Exit Strategies and Trade Dynamics in Repo Markets*

Aleksander Berentsen
University of Basel and Federal Reserve Bank of St. Louis

Sébastien Kraenzlin
Swiss National Bank

Benjamin Müller
Swiss National Bank and University of Basel

March 15, 2015

Abstract

How can a central bank control interest rates in an environment with large excess reserves? In this paper, we develop a dynamic general equilibrium model of a secured money market and calibrate it to the Swiss franc repo market to study this question. The theoretical model allows us to identify the factors that determine demand and supply of central bank reserves, the money market rate and trading activity in the money market. In addition, we simulate various instruments that a central bank can use to exit from unconventional monetary policy. These instruments are assessed with respect to the central bank’s ability to control the money market rate, their impact on the trading activity and the operational costs of an exit. All exit instruments allow central banks to attain an interest rate target. However, the trading activity differs significantly among the instruments and central bank bills and reverse repos are the most cost-effective.

JEL Classification: E40, E50, D83.

Keywords: exit strategies, money market, repo, monetary policy, interest rates

1 Introduction

Prior to the financial crisis of 2007/2008, all major central banks created an environment in which the banking system was kept short of reserves, a so-called structural liquidity deficit.\(^1\) In such an environment, the central bank provides just enough reserves to ensure that financial intermediaries are able to meet their minimum reserve requirements. Consequently, reserves are scarce and the central bank can achieve the desired interest rate simply by changing the stock of reserves by a small amount.

*The views expressed in this paper are those of the authors and do not necessarily represent those of the Swiss National Bank. Berentsen: aleksander.berentsen@unibas.ch. Kraenzlin: sebastien.kraenzlin@snb.ch. Müller: benjamin.mueller@snb.ch.

\(^1\)In a structural liquidity deficit, the banking system has net liabilities towards the central bank. Financial intermediaries are thus forced to participate in the central bank’s reserve providing operations in order to roll-over their net liabilities. Monetary policy is thus implemented via the asset side of the central bank’s balance sheet. In a structural liquidity surplus, the banking system has net claims towards the central bank.
In response to the financial crisis of 2007/2008 and the subsequent sovereign debt crisis, all major central banks decreased interest rates to the zero lower bound and created large excess reserves via asset or foreign currency purchases (quantitative easing or QE). This has led to a situation in which the banking system holds ample reserves and minimum reserve requirements are no longer relevant. The banking system has thus moved from a structural liquidity deficit to a structural liquidity surplus. The key question that central bankers and academics currently discuss is how to control interest rates in such an environment and the term exit strategy is used for various policies that allow central banks to control interest rates in a structural liquidity surplus.

To study these policies, we construct a dynamic general equilibrium model of a secured overnight money market and use it as a laboratory to conduct monetary policy experiments. Our goal is threefold: First, we want to identify the factors that determine demand and supply of central bank reserves, the money market rate, and the trading activity in the money market; i.e., the trade dynamics. Second, we want to analyze the policy instruments central banks can use to exit from unconventional monetary policy. These instruments include interest on reserves, term deposits, central bank bills, and reverse repos. We evaluate these instruments according to the following criteria: The ability to control the money market rate, the impact on the money market trading activity, and the operational costs of an exit. Third, since many central banks will be entering uncharted waters when they start to exit, our theoretical model and calibration allow to assess the impact and the effectiveness of these instruments in a controlled environment.

The theoretical model is a dynamic general equilibrium model of a secured money market developed in Berentsen et al. (2014). The model is adapted to account for the key characteristics of monetary policy implementation in secured money markets and is based on explicit microfoundation: Financial intermediaries face liquidity shocks which determine whether they borrow or lend reserves overnight in the money market or at the central bank’s standing facilities. Since trading in the money market is secured, we explicitly model the role of collateral. In practice, most central banks implement monetary policy by targeting an unsecured money market rate. However, in order to manage the money market rate to be on target, central banks conduct secured transactions. That is, central banks lend or borrow against collateral, only. Hence, we believe that having a model that explicitly takes into account the role of collateral is important for understanding the transmission mechanism of monetary policy.

The model is adapted to replicate the elementary features of the Swiss franc repo

---

2 In the case of Switzerland, the Swiss National Bank increased reserves via foreign exchange purchases from CHF 5.62 bn in 2005 to CHF 370 bn in 2013.

3 The Federal Reserve, the Bank of England, the Bank of Japan, and the Swiss National Bank are currently in a situation where the banking system is in a structural liquidity surplus.

4 For instance, the Swiss National Bank has a target range for the three-month Libor, an unsecured money market rate, and manages the three-month Libor usually via daily repo operations. The European Central Bank’s key policy rate is the EONIA, an unsecured overnight money market rate, which is managed via repo operations, too. Finally, in case of the Federal Reserve, the key policy rate is the Federal Funds Effective Rate, an unsecured overnight interest rate. The Federal Reserve also manages its key policy rate via repo operations.
market and monetary policy implementation by the Swiss National Bank (SNB). In contrast to a growing body of literature, which models money markets as over-the-counter (OTC) markets that are characterized by search and bargaining frictions, we model the money market as a competitive market. We opted for this modelling strategy after carefully inspecting the institutional details of trading in the Swiss franc repo market. In particular, we find that few informational frictions exist in the Swiss franc repo market and counterparty risks are negligible. Our study and findings also apply to other currency areas, since there is a trend towards shifting money market trading onto transparent (centrally cleared) electronic trading platforms that reduce informational frictions.

The following results emerge from our model: First, all four exit instruments allow central banks to achieve an interest rate target. Second, the role of collateral is important for understanding the trade dynamics in the secured money market. For example, we find that an exit via central bank bills or an exit via term deposits differs because the former affects collateral holdings of financial intermediaries while the latter does not. Third, although all exit instruments allow the central bank to achieve a given interest rate target, the money market trading activity differs significantly among the instruments. For example, with interest on reserves, trading activity will be close to zero, while with term deposits, central bank bills or reverse repos, trading activity returns to pre-crisis levels. Fourth, central bank’s operational costs differ significantly among the instruments. For example, our simulation suggests that if the SNB defines a one percent interest rate target, the cost of implementing this target via interest on reserves is CHF 80 million higher per year than with central bank bills.

**Literature.** Our paper is related to Afonso and Lagos (2014) who develop a model of the federal funds market — an unsecured money market for central bank reserves. In their modeling approach, they explicitly take into account the search and bargaining frictions that are key characteristics of this market. With the calibrated model at hand, they evaluate the effectiveness of interest on reserves in controlling the overnight money market rate. Another related paper is Bech and Monnet (2014) which also studies the trade dynamics in an unsecured OTC money market. The authors compare different trading protocols and find that a trading arrangement that allows financial intermediaries to direct their search for counterparties replicates the stylized facts of many unsecured OTC money markets best.

Related literature on general equilibrium models include Berentsen and Monnet

---

5 We model the Swiss franc repo market because this allows us to benefit from outstanding data quality, featuring detailed information on more than 100,000 overnight transactions. In contrast to many other studies, there is no need to identify transactions from payment system data applying the Furine (2000) algorithm which has known caveats (Armantier and Copeland, 2012).

6 For an OTC modeling strategy for a money market, see, for example, Afonso and Lagos (2014). They develop a model of the federal funds market which is a typical OTC market with search and bargaining frictions. Other literature that studies the dynamics of OTC markets include Duffie, Garleanu and Pedersen (2005), Ashcraft and Duffie (2007), Lagos and Rocheteau (2009).

7 See our extensive discussion in Section 2.

8 See ICMA (2014).

9 See Bech and Monnet (2013) for an overview.
(2008) and Martin and Monnet (2011). The former develops a framework for studying the optimal policy when monetary policy is implemented via a channel, and the latter compares feasible allocations when central banks implement monetary policy via channel or floor systems. Curdia and Woodford (2011) extend a New Keynesian model of monetary policy transmission to analyze monetary policy implementation issues, such as the central bank’s balance sheet or interest on reserves as a tool for conducting monetary policy.\footnote{Partial equilibrium models to study monetary policy implementation go back to Poole (1968) and include Campbell (1987), Ho and Saunders (1985), Orr and Mellon (1961), Furune (2000), Woodford (2001), Whitesell (2006).}

This paper is organized as follows: In Section 2, the institutional details of the Swiss franc repo market are discussed. Section 3 develops the theory and Section 4 presents the quantitative analysis. Sections 5 discusses monetary policy implementation before and during the crisis. Section 6 analyses exit strategies and Section 7 concludes.

## 2 The Swiss franc repo market

The Swiss franc repo market (SFRM) is the secured money market for central bank reserves. Financial intermediaries trade in this market to fulfill minimum reserve requirements and in response to liquidity shocks. Trades are concluded on an electronic trading platform with a direct link to the real-time gross settlement payment system (RGTS) called Swiss Interbank Clearing (SIC) and the central securities depository (CSD) called Swiss Security Services (SIS). Transactions concluded on the platform are settled by SIC and SIS where the latter also serves as the triparty-agent.\footnote{The triparty agent manages the collateral selection, the settlement, the ongoing valuation of the collateral and the initiation of margin calls.} On the same platform, the SNB conducts its open market operations and offers its standing facilities. The SFRM represents the relevant money market in Swiss franc in terms of volume and participation.\footnote{This is especially true since the financial crisis, when the unsecured money market collapsed. See Guggenheim, Kraenzlin and Schumacher (2011) for a comparison of the two markets. Repos agreed upon bilaterally and outside the platform are rare.}

Domestic banks, insurances and federal agencies, as well as banks domiciled abroad, may access the SFRM: currently, 152 financial intermediaries have access.\footnote{Among these, 150 also have access to the SNB’s open market operations and standing facilities. See Kraenzlin and Nellen (2014) for a summary of SNB’s access policy.} Tradable maturities range from overnight to twelve months. In this paper, we focus on the overnight maturity since approximately two-thirds of the daily turnover is overnight.\footnote{The overnight market is the origin of the term structure of interest rates. It is the most important interest rate for the pricing of many financial products.} Approximately 99% of all transactions on the platform are secured by securities that belong to a general collateral (GC) basket, the so-called ‘SNB GC’ basket. This is the same collateral basket that the SNB accepts in its open market operations and standing facilities. The collateral standard within the SNB GC is homogenous because the SNB sets high requirements with respect to the rating and the market liquidity of eligible
securities.\footnote{For SNB GC eligible securities, see http://www.snb.ch/en/ifor/fimmkt/operat/snbgc/id/fimmkt_repos_baskets}

