Endogenous Intermediation in Over-the-Counter Markets

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April 9, 2015
Preliminary and incomplete. Not for circulation.

Abstract

We provide a theory of trading through intermediaries in over-the-counter markets. In our model, the role of intermediaries is to sustain unsecured trade. When agents borrow funds without pledging collateral, they can invest an amount in risky projects and increase total surplus. We propose a set-up in which traders in the OTC market are connected through a network. Agents are informed about the actions of their neighbors, and, at the same time, can trade with their counterparty through a path of intermediaries in the network. Intermediaries that facilitate unsecured trade receive, in exchange, a fee. In addition, links are costly. Both borrowers and intermediaries can default on their obligations when trade is unsecured. We show that trading through a network is essential to support investment levels that generate positive gains from unsecured trade, when agents are unlikely to meet the same counterpart again in the market. While unsecured trade can be sustained in various classes of network, the central agent in a star network can receive a higher share of the surplus as he accures higher fees. Nevertheless, we show that the star network is the constrained efficient configuration.
1 Introduction

Many financial transactions take place in the over-the-counter (OTC) market in a decentralized fashion. One prominent feature of OTC market is its concentrated intermediation structure. Li and Schurhoff (2014) show that dealers intermediate 94% of the trades in the municipal bond market, with most of the intermediated trades representing customer-dealer-customer transactions. Another documented feature is the prevalence of relationship trading. Afonso et al. (2014) document evidence that agents in the OTC market frequently choose to interact with the same counterparty over time in the Fed Funds market. Approximately 60% of the funds an individual bank borrows in one month persistently come from the same lender. These findings lead to questions about the role of intermediation and its connection to relationship trading in OTC markets. While there is a growing literature on OTC markets, these questions are rather unexplored. In particular, the optimality of the concentrated intermediation structure is a central issue to policy-makings regarding the “too-big-to-fail” phenomenon.

This paper proposes a theory of dynamic intermediation in OTC markets for unsecured trade. We study the interaction between intermediation and trading relationships by introducing informational frictions and limited commitment. Our model features endogenous intermediation and endogenous use of collateral, and we study the welfare implications of different intermediation structures. In contrast to many papers in the network literature, we consider a finite number of agents and investigate the role of economy size in explaining the rise of intermediation.

We consider a dynamic setting where agents trade bilaterally. Some agents have liquidity surplus and others have investment opportunities. Every period, an agent with a liquidity surplus is randomly paired with an agent with an investment opportunity. The investment opportunity represents a risky asset that yields a high return in the good states of the world and nothing in the bad states of the world. To finance the investment the agent with an investment opportunity borrows cash from the agent with a liquidity surplus. In exchange, he offers a contract that specifies a repayment to be received at the end of the period. The transaction can take place without or against collateral. Since there
is limited commitment and an agent with an investment opportunity may be tempted to renege on payments, collateral may be necessary to secure a transaction. However, unsecured trading is desirable as pledging collateral is costly for the agent with an investment opportunity.

In this setting, we introduce networks that serve two functions to overcome the informational frictions. The first function is informational. Absent the network, any trader can only observe other traders’ past dealings with himself but not their actions with others. With network, each agent can observe their neighbors’ actions and hence it can enhance monitoring. The second function is transactional, and traders may transfer funds through the network. Both functions are costly: the informational cost occurs at each period as long as agents are linked, while the transactional cost occurs only if the link is used to make transfers.

We obtain three main results. First we show that without any network any unsecured trade is not sustainable for large economies. This result follows from standard arguments in repeated games but it shows that the crucial role network provides is to overcome the informational friction. Moreover, the size of the economy, and hence the trading frequency, matters: for a small economy when agents trade with each other frequently, information about trading partners’ past actions is not essential; for large economies, however, to self-enforce repayments on unsecured trades the information provided by the networks is crucial.

Second, we provide conditions under which a star network (i.e. a network in which one agent intermediates all transactions) can sustain unsecured trades, regardless of the population size. It turns out that the incentive compatibility constraint for the star network is also a necessary condition for many networks, including all minimally connected networks. In fact, we show that networks in which some matched traders require many intermediaries cannot sustain any unsecured trade for large populations, and hence, some degree of concentration in intermediation is necessary.

We also find that to satisfy the incentive constraints for the intermediaries, fees for their services are necessary. Since the funds are transferred through them, the fees are the only means to check their temptations to renege on payments. When intermediation is
concentrated, this implies that few dealers receive most of the fees. The upper bound for the fees is determined by the incentive for agents who use the intermediation service, and is given by the relative gains from unsecured trades relative to those from secured trades. The lower bound for the fees is determined by the incentive for the intermediaries, and is given by the temptation to renege on their intermediated repayments. Thus, the fee structure in our model is endogenously determined by incentive compatibility, instead of driven by exogenously assumed bargaining power or monopoly rents.

Finally, we show that the star network is the unique constrained efficient arrangement when it can sustain the first-best level of investment, if the linking costs are small and the economy is large. The social welfare is composed of three elements in any arrangement: first, a higher level of investment is better but is more difficult to implement; the second and the third are determined by the network—a larger average number of links and a larger average number of intermediaries are both costly due to the informational and transaction costs. When the star can sustain the first best level of investment, we show that it minimizes the sum of informational and transaction costs for large economies. Moreover, when the costs are small, this arrangement is better than secured trades without any network.

We also consider the case where the star cannot implement the first best level of investment, and hence there may be a trade-off between linking costs and higher levels of investment. It turns out that for networks without too many average links and without too many intermediaries between average pairs compared to the stars, the set of implementable allocations does not change, at least asymptotically. Thus, the star is still a constrained efficient arrangement for large economies when it can implement investment levels that are close to the first best.

Related Literature

This paper relates to several strands of literature. The more relevant studies are those on contract enforcement, bilateral trading in OTC markets and dynamic network formation games.

The literature on contract enforcement is substantial. The general aim of this literature
is to show that repeated interactions alleviate problems that arise when there is limited enforcement of contracts. Allen and Gale (1999), Kletzer and Wright (2000), and Levin (2003) propose models where contracts are incomplete, either because transaction costs make it too costly to write explicit contracts or simply because the terms of the contract are not verifiable by a third party (i.e. a court). However, when two parties interact repeatedly, they can implement the first-best contract. Several other papers depart from the assumption that the same two parties interact with each other, and consider a large population of agents that are matched at random to interact every period. In this case, whether contracts can be enforced or not depends crucially on how much information is available to each agent. Klein (1992) approaches this issue in a model of repeated interaction between businesses that decide whether to give credit, and consumers who decide whether to pay her bill, and suggests that a credit bureau can hold a record of whether each consumer has ever defaulted or not. Greif (1993) and Tirole (1996) propose an enforcement mechanism based on community reputation. In this paper we also study whether it is possible to enforce first-best contracts through repeated interactions when agents are randomly matched to trade. However, we consider that agents have access to information via network of bilateral relationships, and characterize the constrained efficient network.

In the past decade, a series of papers has studied trading in over-the-counter markets. Most of these studies have been concerned with explaining asset prices through trading frictions. Acharya and Pedersen (2005) study the effect on asset prices of an exogenously specified trading cost. Duffie, Garleanu and Pedersen (2005, 2007) endogenize the trading frictions arising through search and bargaining, and show their effects on asset prices. Some other papers look at trading in exchanges and analyze how information is transmitted through a network and embedded in prices (Colla and Mele (2010) and Ozsoylev and Walden (2009)). Complementary to this literature, we propose a model where agents can overcome frictions that arise from search by trading through a network of relationships.

Methodologically, this model draws from the literature on networks. The general concept of a network is quite intuitive: a network describes a collection of nodes and the links between them. Situations, such as the one we study, where agents form or severe
connections depending on the benefits they bring are modeled through a game of network formation. A rapidly growing literature on network formation games has developed in the past decade, introducing various approaches to model network formation and proposing several equilibrium concepts (Bala and Goyal, 2000, Bloch and Jackson, 2007, Jackson and Wolinsky, 1996). In the context of these initial models, Goyal and Vega-Redondo (2007) analyze the role of intermediaries in bridging structural holes in social networks.

