Computers, Obsolescence, and Productivity

Karl Whelan
Division of Research and Statistics
Federal Reserve Board *

Preliminary Draft, Not for Quotation, September 1999

Abstract

This paper examines the effects on U.S. economic growth of the capital accumulation due to the explosion in investment in computing equipment, emphasizing the important role played by technological obsolescence; this occurs when a machine that is still productive is retired because it is no longer near the technological frontier. The paper shows that standard growth accounting methods are inappropriate in the presence of technological obsolescence and a new theoretical approach is developed. Obsolescence is shown to be important for computing equipment and an empirical growth accounting exercise is presented which suggests the contribution of computer capital accumulation to economic growth has been substantially larger than standard estimates. This effect, combined with the direct effect of improved productivity in the computer industry, explains almost all of the recent improvement in U.S. labor productivity.

*Mail Stop 80, 20th and C Streets NW, Washington DC 20551. Email: kwhelan@frb.gov. I wish to thank Steve Oliner for providing me with access to results from his computer depreciation studies. The views expressed in this paper are my own and do not necessarily reflect the views of the Board of Governors or the staff of the Federal Reserve System.

1 Introduction

The explosion in real business investment in computing equipment has been one of the most impressive and pervasive features of the current U.S. economic expansion. Fueled by rapid declines in quality-adjusted prices, real outlays on computing power have grown at an average rate of 41 percent a year since 1991. This boom in computer investment has accelerated over the past few years and has coincided with a pickup in the productivity performance of the U.S. economy: Private business output per hour grew 2.2 percent per year over the period 1996-98, a rate of advance in productivity not seen late into an expansion since the 1960s (see Chart 1). Not surprisingly, this confluence of events has raised the tantalizing prospect that the investment associated with the Information Technology revolution is leading to an acceleration in aggregate productivity, finally resolving the now-famous Solow Paradox that the impact of computers is seen everywhere except in the productivity statistics.

Economists, however, are a skeptical lot and business-page talk of a "New High-Tech Economy" tends to provoke a cool reaction. Perhaps most influential among the dissenting opinions that the high-tech investment boom could be underlying a productivity revival have been the growth accounting calculations of Steve Oliner and Dan Sichel (1994), updated in Sichel (1997, 1999). Oliner and Sichel argue it is unlikely that computer investment has had much impact on aggregate productivity because computers are a relatively small part of aggregate capital input. Their calculations suggest that prior to 1995, growth in the computer capital stock contributed only about 0.2 percentage points per year to output growth, and while Sichel (1999) suggests this contribution picked up over the period 1996-98 his figures still imply a relatively modest effect on productivity relative to the observed acceleration over this period. Dale Jorgenson and Kevin Stiroh (1999), using a slightly different framework, have come to qualitatively similar conclusions.

This paper presents a new approach to calculating the effect of computer capital accumulation on economic growth, that starts from the observation that one of the most obvious implications of the rapid decline in the price of computing power is *technological obsolescence*, which I define as occurring when a machine that is still productive is retired because it is no longer near the technological frontier.[1] I am sure that most people can relate to

[1] This is a stronger definition of obsolesence than is often used in the capital stock literature, where obsolescence is usually defined as the loss in value of a capital good due to the introduction of new and

this phenomenon, since many of us have owned Intel 386 or 486-based PCs that were made obsolete by the availability at cheap prices of much more powerful Pentium-based machines. One obvious implication of technological obsolescence is that there is no direct relationship between the value of a machine and its productive capacity: Even though my old 486 machine is still as productive as it was the day I bought it a few years ago for $1500, it is now essentially worthless. While this insight may be fairly obvious, I show that it has important implications for growth accounting exercises, which attempt to divide output growth into growth in inputs and growth in total factor productivity (TFP).

The traditional approach to growth accounting has relied extensively on exploiting the relationship between the value of an asset and its productive capacity. In particular, the standard approach relies on estimates of *economic* depreciation - the observable age-related decline in value of a machine - to construct the capital stocks for productivity analysis. When the technology embodied in machines improves over time, then this economic depreciation rate combines both the rate at which the asset loses productive capacity (which we will term the physical decay rate) and the decline in value due to the introduction of new and improved assets. The traditional approach to dealing with embodied technological progress is to use the vintage capital model of Solow (1959) which predicts that the rate of economic depreciation is simply the sum of the rate of physical decay and the rate of embodied technological progress. Given estimates of the rate of embodied technological progress, this approach uses the resulting estimates of the rate of physical decay to construct "quality-adjusted" capital stocks.

The theoretical model in this paper describes the process of economic growth when there is both disembodied and embodied technological change, and those assets that are subject to embodied technological change experience technological obsolescence. I show that when technological obsolescence is important for an asset the traditional approach fails to capture either the magnitude of the productive capital stock for the asset or the effect of this stock on aggregate capital input. In particular, the traditional approach overestimates the rate of physical decay and correspondingly underestimates the level of the productive capital stock. Applying this method to the problem of accounting for the effect of computers on growth, I present estimates that suggest the standard growth accounting methods have substantially underestimated the contribution of computer capital accumulation to economic growth. In

superior goods. Of course, as the theoretical model introduced in this paper will make clear, these two concepts are related.

fact, I show that the combined effects of the step up in computer capital accumulation and the direct contribution of increased productivity in the computer-producing sector account for almost all of the improvement in productivity over the 1996-98 period relative to the previous 20 years.

The paper relates to a number of existing areas of macroeconomic research. Firstly, and most obviously, the paper contributes to the debate on the effect of computer investment on productivity. While the paper's distinction between value-based and productive stocks of computing equipment is not a new one - Oliner's (1994) study of computer peripherals presented separate estimates for wealth and productive stocks while Jorgenson and Stiroh (1995) used similar capital stock estimates to those in this paper - its consistent theoretical and empirical integration of technological obsolescence into the standard approach and the substance of the results are both new. Secondly, in suggesting an alternative empirical growth accounting methodology, the paper begins with a review of the existing methodology for growth accounting in the presence of embodied technological change as developed by Jorgenson and Grilliches (1967). By placing a particular emphasis on how empirical growth accounting exercises have used information on observable prices and quantities to translate theoretical concepts into quantitative estimates, this review may help to bridge the often-wide gap between growth theorists and practitioners of empirical growth accounting. Thirdly, the paper's focus on the retirement of capital goods as an endogenous decision and the argument that explicit modelling of this decision can improve our understanding of the evolution of productivity, echoes the conclusions of Feldstein and Rothschild (1974), and, more recently, the contributions of Caballero and Hammour (1996) and Goolsbee (1998). Finally, the paper helps to shed new light on the recent productivity performance of the U.S. economy, a topic also explored by Gordon (1999).

The contents of the paper are as follows. Section 2 reviews the standard growth accounting methodology in the absence of embodied technological change. Section 3 shows how this methodology can be extended, theoretically and empirically, to account for embodied technological change. Section 4 examines the evidence on computer depreciation presented by Oliner (1989, 1994) and argues that this evidence is consistent with an important role for technological obsolescence, which has been omitted from the standard growth accounting methodology. Section 5 develops an alternative theoretical approach to growth accounting that allows for technological obsolescence. Section 6 presents an empirical implementation of this new approach and compares it with results from a standard growth accounting exercise as well as the results from previous studies. Section 7 looks beyond the effect of computer capital accumulation on growth to the broader question of the total impact of the computer sector on output growth. In particular, I discuss the implications for growth accounting of the U.S. National Income and Product Accounts' quality adjustment of computer output, a topic recently explored by Greenwood, Hercowitz, and Krusell (1997). Section 8 discusses the recent productivity performance of the U.S. economy in the light of the paper's empirical results. Section 9 concludes.

2 Traditional Growth Accounting

Solow (1957) started from a general production function:

$$Q(t) = F(X_1, X_2, ..., X_n, t) \tag{1}$$

Taking derivatives with respect to time we get

$$\dot{Q}(t) = \sum_{i=1}^{n} \frac{\partial F(X_1, X_2, ..., X_n, t)}{\partial X_i} \dot{X_i}(t) + \frac{\partial F(X_1, X_2, ..., X_n, t)}{\partial t} \tag{2}$$

Defining the contribution of technological progress to output growth as that proportion of the change in output that cannot be attributed to increased inputs

$$\frac{\dot{A}(t)}{A(t)} = \frac{1}{Q(t)} \frac{\partial F(X_1, X_2, ..., X_n, t)}{\partial t} \tag{3}$$

we get

$$\frac{\dot{Q}(t)}{Q(t)} = \frac{\dot{A}(t)}{A(t)} + \sum_{i=1}^{n} \frac{X_i(t)}{Q(t)} \frac{\partial F(X_1, X_2, ..., X_n, t)}{\partial X_i} \frac{\dot{X_i}(t)}{X_i(t)} \tag{4}$$

Thus, the contribution to growth of technological progress, known also as Total Factor Productivity, is calculated by subtracting a weighted average of growth in inputs from output growth, where each input's weight (or factor share) is determined by the quantity of the input used times the marginal productivity of the input.

For simplicity, consider a simplified version of this general approach in which output is a Cobb-Douglas function of labor input and the stocks of two types of capital:

$$Q(t) = A(t) L(t)^{\alpha(t)} K_1(t)^{\beta(t)} K_2(t)^{1-\alpha(t)-\beta(t)} \tag{5}$$

In this case, the factor shares are constant and the growth rate of output can be expressed as

$$\frac{\dot{Q(t)}}{Q\left(t\right)}=\frac{\dot{A(t)}}{A\left(t\right)}+\alpha\left(t\right)\frac{\dot{L(t)}}{L\left(t\right)}+\beta(t)\frac{\dot{K_1(t)}}{K_1\left(t\right)}+\left(1-\alpha\left(t\right)-\beta(t)\right)\frac{\dot{K_2(t)}}{K_2\left(t\right)} \tag{6}$$

Empirical implementation of this equation is complicated by the fact that, in reality, we do not observe the factor shares $(\alpha\left(t\right), \beta(t))$. Similarly, we do not directly observe the capital stocks; rather we observe the time series of investment flows for different types of capital and must impute the rate at which each vintage of capital loses productive capacity. The solution to these challenges, pioneered by Solow (1957) and extensively developed by Jorgenson and Grilliches (1967), has been to use data on factor prices to impute values for factor shares and capital stocks. What I wish to stress here is that these imputations require theoretical assumptions and if we use an inappropriate theoretical approach then our calculated factor shares and capital stocks will be incorrect.

Consider the simple case in which there is no embodied technological change and capital goods of type i physically decay at a geometric rate δ_i. The price at time t of a unit of capital of type i produced in period v is $p_v^i(t)$ and there is an efficient rental market for new and used capital goods, so that a productive unit of capital of type i is available for rent at rate $r_i(t)$ where this equals its marginal productivity.

Calculation of Capital Stocks

This calculation has relied on a theoretical relationship between the value of assets and their productive capacity. No-arbitrage in the capital rental market requires that the present value of the stream of rental payments for a capital good should equal the purchase price of the good. This implies that

$$p_{t-v}^i(t)=\int_t^{\infty} r_i(s)e^{-\delta_i(t-v)}e^{-(r+\delta_i)(s-t)}ds=e^{-\delta_i(t-v)}p_t^i(t) \tag{7}$$

where r is the required net rate of return on capital investments, determined by financial market conditions and risk premia. Note that this net rate of return is assumed to constant across all types of capital investment.[2]

So, by observing a cross-section of prices of similar capital goods of different age, we can estimate the so-called *economic* depreciation rate, which is the rate of at which an asset

[2] Throughout this paper I will take r as fixed and exogenous: It could be endogenized by adding consumers and specifying preferences, but this would merely distract from the principal points of the paper.

loses value due to aging. Equation 7 tells us that the rate of economic depreciation equals the rate of physical decay but this identity relies on the assumption of geometric decay. However, empirical studies based on used asset prices such as Hulten and Wykoff (1981a,b) have shown economic depreciation rates to be approximately geometric for most assets, and so these depreciation rates have been widely used to construct productive capital stocks.[3]

Calculation of Factor Shares ($\alpha(t)$ and $\beta(t)$)

Equation 4 tells us that the factor share for an input is determined by the amount of the input used in production times its marginal productivity. Solow's insight was that if the production function displays constant returns to scale with respect to inputs and factors are being paid their marginal products then the growth accounting weight for a factor will equal its share of aggregate income. Solow's original growth accounting exercise used a two-factor (capital and labor) approach, so the use of the labor share of income, an observable parameter, to proxy $\alpha(t)$ was sufficient for him to calculate his TFP series. However, Jorgenson and Grilliches (1967) argued convincingly that the assumption that all units of capital have the same marginal productivity is a poor one; for instance, one would not expect the marginal productivity of a long-lived asset such as an office building to be the same as a short-lived piece of equipment.

To derive factor shares for each asset, Jorgenson and Grilliches proposed exploiting the capital good pricing arbitrage conditions to derive the marginal productivity of each type of capital. Differentiating the equation for the price of new capital goods with respect to time and re-arranging gives the familiar Hall-Jorgenson rental rate formula:

$$r_i(t) = p_t^i(t)\left(r + \delta_i - \frac{\dot{p}_t^i(t)}{p_t^i(t)}\right) \tag{8}$$

In equilibrium, total capital income should equal the Hall-Jorgenson rental rate for each type of capital multiplied by the stock of such capital. Since capital income is observable we can complete our calculation of the rental rate by solving for the value of r that is consistent with the equilibrium relationship.[4] Armed with estimates of capital-specific rental rates,

[3] See Jorgenson (1973) for the general theory on the relationship between economic depreciation and physical decay. Hulten and Wykoff (1996) and Triplett (1996) are two other papers that articulately explain the distinctions between physical decay and economic depreciation and thus between the wealth and productive concepts of the capital stock.

[4] In principle we could solve for a separate r for each period; in practice, r is often calibrated to make

we can estimate the factor share $\beta(t)$ by allocating capital's share of income to each type of capital according to relative size of $r_i(t)K_i(t)$:

$$\beta(t) = \frac{(1-\alpha(t))\, r_1(t)K_1(t)}{r_1(t)K_1(t) + r_2(t)K_2(t)} \tag{9}$$

A major issue usually neglected by growth accounting, and by this paper, is utilization. Growth accounting uses measures of capital stocks to construct the growth rate of capital input. However, that the stock of capital does not decline during recessions does not mean that utilized capital does not decline. Cyclical utilization is probably the principal reason that the TFP measures produced by standard growth accounting methods are procyclical.[5] For this reason, these calculations are best averaged over a number of years to smooth out cyclical effects.

3 Embodied Technological Change

That the model just described is probably a poor representation of reality was recognized by Solow as early as 1959. Solow acknowledged that an important part of the process of technological progress was the replacement of old capital goods by new capital that embodied the latest improvement in technology. In this section I present a slightly altered version of Solow's model of embodied technology. I have added a couple of additional features, such as disembodied technological change and multiple types of capital to help shed light on some issues in empirical growth accounting, but the logic of the model is from Solow.

Technology

There are two types of capital good, one of which, computers, features embodied technological change and another ("ordinary capital") which does not. Aggregate labor input is fixed and equal to one ($L(t) = 1$). Computers physically decay at rate δ while the technology embodied in new computers improves each period at rate γ, meaning that associated with each vintage of computers is a production function of the form

$$Q_v(t) = A(t)\, L_v(t)^{\alpha(t)}\, K_v(t)^{\beta(t)} \left(I(v)e^{\gamma v}e^{-\delta(t-v)}\right)^{1-\alpha(t)-\beta(t)} \tag{10}$$

"required capital income" equal realized capital income on average. This will be the approach followed in the empirical calculations in this paper

[5]See Basu (1996), Fernald and Basu (1999).

where $I(v)$ is investment in computers at time v, $L_v(t)$ and $K_v(t)$ are the quantities of labor and other capital that work with computers of vintage v at time t, and $A(t)$ is a term representing disembodied technological change. Technology is of the putty-putty form implying that *ex post* factor proportions are flexible, an assumption that I will return to in Section 5.[6]

Prices

All valuations are relative to the price of output, which is assumed to be constant and equal to one.[7] All relative prices equal one at time $t = 0$; thereafter, the price of computers (without adjusting for the value of embodied features) grows at rate g $(< \gamma)$ while the price of ordinary capital does not change. Finally, labor and ordinary capital are obtained from spot markets with all labor being paid the same wage $w(t)$ and ordinary capital being rented at a price of $r_o(t)$.

