# Collateral, Rehypothecation, and Efficiency*

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**Abstract**

This paper studies rehypothecation, a practice in which banks or broker-dealers re-use the collateral pledged by their clients for their own trades and borrowing. The model explains how rehypothecation arises and creates a collateral chain in the system, and what benefits and costs it produces in the economy where collateral is in the form of a repurchase agreement. Rehypothecation helps more funds to flow into the system by providing the receiver of collateral with a flexibillity to re-use it, while at the same time it introduces an additional risk that the collateral may not be returned to the pledgor to whom the asset might be more valuable than to others. This failure of rehypothecation thus incurs deadweight costs of misallocating the asset when there is a trading friction between the initial collateral provider and the final cash lender. The model specifies conditions under which rehypothecation is socially efficient, and also asks the question whether each agent's decision to participate in rehypothecaion achieves an optimal outcome. The model shows that in some cases, individuals may participate in socially inefficient rehypothecation, and regulations are needed.

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# 1 Introduction

Most financial contracts are in the form of promises to pay a certain amount of money or exchange assets in a later date at pre-arranged terms. But often these promises cannot be warranted themselves, and they need to be backed by an eligible asset or property, called collateral, such as Treasury bills in repo transactions and residential houses in mortgage contracts.[1] Generally, collateral in the finanancial contracts plays two crucial roles as emphasized in Mills and Reed (2012): (i) first, collateral provides a borrower with incentives to repay to avoid forfeiting it; (ii) second, collateral provides a lender with some insurance allowing him to collect some revenue by liquidating it in the event that the borrower defaults. In order that a certain asset can be used as collateral, however, it has to be sufficiently valuable especially to the borrower so that the lender can be assured that the borrower will repay the loan to get back the collateral.

Nonetheless, such assets that can be used as collateral are scarce in the economy and the cost of generating these assets are also non-negligible. In particular, as the volume of financial transactions has sharply increased over the last few decades, the demand for collateral has also been significantly increased, and economizing on the existing limited amount of collateral has become an important issue for market participants.[2]

A simple and probably the easiest way to save on collateral would be by re-using it. In most cases, collateral sits idle in the lender's account until the borrower repays the loan to get it back. Clearly, during the time that the collateral deposited in the lender's account, it ties up capital that the lender might have other profitable uses for. In that case, one way that the lender can access to that capital is to make a loan by re-pledging the collateral (initially pledged by his borrower) to another party. From the view of liquidity provision, this re-using collateral is socially beneficial because it reduces the cost of holding collateral for the lender, and ultimately it would benefit the borrower since the lender would be willing to provide more funding against the same unit of the collateral posted by the borrower. From the view of the economy as a whole, the same collateral is used to support more than one transaction, and it creates a 'collateral chain' in the system which increases interdependence among the agents.

This paper addresses some basic, but not yet completely answered questions about this practice of re-using collateral: under what circumstances 'rehypothecation' – the practice in which the receiver of collateral re-uses, re-pledges, or sometimes even sells the collateral to

[1]The oldest form of collateralized lending is the pawn shop that Holmström (2015) illustrates as: "The earliest documents on pawning date back to the Tang Dynasty in China (around 650 AD)... The borrower brings to the pawn shop items against which a loan is extended. The pawn shop keeps the items in custody for a relatively short (negotiable) term, say one month, during which the borrower can get back the item in return for repayment of the loan. It sounds simple, but it is a beautiful solution to a complex problem." For other insightful dicussion on the origin of collateralized lending, see Geanakoplos (1996).

[2]Krishnamurthy and Vissing-Jorgensen (2012) estimated the liquidity and safety premium on Treasuries paid by the investors on average from 1926 to 2008 was 72 basis points per year, which supports the idea that there has been a large and persistent demand for safe and liquid assets in the economy. Similarly, Greenwood, Hanson, and Stein (2012) emphasize the monetary premium embedded in short-term Treasury bills, and it have a lower yield than would be in a conventional asset-pricing literature.

another party for its own trading or borrowing – arises; how it creates a collateral chain in the system; what benefits and costs it produces; and whether decentralized decisions made by each individual to participate in rehypothecaion achieves a socially efficient outcome.

Inarguably, rehypothecation has been one of the most popular devices for many broker-dealder banks to serve their own funding liquidity needs before the crisis. After the failure of Lehman Brothers in 2008, however, hedge funds (most of them were the clients of those investment banks) became wary of losing access to their collateral, and limited the amount of the assets that are permitted to be re-pledged. At the same time, regulation on rehypothecation has also been advocated by legislators and policy-makers.[3] Nevertheless, understaning of the economics underlying this practice is still incomplete, and there are still considerable debates on how to regulate rehypothecation as being made clear by the asymmetry of the rules on rehypothecation across different nations.[4]

To answer these questions, we adopt the framework of Bolton and Oehmke (2014) that is in turn based on Biais, Heider, and Hoerova (2012), in which a borrower who is subject to a moral hazard problem and required to post collateral to prevent him from engaging in risk-taking actions. Not surprisingly, within this basic framework, we show that a positive NPV investment of the borrower with limited liability cannot be undertaken without posting the borrower's asset as collateral, which was already demonstrated in many previous literature on collateralized borrowing.[5] These models, however, do not consider the risk on the other side that the lender might fail to return the collateral as well as any incentives to use it for their own purposes. In contrast, our model incorporates the possibility of re-using collateral by the counterparty and the risk associated with it, thereby offering the first formal welfare analysis on rehypothecation.

Another important feature of our model is that the borrower transfers collateral to the lender at the time of the beginning of the contract. In other words, collateral in our model is alike a repurchase agreement in Mills and Reed (2012): the borrower transfers his asset to the lender at the time a contract is initiated and buys it back at a later point.[6] This contrasts to most of the

[3]Singh (2010) estimated that in 2007, in the run up to the crisis, the value of collateral held by the largest U.S. investment banks, Lehman Brothers, Bear Stearns, Morgan Stanley, Goldman, Merrill and JPMorgan, that was permitted to be rehypothecated was around \$ 4.5 trillion. Post the crisis, in 2009, the value of collateral held by the U.S, investment banks that was permitted to be rehypothecated dropped to \$2 trillion, which is less than half its former size. On the regulatory side, the Dodd-Frank Act requires in most swap contracts, the collateral be held in a segragated account of a central counpterparty.

[4]Under SEC rule 15c3-3, a prime broker may rehypothecate assets to the value of 140% of the client's liability to the prime broker. In the U.K., there is no limit on the amount that can be rehypothecated. See Monnet (2011) for more detailed explanation on the difference in regulatory regimes on rehypothecation across countries.

[5]Holmström and Tirole (1998, 2011) shows that the moral hazard problem of the borrower makes the firm's pledgeable income less than its total value, which leads to a shortage of liquidity for its investment in some states. Also, Shleifer and Vishny (1992), Bernanke, Gertler, and Gilchrist (1994), and Kiyotaki and Moore (1997) concern a firm's financing problem constrained by its net wealth.

[6]According to Mills and Reed (2012), this is relevant especially in shadow banking sectors in that the loan is short-term and has large value, which makes enforcing a transfer of collateral after bankruptcy of borrowers highly costly. In constrast, small value loans between a bank and a consumer, it is relatively easy to seize collateral from borrowers at a later point, for example, in a mortgage constract, a house as collateral is not transferred to the

previous works on collateral in which collateral is transferred to the lender after final pay-offs are realized, or at the time when the default of the borrower actually occurs.

This early transfer of collateral, however, introduces another risk that the lender may not be able to return collateral at the time when the borrower wants to repurchase it.[7] Indeed, as observed from the failure of Lehman Brothers in 2008 and MF Global in 2011, this is not simply a theoretical possibility. In consideration of this, we introduce the counterparty risk – the lender might lose collateral too frequently – into the baseline framework, and we show that if the risk is too high, it makes too costly for the borrower to post its asset as collateral, and he may not want to post his asset as collateral upfront. As a result, the positive NPV project of the borrower cannot be undertaken in this case since non-collateralized borrowing is not feasible when the borrower is subject to moral hazard.

Building this basic intuition in the two-player model, we extend it into the three-player model to more explicitly describe how rehypothecation introduces the risk of counterparty failure and specifies the condition under which rehypothecation is socially efficient. Our results show that the efficiency of rehypothecation is determined by the relative size of the two fundamental effects that have already been emphasized by many policy makers and academic researchers: Clearly, rehypothecation lowers the cost of holding collateral and makes the illiquid collateral more liquid thereby provides more funding liquidity into the market; Whereas, the rehypothecation failure – the counterparty failure to return the collateral to the borrower who posted it – may incur deadweight costs in the economy.

One difficulty in this general argument is that it is not obvious by which channel the rehypothecation failure incurs deadweight costs, and this was not been clearly answered in most of the previous works on rehypothecation (we discuss further on those papers in the literature review). Of course, there could be several channels that the rehypothecation failure incurs deadweight costs in the economy, this paper especially focuses on the channel that the rehypothecation failure leads to misallocation of the collateralized assets.

This misallocation of the assets crucially depends on the following two types of market frictions: (i) we assume that the collateralized asset is 'illiquid' in the sense that the asset is likely to be more valuable to the initial owner than to the other agents – think of a collateralized asset as an intermediate good that the initial owner uses it for its own production and he has a better skill to manage it than do the other agents in the economy; (ii) we also consider a possibility that some traders may not have access to some parts of the markets, and they can trade indirectly each other only through the intermediary (who has an access to all the markets). In the model, this appears that the asset provider and the cash lender makes a separate contract with the intermediary who transfers the collateral between them. Taken together, if the intermediary fails, the asset ends up being in the wrong hands: the asset cannot be returned to the initial

bank until the borrower defaults on the loan.

[7]Mills and Reeds (2012) discuss the effect of this counterparty risk on the form of the optimal contract in a different context.

owner (the asset provider) who values it the most, but instead it is seized by the third party (the cash lender) to whom the collateral may not be much useful.

Finally, we ask a question whether an individual agent's decision to participate in rehypothecation achieves a socially optimal outcome. In practice, the initial owner of collateral has the right to permit (or not) rehypothecation of his asset. The model shows that there exists an externality in the initial owner's decision making, and sometimes he may prefer socially inefficient rehypothecation, and thus there might be an excessive participation in rehypothecation from the perspective of the society as a whole. Intuition behind this result is that the gain from the investments with collateralized borrowing diminished as collateral is re-used, and in general, the initial pledgor enjoys more benefits from rehypothecation than does the second pledgor and so forth. The initial pledgor does not internalize the payoff of the follow-up pledgors, however, he may want to allow rehypothecation of his asset even in the case that prohibiting it increases others' payoffs and also the total social welfare.[8]

## 2 Related Literature

This paper relates to the literature that consider collateral as an incentive device to deal with a borrower's moral hazard problem, for example, Holmström and Tirole (1998), Biais, Heider, and Hoerova (2012), and Bolton and Oehmke (2014). First, in their pioneering work, Holmström and Tirole (1998) consider a borrower with limited liability who has to prepare himself against uncertain liquidity shocks tomorrow by making state contingent contracts in advance to provide liquidity in a state of liquidity shortage tomorrow. A fundamental assumption in their analysis is that there is a wedge between the value of the total income of the borrower and the pledgeable income to the lender (in their terminology, inside liquidity or collateral), and as a microfoundation to it, they prove that the wedge arises as the optimal contract when the borrower is subject to the moral hazard problem.