Although there are numerous applications of these models in the social science context, the research on financial networks is still at an early stage. Allen and Babus (2008) provide a comprehensive survey of this literature. Most of the existing research using network theory concentrates on issues such as financial stability and contagion. For instance, Leitner (2007) investigates the possibility of private bail-outs organized by a social planner. Allen, Babus and Carletti (2010) and Zawadowski (2013) concentrate on the interaction between financial connections due to overlapping portfolio exposures and systemic risk.

The role of intermediaries in financial networks has been analyzed thus far by Gale and Kariv (2007), Gofman (2011) and Fainmesser (2011). The first two papers study how the presence of intermediaries affects the efficient allocation of assets for a fixed network. In Gale and Kariv intermediation creates inefficiencies as it delays trade, while in Gofman inefficiencies arise when agents bargain through intermediaries. In contrast, we show that intermediation can alleviate inefficiencies in over-the-counter markets. In a very recent work, Fainmesser (2011) also studies how intermediaries can informally enforce the repayment of loans by borrowers, and identifies which networks sustain trade. However, which agents are intermediaries remains exogenous. Conversely, in the model we provide, certain agents endogenously assume the role of intermediaries to facilitate repeated interactions between traders in the market.

2 The Environment

Consider an infinite-horizon economy in which a set $N = \{1, ..., 2n\}$ of agents participate in the market at each date $t$. All agents are risk-neutral, infinitely lived, and discount the future at the constant rate $\beta = 1/(1 + \phi)$, where $\phi$ is the implicit real interest rate. At
the beginning of each period, uniformly at random, half the agents are assigned a liquidity surplus, and half the agents are assigned an investment opportunity. Let $\mathcal{L}^t$ be the set of agents with liquidity surpluses in period $t$ (henceforth, liquidity agents), and $\mathcal{I}^t$ be the set of agents with investment opportunities in period $t$ (henceforth, investment agents). A liquidity agent is endowed with one unit of cash, which can be stored at no cost until the end of the period. An investment agent is endowed with a riskless asset which yields a return of $r > 1$ at the end of every period. In addition, an investment agent has an opportunity to invest in a risky asset. The investment in the risky asset is scalable: if an amount $q \in [0, 1]$ is invested, the risky asset yields a return $\theta_i \in \{ R(q), 0 \}$ by the end of the period with probability $p$ and $(1 - p)$, respectively. The returns of the risky asset are independently distributed across agents, as well as over time. We assume that $R' > 0$ and $pR'(1) \geq 1, R'' < 0, R(0) = 0, R(1) = \overline{R}$ with $p\overline{R} > r$.

To exploit the investment opportunity, an investment agent $i \in \mathcal{I}^t$ needs to borrow funds from some liquidity agent $\ell \in \mathcal{L}^t$ at the beginning of each period, $t$. We assume that there is limited commitment, but that transactions can take place against collateral. A transaction is secured when the agent with an investment opportunity pledges the riskless asset as collateral at the beginning of the period. In this case, if the risky project fails ($\theta_i = 0$) then the agent with an investment opportunity cannot make any repayments. At the same time, whenever the agent with a liquidity surplus liquidates the collateral at the end of the period, he only obtains a return of one.

Typically, in OTC markets parties trade customized contracts. To capture this feature, we assume that once agents have been assigned a liquidity surplus or an investment opportunity, liquidity and investment agents are matched uniformly at random, and each liquidity agent can only lend to the investment agent he is matched to.

Formally, a matching $m^t$ is a subset of $\mathcal{L}^t \times \mathcal{I}^t$ such that for each liquidity agent $\ell \in \mathcal{L}^t$, there is a unique investment agent $i \in \mathcal{I}^t$ for which the pair $m^t = (\ell, i) \in m^t$. At each date $t$, a matching $m^t$ is randomly drawn from the set of all possible matches at date $t$.

The implicit assumption is that liquidating the riskless asset at the beginning of the period to self-finance the investment is too costly.
Then, the probability that a pair of agents \((k, k') \in \mathcal{N} \times \mathcal{N}\) is matched at date \(t\) is\(^2\)
\[
\Pr[(k, k') \in \mathbf{m}^t] = \frac{1}{2(2n - 1)}.
\]

For the remainder of the paper, we refer to a pair of agents before any uncertainty is realized as \((k, k')\), and to a matched pair of liquidity and investment agents as \((\ell, i)\).

Although the matching \(\mathbf{m}^t\) determines which pairs have the opportunity to trade at date \(t\), a matched pair \((\ell, i)\) needs not trade directly. In particular, we assume that trade can take place in a network through a path of intermediaries. A network, \(g^t\), is a graph \((\mathcal{N}, \mathcal{E}^t)\), where \(\mathcal{N}\) is the set of nodes, and \(\mathcal{E}^t \subset \mathcal{N} \times \mathcal{N}\) is the set of links that exist between agents at date \(t\). The set of agents that have a link with agent \(k\) in the network \(g^t\) is denoted by \(\mathcal{N}^t_k\). A path of intermediaries between a pair \((k, k') \in \mathcal{N} \times \mathcal{N}\) in a network \(g^t\) is a sequence of agents \((j_1, j_2, ..., j_v)\) such that the links \((k, j_1), (j_1, j_2), ..., (j_v, k') \in \mathcal{E}^t\). We use \(\mathbb{P}^t(k, k')\) to denote the set of paths from \(k\) to \(k'\) in the network \(g^t\), and \(\mathcal{P}^t(k, k')\) to denote a generic path. Similarly, once the matching \(\mathbf{m}^t\) is realized, we use \(\mathbb{P}^t(\mathbf{m}^t)\) to denote the set of paths that can be used to intermediate trade between a matched pair \(m^t = (\ell, i)\), and \(\mathcal{P}^t(\mathbf{m}^t)\) to denote a generic path.

Links in the network are costly. In particular, each agent, \(k\), incurs a cost for each link he has in the network that has two components: a recurrent component, \(c_l\), that is paid every period, and an idiosyncratic component, \(c_m\), that is paid only in the periods in which the link is used in a transaction. A link can be used in a transaction when it connects a pair of matched agents, or when it connects agents that intermediate trade between a matched pair. Thus, the total cost that an agent pays in any given period \(t\) depends not only on his position in the network, but also on the realized matched \(\mathbf{m}^t\) and the path of intermediaries used to trade.

The network has both a trading and an informational function. First, a matched pair \((\ell, i)\) that is not directly connected by a link may, nevertheless, transfer funds to each other through a path of intermediaries. Second, an agent can observe the unilateral actions that his neighbors take in the network. We assume the realization of the risky project is\(^2\)

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\(^2\)This is because the probability that \(k\) is a liquidity agent is \(\frac{1}{2}\). Then, conditional on being a liquidity agent, the probability that he is matched with \(k'\) as an investment agent is \(1/(2n - 1)\).
private information and is observable only to the specific investment agent. The motivation behind the structure of the linking costs is related to these two functions: the idiosyncratic component, \( c_m \), can be interpreted as a transaction cost, while the recurrent component, \( c_l \), can be interpreted as a cost to access information. Conceptually, the two functions need not be associated to the same network structure, and can be studied separately. However, in our framework they interact to alleviate limited commitment frictions and affect agents’ incentives to repay their obligations.

We study when the first-best allocation can be decentralized, and characterize second-best outcomes as well.

3 The (Repeated) Trading Game

In this section we assume that the trading network \( g = (\mathcal{N}, \mathcal{E}) \) is fixed and remains constant for all periods. Given a network \( g \), we investigate whether there exists a set of financial contracts for which trade between any matched pair takes place without collateral, when the level of investment is \( q \in [0, 1] \). We consider that the level of investment is \( q \) when each investment agent borrows an amount \( q \) from the liquidity agent with whom he is matched, and invests it in the risky asset.