3.1 What Happens to a Vintage Over Time?

The flow of profits obtained from operating computers of vintage v is

$$\begin{aligned} \pi_v(t) &= A(t) L_v(t)^{\alpha(t)} K_v(t)^{\beta(t)} \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right)^{1-\alpha(t)-\beta(t)} \\ &\quad - r_o(t) K_v(t) - w(t) L_v(t) \end{aligned} \tag{11}$$

Firms choose how much labor and ordinary capital should work with vintage v so as to maximize the profits generated by the vintage. Re-arranging the first-order conditions for profit maximization, we get the optimal allocation of labor and ordinary capital to vintage v at time t is

$$L_v(t) = \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right) A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{1-\beta(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r_o(t)} \right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} \tag{12}$$

$$K_v(t) = \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right) A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r_o(t)} \right)^{\frac{1-\alpha(t)}{1-\alpha(t)-\beta(t)}} \tag{13}$$

So output from vintage v is

$$Q_v(t) = \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right) A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r_o(t)} \right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} \tag{14}$$

[6] Caballero and Hammour (1996), Gilchrist and Williams (1998), and Struckmeyer (1986) present similar vintage capital models that incorporate the putty-clay assumption of fixed *ex post* factor proportions.

[7] Output price inflation could be introduced to the model without adding any extra insight.

Equation 14 tells us there are three factors that determine what happens to the output produced by a particular vintage of computers over time. First, is physical decay which leads to less output from the vintage over time. Second, disembodied technological progress leads to increased productivity from what capital remains. Third is factor prices, wages and the rental rate for ordinary capital. As wages rise over time, it becomes less profitable to allocate labor to work with the vintage. We now show how together these forces dictate the path for the rental rate, price, and output produced over time by each vintage of computer capital.

The key to what happens to a vintage over time comes observing that at any point in time new vintages of capital rent for higher rates than older vintages and that once we know the rental rate for a new unit of capital we can calculate rates for all existing vintages. This initial rental rate is calculated using the standard methodology. Denoting the rental rate for a unit of capital of vintage v at time t as $r_v(t) = \frac{\partial Q_v(t)}{\partial\left(I(v)e^{-\delta(t-v)}\right)}$, and using the capital good arbitrage pricing formula, we have that at purchase time, v, the price of a new unit of capital is

$$p_v(v) = \int_v^\infty r_v(s)e^{-(r+\delta)(s-v)}ds \tag{15}$$

Differentiating this expression with respect to v we get

$$\dot{p}_v(v) = (r+\delta)p_v(v) + \int_v^\infty e^{-(r+\delta)(s-v)}\frac{dr_v(s)}{dv}ds - r_v(v) \tag{16}$$

Using the cross-sectional property of rental rates ($\frac{d}{dv}r_v(s) = \gamma r_v(s)$), we have

$$\dot{p}_v(v) = (r+\delta+\gamma)p_v - r_v(v) \tag{17}$$

which re-arranges to give

$$r_v(v) = p_v\left(v\right)\left(r+\delta+\gamma - \frac{\dot{p}_v(v)}{p_v\left(v\right)}\right) \tag{18}$$

Since computer prices change at rate g, this expression simplifies to:

$$r_t(t) = \left(r+\delta+\gamma-g\right)e^{gt} \tag{19}$$

Applying the same logic to ordinary capital gives

$$r_o\left(t\right) = r+\delta \tag{20}$$

Compared with the no-embodiment case (equation 8), equation 18 tells us that the introduction of embodied technological progress implies a higher initial marginal productivity of capital than when there is no embodied progress. However, note that the net present value rate of return on all capital investments must still equal r. This tells us that this higher initial rental rate must come at the expense of lower rental rates as the vintage ages. Combining the information that the rental rate for new vintages changes at rate g each period, and that, at any point in time, rental rates decline at rate γ with age tell us that

$$\frac{\dot{r_v(t)}}{r_v\,(t)} = g - \gamma \tag{21}$$

These calculations imply that the output of a vintage will decline faster than the rate at which the vintage decays physically. Interestingly, this result is obtained no matter how fast is the rate of disembodied technological progress.

Similarly, the price of a unit of vintage v capital also changes over time at rate $g-\gamma-\delta$ and, in a cross-section, declines with age at rate $\gamma+\delta$:

$$p_v(t) = \int_t^\infty r_v(s)e^{-(r+\delta)(s-t)}ds = e^{gt}e^{-(\gamma+\delta)(t-v)} \tag{22}$$

Thus, with embodied technological change, the rate of economic depreciation is $\gamma + \delta$: Computers decline in price as they age not only because of physical decay but also because the introduction of new and improved computers raises wages and reduces the profitability of existing vintages.

3.2 Aggregate Economic Growth

Define the aggregate stock of computing equipment as

$$C\,(t) = \int_{-\infty}^t I(v)e^{\gamma v}e^{-\delta(t-v)}dv \tag{23}$$

Aggregating over vintages, we get

$$L\,(t) = \int_{-\infty}^t L_v\,(t)\,dv = A\,(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha\,(t)}{w\,(t)}\right)^{\frac{1-\beta(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta\,(t)}{r_o\,(t)}\right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} C\,(t) \tag{24}$$

$$K\,(t) = \int_{-\infty}^t K_v\,(t)\,dv = A\,(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha\,(t)}{w\,(t)}\right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta\,(t)}{r_o\,(t)}\right)^{\frac{1-\alpha(t)}{1-\alpha(t)-\beta(t)}} C\,(t) \tag{25}$$

$$Q\,(t) = A\,(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha\,(t)}{w\,(t)}\right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta\,(t)}{r_o\,(t)}\right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} C\,(t) \tag{26}$$

Re-arranging this expression we get:

$$Q(t) = A(t) L(t)^{\alpha(t)} K(t)^{\beta(t)} C(t)^{1-\alpha(t)-\beta(t)} \tag{27}$$

This is an important result: The aggregate economy can be modelled using a similar Cobb-Douglas production function to that associated with each vintage, replacing the vintage-specific computer capital with an aggregate "quality-adjusted" stock of computer capital that adds up units of quality-adjusted capital $I_v e^{\gamma v}$ using the depreciation schedule given by the pattern of physical decay, $e^{-\delta(t-v)}$. Once we use this quality-adjusted stock of computing equipment, we can apply the standard aggregate Solow growth accounting equation:

$$\frac{\dot{Q}(t)}{Q(t)} = \frac{\dot{A}(t)}{A(t)} + \alpha(t)\frac{\dot{L}(t)}{L(t)} + \beta(t)\frac{\dot{K}(t)}{K(t)} + (1-\alpha(t)-\beta(t))\frac{\dot{C}(t)}{C(t)} \tag{28}$$

Beyond growth accounting, we can use the relationship between the wage rate and the marginal productivity of computer capital to show that the growth rate of the quality-adjusted stock of computer capital is

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\alpha(t)}\frac{\dot{A}(t)}{A(t)} + \frac{1-\beta(t)}{\alpha(t)}(\gamma - g) \tag{29}$$

and that aggregate output grows according to

$$\frac{\dot{Q}(t)}{Q(t)} = \frac{1}{\alpha(t)}\frac{\dot{A}(t)}{A(t)} + \left(\frac{1-\alpha(t)-\beta(t)}{\alpha(t)}\right)(\gamma - g) \tag{30}$$

Ultimately, all growth is generated by either embodied or disembodied technological progress.

3.3 Empirical Growth Accounting with Embodied Technological Change

Equation 28 describes the growth rate of output in terms of growth in quality-adjusted inputs and improvements in disembodied technology. How do we calculate the contribution to aggregate growth of computer capital accumulation, the $(1-\alpha(t)-\beta(t))\frac{\dot{C}(t)}{C(t)}$ term? Again, empirical implementation of this equation has two components: Calculation of capital stocks and factor shares. Now, though, the rate of economic depreciation for computers is not δ but rather $\gamma + \delta$. Since we need estimates of γ to construct the "quality-adjusted" units of capital and estimates of δ to appropriately weight the past values of these adjusted units, this implies that in addition to the rate of economic depreciation we need to have independent estimates of either γ or δ.

The solution to this problem, proposed by Hall (1971), has been to again use a vintage asset price approach, this time to assess the rate of embodied technological change by valuing identifiable aspects of the technology embodied in a machine. In the case of computers, we can observe a vector, X_v, of characteristics associated with a vintage, such as memory, processor speed, and disk space. Suppose that together these characteristics describe the technology embodied in a machine, so that $e^{\gamma v} = X_v^{\theta}$. In this case, by inserting these observable variables into the cross-sectional asset price regression (based on equation 22), we change the regression from

$$\log\left(p_v(t)\right) = gt - (\gamma + \delta)\,(t - v) \tag{31}$$

to

$$\log\left(p_v(t)\right) = (g - \gamma)\,t - \theta \log\left(X_v\right) - \delta\,(t - v) \tag{32}$$

This so-called hedonic regression is used to estimate the value of the observable features of the computer, the θ vector, and once these features are controlled for, the coefficient on the age of the machine equals δ. To contrast the depreciation rate estimated from equation 32 with the economic depreciation rate estimated from the Hulten-Wykoff regression, equation 31, Oliner (1989) has labeled this a "partial depreciation rate". Thus, the hedonic vintage price approach produces two useful pieces of information. First, it allows us to estimate γ and so define a quality-adjusted vintage price index:

$$\log\left(q_v(t)\right) = \log\left(p_v(t)\right) - \theta \log\left(X_v\right) = (g - \gamma)\,t - \delta\,(t - v) \tag{33}$$

to go along with the quality-adjusted quantity, $I_v e^{\gamma v}$. Second, it allows us to estimate the rate of physical decay, δ.

Consider now the calculation of factor shares for capital. Since our vintage production functions display constant returns to scale with respect to inputs, we can still calculate these shares by estimating the share of total capital income due to each type of capital. Thus, we can estimate that

$$\beta(t) = \frac{(1 - \alpha(t))\, r_o\,(t)\, K\,(t)}{\int_{-\infty}^{t} r_v\,(t)\, I(v) e^{-\delta(t-v)} dv + r_o\,(t)\, K\,(t)} \tag{34}$$

This calculation can be simplified by noting that vintage rental rates decline at rate γ as age rises, and so does the quality adjustment for computers. Thus, a cross-sectionally variable rental rate on the non-quality-adjusted computers is consistent with a constant rental rate across all units of quality-adjusted computer capital at any point in time. Algebraically, this insight can be expressed as:

$$\int_{-\infty}^{t} r_v(t)\, I(v) e^{-\delta(t-v)} dv = r_t(t) \int_{-\infty}^{t} I(v) e^{-\gamma(t-v)} e^{-\delta(t-v)} dv = r_t(t)\, e^{-\gamma t} C(t) \tag{35}$$

Inserting the rental rate for new computers, we get

$$\int_{-\infty}^{t} r_v(t)\, I(v) e^{-\delta(t-v)} dv = (r + \delta + \gamma - g)\, e^{(g-\gamma)t} C(t) = q_t(t) \left(r + \delta - \frac{\dot{q_t(t)}}{q_t(t)} \right) C(t) \tag{36}$$

This is a very important and elegant result: If we use quality-adjusted prices, quantities, and depreciation rates, we can simply apply the standard growth accounting formulas and effectively ignore the maze of issues associated with rental rates and embodied technology. This, of course, should be a great relief to empirical practitioners. Unfortunately, though, I will show that the evidence on economic depreciation for computers is not consistent with this model and that an understanding of the details of vintage capital models is crucial for a correct implementation of growth accounting.

4 Evidence on Computer Depreciation

The principal source of evidence that has been used to construct quality-adjusted stocks of computing equipment has been the detailed studies of Steve Oliner (1989, 1994). These studies employed the hedonic vintage price approach described in equation 32, augmenting this basic equation with additional terms that allowed the decline in value due to age to take a non-geometric form and also to vary over time. Since 1997, Oliner's findings have formed the basis for the National Income and Product Accounts (NIPA) estimates of the capital stock published by the Bureau of Economic Analysis (BEA).

Oliner's studies concentrated separately on four categories of computing and computer peripheral equipment: Mainframes, storage devices, printers, and terminals. Chart 2 shows the depreciation schedules for these four categories that the BEA have used to construct their estimates of the quality-adjusted stock of computing equipment.[8] Chart 3 shows depreciation rates implied by these schedules. These so-called "partial" depreciation rates

[8]The depreciation schedules used to construct the stock of computing equipment are available on a CD-ROM that can be ordered from BEA at http://www.bea.doc.gov/bea/uguide.htm. I have made one simple change to these schedules for charting purposes. BEA's stocks are at an annual frequency and measure the

correspond to the δ term in equation 32 and so, if the Solow-style vintage capital model just presented was correct, then they represent the rate of physical decay. However, these estimates seem unlikely to be capturing physical decay for computers. I will note three anomalies that seem inconsistent with a physical decay interpretation, in ascending order of importance.

First, in contrast to the Hulten-Wykoff results, Oliner's schedules show a marked non-geometric pattern, with the exception of printers which display only a weak non-geometric shape. Thus, these schedules seem inconsistent with conventional wisdom on the shape of physical decay. Of course one potential explanation is that physical decay for computers is simply different than for other types of equipment and takes a non-geometric pattern. At this point, I am merely noting this non-geometric shape as a pattern to be explained.

Second, each of the depreciation schedules has shifted down over time. This seems inconsistent with a physical decay interpretation since one would expect that, if anything, computing equipment has probably become more reliable over time, not less.

The third problem with the physical decay interpretation of these depreciation rates is that they appear to be too high. Table 1 shows disaggregated NIPA estimates of 1997 depreciation rates. Remarkably, these supposed physical decay rates for the four categories studied by Oliner (I will discuss the NIPA treatment of personal computers in Section 6) are higher than the depreciation rates for all other categories of equipment except motor vehicles. This pattern is all the more remarkable when one considers that for all other types of equipment, the NIPA depreciation rates are not based on a quality-adjusted partial depreciation approach and hence they combine both physical decay rates and the rate of embodied technological change. Casual observation suggests it is very unlikely that physical decay rates for computers are so much higher than for other types of equipment. An even more serious argument against a physical decay interpretation of these data relates to the nature of the computing equipment used in Oliner's studies. These studies focused on IBM equipment which was sold with pre-packaged service maintenance contracts: IBM guaranteed to repair or replace any damage due to equipment due to wear and tear. Thus, for the types of equipment in these studies, the effective physical decay rates *should have*

---

year-end stock assuming that some of the investment installed during the year has depreciated by year-end. Thus, they do not place a weight of 1 on this year's investment when constructing the stock. Because of this, I have re-scaled the depreciation schedules.

Computing Equipment:		Household Furniture	0.14
Mainframes	0.30	Other Furniture	0.12
Terminals	0.27	Farm Tractors	0.15
Storage Devices	0.28	Construction Tractors	0.16
Printers	0.35	Agricultural Machinery	0.13
Personal Computers	0.11	Construction Machinery	0.16
Other Office Equipment	0.31	Mining and Oilfield Machinery	0.16
Communications Equipment	0.11	Service Industry Machinery	0.15
Instruments	0.14	Other Electrical Equipment	0.18
Photocopying Equipment	0.18	Miscellaneous Equipment	0.15
Fabricated Metals	0.11	Industrial Buildings	0.03
Steam Engines and Turbines	0.05	Office Buildings	0.03
Internal Combustion Engines	0.19	Commercial Buildings	0.03
Metalworking Machinery	0.11	Religious Buildings	0.02
Special Industrial Machinery	0.10	Educational Buildings	0.02
General Industrial Machinery	0.10	Hospitals	0.02
Electrical Transmission	0.06	Other Buildings	0.02
Autos	0.28	Utilities Structures	0.02
Trucks, Buses, and Trailers	0.18	Farm Structures	0.02
Aircraft	0.08	Mining Shafts and Wells	0.07
Ships and Boats	0.07	Other Structures	0.02
Railroad Equipment	0.06	Tenant-Occupied Residencies	0.02

Table 1: NIPA Depreciation Rates

been zero since IBM absorbed the cost of physical decay.[9]

Together, I believe these arguments provide a compelling case that the rate of economic depreciation for computers is higher than the sum of the rate of embodied technological change and the rate of physical decay, as suggested by the Solow-style vintage model. I will now present a simple extension to the Solow vintage model that will explain all three of the patterns noted here: Non-geometric partial depreciation schedules, downward shifts over time in the schedules, and high apparent rates of physical decay.

5 Computing Support Costs and Endogenous Retirement

The model presented in this section is motivated by two observations. The first is that the Solow-style vintage capital model is inconsistent with technological obsolescence as defined in the introduction. It predicts that computers will lose value at a rate that is faster than their decline in productive capacity, but firms will never choose to retire a machine. Rather, it suggests the optimal strategy is simply to let the flow of income from a computer gradually erode over time. The second observation is that computer systems are usually complex in nature and can only be used successfully in conjunction with technical support and maintenance. The explosion in demand for Information Technology positions such as PC network managers is a clear indication of the need to back up computer hardware investments with outlays on maintenance and support. Indeed, research by the Gartner Group (1999), a private consulting firm, shows that, as of 1998, for every dollar firms spend on computer hardware there is another 2.3 dollars spent on wages for Information Technology employees and computing consultants.