In the context of derivative trading, Biais, Heider, and Hoerova (2012) discuss the role of collateral (margin) to mitigate the moral hazard problem of the derivative providers. The key feature of their model is that the agent is required to post collateral 'before' that final payoffs are realized. This contrasts to the most previous works on collateral[9] which assume that collateral is seized by creditors in the event of default ex post. That way, they assure collateral posted upfront in the lender's account not being affected by the borrower's risk taking behaviors as new information arrives after the contract begins. In a similar vein, Bolton and Oehmke (2014), based on the framework of Biais, Heider, and Hoerova (2012), analyze current

[8]The second party (the receiver of the initial pledgor's collateral) may be able to decide not to participate in rehypothecation, and that way, the socially inefficient rehypothecaiton does not occur. The model shows that, however, under some conditions, this is not possible as the second party participates in rehypothecation *ex post* whenever it is allowed by the initial pledgor.

[9]See, for example, Holmström and Tirole (1998), Krishnarmurthy (2003), Kehoe and Levine (2006), Rampini and Viswanathan (2010), Fostel and Geanakoplos (2008), Geanakoplos (2010), Simsek (2013).

previledged bankruptcy treatment of derivatives and show that the seniority of derivatives can be inefficient by transfering default risk to creditors in the debt market, even if the default risk can be born more efficiently in the derivative market.

This paper is closely related to those models in that the borrower has to transfer his asset as collateral to the lender at the time when the contract begins, and buys it back in a later point. However, there are substantial differences between these models and ours in the following two aspects: (i) In these models, posting collateral itself is a costly behavior as it transfers a 'liquid' capital to the lender's account which yields a relatively low return than does the borrower's account, thereby incurring deadweight costs in the economy. In contrast, this paper assumes that collateral is an 'illiquid' asset, and thus posting collateral does not incur any costs at the time when the borrower transfers collateral to the lender, but it may incur some costs if the borrower fails to repay and cannot recover it in a later period.[10] (ii) This paper also considers a default risk by the receiver of collateral which was absent in those models; they assume a central counterparty (CCP) sitting between the borrower and the lender, and collateral deposited in the CCP's margin account is ring fenced not only from the pledgor's moral hazard but also from any other credit risks of the receiver. In contrast, in our setting, rehypothecation renders the collateral open to the receiver's default risk, that is, the collateral may not be returned to the pledgor if the receiver defaults having re-pledged the collateral to another party.

This paper also relates to the literature in which collateral is in the form of a repurchase aggrement, for example, Shi (1996), Mills (2004, 2006), Mills and Reed (2012), Oehmke (2014). First, our assumption that collateralized asset is most valuable to the initial owner is similar to that of Shi (1996) who shows that useless assets except for the owner can be used as collateral. In particular, Mills and Reed (2012) describe collateral as a repurchase agreement when the lender lacks an enforment technology to seize collateral in the event of default. This early transfer of collateral, however, introduces an additional incentive constraint to the lender that he may not return the collateral to the borrower. With this double lack of commitment, they discuss how the default risk of lenders (failure to return collateral to their borrowers) affects the optimal allocation, and they show that actual defaults by lenders will not happen at the optimum. On the other hand, in our model, the defaults by lenders may occur exogenously as long as they participate in rehypothecaion, and thus the defaults can still occur at the optimum. In addition, we assume that the lender is risk-neutral, and do not consider the insurance role of collateral as in their model.

Another interesting feature in our setting is that some traders (in practice, banks and broker-dealers) have dual positions as a lender and a borrower when they re-pledge collateral received from their borrowers. A similar concept also appears in the literature on business cycles and collateral constraints, for example, Moore (2011), Getler and Kiyotaki (2010), and Gertler, Kiyotaki, and Queralto (2011) in that banks not only plays a role as an intermediary between

[10] According to Mills and Reed (2012), it can be alternatively interpreted that costs are born when posting collateral, but may be recovered if the borrower buys it back from the lender at a later point.

capital producing firms (outside borrowers) and households (outside lenders), but they also borrow and lend each other, that is, they mutually hold gross positions. In particular, Moore (2011) addresses questions why banks hold gross positions in the current financial system and whether these mutual gross positions give rise to a systemic risk in the economy. His analysis shows that those mutual gross positions among banks help make more funds flow in the system, thereby increasing investment activities, while at the same time, they make the system more susceptible against a shock which might result in a systemic failure.

More broadly, this paper is related to the literature on the supply and demand for safe and liquid assets, such as Gorton and Pennachi (1990), Dang, Gorton, and Holmström (2013), Gorton and Ordoñez (2013), Caballero and Farhi (2013). In these literature, safe assets are provided by the financial intermediary when there is a demand for such assets in the economy, possibly due to informational problems. Similarly, in this paper, rehypothecation plays a role to provide liquidity to the system by circulating the limited amount of collateral. For the empirical analysis, Krishnamurthy and Vissing-Jorgensen (2013) show the existence of a large and persistent demand for safe and liquid assets, and explains this as the key driver of the prevalence of short-term debt in the economy. Also, Gorton, Lewellen, and Metrick (2012) and Aitken and Singh (2010) discuss the role of the shadow banking system to provide safe and liquid assets.

Finally, after the financial crisis in 2007, there has been growing interests in rehypothecation from both policy groups and academic researchers. Monnet (2011) discusses the possible pros and cons of rehypothecation as well as the current debates on the regulations on rehypothecaiton. Bottazi, Luque, and Pascoa (2012) develop the equilibrium model of repos and demonstrate that prices of securities in the repo markets increase due the leverage built up along the process of rehypothecation. Andolfatto, Martin, and Zhang (2014) analyze the effect of rehypothecation on the monetary policy and argue that restrictions on rehypothecation generally improve social welfare by increasing the value of cash which could be the only accepted means of exchange in some countries. Maurin (2014), based on the general equilibrium model with collateral constraint of Geanakoplos (2010), discusses the effectiveness of rehypothecation compared with other trading techniques such as tranching and pyramiding. He show that rehypothecation has no effect on trading outcomes in complete markets, and thus the effectiveness of rehypothecation depends on the market structure.

In the context of a repo market, Lee (2015) discusses a tradeoff of rehypothecation between economic efficiency and financial stability. She emphasizes that a sudden decline of rehypothecation can lead to an inefficient repo run by creating a positive feedback loop between the repo spread and fire-sale discounts. Our model also concerns a tradeoff of rehypothecation, but our focus is on the welfare effect of the misallocation of collateral after rehypothecation fails, whereas Lee (2015) focuses on the fragility that may occur when the collateral circultation rate suddenly drops. Eren (2014) and Infante (2014) consider the repo market, but focus on the practice in which a dealer bank earns (free) liquidity by using its position as an intermediary between collateral providers and cash lenders. They show that, through this rehypothecation process, the dealer earns liquidity by setting larger margins to the collateral providers than to the cash lenders. In contrast, in our model, there is no uncertainty in collateral value (collateral is riskless assets), and thus no haircuts or margins are needed.

The paper is organized as follows. The basic model with two players is presented in the next section. The basic model is extended to three players to allow for rehypothecation in section 4. Section 5 analyzes the welfare of equilbrium under rehypothecation. Section 6 concludes.

# 3 A Two-Player Model

In this section we build a simple two-player model using the framework of Bolton and Oehmke (2014). They consider the derivative contract setting in which derivative providers (or derivative counterparties) are subject to a moral hazard problem, and are required to post collateral to avoid the borrower's risk-taking behavior (a margin requirement).

In our model, posting collateral provides the borrower with the incentive to work hard to avoid default as in Bolton and Oehmke (2014). A notable difference from their model is that we assume that the asset posted by the borrower is an illquid asset, and posting it as collateral means transfering liquid assets (cash or capital) sitting idle in the lender's hands to be spent for the more productive investment activities made by the borrower.[11] Thus, in our framework, posting collateral itself does not incur the costs, but rather it is generally welfare improving.

After that, we add a risk to the model that the receiver of collateral might lose the collateral, and cannot return it to the borrower (to whom the collateral is probably more valuable than to others). If this risk is too high, that is, the counterparty loses the collateral too frequently, posting his own asset as collateral is too costly for the borrower, and he may not want to involve in collateralized borrowing at all. In this section, we illustrate these results more formally within the two-player model and in the next section we extend it into the three-player model to analyze how these observations are applied to rehypothecation.

## 3.1 Setup

There are three periods, $t = 0, 1, 2$, and two types of agents in the economy: one firm and a large number of outside investors. Agents are risk-neutral and do not discount.

In period 0, the firm has access to an investment project. The project requires an input in period 0 and it produces an uncertain outcome, $\rho$, per unit of inputs in period 2: it delivers $R > 1$ units of consumption good if it succeeds, or zero units of the good if it fails. (We assume

[11] As mentioned earlier, this differs from Bolton and Oehmke (2014) in that posting collateral in their model incurs deadweight costs since they assume collateralized assets are liquid, and yield a low return when deposited in the margin account.

that the price of consumption good in period 2 is normalized to 1.)

$$\rho = \begin{cases} R > 1 & \text{if the project succeeds} \\ 0 & \text{if the project fails} \end{cases} \tag{1}$$

The firm, however, does not have its own resources that can be immediately used for the project, and it is endowed only with one unit of indivisible asset. We assume that the asset is illiquid in the following two senses: (i) it produces $Z > 0$ units of consumption goods in period 2, and yields no outcomes if it is liquidated before the maturity; (ii) the asset is productive only when it is in the hands of the firm – it can be thought as the firm has the special managerial skill or technology to use the asset that is not accessible to other agents in the economy or simply the asset has some special value only to the initial owner. Thus, the value of the asset is different across the agents: the asset is more valuable to the firm than to the others. Each agent knows, however, each other's valuation of the asset, that is, the outside investors agree that the asset is worth $Z$ to the firm and worthless to themselves.[12]

On the other hand, the outside investors are endowed with a large amount of cash (capital) that can be used as an input for the firm's project. Thus, the firm has to borrow funding for the project from the outside investors. We assume that the firm borrows funding by issuing a simple debt[13] the firm promises to pay back a certain amount $X_0$ to the investors in period 2, and if it fulfills the promise it collects the remaining return from the project, but if it does not repay, its project is liquidated by its creditors, and the firm receives zero liquidation value in that case. Also we assume there are a large number of investors who do not discount the future, and the risk-free interest rate is zero.

