# Interest on Reserves, Interbank Lending, and Monetary Policy: Work in Progress

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## 1 Introduction

When a central bank operates under a floor system in which there are reserves outstanding in the financial system each night, arbitrage should dictate that the overnight interest rate is determined by the interest rate on reserves set by the central bank. Indeed, during the period April 2009 to May 2010, the Bank of Canada successfully pegged the overnight interest rate to 0.25%, which was the interest rate on Bank of Canada reserves at the time, by targeting overnight aggregate reserve balances at $3 billion (Canadian).

However, in the United States, where the quantity of reserves in the financial system has been verylarge since the financial crisis, and has grown to almost 2/3 of total Fed liabilities, the fed funds rate has been significantly less than the interest rate on reserves over the entire period that the Fed has operated under a floor system. Further, the margin between the interest rate on reserves and the fed funds rate has been large, and variable.

The goal of this paper is to construct a model that can explain these observations. As well, we want to show that the reasons for the overnight interest rate differential (the difference between the interest rate on reserves and the fed funds rate) in the United States have important implications for the effects of monetary policy actions, and for how policy should be conducted.

Though there seem to be no good models to explain why the overnight interest rate differential exists, a potential explanation (see work by Martin et al.) is related to costly arbitrage. In particular, (i) not all financial institutions can hold reserve accounts; (ii) for financial institutions that hold reserves, there are "balance sheet costs" associated with doing so; (iii) some financial institutions that hold reserve accounts cannot receive interest on reserves. The financial institutions that do not have reserve accounts include money market mutual funds (a large share of financial intermediation activity in the United States), and the financial institutions that hold reserves accounts but do not receive interest are the government sponsored enterprises (GSEs) including Fannie Mae, Freddie Mac, and the Federal Home Loan Banks (FHLBs). The balance sheet costs of holding reserves are mainly deposit insurance premia (tied to total assets) and capital requirements.

Currently, the majority of federal funds market activity consists of attempts by FHLBs to obtain interest on reserve balances by lending overnight to branches of foreign banks operating in the United States, which tend to have low balance sheet costs. Thus, perhaps the FHLBs lend to banks at an overnight rate less than the interest rate on reserves to compensate these banks for balance sheet costs. But why should these costs be variable, which is what we would need to explain the variable overnight interest rate differential that we see in practice?

The model constructed in this paper has two types of financial intermediaries: retail banks, and unconventional banks. The retail banks are meant to stand in for the regulated commercial banking sector, while unconventional banks stand in for unregulated financial intermediaries, including money market mutual funds and the shadow banking sector. In the model, retail banks serve small-transaction depositors who sometimes need currency, and these banks provide a deposit contract that permits currency withdrawals. Unconventional banks serve large-transaction depositors who sometimes need government securities to make repo transactions. There exists an interbank market on which banks can borrow and lend.

In the model, only retail banks can hold reserves, but these retail banks also have capital requirements, which will imply balance sheet costs associated with borrowing on the interbank market. The central bank can conduct asset swaps of central bank liabilities (currency and reserves) for government debt, and can set an interest rate on reserves. There also exist some private assets in the model – Lucas trees – which are intermediated by banks.

Depending on the relative supplies of assets available – determined by the quantity of private assets in existence, and by monetary policy – the interbank market may be inactive or active. If it is active, then the interbank market serves as an indirect means for unconventional banks to hold reserves. In a baseline case, there is a large enough supply of private assets and government debt that private assets are held by both retail and unconventional banks, and there is more than enough government debt to finance all repo transactions by large-scale depositors. The interbank market is inactive, and interest rates are equal for reserves, the interbank market, and government debt. But, if either asset stock is low, the interbank market becomes active. In the extreme case where government debt and private assets are sufficiently scarce, the interest rate on reserves is greater than the interbank interest rate, which is greater than the interest rate on government debt. This looks like the current configuration of interest rates in the short term U.S. financial market.

We get interesting implications of monetary policy actions in this setting. For example, in the extreme case where government debt and private assets are scarce, an expansion of the central bank's balance sheet (a swap of reserves for government debt by the central bank) expands the retail banking sector, contracts the unconventional banking sector, and makes everyone worse off. Basically, this action tightens the collateral constraints of banks (because of capital requirements in the retail banking sector), and reduces the quantity of government bonds available for repo transactions.