We begin by specifying the contracts and the trading game, and define strategies and equilibrium. Then, we characterize the level of investment that is implementable in equilibrium. We proceed to analyze the efficiency of financial networks and propose a robustness exercise.

3.1 Financial contracts and trading procedure

The financial contract that determines the trade between a matched pair has two components. The first component specifies a amount, \( d \in [q, r] \), that an investment agent should repay a liquidity agent with whom he is matched in exchange for borrowing \( q \) units of funds. The second component allocates a fee \( f \in \mathbb{R}_+ \) to any agent that can be an intermediary. Thus, if a pair \((k, k')\) is matched and trade through a path \( \mathcal{P}(k, k') = (j_1, j_2, ..., j_v) \) without collateral, then the investment agent should repay in total \( d + \sum_{s=1}^{v} f \), such that
an intermediary \( j_s \) receives \( f \), for any \( s = 1, ..., v \).

We consider financial contracts, \((d, f)\), that are independent of the position of the agents in the network. However, a crucial feature of our analysis is that the financial contract depends on the network structure \( g \). Thus, an agent’s position in the network is only reflected in the total payoff he expects to receive in a given period. However, by comparing different network structure we highlight the relative advantage that network positions offer some agents over others. We also allow the financial contract to depend, on the amount, \( q \), that an investment agent borrows from the liquidity agent with whom he is matched. Moreover, we assume that the contract is independent of the date, as long as the network is constant over time.

Because of limited commitment, intermediaries need some future benefits to motivate them to transfer the repayments to the next agent. In particular, for an agent with a liquidity surplus who is an intermediary, without such future benefits it is optimal for him to keep the repayments for himself. However, for the fees to serve this purpose agents must use the information obtained from the network adequately.

Given the contract \((d, f)\), we define the trading procedure at date \( t \), as follows. First each agent is assigned a type (liquidity or investment), and the matching \( m^t \) realizes. These realizations are common knowledge among all agents.

Then, for each matched pair \( m^t = (\ell, i) \in m^t \), the investment agent \( i \) proposes a path \( P(m^t) = (j_1, j_2, ..., j_v) \) through which to trade with \( \ell \) (including the empty path, i.e. trade directly with \( \ell \)). We assume that this proposal is common knowledge to all agents. Each agent on the path then sequentially responds with an yes or no, starting from \( j_1 \) and ending with \( \ell \). If all agents on the path respond with yes, then the liquidity agent, \( \ell \), transfers \( q \) units of cash to the investment agent, \( i \), through the path without asking collateral. Otherwise, the liquidity agent, \( \ell \), transfers directly one unit of cash to the investment agent, \( i \), and, in exchange, the investment agent transfers the riskless asset as collateral to the liquidity agent.\(^3\)

\(^3\)Note that when trade is secured, we assume that the liquidity agent lends one unit of fund to the investment agent instead of \( q \). This assumption captures the idea that no gains from trade should be left on the table. Indeed, when trade is secured, the first best quantity to be invested is 1, which is also incentive compatible. However, all our results are robust to this assumption. That is, we can allow for agents to borrow \( q \) units of funds and to pledge a fraction of the riskless asset as collateral.
If all agents agree, then each agent on the path has a debt obligations to the next one according to the financial contract \((d, f)\), as follows. The agent \(i\) is obligated to repay \([d + v \cdot f]\) to \(j_1\). Further, each intermediary \(j_{i'}\) is supposed to receive \([d + (v - s' + 1)]\) from \(j_{i'-1}\) and is obligated to repay \([d + (v - s') \cdot f]\) to \(j_{i'+1}\), with \(j_0 = i\) and \(j_{v+1} = \ell\).

After the risky project realizes its payoff, then each agent on the path decide whether he or she repays his/her debt obligation. We assume that the agent either repays it in full or repay nothing. This assumption will simplify our notation without losing any insights.

Next, we explain the information structure in detail. As we discussed earlier, an agent \(j\) can observe each of his neighbor’s unilateral actions, as well as information that is common knowledge, which is the type of the agents, the matching, and the proposed paths by each investment agent. For each agent \(k\), his unilateral actions in the network \(g\) at date \(t\), denoted by \(a^t_k\), include the following elements: (i) his responses on the proposed trading paths that he is involved; (ii) whether he repays in full to each of his neighbors, if he is either an intermediary and/or an agent with an investment opportunity. If two agents trade directly, their repayment decision is not observed by other agents. Let \(a^t_k = (a^0_k, \ldots, a^t_k)\) be the unilateral actions taken by agent \(k\) up to date \(t\), and let \(a^0_k = (a^0_0, \ldots, a^0_t)\) be the commonly known information up to date \(t\). Then, the history that an agent \(k\) observes at date \(t\) is given by \(h^t_k = \{a^t_j : (j, k) \in E\} \cup \{a^0_i\}\).

Because an agent may be involved in multiple trading paths, we need to specify a timing for their responses and repayments. For each proposed trading path \(P = (j_1, j_2, \ldots, j_v)\) between a matched pair \(m = (i, \ell)\), agents in position \(j_1\) respond simultaneously first, then agents in position \(j_2\), etc. Similarly, for repayment decisions, investment agents decide first simultaneously, and then agents in position \(j_1\), depending on the resources repaid by investment agents, and then agents in position \(j_2\), etc.

Next we introduce strategies and the equilibrium concept. First we define strategies. For each agent \(k\), his strategy in period \(t\), denoted by \(s^t_k\), has three components:

- \(s^{t,1}_k\) maps the history \(h^{t-1}_k\) he observes, the realization of agents’ type, and the matching \(m^t\) to a proposed path, if he is an investment agent;
- \(s^{t,2}_k\) maps the history \(h^{t-1}_k\) he observes, the commonly known information \(a^0_t\), and the
responses of his neighbors before him on the paths that involve him to his responses, if he is a liquidity agent and/or an intermediary;

- \( s_{k}^{t,3} \) maps the history \( h_{k}^{t-1} \) he observes, the commonly known information \( a_{0}^{t} \), and the repayments of his neighbors before him on the paths that involve him to his repayment decisions on all trading paths he is involved, if he is an investment agent and/or an intermediary. Note that his repayment decision is constrained by repayment decisions of agents before him on the trading paths.

We use Perfect Bayesian Equilibrium (PBE) in pure strategies as the solution concept. We restrict attention to equilibria that satisfy the following properties.

(A1) **No default.** Every agent consents to trade the contract \((d, f)\) without collateral and there is no default in equilibrium plays.

(A2) **Shortest path.** The shortest paths in the network \( g \) are always proposed in equilibrium. When there are multiple shortest path between a matched pair, they are proposed with equal probabilities in equilibrium.

(A3) **Stationary equilibrium allocation.** The level of investment, \( q \), is constant across realized matches and across periods.

**Definition 1** A PBE equilibrium satisfying (A1)-(A3) is called a **simple equilibrium**.

Condition (A1) is a symmetry requirement, as it rules out the possibility that trading without collateral happens for a subset of agents. Similar considerations motivate condition (A3). Condition (A2) requires that the equilibrium trading paths are the shortest ones. This assumption simplifies our analysis, since in general networks multiple paths may be used to trade, but only the shortest one minimizes the expected transaction cost, \( c_{m} \). For most of our results, this requirement is not binding.

### 3.2 Implementation

In this section we explain in detail why trading without collateral is desirable, and explore the role of networks in supporting unsecured trade in equilibrium. We first introduce the gains from unsecured trading relative to trading against collateral. We then characterize
the investment level, $q$, that is implementable in a given network $g$. The investment level, $q$, provides a rich metric to compare different network structures that can be more informative than simply showing whether unsecured trade can be sustained in a given network.

**Definition 2** A level of investment, $q$, is implementable in a network $g$ if it is supported in a simple equilibrium given a financial contract $(d, f)$.