The model presented here uses the existence of these support costs to motivate the phenomenon of technological obsolescence: Once the marginal productivity of a machine falls below the support cost, the firm will choose to retire it. This is not the only way to endogenize the retirement decision. The putty-clay assumption of fixed *ex post* factor proportions, as assumed by Gilchrist and Williams (1998) also leads to endogenous retirement. Jovanovic and Rob (1997) present a model of endogenous retirement based on the assumption that a firm can only use one machine at a time for production. I use the

[9]Oliner (1994) also makes this argument and presents separate estimates of the *wealth* stock, using the partial economic depreciation rate,and the *productive* stock, using a physical decay rate of zero.

support cost formulation of the retirement decision because it appears to fit well with the reality of how computer investments are being implemented and also because the assumed lack of flexibility implied by these alternative models does not seem to capture the role of computing equipment in the production process very well. While putty-clay technology may be a reasonable assumption for an industrial plant, it is distinctly unreasonable for a flexible piece of equipment such as a computer, since there is little that prevents firms from allocating less labor to work with old computing technologies after new and improved technologies are introduced.[10]

5.1 Theory

Suppose now that for a vintage of computers purchased at time v, firms need to incur a support cost equal to a fraction s of the purchase price of the piece of capital, $p_v(v)$, and that this cost declines at rate δ throughout the lifetime of the vintage. This latter assumption is best motivated by thinking of the initial investment as relating to a bundle of machines, a proportion δ of which "explode" each period. In this case our assumption implies that the support cost per machine in use stays constant throughout the life of the vintage.

The firm's profit function can now be expressed as

$$\begin{aligned} \pi_v(t) &= A(t) L_v(t)^{\alpha(t)} K_v(t)^{\beta(t)} \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right)^{1-\alpha(t)-\beta(t)} \\ &\quad - r_o(t) K_v(t) - w(t) L_v(t) - s p_v(v) I(v) e^{-\delta(t-v)} \end{aligned} \tag{37}$$

In this case, instead of allowing the marginal productivity of a unit of capital to simply gradually erode to zero, once the machine of vintage v reaches the age, T, where it cannot cover its support cost it is considered *obsolete* and is scrapped.[11] Thus, scrappage age T is defined by

$$\frac{\partial Q_v(T)}{\partial \left(I(v) e^{-\delta T} \right)} = s p_v(v) \tag{38}$$

[10] To give two examples, as personal computers grew in speed and user-friendliness, firms were able to allocate workers away from clunky mainframe-based computing systems towards newer Windows-based PC networks while the emergence of cheap high-quality laser printers allowed older Inkjet-style printers to be used as backups.

[11] We could assume that the computer has some "scrappage value" and so retirement would occur before the point where the machine is only covering maintenance cost. However, this would add an extra parameter without adding much insight.

How does this affect our model? The additive computing support cost has no effect on the marginal productivity of the labor and ordinary capital that works with a vintage of computer capital. Crucially, this implies that all the first order conditions for the allocation of labor and ordinary capital are unchanged, except obviously that no factors will be allocated to units that have been scrapped. The expression for the computer capital stock is changed to

$$C(t) = \int_{t-T}^{t} I(v) e^{\gamma v} e^{-\delta(t-v)} dv \tag{39}$$

However, given this new expression for the computer capital stock, all the other aggregate equations, including the aggregate growth rate, are exactly as before.

What does behave very differently with this new assumption is the rental rate for a unit of capital and consequently its price. Given that firms now have to pay a maintenance cost to operate the unit of capital, the usual equality between the rental rate and the marginal productivity of capital needs to be amended to

$$r_v(t) = \frac{\partial Q_v(t)}{\partial \left(I(v) e^{-\delta(t-v)}\right)} - s p_v(v) \tag{40}$$

Vintage Rental Rates and the Scrapping Age

We can first describe the scrappage decision. The computer price arbitrage formula is now

$$p_v(t) = \int_t^{v+T} \left(\frac{\partial Q_v(n)}{\partial \left(I(v) e^{-\delta(n-v)}\right)} - s p_v(v) \right) e^{-r(n-t)} e^{-\delta(n-v)} dn \tag{41}$$

Denoting the marginal productivity of a unit of capital as

$$r_v^*(t) = \frac{\partial Q_v(t)}{\partial \left(I(v) e^{-\delta(t-v)}\right)} \tag{42}$$

We get the following formula for the purchase price

$$p_v(v) = \frac{1}{1 + \frac{s}{r+\delta}\left(1 - e^{-(r+\delta)T}\right)} \int_v^{v+T} r_v^*(n) e^{-(r+\delta)(n-v)} dn \tag{43}$$

Now, differentiating the price of new computers with respect to v we get

$$\dot{p}_v(v) = (r + \delta + \gamma) p_v(v) - \frac{r_v^*(v)}{1 + \frac{s}{r+\delta}\left(1 - e^{-(r+\delta)T}\right)} + \frac{r_v^*(v+T) e^{-(r+\delta)T}}{1 + \frac{s}{r+\delta}\left(1 - e^{-(r+\delta)T}\right)} \tag{44}$$

At the time of scrappage, the computer must be just covering the support cost, implying $r_v^*(v+T) = sp_v(v)$. Making this substitution, using $p_t(t) = e^{gt}$ and re-arranging gives us the marginal productivity of new computer capital:

$$r_v^*(v) = \left[(r+\delta+\gamma-g) + s\left(1 + \frac{\gamma-g}{r+\delta}\left(1-e^{-(r+\delta)T}\right)\right)\right]e^{gv} \tag{45}$$

This equation shows that the introduction of computing support costs implies a higher marginal productivity of computer capital than in our previous model. One can note that this helps to explain the finding of apparently very large gross returns on computer investments, as found for instance by Brynjolfsson and Hitt (1996): Given the short time period over which firms make use of the computing equipment and the need to cover support costs, it will be necessary that such investments be extremely productive.

Since all the conditions for the allocation of other factors to each vintage are as before, the marginal productivity of a unit of computer capital still declines over time at rate $g-\gamma$, so we can now define the scrappage age T from

$$\left[(r+\delta+\gamma-g) + s\left(1 + \frac{\gamma-g}{r+\delta}\left(1-e^{-(r+\delta)T}\right)\right)\right]e^{(g-\gamma)T}e^{gv} = se^{gv} \tag{46}$$

Re-arranging, we get

$$e^{(r+\delta+\gamma-g)T} = (r+\delta+\gamma-g)\left(\frac{1}{s}+\frac{1}{r+\delta}\right)e^{(r+\delta)T} - \frac{\gamma-g}{r+\delta} \tag{47}$$

From this nonlinear equation we can calculate the service life, T. While the solution to the equation will in general require numerical methods, it has the intuitive property that the faster is the rate of quality-adjusted price decline for new computers, $\gamma - g$, the shorter is the time to scrappage. Chart 4 illustrates for the case $\gamma = .2, r = .03, \delta = .09$ how the introduction of a support cost of $s = .07$ affects the marginal productivity of capital.

Estimating Economic Depreciation

Given a path for the marginal productivity of a unit of computer capital, we can now explain the pattern of economic depreciation implied by this path.

$$p_v(t) = \int_t^{v+T} r_v^*(n)\, e^{-r(n-t)}e^{-\delta(n-v)}dn - sp_v(v)\int_t^{v+T} e^{-r(n-t)}e^{-\delta(n-v)}dn \tag{48}$$

To keep this calculation simple, we will break it into two, defining

$$p^*(t) = \int_t^{v+T} r_v^*(n)\, e^{-r(n-t)}e^{-\delta(n-v)}dn \tag{49}$$

$$= \quad r_v^*(v)\, e^{rt} e^{(\delta+\gamma-g)v} \int_t^{v+T} e^{-(r+\delta+\gamma-g)n} dn \tag{50}$$

$$= \quad r_v^*(v)\, e^{-(\delta+\gamma-g)(t-v)} \frac{\left(1 - e^{-(r+\delta+\gamma-g)(T-t+v)}\right)}{r+\delta+\gamma-g} \tag{51}$$

Now, we use the fact that $r_v^*(v) = se^{gv}e^{(\gamma-g)T}$. Inserting this, re-arranging, and defining the age of the vintage as $\tau = t - v$, we get

$$p_v^*(t) = e^{gt}e^{-(\delta+\gamma)\tau}\left(\frac{se^{-(r+\delta)T}}{r+\delta+\gamma-g}\right)\left(e^{(r+\delta+\gamma-g)T} - e^{(r+\delta+\gamma-g)\tau}\right) \tag{52}$$

Finally, inserting the expression for $e^{(r+\delta+\gamma-g)T}$ into equation 47 we get

$$p_v^*(t) = e^{gt}e^{-(\delta+\gamma)\tau}\left[1 + \frac{s}{r+\delta} - \left(\frac{se^{-(r+\delta)T}}{r+\delta+\gamma-g}\right)\left(\frac{\gamma-g}{r+\delta} + e^{(r+\delta+\gamma-g)\tau}\right)\right] \tag{53}$$

Thus

$$\begin{aligned} p_v(t) \quad = \quad & e^{gt}e^{-(\delta+\gamma)\tau}\left[1 + \frac{s}{r+\delta} - \left(\frac{se^{-(r+\delta)T}}{r+\delta+\gamma-g}\right)\left(\frac{\gamma-g}{r+\delta} + e^{(r+\delta+\gamma-g)\tau}\right)\right] \\ & -e^{gv}e^{-\delta\tau}\left(\frac{s}{r+\delta}\right)\left(1 - e^{-(r+\delta)(T-\tau)}\right) \end{aligned} \tag{54}$$

And the partial depreciation schedule calculated from an Oliner-style study by comparing the price of an old vintage with the price of new computers and then subtracting off the quality-improvement in the new computers gives

$$\begin{aligned} d_v(t) \quad = \quad & e^{-\delta\tau}\left[1 + \frac{s}{r+\delta} - \left(\frac{se^{-(r+\delta)T}}{r+\delta+\gamma-g}\right)\left(\frac{\gamma-g}{r+\delta} + e^{(r+\delta+\gamma-g)\tau}\right)\right] \\ & e^{-(\delta+\gamma-g)\tau}\left(\frac{s}{r+\delta}\right)\left(1 - e^{-(r+\delta)(T-\tau)}\right) \end{aligned} \tag{55}$$

5.2 Implications for Growth Accounting

The model just presented can explain all three of the anomalies noted in our discussion of Oliner's evidence on computer depreciation. The non-geometric depreciation schedule comes straight from equation 55: The rate at which machines lose value speeds up as the time when they become technologically obsolete approaches. In terms of Chart 4, this occurs because the area above the support cost and below the marginal productivity of capital falls off rapidly as the machine approaches scrappage. The downward shift in the depreciation schedules over time is consistent with an increase in the pace of embodied technological progress, a pattern which seems to fit with the apparent increase in the pace of technological change in the computer industry since the early 1980s. Finally, the partial, quality-adjusted, depreciation rates estimated from Oliner's procedure will be significantly higher than the rate of physical decay. Combined with significant anecdotal evidence, these patterns point towards the need to explicitly incorporate technological obsolescence into the theoretical framework used for growth accounting.

An important implication of this conclusion is that we cannot use the traditional hedonic vintage price method to estimate stocks for use in productivity analysis. While the economic depreciation rates can be used to construct the value of the stock for use as a measure of *wealth*, they cannot be used to construct the stock appropriate for an aggregate production function. Indeed, Oliner (1994) explicitly assumed that the rate of physical decay for computer peripherals was zero and presented estimates of quality-adjusted productive stocks for these categories based upon this assumption along with estimates of quality-adjusted wealth stocks that used the partial depreciation rates.

Another implication is that we cannot use the Hall-Jorgenson rental rate formula with quality-adjusted prices to estimate the marginal productivity of capital. Technological obsolescence implies a starkly different formula for the marginal productivity of capital. Note though, that the marginal productivity of capital still declines at rate γ, and so once we have an estimate of the marginal productivity of new capital, from equation 45, then the estimation of factor shares is the same as in Section 3.3. The marginal-productivity-weighted stock is

$$\int_{t-T}^{t} \frac{\partial Q_v(t)}{\partial\left(I(v)e^{-\delta(s-v)}\right)} I(v)e^{-\delta(t-v)}dv = \int_{t-T}^{t} r_v^*(t)\, I(v)e^{-\delta(t-v)}dv \tag{56}$$

$$= r_t^*(t) \int_{t-T}^{t} I(v)e^{-\gamma(t-v)}e^{-\delta(t-v)}dv \tag{57}$$

The integral thus re-arranges to

$$\int_{t-T}^{t} \frac{\partial Q_v(t)}{\partial\left(I(v)e^{-\delta(s-v)}\right)} I(v)e^{-\delta(t-v)}dv = \left[(r+\delta+\gamma-g) + s\left(1+\frac{\gamma-g}{r+\delta}\left(1-e^{-(r+\delta)T}\right)\right)\right] e^{(g-\gamma)t} C(t) \tag{58}$$

6 Empirical Growth Accounting

We have developed two different methods for calculating the contribution of computer capital accumulation to economic growth based on two interpretations of the evidence on economic depreciation for computing equipment. The first interpretation, following the Solow-style model of Section 3, views the rate of economic depreciation as $\gamma + \delta$, the sum of the rate of embodied technological change and the physical decay rate. For computing equipment, this approach interprets Oliner's partial depreciation rates, obtained by subtracting the rate of quality-adjusted price decline from the rate of economic depreciation, as measures of the rate of physical decay. As developed in Section 3.3, this interpretation suggests an empirical growth accounting approach that consists of applying the standard no-embodiment model of Section 2 to quality-adjusted data on real investment, prices, and depreciation rates. Thus, I will label this the "Quality-Adjustment Approach". The second interpretation, developed in the previous section, views the evidence on economic depreciation of computers as consistent with an important role for technological obsolescence and suggests an alternative approach to empirical growth accounting. This section applies both approaches to calculating the contribution of computer capital accumulation to growth.

6.1 Quality-Adjustment Approach

Our growth accounting calculations are for the U.S. private business sector which, as defined by the Bureau of Labor Statistics (BLS), equals GDP minus output from government and nonprofit institutions and the imputed income from owner-occupied housing. The contribution of capital input to growth is calculated using a detailed level of disaggregation. I use stocks of equipment, nonresidential structures, and residential structures at the level of disaggregation shown in Table 1: 5 types of computing equipment (Mainframes, PCs, Storage Devices, Printers, and Terminals), 26 types of non-computing equipment, 11 types of nonresidential structures, and tenant-occupied housing (rental income from such housing is part of business output).[12]

In applying the quality-adjustment approach, I have started with the NIPA capital

[12] This list of assets is not necessarily exhaustive. For instance, the BLS Multifactor Productivity program also includes land and inventories as inputs into the production process. Including extra assets may change the exact results of the growth accounting exercises presented here, but would not qualitatively change the conclusions.

stocks for all categories of equipment and structures except one. Since 1997, these capital stock estimates, documented by Fraumeni (1997) and Katz and Herman (1997), have used the Hulten-Wykoff estimates of geometric economic depreciation rates for most types of capital.[13] For computing equipment, for which NIPA real investment data are based on a quality-adjusted price deflators, the NIPA stocks use Oliner's time-varying non-geometric depreciation schedules (shown on Chart 2) to construct quality-adjusted real stocks.[14] The capital stock data are published through 1997; I have extended them through 1998 using published series on investment. The NIPA stocks have been constructed as year-end values. Since our growth accounting calculations attempt to explain year-average output growth, we use the average of current and lagged NIPA stocks.

The exception for which I do not use the NIPA capital stock is Personal Computers. As can be seen on Table 1, the 1997 NIPA depreciation rate for PCs is far lower than for the other categories of computing equipment. Somewhat surprisingly, there has not been a study of economic depreciation for PCs. In the absence of evidence for this category, and thus evidence that PCs are depreciating faster over time, BEA chose to set the depreciation schedule for this category according to a schedule for mainframes estimated by Oliner (1989) that did not allow the pace of depreciation to vary over time. Since the schedules for each of the other categories of computing equipment have shifted down over time, this has left PCs as the slowest depreciating category. If, as seems likely given the rapid pace of price decline for these machines in recent years, the depreciation schedule for PCs has also shifted down over time, then the NIPA stocks would be severely mis-measured. In my implementation of the quality-adjustment approach, I have constructed the stock of PCs using the same time-varying decay rates that BEA use to construct the stock of mainframes.