## 2 Model

There exists a continuum of small-transaction buyers with mass $\alpha$, and a continuum of large-transaction buyers with mass $1-\alpha$, where $0<\alpha<1$. Each buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ -H_t + \theta^i u(x_t) \right],$$

where $H_t$ is labor supply in the $CM$, $x_t$ is consumption in the $DM$, $0<\beta<1$, and $i=s,l$ denotes the buyer's type, where $i=s,l$ denotes, respectively, a small-transaction and large-transaction buyer. Assume that $\theta^s=1$ (a normalization) and $\theta^l>1$. There also exists a continuum of sellers with unit mass, each of whom has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ -X_t + h_t, \right]$$

where $X_t$ denotes consumption in the $CM$, and $h_t$ denotes labor supply in the $DM$. Finally, there exists a continuum of bankers with unit mass, each of whom has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ -X_t + H_t. \right]$$

The consolidated government issues one-period nominal government bonds, each of which sells for $z_t^b$ units of money in the $CM$ and is a claim to one unit of money in the next $CM$; reserves, which sell at the price $z_t^m$ in the $CM$ and represent claims to money in the next $CM$; and currency, which sells at price $\phi_t$ in the $CM$. There also exists a unit mass of Lucas trees, each of which pays off $y$ units of consumption goods in the $CM$ and trades in the $CM$ at the price $\psi_t$.

In the $CM$, all agents meet at the beginning of the period, debts are paid off, and the holders of Lucas trees receive their payoffs. Then, production and consumption takes place, assets are traded, and economic agents write contracts with financial intermediaries. At the end of the period, each buyer can contact one banker.

In the $DM$, each buyer is randomly matched with a seller, and the buyer makes a take-it-or-leave-it offer to the seller. Assume limited commitment (no one can be forced to work), and no memory, so that the environment does not support unsecured personal credit. For small-transaction buyers, there is a probability $\rho$ that the buyer meets a seller who cannot verify what is in the buyer's portfolio, so the small transaction buyer must then trade currency for goods. With probability $1-\rho$ the seller can verify the existence of all assets in the buyer's portfolio, and it is possible for the buyer to transfer ownership of these assets to the seller. Similarly, for a large-transaction buyer, there is a probability $\pi$ that the buyer meets a seller who cannot verify the existence of assets in the buyer's portfolio, other than liabilities of the consolidated government. Further, the seller will not accept currency in large-scale transactions, as this is too costly, so in this $\pi$ fraction of meetings, only nominal government bonds or reserves are accepted. In the other $1-\pi$ fraction of meetings, the seller can verify what assets are in the buyer's portfolio, and will accept anything. At the beginning of the $CM$, buyers do not know what type of seller they will meet in the next $DM$, i.e. what assets will be accepted by the seller in exchange. However, each buyer learns this at the end of the $CM$.

# 3 Banks' Problems

In equilibrium, banks engage in two different types of intermediation activity. *Retail banks* offer deposit contracts to small-transaction buyers, while *unconventional banks* offer deposit contracts to large-transaction buyers. Retail banks are permitted by the government to hold reserves, while unconventional banks cannot. A retail bank offers a deposit contract $(k^s, c, d^s)$ to small-transaction buyers, which requires the depositor to deposit $k^s$ units of the consumption good with the bank at the beginning of the current $CM$, in return for an option either to withdraw $c$ units of currency at the end of the current $CM$, or receive a tradeable claim to $d^s$ units of consumption in the next $CM$. We will confine attention to stationary equilibria in which prices are constant for all $t$, and $\mu$ denotes the gross inflation rate. Given take-it-or-leave it offers by the small-transaction buyers in the $DM$, the expected utility of a small-transaction buyer from the deposit contract is

$$U^s = -k^s + \rho u\left(\frac{\beta c}{\mu}\right) + (1-\rho)u\left(\beta d^s\right). \tag{1}$$

The retail bank takes in deposits, borrows on the interbank market, and invests in a portfolio of currency, government bonds, reserves, and Lucas trees. The bank diversifies across a positive mass of depositors, exploiting the law of large numbers, and must earn a nonnegative expected return on each deposit contract, i.e.

$$k^s + z^f f^s - z^b b^s - z^m m - \psi q^s - \rho c - \beta(1-\rho)d^s + \frac{\beta(m + b^s - f^s)}{\mu} + \beta q^s(\psi + y) \geq 0 \tag{2}$$

In (2), $f^s$ denotes the quantity of interbank loans the retail bank receives, where each loan is a promise to pay one unit of money in the next $CM$, and $z^f$ denotes the price of a loan in terms of money in the current $CM$. In addition, in (2), $b^s$, $m$, and $q^s$ denote, respectively, the quantity of government bonds, reserves (both in real terms), and Lucas trees acquired by the bank in the current $CM$. Bank liabilities are subject to limited commitment, but if a bank defaults the creditors can seize the bank's assets. Further, the government imposes a capital requirement on the retail bank, i.e. the maturity value of the bank's liabilities cannot exceed a fraction $1-\delta$ of the maturity value of the bank's assets, where

$0 < \delta < 1$. This requires that the bank finance part of its portfolio of assets by working and producing goods in the $CM$, so that the bank effectively supplies its own capital. Thus, the incentive constraint that guarantees that the bank does not default in equilibrium is

$$-(1-\rho)d^s - \frac{f^s(1-I)}{\mu} + \frac{(m+b^s-f^sI)(1-\delta)}{\mu} + q^s(\phi+y)(1-\delta) \geq 0. \quad (3)$$

In equilibrium, a retail bank chooses $k^s$, $c$, $d^s$, $f^s$, $b^s$, $m$, and $q^s$ to maximize $U^s$ subject to (2) and (3). If $f^s > 0$ (the retail bank borrows on the interbank market), then $I = 0$, whereas if $f^s < 0$ (the retail bank lends on the interbank market), then $I = 1$.

