the countries in the European Monetary Union.

The interesting question is what happens when the central bank targets the price level instead of the inflation rate. Under an inflation-targeting regime, the equilibrium results in a price level that has a random-walk component. In statistical jargon, the time-series for the logarithm of the price level has a unit root. With a price-level objective, the equilibrium results in a time series for the price level that is stationary about a deterministic trend—which may or may not be growing, depending on the underlying desired inflation rate. With a price-level objective, the problem becomes more complicated. We revise the loss function to reflect the central bank's preference for a price-level objective:

$$(12) \qquad L^B = \sum_{t=0}^{\infty} \beta^t \left(\lambda\, y_t^{\,2} + (p_t - p_t^*)^2\right),$$

where the price level, p, has replaced the inflation rate and the superscript B denotes a loss function in the price level rather than the inflation rate.

Determining decision rules for the bank in the case of price-level targeting proceeds in a similar manner as in the case of inflation targeting, but is complicated by the presence of two lagged-state variables in the bank's Phillips Curve, y_{t-1} and p_{t-1}. The bank's first-order condition in this case will involve infinite sums of future price levels and output gaps. To simplify the derivation of these conditions, we first define the new variable $\tilde{p}_t = p_t - p_t^*$.

The New-Keynesian Phillips Curve will take the form

$$(13) \qquad y_t = \rho\, y_{t-1} + \alpha\,(-\tilde{p}_{t-1} + 2\,\tilde{p}_t - \tilde{p}_{t+1}^e) + \varepsilon_t,$$

in the transformed price variable. We can now form the bank's Lagrangian as:

$$(14) \qquad E_t\left\{\sum_{i=0}^{\infty} \beta^t \left[\left(\lambda\, y_{t+i}^2 + (\tilde{p}_{t+i})^2\right) - \mu_{t+i} \left(y_{t+i} - \rho y_{t-1+i} - \alpha\,(-\tilde{p}_{t-1+i} + 2\tilde{p}_{t+i} - \tilde{p}_{t+1+i}^e) - \varepsilon_{t+i}\right)\right]\right\},$$

with, once again, the μ_t's being a sequence of random multipliers. First-order conditions when taken with respect to the sequence of y_t's now take the form

$$(15) \qquad 2\lambda y_t - \mu_t + \beta\rho E_t \mu_{t+1} = 0,$$

and when taken with respect to the sequence of $\tilde{p}_t$'s now take the form

$$(16) \qquad 2\,\tilde{p}_t + 2\alpha\,\mu_t + \beta\alpha E_t\,\mu_{t+1} = 0.$$

We get a sequence of first-order conditions expressed in terms of state variables. At each point in time i we get the form

$$(17) \qquad 2\lambda \sum_{j=0}^{\infty} (\beta\rho)^j E_t y_{t+j+i} = -\frac{1}{\alpha} \sum_{j=0}^{\infty} (\frac{\beta}{2})^j E_t \tilde{p}_{t+j+i}.$$

Calculating decision rules for the bank now proceeds in a similar manner to the calculation of decision rules for an inflation-targeting central bank. Details are in the appendix.

[2] See Gray and Spencer (1990) for estimates of the effect of price surprises on output using annual data. See Rudebusch and Svensson (1999) and Orphanides (1998) for estimates of the slope of the Phillips Curve.

