Asymmetric Labor Market Fluctuations in an Estimated Model of Equilibrium Unemployment

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May 2016

The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of San Francisco or the Federal Reserve System.
Introduction

- Cycles can be asymmetric: contractions are often deep, peaks moderate
  - Neo-classical model - central to many policy discussions: symmetric cycles
  - DSGE model estimation - often solved by linear approximation of equilibrium conditions

![Unemployment rate, percent](image)
Introduction

This paper:

- Estimate an model with search frictions labor market incorporating higher order moments
  - Simulated Method of Moments estimation of model solved by projection
  - Model fits skewness and kurtosis of unemployment rate times series

- Particle filter recovers history of shocks to productivity, matching efficiency, job separation
  - Realizations of the states are normally distributed: time series asymmetry of the data generated by the model
  - Focus counterfactuals on the Great Recession: Matching efficiency plays a part in peak unemployment, not the ensuing slow recovery
Take away: Estimation

- Method of solving the model matters: bias in parameter estimates
Take away: State Dependence

- Flip side of deepness: response to shock depend on current state of the economy

![Graph showing response to shock](image-url)
Take away: State Dependence

- Flip side of deepness: response to shock depend on current state of the economy
Take away: State Dependence

- Flip side of deepness: "Stimulating when the labor market is tight is difficult" - Bai, 20 May 2016
Take away: Particle Filter

- Observed time series asymmetry in unemployment model generated
  - Recovered series of exogenous states normally distributed around means

- Productivity shocks drive bulk of business cycle
  - Matching efficiency plays a role in outlier peaks and troughs
  - Little evidence of efficiency explaining the slow recovery
Literature

- Business Cycle Asymmetries: Steep, **deep**, or delayed?
  Focus here on deepness: asymmetry in levels
  - Capacity constraints: limit booms (Hanson and Prescott, 2005)
  - Financial constraints: amplify downturns (Kocherlakota, 2000)
  - Congestion in matching markets: constrain booms and amplify downturns (Petrosky-Nadeau and Zhang, 2013)

- Estimation of Non-linear DSGE models
  - Ruge-Murcia (2012): skewness in the time series is generated by skewness of the shocks
  - Models with search frictions: Cooper, Haltiwanger, Willis (2007)
Model
Model Overview

Standard discrete time model of equilibrium unemployment

- Representative household with a unit measure of workers, employed \((N_t)\) or unemployed \((U_t)\)
- Representative firm with a continuum of jobs
- Random matching between unemployed and vacant jobs \(V_t\)
- Stochastic productivity, matching efficiency, separation rate
Matching

- Number of matches: \( M(U_t, V_t) = \chi_{Lt} U_t^{\eta_L} V_t^{1-\eta_L} \)
  - Stochastic matching efficiency: \( \chi_{Lt} \)

- Exogenous separation
  - Fixed component, \( \delta_C \)
  - Stochastic component, \( \delta_{St} \)

- Law of motion for employment and unemployment
  \[
  U_{t+1} = U_t + (\delta_C + (1 - \delta_C)\delta_{St})N_t - M(U_t, V_t)
  \]
  - \( \partial M(U_t, V_t)/\partial V_t > 0 \) and increasing in \( U_t \)
  - Generates business cycle asymmetry and state dependence
Firm’s Problem

\[
S(N_t, \gamma_t) = \max_{V_t} \left( X_t N_t - W_t N_t - \kappa_t V_t + \beta E_t [S(N_{t+1}, \gamma_{t+1})] \right)
\]

subject to

\[
N_{t+1} = (1 - \delta_C)(1 - \delta_{St})N_t + q(\theta_t)V_t
\]

\[
V_t \geq 0
\]

- Labor market tightness: \( \theta_t = V_t / U_t \)
- Probability of filling vacancy: \( q_t = M(U_t, V_t) / V_t = q(\theta_t) \), \( q'(\theta) < 0 \) and \( q''(\theta) > 0 \)
- \( X_t \): stochastic productivity
- \( \kappa_t = \kappa_0 + \kappa_1 q_t \): vacancy cost
- \( \gamma_t = [X_t, \delta_{St}, \chi_{Lt}] \): vector of stochastic state variables
Job Creation and Wage Conditions