An unconventional bank has no capital requirement, it cannot hold reserves, and it offers deposit contracts to large-transaction depositors. Then, in a manner similar to how we characterize an equilibrium deposit contract and portfolio allocation for a retail bank, an unconventional bank solves

$$\max_{k^l,b^l,b',d^l,f^l,q^l} \left[ -k^l + \pi\theta u\left(\frac{\beta b^l - \beta b'}{\mu\pi}\right) + (1-\pi)\theta u\left(\beta d^l\right) \right] \quad (4)$$

subject to

$$k^l + z^f f^l - z^b b^l - \psi q^l - \beta(1-\pi)d^l - \frac{\beta f}{\mu} + \frac{\beta b'}{\mu} + \beta q^l(\psi+y) \geq 0 \quad (5)$$

$$-(1-\pi)d^l - \frac{f^l}{\mu} + \frac{b'}{\mu} + q^l(\psi+y) \geq 0. \quad (6)$$

Here, note that $b'$ is the quantity of government bonds that are acquired in the current $CM$, and are not exchanged in the $DM$ by depositors who meet sellers who will accept only government bonds.

# 4 Equilibrium

In solving the retail bank's problem, maximizing (1) subject to (3) and (4), note first that (3) holds with equality. Then, in equilibrium, the following must hold:

$$\beta u'\left(\beta d^s\right) - \beta - \lambda^s = 0 \quad (7)$$

$$\frac{\beta}{\mu}u'\left(\frac{\beta c}{\mu}\right) - 1 = 0 \quad (8)$$

$$z^f \leq \frac{\beta}{\mu} + \frac{1}{\mu}\lambda^s \quad (9)$$

$$z^f \geq \frac{\beta}{\mu} + \frac{(1-\delta)\lambda^s}{\mu} \quad (10)$$

$$z^b \geq \frac{\beta}{\mu} + \frac{(1-\delta)\lambda^s}{\mu} \quad (11)$$

$$z^m = \frac{\beta}{\mu} + \frac{(1-\delta)\lambda^s}{\mu} \tag{12}$$

$$-\psi + \beta(\psi + y) + \lambda^s(\psi + y)(1-\delta) \leq 0 \tag{13}$$

Equations (7) and (8) characterize the optimal deposit contract offered by the retail bank, where $\lambda^s$ denotes the multiplier on the incentive constraint (3). In equilibrium, the stock of reserves must be held by retail banks, so (12) must hold, which states that the next discounted payoff to the retail bank to holding reserves must be zero. However, a retail bank may or may not be active on the interbank market (as a lender or borrower), and it may or may not hold government bonds or Lucas trees. Thus, (9), (10), (11), and (13) are weak inequalities, i.e. the net payoff to borrowing or lending on the interbank market, to holding government debt, and to holding Lucas trees, must not be strictly positive.

Similarly, for unconventional banks, from the problem (4)-(6), the following must hold in equilibrium:

$$\beta\theta u'\left(\beta d^l\right) - \beta - \lambda^l = 0 \tag{14}$$

$$\frac{\beta}{\mu}\theta u'\left(\frac{\beta\left(b^l - b'\right)}{\mu\pi}\right) - z^b = 0 \tag{15}$$

$$u'\left(\frac{\beta\left(b^l - b'\right)}{\mu\pi}\right) = u'\left(\beta d^l\right), \text{ if } b' > 0, \tag{16}$$

$$u'\left(\frac{\beta\left(b^l - b'\right)}{\mu\pi}\right) \geq u'\left(\beta d^l\right), \text{ if } b' = 0, \tag{17}$$

$$z^f = \frac{\beta}{\mu} + \frac{1}{\mu}\lambda^l \tag{18}$$

$$-\psi + \beta(\psi + y) + \lambda^l(\psi + y) = 0 \tag{19}$$

Equations (14)-(17) determine the optimal deposit contract for an unconventional bank. Equation (18) states that the unconventional bank must be indifferent in equilibrium between borrowing and lending on the interbank market, while equation (19) must hold as the unconventional bank must hold Lucas trees in any equilibrium.

