Testing the Expectations Hypothesis: Some New Evidence

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Abstract

This paper applies a new test of the expectations hypothesis proposed by Bekaert and Hodrick (2000) to data used to test the expectations hypothesis in the influential paper by Campbell and Shiller (1991). We argue that Bekaert-Hodrick test is more powerful than conventional tests of the expectations hypothesis in that it is based on a specification than encompasses a broader range of data generating processes under the alternative hypothesis. The results suggest that the expectations hypothesis is easily rejected at the shorter-end of the maturity spectrum, where is commonly thought to be most valid. Moreover, the results using the Bekaert-Hodrick test conflict the conclusion reached by Campbell and Shiller using the most commonly used test of the expectations hypothesis.

JEL Classification: G1, E4, E10
Key Words: expectations hypothesis, lagrange multiplier test, vector autoregression

The views expressed here are the author’s and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System or the Federal Reserve Bank of St. Louis. We would like to thank Robert Hodrick for useful comments on an earlier draft of this paper.
1. Introduction

The expectations hypothesis (EH) of the term structure of interest rates is the proposition that the long-term rate is determined by the market’s expectation for the short-term rate plus a constant risk premium. The EH plays an important role in economics and finance, especially in analyses of monetary policy, where longer term rates are thought to be determined by the market’s expectation for the overnight federal funds rate which the Fed controls. It is not surprising, therefore that the EH has been thoroughly investigated using a variety of tests (e.g., Campbell and Shiller, 1991, Frankel and Froot, 1987 and Froot, 1989). In virtually every instance the EH is easily rejected.

Several of the tests of the EH are obtained by assuming that the EH is true and deriving an equation based on this assumption. The EH is then tested by testing whether the data obeys restrictions on the resulting equation required under the null hypothesis. While this approach is widely used (e.g., the Dickey-Fuller unit root test), the equations so derived typically admit a very limited number of alternative hypotheses. As a result, the evolution of such tests is to widen the array of admissible possibilities under then alternative. This has certainly happened in the unit root testing literature, with each successive variant admitting a wider array of potential data generating processes (e.g., Perron, 1989). Because of this, tests derived in this fashion may have relative low power in that the null hypothesis might not because it is true, but because the specification does not admit a wide enough array of alternatives. In particular, it may not admit the true data generating process.

An alternative test of the EH suggested by Hodrick and Bekaert (2000) attempts to mitigate this by specifying a model that admits a wider array of potential general data
generating processes. Specifically, a very general vector autoregression (VAR) for a long-term and short-term rate is specified and restrictions imposed by the EH are tested. It is well known that VARs encompass a wide range of potential data generating processes (e.g., Stock and Watson, 2001). Consequently, the unrestricted VAR is more likely to encompass the true data generating process.

The outline of the paper is as follows. In Section 2, we show that by the way the are derived how standard tests of the EH can yield unusual and, frequently, conflicting results. Section 3 then presents the test applied here. Section 4 present the results of this test applied to yields on a variety of assets ranging in maturity from one to 120 months.

2. Standard Tests of the Expectations Hypothesis

The EH asserts that the long-term rate is determined by the market’s expectation for the short-term rate over the holding period of the long-term assets plus a constant risk premium, i.e.,

\begin{equation}
    r^n_t = \left( \frac{1}{k} \right) \sum_{i=0}^{k-1} E_t r^m_{t+i} + \pi,
\end{equation}

where \( r^n_t \) is the \( n \)-period (long-term) rate, \( r^m_t \) is the \( m \)-period (short-term), \( E_t \) is the expectations operator and \( k = n/m \) is an integer.\(^1\)

Many of the tests of the EH are derived under the assumption that EH is true and that expectations are rational, i.e.,

\begin{equation}
    E_t r^m_{t+i} = r^m_{t+i} + \nu_{t+i}, \quad i = 0, 1, \ldots, k-1,
\end{equation}

\(^1\) Shiller, Campbell and Schoenholtz (1983) argue that Equation 1 is exact in some special cases and that it can be derived as a linear approximation to a number of nonlinear expectations theories of the term structure.
where $\nu_{t+m_i}$ is a mean-zero, iid white noise error. The most widely used of such tests, which we will call the conventional test, is obtained by substituting Equation 2 into Equation 1, which yields

$$r_t^n = (1/k)\sum_{i=0}^{k-1} r_{t+m_i} + (1/k)\sum_{i=0}^{k-1} \nu_{t+m_i} + \pi.$$  