## 3.2 A Moral Hazard Problem of a Firm

In this section, we consider the case that the firm is subject to the moral hazard problem and verify that posting collateral mitigates the firm's incentive towards risk-taking as already shown in Bolton and Oehmke (2014). We assume that the outcome of the firm's investment is observable to the lender, that is, the firm cannot falsely claim that the investment fails when it actually succeeds.

The probability of success of the investment, however, depends on unobserved actions taken by the firm, denoted by $a \in \{0, 1\}$. Each action $a$ can be interpreted as a level of efforts made by the firm to manage the risk of the project: $a = 1$ represents a safe action that increases the possibility of success of the project; $a = 0$ represents a risky action that decreases the possibility

[12] In effect, we do not consider the role of collateral as an insurance for lenders in this paper. Or, the model could be generalized to the case in which the insurance provided by collateral is not complete.

[13] Showing that debt is the optimal contract is beyond the scope of this paper. The list of literature (but not exhaustive) on the optimality of debt include Townsend (1979), Myers and Majluf (1984), Gale and Hellwig (1985), Aghion and Bolton (1992), Hart and Moore (1998), DeMarzo and Duffie (1999), DeMarzo et al (2005), and most recently, Dang et al (2012).

of success of the project. Specifically, if the firm chooses the safe action $a = 1$, the project succeeds with probability 1. If it chooses the risky action $a = 0$, the project succeeds with a lower probability, $p < 1$, while at the same time, it gives the firm a private benefit $b > 0$ (which is measured per unit of projects). We also assume that the outcome of the firm's project is verifiable to the lenders.

We make two assumptions about the firm's project. First, we assume that the investment is socially efficient if the firm chooses $a = 1$, but inefficient if it chooses $a = 0$,

**Assumption 1.**

$$R > 1 > pR + b. \tag{2}$$

The first inequality shows that the expected unit return of the project is greater than the cost of funds (cost of holding cash which is 1) if the firm chooses the safe action, $a = 1$. The second inequality shows that the expected unit return of the project is less than the cost of funds if the firm chooses the risky action, $a = 0$.

Recall that the firm has no resources on its own and has to borrow funds from the outside investors. When the firm invests with borrowed money, however, it may find it advantageous to choose the risky action, $a = 0$, rather than the safe action if the following condition is satisfied,

**Assumption 2.**

$$R - 1 < p(R - 1) + b. \tag{3}$$

To understand the implication of this assumption, it is useful to consider the example: suppose the firm borrows a certain amount of funds $I$ from the investors by promising to pay back $X$ in period 2. If the investors expects that the firm will choose the safe action, $a = 1$, the project succeeds with certainty (and the risk-free interest rate is simply 1). Thus, the investors are willing to lend funding to the firm as much as the firm promises to repay, that is, $I = X$ (each investor earns zero net expected profit due to competitiveness in lending).

This investors' belief, however, is not consistent with the firm's actual choice. To see this, consider first the case in which the firm chooses the safe action $a = 1$. In this case, the expected payoff of the firm is given by

$$RI - X = (R - 1)I. \tag{4}$$

The equality holds by the previous argument that $I = X$.

Next, given the same terms of contract, $(X, I)$, suppose the firm chooses $a = 0$. Then, the expected payoff is given by

$$p(RI - X) + bI = p(R - 1)I + bI. \tag{5}$$

Comparing the right hand sides of Equation 4 and 5, Assumption 2 implies that it is profitable for the firm to choose the risky action, $a = 0$, for any given $I > 0$,

$$(R - 1)I < p(R - 1)I + bI$$

Thus, the firm's expected repayment is $pX$ when the investors lend $I = X$ with the belief that the firm will choose the safe action. This implies that the investor's belief cannot be sustainted in equilibrium.

The only remaining possibility is thus the case that the investors expect that the firm will choose the risky action, $a = 0$, and lends $I = pX$ (because that the probability of success of the project decreases to $p < 1$ and the investor's zero profit condition leads to $I = pX$). However, in this case, the loan is too expensive for the firm (the risk-free interest rate is now $\frac{1}{p} > 1$), and the firm is better off not to invest with borrowing, that is,

$$p(RI - X) + bI = (pR - 1 + b)I < 0$$

where the first equality uses the zero profit condition of the investors, $X = \frac{1}{p}I$, and the last inequality is by Assumption 1. Therefore, if the lender expects that the firm will choose the risky action, the firm will not want to borrow at all.

Taken together, the investment cannot be undertaken in any cases.

**Lemma 1.** *Suppose Assumption 1 and 2 hold. In this case, uncollateralized debt financing for the investment is not feasible.*

## 3.3 Collateralized Borrowing

As discussed in the previous section, the firm may prefer to take the risky action rather than the safe action when he invests with the borrowed money, and the socially efficient investmet may not occur due to this limited liability of the firm. In this case, requiring the firm to post collateral (a margin) can prevent the firm to take a risky action, and makes the socially efficient investment possible. The intuition is that posting collateral in the counterparty's account gives the firm the incentive to choose the safe action by increasing the payoff from making the repayment.

### 3.3.1 Firm's Incentive Constraint with Collateralized Borrowing

Suppose now the firm is required to post its asset (that is worth $Z$ to the firm) into the counterparty's account. This changes the firm's incentive constraint as follows,

$$RI - X + Z \geq p\,[RI - X + Z] + bI. \tag{6}$$

where $I$ is the size of borrowing and $X$ is the promised repayment. The left hand side states that when the firm chooses the safe action, it receives a return $RI$ from the investment and pays $X$ in exchange for getting the collateral with the private value of $Z$. The right hand side states that when it chooses the risky action, however, it receives an expected return $pRI + bI$ from the investment and pays $X$ to get back the collateral worth $Z$ only when the project succeeds with probability of $p$.

Note that, when the firm is required to post collateral, the left hand side can be greater than the right hand side. This is because that the safe action increases the probability of getting back the collateral, and the expected payoff from getting back the collateral is greater for the safe action, $Z$ on the left hand side than that for the risky action, $pZ$ on the right hand side. Thus, posting collateral relaxes the firm's incentive constraint and induces the firm to choose the safe action, which is socially efficient.

To solve for the level of the investment, note that as long as the firm's incentive constraint is satisfied, the investors are willing to lend at the risk-free interest rate of 1, that is, $I = X$. Thus, by substituting $I = X$ into the firm's incentive constraint, we can characterize the maximum level of the firm's investment scale as follows.

$$I = \frac{1}{1-\mathcal{B}} Z \tag{7}$$

where $\mathcal{B} \equiv R - \frac{b}{1-p} \in (0,1)$ (the range of $\mathcal{B}$ is determined by Assumption 1 and 2). Note that the level of the firm's investment scale depends on the value of the collateralized asset, $Z$.

**Remark.** We can rewrite the firm's incentive constraint,

$$\mathcal{B}I + Z \geq X. \tag{8}$$

Then, we may interpret this as the budget constraint of the firm's investment where the left hand side is the firm's pledgeable income which consists of a fraction of the return of the investment $\mathcal{B}I$ and a value of collateral $Z$. Note that if the firm is fully trustworthy, $\mathcal{B}$ is close to 1. As long as $X = I$, then the firm can borrow infinitely large by raising the promised repayment $X$ as large as possible, since $\mathcal{B}I$ increases at the same rate as $X$.

In general, however, the firm has a limited liability, and the pledgeable return of the investment satisfies $\mathcal{B} << 1$. Therefore, the firm has to cover the remaining cost of investment, $I - \mathcal{B}I$, with its own funds or capital. In our setting, the firm fills this wedge by posting its asset with value of $Z$. This is reminiscent of the fundamental assumption in Holmstrom and Tirole (2011) that a firm has a pledgeable income less than the expenses need for the investment, and firms with low initial capital are credit rationed.

Plugging the results in Equation 7 obtained so far into the firm's payoff function, the firm's payoff after posting collateral is given by,

$$RI - X + Z = \frac{R - \mathcal{B}}{1-\mathcal{B}} Z > Z. \tag{9}$$

Note that the payoff after collateralized borrowing is greater than the payoff when holding on to the asset by the assumption that $R > 1$. This shows that posting collateral makes the investmet feasible, and it improves the social welfare.[14]

[14] Of course, all the social surplus is captured by the firm's payoff.

We summarize this result in the following lemma.

**Lemma 2.** *Suppose Assumption 1 and 2 hold. In that case, posting collateral makes financing for the socially efficient investment feasible.*

### 3.4 Cost of Posting Collateral

So far, we have assumed that the firm's counterparty (a receiver of the collateral) is fully trustworthy, and the return of the collateral to the firm is guaranteed as long as the firm makes the obligated payment. In practice, however, sometimes the counterparty might not be able to return the collateral to the borrower. The cost associated with this counterparty failure is that the initial borrower's utility after getting back the collateral is lost altogether in that process, which would never arise if she does not post the asset as collateral and holds on to it.

In this section, we introduce the risk associated with posting collateral into the previous model (For now, we do not specify the reasons why the counterparty lose the collateral. One of the reasons is due to rehypothecaiton which will be clearer when we discuss the three player model later on). The main purpose of this analysis is to show that if the risk is too high (that is, there are too frequent losses of collateral by the lenders), collateralized borrowing becomes too expensive for the borrower, and as in noncollateralized borrowing case, the investment will not be undertaken as the borrower prefers not to post collateral at all than to invest with collateralized borrowing.

#### 3.4.1 Counterparty Risk of Losing Collateral

Now, the firm's counterparty might lose the firm's collateral, and the probability of such event is given by $\delta \in (0,1)$. For simplicity, we assume that in that case, both the firm and the counterparty cannot recover the collateral, or the cost of recovering collateral is extremely high. Furthermore, we assume that the counterparty cannot claim the repayment from the firm without returning the collateral to the firm, and thus the expected payoff from lending is $\delta X$. Then, the competitiveness of collateralized lending market and the counterparty's zero profit condition leads to $I = \delta X$.

Also, note that the firm's incentive constraint is changed to

$$RI - (1-\delta)X + (1-\delta)Z \geq p[RI - (1-\delta)X + (1-\delta)Z] + bI. \tag{10}$$

Note that compared to the previous case without the counterparty risk, the firm's expected return from getting back the collateral decreases from $Z$ to $(1-\delta)Z$ (and at the same time, the firm's expected payment decreases from $X$ to $(1-\delta)X$) because of the risk that the counterparty fails to return the collateral with probability $\delta$.