The ‘Swiss Average Rate Overnight’ (SARON) is the money market rate for the overnight maturity which is calculated as a volume weighted interest rate based on the overnight trading activity in the SFRM.\footnote{The SARON is continuously calculated in real time and published every ten minutes. In addition, there is a fixing at 12.00 noon, 4.00 p.m. and at the close of the trading day. Successful trades and quotes are included in the calculation of the SARON. A detailed description of how the SARON is calculated can be found on http://www.six-swiss-exchange.com/downloads/indexinfo/online/swiss_reference_rates/swiss_reference_rates_rules_en.pdf} The ‘Overnight SNB Special Rate’ is the interest rate in SNB’s lending facility and is calculated based on the SARON plus 50 basis points.\footnote{Until 2009, the Overnight SNB Special Rate was calculated based on the SARON plus 200 basis points.}

Figure 1 displays the SARON, the Overnight SNB Special Rate, and the 20-day moving average of the overnight turnover for the period 2005 to 2013. For that period, the average daily overnight turnover was CHF 3.2 bn and 30 financial intermediaries were active on an average day. In total, 107,517 overnight trades were concluded.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Stylized facts}
\end{figure}

Although, SNB’s key policy rate is not the SARON, but a target range of the Swiss franc three-month Libor, the SARON reflects SNB’s monetary policy stance, since the SNB controls Libor via daily repo auctions in the SFRM. Furthermore, in order to keep track of prevailing monetary conditions, the SNB monitors the intraday development of the SARON and, if needed, conducts fine-tuning operations in the SFRM by placing or accepting overnight quotes.

\textbf{Trading protocol.} Trades in the SFRM are initiated by placing or accepting binding offers (so-called quotes) or by sending offers (so-called addressed offers, AOs) to counterparties. Quotes are entries that are placed on the electronic trading platform which indicate the maturity, the interest rate, the trade volume, the collateral basket, and the identity of the financial intermediary that has entered the quote. Quotes are collected in an order book which lists bid- and ask quotes for all maturity segments and collateral...
baskets. A trade upon a quote can be executed by accepting a quote via a click.\footnote{Theoretically, financial intermediaries can choose to reveal their quotes only to a restricted group of counterparties. However, this is very rarely done in practice.} AOs are price offers that can be sent to selected counterparties and hence are not visible for other financial intermediaries. As in the case of quotes, AOs specify the maturity, the interest rate, the trade volume, and the collateral basket. AOs can be negotiated upon by sending a counteroffer to the AO sender.

The terms-of-trades of all past trades (based on quotes and AOs) are viewable on the platform. The platform thus guarantees that all financial intermediaries have the same information set. In particular, at any time during the day, they can ascertain the maturities, interest rates, traded volumes, and collateral baskets used in all past trades. Current market conditions are likewise common knowledge thanks to the order book.

**Competitive market.** For several reasons, the SFRM is not an OTC market with search and bargaining frictions. First, an analysis of all overnight trades between 2005 and 2013 reveals that three-quarters of overnight trades are based on quotes, and hence, no bargaining on terms-of-trades takes place.\footnote{A comparison to longer maturities suggests that the relative number of quote based trades is largest in the overnight maturity and decreases the longer the term of the transaction. In the case of the one-week (one-month, six-month) maturity, 65% (50%, 43%) are based on quotes.} Second, in an OTC market, traders meet bilaterally and the amount borrowed must be equal to the amount lent in each match. In contrast, in the SFRM, on an average day 13 borrowing and 17 lending financial intermediaries are active on the platform. This implies asymmetric trading volumes: the average borrower borrows more than the average lender lends.\footnote{One way to capture this stylized fact in an OTC market would be to introduce sequential matching; i.e., financial intermediaries are matched multiple times in one period.} Third, deviations of the interest rates of individual overnight transactions from the SARON are very small — the average daily absolute deviation between 2005 and 2013 is 0.042%.\footnote{The comparison to other maturities shows that the deviation is smallest in the overnight maturity and increases the longer the term of the transaction. The respective figure for the one-week (one-month, six-month) maturity is 0.07% (0.1%, 0.27%).} Fourth, for the same period, the average daily bid and ask volume in the order book is CHF 5.5 bn which suggests that an individual financial intermediary is not able to affect the overnight rate substantially. Fifth, the access to the platform is open to many financial intermediaries. In other words, even though on an average day only 30 banks are active, many financial intermediaries continuously monitor the market and are ready to step in if the market conditions provide attractive borrowing and lending opportunities. Sixth, all loans are secured. Consequently, counterparty risk is negligible.

In our view, the six reasons discussed above clearly indicate that the SFRM is best modeled as a competitive market and not as an OTC market. There are no informational frictions since all financial intermediaries have the same information on past market activities and current market conditions. Furthermore, the large number of market participants and the small price dispersion suggest that no financial intermediary has market power. Financial intermediaries also tend to be indifferent with whom they trade which is explained by the high collateral standard and the absence of counterparty risk.
3 Theory

Our theoretical model is motivated by the elementary features of the SFRM and SNB’s monetary policy implementation. First, at the beginning of the day all outstanding overnight loans are settled.\textsuperscript{22} Second, the SFRM operates between 7 am and 4 pm.\textsuperscript{23} Third, the SNB controls the stock of reserves by conducting open market operations, typically at 9 am.\textsuperscript{24} Fourth, after the money market has closed, the SNB offers its lending facility for an additional 15 minutes. This is the last opportunity for financial intermediaries to acquire overnight reserves for the same business day in order to settle outstanding short positions in the payment system.\textsuperscript{25} The SFRM stays open until 6 pm but new trades concluded after 4 pm will not be settled on the same day.

3.1 Environment

To reproduce the above sequence we assume that in each period three perfectly competitive markets open sequentially (see Figure 2):\textsuperscript{26} a settlement market, where credit contracts are settled and a general good is produced and consumed; a money market, where financial intermediaries can borrow and lend reserves on a secured basis; and a goods market, where production and consumption of a specialized good take place. All goods are perfectly divisible and nonstorable, which means that they cannot be carried from one market to the next.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{sequence_of_markets.png}
\caption{Sequence of markets}
\end{figure}

There are two types of agents: firms and financial intermediaries (FIs). Both agent types are infinitely-lived and each of them has measure 1. The focus of our attention will be on the FIs, since firms play a subordinate role in the model. We only need them to obtain a first-order condition in the goods market.

Time is discrete and the discount factor across periods for both agent types is \( \beta = (1 + r)^{-1} < 1 \), where \( r \) is the time rate of discount. There are two perfectly divisible financial assets: reserves and one-period, nominal discount bonds. One bond pays off one

\begin{footnotesize}
\textsuperscript{22}At 7:50 a.m. the repayment of all outstanding overnight transactions is automatically triggered.
\textsuperscript{23}Transactions are rarely concluded between 7 am and 8 am (see Kraenzlin and Nellen, 2010).
\textsuperscript{24}Usually via fixed rate tender auctions. See Kraenzlin and Schlegel (2012) for an overview.
\textsuperscript{25}Short positions remaining at the end of the day must be settled the following business day and are subject to a penalty that is agreed upon bilaterally on the basis of the SARON. The stigma associated with non-settled payments imposes a further penalty which became very pronounced during the financial crisis.
\textsuperscript{26}The theoretical model presented in Section 3 is adapted from Berentsen et al. (2014). Here, we follow their presentation, closely.
\end{footnotesize}
unit of reserves in the settlement market of the following period. Bonds are default-free and book-keeping entries — no physical object exists.

We now discuss the three markets backward. In the goods market, the specialized good is produced by firms and consumed by FIs.\(^\text{27}\) Firms incur a utility cost \(c(q_s) = q_s\) from producing \(q_s\) units and FIs get utility \(u(q)\) from consuming \(q\) units, where \(u(q) = \log(q)\), and \(\varepsilon\) is a preference shock that affects the liquidity needs of FIs.\(^\text{28}\) The preference shock has a continuous distribution \(F(\varepsilon)\) with support \((0, \infty)\), is i.i.d. across FIs and is serially uncorrelated.

In order to introduce a microfoundation for the demand for reserves, we assume that reserves are the only medium of exchange in the goods market. This is motivated by the assumption that FIs are anonymous in the goods market and that none of them can commit to honor intertemporal promises.\(^\text{29}\) Since bonds are intangible objects, only reserves can be used as media of exchange in the goods market.\(^\text{30}\) In other words, bonds are illiquid.\(^\text{31}\)

At the beginning of the money market, FIs hold a portfolio of reserves and bonds and then learn the current realization of the shock. Based on this information, they adjust their reserve holdings by either trading in the money market or at the standing facilities. The central bank is assumed to have a record-keeping technology over bond trades. This implies that FIs are not anonymous to the central bank. Nevertheless, despite having a record-keeping technology over bond trades, the central bank has no record-keeping technology over goods trades.

In the settlement market, a generic good is produced and consumed by firms and FIs. Firms and FIs have a constant returns to scale production technology, where one unit of the good is produced with one unit of labor generating one unit of disutility. Thus, producing \(h\) units of goods implies disutility \(-h\). Furthermore, we assume that the utility of consuming \(x\) units of goods yields utility \(x\). As in Lagos and Wright (2005), these assumptions yield a degenerate distribution of portfolios at the beginning of the money market.

**Monetary policy.** In the settlement market, the central bank controls the stock of reserves and issues one-period bonds. In the goods market, it operates two standing facilities.\(^\text{32}\)

\(^{27}\)In practice, households consume and hold money on accounts at financial intermediaries. The \(\varepsilon\)-shock can be interpreted as a liquidity shock for FIs which originates from preference or technology shocks experienced by their customers. In order to simplify the model, we abstract from this additional layer, by assuming that our FIs are endowed with the same preferences as potential households.

\(^{28}\)It is routine to show that the first-best consumption quantities satisfy \(q_\varepsilon^* = \varepsilon\) for all \(\varepsilon\).

\(^{29}\)In practice, households and firms operate in the goods market and the demand for reserves arises because they are anonymous to each other (see also Footnote 27).

\(^{30}\)Furthermore, claims to collateral (bonds) cannot be used as a medium of exchange, since we assume that agents can perfectly and costlessly counterfeit such claims, which prevents them from being accepted as a means of payment in the goods market (see Lester et al., 2012).

\(^{31}\)One can show that in our environment it is socially beneficial for bonds to be illiquid. See Kocherlakota (2003), Andolfatto (2011), and Berentsen and Waller (2011).