The marginal productivities of the stocks are estimated using the Hall-Jorgenson rental rate, with the formula being applied to computers as described in Section 3.3:

$$\frac{\partial Q(t)}{\partial C(t)} = q_t(t)\left(r + \delta^{PD} - \frac{\dot{q_t}(t)}{q_t(t)}\right) \tag{59}$$

[13]BLS use slightly different capital stock measures in their Multifactor Productivity calculations, based on a somewhat non-geometric pattern of decay. Since the weight of the evidence for most types of capital favors geometric decay and this pattern is consistent with our theoretical model, I have chosen to use the NIPA stocks instead. The results here are not much affected by using the BLS stocks instead.

[14]The data for the subcategories of computing equipment have not been published in the *Survey of Current Business*. They are obtained from a CD-Rom that can be ordered from the Bureau of Economic Analysis at http://www.bea.doc.gov/bea/uguide.htm.

where $q_t(t)$ is the quality-adjusted price of new computers, δ^{PD} is the partial depreciation rate. The real rate of return on capital, r, is set equal to 6.75 percent: This produces a series for the "required" income flow from capital that, on average, tracks with the observed series for business sector capital income over our sample. The $\frac{\dot{q_t(t)}}{q_t(t)}$ term is calculated for each type of capital as a three-year moving average of the rate of change of the price of capital relative to the price of output. I also controlled for the effect of the tax code on the marginal productivity of capital by multiplying the rental rate for each type of capital by the well-known Hall-Jorgenson tax term $\frac{1-ITC-\tau z}{1-\tau}$, where τ is the marginal corporate income tax rate, z is the present discounted value of depreciation allowances per dollar invested, and ITC is the investment tax credit. These tax terms were calculated for each type of capital using the information on tax credit rates and depreciation service lives presented in Gravelle (1994).

One potential concern about this exercise is that there is an apples-and-oranges issue with the NIPA capital stock data. The computer capital stocks explicitly acknowledge quality change while for all other types of capital there is essentially no adjustment for quality.[15] This is despite evidence from Gordon (1990) that the NIPA price deflators have also overstated the rate of quality-adjusted price increase for non-computing equipment, and similar evidence from Gort, Greenwood, and Rupert (1999) for structures. Perhaps, surprisingly, it turns out that while mis-measurement of quality is important for calculating TFP growth, it has no effect on our calculation of the contribution to growth of computer capital accumulation, $(1-\alpha(t)-\beta(t))\frac{\dot{C(t)}}{C(t)}$.[16] This is because quality-adjustment of the computer capital stock will provide the correct value for $\frac{\dot{C(t)}}{C(t)}$, while mis-measurement of quality improvement for other types of capital has equal and offsetting effects on the estimated flow of income from such capital and thus does not affect our estimated value of $1-\alpha(t)-\beta(t)$: Quality mis-measurement leads to an overstatement of the flow of income from a unit of old capital but an understatement of the stock of such capital remaining.

[15] I write "essentially" no adjustment because although there is almost no explicit quality adjustment apart from for computers, the NIPA deflators are based on BLS Producer Price Indexes, and the PPI does reflect some attempts to deal with the introduction of new and superior products. See Moulton and Moses (1997) for a discussion of these methods in the context of the CPI program.

[16] Hulten (1992) and Greenwood, Hercowitz, and Krusell (1997) are two papers that attempt to re-calculate TFP growth using quality-adjusted stocks of equipment based on Gordon's studies; the papers come to quite different conclusions on the importance of the quality adjustment.

6.2 Obsolescence Approach

The model of technological obsolescence presented in the previous section suggests an alternative empirical strategy. Equation 55 describes a non-geometric schedule of economic depreciation for computers as a function of s, δ, r, and $\gamma - g$. Again using $r = .0675$ and using the average rate of quality-adjusted relative price decline for computers to estimate $\gamma - g$, we can calibrate the obsolescence model by estimating the values of s and δ that are most consistent with the observed schedules.

Empirical implementation of the obsolescence approach is complicated by one important fact not captured by the model. The model predicts that all machines of a specific vintage are retired on the same date. Reality is never quite that simple and so in practice there is a distribution of retirement dates. Given a survival probability distribution, $d(\tau)$ that declines with age, the appropriate expression for the productive stock needs to be changed from equation 39 to

$$C(t) = \int_{-\infty}^{t} d(t-v)\, I(v) e^{\gamma v} e^{-\delta(t-v)} dv \tag{60}$$

This problem also needs to be confronted in the construction of economic depreciation schedules. If these schedules are constructed using only information on prices of assets of age τ, they will underestimate the average pace of depreciation: There is a "censoring" bias because we do not observe the price (equal to zero) for those assets that have already been retired. Hulten and Wykoff's (1981a,b) methodology corrects for this censoring problem by multiplying the value of machines of age τ by the proportion of machines that remain in usage up to this age. Oliner's depreciation studies followed the same approach and I have used Oliner's retirement distributions to construct estimates of productive stocks for computing equipment that are consistent with equation 60.

Table 2 shows the estimated values for s and δ for the five types of computing equipment. These estimated values are based on the most recent depreciation schedules for each type of equipment. The table shows that for mainframes, storage devices, and terminals, the obsolescence model's depreciation schedules fit far better than any geometric alternative: Root-Mean-Squared-Errors of the predicted depreciation profiles relative to the actual profile are far lower for the obsolescence model. Also, for mainframes and terminals, the parameter combinations that fit best are those that have a physical decay rate of zero. An exception to these patterns is printers, which as seen on Chart 3, show an approximately geometric pattern of decay. I have interpreted this as a rejection of the obsolescence model for printers and have applied the quality-adjustment approach to this category, even though the increase in depreciation rates over time for printers seems likely to be connected to an increase in the pace of technological change. The estimates of s and δ for PCs were set equal to the estimates for mainframes.

These results suggest that instead of the rapid pace of physical decay implicit in the standard methodology, the evidence on computer depreciation is more consistent with low rates of physical decay with accelerated retirement driven by moderate support costs and rapid technological change. I describe these estimates of support costs as moderate because they are well below those suggested by other studies. Applying our value of $s = 0.17$ for PCs and mainframes, our model predicts nominal maintenance costs in 1998 of $53.1 billion, only 3% more than the nominal hardware expenditures on these two categories, well below the 230% figure suggested by the Gartner Group research.[17]

	Mainframes	PCs	Storage	Printers	Terminals
s	0.17	0.17	0.05	0	0.15
δ	0	0	0.06	NIPA	0
RMSE	0.01	NA	0.04	0.02	0.01
RMSE - Geometric	0.06	NA	0.10	0.02	0.06

Table 2: Calibrating The Obsolescence Model

Given these estimates of s and δ, the obsolescence model is implemented by using equation 60 to construct the quality-adjusted productive capital stocks, and using s and δ

[17] One potential resolution of the low estimates of support costs derived here relative to the Gartner Group evidence is that firms may not view all of the expenditures on IT employees and consultants as simply costs aimed at keeping the current computer system working. Rather, some of these expenditures may represent investments in human capital though training. Brynjolfsson and Yang (1999) show that market valuations of firms undertaking IT investments seem to rise by more than would be warranted by the capital investments alone.

to estimate the marginal productivity of these stocks:

$$\frac{\partial Q(t)}{\partial C(t)} = q_t(t)\left[\left(r+\delta-\frac{\dot{q_t(t)}}{q_t(t)}\right)+s\left(1-\frac{1-e^{-(r+\delta)T}}{r+\delta}\frac{\dot{q_t(t)}}{q_t(t)}\right)\right] \tag{61}$$

6.3 Results

Chart 5 displays the capital stocks implied by the quality-adjustment and obsolescence models for mainframes, PCs, storage devices, and terminals. The low estimated rates of physical decay for the obsolescence model imply stocks that, in 1997, ranged from 24 percent (for storage devices) to 72 percent (for mainframes) higher than their quality-adjusted counterparts. The wide range in these ratios comes in part from the variations in the average ages of these stocks: The quality-adjustment approach places far lower weights on old machines than does the obsolescence model, and the stock of mainframes contains more old equipment than the stock of storage devices. For PCs, by far the largest category, the obsolescence model implies a stock that is 44 percent larger than that implied by the quality-adjustment approach.[18]

Table 3 shows the 1997 values for the gross required rates of return on computer capital, where this is defined as the marginal productivity of capital divided by the quality-adjusted price index, $q(t)$. The marginal productivities implied by the quality-adjustment and obsolescence models are similar in magnitude. The reason for this is intuitive. Based on high rates of economic depreciation, the quality-adjustment approach correctly calculates that computer investments need to have a high marginal productivity; it just arrives at this conclusion via the wrong interpretation - that computers have a high rate of physical decay. Note that the obsolescence model's formula for the marginal productivity of computer capital rises more with the pace of quality-adjusted price decline, $\gamma - g$, than does the Hall-Jorgenson formula; this is because a faster pace of price decline leads to early retirement. For this reason, the marginal productivity for PCs, the largest category and also the category with the fastest rate of price decline, is higher for the obsolescence model than the quality-adjustment model.

[18]Note, however, that because the NIPA capital stocks use a very low depreciation rate for PCs, the obsolescence model's stock for this category is similar in size to the NIPA stock.

	Mainframes	PCs	Storage	Printers	Terminals
Quality-Adjustment	0.61	0.64	0.31	0.62	0.39
Obsolescence Model	0.67	0.76	0.20	0.62	0.33

Table 3: 1997 Values for Gross Required Rate of Return on Computer Capital

The substantially higher capital stocks and slightly higher marginal productivities implied by the obsolescence model translate into higher factor shares for computing equipment. On average these factor shares are about 50 percent higher than for the quality-adjustment model. These higher factor shares translate directly into higher estimates of the contribution of computers to growth since despite very different levels, the capital stock growth rates implied by our two approaches are very similar (see Chart 6). Chart 7 shows our two estimates of the contribution of computer capital accumulation to business sector growth. These figures for the obsolescence model suggest that computers have probably been providing more of a stimulus to economic growth than was previously supposed. Moreover, both approaches agree that the contribution of computer capital accumulation has picked up remarkably over the past few years, with the obsolescence model suggesting that by 1998, this contribution was worth 1.14 percentage points for economic growth. For 1998, the difference between the two contribution series is 0.36 percentage points, an economically significant figure.

6.4 Comparison with Previous Studies

Having explained the differences between the results I obtained for the quality-adjustment and obsolescence models, I should note that my calculations of contribution of computer capital accumulation to growth using the quality-adjustment approach are higher, and in some cases much higher, than those presented by previous researchers such as Oliner and Sichel (1994), Sichel (1999), and Jorgenson and Stiroh (1999) (see Table 4). These higher contribution estimates can be explained by some simple differences in methodology.

One difference is the level of aggregation. I used separate capital stocks and rental rates for each of the five types of computing equipment, whereas the other studies used the published NIPA aggregate for all computing equipment, basing the rental rate on a depreciation rate obtained from re-arranging a perpetual inventory formula ($K_t = (1-\delta)\, K_{t-1} + I_t$) with the aggregate data. This choice of aggregation does not have much effect on the results of the quality-adjustment approach but is theoretically superior. This is because the chain-weighted aggregation methodology now used to construct all NIPA real aggregates implies that the depreciation rate obtained from re-arranging an aggregate perpetual inventory equation (currently about 31 percent for computing equipment) is not a good measure of the average depreciation rate for the underlying categories. In an appendix, I show that under standard steady-state conditions, the presence of relative price movements within a bundle of capital goods (in this case PCs decline in price faster than the other types of computing equipment) will imply that a real chain-aggregate for investment grows faster than the real chain-aggregate of corresponding capital stocks. Hence the estimated depreciation rate from an aggregate perpetual inventory equation ($\delta_t = \frac{I_t}{K_{t-1}} - \frac{\Delta K_t}{K_{t-1}}$) will trend upwards over time even when all of the components of the capital stock depreciate at the same constant rate.

The more substantive differences between my calculations and those of Jorgenson-Stiroh and Oliner-Sichel revolve around differences in the measurement of inputs and in the calculation of rental rates:

- Jorgenson and Stiroh (1999) use a much broader concept of private output, including additional imputations for the service flows from owner-occupied housing and consumer durables. Thus, they add a large amount of output that cannot be accounted for as resulting from computer capital. The use of a comparable concept of output would substantially increase Jorgenson and Stiroh's estimates of the contribution to output growth of computer capital accumulation.
- Oliner and Sichel (1994) use a constant value of 0.24 to represent the sum of the physical decay (δ) and the loss in value due to obsolescence ($\gamma - g$); Sichel (1999) uses 0.31 for this sum. These figures are significantly below what is obtained from summing recent quality-adjusted rates of price decline with the partial depreciation rates used in the NIPAs. Re-doing my calculations using $\delta + \gamma - g = 0.31$, the quality-adjustment approach gives results similar to those reported in Sichel (1999).

One study that reported effects for computer capital accumulation that were of similar magnitude to those generated by my obsolescence model is Jorgenson and Stiroh (1995). Like this paper, Jorgenson and Stiroh used Oliner's retirement schedules to calculate productive stocks for the disaggregated categories of computing equipment. However, they used a very different approach to calculating rental rates. Jorgenson and Stiroh treat the retirement process for each "technology class" (about every 10 years for mainframes, every 5 years for computer peripherals) as occurring according to an exogenous distribution, so that at time t a proportion $d_v(t)$ of vintage v is still in use. This implies the following arbitrage formula for the price of the vintage at time t.

$$p_v(t) = \int_t^\infty \frac{d_v(z-t)}{d_v(t)} r_v(z) e^{-r(z-t)} dz \tag{62}$$

Differentiating with respect to time gives the rental rate:

$$r_v(t) = p_v(t) \left(r - \frac{\dot{d_v(t)}}{d_v(t)} - \frac{\dot{p_v(t)}}{p_v(t)} \right) \tag{63}$$

Jorgenson and Stiroh argue that the quality-adjusted prices of all units of computing equipment should be the same and thus use the quality-adjusted investment price deflator to represent $p_v(t)$. The $\frac{\dot{d_v(t)}}{d_v(t)}$ term in the rental rate represents the fact that firms require a higher marginal productivity of capital to compensate for the "risk" that the machine may be retired.

This formulation is somewhat unsatisfactory because it treats the retirement of old computers as an exogenous process not connected to the decline in quality-adjusted prices. This approach has the implication that, because they are being retired at a faster rate, marginal productivity is higher for older technology classes, which, of course, begs the question as to why these machines are being retired. The obsolescence model presented in this paper gets around these problems by endogenizing the retirement decision and is sufficiently different in its approach that the similarity between the Jorgenson-Stiroh calculations for the 1980-85 and 1986-92 periods and those of the obsolescence model is probably a coincidence.

Authors	Period	Previous Estimates	Quality Adjustment	Obsolescence Model
Sichel (1999)	96-98	0.35	0.66	0.95
	90-95	0.17	0.24	0.38
	80-89	0.22	0.29	0.47
Oliner-Sichel (1994)	80-92	0.21	0.26	0.42
	70-79	0.09	0.16	0.24
Jorgenson-Stiroh (1999)	91-96	0.12	0.30	0.45
	73-90	0.09	0.23	0.36
Jorgenson-Stiroh (1995)	86-92	0.38	0.23	0.39
	80-85	0.52	0.30	0.47

Table 4: The Contribution of Computer Capital Accumulation to Growth

7 The Computer Sector and Aggregate TFP

Thus far, I have focused on the role that computer capital accumulation has played in boosting output growth. Note, though, that the models presented have been silent on the cause of this massive accumulation of computing power: Why has the price of computing power fallen so rapidly?

Our models have featured an aggregate production function, which suggests two possibilities: Either computers are produced using the same technology as all other goods (in which case why would their relative price decline?) or else the economy does not produce computers (so all computers have been imported). Since neither of these cases seem to fit the reality of the U.S. economy, an alternative approach seems appropriate which recognizes that computers may be produced using a different technology to other goods. In this section, I discuss the direct effects of technological progress in the computer sector on output growth using a simple two-sector model. How this technological progress affects our measure of aggregate output turns out to depend on whether or not we also apply a quality-adjustment to the output of the computer sector. I briefly discuss the objection of

Greenwood, Hercowitz, and Krusell (1997) to growth accounting using the NIPA measure of real GDP, which applies such a quality adjustment.

7.1 Introducing Computer Production

Suppose now that the economy can be broken into two sectors, with Sector 1 producing consumption and ordinary capital according to the aggregated production function, derived from a vintage structure as in previous sections:

$$Q_1(t) = A(t)\,L(t)^{\alpha(t)}\,K(t)^{\beta(t)}\,C(t)^{1-\alpha(t)-\beta(t)} \tag{64}$$

and sector 2 producing computers according to a production function that is identical up to the multiplicative disembodied technology term:

$$Q_2(t) = I(t) = B(t)\,L(t)^{\alpha(t)}\,K(t)^{\beta(t)}\,C(t)^{1-\alpha(t)-\beta(t)} \tag{65}$$

Again we assume that these computers increase in quality at rate γ each period.