Figure 2

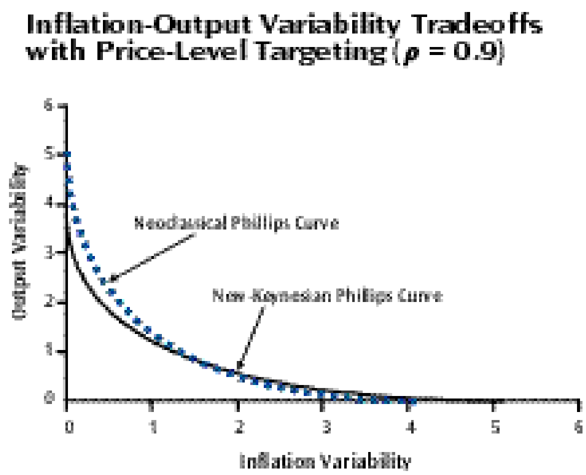


Figure 3

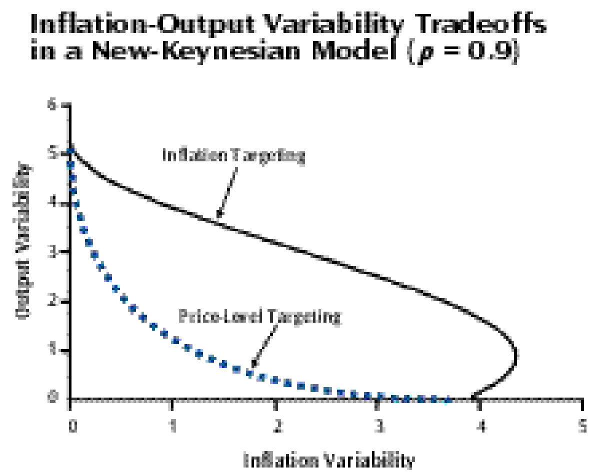


Figure 4

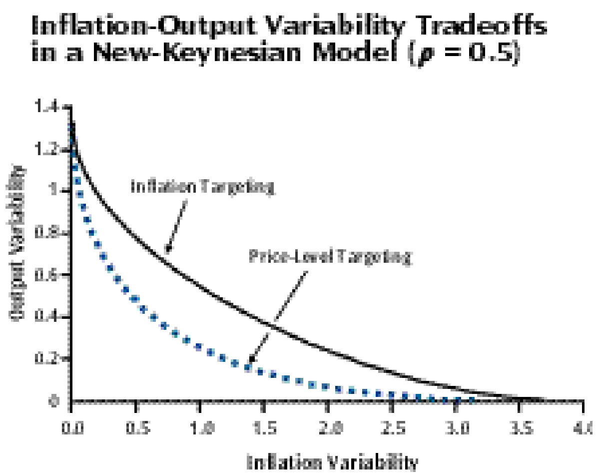
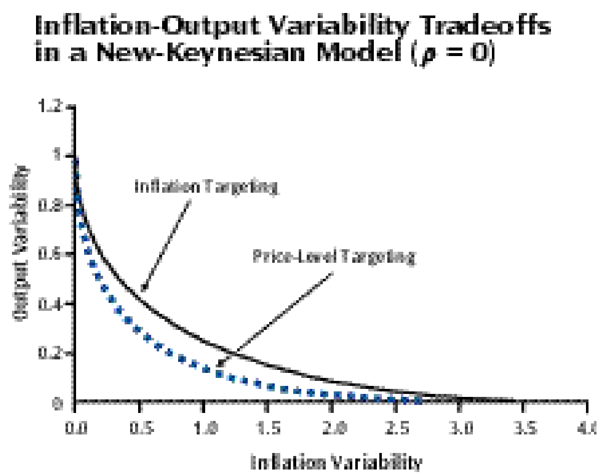


Figure 5



We then calculated the inflation-output tradeoff implied by varying λ, the bank's relative preference for output stability, between zero and infinity. Figure 2 shows that the inflation-output variability tradeoffs almost are identical for the two versions of the Phillips Curve when there is a high degree of persistence in the output gap. We are not interested really in distinguishing between these alternative views of the Phillips Curve. We want to respond to the suggestion that price-level targeting would not work well under the New-Keynesian specification. Figures 3 through 5 show the inflation-output variability tradeoffs implied by the New-Keynesian specification for three alternative values of ρ, our measure of persistence in the output gap. The first case compares inflation targeting with price-level targeting for what we believe is a realistic amount of persistence, $\rho = 0.9$. This case is shown in Figure 3, which shows that the price-level target results in a better inflation-output variability tradeoff than does an inflation target. Figure 4 shows that price-level targeting still dominates inflation targeting when $\rho = 0.5$. In the Neoclassical case, the tradeoffs were identical in this case. As ρ falls below 0.5, the inflation target came to dominate the price-level target in our earlier analysis. In contrast, Figure 5 shows that, under the New-Keynesian specification, inflation targeting results in a worse tradeoff between inflation and output variability even when $\rho = 0$.

In summary, Dittmar et al. (1999a), assuming a Neoclassical Phillips Curve, found that price-level targeting dominated inflation targeting for cases where the output gap was relatively persistent; that is, when $\rho > 0.5$. In this article, we find that when we use a New-Keynesian Phillips Curve, price-level targeting dominates inflation targeting for all values of λ, even if we omit the lagged output gap from the aggregate supply function.