Substituting $I = \delta X$ into the firm's incentive constraint and rearranging the terms for $I$, we obtain the maximum level of the firm's investment scale,

$$I = \frac{1-\delta}{1-\mathcal{B}} Z. \tag{11}$$

where $\mathcal{B} \equiv R - \frac{b}{1-p} \in (0,1)$. Plugging this result into the firm's payoff function, we have

$$RI - X + (1-\delta)Z = (1-\delta)\frac{R-\mathcal{B}}{1-\mathcal{B}} Z \tag{12}$$

Comparing this result with the previous case without the counterparty risk, it follows that the firm's payoff is lower than when there is no counterparty risk of losing collateral,

$$(1-\delta)\frac{R-\mathcal{B}}{1-\mathcal{B}} Z < \frac{R-\mathcal{B}}{1-\mathcal{B}} Z. \tag{13}$$

It is noteworthy that if the counterparty risk $\delta$ is too high, the firm's payoff from the investment with collateralized borrowing is close to zero,

$$\lim_{\delta \to 1} (1-\delta)\frac{R-\mathcal{B}}{1-\mathcal{B}} Z = 0.$$

In this case, it is more profitable for the firm not to borrow at all and holds on to the asset, which delivers a payoff $Z$ with certainty. This simple example highlights that in order that the collateralized borrowing works properly, the credit of the receiver of collateral is important as much as the value of collateral posted by the borrower. We summarize the result in the following lemma.

**Lemma 3.** *Suppose Assumption 1 and 2 hold. Too frequent losses of collateral makes collateralized borrowing too costly for the borrower, and the socially efficient investment may not occur.*

# 4 A Three-Player Model: Collateral Chain

In this section, we extend the previous two-player model into three players. The purpose of this extension is to describe rehypothecation in which a receiver of collateral re-uses or re-pledges it to the third party for the purpose of their own trading and borrowing, and show that the same collateral is used to support more than one transaction, creating a collateral chain in the system.

As before, there are three periods, $t = 0, 1, 2$, but now there are three players: a firm (A), a firm's counterparty (B), and a creditor of the counterparty (C). We assume that there is a friction in this economy that A and C cannot meet each other, but they are connected only through B. This implies that there are two contracts in the economy: one between A and B, another between and B and C. Furthermore, we assume that these contracts are made sequentially. In period 0, A and B enter the contract first, and in period 1, B and C enter another contract.

As in the previous two player model, the asset initially owned by A is illiquid in the sense that: (i) the asset delivers an outcome only in period 2 and yields zero return if it is liquidated earlier; (ii) the outcome generated from the asset is more valuable to A than to the other agents in the economy. We assume the final outcome of the asset is worth $Z$ to A, but worth $\delta Z$ to B and C, where $\delta \in [0, 1)$. From now on, we consider $\delta = 0$ for analytical convenience.

As in the previous two player model, in period 0, A has the opportunity to invest but no other liquid assets except for the endowment of one (indivisible) unit of the illiquid asset. On the other hand, B has the resources for the investment but has no access to A's investment technology, and A has to borrow funding for his own project from B.

In period 1, suppose B has another profitable investment opportunity which requires an immediate input to produce an outcome one period later. However, B has no resources on its own that can be spent immediately for its investment at that time. This would be the case if all the endowment of B in period 0 cannot be stored and B does not have any additional endowment in the other periods except for period 0. Therefore, B has to raise funding for its investment from the outside investor, C, who has the resources for B's project at that time but has no access to B's investment technology.

In particular, we are interested in the case that B can raise funding from C *only* by re-pledging the collateral initially posted by A. Think of the case in which the pledgeable income of B's investment is sufficiently small, and it is not possible to raise funding needed to initiate the project solely backed by the future return of the investment. In such case, re-pledging the collateral originally pledged by A to C can be helpful to raise B's pledgeable income to C: B is willing to repay at least $Z$ (A's private value of the asset) to C to recover the collateral whenever B is solvent, otherwise B cannot receive the payment from A in exchange for returning it.[15]

This final reallocation of the collateralized asset, however, crucially depends on the existence of the intermediary, B. To get an idea, suppose B defaults having re-pledged A's asset to C. Then, there is no way that A and C can meet each other, and the asset cannot be transferred from C to A even if it is welfare improving. As a result, when there is a trading friction in the market, rehypothecation failure may incur the deadweight cost by misallocating the collateralized asset – the initial owner of the asset is likely to value it higher than do other agents, but the asset cannot be transferred without the intermediary. The model emphasizes that the deadweight cost from the misallocation of the collateralized asset is implicit but significant, and this needs to be considered when evaluating the efficiency of rehypothecaiton.

**Remark.** Before we proceed to the main analysis, let us briefly discuss fungibility of collateral. When collateral is a fungible asset, B does not have to return the exactly same collateral, but he can return other assets of the same value to A. In that case, if the cost of repurchasing the

[15]The promised repayment from A does not necessarily the same as $Z$ (A's valuation of the collateral). To understand this, note that we assume the collateral is riskless, and the total promised repayment from A is always at least $Z$. Then, the total promised repayment from A can exceed $Z$ if a part of the future return of the investment is also included as pledgeable income other than the asset posted up-front.

collateral from C is higher than the cost of buying other equivalent assets from elsewhere, B can choose the option not to recover it from C, but instead buy the other assets. On the other hand, when collateral is non-fungible, B has to return the exactly the same collateral originally posted by A. In that case, if C knows how much B is going to receive from A in return of the collateral to A, B can credibly promise to repay (at most) that much to C when re-pledging the collateral. Therefore, fungibility of collateral will affect the amount of funding transferred when the collateral is rehypothecated. From now on, we assume that collateral is non-fungible to avoid unnecessary complications. The fungible collateral case can be dealt with in a similar way, and it reaches to almost the same results as in the non-fungible collateral case.

## 4.1 Terms of Contracts

Let us describe more formally the terms of contracts among the traders in this economy. There arises a sequence of two contracts under rehypothecation: (i) A and B enter the contract in period 0; (ii) B and C enter the contract in period 1. More formally, timing of the model is described as follows.

- In period 0, A posts his asset as collateral and borrow funding $I_A$ for the investment from B by promising to repay $X_A$, provided that B returns the collateral to A in period 2. The basic environment about the investment of A and the terms of contract are analogous to the previous two player setting.

- In period 1, B is given a profitable investment opportunity that requires an immediate input to produce an outcome in period 2. B has no pledgeable income or liquid assets on its own to be spent for the investment, however, B can borrow funding $I_B$ for the investment from C only by re-pledging A's collateral to C. We assume that B borrows funding by issuing a simple debt and denote the promised repayment from B to C by $X_B$. We assume that the terms of contract between A and B is known to C.[16] For simplicity, we also assume that C does not re-use or re-pledge the collateral, that is, there is no further rehypothecation beyond B (C holds on to the collateral until he returns it to B in exchange for the repayment from B).

- In period 2, both A's and B's investments produce an outcome, and A pays off the loan $X_A$ to B provided that B returns the collateral to A. There are two cases: (i) if B does not re-pledge A's asset, it is stored safe in the segregated account of B, and it is returned

[16] C knows that B has to return the collateral to A in order to receive the repayment $X_A$ from A. It might be the case that A and C does not have full knowledge of each other's contract with B. This lack of transparency in a setting where a single agent makes a deal with multiple counterparties are discussed in Acharya and Bisin (2014). In this paper, A's asset is assumed to be worthless to C, and thus without the knowledge of the contract between A and B – especially, about $X_A$ –, C may not be willing to lend money to B only against the collateral. That is to say, it does not matter that the collateral is useless to the non-owners. More important is that the creditor 'knows' that the collateral is so much valuable to the borrower that he is willing to repay the loan to get it back.

to A with certainty; (ii) if B re-pledges A's asset to C, however, B has to first recover it from C in order to return it to A. In some cases, B may not have enough cash to recover the collateral from C if the investment yields a low return, and the collateral is seized by C. Then, A is also exempt from the obligation to repay the loan $X_A$ to B.

## 4.2 Optimal Contract

In this section, we consider the optimal contract in this three player model. By the backward induction, we begin by analyzing the last period and then move backward to period 1 and 0. In period 2, the investment of both A and B produce outcomes and the contracts are carried out according to the terms made in the preceeding periods.

For analytic convenience, let us focus on the case in which A does not engage in risk taking (that is, the incentive constraint for A is satisfied) in the optimal contract, or equivalently called the incentive contract following the terminology in Koeppl (2013). It is also possible that A engage in risk taking in the optimal contract. The main results will remain the same in that case, however, and this is discussed in the Appendix A. From now on, we presume that the optimal contract is in the form of the incentive contract.

### 4.2.1 Optimal Contract in Period 1

In period 1, B has the opportunity of a profitable investment which requires an immediate input to produce an outcome in the next period, $t = 2$. The investment is risky in the sense that the outcome of the investment is uncertain. For simplicity, we assume that the outcome per unit of projects, $Y$, can take two values,

$$Y = \begin{cases} \bar{Y} > 0 & \text{with prob. } \theta \\ 0 & \text{with prob. } 1 - \theta \end{cases} \tag{14}$$

We assume the investment is efficient, $\theta\bar{Y} > 1$, which implies that it is socially optimal to invest as much capital as possible into the project.

The problem is that B has no resources of its own at the time when he faces the opportunity of the investment. We assume that all the endowment of B in period 0 is not storable to the next period and B does not have any other endowments in the other periods except for period 0. Morevoer, the outcome of the investment of B is non-verifiable, and B cannot borrow any funding against the future outcome of the investment. To understand this, first suppose the investment produces the high return, $\bar{Y}$. Then, since the outcome is not verifiable to C, B would want to falsely claim that it produces the low return, 0, and try to avoid repaying the loan, $X_B$. Next, suppose the investment produces a low return, then again B will not repay the loan since he has no wealth in that state. Taken together, with no pledgeable income on its own, B cannot borrow any funds from C.

The collateral posted by A can be useful in that case. Note that it can be pledgeable to C since he knows that B has to recover it in order to receive the payment from A. Recall that the collateral is non-fungible and B's opportunity cost of giving up the collateral is $X_A$. This implies that B can borrow at most $X_A$ by re-pledging the collateral.

**Lemma 4.** *In period 1, when B borrows funding from C, B faces a collateral constraint such that*

$$X_B \leq X_A. \tag{15}$$

For simplicity, we assume that there is a continuum of C, and the lending market in period 1 is competitive.[17] Then, given the terms of contract in period 0, $(I_A, X_A)$, we can define the optimal contract in this period, $(I_B, X_B)$, as a solution of the maximation problem of B as follows.

$$\max_{I_B, X_B} \quad \theta(\bar{Y} I_B - X_B + X_A) \tag{16}$$

subject to

$$I_B \leq \theta X_B + (1-\theta)\delta Z \tag{17}$$

$$X_B \leq X_A \tag{18}$$

The objective function is the expected net payoff of B from the investment when B rehypothecated A's collateral to C. As we mentioned before, since A does not engage in risk taking when the contract is written as the incentive contract, B expects to receive $X_A$ with certainty as long as he gets back the collateral from C. The return from the investment is either $\bar{Y} I_B$ (success) with probability of $\theta$ or 0 (failure) with probability of $1-\theta$. Also, recall that the investment is efficient, $\theta\bar{Y} > 1$. If the investment succeeds with probability $\theta$, B can repay the loan $X_B$ and return the collateral to A in exchange for receiving the repayment, $X_A$. If the investment fails with probability $1-\theta$, B has no cash to pay the loan to C, and the collateral is seized by C. Hence, B cannot receive $X_A$ from A, and B's payoff is zero in that case.