We can express the consolidated government's budget constraints in a stationary equilibrium as

$$z^m\bar{m} + z^b\bar{b} + \bar{c} = \tau_0, \tag{20}$$

$$\left(z^m - \frac{1}{\mu}\right)\bar{m} + \left(z^b - \frac{1}{\mu}\right)\bar{b} + \left(1 - \frac{1}{\mu}\right)\bar{c} = \tau, \tag{21}$$

where $\tau_0$ is the lump sum transfer to each buyer in the $CM$ in period 0, and $\tau$ is the lump sum transfer to each buyer in the $CM$ of each succeeding period. As well, $\bar{m}$, $\bar{b}$, and $\bar{c}$ denote, respectively, the real quantities of reserves, government bonds, and currency outstanding. We will assume a particular fiscal policy rule, which is that the government sets taxes so that

$$z^m \bar{m} + z^b \bar{b} + \bar{c} = V, \tag{22}$$

where $V > 0$ is constant, i.e. the fiscal authority pegs the real value of the outstanding consolidated government debt to a constant forever. This implies that $\tau_0 = V$ is exogenous, but $\tau$ is endogenous, and from (21) and (22) it is determined by

$$V\left(1 - \frac{1}{\mu}\right) + \frac{\bar{m}}{\mu}(z^m - 1) + \frac{\bar{b}}{\mu}(z^b - 1) = \tau \tag{23}$$

**Definition 1** *A stationary equilibrium with binding collateral constraints consists of quantities* $(\bar{m}, \bar{b}, \bar{c}, d^s, d^l, c, m, f^s, b^s, q^s, b^l, b', q^l, f^l)$, *prices* $(z^f, \psi)$, *multipliers* $(\lambda^s, \lambda^l)$, *and gross inflation rate* $\mu$, *satisfying (3) and (6) with equality, (7)-(19), (22), and market clearing,*

$$\alpha b^s + (1-\alpha) b^l = \bar{b}, \text{ [government bond market clears]} \tag{24}$$

$$\alpha \rho c = \bar{c}, \text{ [market in currency clears]} \tag{25}$$

$$\alpha q^s + (1-\alpha) q^l = 1, \text{ [market in Lucas trees clears]} \tag{26}$$

$$\alpha m = \bar{m}, \text{ [market in reserves clears]} \tag{27}$$

$$\alpha f^s + (1-\alpha) f^l = 0, \text{ [interbank market clears]} \tag{28}$$

*given fiscal policy* $V$ *and monetary policy* $(z^m, z^b)$.

The equilibria we will be focusing on are ones in which the collateral constraints of banks bind, so the multipliers $\lambda^s$ and $\lambda^l$associated respectively with the collateral constraints (3) and (6) are strictly positive. For such equilibria to exist requires that $V$ be sufficiently small. Thus, the value of the consolidated government debt needs to be small enough so that the assets that are needed to support exchange are in short supply. We will investigate this further when we construct equilibria.

As well, we have specified monetary policy in terms of $(z^m, z^b)$, with the asset quantities $\bar{m}$, $\bar{b}$, and $\bar{c}$ endogenous. There is thus an underlying set of central bank actions that are required to support the equilibrium given $(z^m, z^b)$. At the beginning of the $CM$ in any period, the total value of central bank liabilities is $z^m \bar{m} + \bar{c}$, i.e. the current value of reserves and currency outstanding, and the value of government bonds held by the central bank is $V - z^b \bar{b}$. Given the consolidated government budget constraint, central bank liabilities are equal to central bank assets, of course.

In equilibrium any exchange on the interbank market will be between retail banks and unconventional banks. As we will see, under some conditions, trade on the interbank market will be zero, but when that is not the case unconventional banks will be lenders and conventional banks will be borrowers. Basically, trade on the interbank market will be a means for unconventional banks to hold reserves indirectly. In what follows, we will examine the four different types of equilibria that can arise. In the first two the interbank market is inactive, and in the latter two it will be active.

# 5 Baseline Case: Inactive Interbank Market; Retail and Unconventional Banks Hold Government Bonds and Lucas Trees

In the baseline case (as in all the cases we examine), assets will be scarce in the sense that there are insufficient quantities of government liabilities, central bank liabilities, and private assets (Lucas trees) to finance efficient exchange. But financial markets will not be segmented, in that all financial intermediaries hold government bonds and Lucas trees as liabilities, and some government bonds are used to back the deposits of unconventional banks. In this equilibrium, the fact that retail banks and unconventional banks participate in the same asset markets will imply that the interbank market is inactive, as we will show.

It proves convenient, as in Williamson (2014a, 2014b), to solve for an equilibrium in terms of quantities traded in the $DM$. Let $x_1^s$ and $x_2^s$ denote the consumption of small-transaction buyers (depositors in retail banks), in $DM$ trades where, respectively, sellers accept only currency and accept bank deposits or currency. The quantities $x_1^l$ and $x_2^l$ represent similar consumption quantities for large-transaction buyers, who sometimes meet sellers who will accept only government bonds in exchange.

First, from (7) and (14), we can write the multipliers associated with banks' collateral constraints as

$$\lambda^s = \beta[u'(x_2^s) - 1], \tag{29}$$

$$\lambda^l = \beta[\theta u'(x_2^l) - 1]. \tag{30}$$

Therefore, binding collateral constraints are associated with inefficiency in $DM$ transactions involving bank deposits, as surplus-maximizing transactions imply $u'(x_2^s) = 1$ and $\theta u'(x_2^l) = 1$.