The marginal cost of producing non-quality-adjusted computers must change relative to the price of consumption and ordinary capital according to the difference in the growth rates of disembodied technology between the two sectors. Assuming competitive pricing, this endogenizes the relative price of non-quality-adjusted computers, introduced earlier as the exogenous parameter g:

$$g = \frac{\dot{A}(t)}{A(t)} - \frac{\dot{B}(t)}{B(t)} \tag{66}$$

Since the relative price change for computers offsets the productivity differential, this implies that a factor allocation $(L(t), K(t), C(t))$ produces the same flow of nominal revenue in the computer and non-computer sectors. This implies that nominal output must grow at the same rate in each sector, and thus that real output in the computer sector is given by

$$\frac{\dot{Q}_2(t)}{Q_2(t)} = \frac{\dot{Q}_1(t)}{Q_1(t)} - g \tag{67}$$

This simple two-sector model suggests two alternative ways to aggregate the output of these two sectors, one which adjusts the output of the computer industry to reflect the improvement in quality, and one which does not. We will examine these in turn.

Aggregate Output: No Quality-Adjustment

Consider now the behavior of a Tornqvist aggregate of $Q_1(t)$ and $Q_2(t)$. This aggregation procedure, which is a close theoretical approximation to the Fisher chain-aggregate that BEA has used to construct real GDP since 1996[19], weights the real growth rates for each category according to its share in nominal output. Note that the ratio of nominal computer output to total computer output is constant; call this ratio μ. Thus, the growth rate of a Tornqvist aggregate of real output will be:

$$\frac{\dot{Q}(t)}{Q(t)} = (1-\mu)\frac{\dot{Q_1}(t)}{Q_1(t)} + \mu\frac{\dot{Q_2}(t)}{Q_2(t)} = \frac{\dot{Q_1}(t)}{Q_1(t)} - \mu g \tag{68}$$

and measured aggregate TFP growth will be

$$\frac{\dot{Q}(t)}{Q(t)} - \alpha(t)\frac{\dot{L}(t)}{L(t)} - \beta(t)\frac{\dot{K}(t)}{K(t)} - (1-\alpha(t)-\beta(t))\frac{\dot{C}(t)}{C(t)} = (1-\mu)\frac{\dot{A}(t)}{A(t)} + \mu\frac{\dot{B}(t)}{B(t)} \tag{69}$$

If non-quality-adjusted computers are produced using the same production function as other goods $(B(t) = A(t))$ then $g = 0$ implying that the growth rate of this aggregate will be identical to that just derived from our Solow-style vintage model.

Aggregate Output: With Quality-Adjustment

A problem with this simple measure of aggregate output is that $Q_2(t)$ is a poor measure of the true output of the computer sector. Since the technology embodied in each generation of computers is improving, measuring computer output by adding up the number of boxes leaving the factory seems inappropriate. The alternative is to define the real output of the computer sector in terms of quality-adjusted units, implying a simple transformation:

$$Q_2^*(t) = e^{\gamma t} I(t) = e^{\gamma t} B(t) L(t)^{\alpha(t)} K(t)^{\beta(t)} C(t)^{1-\alpha(t)-\beta(t)} \tag{70}$$

This re-formulation sees the improvement in the quality of computers for a given production function as a form of disembodied technology growth. Now, the Tornqvist aggregate of real GDP grows at rate

$$\frac{\dot{Q^*}(t)}{Q^*(t)} = \frac{\dot{Q_1}(t)}{Q_1(t)} + \mu(\gamma - g) \tag{71}$$

and measured total factor productivity is

$$\frac{\dot{Q^*}(t)}{Q^*(t)} - \alpha(t)\frac{\dot{L}(t)}{L(t)} - \beta(t)\frac{\dot{K}(t)}{K(t)} - (1-\alpha(t)-\beta(t))\frac{\dot{C}(t)}{C(t)} = (1-\mu)\frac{\dot{A}(t)}{A(t)} + \mu\left(\gamma + \frac{\dot{B}(t)}{B(t)}\right) \tag{72}$$

[19]See Diewert (1976) for the properties of the Tornqvist index. See Landefeld and Parker (1997) for a discussion of the properties of the Fisher chain-type indexes used to construct real GDP.

Since 1985 the NIPAs have followed this second approach, applying a quality adjustment to computer output. Thus, while the nominal output of the computer industry has fluctuated around 1.5 percent of business output since 1983 (see the upper panel of Chart 8), measured in terms of computing power the real output of the computing industry grew an astonishing 5000 percent between 1983 and 1998.[20]

The NIPAs do not include an estimate of non-quality-adjusted computer prices that we could use to calibrate this simple model with an empirically-based estimate of the relative growth rates of $B(t)$ and $A(t)$. A conservative assumption likely to produce a lower bound on the contribution of the computer sector to aggregate TFP growth is that the price of non-quality-adjusted computers has grown in line with output prices ($B(t) = A(t)$). This is a lower bound because it equates the productivity differential between the computer and non-computer sectors with the decline in quality-adjusted prices. However, even if we ignored quality improvement and looked only at the computer industry's ability to produce nominal output, one can still see productivity improvements. Perhaps surprisingly, despite maintaining its share in aggregate nominal output, employment in SIC Industry 357 (computer and office machinery) has declined almost continuously from a high of 522,000 in 1985 to about 380,000 in 1998.[21]

The bottom panel of Chart 8 shows the series for the contribution of the computer industry to raising business sector TFP growth under this assumption that $B(t) = A(t)$: This series equals the nominal share of computers in output multiplied by the rate of decline of quality-adjusted computer prices relative to output prices. This contribution, which had fluctuated around 0.25 percentage points a year between 1978 and 1995 has picked up considerably since 1995, averaging almost 0.5 percentage points a year in 1997 and 1998 as a result of a significant acceleration in price decline and an increase in the nominal share of computer output.

[20]There is no official measure of the output of the computer industry. The measure of nominal computer output used here is the sum of consumption, investment, and government expenditures on computers plus exports of computers and peripherals and parts minus imports for the same category. The measure of real output is the Fisher chain-aggregate of these 5 components.

[21]Source: Bureau of Labor Statistics, *Employment and Earnings*.

7.2 In Defense of Growth Accounting with NIPA Data

Greenwood, Hercowitz, and Krusell (1997) have criticized the use of NIPA data for growth accounting because of the quality-adjustment applied to the output of the computer sector and, in a more limited way, to other sectors. Greenwood et al describe two alternative theoretical models for capturing embodied technological change in investment goods. Letting c represent consumption, i be the quantity of non-quality-adjusted investment goods, q be the level of technology embodied in capital goods, and k be the stock of quality-adjusted capital, they contrast the following two aggregate models:

$$y_t = c_t + i_t = z_t f(k_t, l_t) \tag{73}$$

$$y_t = c_t + i_t q_t = z_t f(k_t, l_t) \tag{74}$$

The first model assumes that consumption goods and non-quality-adjusted investment goods are produced using the same technology. Since this corresponds to the case $B(t) = A(t)$ just presented above, we know that the output aggregate defined in equation 73, which sums consumption goods and non-quality-adjusted investment, grows according to the formula obtained from the simple Solow-style model derived in Section 3.2 above. For this reason Greenwood et al label this first equation the Solow model. The second model assumes that consumption goods and quality-adjusted investment goods are produced using the same technology: Greenwood et al attribute this approach to Evsey Domar (1963) and Dale Jorgenson (1966). In terms of our two-sector model, this is the case $B(t) = A(t)e^{-\gamma t}$. Hercowitz (1998) points out that this case does not match the empirical evidence. If consumption goods and quality-adjusted investment goods are produced with the same technology, then we should not see quality-adjusted prices for investment goods decline over time relative to consumption goods, an implication that is obviously counterfactual.

Greenwood et al view the decision to quality adjust investment goods when measuring output as an endorsement of the incorrect Domar-Jorgenson model. To quote: "the Domar-Jorgenson and Solow models call for output to be measured in different ways. The key issue is whether or not to adjust output for quality change. The Domar-Jorgenson model demands that you do. The Solow model dictates that you do not". This argument seems to be based on a theoretical misconception that comes from interpreting the quality adjustment using these one-sector models of production. As we have just described it, the quality-adjusted measure of aggregate output has a clear and intuitive interpretation and its validity does not depend on a particular assumed form for the relative growth rates of $A(t)$ and $B(t)$ and thus a choice between the two extreme cases given by equations 73 and 74. Rather, the essence of the argument in favor of applying a quality adjustment to simple measures of a sector's output is that before we can add up the output of each sector, we must decide *what* it is that the sector produces. Quality-adjustment of the computer industry is based on a judgement, which I believe to be correct, that the industry produces *computing power*, not simply computers.[22] That computing power is considered the correct economic concept for use in construction of the stock of productive capital simply further reinforces the choice of quality-adjusted units as the appropriate metric for measurement of this sector's output.

To put this argument another way, in terms of our two-sector model, one can believe that $B(t) = A(t)$ and thus that equation 73 (Solow) is correct, but still believe that $c_t + i_t q_t$ is a preferable measure of economic output.[23] Introducing the quality-adjustment has the effect of raising measured aggregate TFP relative to the non-quality adjustment case, a feature that Greenwood et al object to because it implies that "the use of NIPA data in conventional growth accounting will cause investment-specific technological change to appear as neutral" (they recommend using nominal GDP divided by the deflator for consumer nondurables). However, our two-sector model makes clear that as long as this "investment-specific" technological change results from "neutral" technological change in the computer sector, then this should be reflected in aggregate measures of neutral technology, which is what the quality-adjustment approach achieves.

---

[22]There are of course, a number of tricky theoretical issues surrounding quality adjustment of the output of investment industries. Whether or not such an approach is appropriate for the NIPAs was the subject of a somewhat rancorous debate during the 1980s. See Young (1989) for an explanation of the landmark decision to introduce quality adjustment of computers into the NIPAs. See Denison (1989), Young's predecessor as Director of BEA, for arguments against the quality-adjustment of computer output.

[23]Greenwood, Hercowitz, and Krusell also present a two-sector, consumption and investment model, model. However, they do so to show that a two-sector model *without* embodied technological change could not account for the observed decline in the relative price of equipment. My argument here is that the NIPA quality adjustment makes sense within a two-sector model *with* embodiment.

8 Computers and The Acceleration in Productivity

The results from our obsolescence model suggest that the effects on growth of both accumulation of computer capital and total factor productivity growth in the computer sector increased substantially over the period 1996-98 (See Chart 9). This period also saw a notable step-up in the growth rate of labor productivity, an unusual development late into an expansion: Business sector labor productivity averaged 2.15 percent during this period, a full 1 percentage point more than the average rate over the previous 22 years.

We can calculate the role of computer-related factors in the acceleration in labor productivity by dividing both sides of our growth accounting equation by hours:

$$\begin{aligned}\frac{\dot{Q(t)}}{Q(t)} - \frac{\dot{H(t)}}{H(t)} &= \frac{\dot{A}_C(t)}{A_C(t)} + \frac{\dot{A}_{NC}(t)}{A_{NC}(t)} + \alpha(t)\left(\frac{\dot{L(t)}}{L(t)} - \frac{\dot{H(t)}}{H(t)}\right) \\ &+\beta(t)\left(\frac{\dot{K(t)}}{K(t)} - \frac{\dot{H(t)}}{H(t)}\right) + (1-\alpha(t)-\beta(t))\left(\frac{\dot{C(t)}}{C(t)} - \frac{\dot{H(t)}}{H(t)}\right) \quad (75)\end{aligned}$$

Productivity growth is a function of TFP growth (here divided into the contributions of the computer and non-computer sectors, labeled C and NC), of computer and non-computer capital accumulation, and of improvements in the quality of labor input (represented as an increase in labor input relative to hours). I will focus on the two computer-related elements of productivity growth, $\frac{\dot{A}_C(t)}{A_C(t)}$ and $(1-\alpha(t)-\beta(t))\left(\frac{\dot{C(t)}}{C(t)} - \frac{\dot{H(t)}}{H(t)}\right)$, and represent the productivity growth due to all other factors as a residual.

Table 5 shows the results of this decomposition while Chart 10 gives a graphical illustration. The combined effects of computer capital accumulation and computer sector TFP growth can account for a remarkable 1.36 percentage points of the 2.15 percent a year growth in business sector productivity over this period. Moreover, a remarkable 0.8 percent of the 1 percent increase in labor productivity growth over 1996-98 can be explained by computer-related factors.[24] In fact, the calculated two-tenths acceleration due to other factors probably overstates the true effect of these factors since, as Gordon (1998) has discussed, methodological changes in price measurement introduced into the GDP statistics that were not fully "backcasted" to earlier periods have probably contributed around

[24]Stiroh (1998) is another paper that examines the combined effects of computer capital accumulation and computer-sector TFP growth. For his sample, which ends in 1991, Stiroh reports total computer-related effects on growth that are notably lower than those in this paper, largely because of the differences in the treatment of computer capital accumulation.

three-tenths a year to the acceleration in productivity growth over this period. Outside the computer-producing sector of the economy, there has been no apparent pickup in productivity growth due to improved rates of TFP growth (or indeed improved labor quality, accumulation of non-computer capital, or cyclical utilization).

These results indicate that computing equipment has played a crucial role in the recent pickup in productivity growth. However, while the results appear to endorse the popular belief that there is a connection between high-tech investments and improved productivity, they also contradict the position of some of the more enthusiastic believers in the benefits of technology investments. In particular, the common belief that high-tech investments have supernormal returns, would if correct, show up here under "All Other Factors" because we have assumed that all capital investments earn the same net rate of return. It seems that, as of yet, the hope that computer technology can revolutionize business practices throughout the economy in a way that makes all other factors of production more efficient has not panned out.

	1974-95	1996-98
Growth in Labor Productivity	1.16	2.15
Effect of Computer Capital Accumulation	0.35	0.89
Effect of Computer TFP Growth	0.20	0.47
Total Computer-Related Effect	0.56	1.36
All Other Factors	0.60	0.79

Table 5: Computers and Business Sector Productivity

The calculations in Table 5 are similar to those of Robert Gordon (1999) in stressing the important role that increased productivity in the computer sector has played in directly boosting aggregate productivity growth in recent years. However, they differ starkly from Gordon's calculations in attributing an even more important role to the effect of computer capital accumulation on productivity throughout the economy. Unlike my calculations, which assume that investments in computers have the same net return as all other capital investments, Gordon explicitly rejects the idea that computers are having an important positive effect on productivity, instead attributing more of the recent productivity improvement to cyclical utilization effects.[25]

Some Crystal Ball Gazing

What of the outlook for future productivity growth? Moving from growth accounting to forecasting is an inherently dangerous activity and all I have to offer are upside and downside risks relating to whether the recent productivity acceleration will persist or evaporate. The downside risk is that the recent period of spectacular rates of productivity improvement in the computer sector, and the associated acceleration in quality-adjusted price declines, turns out to be a flash in the pan. Indeed, it seems unlikely that the recent pace of computer-related technological advance can be sustained. Given that there is no evidence of an acceleration in multifactor productivity outside the computer sector, a slowdown in aggregate productivity growth would be the most likely outcome.

The upside potential has two elements. First, thus far, it does not seems as though the computer industry has fully exhausted the potential for producing rapid improvements in computing speeds and efficiency. Second, the very fact that the economy does not yet seem to be benefitting from "spillover" effects of computing technology on TFP has implicit in it the potential promise that these effects may start to have an impact in the coming years. The well-known historical analogies of Paul David (1990, 1999) suggest that such lagged productivity effects are not without precedent.[26] Moreover, while the ubiquitous comparisons with the Interstate Highway system are probably hype, the Internet, as its name suggests, seems a likely source of positive network externalities that can boost productivity.

[25] See Gordon (1998) for arguments against computers having a positive effect on productivity. I should note, however, that a number of these arguments (unlike previous technology revolutions, computer capital accumulation does not boost TFP; the marginal productivity of computer investments has declined) are consistent with the growth accounting analysis in this paper.

[26] Greenwood and Jovanovic (1998) take a wide-ranging look at the theoretical implications of learning about new technologies for the process of economic growth.

9 Conclusions

The starting point for this paper was the observation that the recent explosion in high-tech investment has coincided with a notable acceleration in U.S. labor productivity. The paper investigated the traditional approach to growth accounting and concluded that it does not provide satisfactory estimates of the contribution to output growth of assets such as computers that are subject to obsolescence due to rapid technological change. A new growth accounting methodology was introduced which suggested that investment in computing equipment is having a substantially larger payoff than previously thought, principally because the productive stock of such equipment is larger than previous studies had calculated. One interpretation of the results in this paper is that they provide a partial reversal of Oliner and Sichel's resolution of the Solow paradox: Computers may not be everywhere but they are more prevalent than the NIPA capital stocks suggest. Moreover, the combined effects of computer capital accumulation and increased productivity in the computer sector are currently accounting for a significant proportion of U.S. productivity growth and account for almost all of the acceleration in productivity over the past few years.