The first constraint in this problem is the participation constraint for C. Plugging $\delta = 0$, it reduces to

$$I_B \leq \theta X_B.$$

The right-hand side is the expected return from lending: C receives $X_B$ from B with probability $\theta$ or seizes the collateral which is worthless to him with probability $1-\theta$ (due to the friction, C cannot recover any positive revenue by trading the asset with A). Also note that this constraint determines the level of the investment scale, $I_B$, and this can be interpreted as the funding constraint of the investment. Finally, the second constraint is the collateral constraint for B in Lemma 4.

To avoid a trivial case, we need a following assumption on the investment of B,

[17] We may assume a more general market structure in period 1, and C has some bargaining power. The main result of this paper, however, will not change with this assumption.

**Assumption 3** (The participation constraint of B)**.**

$$\theta^2 \bar{Y} > 1$$

Intuitively, this assumption means that the investment made by re-pledging the asset is profitable from the perspective of B in period 1 for any given period 0 contract $(I_A, X_A)$. Thus, as long as rehypothecation is allowed by A, B always wants to participate in rehypothecation ex post. For the same reason, if this condition fails, B is not willing to re-pledge the collateral at all, and holds on the collateral until he receives the payment $X_A$ from A. Thus, this condition can be interpreted as the participation constraint of B.

Based on Assumption 3, the linearity of the Lagrangian of the problem ensures that it is optimal to choose $X_B$ as highest as possible,

$$X_B^* = X_A. \tag{19}$$

And the competitiveness ensures that the participation constraint of C binds,

$$I_B^* = \theta X_B^* = \theta X_A. \tag{20}$$

Plugging these values to the objective function, we can represent B's net payoff from re-pledging the collateral as a function of the contract in period 0, $(I_A, X_A)$. We summarize these results in the following lemma.

**Lemma 5.** *Suppose Assumption 3 holds. Taking the contract in period 0,* $(I_A, X_A)$*, as given, the optimal contract in period 1,* $(I_B^*, X_B^*)$*, is given by*

$$(I_B^*, X_B^*) = (\theta X_A, X_A). \tag{21}$$

*At these values, the profit of B from rehypothecation is written as a function of the period 0-contract,* $(I_A, X_A)$*,*

$$\theta(\bar{Y} I_B^* - X_B^* + X_A) - I_A = \theta^2 \bar{Y} X_A - I_A. \tag{22}$$

### 4.2.2 Optimal Contract in Period 0

Next, we consider the optimal contract between A and B in period 0. In contrast with the competitive lending market in period 1, we assume a various market structure in period 0. Thus, we define the optimal contract in period 0 as a solution to the Nash bargaining problem between A and B where $\beta \in [0, 1]$ and $1 - \beta$ measures the bargaining power of A and B, respectively.

$$\max_{I_A, X_A} \quad (RI_A - \theta X_A + \theta Z - Z)^\beta (\theta^2 \bar{Y} X_A - I_A)^{1-\beta} \tag{23}$$

subject to

$$-Z + RI_A - \theta X_A + \theta Z \geq -Z + p\left[RI_A - \theta X_A + \theta Z\right] + bI_A. \tag{24}$$

The first term in the objective function is the expected return to A from the investment with the borrowed money from B by posting his asset as collateral: A obtains the return $RI_A$ from the investment with probability 1 since A choose the safe action at the optimum. With probability $\theta$, A get back the collateral from B (which is worth $Z$ to A himself) and at the same time, A pays the loan $X_A$ to B. The second term in the objective function is the expected payoff of B received from the collateralized lending under the expectation that B will re-pledge the collateral for his investment in the next period. Lastly, the constraint ensures that A chooses the safe action in the optimum, called the incentive constraint for A.

For analytical convenience, we rewrite the incentive constraint,

$$\mathcal{B}I_A + \theta Z \geq \theta X_A \tag{25}$$

where $\mathcal{B} \equiv R - \frac{b}{1-p} \in (0,1)$. From this, we derive a condition for the existence of the equilibrium,

**Assumption 4.**

$$\mathcal{B}\theta\bar{Y} < 1$$

An intuition of this assumption is that the pledgeable income of A (which is positively correlated with $\mathcal{B}$) is sufficiently low so that A cannot raise funding needed for the investment without pledging his asset as collateral.

Then, the linearity of the problem ensures the incentive constraint binds in the optimum,

$$\mathcal{B}I_A^* + \theta Z = \theta X_A^*. \tag{26}$$

Substituting this equation for $X_A$ into the objective function and maximizing it with respect to $I_A$, we can obtain the optimal contract in period 0, $(I_A^*, X_A^*)$,

$$I_A^* = \left[\beta\frac{\theta^2\bar{Y}}{1-\mathcal{B}\theta\bar{Y}} + (1-\beta)\frac{1}{R-\mathcal{B}}\right]Z \tag{27}$$

$$X_A^* = \left[1 + \beta\frac{\mathcal{B}\theta\bar{Y}}{1-\mathcal{B}\theta\bar{Y}} + (1-\beta)\frac{\mathcal{B}}{\theta(R-\mathcal{B})}\right]Z \tag{28}$$

Then, plugging these values into the objective function, we represent the (net) payoff of A from pledging the asset as summarized in the following lemma.

**Lemma 6.** *Suppose the asset posted by A is non-fungible and Assumption 3 and 4 hold. The optimal contract in period 0, $(I_A^*, X_A^*)$, is given by*

$$(I_A^*, X_A^*) = \left(\left[\beta\frac{\theta^2\bar{Y}}{1-\mathcal{B}\theta\bar{Y}} + (1-\beta)\frac{1}{R-\mathcal{B}}\right]Z,\ \left[1 + \beta\frac{\mathcal{B}\theta\bar{Y}}{1-\mathcal{B}\theta\bar{Y}} + (1-\beta)\frac{\mathcal{B}}{\theta(R-\mathcal{B})}\right]Z\right). \tag{29}$$

### 4.3 Equilibrium

Based on all the results obtained so far, we define the equilibrium in this economy as follows.

**Definition 1.** *We define an equilibrium as a profile of* $(I_A, X_A, I_B, X_B)$ *given* $\theta \in (0,1)$ *and* $\beta \in [0,1]$ *where*

- $I_A$ *is the scale of the investment by A which equals to the amount of borrowing from B in period 0.*
- $X_A$ *is the contractual repayment from A to B in period 2.*
- $I_B$ *is the scale of the investment by B which equals to the amount of borrowing from C in period 1.*
- $X_B$ *is the contractual repayment from B to C in period 2.*
- $1-\theta$ *is the probability of the default by B.*[18]

*which satisfy:*

*(i)* $(I_A, X_A)$ *split the surplus between A and B according to the bargaining power,* $\beta$ *and* $1-\beta$, *by solving the Nash bargaining problem in period 0.*

*(ii)* $(I_B, X_B)$ *maximize the expected payoff of B in period 1 and makes C break even.*[19]

Then, combining the results in Lemma 5 and 6 leads to a characterization of the equilibrium in the three player model as follows.

**Proposition 1.** *Suppose the asset posted by A is not fungible and Assumption 3 and 4 hold. The equilibrium is characterized by a profile of* $(I_A^*, X_A^*, I_B^*, X_B^*)$ *where* $I_A^*, X_A^*$ *are as in Lemma 6 and* $I_B^*, X_B^*$ *are as in Lemma 5. In the equilibrium, the payoff of A is*

$$[RI_A^* - \theta X_A^* + \theta Z] - Z = \beta \left[ \frac{\{\theta R + (1-\theta)\mathcal{B}\}\theta\bar{Y} - 1}{1 - \mathcal{B}\theta\bar{Y}} \right] Z \tag{30}$$

*and the payoff of B is*

$$\theta(\bar{Y} I_B^* - X_B^* + X_A^*) - I_A^* = (1-\beta) \left[ \frac{\{\theta R + (1-\theta)\mathcal{B}\}\theta\bar{Y} - 1}{R - \mathcal{B}} \right] Z \tag{31}$$

Note that each agent's payoff is positively correlated with its bargaining power, $\beta$ and $1-\beta$, respectively, and also the value of the collateralized asset, $Z$.

[18] There is a sixth variable, the probability of the default by A which turns out to be 0 because the incentive constraint of A binds in the optimal contract.

[19] This is because that the lending market is competitive in period 1.

# 5 Welfare Analysis

In this section, we calculate the social welfare in the equilibrium that we obtained in the previous section and discuss whether rehypothecation is welfare improving or not compared with the case where rehypothecation is not allowed. As we briefly mentioned in the introduction, rehypothecation has a beneficial effect of providing more funding liquidity to the positive NPV projects, while, at the same time, it introduces the risk of losing collateral by the countera-party who re-pledges the collateral. The result shows that the efficiency of rehypothecation is determined by the relative size of these trade-offs.

### 5.0.1 Benchmark: Non-Rehypothecation Case

As a benchmark, we start by considering the welfare in the non-rehypothecation case in which B is not allowed re-pledge the collateral posted by A. This corresponds to the previous two player model, and it is straightforward that the expected return to B from collateralized lending is the expected repayment from A deducted by the amount of lending to A,

$$X_A - I_A. \tag{32}$$

Note that in the optimum, there is no default by A, and B's expected revenue from lending is $X_A$.

On the other hand, A can recover the collateral from B with certainty since B does not participate in rehypothecaion, and holds on to the asset until A repays the loan, and the (net) payoff of A from pledging the asset is given by,

$$RI_A - X_A. \tag{33}$$

Then, as in the previous sections, we define the optimal contract between A and B in period 0 as the solution to the Nash bargaining problem,

$$\max_{I_A, X_A} \quad (RI_A - X_A)^{\beta}(X_A - I_A)^{1-\beta} \tag{34}$$

subject to the incentive constraint of A,

$$-Z + RI_A - X_A + Z \geq -Z + p\left[RI_A - X_A + Z\right] + bI_A \tag{35}$$

Note that with no rehypothecation, A can get back his asset from B whenever he repays the loan, and thus there is no risk of losing collateral ($\theta < 1$ is removed from Equation 24) in the incentive constraint.