\(^{32}\)Strictly speaking, the SNB does not operate a deposit facility: rather, FIs hold reserves on a reserve account. Other central banks differentiate between the deposit facility and the reserve account. For ease of reference, we do not differentiate between the two and just call it deposit facility. Finally, we do not
At the lending facility, the central bank offers nominal loans \( \ell \) at an interest rate \( i_\ell \) and at the deposit facility it pays interest rate \( i_d \) on nominal deposits \( d \) with \( i_\ell \geq i_d \). Since we focus on the overnight market, we restrict financial contracts to overnight contracts. A FI that borrows \( \ell \) units of reserves in the lending facility in the goods market in period \( t \) repays \( (1 + i_\ell) \ell \) units of reserves in the settlement market of the following period. Also, a FI that deposits \( d \) units of reserves at the deposit facility in the goods market of period \( t \) receives \( (1 + i_d) d \) units of reserves in the settlement market of the following period. Finally, the central bank operates at zero cost.

The law of motion for the stock of reserves satisfies

\[
M^+ = M + (B - \rho B^+) + (1/\rho_d - 1) D - (1/\rho_\ell - 1) L - T,
\]

where \( M \) is the stock of reserves at the beginning of the current-period settlement market and \( M^+ \) the stock of reserves at the beginning of the next-period settlement market.\(^{33}\) The quantity \( B \) is the stock of bonds at the beginning of the current-period settlement market and \( B^+ \) the stock of bonds at the beginning of the next-period settlement market, and \( \rho = 1/(1 + i) \) the price of newly issued bonds in the settlement market, where \( i \) denotes the nominal interest rate. Since in the settlement market total loans, \( L \), are repaid and total deposits, \( D \), are redeemed, the difference \( (1/\rho_\ell - 1) L - (1/\rho_d - 1) D \) is the central bank’s revenue from operating the standing facilities. Finally, \( T = \tau M \) are lump-sum taxes (\( T > 0 \)) or lump-sum subsidies (\( T < 0 \)).

3.2 Agents’ decisions

In this section, we study the decision problems of FIs and firms. For this purpose, let \( P \) denote the price of goods in the settlement market and define \( \phi \equiv 1/P \). Furthermore, let \( p \) denote the price of goods in the goods market.

**Settlement market.** \( V_S(m, b, \ell, d, z) \) denotes the expected value of entering the settlement market with \( m \) units of reserves, \( b \) bonds, \( \ell \) loans from the lending facility, \( d \) deposits from the deposit facility, and \( z \) loans from the money market. \( V_M(m, b) \) denotes the expected value from entering the money market with \( m \) units of reserves and \( b \) collateral prior to the realization of the liquidity shock \( \varepsilon \). For notational simplicity, we suppress the dependence of the value function on the time index \( t \).

In the settlement market, the problem of an agent is

\[
V_S(m, b, \ell, d, z) = \max_{h, x, m', b'} x - h + V_M(m', b')
\]

s.t. \( x + \phi m' + \phi p b' = h + \phi m + \phi b + \phi d/\rho_d - \phi \ell/\rho_\ell - \phi z/\rho_m - \phi \tau M \),

where \( h \) is hours worked in the settlement market, \( x \) is consumption of the generic good, and \( m' (b') \) is the amount of reserves (bonds) brought into the money market. Using the

\(^{33}\)Throughout the paper, the plus sign is used to denote the next-period variables.

Consider the intraday facility since intraday liquidity is not considered for the fulfilment of minimum reserve requirements and hence has no role in our framework.
budget constraint to eliminate \( x - h \) in the objective function, one obtains the first-order conditions

\[
V_M^{m'} \leq \phi (= \text{ if } m' > 0) \quad (2) \\
V_M^{b'} \leq \phi \rho (= \text{ if } b' > 0).
\]

\( V_M^{m'} \equiv \frac{\partial V_M(m', b')}{\partial m'} \) is the marginal value of taking an additional unit of reserves into the money market. Since the marginal disutility of working is one, \( -\phi \) is the utility cost of acquiring one unit of reserves in the settlement market. \( V_M^{b'} \equiv \frac{\partial V_M(m', b')}{\partial b'} \) is the marginal value of taking additional bonds into the money market. The term \( -\phi \rho \) is the utility cost of acquiring one unit of bonds in the settlement market. The implication of (2) and (3) is that all FIs enter the money market with the same amount of reserves and the same quantity of bonds. The same is true for firms, since in equilibrium they will bring no reserves into the money market.

The envelope conditions are

\[
V_S^m = V_S^b = \phi; V_S^d = \phi / \rho_d; V_S^e = -\phi / \rho_e; V_S^z = -\phi / \rho_m; \quad (4)
\]

where \( V_S^j \) is the partial derivative of \( V_S(m, b, d, z) \) with respect to \( j = m, b, d, z \).

**Money and goods markets.** The money market is perfectly competitive so that the money market interest rate \( i_m \) clears the market. Let \( \rho_m \equiv 1/(1 + i_m) \). We restrict all transactions to overnight transactions. A FI that borrows one unit of reserves in the money market repays \( 1/\rho_m \) units of reserves in the settlement market of the following period. Also, a FI that lends one unit of reserves receives \( 1/\rho_m \) units of reserves in the settlement market of the following period.

Firms produce goods in the goods market with linear cost \( c(q) = q \) and consume in the settlement market, obtaining linear utility \( U(x) = x \). It is straightforward to show they are indifferent as to how much they sell in the goods market if

\[
p \beta \phi^+ / \rho_d = 1, \quad (5)
\]

where \( \phi^+ \) is the value of reserves in the next-period settlement market. Since we focus on a symmetric equilibrium, we assume that all firms produce the same amount. With regard to bond holdings, it is straightforward to show that, in equilibrium, firms are indifferent to holding any bonds if the Fisher equation holds and that they will hold no bonds if the yield on bonds does not compensate them for inflation or time discounting. Thus, for brevity of analysis, we assume firms carry no bonds across periods.

Note that we allow firms to deposit their proceeds from sales at the deposit facility which explains the deposit factor \( \rho_d \) in (5).\(^{34}\) Furthermore, it is also clear that they will never acquire reserves in the settlement market, so for them \( m' = 0 \).

A FI can borrow or lend at the money market rate \( i_m \) or use the standing facilities. For a FI with preference shock \( \varepsilon \), which enters the money market with \( m \) units of reserves

\(^{34}\)This assumption reflects the fact that, in practice, firms hold cash from the proceeds of sales on their deposit account at FIs. FIs, in turn, hold these deposits on the reserve account at the central bank.
and $b$ units of bonds, the indirect utility function $V_M(m, b|\varepsilon)$ satisfies

$$V_M(m, b|\varepsilon) = \max_{q, z, d, \ell} \varepsilon u(q) + \beta V_S(m + \ell - z - pq - d, b, \ell, d, z)$$

s.t. $m + z + \ell - pq - d \geq 0$, $\rho_m b - z \geq 0$, $\rho_m b - z - (\rho_m / \rho_e) \ell \geq 0$, $d \geq 0$.

The first inequality is the FI's budget constraint in the goods market. The second inequality is the collateral constraint in the money market, and the third inequality is the collateral constraint at the lending facility. It is clear that the latter is binding first since $\ell \geq 0$ and so we can ignore the second one without loss in generality. The last inequality reflects the fact that deposits cannot be negative. Let $\lambda_1$ denote the Lagrange multiplier for the first inequality, $\lambda_2$ denote the Lagrange multiplier for the third inequality, and $\lambda_3$ denote the Lagrange multiplier for the last inequality.

In the above optimization problem, we set $d = 0$ and $\ell = 0$ when $\rho_d > \rho_m > \rho_e$ since FIs use the standing facilities if and only if $\rho = \rho_m$ or $\rho_d = \rho_m$. For brevity of our analysis, in the characterization below, we ignore these two cases by assuming $\rho_d > \rho_m > \rho_e$.

Using (4), the first-order condition for $z$ is

$$1 + \lambda_1 = \lambda_2 + \frac{1}{\rho_m}. \tag{6}$$

If $\rho_d > \rho_m > \rho_e$, we can use (4) and (5) to write the first-order conditions for $q$ as follows:

$$\varepsilon u'(q) - \rho_d / \rho_m = \rho_d \lambda_3. \tag{7}$$

Lemma 1 characterizes the optimal borrowing and lending decisions and the quantity of goods obtained by an $\varepsilon$-FI:

**Lemma 1** There exist critical values $\varepsilon_1$, $\varepsilon_2$, with $0 \leq \varepsilon_1 \leq \varepsilon_2$, such that the following is true: if $0 \leq \varepsilon \leq \varepsilon_1$, a FI lends reserves in the money market; if $\varepsilon_1 \leq \varepsilon \leq \varepsilon_2$, a FI borrows reserves and the collateral constraint is nonbinding; if $\varepsilon_2 \leq \varepsilon$, a FI borrows reserves and the collateral constraint is binding. The critical values in the money market solve

$$\varepsilon_1 = \frac{\rho_d m}{\rho_m p}, \text{ and } \varepsilon_2 = \varepsilon_1 \left(1 + \frac{b}{\rho_m m}\right). \tag{8}$$

---

As discussed, in the case of the SNB, $i_\ell$ is determined based on the SARON plus a spread. Here, $i_\ell$ is assumed to be exogenous and constant for the following reasons. First, it simplifies the theoretical analysis considerably. Without this assumption, FIs would have to form expectations about the future SARON. Moreover, an individual FI would need to take into account that his borrowing or lending decision might affect the SARON. Since we assume perfect competition, such strategic considerations play no role but they would certainly be important if, instead, we would model the money market as an OTC market. Second, from an individual FI's point of view, the current SARON is exogenously given since it is determined in the past. Third, although we cannot solve the model analytically if we assume that today's $i_\ell$ is equal to the previous day money market rate plus a fixed spread, we have calibrated and simulated the model under this assumption. Our numerical results indicate that it does not affect our results in an important way.
Furthermore, the amount of borrowing and lending by a FI with a liquidity shock $\varepsilon$ and the amount of goods purchased by the FI satisfy:

\[
\begin{align*}
q_\varepsilon &= \varepsilon p_m / \rho_d, \\
q_\varepsilon &= \varepsilon p_m / \rho_d, \\
q_\varepsilon &= \varepsilon_2 p_m / \rho_d, \\
z_\varepsilon &= p (p_m / \rho_d) (\varepsilon - \varepsilon_1), & \text{if } 0 \leq \varepsilon \leq \varepsilon_1 \\
z_\varepsilon &= p (p_m / \rho_d) (\varepsilon - \varepsilon_1), & \text{if } \varepsilon_1 \leq \varepsilon \leq \varepsilon_2, \\
z_\varepsilon &= \rho_m b, & \text{if } \varepsilon_2 \leq \varepsilon.
\end{align*}
\]

(9)

**Proof of Lemma 1.** For unconstrained FIs, the quantities $q_\varepsilon$ are derived from the first-order condition (7) by setting $\lambda_z = 0$. Since $q_\varepsilon$ is increasing in $\varepsilon$, there exists a critical value $\varepsilon_2$ such that the FI is just constrained. Since in this case, (7) holds as well, we have $q_\varepsilon = \varepsilon p_m / \rho_d$ for $\varepsilon \leq \varepsilon_2$.