Then, since government bonds are held to back the deposits of retail banks and unconventional banks, from (29), (30), (11), (15), and (16), we get

$$z^b = \frac{\beta}{\mu}\left[(1-\delta)u'\left(x_2^s\right) + \delta\right] = \frac{\beta}{\mu}\theta u'\left(x_2^l\right) = \frac{\beta}{\mu}\theta u'\left(x_1^l\right) \tag{31}$$

Equation (8) gives

$$1 = \frac{\beta}{\mu}u'(x_1^s). \tag{32}$$

From (19) and (30), we can solve for the price of Lucas trees,

$$\psi = \frac{\beta\theta u'\left(x_2^l\right)y}{1-\beta\theta u'\left(x_2^l\right)}. \tag{33}$$

Then, from (3), (6), (22), (24)-(27), and (31)-(33), we obtain

$$\frac{\alpha(1-\rho)x_2^s\left[(1-\delta)u'\left(x_2^s\right)+\delta\right]}{1-\delta} + (1-\alpha)x_2^l\theta u'(x_2^l) + \alpha\rho x_1^s u'(x_1^s) = V + \frac{\beta\theta u'\left(x_2^l\right)y}{1-\beta\theta u'\left(x_2^l\right)} \tag{34}$$

Further, from (31) and (32), we have

$$z^b = \frac{\theta u'\left(x_2^l\right)}{u'(x_1^s)} \tag{35}$$

$$x_1^l = x_2^l \tag{36}$$

$$(1-\delta)u'\left(x_2^s\right) + \delta = \theta u'\left(x_2^l\right). \tag{37}$$

Then, (34)-(37) solve for $x_1^s$, $x_2^s$, $x_1^l$, and $x_2^l$, given fiscal policy $V$ and monetary policy $z^b$. For the remaining interest rates, $z^m = z^f = z^b$, so the nominal interest rates on reserves, the interbank market, and government bonds are identical in the baseline case. Then, from (31) and (29), equations (9) and (10) are satisfied, so banks choose not to trade on the interbank market given prices.

Finally, open market operations need to support the central bank's interest rate policy. We can work backward from the solution to (34)-(37) to determine what the outstanding asset quantities need to be. That is, in this case outstanding reserves, government bonds, and Lucas trees must finance purchases in the $DM$ of small-transaction buyers who make purchases using bank deposits, and of all large-transaction buyers, so

$$\frac{\beta}{\mu}\left(\bar{m} + \bar{b}\right) + \frac{\beta y}{1 - \beta\theta u'\left(x_2^l\right)} = \alpha(1-\rho)x_2^s + (1-\alpha)x_2^l \tag{38}$$

Then, we can use (31) to rewrite (38) as

$$z^b\left(\bar{m} + \bar{b}\right) = \alpha(1-\rho)x_2^s\left[(1-\delta)u'\left(x_2^s\right) + \delta\right] + (1-\alpha)x_2^l\theta u'\left(x_2^l\right) - \frac{\beta y\theta u'\left(x_2^l\right)}{1 - \beta\theta u'\left(x_2^l\right)} \tag{39}$$

The right-hand side of (39) is strictly increasing in $x_2^s$ and in $x_2^l$, if $-x\frac{u''(x)}{u'(x)} < 1$, which will be our maintained assumption. Thus (39) is useful, as it states that, in equilibrium, the value of reserves and government bonds outstanding at the beginning of the period is strictly increasing in consumption in the $DM$ by buyers using deposit liabilities in exchange.

Similar to (38), the quantity of currency needs to finance purchases by small-transaction buyers meeting sellers who accept only currency, so using (32),

$$\bar{c} = \alpha\rho x_1^s u'(x_1^s). \tag{40}$$

Finally, the stock of government debt outstanding needs to be large enough to support exchange by large-transaction buyers who meet sellers who will only accept government debt or, using (31)

$$z^b\bar{b} \geq (1-\alpha)\pi x_2^l\theta u'(x_2^l). \tag{41}$$

As well, there needs to be enough government debt and Lucas trees, such that all transactions by non-conventional bank depositors can be supported, or

$$z^b\bar{b} \geq (1-\alpha)x_2^l\theta u'\left(x_2^l\right) - \frac{\beta y\theta u'\left(x_2^l\right)}{1 - \beta\theta u'\left(x_2^l\right)}. \tag{42}$$

Suppose then that we think of the mechanics of government debt issue and central banking working as follows. Each period, the fiscal authority issues government debt with a total value of $V$. Then, the central bank purchases government debt with a total value of $z^m\bar{m}+\bar{c}$ to hold in its portfolio, and purchases this government debt by issuing $z^m\bar{m}+\bar{c}$ in reserves to retail banks. These retail banks then withdraw $\bar{c}$ of these reserves as currency, which is in turn withdrawn from retail banks by depositors to make purchases in the $DM$. Equations (39), (40), (41) and (42), then put bounds on the size of the central bank's balance sheet, $z^m\bar{m}+\bar{c}$, i.e.

