A Markov-Switching Model of Business Cycle Dynamics with a Post-Recession “Bounce-Back” Effect*

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Abstract: This paper presents a nonlinear model of U.S. GDP growth dynamics that allows for a post-recession “bounce-back” effect in the level of GDP. While a number of studies have attempted to capture such an effect using ad hoc recession-based dummy variable methods, we endogenously estimate this business cycle asymmetry using an extended version of Hamilton's (1989) Markov-switching model. Like Hamilton, we find model regimes that correspond closely to NBER-dated recession and expansions. We also find a large “bounce-back” effect that, according to our Monte Carlo analysis, is statistically significant and implies a relatively small permanent effect of recessions.

* We would like to thank Mrinalini Lhila for providing research assistance. Responsibility for all errors is our own. Morley acknowledges support from the Weidenbaum Center on the Economy, Government, and Public Policy. The views expressed in this paper should not be interpreted as those of the Weidenbaum Center, the Federal Reserve Bank of St. Louis, or the Federal Reserve System.
1. Introduction

In his seminal paper, Hamilton (1989) captures asymmetry in U.S. business cycles using an endogenous regime-switching model of real output. His model portrays the short, violent nature of recessions relative to expansions. However, other studies have emphasized another distinctive feature of U.S. business cycles that is not captured by Hamilton’s model: output growth tends to be relatively strong following recessions. This feature has traditionally been modeled in a somewhat ad hoc way by allowing growth dynamics to change in the quarters immediately after a decline in output below its historical maximum. In this paper, however, we show that Hamilton’s model can be extended in a very simple way to allow for a post-recession “bounce-back” effect while maintaining endogenous estimation of the underlying recessionary shocks. Our model provides a simple test of the “bounce-back” effect and produces a straightforward measure of the long-run effects of recessions on the level of output. We find that a post-recession “bounce-back” has been an important feature of U.S. business cycle dynamics and that the permanent effects of recessions are substantially less than suggested by Hamilton’s original model.

2. Background

The idea of inherently different dynamics in expansions and recessions has a long history in business cycle analysis, dating back at least to Mitchell (1927) and Keynes (1936). Recent advances in econometrics have allowed this idea to be formally modeled and tested. Hamilton (1989) captures asymmetric dynamics using a Markov-
switching model that estimates two regimes in U.S. GNP growth behaviour. Notably, even though the timing of the regimes is endogenously estimated, he finds that the regimes correspond closely to NBER-dated recessions and expansions.

While the statistical significance of the Markov-switching behaviour in output is clouded by nonstandard test conditions (see Hansen, 1992, and Garcia, 1998), one implication of Hamilton’s estimates is clear: recessions have large permanent effects on the level of output. By one measure discussed in his paper and employed here, the expected level of output is permanently lowered by as much as 4.5% as a result of a transition into recession. However, one reason this estimate may be so large is that Hamilton’s original model is unable to capture the high growth recovery phase typical of post-recession dynamics. We consider this possibility in this paper.

One approach to modeling the high growth recovery phase is to add a third regime to Hamilton’s model, as in Sichel (1994). However, there is much evidence that a recovery is not independent of the preceding recession, as would be implied by a three regime model, but rather the magnitude of the “bounce-back” is closely related to the severity of the recession (see Friedman, 1964, 1993, and Wynne and Balke, 1992, 1996). Kim and Nelson (1999a) allow for this type of business cycle asymmetry by modeling regime switching in the cyclical component of output only. While this relates the “bounce-back” to the severity of a recession, it constrains the effects of recessionary shocks to be completely transitory, a priori. Thus, we cannot use this approach to examine the permanent effects of recessions on the level of output. Kim and Murray (in press) combine the Hamilton (1989) and Kim and Nelson (1999a) approaches in a multivariate model with regime switching in both the trend and cyclical component of
output. While this approach is capable of providing a measure of the permanent effects of recessions, it comes at the price of considerable added complexity and the need for strong identification assumptions.

A related literature models the “bounce-back” effect using nonlinear ARMA processes in which dynamics change when an observed indicator variable exceeds a given threshold. In an important paper, Beaudry and Koop (1993) augment a standard ARMA model of output growth with a “current-depth-of-recession” dummy variable that measures the distance output has fallen below its historical maximum. They find that this additional variable is highly significant using a standard $t$-test and that typical recessions have no significant permanent effect on the level of GDP. However, Hess and Iwata (1997) argue that the dummy variable is nonstationary and the $t$-test overstates the significance of the “bounce-back” effect. The Beaudry and Koop model has been extended and modified by several authors, most notably Pesaran and Potter (1997) who endogenize the threshold.