Next, we derive the cut-off value $\varepsilon_1$. From (5) and (7), the consumption level of a FI that is unconstrained satisfies

\[
q_\varepsilon = \frac{\varepsilon p_m}{\rho_d}
\]

(10)

The consumption level of a FI, who neither deposits nor borrows is

\[
q_0 = \frac{m}{p}
\]

(11)

Since (10) is increasing in $\varepsilon$, there exists an $\varepsilon_1$ such that

\[
\varepsilon_1 = \frac{\rho_d m}{\rho_m p}.
\]

(12)

At $\varepsilon = \varepsilon_1$, the FI is indifferent between depositing or borrowing. The quantity consumed by such a FI is $q_{\varepsilon_1} = \varepsilon_1 p_m / \rho_d = m$.

We now calculate $\varepsilon_2$. At $\varepsilon = \varepsilon_2$, the collateral constraint is just binding. In this case, we have the following equilibrium conditions: $q_{\varepsilon_2} = \varepsilon_2 p_m / \rho_d$ and $pq_{\varepsilon_2} = m + \rho_m b$. Eliminating $q_{\varepsilon_2}$ we get

\[
\varepsilon_2 = \varepsilon_1 \left(1 + \frac{\rho_m b}{m} \right).
\]

It is then evident that

\[
0 \leq \varepsilon_1 \leq \varepsilon_2.
\]

Finally, for $\varepsilon < \varepsilon_2$, the quantities deposited and borrowed are derived from the budget constraints $pq_\varepsilon = m + z_\varepsilon$. Using (10) yields:

\[
z_\varepsilon = p (p_m / \rho_d) (\varepsilon - \varepsilon_1).
\]

For $\varepsilon \geq \varepsilon_2$, we have $z_\varepsilon = \rho_m b$. ■

Figure 3 illustrates consumption quantities by FIs. The black dotted linear curve (the 45-degree line) plots the first-best quantities. Consumption quantities by FIs are increasing in $\varepsilon$ in the interval $\varepsilon \in [0, \varepsilon_2]$ and are flat for $\varepsilon \geq \varepsilon_2$. Note that initially the slope of the green curve is equal to $\rho_m / \rho_d \leq 1$, which means that the quantities consumed by FIs are always below the first-best quantities, unless $\rho_m = \rho_d$.  

12
Figure 3 also illustrates the borrowing and lending decisions by the FIs. FIs with a low liquidity shock $\varepsilon$ are lenders. Furthermore, there are two types of borrowers. FIs with an intermediate liquidity shock borrow small amounts of reserves so that the collateral constraint is nonbinding. FIs with a high liquidity shock would like to borrow large amounts of reserves, but their collateral constraint is binding.

### 3.3 Equilibrium

We focus on symmetric stationary equilibria with strictly positive demand for nominal bonds and reserves. Such equilibria meet the following requirements: (i) FIs' and firms' decisions are optimal, given prices; (ii) The decisions are symmetric across all firms and symmetric across all FIs with the same preference shock; (iii) All markets clear; (iv) All real quantities are constant across time; (v) The law of motion for the stock of reserves (1) holds in each period.

Let $\gamma \equiv M^+/M$ denote the constant gross reserves growth rate, let $\eta \equiv B^+/B$ denote the constant gross bond growth rate, and let $B \equiv B/M$ denote the gross bonds-to-reserves ratio. We assume there are positive initial stocks of reserves $M_0$ and bonds $B_0$. A stationary equilibrium requires a constant growth rate for the supply of reserves. Furthermore, in any stationary equilibrium the stock of reserves and the stock of bonds must grow at the same rate. In what follows we therefore assume $\gamma = \eta$, where $\eta$ is exogenous to the central bank. It then follows that the remaining policy variables of the central bank are $\rho_d$ and $\rho_t$.

Market clearing in the goods market requires

$$q_s - \int_0^{\infty} q_\varepsilon dF(\varepsilon) = 0,$$

where $q_s$ is aggregate production by firms in the goods market.

---

$^{36}$Since the assets are nominal objects, the government and the central bank can start the economy off with one-time injections of cash $M_0$ and bonds $B_0$. 

---
Market clearing in the money market is affected by the presence of the central bank’s standing facilities. To understand their role, let \( \rho_m^u \) denote the rate that would clear the money market in the absence of the standing facilities. We call this rate the unrestricted money market rate. From Lemma 1, the supply and demand of money satisfy

\[
S(\rho_m^u) = \int_0^{\varepsilon_1} p(\rho_m^u/\rho_d)(\varepsilon_1 - \varepsilon) \, dF(\varepsilon)
\]

\[
D(\rho_m^u) = \int_{\varepsilon_1}^{\varepsilon_2} p(\rho_m^u/\rho_d)(\varepsilon - \varepsilon_1) \, dF(\varepsilon) + \int_{\varepsilon_2}^{\infty} \rho_m^u b \, dF(\varepsilon),
\]

respectively, where \( \varepsilon_1 = \frac{m}{p} \rho_d \) and \( \varepsilon_2 = \left( \frac{m}{p} \rho_d \right) \left( 1 + \rho_m^u \frac{b}{m} \right) \). Money market clearing requires \( S(\rho_m^u) = D(\rho_m^u) \), which can be written as follows:

\[
\int_0^{\varepsilon_1} (\varepsilon_1 - \varepsilon) \, dF(\varepsilon) = \int_{\varepsilon_1}^{\varepsilon_2} (\varepsilon - \varepsilon_1) \, dF(\varepsilon) + \int_{\varepsilon_2}^{\infty} (\varepsilon_2 - \varepsilon_1) \, dF(\varepsilon). \tag{14}
\]

Suppose (14) yields \( \rho_m^u > \rho_d \); i.e., the deposit rate is higher than the unrestricted money market rate. In this case, FIs prefer to deposit reserves at the central bank, which reduces the supply of reserves until \( \rho_m = \rho_d \). Thus, if \( S(\rho_d) > D(\rho_d) \), we must have \( \rho_m = \rho_d \). Along the same lines, suppose (14) yields \( \rho_m^u < \rho_f \). In this case, FIs prefer to borrow reserves at the central bank’s lending facility, which reduces the demand for reserves until \( \rho_m^u = \rho_f \). Thus, if \( S(\rho_f) < D(\rho_f) \), we must have \( \rho_m = \rho_f \). Finally, if \( \rho_d > \rho_m^u > \rho_f \), FIs prefer to trade in the money market, so \( \rho_m = \rho_m^u \).

Accordingly, we can formulate the market-clearing condition as follows:

\[
\rho_m = \begin{cases} 
\rho_d & \text{if } S(\rho_d) < D(\rho_d) \\
\rho_f & \text{if } S(\rho_f) > D(\rho_f) \\
\rho_m^u & \text{otherwise.}\end{cases} \tag{15}
\]

**Proposition 2.** A symmetric stationary equilibrium with a positive demand for reserves and bonds is a policy \((\rho_d, \rho_f)\) and endogenous variables \( (\rho, \rho_m, \varepsilon_1, \varepsilon_2) \) satisfying the money market clearing condition (15) and

\[
\rho_d \eta / \beta = \int_0^{\varepsilon_2} (\rho_d/\rho_m) \, dF(\varepsilon) + \int_{\varepsilon_2}^{\infty} (\rho_d/\rho_m)(\varepsilon/\varepsilon_2) \, dF(\varepsilon) \tag{16}
\]

\[
\rho_f \eta / \beta = \int_0^{\varepsilon_2} \rho_f \, dF(\varepsilon) + \int_{\varepsilon_2}^{\infty} (\varepsilon/\varepsilon_2) \, dF(\varepsilon) \tag{17}
\]

\[
\varepsilon_2 = \varepsilon_1 (1 + \rho_m B). \tag{18}
\]

**Proof of Proposition 2.** The proof involves deriving equations (16) to (18). Equation (18) is derived in the proof of Lemma 1. To derive equation (16), differentiate \( V_M (m, b) \)
with respect to $m$ to get

$$V_M^m (m, b) = \int_0^\infty \left[ \beta V_S^m (m + z^\varepsilon + \ell^\varepsilon - pq^\varepsilon \lambda^\varepsilon, b, \ell^\varepsilon, d^\varepsilon | \varepsilon) + \beta \phi^+ \lambda^\varepsilon \right] dF (\varepsilon).$$

Then, use (4) to replace $V_S^m$ and (7) to replace $\beta \phi^+ \lambda^\varepsilon$ to obtain

$$V_M^m (m, b) = \int_0^\infty \frac{\varepsilon u'(q^\varepsilon)}{p} dF (\varepsilon). \quad (19)$$

Use the first-order condition (5) to replace $p$ to get

$$V_M^m (m, b) = (\beta \phi^+ / \rho_d) \int_0^\infty \varepsilon u'(q^\varepsilon) dF (\varepsilon).$$

Use (2) to replace $V_M^m (m, b)$ and replace $\phi / \phi^+$ by $\eta$ to get

$$\frac{\rho d \eta}{\beta} = \int_0^\infty \varepsilon u'(q^\varepsilon) dF (\varepsilon).$$

Finally, note that $u'(q) = 1/q$ and replace the quantities $q^\varepsilon$ using Lemma 1 to get (16), which we replicate here:

$$\frac{\rho d \eta}{\beta} = \int_0^{\varepsilon_2} \frac{\rho d}{\rho_m} dF (\varepsilon) + \int_{\varepsilon_2}^\infty \frac{\rho d}{\varepsilon^2 \rho_m} dF (\varepsilon). \quad (20)$$