Abstracting from linking and transaction costs, unsecured trading is beneficial relative to trading against collateral. This is because when trade is secured, agents forego some of the return of the riskless asset in those states of the world in which the risky project fails. Indeed, suppose that the level of investment is $q$. Then, the average surplus generated at each date by trading without collateral is in expectation

$$pR(q) - q + r.$$ 

In contrast, the average surplus generated at each date by trading against collateral is in expectation

$$pR(1) - 1 + pr + 1 - p.$$ 

Therefore, the relative gains from unsecured trading are given by the following function

$$\Delta(q) = [pR(q) - q] - [pR(1) - 1] + (1 - p)(r - 1).$$

Since the return $R(\cdot)$ is a concave and increasing function, the condition $pR'(1) \geq 1$ ensures that $\Delta(\cdot)$ is increasing in $q$. The relative gains from unsecured trade are maximized and positive when $q = 1$. This implies that $q = 1$ represents the first-best level of investment. At the same time, since $\Delta(1) > 0$, it follows that there are positive gains from trading without collateral even for a level of investment $q < 1$.

Although trading without collateral may generate a higher expected surplus than secured trade, it is not necessarily the case that it can be supported in equilibrium. Even in the least restrictive case of complete information, when all histories are publicly observable, unsecured trade can be supported in equilibrium for an investment level $q$ if and
only if
\[
\phi q \leq \frac{1}{2} \Delta(q).
\]  
(1)

The intuition is simple. When trade is unsecured, agents weigh the long-term benefit from trading without collateral against the one time gain of retaining all the return of the assets and paying 0. In particular, when an investment agent decides whether to repay at the end of the period, he takes into consideration he will be required to pledge collateral at all future dates as an investment agent, if he defaults on his obligations. For the remainder of the paper we assume that condition (1) holds.

When there is incomplete information, condition (1) is no longer sufficient. In this case, the frequency with which an agent trades with counterparties that have access to his private history affects his incentives to default or not on his obligations. Thus, whether unsecured trade can be supported in equilibrium may depend on the population size. Since when trading in a network agents can access the private histories of their neighbors, networks may facilitate the implementation of an investment level \( q \) if there are positive gains from unsecured trade, particularly when the population grows large.

To understand the role of networks in supporting trade without collateral, we first explore the empty network benchmark. The trading procedure in the empty network is nested in the trading procedure described in Section 3.1. In particular, once the agents’ type has been assigned and the matching has been realized, an investment agent can only propose to trade directly (i.e. the empty path) with the liquidity agent he has been matched with. The liquidity agent can accept, and trade without collateral, or reject, and trade against collateral. Clearly, no agent intermediate trades in the empty network. The following lemma gives a full characterization of the level of investment that is implementable in the empty network.

**Lemma 1** Let agents trade in an empty network.

**Proposition 1** 1. A level of investment, \( q \), is implementable if

\[
\phi q \leq \frac{1}{2(2n - 1)} \Delta(q).
\]  
(2)

\(^4\) We do not provide a proof for this statement, as the result is standard.
2. For any level of investment \( q > 0 \), there exists \( \bar{n} \) such that \( q \) is not implementable for all \( n \geq \bar{n} \).

The lemma shows that the level of investment that is implementable when no information is observable depends on how large the number of agents is. This is because the population size affects how likely it is that two counterparties who trade at date \( t \), meet again in a given future period. Moreover, as the population size increases, no level of investment is implementable in an empty network. In other words, there is no \( q \) for which trade takes place without collateral as \( n \) grows large, even though there may be positive gains from unsecured trade.

A stark contrast with the empty network in terms of the level of investment that is implementable is provided when we analyze a star network. A network \( g \) is a star if there exists an agent \( k \) such that

\[
E = \{(k, j) : j \in N, j \neq k\}.
\]

We call the agent \( k \) in a star network the center agent. The next proposition gives a full characterization of the level of investment that can be implemented under a star network.

To characterize equilibria in networks for the remainder of the paper, we restrict our attention to financial contracts with the property that \( d \geq q + c_m \). We use this restriction for simplicity, as it ensures that the liquidity agent is willing to lend to the investment agent through the network, provided that he believes that his counterparties will repay their debts.

**Proposition 2** Let agents trade in a star network. Then, a level of investment, \( q \), is implementable if

\[
\phi(q + c_m) + 2c_m \leq \frac{1}{1/2 + \phi} \left[ -\phi(q + c_m) + \frac{1}{2} \Delta(q) - c_m \right].
\]  

(3)

Proposition 2 provides the necessary condition for a star network to implement a given level of investment \( q \). Condition (3) shows that the level of investment that is implementable in a star network is independent of the number of market participants.
Moreover, as the transaction cost, $c_m$, goes to 0, the condition becomes

$$\left(\frac{3}{2} + \phi\right) \phi q \leq \frac{1}{2} \Delta(q).$$

While this is more restrictive than the condition necessary in the complete information case, the first-best level of investment can be implemented in the star network for a nearly as large set of parameters.

A star network represents an improvement relative to the empty network. However, not all networks can implement the first-best, or even lower investment levels. The following result formalizes this idea.

**Lemma 2**  Let agents trade in a connected network $g$. Suppose that $n > 3$, and let $v_{\text{max}}$ be the maximum number of intermediaries between any pair of agents. Then, the investment level $q$ is implementable only if

$$\frac{v_{\text{max}}^2}{4n} \leq \frac{\Delta(q)}{\phi(q + c_m)}.$$

This result shows that networks in which intermediation chains are too long cannot sustain unsecured trade. When many intermediaries are involved in a transaction between a matched pair, the investment agent needs to pay a large amount in fees, which distorts his incentives to repay when trading without collateral.

Lemma 2 together with Proposition 2 suggest that some degree of concentration in intermediating trades is necessary. While the star network is not the only network that can sustain unsecured trade, the condition (3) is sufficient for a level of investment $q$ to be implemented in other two classes of networks, at least asymptotically.

**Definition 3**  Let $\{g_n\}$ be a sequence of networks, and for each network $g_n$ consider a financial contract $(d_n, f_n)$. Then, a level of investment, $q$, is **asymptotically implementable** if there exists $\bar{n}$ such that $q$ is supported in a simple equilibrium given the contract $(d_n, f_n)$ for all $n \geq \bar{n}$.

The first class of networks we consider are connected networks that have the same number of links as a star, namely **minimally connected networks**. In a minimally connected
network there exists a unique path between any pair of agents. Such networks have the property that the average number of links in a network \( g \) is \( \eta_g \approx 1 \). The following proposition characterizes asymptotically implementable investment levels in minimally connected networks.

**Proposition 3** Let \( \{g_n\} \) be a sequence of minimally connected networks. Then, the level of investment \( q \) is asymptotically implementable only if (3) holds.

The second class of networks we consider are connected networks that have on average more links and more agents can intermediate trade than in a star network. We assume that a network \( g \) in this class is nevertheless *small*, in the sense that both the average number of links, \( \eta_g \), and the average number of intermediaries, \( v_g \), are bounded. The following proposition characterizes asymptotically implementable investment levels in this class of networks.

**Proposition 4** Let \( \{g_n\} \) be a sequence of networks. There exist \( \bar{\eta} > 1 \) and \( \bar{v} > 1 \) such that, if \( \eta_{g_n} \leq \bar{\eta} \) and \( v_{g_n} \leq \bar{v} \) for all \( n \), then the level of investment \( q \) is asymptotically implementable only if (3) holds.

Proposition 3 and Proposition 4 show that if a level of investment, \( q \), is implementable in a network belonging to either the small or minimally connected class, it must be implementable in a star network as well. This implies that a star network can implement the highest level of investment, at least relative to small and minimally connected networks. Moreover, the center agent in a star can accrue higher fees than any other intermediary in small or minimally connected networks.