Our approach in this paper is to directly augment Hamilton’s original model with a new term that is able to capture the length and severity of a recession. In this way, our model is like Beaudry and Koop’s (1993). However, unlike the “current-depth-of-recession” variable used in their paper, our “bounce-back” term is directly related to the underlying recessionary regimes and is, therefore, endogenously estimated. It is also stationary by construction and so does not suffer from the Hess and Iwata (1997) critique. Meanwhile, our model places no constraints \textit{a priori} on the permanent effects of a typical recession and, like Hamilton’s original model, yields a straightforward measure of this effect.
3. Model

Our extended version of Hamilton’s model, augmented to allow for a post-recession “bounce-back” effect, is given as follows:

\[
\phi(L) \left( \Delta y_t - \mu_0 - \mu_1 S_t - \lambda \sum_{j=1}^{m} S_{t-j} \right) = \epsilon_t, \quad \epsilon_t \sim i.i.d. N(0, \sigma^2),
\]

where the lag operator \( \phi(L) \) is \( p \)-th order with roots outside the unit circle, \( \Delta y_t \) is the first difference of log U.S. real GDP, and \( S_t \) is an unobserved Markov-switching state variable that takes on discrete values of 0 or 1 according to transition probabilities \( \Pr[S_t = 0 | S_{t-1} = 0] = q \) and \( \Pr[S_t = 1 | S_{t-1} = 1] = p \). We normalize the states by restricting \( \mu_i < 0 \). That is, \( S_t = 1 \) corresponds to a “lower growth” regime or, if \( \mu_0 + \mu_1 < 0 \), a “contractionary” regime.

The innovation in our model is the summation term, which for future convenience we denote \( \bar{S}_t(m) \equiv \sum_{j=1}^{m} S_{t-j} \). This term is the only addition to Hamilton’s model, as his model obtains if \( \lambda = 0 \). The term reflects the length and severity of the most recent “lower growth” or “contractionary” regime. In practice, we set \( m = 6 \), which is equal to the length of the longest postwar U.S. recessions (1973-75 and 1981-82).

In terms of our model, a “bounce-back” effect occurs if \( \lambda > 0 \). Figure 1 shows this effect by simulating stylized versions of our model and Hamilton’s original model.
For both models, we set the underlying growth rate parameters to be $\mu_0 = 1$ and $\mu_1 = -2$. For our model, we set the “bounce-back” coefficient to be $\lambda = 0.2$. For Hamilton’s model, $\lambda = 0$. We ignore the autoregressive parameters since for the simulation we assume that there are no “regular” shocks (i.e., $\varepsilon_t = 0$ for all $t$). In the bottom of the figure, the thick line represents a hypothetical time path for the state variable $S_t$. The shift in $S_t$ from 0 to 1 represents a movement of the economy into a “contractionary” regime for $l = 4$ quarters, denoted by the shading. As the regime hits in period 0 and persists until period 4, output falls both for our model and for Hamilton’s model.

Meanwhile, the summation term $\tilde{S}_t(m)$ increases up to the $\min\{m,l\}$, which is $l = 4$ in this case. The $\tilde{S}_t(m)$ term behaves in a similar fashion as the “current-depth-of-recession” variable in Beaudry and Koop (1993). However, again, it is not an ad hoc dummy variable, but is endogenously determined by the underlying states. For our model, the effect of the $\tilde{S}_t(m)$ term begins to offset the effect of the $S_t$ term as the recession persists, and output levels off. After $S_t$ returns to 0 and the economy moves back into expansion, the $\tilde{S}_t(m)$ term reaches its maximum, and the level of output rises dramatically due to $\lambda > 0$. This “bounce-back” in the level of output continues as the expansion persists, but its effect diminishes as the $\tilde{S}_t(m)$ term eventually falls back to its minimum of 0. By contrast, for Hamilton’s model with $\lambda = 0$, output rises from its trough at its regular “expansionary” growth rate only, implying a much larger permanent effect of the recession on the level of output.

Estimation of our model is a straightforward application of Hamilton’s (1989)
filter. The only new wrinkle is that, due to the $\tilde{S}_t(m)$ term, we need to keep track of $2^{p+m}$ states in each period, whereas Hamilton only needed $2^p$ states. See Hamilton (1989) for estimation details.

4. Estimates

The data for $y_t$ are 100 times the log of real U.S. GDP over the sample period of 1952:Q1 to 2001:Q2. Given a maximum lag order of $p = 4$, both the AIC and BIC pick $p = 1$. Table 1 reports model estimates for this case. The first important result to notice is that $\mu_0 + \mu_1 < 0$, implying that $S_t = 1$ corresponds to a “contractionary” regime. The transition probabilities also suggest that expansions are much more persistent than contractions, much like the NBER reference cycle.

Figure 2 reveals a strong correspondence between the smoothed probability of being in a contractionary regime and the NBER recession dates. This result is particularly notable since it has been widely reported that Hamilton’s original model does not capture NBER recession dates when applied to the longer data sample employed here (see, for example, Kim and Nelson, 1999b, and McConnell and Perez-Quiros, 2000). The figure also displays the smoothed estimate of $\tilde{S}_t(m)$. As with the previous figure, this term increases as the length of each contraction progresses, and declines soon after the recession is over. Again, this term and its coefficient $\lambda$ determine the size of the “bounce-back” effect. Our estimate of $\lambda$ is positive, corresponding to faster growth during post-recession recoveries. The $t$-statistic for $H_0 : \lambda = 0$ is 4.2, which is highly
significant using standard asymptotic critical values.