$$\begin{aligned}\alpha\rho x_1^s u'(x_1^s) &\leq z^m\bar{m}+\bar{c}\leq \alpha\rho x_1^s u'(x_1^s)+\alpha(1-\rho)x_2^s\left[(1-\delta)u'\left(x_2^s\right)+\delta\right] \\ &+\min\left[0,(1-\alpha)(1-\pi)x_2^l\theta u'\left(x_2^l\right)-\frac{\beta y\theta u'\left(x_2^l\right)}{1-\beta\theta u'\left(x_2^l\right)}\right]\end{aligned} \tag{43}$$

## 5.1 Effects of Monetary Policy

In this case, open market operations are irrelevant, at the margin, holding constant the one-period nominal interest rate, which is determined by $z^b=z^m=z^f$. If the central bank swaps reserves for government debt, then this affects only the portfolios of retail banks, which are indifferent between reserves and government debt in equilibrium. Effectively, this case is a floor system for the central bank, since the nominal interest rate on reserves determines the one-period nominal interest rate, so long as interest-bearing reserves are outstanding. Monetary policy consists of setting $z^m$, which by arbitrage then determines $z^b$ and $z^f$.

What are the effects of a change in $z^m$ in this case? To analyze this, it is useful to assume $u(x)=\frac{x^{1-\gamma}-1}{1-\gamma}$, where $\gamma>0$ is the coefficient of relative risk aversion. Further, assume that $0<\gamma<1$, which will imply that the demand for an asset is strictly increasing in its rate of return (substitution effects dominate income effects). We can the rewrite the equilibrium conditions (34)-(37) as

$$\begin{aligned}&\alpha(1-\rho)\left(1-\delta\right)^{\frac{1}{\gamma}-1}\left[\theta\left(x_2^l\right)^{-\gamma}-\delta\right]^{-\frac{1}{\gamma}}\theta\left(x_2^l\right)^{-\gamma}+(1-\alpha)\theta\left(x_2^l\right)^{1-\gamma}+\alpha\rho(x_1^s)^{1-\gamma} \\ &= V+\frac{\beta\theta\left(x_2^l\right)^{-\gamma}y}{1-\beta\theta\left(x_2^l\right)^{-\gamma}}\end{aligned} \tag{44}$$

$$x_2^l=\left(\frac{\theta}{z^m}\right)^{\frac{1}{\gamma}}x_1^s \tag{45}$$

$$x_1^l=x_2^l \tag{46}$$

$$x_2^s=\left[\frac{1-\delta}{\theta\left(x_2^l\right)^{-\gamma}-\delta}\right]^{\frac{1}{\gamma}} \tag{47}$$

Then, (44) and (45) solve for $x_1^s$ and $x_2^l$, and then we can solve in turn for $x_1^l$ and $x_2^s$ from (46) and (47), respectively.

The left-hand side of (44) is strictly increasing in $x_1^s$ and in $x_2^l$, while the right-hand side is decreasing in $x_2^l$. Thus, from (44) and (45), the equilibrium is unique if it exists, and $x_1^s$ and $x_2^l$ are determined, given $z^m$, as in Figure 1 (to be added). Then, if the central bank reduces $z^m$, which serves to increase all nominal interest rates, as in Figure 2 (to be added) this causes $x_1^s$ to fall, and $x_2^l$ to rise. As well, from (46), $x_1^l$ rises, and from (47, $x_2^s$ rises. Further, from (32) the gross inflation rate $\mu$ rises.

As long as there are reserves outstanding before the reduction in $z^m$, no open market operation is required to support the change in the nominal interest rate on reserves. When $z^m$ goes down, if the central bank's balance sheet is held constant in real terms, then there is substitution from currency to reserves, which serves to expand the assets of retail banks, which can then support more transactions by small-transaction buyers who use bank deposits in exchange. Consumption falls in the $DM$ for small transaction buyers who use only currency in transactions. The fact that retail banks are holding more reserves frees up bonds for use in transactions by non-conventional banks, so that $x_1^l$ and $x_2^l$ increase.

# 6 Scarce Government Bonds: Inactive Interbank Market; Retail and Unconventional Banks Hold Lucas Trees; Government Bonds Do Not Back Bank Deposits

In this case, government bonds are in short enough supply that they are used only by large-transaction buyers when they meet a seller who will accept only government bonds. But the interbank market is inactive in this case, as Lucas trees are not traded in a segmented market. That is, Lucas trees are held as assets by retail banks and non-conventional banks, so these two types of institutions face the same rates of return on assets backing deposit liabilities, and so have no incentive to trade on the interbank market.

In (15) and (17), $b' = 0$, so given (30),

$$z^b = \frac{\beta}{\mu}\theta u'(x_1^l). \tag{48}$$

From (12), (13), (19), (29), and (30), we have

$$z^m = \frac{\beta}{\mu}\left[(1-\delta)u'\left(x_2^s\right)+\delta\right] = \frac{\beta}{\mu}\theta u'\left(x_2^l\right), \tag{49}$$

and (33). As well, (32) holds.