The EH cannot be tested using Equation 3 because the interest rates are unit root, or perhaps more correctly, near unit root processes. Because of this, stationarity is achieved by subtracting a variable, $Z_t$, from both sides of the above equation that such that the resulting variables are stationary. Subtracting $Z_t$ from both sides of Equation 3 and rewriting yields

$$r_t^n = (1/k)\sum_{i=0}^{k-1} r_{t+m_i} - Z_t = -\pi + (r_t^n - Z_t) + \omega_t.$$  

The conventional test of the expectations theory is obtained by parameterizing the equation to yield

$$r_t^n = (1/k)\sum_{i=0}^{k-1} r_{t+m_i} - Z_t = \alpha + \beta (r_t^n - Z_t) + \sigma_t.$$  

If the EH is true the null hypothesis that $\beta = 1$ will not be rejected.

The ordinary least squares estimate of $\beta$ is

$$\hat{\beta} = \frac{\sum_{t=1}^{T} \left[ (1/k) \sum_{i=0}^{k-1} (r_{t+m_i} - \bar{r}_t)(r_t^n - \bar{r}_t) \right]}{\sum_{t=1}^{T} (r_t^n - \bar{r}_t)^2},$$

where the over bar indicates that the variable is adjusted for the mean. Note that if the null hypothesis is true, i.e.,

$$r_t^n = (1/k)\sum_{i=0}^{k-1} r_{t+m_i} + (1/k)\sum_{i=0}^{k-1} \nu_{t+m_i},$$

\footnote{Here, $\omega_t = (1/k)\sum_{i=0}^{k-1} \nu_{t+m_i}$.}
\( E\hat{\beta} = 1 \) independent of the choice of \( Z_t \).

A problem arises, however, when the EH is not true, in that the estimate of \( \beta \) need not equal zero. Indeed, it is easy to show that there are circumstances such that the test may not reject the null hypothesis that \( \beta = 1 \). The problem arises because the estimate of \( \beta \) depends on the variance of \( Z_t \) and the covariance between \( Z_t \) and \( r_t^m \) and \( r_t^n \). Consequently, the power of the test will be sensitive to the choice of \( Z_t \).

In the EH hypothesis testing literature, \( Z_t \) is typically taken to be \( r_t^m \). In this case, Thornton (2002) has shown that the estimate of \( \beta \) will be positive so long as the variance of the short-term rate is greater than that of the long-term rate. Moreover, circumstance can arise where the test is unable to reject the null hypothesis even though the EH is false.\(^3\)

2. An Alternative Test

Hodrick and Bekaert (2000) have suggested a test that mitigates the problem described above. Specifically, the EH is tested by estimating a VAR of the short-term and long-term interest rates and directly testing the restrictions implied by the EH. This test has the advantage that the VAR is very general and therefore encompasses a wide range of potential alternative data generating processes.

The restrictions implied by the EH are highly nonlinear. Consequently, Bekaert and Hodrick (2000) suggest using a Lagrange Multiplier (LM) test. Their Monte Carlo

\(^3\) Of course, it is also possible that estimates of \( \beta \) can be biased away from 1. For example, it is well known that the \( \beta \) can be biased down for variety of reasons (e.g., Campbell and Shiller, 1991).\(^3\) Indeed, the most common ‘explanation’ for the EH is rejected when this test is used is that there is a time varying risk premium that is positively correlated with the spread between the long-term and short-term rates. This explanation is frequently given despite the fact that it and others have received little empirical support (e.g.,
evidence suggested that while the Wald and LM both exhibited size distortions, the LM did considerably much better. For a 5 percent nominal test, the empirical size of the LM tests varied between 0.7 percent and 7.9 percent and in half of the cases, the empirical size was within 1 percentage point of the nominal size. Moreover, the LM test has reasonable power compared with the Wald test.

To fit a VAR to interest rate data that satisfies the restrictions imposed by the EH, we use a constrained GMM estimation described in Bekaert and Hodrick. A constrained VAR estimate in turn can be used in a Lagrange multiplier test of the EH. The VAR take the following form

$$ (I - \Theta(L)y_t = \eta_t $$

for $$ y_t = (r^m_t, r^n_t)' $$. GMM estimation imposes orthogonality conditions of the form

$$ g(z_t, \theta) \equiv \eta_t \otimes x_{t-1}, $$

where $$ x_{t-1} $$ is a vector formed from stacking lagged values of $$ y_t $$, possibly with a constant, $$ z_t $$ is defined as $$ (y_t, x_{t-1})' $$, and $$ \theta $$ is a vector formed from the parameters in $$ \Theta(L) $$. Using the sample moment condition

$$ g_T(\theta) \equiv \frac{1}{T} \sum_{t=1}^{T} g(z_t, \theta), $$

GMM estimation proceeds by choosing $$ \theta $$ to maximize the following objective:

$$ J_T(\theta) \equiv g_T(\theta)'Wg_T(\theta). $$

The optimal weighting matrix, $$ W $$, is a consistent estimator of the inverse of

$$ \Omega \equiv \sum_{k=-\infty}^{\infty} E\left[g(z_t, \theta)g(z_{t-k}, \theta)'ight]. $$