### 3.3 Efficient Networks

In this section we study issues related to welfare and efficiency. Given a network \( g \), an investment level \( q \), and aggregate welfare is given by

\[
W(g, q) = \sum_{t=0}^{\infty} \beta^t n \left\{ pR(q) - q + (1 + r) - 4\eta_g c_l - 2(v_g + 1)c_m \right\}
\]  

(4)
where $\eta_g$ and $v_g$ have been defined above as the average number of links and the average number of intermediaries between pairs of agents in $g$, respectively. As it is evident from the welfare function, a network $g$ can be summarized thus only by two variables, $\eta_g$ and $v_g$.

**Definition 4** A network $g$ and an investment level $q$ is a constrained efficient arrangement if it maximizes $W(g,q)$ over the space of connected networks and investment levels such that $q$ is implementable in $g$.

As it was anticipated by Proposition 3 and Proposition 4, a star network is a good candidate for a constrained efficient arrangement as it can implement higher investment levels than small and minimally connected networks. The result is in fact more general, even when taking account both informational costs, $c_l$, and transaction costs, $c_m$, as the following proposition shows.

**Proposition 5** Suppose that (3) holds when $q = 1$. Then, for sufficiently large $n$, the star network and $q = 1$ is the unique constrained efficient arrangement.

Proposition 5 shows that when the first-best level of investment is implementable in a star network, then this is the most efficient network. In other words, concentrated intermediation maximizes social welfare.

The star network yields higher welfare even when compared to the empty network, if informational and transaction costs are small. This is because, the welfare loss induced by linking costs is smaller than the welfare loss incurred when trade takes place against collateral. As we have learned in Proposition 1, this is the case in the empty network if the number of market participants is sufficiently large.

If the first-best is not implementable in a star network, then the trade off between the level of investment and linking costs that the welfare function (4) embeds settles asymptotically also in the favor of the star network. To show this, we first introduce the following definition.

**Definition 5** The sequence $\{g_n\}$ and the investment level $q$ is an asymptotically constrained efficient arrangement if for any sequence of connected networks $\{g'_n\}$ and any
$q'$ asymptotically implementable under \( \{g'_n\} \),

\[
W(g_n, q) \geq W(g'_n, q') \quad \text{for all large } n.
\]

Since the inequality (3) does not hold for \( q = 1 \), we consider the second-best level of investment

\[
\hat{q} = \arg \max \Delta(q)
\]

\[
\text{s.t.} \quad \left( \frac{3}{2} + \phi \right) \phi(q + c_m) + 2c_m (1 + \phi) \leq \frac{1}{2} \Delta(q).
\]

Then we have the following result.

**Proposition 6** Suppose that \( W(1, 1, \hat{q}) \geq \max\{W(1, \bar{v}, 1), W(\bar{q}, 1, 1)\} \). Then, \( \hat{q} \) and the star networks are asymptotically constrained efficient.

Proposition 6 shows that if \( \hat{q} \) is sufficiently close to 1, then the star is still the constrained efficient network. However, for lower \( \hat{q}'s \), secured trade in the empty network dominates.
A Appendix

Proof of Lemma 1. First we prove sufficiency. Set \( d = q \). We construct a strategy profile and show that it is a simple equilibrium, as follows. For each possible match \( m = (m_i, m_\ell) \), we summarize the observed history of the match at the end of period \( t \) (which is observable to the match) with a state \( s_{m,t} \in \{G, B\} \). We use \( m^r \) to denote the match with the same pair of agents but with their roles reversed, i.e., \( m^r_\ell = m_\ell \) and \( m^r_i = m_i \). The state is such that \( s_{m,t} = s_{m^r,t} \) for all \( t \), and it evolves as follows:

\[
s_{m,0} = s_{m^r,0} = G; \quad s_{m,t+1} = G \text{ if } s_{m,t} = G \text{ and if either one of the following two conditions holds: (a) neither } m \text{ or } m^r \text{ realizes at period } t + 1, \text{ (b) either } m \text{ or } m^r \text{ is realized, and the agent assigned to the investment role repays his debt if traded without collateral;}
\]

\[
s_{m,t+1} = B \text{ otherwise. Note that for any match } m = (m_i, m_\ell) \text{ at period } t, \text{ the pair’s actions have no effect on states } s_{m',t} \text{ with } m' \text{ having agents other than the pair.}
\]

For any realized match \( m = (m_i, m_\ell) \) at period \( t \), the strategy for the pair only depends on \( s_{m,t-1} \) as follows: \( m_\ell \) accepts the proposed trade without collateral from \( m_i \) if \( s_{m,t-1} = G \) and rejects it otherwise; \( m_i \) repays his debt if \( s_{m,t-1} = G \) and does not repay otherwise.

Now we show that this strategy is sequentially optimal. Consider a realized match \( m = (m_i, m_\ell) \) at period \( t \). Because state \( B \) is self-absorbing, if \( s_{m,t-1} = B \), \( m_i \) has no incentive to repay his debt and hence it is optimal for \( m_\ell \) to reject his proposal. Now, suppose that \( s_{m,t-1} = G \). By the equilibrium strategy of \( m_i \), he will repay if his trade if accepted. Moreover, accepting or rejecting the trade has no impact on future states of the match. Thus, the current-period payoff for \( m_\ell \) to accept the trade without collateral is \((d + (1 - q))\) while the current-period payoff to reject the trade is \( p(d - 1) + 1 \leq d + (1 - q) \) since \( d \geq q \). Hence, it is optimal for agent \( m_\ell \) to accept the trade without collateral. Finally, assuming that the proposed trade without collateral from \( m_i \) was accepted by \( m_\ell \), by not repaying the debt, \( m_i \)'s equilibrium strategy follows the deviation considered in the proof of necessity. Thus, by (2), it is optimal for him to repay his debt.

Now we show that, for any given \( q > 0 \), it is not implementable for large \( n \)'s. Let \( \pi(q) = 1/2\{p[R(q) + r] + (1 - p)r + (1 - q)\} \), the expected surplus from trades without collateral. Here we assume that \( \beta > 1/2 \); the other case can be proved in a similar fashion.
Let $N$ be so large that if $K = \log_2(2N - 1) - 1$, then
\[
\frac{\beta^K}{2\beta - 1} + \frac{\beta^K}{1 - \beta} < \frac{q}{\pi(q)}.
\] (5)

Suppose, by contradiction, that $q$ is implementable with $2n \geq 2N$ agents. Now, at period zero, consider an agent with the investment role at the end of period 0 and is supposed to repay his promise, $d \geq q$. Consider the deviation to default now and, in all future period, behave as a non-defector. The worst scenario for this deviation would be that his current trading partner defects in all future periods, and all who are defected also defect. Thus, at period $t$, the probability of meeting a defector is at most
\[
p_t = \frac{\min\{2^{t-1}, 2n - 1\}}{2n - 1}.
\]

Hence, the expected payoff is at least $\sum_{t=1}^{\infty} \beta^t (1 - p_t) \pi(q)$, and, for the agent to prefer the equilibrium action than this deviation, it must be the case that
\[
-d + \sum_{t=1}^{\infty} \beta^t \pi(q) \geq \sum_{t=1}^{\infty} \beta^t (1 - p_t) \pi(q),
\]
that is,
\[
d \leq \sum_{t=1}^{\infty} \beta^t p_t \pi(q). \tag{6}
\]

Now, for any $n \geq N$ (recall that $N$ is defined by (5)) and for $k = \log_2(2n - 1) - 1$, we have
\[
\sum_{t=1}^{\infty} \beta^t p_t \leq \frac{(2n - 1)\beta^k - 1}{(2n - 1)(2\beta - 1)} + \frac{\beta^k}{1 - \beta} \\
\leq \frac{\beta^K}{2\beta - 1} + \frac{\beta^K}{1 - \beta} < \frac{q}{\pi(q)} < \frac{d}{\pi(q)},
\]
a contradiction to (6).
Proof of Proposition 2. We claim that \( q \) is implementable under star if and only if

\[
\min \left\{ \frac{1}{n-1} \left[ n\phi(q + c_m) - \frac{1}{2} \Delta(q) + 2nc_m \right], 0 \right\} \leq \frac{1}{n-1} + \phi \left[ -\phi(q + c_m) + \frac{1}{2} \Delta(q) - c_m \right].
\]

Note that (3) implies (7) for any \( n > 0 \): first,

\[
\frac{1}{2} + \phi \left[ -\phi(q + c_m) + \frac{1}{2} \Delta(q) - c_m \right] \leq \frac{1}{2n-1} + \phi \left[ -\phi(q + c_m) + \frac{1}{2} \Delta(q) - c_m \right]
\]
since \( \frac{1}{2} + \phi \geq \frac{n-1}{2n-1} + \phi \) for any \( n \); second,

\[
n\phi(q + c_m) - \frac{1}{2} \Delta(q) + 2nc_m \leq (n - 1) [\phi(q + c_m) + 2c_m]
\]
since \( -\phi(q + c_m) + \frac{1}{2} \Delta(q) - c_m \geq 0 \).