Solving this problem then leads us to characterize the optimal contract and the expected payoff of each agent as in the following lemma.

**Proposition 2.** *The optimal contract without rehypothecation, $(I_A^{**}, X_A^{**})$, is given by*

$$(I_A^{**}, X_A^{**}) = \left( \left[ \beta \frac{1}{1-\mathcal{B}} + (1-\beta) \frac{1}{R-\mathcal{B}} \right] Z, \ \left[ 1 + \beta \frac{\mathcal{B}}{1-\mathcal{B}} + (1-\beta) \frac{\mathcal{B}}{R-\mathcal{B}} \right] Z \right). \tag{36}$$

*At these values, the net payoff of A is given by*

$$[RI_A^{**} - X_A^{**} + Z] - Z = \beta \left( \frac{R-1}{1-\mathcal{B}} \right) Z \tag{37}$$

*and the net payoff of B is given by*

$$X_A^{**} - I_A^{**} = (1-\beta) \left( \frac{R-1}{1-\mathcal{B}} \right) Z. \tag{38}$$

Note that as in the previous analysis, each agent's payoff is positively correlated with its bargaining power, $\beta$ and $1-\beta$, respectively, and also with the value of the collateralized asset, $Z$. For comparison, we refer to this as the non-rehypothecation equilibrium.

#### 5.0.2 Efficiency of Rehypothecation

In this section, we compare the welfare in the non-rehypothecation equilibrium $(I_A^{**}, X_A^{**})$ with that in the rehypothecation equilibrium $(I_A^*, X_A^*, I_B^*, X_B^*)$. First, note that as $\theta$ is high (equivalently, the risk of losing collateral by the counterparty is low), the scale of the positive NPV investments of A is likely to higher under rehypothecation,

$$\lim_{\theta \to 1} (I_A^* - I_A^{**}) = \beta \left[ \frac{\bar{Y}}{1-\mathcal{B}\bar{Y}} - \frac{1}{1-\mathcal{B}} \right] Z > 0.$$

the inequality is due to the assumption that $\bar{Y} > 1$. This verifies that rehypothecation helps more funding liquidity into the system.[20] At the same time, however, rehypothecation introduces the risk that A may not get back his collateral from B in case that B defaults. Thus, the efficiency of rehypothecation is determined by the relative size of these tradeoffs: improving liquidity provision versus counterparty risk of losing collateral.

To show this more formally, we first need to clarify the definition of the social welfare in the economy.

**Definition 2.** *The welfare gain, $W$ under collateralized borrowing is defined by the sum of (i) the net surplus from A's investment, $(R-1)I_A$ and (ii) the net surplus from B's investment, $(\theta\bar{Y}-1)I_B$, which is deducted by (iii) the deadweight cost from misallocation of the collateral in*

[20]More precisely, we need to show that $I_A^* \geq I_A^{**}$ for all the possible values of $\theta$ and $\bar{Y}$. Note that, however, at the optimum, this will be satisfied since B participates in rehypothecation only when $\theta^2\bar{Y}$ as shown in Assumption 3.

*case that B defaults having re-pledged it to C, $(1-\theta)(1-\delta)Z$,*[21]

$$W \equiv (R-1)I_A + (\theta\bar{Y}-1)I_B - (1-\theta)(1-\delta)Z. \tag{39}$$

Using the results obtained in the preceeding sections, the equation in Definition 2 can be rewritten as the sum of the net payoff of each agent, A, B, and C, respectively

$$\begin{aligned} W &\equiv (R-1)I_A + (\theta\bar{Y}-1)I_B - (1-\theta)Z \\ &= \underbrace{[RI_A - \theta X_A + \theta Z - Z]}_{\text{net payoff of A}} + \underbrace{[\theta\bar{Y}I_B - \theta X_B + \theta X_A - I_A]}_{\text{net payoff of B}} + \underbrace{[\theta X_B + (1-\theta)\delta Z - I_B]}_{\text{net payoff of C}} \qquad (40) \end{aligned}$$

where the last term is zero in the competitive lending market in period 1, $\theta X_B + (1-\theta)\delta Z - I_B = 0$.[22]

Then, plugging the equilibrium obatained in the preceeding section into this equation, we calculate the social welfare in each case; non-rehypothecation and rehypothecation as in the following lemma.

**Lemma 7.** *The welfare gain from collateralized borrowing in the non-rehypothecation equilibrium is given by*

$$W^{**} = [RI_A^{**} - X_A^{**} + Z - Z] + [X_A^{**} - I_A^{**}] = \left(\frac{R-1}{1-\mathcal{B}}\right) Z. \tag{42}$$

*Suppose Assumption 3 and 4 hold and the market for collateralized lending in the second period (when the collateral is rehypothecated) is competitive. Then, the welfare gain from collateralized borrowing in the rehypothecation equilibrium is given by*

$$\begin{aligned} W^* &= [RI_A^* - \theta X_A^* + \theta Z - Z] + [\theta(\bar{Y}I_B^* - X_B^* + X_A^*) - I_A^*] + [\theta X_B^{**} + (1-\theta)\delta Z - I_B^{**}] \qquad (43) \\ &= \big((\theta R + (1-\theta)\mathcal{B})\theta\bar{Y} - 1\big)\left[\beta\frac{1}{1-\mathcal{B}\theta\bar{Y}} + (1-\beta)\frac{1}{R-\mathcal{B}}\right] Z. \qquad (44) \end{aligned}$$

It is noteworthy that the total welfare under rehypothecation,$W^*$ varies with the bargaining power between A and B,

$$W^* = \underbrace{\beta\left[\frac{\{\theta R + (1-\theta)\mathcal{B}\}\theta\bar{Y} - 1}{1-\mathcal{B}\theta\bar{Y}}\right] Z}_{\text{payoff of A}} + \underbrace{(1-\beta)\left[\frac{\{\theta R + (1-\theta)\mathcal{B}\}\theta\bar{Y} - 1}{R-\mathcal{B}}\right] Z}_{\text{payoff of B}}. \tag{45}$$

This shows that the total welfare increasese as A has more bargaining power than B since

[21] Note that if rehypothecation is not allowed, the investment of B cannot be undertaken and, at the same time, there is no risk of losing collateral, which implies that both the second and the third term are zero.

[22] When rehypothecation is not allowed, the social welfare is simply given by

$$W \equiv (R-1)I_A = \underbrace{[RI_A - X_A + Z - Z]}_{\text{net payoff of A}} + \underbrace{[X_A - I_A]}_{\text{net payoff of B}} \tag{41}$$

$\frac{1}{1-\mathcal{B}\theta\bar{Y}} > \frac{1}{R-\mathcal{B}}$. Intuition behind this result is that the value of the collateral diminishes as it moves on to the next link of the collateral chain. In this model, when B re-pledges the collateral to C, B may not be able to repurchase it from C if she defaults with probability $1-\theta$, and this reduces the amount that can be borrowed against the collateral. That is, there is an additional risk introduced as the collateral is re-used.

Then, by comparing $W^*$ with $W^{**}$ in the Lemma 7, we characterize the condition under which the rehypothecation equilibrium is more efficient than the non-rehypothecation equilibrium.

**Proposition 3.** *Suppose Assumption 3 and 4 hold and the market for lending in the second period is competitive. Then, rehypothecation is welfare improving, i.e., $W^* - W^{**} > 0$, if and only if*

$$[\theta R + (1-\theta)\mathcal{B}\}\theta\bar{Y} - 1]\left(\beta\frac{1}{1-\mathcal{B}\theta\bar{Y}} + (1-\beta)\frac{1}{R-\mathcal{B}}\right) - \frac{R-1}{1-\mathcal{B}} > 0. \tag{46}$$

This proposition chracterizes the condition in which rehypothecation is socially efficient. When the productivity of investments are high ($R$, $\bar{Y}$ high) and the risk of default of B is low ($\theta$ high), the benefit of liquidity provision due to rehypothecation outweighs the cost of rehypothecation failure, and rehypothecation is socially beneficial.

## 5.1 Externality in the Individualized Decision

In this section, we ask a question whether individual agents' decisions to participate in rehypothecation achieves a socially optimal outcome. In practice, the initial provider of collateral (A in the model) has the right to allow or prohibit re-pledging his asset. This paper shows that the externality in the initial owner's decision may lead to excessive rehypothecation than the optimum.

In general, the surplus from collateralized borrowing diminishes as the collateral is re-used as we have seen in the previous section. Other things being equal (that is, each agent's bargaining power held constant both under rehypothecation or non-rehypothecaion), the gain from rehypothecaion is thus greater for A than for B, and it is possible that A is better off after rehypothecaion, while B is worse off after rehypothecation.

In this case, A does not internalize the loss of B, and may prefer to allow rehypothecation even if it is socially inefficient (when the loss of B is greater than the gain of A). It is also noteworthy that B is always willing to participate in rehypothecation ex post as long as Assumption 3 is satisfied, even if it results in a lower payoff than she does not participate in rehypothecation. Therefore, whenever A allows rehypothecation, it always occur.

To show this more formally, consider the following case

$$(1-\mathcal{B}\theta\bar{Y})\left(\frac{R-1}{1-\mathcal{B}}\right) < \{\theta R + (1-\theta)\mathcal{B}\}\theta\bar{Y} - 1 < (R-\mathcal{B})\left(\frac{R-1}{1-\mathcal{B}}\right). \tag{47}$$

This is possible as $R - \mathcal{B} > 1 - \mathcal{B}\theta\bar{Y}$.

Using Lemma 7, the first inequality implies that A's payoff increases after rehypothecation and the second inequality implies that B's payoff decreases after rehypothecation. Thus, A wants to allow rehypothecation even though B is worse off after rehypothecation. Rearranging the terms in Proposition 3, this implies that, in this case, some socially inefficient rehypothecation may occur if the following conditions holds,

$$\underbrace{\beta\left(\frac{\{\theta R+(1-\theta)\mathcal{B}\}\theta\bar{Y}-1}{1-\mathcal{B}\theta\bar{Y}}-\frac{R-1}{1-\mathcal{B}}\right)}_{\text{A's loss from rehypothecation} > 0}+\underbrace{(1-\beta)\left(\frac{\{\theta R+(1-\theta)\mathcal{B}\}\theta\bar{Y}-1}{R-\mathcal{B}}-\frac{R-1}{1-\mathcal{B}}\right)}_{\text{B's gain from rehypothecation} << 0}<0. \tag{48}$$

This conditions says that rehypothecation is socially inefficient as the loss of B outweighs the gain of A after rehypothecation, but it always occur whenever A allows it for the reasons we previously explained.

We summarize this result in the following proposition.