IT'S NOT THE PHILLIPS CURVE, IT'S EXPECTATIONS FORMATION

Our results raise an important issue. Simulations of econometric models typically find that targeting the price level is a bad idea. Economists have attributed this result to the presence of nominal rigidities such as wage contracts or price adjustment costs. Yet in these econometric experiments, inflation expectations almost always are assumed to be formed adaptively. For example, Haldane and Salmon (1995) use a small econometric model with adaptive inflation expectations to examine whether monetary policy targets for price stability should be expressed in levels or rates of change. They find that price-level targeting results in higher short-run variability for both inflation and output growth. These results are typical of econometric model simulations with backward-looking expectations.[3]

There are at least two examples where central bank economists conducted experiments with price-level targets using econometric models modified to include some forward-looking behavior. Black, Macklem, and Rose (1997) look at combination rules that combine a long-term price-level objective with a short-term inflation-targeting rule. The presence of an error-correction term guarantees the eventual return of the price level to its long-run target path. For some values of the error-correction parameter between 0.1 and 0.125, they derive an inflation-output variability tradeoff that is better than with the inflation rule alone. Using a policy model estimated at the Board of Governors of the Federal Reserve System, Williams (1999) finds "interestingly, targeting the price level rather than the inflation rate generates little additional cost in terms of output and inflation variability. Under price-level targeting, the expectations channel helps stabilize inflation, thereby eliminating much of the output stabilization costs that would otherwise be associated with reversing deviations of the price level from its target." Williams confirms our view that the reason price-level targeting fares so badly in econometric simulations is that this is exactly the type of exercise for which the Lucas Critique is likely to be most relevant. The policy rules that were most efficient in reducing inflation and output variability when the model assumes forward-looking expectations, turn out to be the worst when fixed adaptive expectations are assumed. And, vice-versa, policies that are efficient when expectations are assumed to be adaptive do poorly when expectations are forward looking. Assumptions about expectations are critical for the analysis.

We focus on an extreme comparison in our analysis, inflation targeting versus price-level targeting. Our results suggest that targeting the price level in the short run may work better than previously thought. But these results should be put into perspective. We do not have enough confidence in our knowledge about the short-run dynamics of the economy to recommend that any central bank adopt a policy rule that would represent a sharp break with current practice. Rather, the role of the price-level target is to provide a long-term anchor for the monetary system. Dittmar et al. (1999b) showed that a central bank can dramatically reduce the uncertainty about inflation inherent in an inflation-targeting regime by 1) adopting a long-term price-level objective, and 2) using it in an error-correction framework to modify the short-run inflation targets. That analysis was based on an aggregate model including the Neoclassical Phillips Curve. But as we have shown here, the results would not be substantially different if we had started with a New-Keynesian specification.

REFERENCES

Black, Richard, Tiff Macklem, and David Rose. "On Policy Rules for Price Stability," presented at a Bank of Canada Conference, *Price Stability, Inflation Targets and Monetary Policy* on May 3-4, 1997.

Calvo, Guillermo A. "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* (September 1983), pp. 383-98.

Cecchetti, Stephen G., Margaret M. McConnell, and Gabriel Perez Quiros. "Policymakers' Revealed Preferences and the Output-Inflation Variability Trade-off: Implications for the European System of Central Banks," manuscript, Federal Reserve Bank of New York, February 1999.

Clarida, Richard, Jordi Gali, and Mark Gertler. "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature* (December 1999), pp. 1661-707.

[3] See Haldane and Salmon (1995) for an example and further references.

Dittmar, Robert, William T. Gavin, and Finn E. Kydland. "The Inflation-Output Variability Tradeoff and Price-Level Targets," this *Review* (January/February 1999a), pp. 23-31.

________, ________, and ________. "Price-Level Uncertainty and Inflation Targeting," this *Review* (July/August 1999b), pp. 23-33.

Fischer, Stanley. "Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy* (February 1977), pp. 191-205.

Gray, Jo Anna, and David E. Spencer. "Price Prediction Errors and Real Activity: A Reassessment," *Economic Inquiry* (October 1990), pp. 658-81.

Haldane, Andrew G., and Christopher K. Salmon. "Three Issues on Inflation Targets," in *Targeting Inflation: A conference of central banks on the use of inflation targets organised by the Bank of England, 9-10 March 1995,* Haldane, Andrew G., ed., Bank of England, 1995, pp. 170-201.

Kiley, Michael T. "Monetary Policy under Neoclassical and New-Keynesian Phillips Curves, with an Application to Price Level and Inflation Targeting," Board of Governors of the Federal Reserve System, Working Paper 1998-27, May 20, 1998.

King, Robert G., and Mark W. Watson. "The Post-War U.S. Phillips Curve: A Revisionist Econometric History," *Carnegie-Rochester Conference Series on Public Policy* (December 1994), pp. 157-219.

Kydland, Finn E., and Edward C. Prescott. "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy* (June 1977), pp. 473-91.