First we prove necessity. Let \( jC \) be the center agent. Suppose that \((d, f)\) with \( d \geq q+c_m \) implements \( q \) under the star. Consider an agent assigned to the investment role in the periphery, deciding whether to repay his debt, \( f + d \). We consider two choices: (a) repay the debt and follow the equilibrium strategies; (b) do not repay and propose direct trades in all following periods. By (A1), the choice (a) has to be better than the choice (b), and hence we have

\[
-(d + f) + \frac{\beta}{1 - \beta} \frac{1}{2} \left[ (pR(q) + r + (1 - q) - \frac{2n - 2}{2n - 1} f) - 2c_m \right] \geq \frac{\beta}{1 - \beta} \frac{1}{2} [p(R(1) - d + r) + pd + (1 - p)],
\]

that is,

\[
f \leq \frac{1}{n-1} + \phi \left[ -\phi d + \frac{1}{2} \Delta(q) - c_m \right].
\]

Now consider the center agent, \( jC \), assigned to the investment role and who is at the moment deciding whether to repay his debt, \( nd \). Again, we consider two choices: (a) repay all the debts and follow the equilibrium strategies; (b) do not repay (to any debt) and propose direct trades in all following periods. By (A1), the choice (a) has to be better
than the choice (b), and hence we have

\[-nd + \frac{\beta}{1 - \beta} \left\{ \frac{1}{2} [pR(q) + r + (1 - q)] + (n - 1)f - 2nc_m \right\} \geq \frac{\beta}{1 - \beta} \frac{1}{2} [p(R(1) - d - r) + pd + (1 - p)],\]

that is,

\[f \geq \frac{1}{n - 1} \left[ n\phi d - \frac{1}{2} \Delta(q) + 2nc_m \right]. \tag{10}\]

Combining (9) and (10) and the fact that \( d \geq q + c_m \), we obtain (7).

Now we prove sufficiency. Suppose that (7) holds. The financial contracts are given as follows. For unsecured trades on the equilibrium path, let \( d = q + c_m \) and let \( f \geq 0 \) satisfy

\[\frac{1}{n - 1} \left[ n\phi d - \frac{1}{2} \Delta(q) + 2nc_m \right] \leq f \leq \frac{1}{n - 1} + \phi \left[ -\phi d + \frac{1}{2} \Delta(q) - c_m \right].\]

For secured trades on off-equilibrium paths, the investment level is \( q = 1 \), and the investment agent repays 1 to the liquidity agent when the return is \( \theta = R(1) \) (when \( \theta = 0 \), the liquidity agent liquidates the collateral and obtains payoff 1). Given the contracts, the liquidity agent is indifferent between secured and unsecured trades (both of which give him a zero surplus), assuming that the investment agent will repay in unsecured trades.

We construct equilibrium strategies as follows. Each periphery agent can be one of the two states, \( G \) and \( B \). At date 0, all agents are in state \( G \). A periphery agent stays in state \( G \) if and only if he repays all his debts to \( j_C \) when assigned to the investment role in the previous periods; otherwise, he enters state \( B \). An agent who enters state \( B \) stays there forever. The state of these agents is only observable to the center agent, \( j_C \). Note that if a periphery agent trades directly then this choice does not affect his state and a periphery agent’s action in liquidity role does not affect his state. The center agent, \( j_C \), can also be in one of the two states, \( G \) and \( B \). He stays in state \( G \) if and only if he repays all his debts in the previous periods; otherwise, he enters state \( B \). His state is then observable to all agents.

The strategy of a periphery agent \( j \) assigned to the liquidity role in state \( G \) is as follows: if \( j_C \) is in state \( G \), then he accepts any trade through \( j_C \); if \( j_C \) is in state \( B \), he demands collateral for any trade. Moreover, he always asks for collateral if asked to trade directly.
A periphery agent $j$ assigned to the liquidity role in state $B$ always asks for collateral.
The strategy of a periphery agent $j$ in investment role is as follows: if both himself and $j_C$ are in state $G$, then he propose to trade through $j_C$ and repay his debt; if himself or $j_C$ is in state $B$ (or both), he proposes to trade directly, and, if his trade without collateral is accepted (following off-equilibrium behavior), he does not repay anything. Finally, the strategy of $j_C$ is as follows: if he is in state $B$, then he never repays anything; otherwise, he accepts trades from a match $m = (m_i, m_\ell)$ if and only if both $m_i$ and $m_\ell$ are in state $G$ and rejects it otherwise, and he repays all debts if and only if it is feasible and the number of periphery agents in state $G$ who repays at the current period, denoted by $k_1$, and the number of loans $j_C$ has, denoted by $k_2$ (including his own), satisfy

$$-k_2d + \frac{\beta}{1-\beta}\left[pR(q) + r + \frac{k_1(k_1 - 1)}{2n - 1}f\right] \geq \frac{\beta}{1-\beta}\left[p(R(1) - r) + (1 - p)\right].$$

(11)

Note that when there are still $k_1$ periphery agents in state $G$, the expected fees for each such agent is $(k_1 - 1)f/2(2n - 1)$ and since any such fee is paid to $j_C$, the expected fee revenue is $k_1(k_1 - 1)f/2(2n - 1)$.

We also need to construct equilibrium beliefs. As agent $j_C$ has complete information, his belief is the actual history. For a periphery agent $j$, his belief is such that if $j_C$ is in state $G$, then he believes that all other agents are also in state $G$ (and hence, he treats any rejection of unsecured trades from $j_C$ as mistakes). Note that once the center agent $j_C$ enters state $B$, all agents observe that and all coordinate to trade only with collateral.

To show that these strategies form a simple equilibrium, first notice that (A1)-(A3) are satisfied. Moreover, the agents’ beliefs are consistent with equilibrium strategies. In particular, when a proposed trade is rejected with $j_C$ in state $G$, it is believed to be a mistake and agents are all in state $G$ and will continue to accept trades and repay from next period on. We use the one-shot-deviation principle to verify sequential rationality. By (9) and (10) and the previous discussion no agent has incentive to deviate along the equilibrium path. On the off-equilibrium path, the history is summarized by the configuration of states and we verify sequential rationality as follows.

(i) Consider a periphery agent $j$ assigned to the liquidity role. Since $j_C$ will not accept
any trade from an investment agent in state $B$, he is indifferent between accepting a unsecured trade with $j_C$ and having a secured trade so long as $j_C$ is in state $G$ and it is optimal to reject any other unsecured trade (note that a periphery investment agent will not repay any debt incurred through direct trading). Thus, it is optimal for $j$ to accept unsecured trades from $j_C$ when $j_C$ has state $G$ and to reject unsecured trades from $j_C$ when $j_C$ has state $B$.

(ii) Consider a periphery agent $j$ assigned to the investment role. Since $j$’s state only depends on whether they repay $j_C$ and since $j$ will only have secured trades once entering state $B$, his incentive to repay is determined by (9). Note that as they believe all other agents are in state $G$, the continuation payoff is given by the left side of (8).