Then, from (3), (6), (22), (24), (26), and (27), we obtain

$$\begin{aligned}&\frac{\alpha(1-\rho)x_2^s\left[(1-\delta)u'\left(x_2^s\right)+\delta\right]}{1-\delta}+(1-\alpha)(1-\pi)x_2^l\theta u'(x_2^l) \\ &+\alpha\rho x_1^s u'(x_1^s)+(1-\alpha)\pi x_1^l\theta u'(x_1^l) \\ =\ & V+\frac{\beta\theta u'\left(x_2^l\right)y}{1-\beta\theta u'\left(x_2^l\right)}\end{aligned} \tag{50}$$

Also, from (48), (49) and (32), we have

$$z^b=\frac{\theta u'\left(x_1^l\right)}{u'(x_1^s)}, \tag{51}$$

$$z^m=\frac{\theta u'\left(x_2^l\right)}{u'(x_1^s)}. \tag{52}$$

$$(1-\delta)u'\left(x_2^s\right)+\delta=\theta u'\left(x_2^l\right) \tag{53}$$

Equations (50)-(53) then solve for $x_1^s$, $x_1^l$, $x_2^s$, and $x_2^l$, given $V$, $z^b$, and $z^m$. Note that $u'\left(x_1^l\right)\geq u'\left(x_2^l\right)$ in this equilibrium, so $z^b\geq z^m$ (the nominal interest rate on government debt is less than the nominal interest rate on reserves), i.e. the liquidity premium on government bonds is higher than the liquidity premium on reserves, because of the scarcity of government bonds. As well, note that $z^f=z^m$, so from (9), (10), (18), (29), and (30), neither retail banks nor unconventional banks wish to trade on the interbank market in equilibrium.

This case then does not involve a floor system for the central bank, as the interest rate on reserves does not determine the short-term interest rate on government debt. The interest rate on reserves does, however, determine the interbank interest rate, though there is no trade on this market.

We determine outstanding asset quantities in a manner similar to the previous case. Here, (40) holds, and since government bonds finance consumption $x_1^l$ in the $DM$, using (48) we get

$$z^b\bar{b}=(1-\alpha)\pi x_1^l\theta u'(x_1^l) \tag{54}$$

As well, reserves and Lucas trees must finance consumption of buyers who trade bank deposits, or

$$z^m\bar{m}=\alpha(1-\rho)x_2^s\left[(1-\delta)u'\left(x_2^s\right)+\delta\right]+(1-\alpha)(1-\pi)x_2^l\theta u'(x_2^l)-\frac{\beta\theta u'\left(x_2^l\right)y}{1-\beta\theta u'\left(x_2^l\right)} \tag{55}$$

Finally, reserves have to be held only by retail banks, so

$$(1-\alpha)(1-\pi)x_2^l\leq\frac{\beta y}{1-\beta\theta u'\left(x_2^l\right)} \tag{56}$$

must hold for this equilibrium to exist.

Equations (55) and (40) then imply that the size of the central bank's balance sheet is

$$\begin{aligned} z^m\bar{m}+\bar{c} &= \alpha(1-\rho)x_2^s\left[(1-\delta)u'\left(x_2^s\right)+\delta\right] \\ &\quad +(1-\alpha)(1-\pi)x_2^l\theta u'(x_2^l)+\alpha\rho x_1^s u'(x_1^s)-\frac{\beta\theta u'\left(x_2^l\right)y}{1-\beta\theta u'\left(x_2^l\right)} \end{aligned} \quad (57)$$

## 6.1 Effects of Monetary Policy

As for the previous case, assume a constant relative risk aversion utility function, with coefficient of relative risk aversion $\gamma < 1$. Then, rewrite (50)-(53) as

$$\begin{aligned} &\alpha(1-\rho)\left(1-\delta\right)^{\frac{1}{\gamma}-1}\left[\theta\left(x_2^l\right)^{-\gamma}-\delta\right]^{-\frac{1}{\gamma}}\theta\left(x_2^l\right)^{-\gamma} \quad (58) \\ &+(1-\alpha)(1-\pi)\theta\left(x_2^l\right)^{1-\gamma}+\left[(1-\alpha)\pi\left(\frac{\theta}{z^b}\right)^{\frac{1}{\gamma}}+\alpha\rho\right]\left(x_1^s\right)^{1-\gamma} \\ = \quad &V+\frac{\beta\theta\left(x_2^l\right)^{-\gamma}y}{1-\beta\theta\left(x_2^l\right)^{-\gamma}} \end{aligned}$$

$$x_2^l=\left(\frac{\theta}{z^m}\right)^{\frac{1}{\gamma}}x_1^s \quad (59)$$

$$x_1^l=\left(\frac{\theta}{z^b}\right)^{\frac{1}{\gamma}}x_1^s \quad (60)$$

$$x_2^s=\left[\frac{1-\delta}{\theta\left(x_2^l\right)^{-\gamma}-\delta}\right]^{\frac{1}{\gamma}} \quad (61)$$