A possible concern is whether the standard asymptotic distribution applies in this case. Hess and Iwata (1997) argue that Beaudry and Koop’s (1993) “current-depth-of-recession” variable is nonstationary. Thus, the estimate for its coefficient has a nonstandard distribution. In our case, given finite $m$, the $\tilde{S}_t(m)$ term will be stationary since $S_t$ is stationary. However, given the persistence of the $\tilde{S}_t(m)$ term, the small sample distribution may be very different to the asymptotic distribution. To examine this possibility, we conduct a Monte Carlo experiment. For our data generating process, we use Hamilton’s (1989) original estimated model for which $\lambda = 0$. We estimate our model allowing $\lambda \neq 0$ for each simulation and calculate $t$-statistics for the null hypothesis $H_0 : \lambda = 0$. Table 2 reports critical values for our experiment. We consider sample sizes of $T=200$ and $T=500$. The critical values are larger than the standard normal case, reflecting a small-sample distortion. However, the distortion gets smaller as the sample size gets larger. Meanwhile, our estimate of $\lambda$ is still significant at the 5% level using the $T=200$ results.

Given a “bounce-back” effect, the question is whether recessions have permanent effects on the level of output. Hamilton (1989) provides a useful measure of the long-run effects of recessions in the context of a regime switching models. He considers the expected difference in the long-run level of output given a “contractionary” regime versus an “expansionary” regime in period $t$:

$$
\lim_{j \to +\infty} \{ E[y_{t+j} \mid S_t = 1, I_{t-1}] - E[y_{t+j} \mid S_t = 0, I_{t-1}] \},
$$
where \( I_{t-1} = \{y_{t-1}, y_{t-2}, \ldots; S_{t-1}, S_{t-2}, \ldots\} \). For our model, this limit converges to

\[ (\mu_t + m\lambda)/(2 - q - p), \]

which given the estimates in Table 1 is equal to −0.945, or about a 1% permanent drop in the level of GDP, and is not statistically significant. By contrast, Hamilton’s estimates imply a 4.5% permanent drop that is statistically significant. It should be noted that an alternative metric is also reported in Hamilton’s paper that conditions on \( I_t \) rather than \( I_{t-1} \). Instead of giving the dynamic multiplier for a shift in \( S_t \), this alternative metric calculates the forecastable consequences of a recession for future output. For Hamilton, this number is −3%. For our model, the number is actually positive and about 1%, corresponding to a large predicted “bounce-back”.

In addition to very different implications for the permanent effects of recessions, another notable difference between our results and Hamilton’s (1989) relates to the autoregressive dynamics propagating the “regular” \( \varepsilon_t \) shocks. Hamilton reports third and fourth order lags that are large and negative. By contrast, we find that higher order lags are small and insignificant. One possible explanation for this difference is that the negative serial correlation in Hamilton’s specification is better captured by the additional \( \tilde{S}_t(m) \) term than by linear autoregressive dynamics. Thus, our results imply very little serial correlation in output outside of recessions and their recoveries.

5. Conclusions

In summary, we find that postwar recessions have no significant permanent impact on U.S. real GDP. Instead, we find a significant and large “bounce-back” effect
during the recovery phase of the business cycle. Meanwhile, there appears to be little serial correlation in output growth during the regular expansion phase of the business cycle.

A virtue of our model is its simplicity. In particular, it is able to capture a defining feature of the business cycle with only a small modification to Hamilton’s original Markov-switching model of nonlinear dynamics. Again, the modification is the addition of a term that reflects the length and severity of the most recent recession. In this way, our model is reminiscent of Beaudry and Koop’s (1993) model, which also implies small permanent effects of recessions. However, it should be emphasized that our model is able to capture the “bounce-back” effect using an endogenously estimated state variable. Meanwhile, the simplicity of our model suggests that extensions, such as multivariate analysis to capture the co-movement feature of business cycles or allowing for time-varying transition probabilities, should be relatively easy to implement. We leave these extensions to future research.
References


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Tests and estimates of permanent and transitory components, Journal of Money, Credit and Banking 31, 317-34.


Table 1
Maximum Likelihood Estimates

<table>
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<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
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<td>$\mu_0$</td>
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<td>$\mu_1$</td>
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<td>$\lambda$</td>
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<tr>
<td>$q$</td>
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<td>$p$</td>
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<td>$\sigma$</td>
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<tr>
<td>$\mu_0 + \mu_1$</td>
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<td>0.232</td>
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<td>$(\mu_i + m\lambda)/(2 - q - p)$</td>
<td>-0.945</td>
<td>1.013</td>
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### Monte Carlo Results

<table>
<thead>
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<th>$p$-value</th>
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<tr>
<td>0.01</td>
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<td>0.10</td>
<td>2.45</td>
<td>1.85</td>
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Fig. 1
The “Bounce-Back” Effect (Simulated recession is shaded)
Fig. 2
Smoothed Inferences for $S_t$ and $\tilde{S}_t(m)$ (NBER recession dates are shaded)