(iii) Finally, consider the center agent $j_C$. By construction, (11) determines whether he has incentive to remain in state $G$ or not following any history. ■

Proof of Lemma 2. Let $g$ be a given network with $2n > 6$ agents. For any pair of agents, $(j, j')$, let $\text{dist}(j, j')$ be the distance between them. Then, if the pair is matched, by (A2), the number of intermediaries between them is $\text{dist}(j, j') - 1$. Define

$$D = \frac{\max\{\text{dist}(j, j'): j, j' \in \mathcal{N}\}}{2},$$

i.e., either $2D$ or $2D + 1$ is the longest distance between any two nodes in $g$. Thus, $2D - 1 \leq v_{\text{max}} \leq 2D$. Here we assume that the path corresponding to this distance is given by $\mathcal{P} = (j_1, \ldots, j_{2D})$; the other case is similar. Suppose that the investment level $q$ is implementable under $g$ with financial contract $(d, f)$, $d \geq q + c_m$. Note that by (A2), the investment agent always choose the shortest path to intermediate trades, and hence, when $(j_1, j_{2D})$ forms a match, they use the path $\mathcal{P}$ with a positive probability. Moreover, if $(j_k, j_{k'})$ forms a match with $1 \leq k < k' \leq 2D$, they use the path $(j_k, \ldots, j_{k'})$ with a positive probability.

Thus, there exists a realization of matching and proposed paths such that, for each $k = 1, \ldots, D$, the maximum debt the agent $j_k$ has is at least $kd$. By symmetry, the
maximum sum of total debts $j_1, \ldots, j_{2D}$ have are at least

$$2 \sum_{k=1}^{D} kd = (D + 1)Dd.$$  

However, because all the fees cannot exceed the total future gains from trade relative to secured trades, $\frac{1}{\phi} n \Delta(q)$, we have

$$-D(D + 1)d + \frac{1}{\phi} n \Delta(q) \geq 0$$

for the first-best to be implementable, i.e.,

$$\frac{D(D + 1)}{n} \leq \frac{\Delta(q)}{\phi d}.$$  

The result follows from the fact that $d \geq q + c_m$. Now, note that $\nu_{\text{max}} \leq 2D$ and we obtain the desired result. ■

**Proof of Proposition 3.** Let $g$ be a given tree with $2n$ agents under which $q$ is implementable under $g$ with financial contract $(d, f)$, $d \geq q + c_m$. We show that

$$\phi d - \frac{3}{4n} \Delta(q) + 2c_m \leq f \leq \frac{-\phi d + \frac{1}{2} \Delta(q) - c_m}{\frac{n+1}{2n-1} + \phi}. \quad (12)$$

By taking $n$ to infinity in the above inequality and replacing $d$ with $q + c_m$, we obtain (3).

First we show the first inequality in (12). We show this by finding an agent who intermediates “many” unsecured trades.

For each agent $j$ and a neighbor $j'$ to $j$, define $L_{(j,j')}$ as the number of agents who have a path to $j$ through $j'$, and let $L_j = \max_{j'} \text{links to } j L_{(j,j')}$. Let $\chi$ be the largest integer less than $2n/3$.

**Claim 1.** There exists an agent $j$ with $L_j < 2n - \chi$.

**Proof.** We prove the claim by contradiction. Suppose such an agent $j$ does not exist, that is, for any non-leaf agent $j$, $L_j \geq 2n - \chi$. Then we construct a path that is arbitrarily long. Take an arbitrary non-leaf agent $j_1$. Then, there exists an agent $j_2$, a neighbor of
$j_1$, such that $L_{(j_1,j_2)} \geq 2n - \chi$. Then, remove all the agents who have a path to $j_1$ without passing through $j_2$, and call the remaining graph $g_1$. $g_1$ is then still a minimally connected network with at least $2n - \chi > 4n/3$ agents. Moreover, since $L_{(j_1,j_2)} \geq 2n - \chi > n$, and since $L_{j_2} \geq 2n - \chi$, there exists $j_3$ such that $j_3$ is in $g_1$ and $L_{(j_1,j_2)} \geq 2n - \chi$. Then, remove all the agents who have a path to $j_2$ without passing through $j_3$, and call the remaining graph $g_2$. Suppose that we have constructed $j_1, \ldots, j_\nu$ and $j_\nu \in g_{\nu-1}$, with $g_{\nu-1}$ has at least $2n - \chi$ agents. Then, $j_\nu$ has a neighbor $j_{\nu+1}$ with $L_{(j_\nu,j_{\nu+1})} \geq 2n - \chi$. Remove all the agents who have a path to $j_\nu$ without passing through $j_{\nu+1}$, and call the remaining graph $g_\nu$; note that $j_{\nu+1}$ is in $g_\nu$. Therefore, we may continue the process indefinitely, a contradiction to the finiteness of the graph $g$. □

By Claim 1, there exists an agent $j$ with $L_j < 2n - \chi$. Let $(j_1, \ldots, j_\nu)$ be his neighbors, ordered in a way such that

$$L_{(j,j_1)} \geq L_{(j,j_2)} \geq \ldots \geq L_{(j,j_\nu)}.$$  

Then we claim that we can find a realization of matchings such that $j$ intermediates at least $\chi$ matches. Consider two cases.

(a) $2n - \chi > L_{(j,j_1)} \geq \chi$. In this case we can have $\chi$ agents that has a path to $j$ through $j_1$ with investment role, and match each of them to an agent that has a path to $j$ without going through $j_1$ with liquidity role.

(b) $L_{(j,j_1)} < \chi$. Then, we can find $\nu^*$ such that

$$2\chi > L_{(j,j_1)} + \ldots + L_{(j,j_{\nu^*})} > \chi.$$  

Then, we can take $\chi$ agents that has a path to $j$ through $j_1, \ldots, j_{\nu^*}$ with investment role, and match each of them to an agent that has a path to $j$ without going through $j_1, \ldots, j_{\nu^*}$ with liquidity role.

Now we consider the incentive to repay for the agent $j$. Let $K \geq \chi$ be the largest number of matches $j$ intermediates. Then, the expected number of fees for $j$ is at most
K. Hence, we have

\[-Kd + \frac{\beta}{1 - \beta} \left[ \frac{1}{2} \Delta(q) + Kf - 2Kc_m \right] \geq 0,\]

that is,

\[f \geq \phi d - \frac{1}{2K} \Delta(q) + 2c_m > \phi d - \frac{3}{4n} \Delta(q) + 2c_m,\]

which gives the first inequality in (12).

To prove the second inequality in (12), consider the incentive for a leaf agent. Since the leaf agent has degree only one, he has to pay the fee \( f \) with probability \( \frac{n-1}{2n-1} \) and he never serves as intermediaries. Thus, it is optimal to repay his largest possible debt, \( d + f \), when assigned to the investment role, it must be the case that

\[-(d + f) + \frac{\beta}{1 - \beta} \left[ \frac{1}{2} \Delta(q) - c_m - \frac{n-1}{2n-1} f \right] \geq 0,\]

and hence

\[f \leq \frac{-\phi d + \frac{1}{2} \Delta(q) - c_m}{\frac{n-1}{2n-1} + \phi}.\]

This proves the second inequality in (12).

**Proof of Proposition 4.** Here we choose \( \overline{\eta} = 1.05 \) and \( \overline{\nu} = 1.05 \). Let

\[\overline{\Lambda}_n = (2n - 1) \frac{2 - \overline{\nu}}{6\overline{\eta}} - 1.\]  

(13)

For \( n \) large, \( \overline{\Lambda}_n \geq 0.3n \). Let \( g \) be a given network with \( 2n \) agents with \( \eta_g \leq \overline{\eta} \) and \( \nu_g \leq \overline{\nu} \) under which \( q \) is implementable under \( g \) with financial contract \((d, f)\), \( d \geq q + c_m \). We show that, for \( n \geq 30 \),

\[\phi d - \frac{1}{0.15n} \Delta(q) + 2c_m \leq f \leq \frac{-\phi d + \frac{1}{2} \Delta(q) - c_m}{\frac{n-1}{2n-1} + \phi}.\]  

(14)

By taking \( n \) to infinity in the above inequality and replacing \( d \) with \( q + c_m \), we obtain (3).