We can use GMM to estimate restricted VARs by forming a Lagrangian from the usual GMM quadratic objective and a vector of parameter constraints. The Lagrangian is defined

\[ L(\theta, \gamma) = -\frac{1}{2} g_T(\theta)' \Omega_T^{-1} g_T(\theta) - a_T(\theta)' \gamma \]

where \( \gamma \) is a vector of Lagrange multipliers, and the constraints on \( \theta \) have been represented by the vector-valued function, \( a_T(\theta) = 0 \). Here the matrix \( \Omega_T \) is again a consistent estimate of the matrix \( \Omega \) defined above. If we denote the Jacobians of \( g_T(\theta) \) and \( a_T(\theta) \) by \( G_T \) and \( A_T \) respectively, we can write the first order conditions for the maximizing \( \bar{\theta} \) and \( \bar{\gamma} \) as:

\[ \begin{bmatrix} -G_T' \Omega_T^{-1} \sqrt{T} g_T(\bar{\theta}) - A_T' \sqrt{T \bar{\gamma}} \\ -\sqrt{T} a_T(\bar{\theta}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

The asymptotic distribution of the constrained estimator can be derived from these first order conditions by expanding \( g_T(\theta) \) and \( a_T(\theta) \) in Taylor series around the true parameter value, \( \theta_0 \), and substituting these into the first order conditions above. This yields a system of the form:

\[ \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -G_T' \Omega_T^{-1} \sqrt{T} g_T(\theta_0) \\ 0 \end{bmatrix} - \begin{bmatrix} B_T & A_T' \end{bmatrix} \begin{bmatrix} \sqrt{T} (\bar{\theta} - \theta_0) \\ \sqrt{T \bar{\gamma}} \end{bmatrix} \]

for \( B_T \equiv G_T' \Omega_T^{-1} G_T \). Use of the partitioned inverse formula allows one to argue that the constrained estimator, \( \bar{\theta} \), is distributed as \( \sqrt{T} (\bar{\theta} - \theta_0) \rightarrow N(0, \Sigma_T) \) for

\[ \Sigma_T = B_T^{-1} - B_T^{-1} A_T' \left( A_T B_T^{-1} A_T' \right)^{-1} A_T B_T^{-1} \]

and the Lagrange multipliers are distributed asymptotically as
If the constraints have a significant impact on parameter estimation, then the estimated Lagrange multipliers should be significantly different from zero. The asymptotic distributions given above can be used to show that a test that the multipliers are jointly zero can be based on the fact that the statistic

\[(16) \sqrt{T} \tilde{\nu} \rightarrow N\left(0, \left( A_T B_T^{-1} A_T' \right)^{-1}\right).\]

which is asymptotically distributed as a $\chi^2(l)$ random variable, where $l$ is the number of restrictions imposed.

Maximization of the Lagrangian above is often computationally troublesome, so Taylor series approximations to $a_T(\theta)$ and $g_T(\theta)$ can again be used to derive a constrained estimate with similar asymptotic properties. Instead of expanding around the true value, $\theta_0$, we expand around a current estimate of the true value, $\theta_i$, and use it to form a better approximation, $\theta_{i+1}$. Since

\[g_T(\theta_{i+1}) \approx g_T(\theta_i) + G_T(\theta_{i+1} - \theta_i)\]

and

\[a_T(\theta_{i+1}) \approx a_T(\theta_i) + A_T(\theta_{i+1} - \theta_i),\]

we can substitute in the first-order conditions for maximization to derive the following iterative method

\[(18) \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -G_T' \Omega_T^{-1} \sqrt{T} g_T(\theta_i) \\ -\sqrt{T} a_T(\theta_i) \end{bmatrix} - \begin{bmatrix} B_T & A_T' \\ A_T & 0 \end{bmatrix} \begin{bmatrix} \sqrt{T}(\theta_{i+1} - \theta_i) \\ \sqrt{T} \gamma_{i+1} \end{bmatrix}.\]
In constructing our constrained estimates, we began with the unconstrained VAR parameters and iterated until the constraints were satisfied. The moment conditions for VAR estimation should be uncorrelated over time, so we estimated \( \Omega_T \) by

\[
(19) \quad \Omega_T = \frac{1}{T} \sum_{t=1}^{T} g(z_t, \theta_U) g(z_t, \theta_U)',
\]
evaluating the moment conditions at the unconstrained VAR parameters.