I will conclude by pointing to the implications of the paper for future research and for official statistics. First, I wish to clarify that the use of alternative capital stocks for productivity analysis should not be taken as a criticism of the procedures of the Bureau of Economic Analysis in constructing the NIPA capital stocks. It is not commonly realized that the principal use of these stocks is to provide estimates of the loss in the value of the capital stock associated with production, the NIPA variable "Consumption of Fixed Capital" that is subtracted from GDP to arrive at Net Domestic Product. Thus, these stocks are explicitly constructed to be measures of *wealth*. Rather, my point is a variation on an old theme in capital theory, that wealth-based capital stocks are not the same as productive capital stocks, and that this distinction is particularly important for computing equipment. Because of the important role played by obsolescence, researchers interested in estimating the effects of computers on productivity need to construct separate productive stocks of computing equipment, and also account for the effect of obsolescence on the equilibrium marginal productivity of these stocks. For example, in its Multifactor Productivity calculations, the Bureau of Labor Statistics bases its capital stocks of computing equipment on service lives implied by the NIPA depreciation schedules; an alternative procedure along the lines developed in this paper would seem more appropriate.

Second, a detailed study of economic depreciation for Personal Computers, along the lines of Oliner's (1989) study of mainframes, would be very valuable. Given the importance of investment in PCs in recent years, the absence of these estimates has made the results in this, and other papers on computers and productivity, relatively speculative. In addition, such research would help to improve the NIPA capital stock estimates, which currently seem to use a rate of economic depreciation rate that is too low. Such a study should probably top the list of future research projects in this area.

References

[1] Basu, Susanto (1996) "Cyclical Productivity: Increasing Returns or Cyclical Utilization " *Quarterly Journal of Economics*, 111, 719-751.

[2] Brynjolfsson, Erik and Loren Hitt (1996). "Paradox Lost? Firm-Level Information on the Returns to Information Systems Spending", *Management Science*, 42, 541-58.

[3] Brynjolfsson, Erik and Shinkyu Yang (1999). The Intangible Benefits and Costs of Computer Investments: Evidence from Financial Markets, mimeo, M.I.T. Sloan School.

[4] Bureau of Labor Statistics (1983). *Trends in Multifactor Productivity, 1948-1981.* BLS Bulletin No. 2178.

[5] Caballero, Ricardo and Mohomad Hammour (1996). "On the Timing and Efficiency of Creative Destruction", *Quarterly Journal of Economics*, 61, 805-852.

[6] David, Paul (1990). "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox", *American Economic Review*, 80, 355-361.

[7] David, Paul (1999). Digital Technology and the Productivity Paradox: After Ten Years, What Has Been Learned?, paper presented to the Conference "Understanding the Digital Economy: Data, Tools, and Research", held at U.S. Department of Commerce, 25-26 May.

[8] Denison, Edward F. (1989). *Estimates of Productivity Change by Industry*, Washington, DC: The Brookings Institution.

[9] Diewert, Erwin (1976). "Exact and Superlative Index Numbers", *Journal of Econometrics*, 4, 115-145.

[10] Domar, Evsey (1963) "Total Productivity and the Quality of Capital", *Journal of Political Economy*, 71, 586-88.

[11] Feldstein, Martin and Michael Rothschild (1974). "Towards an Economic Theory of Replacement Investment", *Econometrica*, 42, 393-423.

[12] Fernald, John and Susanto Basu (1999). "Why is Productivity Procyclical? Why Do We Care?, Federal Reserve Board, Internatioal Finance Discussion Paper No. 638.

[13] Fraumeni, Barbara M. (1997). "The Measurement of Depreciation in the U.S. National Income and Product Accounts", *Survey of Current Business*, July, 7-23.

[14] Gartner Group (1999). 1998 IT Spending and Staffing Survey Results, report available at www.gartnergroup.com.

[15] Gilchrist, Simon and John C. Williams (1998). Investment, Capacity, and Output: A Putty-Clay Approach", Federal Reserve Board, Finance and Economics Discussion Series No. 1998-44.

[16] Goolsbee, Austan (1998). "The Business Cycle, Financial Performance, and the Retirement of Capital Goods", *Review of Economic Dynamics.*

[17] Gordon, Robert J. (1990). *The Economics of New Goods*, Chicago: University of Chicago Press.

[18] Gordon, Robert J. (1998). Monetary Policy in the Age of Information Technology: Computers and the Solow Paradox, mimeo, Northwestern University.

[19] Gordon, Robert J. (1999). Has the New Economy Rendered the Productivity Slowdown Obsolete?, mimeo, Northwestern University.

[20] Gort, Michael, Jeremy Greenwood, and Peter Rupert (1999). "Measuring the Rate of Technological Progress in Structures", *Review of Economic Dynamics*, 2, 342-62.

[21] Gravelle, Jane (1994). *The Economic Effects of Taxing Capital Income*, Cambridge: MIT Press.

[22] Greenwood, Jeremy, Zvi Hercowitz, and Per Krussell (1997). "Long-Run Implications of Investment Specific Technological Change", *American Economic Review*, 87, 342-362.

[23] Greenwood, Jeremy and Boyan Jovanovic (1998). Accounting for Growth, NBER Working Paper No. 6647.

[24] Hall, Robert E. (1971). "The Measurement of Quality Change from Vintage Price Data", in Z. Grilliches, ed., *Price Indexes and Quality Change*, Cambridge: Harvard University Press.

[25] Hercowitz, Zvi (1998). "The 'Embodiment Controversy': A Review Essay", *Journal of Monetary Economics*, 41, 217-224.

[26] Herman, Shelby and Arnold Katz (1997). "Improved Estimates of Fixed Reproducible Tangible Wealth, 1929-95", *Survey of Current Business*, May, 69-92.

[27] Hulten, Charles (1992). "Growth Accounting When Technical Change is Embodied in Capital", *American Economic Review*, 82, 964-80.

[28] Hulten, Charles and Frank Wykoff (1981a). "The Estimation of Economic Depreciation Using Vintage Asset Prices: An Application of the Box-Cox Power Transformation", *Journal of Econometrics*, 15, 367-396.

[29] Hulten, Charles and Frank Wykoff (1981b). "The Measurement of Economic Depreciation" in Charles Hulten, ed., *Depreciation, Inflation, and the Taxation of Income from Capital*, Washington DC: The Urban Institute Press. 367-396.

[30] Hulten, Charles and Frank Wykoff (1996). "Issues in the Measurement of Economic Depreciation: Introductory Remarks", *Economic Inquiry*, 33, 10-23.

[31] Jorgenson, Dale (1966). "The Embodiment Hypothesis", *Journal of Political Economy*, 74, 1-17.

[32] Jorgenson, Dale (1973). "The Economic Theory of Replacement and Depreciation" in W. Sellekaerts, ed., *Econometrics and Economic Theory*, New York: Macmillan.

[33] Jorgenson, Dale and Zvi Grilliches (1967). "The Explanation of Productivity Change", *Review of Economic Studies*, 34, 249-280.

[34] Jorgenson, Dale and Kevin Stiroh (1995). "Computers and Growth", *Economics of Innovation and New Technology*, 3, 295-316.

[35] Jorgenson, Dale and Kevin Stiroh (1999). "Information Technology and Growth", *American Economic Review*, May, 109-115.

[36] Jovanovic, Boyan and Rafael Rob (1997). Solow vs. Solow: Machine Prices and Development, NBER Working Paper No. 5871.

[37] Landefeld, Steven and Robert Parker (1998). "BEA's Chain Indexes, Time Series, and Measures of Long-Term Economic Growth ", *Survey of Current Business*, May, 58-68.

[38] Moulton, Brent and Karin Moses (1997). "Addressing the Quality-Change Issue in the Consumer Price Index", *Brookings Papers on Economic Activity*, 1, 305-49.

[39] Oliner, Stephen (1989). "Constant-Quality Price Change, Depreciation, and Retirement of Mainframe Computers", in *Price Measurements and Their Uses*, ed. Allan Young, Murray Foss, and Marilyn Manser, Chicago: University of Chicago Press.

[40] Oliner, Stephen (1994). Measuring Stocks of Computer Peripheral Equipment: Theory and Application, mimeo, Federal Reserve Board.

[41] Oliner, Stephen and Daniel Sichel (1994). "Computers and Output Growth Revisited: How Big is the Puzzle?", *Brookings Papers on Economic Activity:2*, 273-317.

[42] Sichel, Daniel (1997). *The Computer Revolution: An Economic Perspective*, Washington DC: Brookings Institution Press.

[43] Sichel, Daniel (1999). "Computers and Aggregate Economic Growth: An Update", *Business Economics*, 18-24.

[44] Solow, Robert (1957). "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics*, 39, 312-320.

[45] Solow, Robert (1960). "Investment and Technical Progress" in *Mathematical Methods in the Social Sciences*, 1959, eds. Kenneth Arrow, Samuel Karlin, and Patrick Suppes, 89-104. Stanford, CA: Stanford University Press.

[46] Stiroh, Kevin J. (1998). "Computers, Productivity, and Input Substitution", *Economic Inquiry*, 36, 175-191.

[47] Struckmeyer, Charles S. (1986). "The Impact of Energy Price Shocks on Capital Formation and Economic Growth in a Putty-Clay Technology", *Southern Economic Journal*, 53, 127-140.

[48] Triplett, Jack (1996). "Depreciation in Production Analysis and in Income and Wealth Accounts", *Economic Inquiry*, 33, 93-115.

[49] Young, Allan H. (1989). "BEA's Measurement of Computer Output", *Survey of Current Business*, 108-115.

A Estimating Depreciation Rates from Chained Aggregates

Since 1996, the U.S. National Income and Product Accounts' estimates of aggregate real GDP have been derived using a Fisher chain-aggregation methodology (Landefeld and Parker, 1997). Since 1997, real capital stock aggregates have been constructed using the same methodology. Rather than aggregating all quantities according to their base-year prices, as in the traditional Laspeyres index, chained aggregates define a growth rate that reflects a mix of old and new prices. Given a series of quantities and prices for n goods, $q_i(t)$ and $p_i(t)$, the gross growth rate for the Fisher chain-aggregate quantity is defined to be

$$G = \sqrt{\frac{\sum_{i=1}^{n} p_i(t)\, q_i(t)}{\sum_{i=1}^{n} p_i(t)\, q_i(t-1)} \frac{\sum_{i=1}^{n} p_i(t-1)\, q_i(t)}{\sum_{i=1}^{n} p_i(t-1)\, q_i(t-1)}} \tag{76}$$

In the base-year, all price indexes are set equal to one and the level of real GDP is set equal to nominal GDP. For all subsequent and previous years, real chain-aggregated "level" series are simply grown forward and backwards using the Fisher chained growth rates.

This procedure helps to reduce biases due to valuing at goods at prices that become irrelevant once we move away from the base-year. However, one complexity it introduces is that aggregate real GDP is no longer the additive sum of its components. Consider now the implications of this for the calculation of depreciation rates based on aggregates for investment and the capital stock.

To keep things simple, we will look at the simplest possible case. Suppose there are two types of capital that depreciate at identical rates δ. Capital of type i can be purchased at price $p_i(t)$ and the price of capital of type 1 falls at rate γ relative to the prices of capital of type 2 and the price of output, $p_Q(t)$, which are equal. Firms combine the two types of capital according to a Cobb-Douglas production function

$$Q(t) = A(t)\, K_1(t)^{\alpha}\, K_2(t)^{1-\alpha} \tag{77}$$

There are no adjustment costs or taxes and so firms accumulate both types of capital according to the first-order conditions

$$K_1(t) = \alpha\left(\frac{p_Q(t)\,Q(t)}{p_1(t)\,(r+\delta+\gamma)}\right) \tag{78}$$

$$K_2(t) = (1-\alpha)\left(\frac{p_Q(t)\,Q(t)}{p_1(t)\,(r+\delta)}\right) \tag{79}$$

Consider first the simplest case of Laspeyres fixed-weight aggregate of these two types of capital. We have

$$\begin{aligned} K^{FW}(t) &= K_1(t) + K_2(t) \\ &= (1-\delta)\left(K_1(t-1) + K_2(t-1)\right) + I_1(t) + I_2(t) \\ &= (1-\delta)\,K^{FW}(t-1) + I(t) \end{aligned}$$

The additivity of the Laspeyres formula implies that we can re-arrange a perpetual inventory equation for the fixed-weight aggregate and uncover the underlying rate at which both types of capital depreciate.

Consider now the behavior of the chain-aggregates. The Fisher chain-aggregate is somewhat cumbersome so we will illustrate using a Tornqvist chain aggregation formula. This procedure weights the growth rates of each category according to their share in the nominal aggregate, and produces aggregates with almost identical properties to the Fisher procedure. These conditions imply that the ratio of nominal capital stocks, valued at replacement cost as in the NIPAs, is constant

$$\frac{p_1(t)\,K_1(t)}{p_2(t)\,K_2(t)} = \frac{\alpha}{1-\alpha}\left(\frac{r+\delta}{r+\delta+\gamma}\right) \tag{80}$$

Letting θ be the nominal share of capital of type 1 in the total nominal capital stock, and g_Q be the growth rate of real output, our assumption of steady-state growth implies that the real stock of type 1 is growing at rate $g_Q + \gamma$ while the real stock of type 2 grows at rate g_Q. Thus, the Tornqvist chain aggregate real capital stock grows at rate

$$g_K^{CW} = \theta\,(g_Q + \gamma) + (1-\theta)\,g_Q = g_Q + \theta\gamma \tag{81}$$

Now consider the behavior of a chain-aggregate for real investment. A simple rearranging of the perpetual inventory equation for each type of capital tells us that

$$\frac{I_1(t)}{K_1(t-1)} = g_Q + \delta + \gamma \tag{82}$$

$$\frac{I_2(t)}{K_2(t-1)} = g_Q + \delta \tag{83}$$

Thus the ratios of nominal investment in capital of type 1 relative to capital of type 2 is

$$\begin{aligned}
\frac{p_1(t) I_1(t)}{p_2(t) I_2(t)} &= \left(\frac{p_1(t) I_1(t)}{p_1(t) K_1(t-1)}\right)\left(\frac{p_2(t) K_2(t-1)}{p_2(t) I_2(t)}\right)\left(\frac{p_1(t) K_1(t-1)}{p_2(t) K_2(t-1)}\right) \\
&= \left(\frac{g_Q+\delta+\gamma}{g_Q+\delta}\right)\left(\frac{p_1(t) K_1(t)}{p_2(t) K_2(t)}\right)\left(\frac{K_1(t-1)}{K_1(t)}\right)\left(\frac{K_2(t)}{K_2(t-1)}\right) \\
&= \left(\frac{g_Q+1}{g_Q+\delta}\right)\left(\frac{g_Q+\delta+\gamma}{g_Q+1+\gamma}\right)\left(\frac{p_1(t) K_1(t)}{p_2(t) K_2(t)}\right) > \frac{p_1(t) K_1(t)}{p_2(t) K_2(t)} \qquad (84)
\end{aligned}$$

Thus the share of capital good 1 in nominal investment is larger than its share in the nominal capital stock. The reason for this is intuitive. The real capital stock of type 1 is growing faster than the real stock of type 2. This means that, measured in today's dollars at replacement cost, there is more investment relative to the stock for type 1 than there is for type 2; as a result the nominal share of investment for type 1 is higher.