To show the first inequality in (14), we find an agent whose incentive is similar to that of the center agent in the star network. We need two claims, the first about existence of
an agent \( j \) with large degrees, and second showing that a proportion of agents connected to \( j \) trade through \( j \).

**Claim 1.** Let \( \Lambda \) be the maximum degree in \( g \). We show that

\[
\Lambda \geq (2n - 1) \frac{2 - \nu_g}{6\eta_g} - 1. \tag{15}
\]

**Proof.** Let \( \delta_j \) be the degree of agent \( j \). Then,

\[
2n(2n - 1)(\nu_g + 1) \geq 3 \times \left[ \sum_{j \in \mathcal{N}} \left( 2n - 1 - \delta_j - \sum_{j' \text{ linked to } j} \delta_{j'} \right) \right]
\]

\[
\geq 3 \times \left\{ 2n[2n - 1] - 2|\mathcal{E}(g_n)| - 2|\mathcal{E}(g_n)|\Lambda \right\}
\]

\[
\geq 3 \times (2n) \times [2n - 1 - 2\eta_g(1 + \Lambda)].
\]

Then, (15) follows directly by rearranging terms. \( \square \)

Since \( \eta_g \leq \eta \) and \( \nu_g \leq \nu \), (15) also implies that \( \Lambda \geq \tilde{\Lambda}_n \). Hence, we can find an agent \( j \) who has degree at least \( \tilde{\Lambda}_n \geq 0.3n \). Now, consider, \( S \), the set of \( j \)'s neighbors. Since by deleting all the links between agents in \( S \) the network is still connected, it follows that the number of those links has to be at most

\[
2n\eta - (2n - 1) = 2n(\eta - 1) + 1 \leq 0.1n + 1.
\]

Thus, there are at least \( 0.15n \) agents in the set \( S \) who has no link with any other agent in \( S \). Thus, the maximum number of intermediations for agent \( j \) is at least \( K \geq 0.15n \). Note that the expected number of fees for \( j \) is less than \( K \). To ensure that a simple equilibrium exists, considering \( j \)'s incentive, it must be the case that

\[
-Kd + \frac{\beta}{1 - \beta} \left[ \frac{1}{2} \Delta(q) + Kf - 2Kc_m \right] \geq 0. \tag{16}
\]

Since \( K \geq 0.15n \), (16) implies the first inequality in (14).

Now we show the second inequality in (14). Since \( |\mathcal{E}(g_n)| = 2n\eta_g \leq 2n\eta = 2.1n \) and hence the sum of all agents’ degrees is less than \( 4.2n \), and since there exists one agent with degree at least \( 0.3n \), there exists an agent with degree less than \( (4.2 - 0.3)/2 = 1.95 \).
Hence, there exists some agent with only one link. Since he has only one link, he cannot serve as an intermediary but has to go through an intermediary with probability at least \( \frac{n-1}{2n-1} \), that is, his incentive to repay is exactly the same as a leaf agent in the proof of Proposition 3. Hence, his incentive requires

\[-(d + f) + \frac{\beta}{1-\beta} \left[ \frac{1}{2} \Delta(q) - \frac{n-1}{2n-1} f - c_m \right] \geq 0. \tag{17}\]

By rearranging terms, (17) implies the second inequality in (14). □

**Proof of Proposition 5.** First note that under the star network, \( g_n^* \) with \( 2n \) agents, \( \eta_{g_n^*} = 1 - 1/2n \) and \( \nu_{g_n^*} = 1 - 1/n \). By Proposition , \( q = 1 \) is implementable. Hence, the average welfare is given by

\[ W^* = \frac{1}{2} \left\{ [R(1) - 1] + (1 + r) - 4 \left( 1 - \frac{1}{2n} \right) c_l - 2 \left( 1 - \frac{1}{n} \right) c_m \right\}. \]

Since the star network already implements the first-best level of investment, it remains to show that it minimizes linking costs (both recurrent and idiosyncratic) among all connected networks.

First it is easy to verify that linking costs are minimized under star among all minimally connected networks.

Next, we show that for any connected network \( g_n \) with \( 2n \) agents,

\[ \nu_{g_n} + 1 \geq 2 - \frac{2}{2n - 1} \eta_{g_n}. \]

To see this, note that for each agent \( i \), any agent who is directed connected to him has distance 1 but every other agent has distance at least 2, and hence

\[
\nu_{g_n} + 1 \geq \frac{\sum_{i \in N} \{ \text{deg}(i) + 2[2n - 1 - \text{deg}(i)] \}}{2n(2n-1)} \\
= \frac{4n(2n - 1) - 2(2n)\eta_{g_n}}{2n(2n-1)} \\
= 2 - \frac{2}{2n - 1} \eta_{g_n},
\]
where \( \text{deg}(i) \) is the degree of agent \( i \).

Thus, the network costs of \( g_n \) per capita, denoted by \( C_n \), satisfies

\[
C_n = 4v_{g_n}c_l + 2\eta_{g_n}c_m \geq 4v_{g_n}c_l + 2 \left[ 2 - \frac{2}{2n - 1} \eta_{g_n} \right] c_m.
\]

Now, let \( C_n^* = 4(1 - \frac{1}{2n})c_l - 2(2 - 1/n)c_m \) be the corresponding cost for the star network, we have

\[
C_n - C_n^* \geq S(\eta_{g_n}, n) \equiv 4 \left\{ \left[ \eta_{g_n} - \left( 1 - \frac{1}{2n} \right) \right] c_l + \left( \frac{1}{2n} - \frac{1}{2n - 1} \eta_{g_n} \right) c_m \right\}.
\]

Now, for each \( n \), \( S_1(\eta_{g_n}, n) = 4 \{ c_l - \frac{1}{2n - 1}c_m \} \). Then, for all \( n > N_2 \), \( S_1(\eta_{g_n}, n) > 0 \) and hence is strictly increasing in \( \eta_{g_n} \). Since we are only concerned with networks other than the minimally connected one, we may assume that \( \eta_{g_n} \geq 1 \). Now, for all \( n > N_2 \),

\[
S(1, n) \equiv 4 \left\{ \frac{1}{2n}c_l + \left( \frac{1}{2n} - \frac{1}{2n - 1} \right) c_m \right\} > 0.
\]

This implies that \( C_n - C_n^* > 0 \).

**Proof of Proposition 6.** Recall that \( \hat{q} \) is defined by

\[
\hat{q} = \arg \max_q \Delta(q)
\]

s.t. \( \phi(q + c_m) - 2c_m \leq \frac{1}{2 + \phi} \left[ -\phi(q + c_m) + \frac{1}{2} \Delta(q) - c_m \right]. \hfill (18)
\]

Let \( \{g_n\} \) be a sequence of networks and let \( q \) be asymptotic implementable with that sequence. Consider two cases. First, suppose that \( \eta_{g_n} \leq \bar{\eta} \) and \( \nu_{g_n} \leq \bar{\nu} \) infinitely often. Then, by Proposition 4, \( q \leq \hat{q} \). Since, by the arguments in Proposition 5, the star minimizes the linking costs among all connected networks for large \( n \)'s, the candidate arrangement dominates that sequence. Next, suppose that \( \eta_{g_n} > \bar{\eta} \) or \( \nu_{g_n} > \bar{\nu} \) for all sufficiently large \( n \). Since \( W(1, 1, \hat{q}) \geq \max \{ W(\bar{\eta}, 1, 1), W(1, \bar{\nu}, 1) \} \), \( \hat{q} \) and the star network performs better for large \( n \)'s.