**Proposition 4.** *Suppose Assumption 1, 2, 3 and 4 hold and the marketing for collateralized lending in period 1 is competitive. If the parameters satisfy the following condition,*

$$(1-\mathcal{B}\theta\bar{Y})\left(\frac{R-1}{1-\mathcal{B}}\right)<\{\theta R+(1-\theta)\mathcal{B}\}\theta\bar{Y}-1<(R-\mathcal{B})\left(\frac{R-1}{1-\mathcal{B}}\right)$$

*and*

$$\beta\left(\frac{\{\theta R+(1-\theta)\mathcal{B}\}\theta\bar{Y}-1}{1-\mathcal{B}\theta\bar{Y}}-\frac{R-1}{1-\mathcal{B}}\right)+(1-\beta)\left(\frac{\{\theta R+(1-\theta)\mathcal{B}\}\theta\bar{Y}-1}{R-\mathcal{B}}-\frac{R-1}{1-\mathcal{B}}\right)<0$$

*then A allows rehypothecation, and it occurs as B is also willing to participate in rehypothecation ex post, which leads to a lower social welfare than without rehypothecation.*

# 6 Conclusion

This paper discusses how rehypothecation arises leading to create a collateral chain in the system and what benefits and costs it produces to the economy. A main reason for rehypothecation is to save on the scarce collateral, or safe and liquid assets as in Krishnamurthy and Vissing-Jorgensen (2012) and Greenwood, Hanson, and Stein (2012). We show that the efficiency of rehypothecation is determined by the relative size of the two trade-off effects. First, rehypothecation makes collateral sitting idle in the receiver's account more liquid, thereby providing more funding liquidity into the market. At the same time, however, it introduces an additional risk that the receiver of collateral fails to return the collateral to the borrower in the event of default. This paper emphasizes that this rehypothecation failure incurs deadweight costs by resulting in the misallocation of assets in the economy: the rehypothecated asset remains in the hands of the third party who values the asset less than the initial owner does.

We also address a question whether an individual agent's decision to participate in rehypothecation achieves a socially optimal outcome. Our analysis shows that there might be too much rehypothecation than the optimum. There are three main reasons behind this results: (i) the gain from rehypothecation diminishes as it moves on to the later link of the collateral chain; and (ii) the the initial owner of collateral, A, does not internalize others' loss from rehypothecation into his decision making; and (iii) under a certain condition, B is always willing to participate in rehypothecation as long as it is allowed by A. Therefore, in some cases, a socially inefficient rehypothecation might occur, and this suggests that restrictions on rehypothecation may be desirable in such cases.

Finally, the model could be extended to the case where: (i) the optimal contract is in the form of the insurance contract in which A involves in risk taking; (ii) rehypothecation occurs in period 0 simultaneously with the tranfer of the collateral from A to B, that is, B lends and borrows at the same time; and (iii) collateral is a fungible asset that B can compensate A with not exactly the same asset posted by A, but other equivalent asset of the same value. These are discussed in the Appendix, and we show that the main results of this paper remain the same in those cases.

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## Appendix A Choice of the Type of Contracts

In this section, we show that under what conditions, the optimal contract between A and B is in the form of the incentive contract, that is A does not engage in risk taking (following the terminology in Koeppl (2013)).

Suppose that Assumption 3 holds. Then, B is always willing to re-pledge A's collateral in period 1 for any given period 0 contract $(I_A, X_A)$. In equilibrium, there are two possibilities: (i) A does not engage in risk taking; (ii) A engages in risk taking. In the previous section, we consider case (i) only. In this section, we consider case (ii). In this case, the probability of the success of A's investment is $p < 1$, and so B receives the payment $X_A$ from A with probability $p < 1$. We assume that B observes whether A succeeds or fails before he repurchases the collateral from C by paying $X_B$. We use the backward induction to solve the equilibrium as in the previous section. Thus, taking the terms of contract in period 0 as given, the optimization problem of B in period 1 is given by

$$\max_{I_B, X_B} \quad p\theta(\bar{Y}I_B - X_B + X_A) + (1-p)\theta\bar{Y}I_B - I_A \tag{49}$$

subject to

$$I_B \leq p\theta X_B \tag{50}$$

$$X_B \leq X_A \tag{51}$$

Note that C is willing to lend up to $p\theta X_B$ because C knows that B does not want to recover the collateral from C in case that A fails with probability $1-p$ even when B's investment succeeds (recall that the outcome of B's investment is not verifiable to C).[23]

At the optimum, all the constraints are binding, and thus the objective function can be written as a function of $I_A$ and $X_A$,

$$p\theta(\bar{Y}I_B^* - X_B^* + X_A) + (1-p)\theta\bar{Y}I_B^* - I_A = p\theta^2\bar{Y}X_A - I_A. \tag{55}$$

It is noteworthy that B is willing to re-pledge the collateral for any given period 0 contract $(I_A, X_A)$ only if $p\theta^2\bar{Y} > 1$. Thus, we make the following assumption

**Assumption 5.** $p\theta^2\bar{Y} > 1$.

Next, we move backward to period 0 to consider the bargaining problem between A and B. For analytical convenience, assume that B has all the bargaining power and makes a take-it-or-leave-it offer $(I_A, X_A)$ to A.

B offers one of two types of contracts to A: (i) the incentive contract in which A does not engage in risk taking; (ii) the insurance contract in which A engages in risk taking (Koeppl (2013) refers to each type of contracts as the incentive- and insurance contract, respectively). Let $\lambda_B \in \{0, 1\}$ denote the choice of contracts by B: $\lambda_B = 0$ indicates the incentive contract and $\lambda_B = 1$ indicates the insurance contract. Furthermore, denote the terms of the incentive contract by $(I_A, X_A)$ and those of the insurance contract by $(\tilde{I}_A, \tilde{X}_A)$.

[23] Alternatively, we may assume that B repurchases the collateral from C before B observes whether A succeeds or fails. In either case, however, the result will be the same. In this case, the optimization problem of B in period 1 is changed to

$$\max_{I_B, X_B} \quad \theta(\bar{Y}I_B - X_B + pX_A) - I_A \tag{52}$$

subject to

$$I_B \leq \theta X_B \tag{53}$$

$$X_B \leq pX_A \tag{54}$$

Then, B's maximiation problem in period 0 is given by

$$\max_{\lambda_B \in \{0,1\}, I_A, X_A, \tilde{I}_A, \tilde{X}_A} \quad (1-\lambda_B)(\theta^2 \bar{Y} X_A - I_A) + \lambda_B (p\theta^2 \bar{Y} \tilde{X}_A - \tilde{I}_A) \tag{56}$$

subject to

$$\mathcal{B} I_A + \theta Z \geq \theta X_A \tag{57}$$

$$R I_A - \theta X_A + \theta Z \geq Z \tag{58}$$

$$\mathcal{B} \tilde{I}_A + \theta Z \leq \theta \tilde{X}_A \tag{59}$$

$$p(R\tilde{I}_A - \theta \tilde{X}_A + \theta Z) + b\tilde{I}_A \geq Z \tag{60}$$

where $\mathcal{B} \equiv R - \frac{b}{1-p} \in (0,1)$. The first and third constraint is the incentive constraint of A and the reverse of it. For the insurance contract, A engages in risk taking, and thus the incentive constraint is not satisfied. The second and fourth constraint is the participation constraint for A when the incentive contract is chosen and when the insurance contract is chosen, respectively. The left hand side of each constraint is the expected payoff of A in each case. These constraints show that A will take the offer only if the expected payoff is at least $Z$, the expected payoff he gets from holding on to the illiquid asset.

First, for the existence of the equilibrium, we need the following assumption,

**Assumption 6.** $\theta \bar{Y}(pR + b) < 1$.

Intuition behind this assumption is similar to that of Assumption 4. This implies that the pledgeable return of A's investment is low enough even under the rehypothecation, and thus A cannot raise the funding needed to initiate the investment without posting his asset as collateral.

First, let us solve for the optimal incentive contract $(I_A, X_A)$. At the optimum, the incentive constraint 57 must be binding, otherwise B can increase his payoff by increasing $X_A$ slightly, while ralaxing the participation constraint 58,

$$\mathcal{B} I_A + \theta Z = \theta X. \tag{61}$$

Plugging this into $\theta X_A$ in the objective function, we can write it as a function of $I_A$,

$$\theta^2 \bar{Y} X_A - I_A = (\theta \bar{Y} \mathcal{B} - 1) I_A + \theta Z. \tag{62}$$

By Assumption 4, this is decreasing in $I_A$. In a similar way, the participation constraint can be written as a function of $I_A$,

$$I_A \geq \frac{Z}{R - \mathcal{B}}. \tag{63}$$

Next, we solve for the optimal insurance contract $(\tilde{I}_A, \tilde{X}_A)$. In this case, the participation constraint must be binding at the optimum, otherwise B can increase his payoff by increasing $\tilde{X}_A$ slightly while relaxing the incentive constraint.

$$\left(R + \frac{b}{p}\right) \tilde{I}_A - \left(\frac{1}{p} - \theta\right) Z = \theta \tilde{X}_A \tag{64}$$

Plugging this into $\theta X_A$ in the objective function, we can rewrite it as a function of $\tilde{I}_A$ only,

$$p\theta^2 \bar{Y} \tilde{X}_A - \tilde{I}_A = [\theta \bar{Y}(pR + b) - 1]\tilde{I}_A - \theta \bar{Y}(1 - p\theta) Z \tag{65}$$

In a similar way, the incentive constraint can be written as

$$\tilde{I}_A \geq \frac{Z}{p(R-\mathcal{B})+b}. \tag{66}$$

Combining the results obtained so far, B's maximization problem in period 0 is given by

$$\max_{\lambda_B\in\{0,1\},I_A,\tilde{I}_A,} \quad (1-\lambda_B)\left((\theta\bar{Y}\mathcal{B}-1)I_A+\theta Z\right)+\lambda_B\left([\theta\bar{Y}(pR+b)-1]\tilde{I}_A-\theta\bar{Y}(1-p\theta)Z\right) \tag{67}$$

subject to

$$I_A \geq \frac{Z}{R-\mathcal{B}} \tag{68}$$

$$\tilde{I}_A \geq \frac{Z}{p(R-\mathcal{B})+b} \tag{69}$$

Then, by Assumption 4 and 6, it is optimal to choose $I_A$ and $\tilde{I}_A$ as low as possible,

$$I_A^* = \frac{Z}{R-\mathcal{B}} \tag{70}$$

$$\tilde{I}_A^* = \frac{Z}{p(R-\mathcal{B})+b} \tag{71}$$

and the optimal choice of the types of contracts is determined by,

$$\lambda_B^* = \begin{cases} 0 & \text{if} \quad \frac{\theta[R+(\bar{Y}-1)\mathcal{B}]-1}{R-\mathcal{B}} > \frac{\theta\bar{Y}[p\theta(pR+b)+(1-p\theta)p\mathcal{B}]-1}{p(R-\mathcal{B})+b} \\ 1 & \text{if} \quad \frac{\theta[R+(\bar{Y}-1)\mathcal{B}]-1}{R-\mathcal{B}} < \frac{\theta\bar{Y}[p\theta(pR+b)+(1-p\theta)p\mathcal{B}]-1}{p(R-\mathcal{B})+b} \end{cases} \tag{72}$$

This shows that the incentive contract ($\lambda_B = 1$) is more attractive as the private benefit that A receives from risk taking, $b$, is lower (since $p\theta^2\bar{Y} > 1$ by Assumption 5) and the probability of success when A engages in risk taking, $p$, is higher, which is consistent with the results in Koeppl (2013).