McCallum, Bennett T. "A Semi-classical Model of Price-Level Adjustment," *Carnegie-Rochester Conference Series on Public Policy* (December 1994), pp. 251-84.

Orphanides, Athanasios. "Monetary Policy Evaluation With Noisy Information," Finance and Economics Discussion Series 1998-50, December 1998.

Roberts, John. "New-Keynesian Economics and the Phillips Curve," *Journal of Money, Credit and Banking* (November 1995 Part I), pp. 975-84.

Rotemberg, Julio J. "Sticky Prices in the United States," *Journal of Political Economy* (December 1982), pp. 1187-211.

Rudebusch, Glenn D., and Lars E. O. Svensson. "Policy Rules for Inflation Targeting," in *Monetary Policy Rules*, John B. Taylor, ed., University of Chicago Press, 1999, pp. 203-46.

Svensson, Lars. "Price Level Targeting vs. Inflation Targeting: A Free Lunch?" *Journal of Money, Credit and Banking* (August 1999 Part I), pp. 277-95.

Taylor, John B. "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* (February 1980), pp.1-23.

Williams, John C. "Simple Rules for Monetary Policy," Finance and Economics Discussion Series 1999-12, February 1999.

Woodford, Michael. "Optimal Monetary Policy Inertia," NBER Working Paper 7261, July 1999.

Appendix

SOLUTION FOR THE CASE OF PRICE-LEVEL TARGETING

The central bank's constrained optimization problem with a price-level objective is given as the Lagrangian (equation 14 in the text):

$$E_t\left\{\sum_{i=0}^{\infty}\beta^t\left[\left(\lambda y_{t+i}^2+(\tilde{p}_{t+i})^2\right)-\mu_{t+i}\left(y_{t+i}-\rho y_{t-1+i}-\alpha(-\tilde{p}_{t-1+i}+2\tilde{p}_{t+i}-\tilde{p}_{t+1+i}^{e})-\varepsilon_{t+i}\right)\right]\right\}, \quad \text{(A1)}$$

where the μ_t's are a sequence of random multipliers. First-order conditions when taken with respect to the sequence of y_t's now take the form

$$2\lambda y_t-\mu_t+\beta\rho E_t\mu_{t+1}=0, \quad \text{(A2)}$$

and when taken with respect to the sequence of p_t's now take the form

$$2\tilde{p}_t+2\alpha\mu_t+\beta\alpha E_t\mu_{t+1}=0. \quad \text{(A3)}$$

We have found the simplest way to eliminate multipliers from the bank's first-order conditions is to regard both sequences above as linear systems in the unknown multipliers. The sequence of first-order conditions derived as a result of differentiating the Lagrangian with respect to the sequence of y_t's can be written as the linear system

$$\begin{pmatrix}-1 & \beta\rho & 0 & 0 & \cdots\\ 0 & -1 & \beta\rho & 0 & \cdots\\ 0 & 0 & -1 & \beta\rho & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix}\begin{pmatrix}\mu_t\\ E_t\mu_{t+1}\\ E_t\mu_{t+2}\\ \vdots\end{pmatrix}=\begin{pmatrix}-2\lambda y_t\\ -2\lambda E_t y_{t+1}\\ -2\lambda E_t y_{t+2}\\ \vdots\end{pmatrix}, \quad \text{(A4)}$$

while the other sequence of first-order conditions can be written as the linear system

$$\begin{pmatrix}2\alpha & -\beta\alpha & 0 & 0 & \cdots\\ 0 & 2\alpha & -\beta\alpha & 0 & \cdots\\ 0 & 0 & 2\alpha & -\beta\alpha & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix}\begin{pmatrix}\mu_t\\ E_t\mu_{t+1}\\ E_t\mu_{t+2}\\ \vdots\end{pmatrix}=\begin{pmatrix}-2\tilde{p}_t\\ -2E_t\tilde{p}_{t+1}\\ -2E_t\tilde{p}_{t+2}\\ \vdots\end{pmatrix}. \quad \text{(A5)}$$