To derive (17), note that in any equilibrium with a strictly positive demand for reserves and bonds, we must have $\rho V_M^m (m, b) = V_M^b (m, b)$. We now use this arbitrage equation to derive (17). We have already derived $V_M^m (m, b)$ in (19). To get $V_M^b (m, b)$ differentiate $V_M (m, b)$ with respect to $b$ to get

$$V_M^b (m, b) = \int_0^\infty \left\{ \beta V_S^b (m + \ell^\varepsilon - pq^\varepsilon \lambda^\varepsilon, b, \ell^\varepsilon, d^\varepsilon | \varepsilon) + \rho_m \beta \phi^+ \lambda^\varepsilon \right\} dF (\varepsilon).$$

Use (4) to replace $V_S^b$ to get

$$V_M^b (m, b) = \beta \phi^+ \int_0^\infty (1 + \rho_m \lambda^\varepsilon) dF (\varepsilon).$$

Use (7) to replace $\lambda^\varepsilon$, and rearrange to get

$$V_M^b (m, b) = \int_0^{\varepsilon_2} \beta \phi^+ dF (\varepsilon) + \int_{\varepsilon_2}^\infty (\rho_m / \rho_d) \varepsilon u'(q^\varepsilon) dF (\varepsilon).$$
Equate $\rho V^m_M (m, b) = V^b_M (m, b)$ and simplify to get

$$\rho \int_0^{\infty} \varepsilon u' (q_\varepsilon) dF (\varepsilon) = \int_0^{\varepsilon_2} \rho_d dF (\varepsilon) + \int_{\varepsilon_2}^{\infty} \rho_m \varepsilon u' (q_\varepsilon) dF (\varepsilon).$$

Note that $\int_0^{\infty} \varepsilon u' (q_\varepsilon) dF (\varepsilon) = \rho_d \eta / \beta$ and rearrange to get

$$\frac{\rho m}{\beta} = \int_0^{\varepsilon_2} dF (\varepsilon) + \int_{\varepsilon_2}^{\infty} (\rho_m / \rho_d) \varepsilon u' (q_\varepsilon) dF (\varepsilon).$$

Finally, use Lemmas 1 to get (17), which we replicate here:

$$\frac{\rho m}{\beta} = \int_0^{\varepsilon_2} dF (\varepsilon) + \int_{\varepsilon_2}^{\infty} (\varepsilon / \varepsilon_2) dF (\varepsilon).$$

Equation (16) is obtained from the choice of reserves holdings (2). Equation (17) is obtained from (2) and (3); in any equilibrium with a strictly positive demand for reserves and bonds, $\rho V^m_M (m, b) = V^b_M (m, b)$. We then use this arbitrage equation to derive (17). Finally, equation (18) is derived from the budget constraints of the FIs.

We postpone the discussion of the model’s predictions regarding the trade dynamics to Section 4.3. This allows us to discuss the trade dynamics based on figures obtained from the calibrated parameters.

4 Quantitative analysis

Our quantitative analysis covers the period from 2005 to 2013. We calibrate our model to the moments of 244 trading days which range from 3 January 2005 to 15 December 2005 (baseline period). During that period, the SNB controlled the stock of reserves via daily repo auctions. The stock was chosen such that FIs were just able to fulfill their minimum reserve requirements. To counter undesired fluctuations in the Saron (money market rate, $i^m$), the SNB conducted fine-tuning operations on an irregular basis. During the baseline period, the SNB kept its key policy rate constant.

In the baseline period the average Saron was 0.6% and the average Overnight SNB Special Rate (lending rate, $i^s_d$) was 2.6%. Since the SNB does not remunerate reserves the deposit rate $i_d$ was 0%. The average overnight turnover amounted to CHF 2.7 bn and 32 FIs were active on average per day. Finally, the average stock of reserves was CHF 5.62 bn.

4.1 Calibration

We choose a model period as one day. The function $u(q)$ has the form $\log(q)$ and the liquidity shock $\varepsilon$ is log-normally distributed with mean $\mu$ and standard deviation $\sigma$.37 Although the distribution of liquidity shock cannot be observed in the data, we are able to assess indirectly, whether the log-normal distribution is a good approximation. This can be done by comparing
The parameters to be identified are (i) the preference parameter $\beta$; (ii) the consumer price index (CPI) inflation $\gamma$; (iii) the policy parameters $\rho_l$ and $\rho_d$; (iv) the bond-to-reserves ratio $B = B/M$ where $M$ denotes the stock of reserves and $B$ the stock of bonds (collateral); and (v) the moments $\mu$ and $\sigma$ of the log-normal distribution. All data sources are provided in Table 6 in the Appendix. Table 1 reports the identification restrictions and the identified parameter values.

**Table 1: Calibration targets**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target description</th>
<th>Parameter value</th>
<th>Target value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Average real interest rate $r$</td>
<td>0.99105</td>
<td>0.00188</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Average inflation rate $\phi_t/\phi_{t+1}$</td>
<td>0.01173</td>
<td>0.01173</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>Average lending rate $i_l$</td>
<td>0.02613</td>
<td>0.02613</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>Average deposit rate $i_d$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$B$</td>
<td>Average money market rate $i^e_m$</td>
<td>0.03992</td>
<td>0.00628</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Average turnover-to-reserves ratio $v^e$</td>
<td>0.04799</td>
<td>0.01566</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Normalized</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 displays the parameters to be identified and their calibrated values. To identify $\beta$, $\gamma$, $\rho_l$ and $\rho_d$, we use data from the baseline period described in Table 6 in the Appendix. The parameters $B$ and $\sigma$ are obtained by matching $i^e_m$ and $v^e$ simultaneously. Finally, parameter $\mu$ is normalized.

The four parameters $\beta$, $\rho_l$, $\rho_d$, and $\gamma$ can be set equal to their direct targets. We set $\beta = (1 + r)^{-1} = 0.99105$ so that the model’s real interest rate matches the average real interest rate in the data, $r = 0.00188$ which is the difference between one year Swiss treasury bond yields and CPI inflation. We set $\rho_l = (1 + i_l)^{-1} = 0.97454$ and $\rho_d = (1 + i_d)^{-1} = 1$ in order to replicate the average lending and deposit rate. In order to match the average CPI inflation we set $\gamma = \phi_t/\phi_{t+1} = 0.01173$. Finally, we normalize $\mu = 1$, since our numerical analysis shows that $\mu$ is not relevant for the calibration of the parameters.

The targets discussed above allow us to explicitly calibrate all parameters but the bonds-to-reserves ratio, $B$, and the standard deviation, $\sigma$. We determine these by simultaneously matching the average money market rate, $i^e_m$, and the average turnover-to-reserves ratio, $v^e$, by minimizing the following weighting function:

$$\min_{\sigma, B} \omega (|i^e_m - i^e_m|) + (1 - \omega) (|v - v^e|),$$

where $\omega = 0.5$.

To map the data to the model we calculate the turnover-to-reserves ratio as follows. We divide the overnight turnover by the number of active FIs per day. We normalize the average turnover per FI by the stock of reserves and call it the turnover-to-reserves ratio. In the baseline period, the average daily turnover-to-reserves ratio $(v^e)$ was 0.016.

The distribution of trades that the model generates with the empirical distribution of trades in our dataset. Our results indicate that log-normally distributed liquidity shocks generate theoretical trading patterns that are similar to the empirical ones.

---

*38* We divide the turnover by the number of active FIs, because in the theoretical model the measure of FIs is normalized to one.
We normalize $M = 5.62$, since the average stock of reserves was CHF 5.62 bn in the baseline period. Note that in the theoretical model only the bonds-to-reserves ratio is relevant for the equilibrium allocation so $M$ can be normalized.

### 4.2 Model fit

In order to assess the model’s fit, we draw a finite number $n^t$ of liquidity shocks from a log-normal distribution with the calibrated moments $\mu$ and $\sigma$. Let $\Omega^t$ denote the set of liquidity shocks $\varepsilon$ drawn in period $t$. For each $\varepsilon \in \Omega^t$ we use Lemma 1 to calculate the net borrowing $z_{\varepsilon}$. Given the various $z_{\varepsilon}$, we then use the market clearing condition (14) to calculate the money market rate $i^t_m$. Since we know each individual trade that occurs under $\Omega^t$, we can also calculate the turnover-to-reserves ratio $v^t$ from (9) that occurs in period $t$.

To generate a sequence of $i^t_m$ and $v^t$, we simply repeat the sampling exercise for $T$ periods. We report the mean and the standard deviation calculated over $t = 1, \ldots, T$ market clearing interest rates and associated turnover-to-reserves ratios denoted as $i_m$ and $v$ and compare them with the empirical counterparts $i^e_m$ and $v^e$ of the baseline period.  

Naturally, the choice of the sample size $n^t$ affects the standard deviation of $i_m$ and $v$. In particular, the standard deviation converges to zero as we increase the sample size to infinity. To pin down $n^t$, we choose $n^t = 4,000$ such that the standard deviation of $i_m$ matches the empirical standard deviation of $i^e_m$.  

The number of $T$ periods is chosen such that it fits the number of trading days in the baseline period. Table 2 summarizes the empirical and simulated moments of $i_m$ and $v$ for $n^t = 4,000$ and $T = 244$.

**Table 2: Empirical and simulated moments**

<table>
<thead>
<tr>
<th></th>
<th>Empirical</th>
<th></th>
<th>Simulated</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>STD</td>
<td>Mean</td>
<td>STD</td>
</tr>
<tr>
<td>Money market rate $i_m$</td>
<td>0.00628</td>
<td>0.00075</td>
<td>0.00620</td>
<td>0.00078</td>
</tr>
<tr>
<td>Turnover-to-reserves ratio $v$</td>
<td>0.01566</td>
<td>0.00540</td>
<td>0.01566</td>
<td>0.00016</td>
</tr>
</tbody>
</table>

*Table 2 displays the empirical and simulated moments for $i_m$ and $v$ for the baseline period. The sample size is $n^t = 4,000$ and we consider $T = 244$ days.*

---

39 When we calibrate the model, the assumption is that all liquidity shocks from the underlying distribution are present. In contrast, when we simulate the model, we draw a finite set of liquidity shocks from the underlying distribution and repeat it for each period. This, of course, leads to variability in the money market rate and the turnover-to-reserves ratio across periods. We have chosen this simulation strategy because it is easy to implement. Alternatively, we could calibrate the model under the assumption that in each period, only a finite set of liquidity shocks is present.  