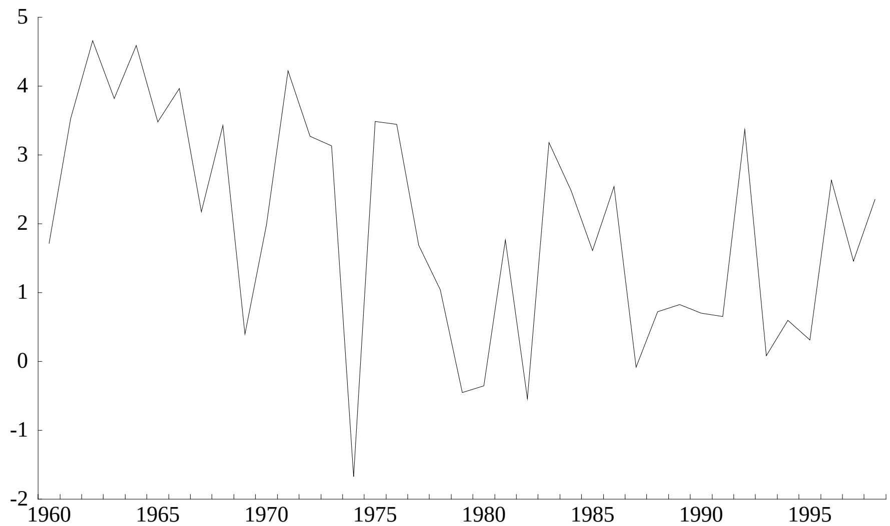
Now, since real investment of type 1 grows at rate $g_Q + \gamma$ while real investment of type 1 grows at rate g_Q, the growth rate of the Tornqvist chain aggregate for real investment places more weight on the faster growing category than does the corresponding growth rate for the aggregate capital stock. Hence our result, the chain aggregate for investment will always grow faster than the chain aggregate for the capital stock. This means that if we try to re-arrange a perpetual inventory equation for aggregate investment and the aggregate capital stock, solving for

$$\delta^{CW}(t) = \frac{I^{CW}(t)}{K^{CW}(t)} - g_K^{CW}(t) \tag{85}$$

Then this value will equal δ only in the base year. Since I^{CW} grows faster than K^{CW} in each period, this "depreciation rate" gets larger each period. More generally, if we allowed the two types of capital to have varying depreciation rates, the depreciation rate estimated from equation 85 would only equal a weighted average of the underlying depreciation rates in the base year, as we move forward from the base year this measure would eventually be higher than each of the underlying depreciation rates. It is this pattern that explains why the depreciation rate for computers and peripherals estimated from equation 85 equals about 0.31 in recent years, even though a very large proportion of the stock is Personal Computers, which are estimated to have a depreciation rate of only 0.11.

Chart 1

Growth in Labor Productivity, U.S. Private Business Sector



Real Investment in Computing Equipment (1992 Dollars)

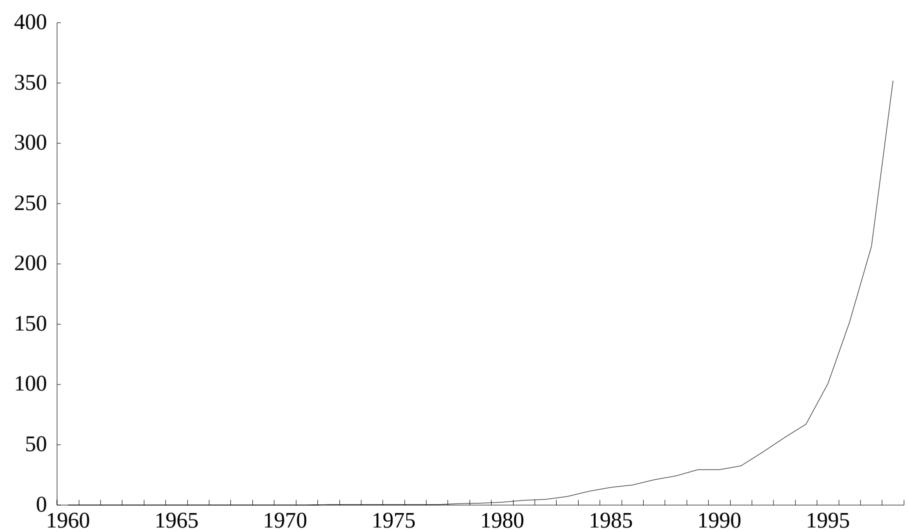


Chart 2
Depreciation Schedules for Computing Equipment

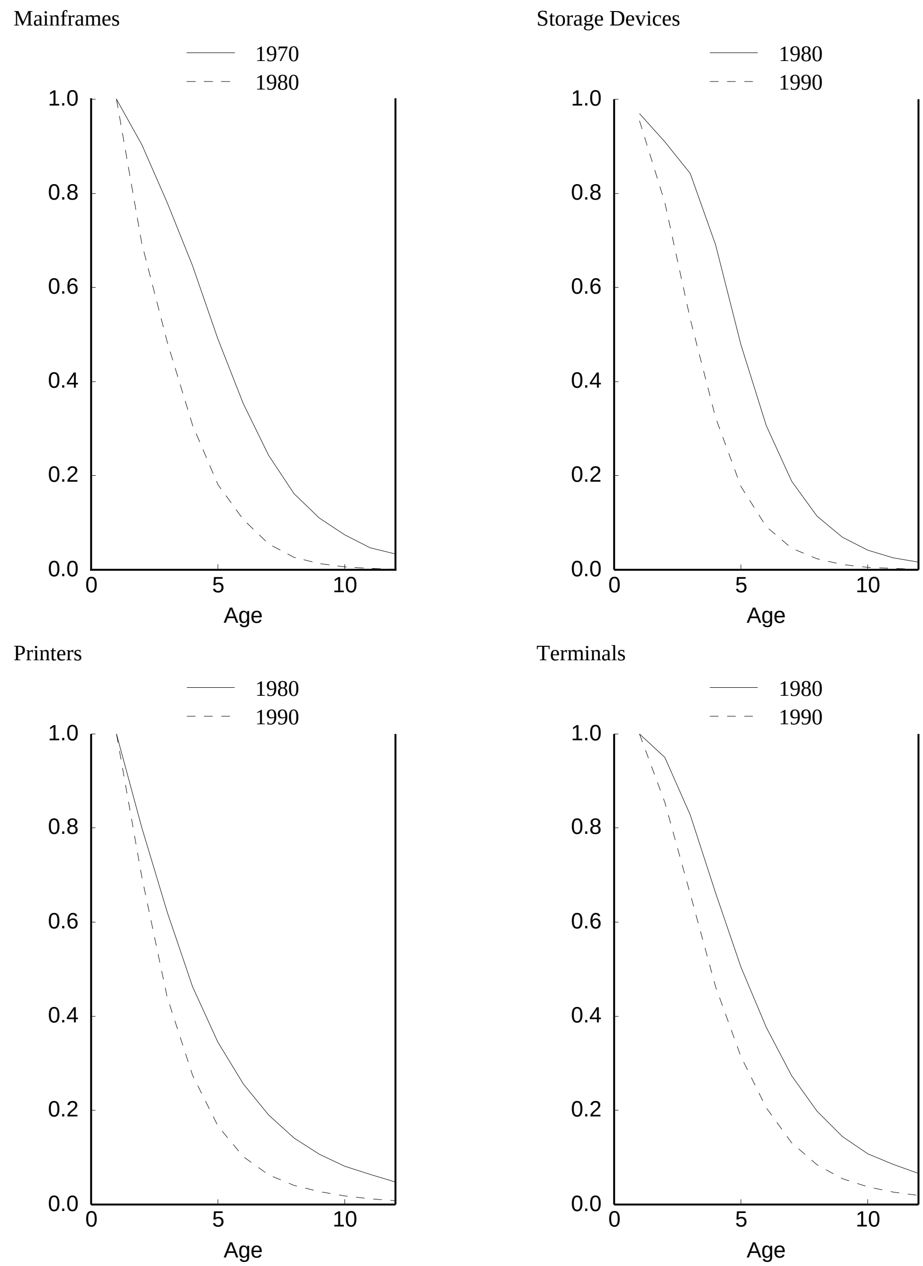


Chart 3
Depreciation Rates for Computing Equipment

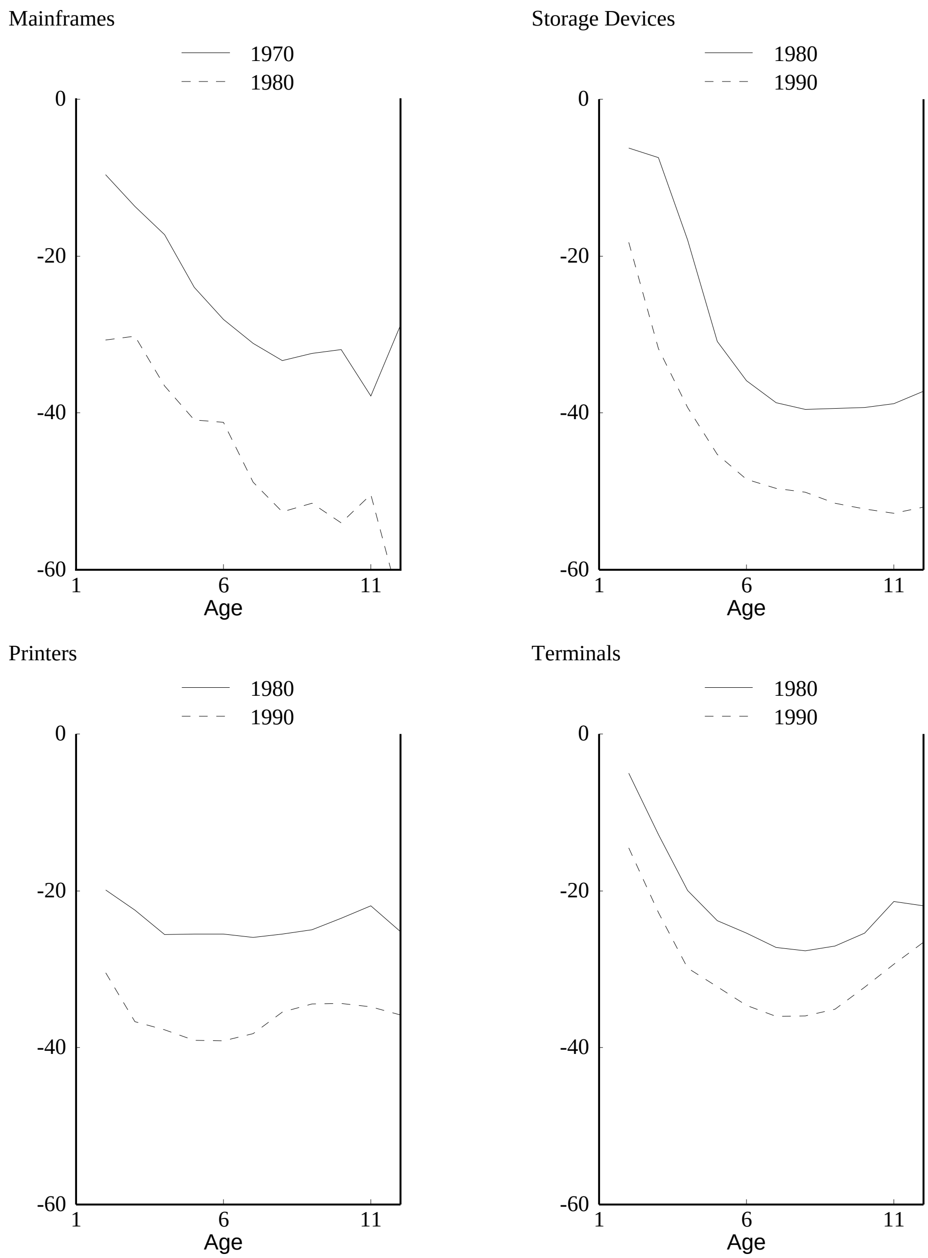
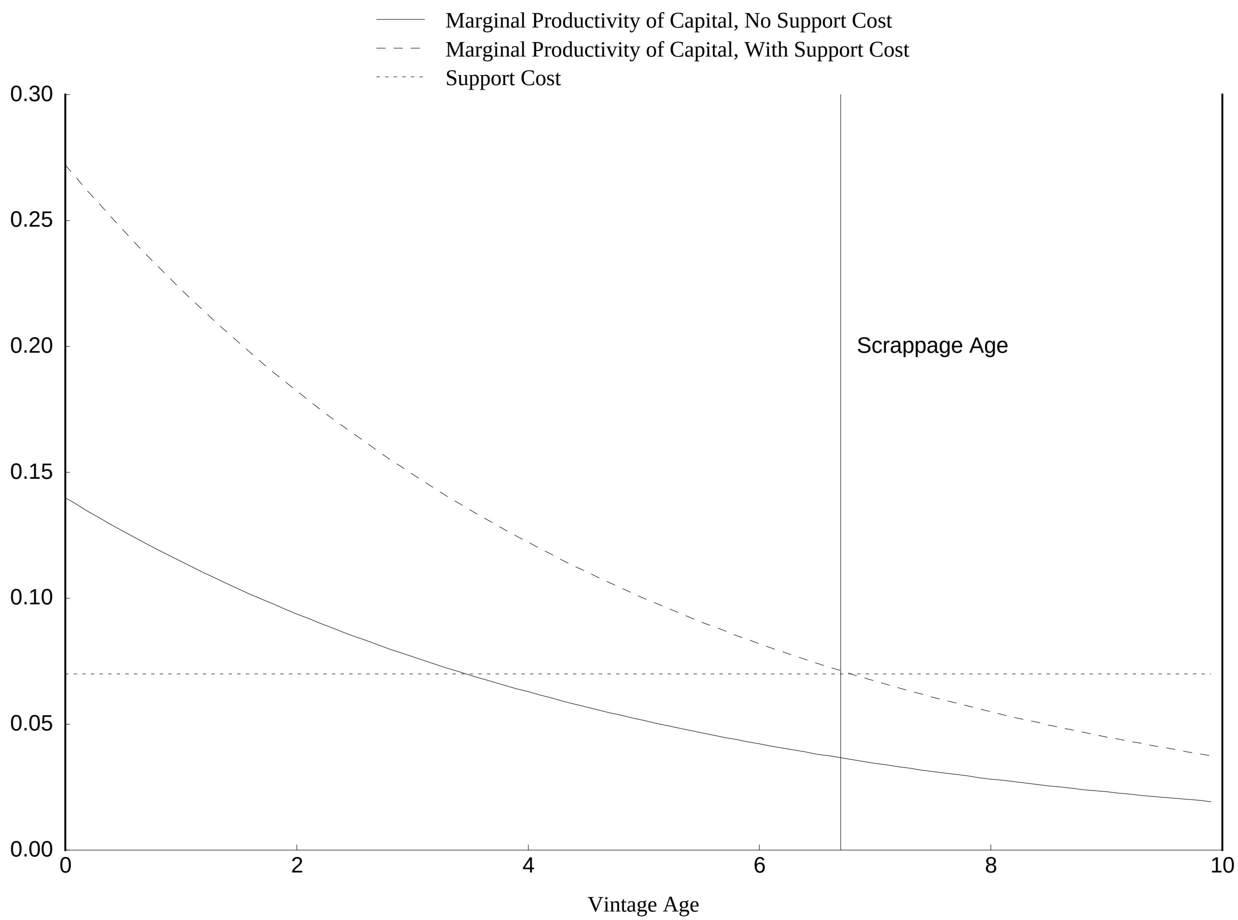


Chart 4
Scrappage and the Marginal Productivity of Capital



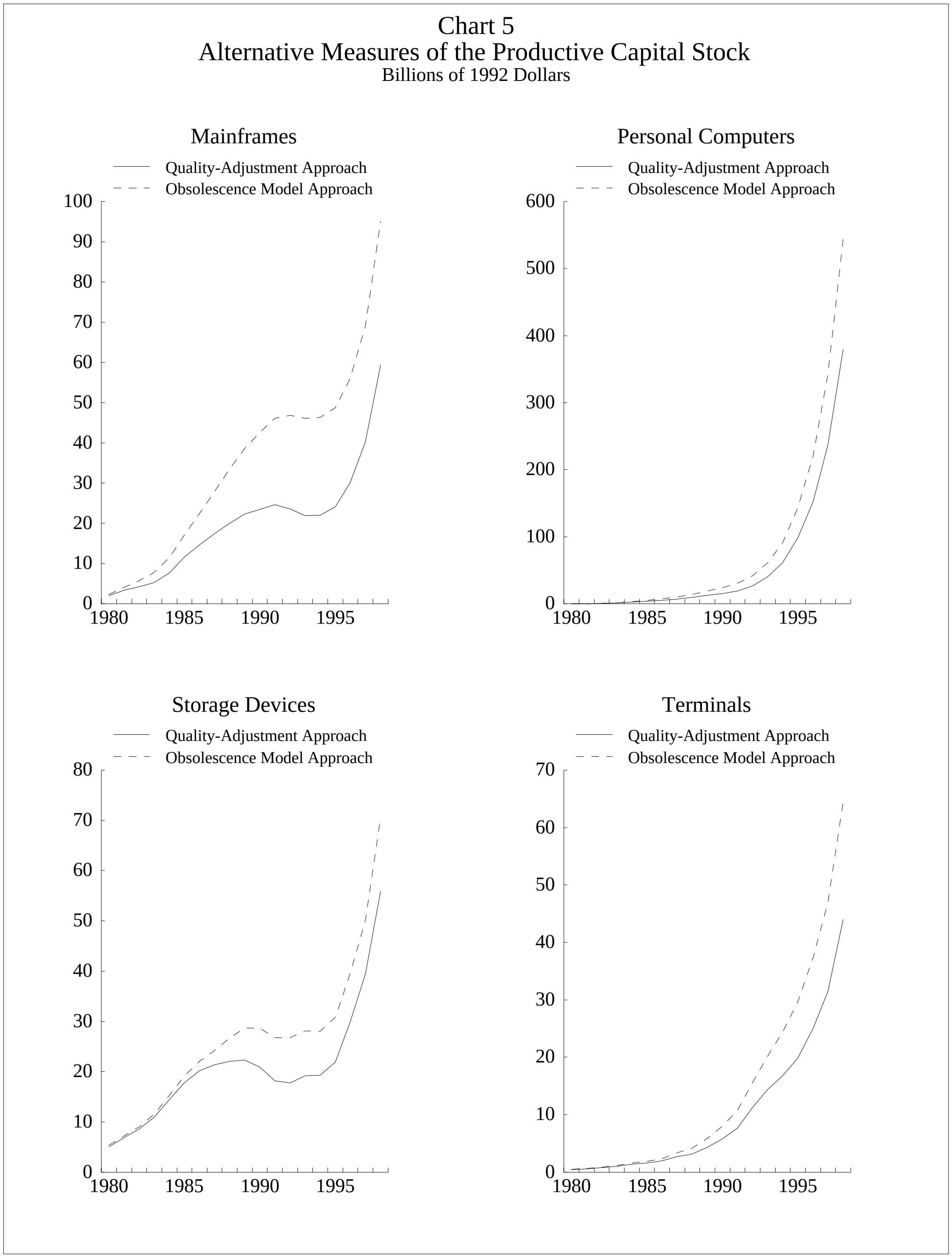


Chart 5
Alternative Measures of the Productive Capital Stock
Billions of 1992 Dollars
Mainframes
Quality-Adjustment Approach
Obsolescence Model Approach
100
90
80
70
60
50
40
30
20
10
0
1980
1985
1990
1995
Personal Computers
Quality-Adjustment Approach
Obsolescence Model Approach
600
500
400
300
200
100
0
1980
1985
1990
1995
Storage Devices
Quality-Adjustment Approach
Obsolescence Model Approach
80
70
60
50
40
30
20
10
0
1980
1985
1990
1995
Terminals
Quality-Adjustment Approach
Obsolescence Model Approach
70
60
50
40
30
20
10
0
1980
1985
1990
1995