These systems can be solved for the unknown multipliers by noting that

$$\begin{pmatrix}-1 & \beta\rho & 0 & 0 & \cdots\\ 0 & -1 & \beta\rho & 0 & \cdots\\ 0 & 0 & -1 & \beta\rho & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix}^{-1}=\begin{pmatrix}-1 & -\beta\rho & -(\beta\rho)^2 & -(\beta\rho)^3 & \cdots\\ 0 & -1 & -\beta\rho & -(\beta\rho)^2 & \cdots\\ 0 & 0 & -1 & -\beta\rho & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix} \quad \text{(A6)}$$

and

$$\begin{pmatrix}2\alpha & -\beta\alpha & 0 & 0 & \cdots\\ 0 & 2\alpha & -\beta\alpha & 0 & \cdots\\ 0 & 0 & 2\alpha & -\beta\alpha & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix}^{-1}=\frac{1}{2\alpha}\begin{pmatrix}1 & \frac{\beta}{2} & \left(\frac{\beta}{2}\right)^2 & \left(\frac{\beta}{2}\right)^3 & \cdots\\ 0 & 1 & \frac{\beta}{2} & \left(\frac{\beta}{2}\right)^2 & \cdots\\ 0 & 0 & 1 & \frac{\beta}{2} & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{pmatrix}. \quad \text{(A7)}$$

Thus, we can conclude from the first set of first-order conditions that

(A8) $$E_t\mu_{t+i} = 2\lambda \sum_{j=0}^{\infty} (\beta\rho)^j E_t y_{t+j+i},$$

for i = 0,...,∞, and we can conclude from the second set that

(A9) $$E_t\mu_{t+i} = -\frac{1}{\alpha} \sum_{j=0}^{\infty} \left(\frac{\beta}{2}\right)^j E_t\tilde{p}_{t+j+i},$$

for i = 0,...,∞. Equating the two expressions for the unknown multipliers gives the following sequence of first-order conditions expressed solely in terms of state variables:

(A10)
$$2\lambda \sum_{j=0}^{\infty} (\beta\rho)^j E_t y_{t+j+i} = -\frac{1}{\alpha} \sum_{j=0}^{\infty} \left(\frac{\beta}{2}\right)^j E_t\tilde{p}_{t+j+i}.$$

Calculating decision rules for the bank now proceeds in a similar manner to the calculation of decision rules for an inflation-targeting central bank. When decisions are made at time t, the bank's state variables are y_{t-1}, $\tilde{p}_{t-1}$, and ε_t. We assume linear decision rules of the form $B_1\tilde{p}_{t-1}+B_2y_{t-1}+B_3\varepsilon_t$ for y_t and $A_1\tilde{p}_{t-1}+A_2y_{t-1}+A_3\varepsilon_t$ for $\tilde{p}_t$. Using the rational expectations condition, $\tilde{p}^e_{t+1}=A_1\tilde{p}_t+A_2y_t$, and the Phillips Curve equation allows us to relate the coefficients of the decision rule for y_t to those in the rule for $\tilde{p}_t$. Iterating these decision rules forward and taking expectations allows us to write both $E_t\tilde{p}_{t+n}$ and E_ty_{t+n} as a linear function of y_{t-1}, $\tilde{p}_{t-1}$ and ε_t. In general, we have

(A11) $$E_t\tilde{p}_{t+n} = A_1^{(n)}\tilde{p}_{t-1} + A_2^{(n)}y_{t-1} + A_3^{(n)}\varepsilon_t, \text{ and}$$

(A12) $$E_t y_{t+n} = B_1^{(n)}\tilde{p}_{t-1} + B_2^{(n)}y_{t-1} + B_3^{(n)}\varepsilon_t,$$

with the coefficients $A_i^{(n)}$ and $B_i^{(n)}$ for i = 1, 2, 3, determined iteratively as:

(A13) $$\begin{pmatrix} A_i^{(n+1)} \\ B_i^{(n+1)} \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ B_1 & B_2 \end{pmatrix}\begin{pmatrix} A_i^{(n)} \\ B_i^{(n)} \end{pmatrix}.$$

Since this is a linear difference equation in $A_i^{(n)}$ and $B_i^{(n)}$, we explicitly solve it in the form

(A14) $$\begin{pmatrix} A_i^{(n)} \\ B_i^{(n)} \end{pmatrix} = \alpha_{1,i}\theta_1^n v_1 + \alpha_{2,i}\theta_2^n v_2,$$

where θ_1, θ_2, v_1, and v_2 are the eigenvalues and eigenvectors respectively of the matrix

(A15) $$\begin{pmatrix} A_1 & A_2 \\ B_1 & B_2 \end{pmatrix},$$

expressed as algebraic functions of the unknown decision rule parameters A_1, A_2, B_1, and B_2. With this representation we can substitute into the first-order condition, explicitly sum the resulting geometric series on the supposition that both θ_1 and θ_2 are less than 1 in absolute value, and finally equate coefficients on state variables. We have found the resulting equations for the coefficients in the decision rules to be too algebraically complex to admit a closed form solution. We have solved them numerically for a range of parameters, however. We found that both eigenvalues, θ_1 and θ_2, were real and inside the unit circle.