40 Note that in the model, a FI receives exactly one liquidity shock. Hence, $n^t$ represents the number of active FIs in the money market at time $t$. In practice, we only observe a limited number of FIs which are active in the market on a specific day. In case of the baseline period, on average 32 FIs were active on a daily basis. Potential reasons why $n^t$ has to be set higher in order to match the empirical standard deviation of $i^e_m$ are SNB’s fine-tuning operations. Fine-tuning operations were conducted when the money market rate deviated too far from an internal target. This, of course, dampened the fluctuation of the money market rate and hence reduced the standard deviation.
Table 2 shows that our model fits the average $i^e_m$ and $v^e$ as well as the standard deviation of $i^e_m$ well. In contrast, the standard deviation of $v$ is too low.

4.3 Comparative statics

Based on the calibrated parameters, we now explore graphically how the demand and the supply of reserves react to exogenous shocks to $M, \mu, B$, and $\sigma$.\footnote{In drawing these figures, we keep the value of reserves $\phi$ constant (see our discussion below).} For each figure, the money market rate $i_m$ is displayed on the horizontal axis and the turnover-to-reserves ratio $v$ is displayed on the vertical axis. Demand and supply are shown for the calibrated parameters (solid lines) and for a variation of the parameter under consideration (dashed lines).

The panel on the left-hand side of Figure 4 displays the effect of a reduction of $M$ by one percent. In this case, the demand for reserves increases (the blue curve shifts up) and the supply decreases (the red curve shifts down). As a result, $i_m$ unambiguously increases. The effect on $v$ is ambiguous, but in the present case the numerical comparison suggests a slight decrease of $v$.

The effect of an increase of $\mu$ is very similar and is shown in the panel on the right-hand side of Figure 4. If the average liquidity shock increases, the demand for reserves increases and the supply of reserves decreases. Consequently, $i_m$ unambiguously increases. The effect on $v$ is ambiguous, but in the present case we find a decrease of $v$.

The panel on the left-hand side of Figure 5 displays the effect of doubling $B$. A change in $B$ has no effect on the supply curve. It only increases the demand for reserves, since fewer FIs are collateral constraint. Here, the comparative statics are unambiguous: both $i_m$ and $v$ increase.

The panel on the right-hand side of Figure 5 displays the effect of a decrease of $\sigma$ to 0.5$\sigma$. If the standard deviation of the liquidity shock decreases, the need for reallocating reserves between FIs decreases. Consequently, the demand for reserves and the supply of reserves decrease. This unambiguously decreases $v$, but the effect on $i_m$ is ambiguous. In the present case we find an increase of $i_m$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure_4.png}
\caption{Comparative statics (I)}
\end{figure}
5 Monetary policy before and during the crisis

The focus of the paper are exit strategies. However, before we study them, we want to discuss monetary policy implementation before and during the financial crisis of 2007/2008. As discussed in the introduction, prior to the crisis, all major central banks created an environment where the banking system was kept short of reserves, a so-called structural liquidity deficit. In such an environment, the central bank provides just enough reserves to ensure that FIs are able to meet their minimum reserve requirements. Consequently, reserves are scarce and a central bank can achieve the desired interest rate simply by changing the stock of reserves by a small amount.

In response to the financial crisis of 2007/2008 and the subsequent sovereign debt crisis, all major central banks decreased interest rates to the zero lower bound and created large excess reserves via asset or foreign currency purchases (QE). This led to a situation in which the banking system holds ample reserves and minimum reserve requirements are no longer relevant.

In the case of Switzerland, the SNB increased reserves via foreign exchange purchases from CHF 5.62 bn in 2005 (baseline period) to CHF 370 bn in 2013 (factor 66). As a result, the banking system holds large excess reserves and is in a structural liquidity surplus. Money market interest rates are at the zero lower bound and money market activity collapsed as shown in Figure 6.

Figure 6: SNB’s response in the crisis
5.1 Structural liquidity deficit: scarcity of reserves

In a structural liquidity deficit, the central bank provides reserves to the banking system on a regular basis, usually through an auction. A temporary and unexpected shock to the stock of reserves $M$ ($M$-shock) occurs due to forecast errors of the demand for reserves which is estimated by the central bank on a daily basis. Such a temporary shock does not affect the value of reserves $\phi$, since $\phi$ is a forward looking variable: that is, it is determined by future monetary conditions only. Thus, for a temporary shock, we keep $\phi$ at the calibrated value.

To study the effect of an unexpected shock to $M$, we simulate $n^t = 4,000$ liquidity shocks and calculate the market clearing money market rate $i^t_m$ and the associated turnover-to-reserves ratio $v^t$. By repeating this procedure we get a sequence of $i^t_m$ and $v^t$ with length $T$ which we display in a box-plot representation. For the subsequent experiments, we choose $T = 40$.

The box-plot provides the following information: First, the means of the simulated $i^t_m$ and $v^t$ are indicated by the horizontal line in the blue area. Second, the width between the 25th to the 75th percentile which is represented by the blue area, and third, the minimum and maximum value which is indicated by the vertical lines at the end of the box-plot. The distribution of $i^t_m$ and $v^t$ represented in the box-plot reflects SNB’s uncertainty in setting $i_m$ on the targeted level.

The simulation results of a temporary $M$-shock are shown in Figure 7. They are based on a variation of $M$ by $+/-5\%$ from the calibrated value $M = 5.62$. For each value of $M$, the simulation results are displayed in a box-plot representation as explained above. The panel on the left-hand side displays the effect of the parameter change on $i_m$, whereas the panel on the right-hand side displays the effect on $v$.

Figure 7: Temporary $M$-shock

---

42 A forecasting error can occur if factors such as the government’s reserve balances or banknotes in circulation unexpectedly change.

43 In the Appendix, we provide a set of other experiments. In particular, we study shocks to $\mu, \sigma, B$.

44 The simulation for each parameter value under consideration is repeated with the identical random sample of liquidity shocks.
Money market rate. The simulation of the model generates the typical relationship between $M$ and $i_m$. A temporary increase of the stock of reserves $M$ reduces the demand for reserves and increases the supply of reserves. Consequently, $i_m$ decreases and ultimately reaches the deposit rate. In contrast, a decrease of $M$ increases the demand for reserves and decreases the supply of reserves. As a result, $i_m$ increases and ultimately reaches the lending rate.\footnote{The SNB conducted fine-tuning operations whenever the deviation of $i_m$ from the internal target was too large. Hence, in practice, the SNB reacted before $i_m$ reached $i_d$ or $i$.}

At the calibrated value $M = 5.62$, $i_m$ is highly elastic to changes in $M$: A change of $M$ by 1\% is associated with a change of $i_m$ by 81\% (i.e., $\varepsilon = 81$).\footnote{The elasticity at $M = 5.62$ is measured as the average elasticity of an increase and decrease of $M$ by 1\%.} As we move away from the calibrated value, $i_m$ becomes less elastic. Eventually, at the extreme values presented in Figure 7, the elasticity drops to $\varepsilon = 0$.\footnote{Qualitatively, our results are in line with Kraenzlin and Schlegel (2012) which estimate elasticities for the demand for Swiss franc reserves covering the period 2000 to 2006.}

The standing facilities play a particular role for this observation. If $M$ increases, $i_m$ decreases and ultimately reaches $i_d$. Since $i_d$ represents the lower bound the dispersion of the simulated $i_m$ is compressed. Similarly, if $M$ decreases, $i_m$ increases until it reaches $i\ell$.

Turnover. The analysis of the simulated turnover-to-reserves ratio suggests that $v$ is increasing in $M$ if $M < 5.62$ and decreasing if $M \geq 5.62$. To understand the relationship between $M$ and $v$, the effect on $i_m$ and the role of the standing facilities have to be considered. If $M$ is small, there is excess demand for reserves, $i_m$ is at $i\ell$, and FIs borrow at the lending facility. These borrowed reserves are not included in the calculation of $v$. The same is true if $M$ is large. In this case, there is excess supply of reserves, $i_m$ is at $i_d$ and the excess supply of reserves is absorbed by the deposit facility. Finally, if $M$ is close to the calibrated value, $v$ is highest since the re-allocation of reserves is exclusively performed via the money market.

5.2 Structural liquidity surplus: large excess reserves

We now move on to study a permanent and large increase of the stock of reserves. In contrast to the temporary shock experiment, we let the value of reserves $\phi$ adjust to its the new equilibrium value. Permanent $M$-shocks include, for example, increases in reserves via QE.\footnote{The central bank’s counterparties in case of QE are usually non-FIs. Therefore, we study a large and permanent increase of the stock of reserves with a permanent $M$-shock but without an additional shock to FIs’ stock of collateral $B$ (see Benford et al. 2009 for a reference).} The simulation results of a permanent increase of $M$ are shown in Figure 8. The simulation results are based on a permanent increase of $M$ from the calibrated value up to factor 5.
Money market rate. Although the qualitative effects of a permanent $M$-shock are identical to a temporary $M$-shock, the quantitative impact of $M$ on $i_m$ is smaller. The simulation results suggest that $i_m$ is decreasing in $M$ and $i_m$ is much less elastic to changes of $M$ compared to a temporary $M$-shock. An increase of $M$ by factor 1.8 is sufficient for $i_m$ to reach the deposit rate. The difference between the temporary and the permanent $M$-shock experiment can be attributed to the adjustment of the value of reserves $\phi$, which is kept at the calibrated value in case of a temporary shock. If $M$ decreases, reserves become scarce and $\phi$ increases. In contrast, if $M$ increases, FIs are satiated with reserves and $\phi$ decreases. By this, the adjustment of $\phi$ offsets the change in $i_m$ that is necessary to ensure market clearing.

Although all prices are fully flexible, a change of $M$ is not neutral. The reason for the non-neutrality is the restriction of the demand for reserves imposed by the collateral constraint. If $M$ increases, $\phi$ decreases but the demand for reserves cannot adjust because the collateral constraint becomes more binding.

Turnover. The analysis of the simulated turnover-to-reserves ratio suggests that $v$ is steadily decreasing in $M$. Along the lines of the argumentation above, the decrease can again be attributed to the collateral constraint.

Table 3: Empirical and simulated means\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>Empirical</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 370$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money market rate $i_m$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Turnover-to-reserves ratio $v$</td>
<td>0.00017</td>
<td>0.00024</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Table 3 displays the empirical and simulated means for $i_m$ and $v$ for $M = 370$, which represents the average stock of reserves in 2013.