- Firms post jobs until cost of marginal hire equals conditional payoff of a filled vacancy:

\[
\frac{\kappa_0}{q(\theta_t)} + \kappa_1 = \beta E_t \left[ X_{t+1} - W_{t+1} + (1 - \delta_C)(1 - \delta_{St+1}) \left( \frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 \right) \right]
\]

- Nash Bargained wage:

\[
W_t = \alpha_L \left( X_t + \kappa_t \theta_t \right) + (1 - \alpha_L)z
\]

- \( \alpha_L \): worker bargaining weight
- \( z \): flow utility from non-employment (reservation wage)
Stochastic variables

\[
\begin{bmatrix}
\log X_t \\
\log \delta_{St} \\
\log \chi_{Lt}
\end{bmatrix} =
\begin{bmatrix}
\rho_x & 0 & 0 \\
\rho_x \delta_S & \rho_{\delta_S} & 0 \\
0 & 0 & \rho_{\chi_L}
\end{bmatrix}
\begin{bmatrix}
\log X_{t-1} \\
\log \delta_{S,t-1} \\
\log \chi_{L,t-1}
\end{bmatrix} +
\begin{bmatrix}
\sigma_x & 0 & 0 \\
0 & \sigma_{\delta_S} & 0 \\
0 & 0 & \sigma_{\chi_L}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{X_t} \\
\varepsilon_{\delta_{St}} \\
\varepsilon_{\chi_{Lt}}
\end{bmatrix}
\]

- All \( \varepsilon \) are iid, standard normal
Estimation
Estimation - Simulated Method of Moments

- Vector of 14 parameters $\omega$

- Vector of 14 moments of interest in the data $\mu$ and model $\mu_s(\omega)$
  - Model solved by projection (Petrosky-Nadeau and Zhang, 2013)
  - Moments averaged over $S = 1000$ simulations

- Minimize the distance $g(\omega) = \left( \mu - \frac{1}{S} \sum_{s=1}^{S} \mu_s(\omega) \right)$:
  $$\hat{\omega} = \arg \min_{\omega} g(\omega)'W^{-1}g(\omega)$$

- Newey-West standard errors, optimal weighting matrix $W$
Estimation - Data

U.S., monthly, April 1976 to Dec. 2015:

- **Unemployment rate**: civilian population over 16 (BLS)

- **Job vacancy rate**: constructed from Conference Board Help-Wanted index, Barnichon, and JOLTS (Petrosky-Nadeau and Zhang, 2013)

- **Job separation rate**: CPS employment to unemployment transition rate, provided by Rob Valleta

- **Wage**: Compensation pour hour (BEA)

- **Job filling rate**: CPS unemployment to employment transition rate $\times$ v-u raito; lines up with JOLTS

- **Labor productivity**: real output per worker (BEA)
Estimation Results - data and model moments

- Transform to quarterly data (averages), HP filter proportional deviations from mean

**Labor market moments**

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<tr>
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<th>Model</th>
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<td>mean(U)</td>
<td>0.064</td>
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<td>Unemployment: volatility</td>
<td>(\sigma_U)</td>
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<td>(V, U) correlation:</td>
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<td>Wage volatility</td>
<td>(\sigma_W)</td>
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<td>Vacancy filling rate: mean</td>
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### Stochastic process moments

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<tr>
<td>Separation: mean</td>
<td>( \text{mean}(\delta_{St}) )</td>
<td>0.044</td>
</tr>
<tr>
<td>Separation: volatility</td>
<td>( \sigma_{\delta_S} )</td>
<td>0.042</td>
</tr>
<tr>
<td>Separation: autocorrelation</td>
<td>( \text{corr}(\delta_{St}, \delta_{St-1}) )</td>
<td>0.970</td>
</tr>
<tr>
<td>(V, ( \delta_S )) correlation</td>
<td>( \text{corr}(V_t, \delta_{St}) )</td>
<td>-0.854</td>
</tr>
<tr>
<td>Productivity: volatility</td>
<td>( \sigma_X )</td>
<td>0.009</td>
</tr>
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<td>Productivity: autocorrelation</td>
<td>( \text{corr}(X_t, X_{t-1}) )</td>
<td>0.735</td>
</tr>
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Estimation Results - parameter estimates