Equations (58) and (59) then solve for $x_1^s$ and $x_2^l$ given $z^m$ and $z^b$, and then (60) and (61) in turn solve for $x_1^l$ and $x_2^s$. In a manner similar to the previous case, we can show that equations (58) and (59) can be depicted as in Figure 1 (to be added). Then, a decrease in $z^m$, i.e. an increase in the interest rate on reserves, with $z^b$ held fixed, implies an increase in $x_2^l$ and a decrease in $x_1^s$, as shown in Figure 3 (to be added). Then, (60) implies that $x_1^l$ must decrease, and (61) implies that $x_2^s$ increases. Therefore, from (54), the value of government bonds outstanding, $z^b\bar{b}$, declines, which implies, from (22), that the size of the central bank's balance sheet, $z^m\bar{m}+\bar{c}$, must increase. As well, from (40), the stock of currency outstanding falls.

If $z^b$ decreases with $z^m$ held fixed – an increase in the nominal interest rate on government debt – then in Figure 4 (to be added), $x_1^s$ and $x_2^l$ both decrease. Therefore, from (61), $x_2^s$ falls. Then, from (55) and (**??**), the quantity of reserves outstanding at the beginning of the $CM$, and the size of the central bank's balance sheet must both decrease. Then, from (22) and (54), $x_1^l$ must increase, since the real value of government debt outstanding rises.

# 7 Active Interbank Market; Plentiful Government Bonds

In this model, activity on the interbank market arises because of the restriction that non-conventional banks cannot hold reserves. If there is any interbank credit, it will take the form of lending from unconventional banks to retail banks so that unconventional banks can effectively earn interest on reserves. But, as we will show, the capital requirements on conventional banks will imply that arbitrage will be imperfect, in that the interest rate on reserves will be greater than the interbank interest rate. Further, a role for interbank lending will only arise if bonds and Lucas trees are sufficiently scarce that these assets are held only by unconventional banks.

First, given (12),

$$z^m = \frac{\beta}{\mu}\left[(1-\delta)u'\left(x_2^s\right)+\delta\right] \tag{62}$$

Next, because retail banks are borrowers in the interbank market, (9) must hold, and since unconventional banks are lenders on the interbank market, (18) holds, so given (29), (30), (15), and (16), we get

$$z^f = \frac{\beta}{\mu}u'(x_2^s) = \frac{\beta}{\mu}\theta u'(x_2^l) = \frac{\beta}{\mu}\theta u'(x_1^l) = z^b. \tag{63}$$

Therefore, as long as $u'(x_2^s) > 1$, so that there is inefficiency in exchange when small-transaction buyers trade bank deposits, from (62) and (63), $z_m < z^f = z^b$, so the interest rate on reserves is greater than the interbank rate, which in turn is equal to the interest rate on government bonds.

Then, similar to our approach in the previous two cases, we obtain a set of equations that solves for $x_1^s$, $x_2^s$, $x_1^l$, and $x_2^l$,

$$\begin{aligned} &\alpha(1-\rho)x_2^s\left[(1-\delta)u'\left(x_2^s\right)+\delta\right]+(1-\alpha)x_2^l\left[\delta+(1-\delta)\theta u'\left(x_2^l\right)\right]+(1-\delta)\alpha\rho x_1^s u'\left(x_1^s\right) \\ = \;& (1-\delta)V+\frac{\beta\left[\delta+(1-\delta)\theta u'\left(x_2^l\right)\right]y}{1-\beta\theta u'\left(x_2^l\right)}+\frac{z^b\bar{b}\delta}{u'(x_2^s)} \end{aligned} \tag{64}$$

$$z^b = \frac{\theta u'\left(x_2^l\right)}{u'(x_1^s)}, \tag{65}$$

$$x_1^l = x_2^l \tag{66}$$

$$u'\left(x_2^s\right) = \theta u'\left(x_2^l\right) \tag{67}$$

Note that, in this case, the central bank cannot set the interest rate on reserves independent of the interest rate on government bonds, which is equal to the interbank rate. As well, here we cannot capture the effect of monetary policy on the allocation of consumption only through nominal interest rates, as the asset quantities $\bar{b}$ and $\bar{m}$ matter. In showing how an equilibrium is determined, we treat the interest rate on government bonds as the central bank's policy rate, but it would be equivalent to use the interest rate on reserves as the policy rate.

## 8 Active Interbank Market; Scarce Government Bonds

This equilibrium will exist if the supply of government bonds is sufficiently small, and the stock of reserves is sufficiently large. As in the previous case, unconventional banks hold all of the Lucas trees, and the interbank market is active, with the unconventional banks lending to the retail banks. What will happen here is that $z^b > z^f > z^m$, so the interest rate on government debt is less than the interbank rate, which is less than the interest rate on reserves. Indeed, this looks like the current configuration of the interest rates in the United States, and we can make the case that the model provides an explanation for this configuration. Results will be forthcoming.