The constraints that the expectations hypothesis imposes on a VAR can be seen by writing the VAR in first-order form, that is

\[
\begin{pmatrix}
  r_t^m \\
  r_t^n \\
  r_{t-1}^m \\
  \vdots \\
  r_{t-k}^m \\
  r_{t-k}^n
\end{pmatrix} =
\begin{pmatrix}
  \theta_1 & \theta_2 & \cdots & \theta_r \\
  I & 0 & & 0 \\
  0 & I & & \vdots \\
  \vdots & \ddots & \ddots & \ddots \\
  0 & I & 0 & \vdots \\
  \end{pmatrix}
\begin{pmatrix}
  r_{t-1}^m \\
  r_{t-1}^n \\
  \vdots \\
  r_{t-k}^m \\
  r_{t-k}^n
\end{pmatrix} +
\begin{pmatrix}
  \eta_t \\
  0 \\
  \vdots \\
  0
\end{pmatrix}
\]

or simply \( x_t = \Theta x_{t-1} + \nu_t \). Note that \( E_t(x_{t+k}) = \Theta^k x_t \), so that \( E_t(r_{t+k}^m) = e_t' \Theta^k x_t \) for \( e_1 = (1,0,\ldots,0)' \). Note too that \( r_t^n = e_2' x_t \) for \( e_2 = (0,1,0,\ldots,0)' \). Consequently, for any two interest rates such that \( k = n/m \) is an integer, the expectations hypothesis implies that

\[
(20) \quad r_t^n = \frac{1}{k} \sum_{i=0}^{k-1} E_t(r_{t+i}^m),
\]

so that the EH can be expressed equivalently as

\[
(21) \quad e_2' x_t = \frac{1}{k} \sum_{i=0}^{k-1} e_1' \Theta^i x_t.
\]

The constraint that satisfy the expectations are given by

\[
(22) \quad a_T(\theta) = e_2' - \frac{1}{k} \sum_{i=0}^{k-1} e_1' \Theta^i = 0.
\]
No simple closed form exists for the Jacobian of these constraints. Consequently, they are calculated numerically for use in the iterative procedure described above.

4. The Results

The data used are an update of the data used by Campbell and Shiller (1991). Specifically, we use continuously compounded yields on riskless pure discount bonds. These yields were calculated by McCulloch and Kwon (1993). They are end-of-month observations for the period 1952.01 to 1991.02. The maturity of the bonds range from 1 to 120 months.

We begin by estimating a general VAR for each possible combination of long-term, \( n \), and short-term, \( m \), maturities such that \( k = n / m \) is an integer. To simplify the test procedure, the data were adjusted for their mean. Following Bekaert and Hodrick (2000), the order of the VAR is determined by minimizing the Schwarz Information Criteria. The Schwarz Criteria selected an order of either 1 or 2, but in most instances 2, so we present the results of both lag orders. There “s” behind the Lagrange multiplier statistic if the lag length is the one chosen by the Schwarz Criteria.

The results for VARs or order 1 and 2 are presented in Tables 1 and 2, respectively. When the null hypothesis is not rejected, the statistics are in bold type. In all but one instance when the order of the VAR was 1, the null hypothesis is easily rejected. But even in this instance the marginal significance level was only slightly above 5 percent.

The results are somewhat more encouraging for the EH when the lag order is 2. In this case, the null hypothesis is not rejected at the longer end of the term structure, but is easily rejected at the shorter-end of the term structure. The results appear to be
anomalous. For example, the null hypothesis is not rejected when the short-term rate is six months and the long-term rate is 60 or 120 months, but is rejected at a very low significance level when short-term rate is six month and the long-rate is 12 or 24 months. Similarly, the null hypothesis is rejected when the short-term rate is 12 and the long-term rate is 36 months or longer, but not when the long-term rate is 24 months.