Chart 6
Alternative Measures of the Productive Capital Stock
Growth Rates

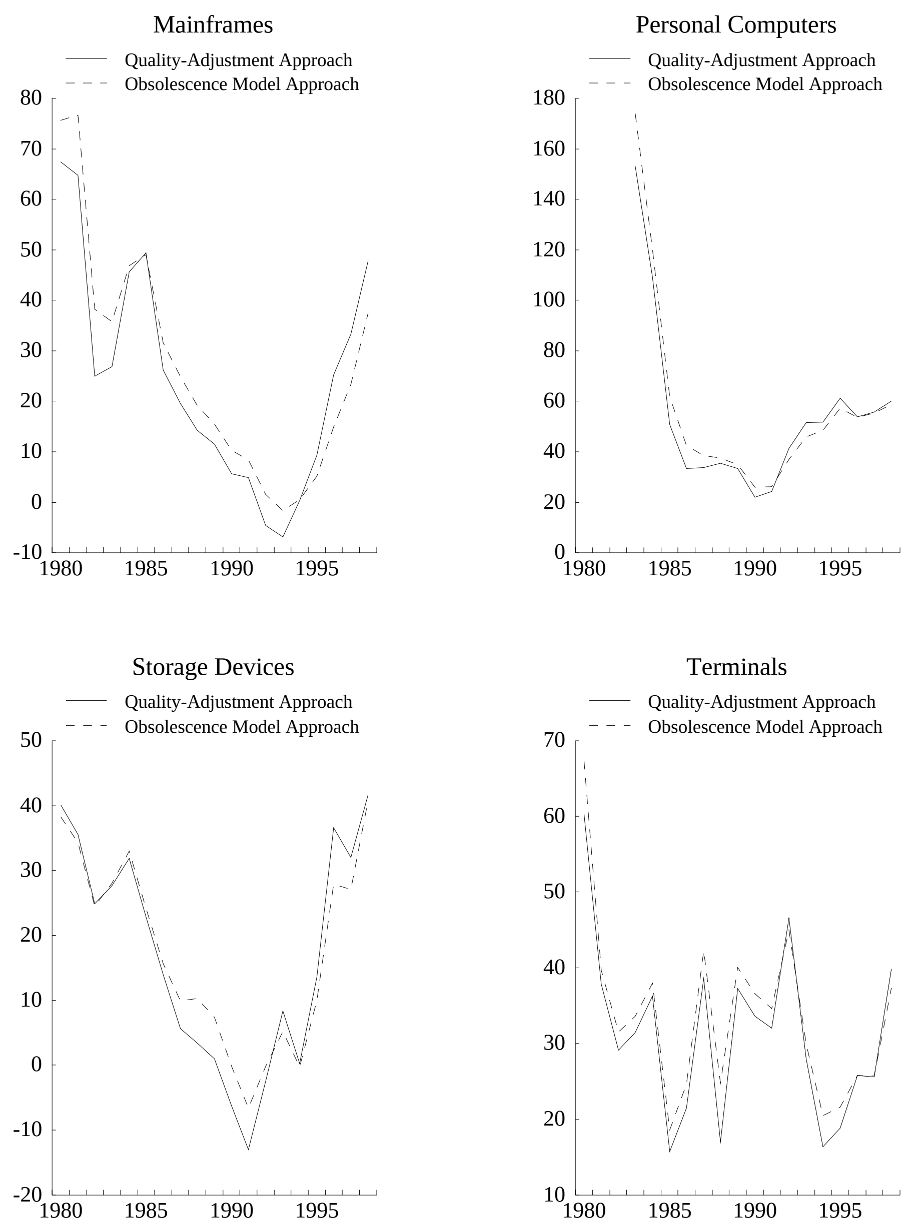


Chart 7
The Contribution of Computer Capital Accumulation to Aggregate Growth

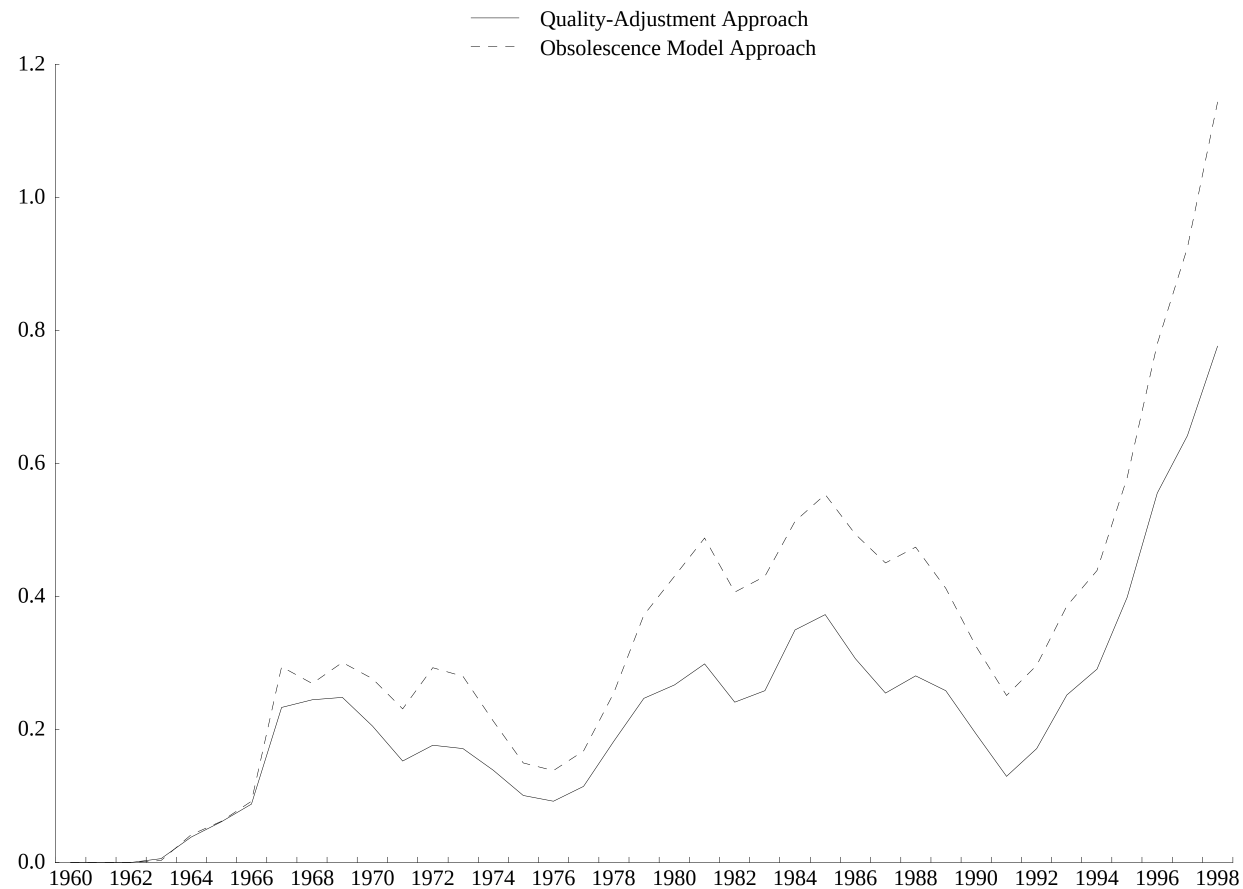


Chart 8
Computers and Aggregate TFP

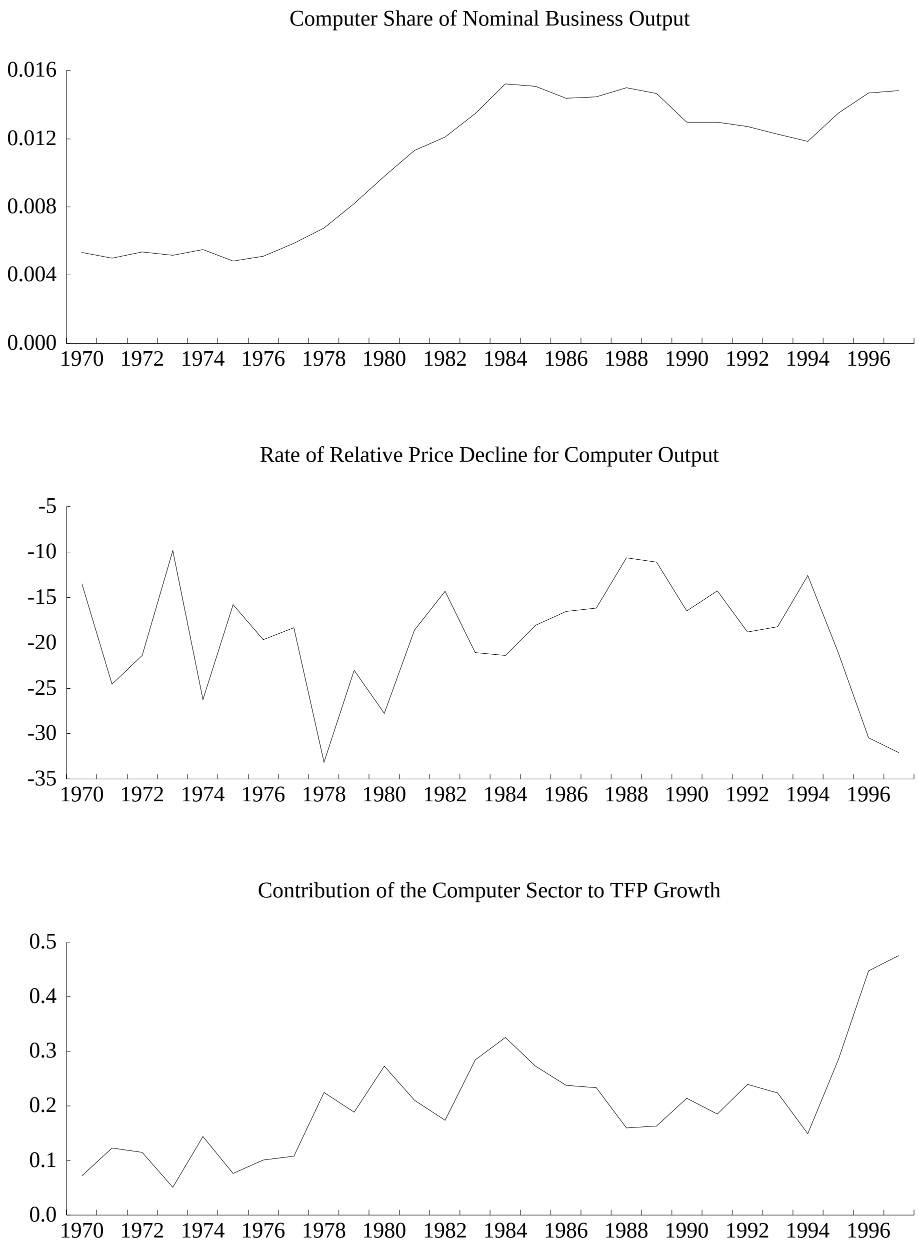


Chart 9
The Contribution of the Computer Sector to Aggregate Growth

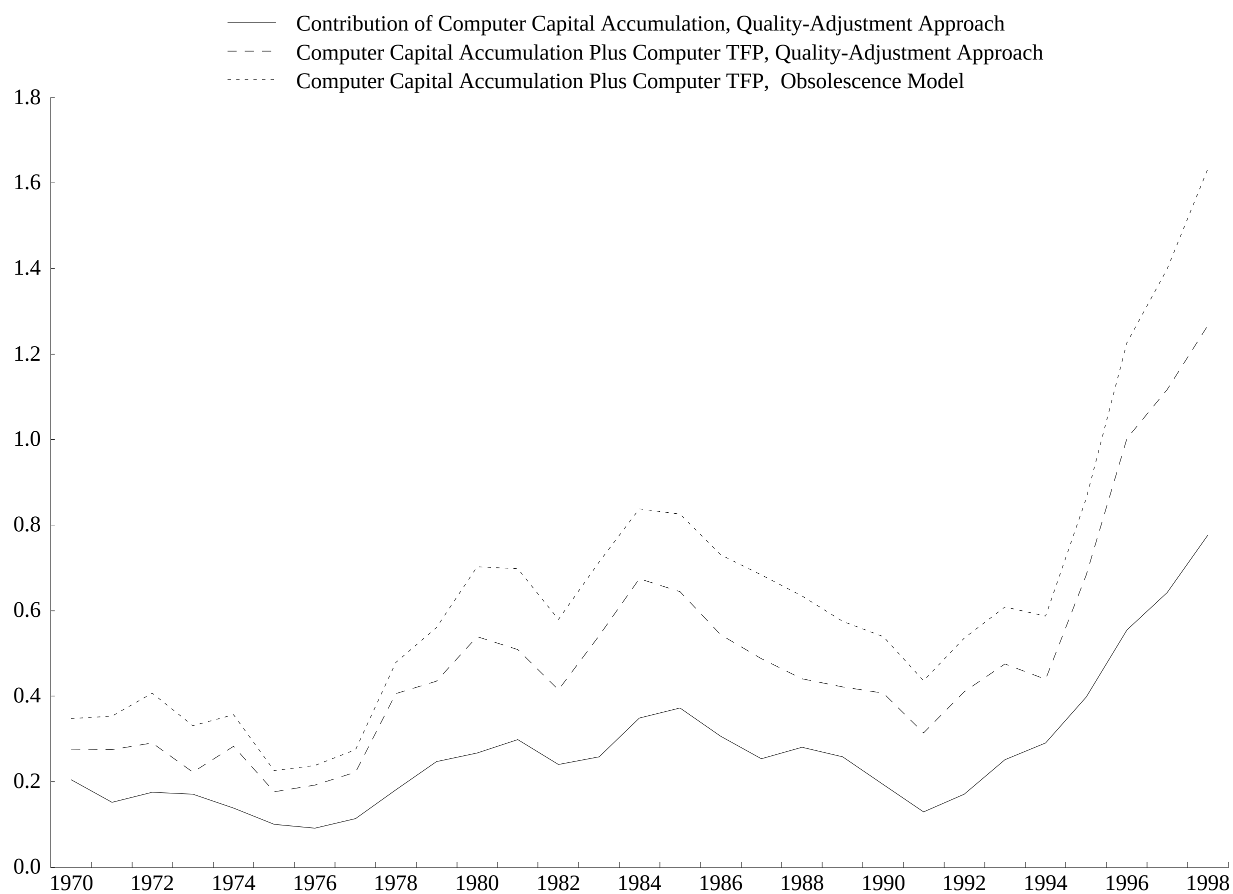


Chart 10
Computers and Productivity Growth