The simulated model matches the stylized facts which involve money market rates at the zero lower bound and subdued money market activity when the stock of reserves is

\textsuperscript{49}In the Appendix, we provide simulation results based on a variation of $M$ by $+/−5\%$ from the calibrated value $M = 1$ which can be compared to the results of the temporary $M$-shock experiment.
increased permanently. In Table 3, we report the money market rate and the turnover-to-reserves ratio if the stock of reserves is increased to CHF 370 bn. The model predicts a money market rate of zero and a turnover-to-reserves ratio of 0.00024. Both simulated values are very close to the empirical counterparts.

6 Exit strategies

The current monetary policy environment is characterized by large excess reserves. As a result, the money market rate is at the deposit rate and the trading activity is close to zero. Central bankers and academics currently discuss how to control interest rates in such an environment and the term exit strategy is used for various policies that allow to do so.\textsuperscript{50}

The following four instruments are widely discussed: interest on reserves, term deposits, central bank bills or reverse repos. In what follows, we analyze these instruments. The focus of our attention will be on how well the central bank can control the money market rate, how the money market trading activity is affected, and how costly these instruments are for a central bank.

6.1 Interest on reserves

By paying interest on reserves (IOR) the central bank remunerates reserves at rate $i_d$ which imposes a floor for the overnight money market rate: no FI would lend to other counterparties at a rate below $i_d$.\textsuperscript{51} In an environment with large excess reserves, $i_m$ will be equal to $i_d$. For that reason, this method of implementing monetary policy is referred to as a floor system. The simulation results of IOR are shown in Figure 9. We simulate IOR by raising $i_d$ from zero up to two percent. All simulation results are based on the average stock of reserves as of 2013 (i.e., CHF 370 bn).

\textsuperscript{50} Central banks can also lower aggregate reserves by selling assets in order to reduce their balance sheets to pre-crisis levels. For various reasons beyond the scope of our model, central banks refrain from this measure. Furthermore, in order to align the demand for reserves with the existing large stock of reserves, central banks could also increase minimum reserve requirements.

\textsuperscript{51} The IOR might represent a binding floor or should at least serve as a magnet for the overnight money market rate. This depends on the degree of segmentation and competition in the money market as well as on the central bank’s access policy to a reserve account and hence for earning the IOR. See Bech and Klee (2011) for an analysis of the federal funds market and Jackson and Sim (2013) for a study of the UK case. For an analysis of the impact of central banks’ access policies, see Kraenzlin and Nellen (2014).
Figure 9: Interest on reserves

The model replicates central bank experience with IOR in an environment with excess reserves. The money market rate satisfies $i_m = i_d$ for all values of $i_d$, and trading activity in the money market is close to zero. This shows that the central bank can control $i_m$ precisely without changing the stock of reserves in the economy. Moreover, in order to control $i_m$, the central bank faces considerable operational costs because it has to pay $i_d$ on the entire stock of reserves.

6.2 Term deposits

With a term deposit, the central bank issues an IOU and sells it to FIs against reserves in order to absorb reserves.\footnote{Term deposits leave the central bank’s balance sheet unchanged since reserves are converted into term deposits which are also claims of FIs towards the central bank. Hence, the banking system remains in a structural liquidity surplus. Counterparties for term deposits are FIs that can participate in central bank’s open market operations. Depending on the central bank’s access policy, the range of potential counterparties may be broader than in the case of IOR.} If sufficient reserves are absorbed, they become scarce again so that the money market rate increases above the deposit rate. The key issue is what quantity of reserves does the central bank need to absorb in order to achieve a given interest rate target? A key characteristic of a term deposit is that it cannot be traded by FIs, and, in particular, it cannot be used as collateral. The simulation results of term deposits are shown in Figure 10. We simulate the effects of term deposits by permanently reducing reserves, while holding the stock of collateral constant.
Note that in Figure 10, the calibrated value of $M = 5.62$ is at the right-hand side of each panel. The simulation results of term deposits suggest that $i_m$ remains at $i_d$ as long as $M > \text{CHF 10 bn}$. For low values of $M$, $i_m$ increases steadily until it reaches the calibrated $i_m$ at the calibrated $M$. The same is true for $v$, although some money market activity emerges at much higher values of $M$.

Our simulation suggests that the central bank can use term deposits to control the money market rate in a structural liquidity surplus. Compared to IOR, the control of the money market rate is less precise, as indicated by the box-plots. In contrast to IOR, money market trading activity returns to pre-crisis levels at $M = 5.62$ which represents the level of minimum reserves at the SNB. Finally, in Table 4 and 5, we show that the operational costs of term deposits are smaller than with IOR because the central bank has to pay interest on a smaller stock of reserves to achieve a given interest rate target.

### 6.3 Central bank bills and reverse repos

With a central bank bill, the monetary authority issues an IOU and sells it to FIs in order to absorb reserves. In contrast to a term deposit, the central bank creates a tradeable security (the IOU) which can be used as collateral by FIs. Another difference to term deposits is that central bank bills can be sold to non-FIs such as institutional investors. Central bank bills held by non-FIs, however, reduce the aggregate banking

---

53 From an operational point of view, a difficulty with term deposits is that the central bank needs to absorb a large quantity of aggregate reserves. In the case of Switzerland, this is roughly CHF 360 bn. Such an absorption operation cannot be done in a short period of time. In contrast, an IOR can be imposed immediately on the entire quantity of reserves.

54 Depending on the design of term deposits, they might also involve regulatory costs. For instance, in the case of the Liquidity Coverage Ratio (LCR), if the maturity of the term deposit is longer than 30 days, FIs’ regulatory liquidity position worsens and FIs want to be compensated for this. In response to this, the Federal Reserve, for instance, has introduced a call option on term deposits. Consequently, term deposits are considered to have a maturity of less than 30 days.

55 Reverse repos and central bank bills held by FIs leave the central bank’s balance sheet unchanged since reserves are swapped into a claim from a reverse repo or a central bank bill but still represent a claim of FIs towards the central bank. Hence, the banking system remains in a structural liquidity surplus.
system’s balance sheet, since non-FIs use deposits they hold at FIs in order to purchase central bank bills.

With a reverse repo, the central bank borrows reserves against collateral which it holds in its own books. Thus, in contrast to central bank bills, no new collateral is created with reverse repos. Nevertheless, collateral holdings of FIs increase which makes this instrument similar to central bank bills. In what follows, we simulate a policy where the central bank absorbs reserves and, at the same time, increases FIs’ collateral holdings.

The simulation results of central bank bills and reverse repos are shown in Figure 11. As explained above, central bank bills may be purchased by non-FIs. Therefore, we need to take a stand to what extent the collateral holdings of FIs increases. In the simulation exercise we assume that FIs’ collateral holdings double.

Figure 11: Central bank bills and reverse repos

Note that in Figure 11, the calibrated value of $M = 5.62$ is at the right-hand side of each panel. The simulation results of central bank bills and reverse repos suggest that $i_m$ remains at $i_d$ as long as $M > \text{CHF 20 bn}$. For low values of $M$, it increases steadily and, at the calibrated value of $M$, reaches an $i_m$ that is larger than the calibrated value of $i_m = 0.6\%$. The same is true for $v$, although some money market activity emerges at much higher values of $M$.

Our simulation suggests that central bank bills and reverse repos are two additional tools that allow to have control of the money market rate in a structural liquidity surplus. As for term deposits, the control of the money market rate is less precise as with IOR which is indicated by the box-plots. Money market trading emerges more quickly than with term deposits and lies above pre-crisis levels. Finally, in Table 4 and 5, we

---

$^56$ A possible disadvantage of reverse repos might be that the central bank has insufficient collateral to absorb the quantity of reserves needed in order to achieve a given money market rate.

$^57$ The SNB used central bank bills and reverse repos in order to absorb reserves in 2010 and 2011. Fuhrer, Gugenheim and Schumacher (2014) provide evidence that FIs did not increase their available collateral by the same amount as the SNB absorbed reserves. This can be attributed to the fact that SNB Bills were mainly purchased by non-FIs.

$^58$ Again, as for term deposits, there are operational issues, since a large amount of reserves has to be absorbed. See our discussion above.
show that the operational costs of central bank bills or reverse repos are even smaller than is the case with term deposits. The reason for this is that central bank bills and reverse repos increase the collateral holdings of FIs (the fraction that is not acquired by non-FIs). This relaxes their collateral constraint and increases the demand for reserves. Consequently, the money market rate increases above the deposit rate earlier with central bank bills and reverse repos than with term deposits. As a result, fewer reserves need to be absorbed in order to achieve a given interest rate target.

6.4 Discussion

We end our analysis by comparing the four exit strategies. In Table 4, we assume that the central bank has a target for the money market rate of 0.5%. With all instruments $i_m$ is on target. The column labelled STD of $i_m$ displays the standard deviation of $i_m$. The standard deviation is zero with IOR and strictly positive with all other instruments. The second column shows the turnover which is close to zero with IOR and CHF 2.7 bn with all other instruments. The third column shows costs per year. Central bank bills and reverse repos are clearly the most cost-effective measures. In comparison to IOR the central bank can economize CHF 70 mn per year. The intuition for the savings is simply that with central bank bills, the monetary authority does not have to remunerate all reserves but only a fraction of it.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>STD of $i_m$</th>
<th>Turnover</th>
<th>Costs per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>IOR ($i_d = 0.5%$)</td>
<td>0</td>
<td>CHF 0.03 bn</td>
<td>CHF 1.86 bn</td>
</tr>
<tr>
<td>Term deposits</td>
<td>0.08</td>
<td>CHF 2.69 bn</td>
<td>CHF 1.82 bn</td>
</tr>
<tr>
<td>CB-bills or reverse repos</td>
<td>0.07</td>
<td>CHF 2.69 bn</td>
<td>CHF 1.79 bn</td>
</tr>
</tbody>
</table>

Table 4: Target $i_m = 0.5\%$a

aTable 4 displays the simulation results of the various exit strategies when the central bank targets a money market rate of 0.5%. The following instruments are considered: interest on reserves (IOR), term deposits, central bank bills and repos. The instruments are evaluated according to the following criteria: the ability to control the money market rate, the impact on the money market trading activity, and the yearly operational costs of an exit.

Table 5 repeats the exercise of Table 4 but for a money market target rate of 1%. The main difference is that the cost are significantly higher. This is no surprise, since, the central bank now pays 1% on the stock of reserves instead of 0.5%. Note that the cost advantage of central bank bills or repos is even higher.