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<th>Parameter Description</th>
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<th>S.E.</th>
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<td>Matching function: elasticity $\eta_L$</td>
<td>0.566</td>
<td>0.115</td>
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<td>Matching function: mean efficiency $\bar{\chi}_L$</td>
<td>0.491</td>
<td>0.267</td>
</tr>
<tr>
<td>Worker bargaining weight $\alpha_L$</td>
<td>0.197</td>
<td>0.111</td>
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<td>Vacancy cost $\kappa_{V_0}$</td>
<td>0.056</td>
<td>0.040</td>
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<td>Fixed hiring cost $\kappa_{V_1}$</td>
<td>0.477</td>
<td>0.393</td>
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<td>Unemployment value: $z$</td>
<td>0.887</td>
<td>0.002</td>
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\log X_t \\
\log \delta_{S_t} \\
\log \chi_{Lt}
\end{bmatrix} =
\begin{bmatrix}
.966 & 0 & 0 \\
-.819 & .765 & 0 \\
0 & 0 & .356
\end{bmatrix}
\begin{bmatrix}
\log X_{t-1} \\
\log \delta_{S_{t-1}} \\
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\end{bmatrix} +
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.005 & 0 & 0 \\
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Model Dynamics and Particle Filter
Impulse Response Functions - market tightness

Standard deviation shock to $X$ at peak

Petrosky-Nadeau, Tengelsen

DMP Estimated
Impulse Response Functions - market tightness

Standard deviation shock to X at trough

Petrosky-Nadeau, Tengelsen

DMP Estimated
Impulse Response Functions - unemployment

Standard deviation shock to X at peak

Petrosky-Nadeau, Tengelsen  
DMP Estimated

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Impulse Response Functions - unemployment

- Negative shock at trough
- Positive shock at trough
- Negative shock at peak
- Negative shock at peak

Standard deviation shock to X at trough

Petrosky-Nadeau, Tengelsen

DMP Estimated
"Stimulating when the labor market is tight is difficult," Bai, May 2016
Particle Filter

- Particle filter determines the most likely sequence of innovations 
  \( v_t = \{\nu_t^x, \nu_t^{\chi_L}, \nu_t^{\delta_s}\} \) in order for the model to generate the observed 
  \( U_t, V_t \) and \( \delta_{st} \)

- The sequence of innovations yields a series of \( \gamma_t = [X_t, \delta_{St}, \chi_{Lt}] \) 
  over 1976 to 2015

- Perform counterfactuals:
  - Remove matching efficiency shocks
  - Focus on the Great Recession
Particle Filter - Results

- Technology series $\{\hat{X}_t\}_{t=1}^T$: normally distributed around mean
- Matching efficiency $\{\hat{\chi}_{Lt}\}_{t=1}^T$: small, symmetric fluctuations
Counterfactuals

- Set $\hat{\chi}_{Lt} = \bar{\chi}_{Lt} \ \forall \ t$, impulse model with $\{\hat{X}_t\}_{t=1}^T$ and $\{\delta_{St}\}_{t=1}^T$
Counterfactuals

- Set $\hat{\chi}_Lt = \bar{\chi}_Lt \ \forall \ t$, impulse model with $\{\hat{X}_t\}_{t=1}^T$ and $\{\hat{\delta}_St\}_{t=1}^T$.
Great Recession

- Fix matching efficiency to its Dec. 2007, pre-Great Recession, level
Great Recession

- Fix matching efficiency to its Dec. 2007, pre-Great Recession, level

![Graph showing the relationship between unemployment rate and job vacancy rate with two lines representing all shocks and Dec. 2007 matching efficiency.](image)
Conclusion - discussion

• A work in progress with promise:
  ◦ Well understood framework fits the non-linear dynamics of the data
  ◦ State dependence with important implications for policy analysis and counterfactuals

• Important elements still to incorporate:
  ◦ Endogenous job separation
  ◦ Endogenous labor productivity and include moments in estimation (cross-correlation)

• Most important: extend to frictional financial market
  ◦ Market well described by search friction
  ◦ Similar (more pronounced) time series asymmetries - returns, spreads, ...