One possible explanation for these anomalous results and, more generally, for the disparity between the results using one or two lags comes from noting that the difference in the Lagrange multiplier statistics using one or two lags are generally quite small. The number of restrictions being tested, however, doubles from two to four. Hence, the difference between not rejecting the null hypothesis, in the case of one lag, and rejecting the null, in the case of two lags, is largely due to the fact that more restrictions are being tested. This suggests that great care must be taken in determining the lag length.

Nevertheless, generally speaking the EH gets more support at the longer end of the term structure. Specifically, at the longer end of the term structure, there is more evidence that the rates are determined by the expectation for the rate on the next shortest-term asset. At the shorter end of the maturity spectrum, however, the EH is easily rejected. Hence, it appears that there is no validity in the EH at the shorter end of the maturity spectrum where it is frequently thought to play its most important role.

It is interesting to compare these results with those obtained using the conventional test, i.e., by estimating the equation

$$\frac{1}{k} \sum_{i=0}^{k-1} (r_{t+mi}^m - r_t^m) = \alpha + \beta (r_t^n - r_t^m) + \sigma_i.$$  

The results using Equation 23 are reported in Table 3. The table reports the estimates of $\beta$, the marginal significance level for the null hypothesis $\beta = 1$, and $\bar{R}^2$. The results are
quantitatively similar to those reported by Campbell and Shiller (1991, Table 2, page 504) over a slightly shorter sample period—1952.02 – 1987.02. Despite the fact that the null hypothesis that $\beta = 1$ is easily rejected, the estimates reveal the standard U-shape pattern, or what Roberds and Whiteman (1999) refer to as the “smile.” Specifically, the estimates of $\beta$ are larger for long-term rates with maturity of 3 months or less and again for those with maturity of two years or more. These results are commonly been interpreted as suggesting that the EH fair somewhat better at the short and long ends of the maturity spectrum but not over the intermediate range. The results using the Bekaert-Hodrick test do not support this interpretation. For a given short-term maturity, the test statistic tends to decline monotonically as the term of the long-term asset lengthens. It is also the case that the for a given long-term maturity, the test statistic tends to decline monotonically as the term of the short-term asset lengthens. Hence, if anything the Bekaert-Hodrick test suggests that the EH most likely to be valid at the longer end of the maturity spectrum.

5. Conclusions

In this paper we apply a new test of the expectations hypothesis proposed by Bekaert and Hodrick (2000) to data used to test the expectations hypothesis in the influential paper by Campbell and Shiller (1991). We argue that this test is more powerful than conventional tests of the expectations hypothesis in that it is based on a specification than encompasses a broader range of data generating processes under the

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4 Roberds and Whiteman (1999, p. 556) note that “Studies of the implications of the yield spreads for movements in short-term rates indicate that the is a ‘predictability smile’ in the term structure for post war U S data. That is, when the maturity of the long-term bond is three months or less, short rate generally move as predicted by the expectations hypothesis; for maturities between about three months and two years, short rates do not on average react to the long-short spreads; and for maturities beyond two years, the long-short spread again predicts future short rate movements.”
alternative hypothesis. The results suggest that the expectations hypothesis is easily rejected at the shorter-end of the maturity spectrum, where is commonly thought to be most valid. Indeed, the null hypothesis is easily rejected when the short-term rate is four months or shorter and the long-term rate is 48 months or shorter. The null hypothesis is generally not rejected when the short-term rate is 12 months or longer and the long-term rate is 36 months or longer, except in the case when the short-term rate is 60 months and the long-term rate is 120 months.

The results from this test are inconsistent with the impression that one gets from using one of the commonly used tests of the expectations hypothesis. While the expectations hypothesis is always rejected using the commonly used test, the quantitative results give the impression that the expectations hypothesis has more validity when long-term rates has a maturity of 3 months or less and again when the short-term rate is two years or longer. The results from the Bekaert-Hodrick test suggest that qualitatively, the expectations hypothesis is less valid the shorter the maturity of the short-term rate.
References


Table 1: Lagrange Multiplier Statistics and Significance Levels

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Entries of 0.0000 indicate a value less than 0.00005.
Table 3: Results from Estimating the Conventional Equation \((1/k) \sum_{i=0}^{k-1} r_{m_{i}} - r_{m} = \alpha + \beta(r_{m} - r_{m}) + \sigma_{i}\)

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Estimates of \(\beta\) appears in the first line of each cell, the Wald statistic on the second, the marginal significance level of the Wald statistic on the third and the estimate of \(\bar{R}^2\) on the fourth (in parentheses). Entries of 0.0000 indicate a value less than 0.00005.