59Note that the simulation of the model provides us with $v$ from which we calculate the aggregate trading volumes in the SFRM.
Table 5: Target $i_m = 1\%$

<table>
<thead>
<tr>
<th>Instrument</th>
<th>STD of $i_m$</th>
<th>Turnover Costs per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>IOR ($i_d = 1%$)</td>
<td>0</td>
<td>CHF 0.02 bn</td>
</tr>
<tr>
<td>Term deposits</td>
<td>0.07</td>
<td>CHF 3.11 bn</td>
</tr>
<tr>
<td>CB-bills or reverse repos</td>
<td>0.08</td>
<td>CHF 3.11 bn</td>
</tr>
</tbody>
</table>

Table 5 displays the simulation results of the various exit strategies when the central bank targets a money market rate of 1%. The following instruments are considered: interest on reserves (IOR), term deposits, central bank bills and repos. The instruments are evaluated according to the following criteria: the ability to control the money market rate, the impact on the money market trading activity, and the yearly operational costs of an exit.

7 Conclusion

In response to the financial crisis of 2007/2008 and the subsequent sovereign debt crisis, all major central banks decreased interest rates to the zero lower bound and created large excess reserves via asset or foreign currency purchases. This paper addresses the question of how to control interest rates in such an environment. We investigate the following four exit instruments: interest on reserves, term deposits, central bank bills, and reverse repos. The evaluation criteria are: the ability to control the money market rate, the impact on the trading activity in the money market, and the operational costs of an exit.

The following results emerge from our model: all four exit instruments allow central banks to achieve an interest rate target. Nevertheless, central bank bills (or reverse repos) have several advantages. First, money market trading activity re-emerges more quickly with central bank bills than with any other instrument. Second, central bank bills are tradable and relax FIs’ collateral constraints. For this reason, fewer reserves need to be absorbed to attain a given interest rate target. Third, this also implies that the operational costs of an exit are lower with central bank bills than with any other instrument. This finding might become relevant as the cost of an exit has the potential to become a political issue. Finally, for all exit instruments, minimum reserve requirements will become less important for controlling interest rates.

Our analysis contributes to the ongoing discussion about exit strategies. For example, the Federal Reserve (Fed) is currently testing a combination of interest on reserves, reverse repos and term deposits and is evaluating the effectiveness to control the federal funds effective rate. Reasons for this particular combination of instruments are the Fed’s access policy and the structure of the US money market. The SNB used central bank bills and reverse repos in 2010 and 2011 to absorb reserves that were created through foreign exchange purchases. As predicted by our model, the SNB was able to manage interest rates with these instruments and trading activity in the money market re-emerged quickly.

Finally, IOR allows for a perfect control of the money market rate but leads to a virtually nonexistent money market trading activity. This finding is relevant for the current economic environment.

---

For further details, see FOMC Minutes, June 2014.
discussions about interest rate benchmark reforms. In order to reduce the potential for manipulation, regulators currently discuss whether references rates should be based on concluded transactions, only (see Financial Stability Board, 2013 and European Central Bank, 2013). If transaction based reference rates are important, then, central bank bills or reverse repos would become the preferred exit instruments.
Appendix I: Data

The data we use for the calibration is described in Table 6 and is provided by Eurex Ltd., the Swiss Federal Statistical Office (SFSO), the SNB and SIX Ltd.

Table 6: Data

<table>
<thead>
<tr>
<th>Description</th>
<th>Period</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>SARON</td>
<td>Jan 2005 - Dec 2013</td>
<td>Daily</td>
</tr>
<tr>
<td>Overnight SNB Special Rate</td>
<td>Jan 2005 - Dec 2013</td>
<td>Daily</td>
</tr>
<tr>
<td>Inflation (year-on-year change)</td>
<td>Jan 2005 - Dec 2013</td>
<td>Monthly</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>Q3 2005 - Q4 2013</td>
<td>Quarterly</td>
</tr>
<tr>
<td>Money market transaction data</td>
<td>Jan 2005 - Dec 2013</td>
<td>Daily</td>
</tr>
<tr>
<td>Central bank reserves</td>
<td>Jan 2005 - Dec 2013</td>
<td>Daily</td>
</tr>
</tbody>
</table>

Table 6 displays the source of the data we use in the quantitative analysis section. The data described in row one to row four is publicly available. The data described in row five and six have restricted access.

Appendix II: Experiments

Subsequently, we discuss temporary and permanent shocks to the parameters $M, \mu, \sigma, B$. We consider parameter values ranging from $+/-5\%$ (unless otherwise indicated) starting from the calibrated parameter value. The simulation results for each parameter value are displayed in a box-plot representation. The box-plot in the middle of each panel represents the simulation based on the calibrated parameter values. The five box-plots to the right (left) display the results if the parameter value is increased (decreased). In the figures displayed below, the panel on the left-hand side displays the effect of the parameter change on $i_m$, whereas the panel on the right-hand side displays the effect on $v$.

**Permanent $M$-shock.** A temporary $M$-shock is presented above. Hence, we only consider a permanent shock to $M$. The simulation results are displayed in Figure 12.

---

61 The simulations for each parameter value under consideration is repeated with the identical random sample of liquidity shocks, except for the experiments where moments $\mu$ and $\sigma$ of the log-normal distribution are varied.
Although the qualitative effects of a permanent $M$-shock are identical to a temporary $M$-shock, the quantitative impact of $M$ on $i_m$ is smaller. The simulation results suggest that $i_m$ is decreasing in $M$ and $i_m$ is much less elastic to changes of $M$ compared to a temporary $M$-shock. The difference between the temporary and the permanent $M$-shock experiment can be attributed to the adjustment of the value of reserves $\phi$, which is kept at the calibrated value in case of a temporary shock. If $M$ decreases, reserves become scarce and $\phi$ increases. In contrast, if $M$ increases, FIs are satiated with reserves and $\phi$ decreases. By this, the adjustment of $\phi$ offsets the change in $i_m$ that is necessary to ensure market clearing. Although all prices are fully flexible, a change of $M$ is not neutral. The reason for the non-neutrality is the restriction of the demand for reserves imposed by the collateral constraint. If $M$ increases, $\phi$ decreases but the demand for reserves cannot adjust because the collateral constraint becomes more binding.

The analysis of the simulated turnover-to-reserves ratio suggests that $v$ is decreasing in $M$. Along the lines of argumentation above, the decrease can again be attributed to the collateral constraint.

**Temporary $\mu$-shock.** A shock to $\mu$ is a shock to FIs’ average reserve needs. The simulation results of a temporary $\mu$-shock are displayed in Figure 13.$^{62}$

---

$^{62}$In contrast to all other experiments, the impact of a temporary or a permanent change to $\mu$ and $\sigma$ on $i_m$ and $v$ cannot be studied based on the identical sample of random liquidity shocks since a variation of $\mu$ and $\sigma$ requires an updated sample of random liquidity shocks.
The money market rate $i_m$ is increasing in $\mu$. At the calibrated value $\mu = 1$, the interest rate $i_m$ is strongly elastic to changes in $\mu$. Very similar to the simulation results of a temporary $M$-shock, the elasticity of $i_m$ decreases, the further away $\mu$ is from its calibrated value and eventually drops to zero at the extreme values presented in Figure 13.

The interpretation of the simulation results is very similar to the temporary $M$-shock experiment since the effect of an increase of the mean of the liquidity shock is similar to those of a temporary decrease of $M$ (and vice versa).

**Temporary $\sigma$-shock.** A shock to $\sigma$ is a shock to FIs’ uncertainty about their reserve needs. The simulation results of a temporary $\sigma$-shock are displayed in Figure 14.

The simulation results suggest that $i_m$ is decreasing in $\sigma$. In contrast to the temporary $M$- and $\mu$-shock experiment, $i_m$ is inelastic to changes in $\sigma$. In contrast to a shock to $\mu$, the $\sigma$-shock does not affect the average liquidity needs but increases the mismatch of reserves. Consequently, an increase in $\sigma$ increases the demand for reserves as well as the supply of reserves. If the increase of the demand and supply is symmetric, it would have no effect on the interest rate. However, the effect is asymmetric because of the
collateral constraint which limits the increase in the demand for reserves. If $\sigma$ increases, more FIs need to borrow larger amounts of reserves but they are collateral constraint. Consequently, the demand for reserves is bounded whereas the supply of reserves increases. The analysis of the turnover-to-reserves ratio suggest that $v$ is increasing in $\sigma$. As described before, this can be attributed to an increased mismatch of reserves which requires more re-allocation if $\sigma$ increases.

**Permanent $\mu$-shock.** The simulation results of a permanent shock to $\mu$ are displayed in Figure 15.

![Figure 15: Permanent shock to $\mu$](image_url)

A permanent $\mu$-shock has neither an impact on $i_m$ nor on $v$. This may be attributed to the adjustment of the value of reserves. In contrast to a temporary shock to $\mu$ the effects on $i_m$ and $v$ are very small because the value of reserves $\phi M$ adjusts.

**Permanent $\sigma$-shock.** The simulation results of a permanent shock to $\sigma$ are displayed in Figure 16.

![Figure 16: Permanent shock to $\sigma$](image_url)
In contrast to a permanent $\mu$-shock, the simulation results suggest that a permanent $\sigma$-shock has an effect on $i_m$ as well as $v$. As previously discussed, this is due to the collateral constraint which limits the demand for reserves.

**Temporary $B$-shock.** A change of $B$ can be interpreted as a shock to FIs’ collateral holdings. The simulation results of a temporary $B$-shock to are displayed in Figure 17.

![Figure 17: Temporary shock to $B$](image)

The simulation results suggest that $i_m$ is increasing in $B$ and $i_m$ is inelastic to changes in $B$. Increasing FIs’ collateral holdings relaxes the constraint and increases the demand for reserves. Since the supply of reserves remains unchanged, $i_m$ as well as $v$ increase. In contrast, a decrease of $B$ lowers $i_m$ and for $B = 0$, $v = 0$.

**Permanent $B$-shock.** The simulation results of a temporary $B$-shock are displayed in Figure 18.

![Figure 18: Permanent shock to $B$](image)

The permanent $B$-shock has the same effect on $i_m$ and $v$ as the temporary $B$-shock. Again, this demonstrates the effect of the collateral constraint in our model. Although the value of reserves adjusts, the collateral constraint restricts the demand for reserves that cannot be compensated for by an endogenous adjustment of the value of reserves.
References