To be more general, we can consider the case in which A also has some positive bargaining power in period 0 for the insurance contract as in the previous analysis of the incentive contract,

$$\max_{\tilde{I}_A,\tilde{X}_A} \quad \left(p(R\tilde{I}_A-\theta\tilde{X}_A+\theta Z)+b\tilde{I}_A-Z\right)^{\beta}\left(p\theta^2\bar{Y}\tilde{X}_A-\tilde{I}_A\right)^{1-\beta} \tag{73}$$

subject to

$$-Z+R\tilde{I}_A-\theta\tilde{X}_A+\theta Z \leq -Z+p(R\tilde{I}_A-\theta\tilde{X}_A+\theta Z)+b\tilde{I}_A. \tag{74}$$

At the optimum, the constraint wil be binding and the optimal insurance contract is given by

$$\tilde{I}_A^* = \left[\beta\frac{p\theta^2\bar{Y}}{1-p\mathcal{B}\theta\bar{Y}}+(1-\beta)\frac{1}{p(R-\mathcal{B})+b}\right]Z \tag{75}$$

$$\tilde{X}_A^* = \left[1+\beta\frac{p\mathcal{B}\theta\bar{Y}}{1-\mathcal{B}\theta\bar{Y}}+(1-\beta)\frac{\mathcal{B}}{\theta(p(R-\mathcal{B})+b)}\right]Z \tag{76}$$

Then, the remaining analysis about welfare and externality is analogous to the previous analysis of the incentive contract.

# Appendix B Simultaneous Rehypothecation

In this section, we consider the case in which B lends and borrows simultaneously: B lends cash to A in exchange for collateral and at the same time, B borrows cash from C by re-pledging the collateral posted by A.

We assume that the investment technologies of A and B are the same as in the previous sequential contracts. Also, A and C cannot meet and trade by themselves, and trade assets only through B. In this simultaneous contract, however, we assume that B has a limited amount of cash denoted by $M$. For simplicity, we assume that there is a continuum of A and C, and the optimal contract is defined as a solution to the maximization problem of B as follows.

$$\max_{I_A, I_B, X_A, X_B} \quad \theta(\bar{Y} I_B - X_B + X_A) - I_A \tag{77}$$

subject to

$$I_A + I_B \leq \theta X_B + M \tag{78}$$

$$X_B \leq X_A \tag{79}$$

$$\mathcal{B} I_A + \theta Z \geq \theta X_A \tag{80}$$

$$R I_A - \theta X_A + \theta Z \geq Z \tag{81}$$

where $\mathcal{B} \equiv R - \frac{b}{1-p} \in (0,1)$. The objective function is the expected payoff of B from lending cash to A and simultaneously borrowing cash from C by re-pledging A's collateral. The first constraint means that B can use her cash in two ways, either lending it to A, $I_A$, or spending it for her project, $I_B$, and she can raise funding either by re-pledging A's asset to C, $X_B$, or by spending her own cash endowment, $M$. The second constraint means that any promised repayment, $X_B$, beyond the repayment from A, $X_A$, is not credible to C. The third constraint is A's incentive constraint. Here, we consider only the case in which A does not engage in risk taking (for the complete analysis, we also have to consider another case in which A engages in risk taking, but the main results of this paper will not be affected by the types of contracts). The last constraint is the participation constraint for A that the payoff of A from the collateralized borrowing must be greater than or equal to the payoff when just holding on to the asset.

At the optimum, the incentive constraint of A will be binding, otherwise B can increase her payoff by increasing $X_A$ slightly, while relaxing A's participation constraint and also, by Assumption 3, the second constraint will be binding. Using these, we can simplify B's maximization problem,

$$\max_{I_A, I_B} \quad \theta \bar{Y} I_B - I_A \tag{82}$$

subject to

$$(1 - \mathcal{B}) I_A + I_B \leq \theta Z + M \tag{83}$$

$$I_A \geq \frac{Z}{R - \mathcal{B}} \tag{84}$$

By the linearity of the problem, all the constraints will be binding at the optimum, and we get the optimal solution to this problem as follows,

$$I_A^* = \frac{Z}{R - \mathcal{B}}, \qquad I_B^* = M + \left(\theta - \frac{1 - \mathcal{B}}{R - \mathcal{B}}\right) Z. \tag{85}$$

Then, the payoff of B under the (simultaneous) rehypothecation is given by

$$\theta \bar{Y} I_B^* - I_A^* = \theta \bar{Y} M + \frac{\theta \bar{Y}[\theta R + (1-\theta)\mathcal{B} - 1] - 1}{R - \mathcal{B}} Z. \tag{86}$$

In order to determine whether rehypothecation is socially efficient, we also need to evaluate the welfare under non-rehypothecation and compare it with that under rehypothecation. Under non-rehypothecation, the maximization problem of B is changed as follows.

$$\max_{I_A, X_A} \quad \theta \bar{Y} I_B + X_A - I_A \tag{87}$$

subject to

$$I_A + I_B \leq M \tag{88}$$

$$\mathcal{B} I_A + Z \geq X_A \tag{89}$$

$$R I_A - X_A + Z \geq Z \tag{90}$$

Again, the linearity of the problem ensures that the constraints are binding at the optimum, and we get the optimal solution to this problem as follows,

$$I_A^{**} = \frac{Z}{R - \mathcal{B}}, \qquad I_B^{**} = M - \frac{Z}{R - \mathcal{B}}. \tag{91}$$

Plugging these results into the objective function, the payoff of B under non-rehypothecation is thus given by

$$\theta \bar{Y} I_B^{**} + X_A^{**} - I_A^{**} = \theta \bar{Y} M + \frac{R - \theta \bar{Y} - 1}{R - \mathcal{B}} Z. \tag{92}$$

Finally, comparing the payoff of B in this case with that without rehypothecation, we show that rehypothecation is more efficient than non-rehypothecation (that is, $\theta \bar{Y} I_B^* - I_A^* > \theta \bar{Y} I_B^{**} + X_A^{**} - I_A^{**}$) if and only if[24]

$$R > \theta \bar{Y}[\theta R + (1-\theta)\mathcal{B}]. \tag{93}$$

This condition shows that rehypothecation is more likely to be efficient as the return of B's investment, $\theta \bar{Y}$ (which is possible only by rehypothecation) and the risk of default of B is lower ($\theta$ is high), which is consistent with the results in the case of the sequential rehypothecation.

[24] To avoid a trivial case, we assume that R is sufficiently large such that

$$\max\{R, \theta \bar{Y}[\theta R + (1-\theta)\mathcal{B}]\} > \theta \bar{Y} + 1$$

and B always finds it profitable to lend some of her cash to A. If this condition is not satisfied, it may be optimal for B to spend all her cash endowment $M$ for her investment and receives the payoff, $\theta \bar{Y} M$.

# Appendix C When Collateral is a Fungible Asset

In this section, we consider the case where the collateral initially posted by A is a fungible asset that is interchangeable with other goods of the same type, for example, stocks, bonds, currencies, gold, oil etc. The terms of contracts are analogous to the case with non-fungible collateral except that B is allowed to return other assets of the same value (called the equivalent assets) rather than the exactly same collateral to A. Therefore, with fungibility, the collateral constraint faced by B changes as follows.

**Lemma 8.** *Suppose A's collateral is fungible. Then, in period 1, the collateral constraint for B is given by*

$$X_B \leq Z. \tag{94}$$

An intuition of this result is that if this condition does not hold, B would find it more profitable to return the other equivalent assets (which costs $Z$ for B) to A rather than recovering the collateral by repaying $X_B$ which is higher than $Z$. As in the previous section, we solve for the optimal incentive contract by the backward induction to discuss the efficiency of rehypothecation and finally ask whether there arises the externality from the individualized decision making.[25]

In period 1, the optimal contract is defined as a solution to B's maximization problem taking the terms of period 0 contract $(I_A, X_A)$ as given,

$$\max_{I_B, X_B} \theta(\bar{Y} I_B - X_B + X_A) - I_A \tag{95}$$

subject to

$$I_B \leq \theta X_B \tag{96}$$

$$X_B \leq Z \tag{97}$$

The linerity of the Lagrangian of the problem ensures that all the constraints will be binding at the optimum, and the optimal contract in period 1 can be written as a function of Z as follows,

$$(I_B^*, X_B^*) = (\theta Z, Z) \tag{98}$$

and the payoff of B can be written as a function of $I_A$ and $X_A$,

$$\theta(\bar{Y} I_B^* - X_B^* + X_A) - I_A = \theta(\theta \bar{Y} - 1)Z + \theta X_A - I_A. \tag{99}$$

Next, we move backward to solve for the optimal contract in period 0. As before, the optimal contract solves the Nash bargaining problem between A and B as follows,[26]

$$\max_{I_A, X_A} (R I_A - \theta X_A + \theta Z - Z)^{\beta} (\theta(\theta \bar{Y} - 1)Z + \theta X_A - I_A)^{(1-\beta)} \tag{100}$$

subject to

$$\mathcal{B} I_A + \theta Z \geq \theta X_A \tag{101}$$

where $\mathcal{B} \equiv R - \frac{b}{1-p} \in (0, 1)$. At the optimum, the incentive constraint will be binding, and the optimal

[25] As we mentioned in Appendix A, we can analyze the optimal insurance contract in a similar way.

[26] Again, we focus on the incentive contract, here.

contract in period 0 is given by,

$$(I_A^*, X_A^*) = \left( \left[ \beta \frac{\theta^2 \bar{Y}}{1-\mathcal{B}} + (1-\beta) \frac{1}{1-\mathcal{B}} \right] Z, \left[ 1 + \beta \frac{\mathcal{B}\theta\bar{Y}}{1-\mathcal{B}} + (1-\beta) \frac{\mathcal{B}}{\theta(1-\mathcal{B})} \right] Z \right) \tag{102}$$

At these values, the payoff of A is

$$RI_A^* - \theta X_A^* + \theta Z - Z = \beta \left[ \frac{(R-\mathcal{B})\theta^2\bar{Y} + \mathcal{B} - 1}{1-\mathcal{B}} \right] Z, \tag{103}$$

and the payoff of B is

$$\theta(\theta\bar{Y} - 1)Z + \theta X_A^* - I_A^* = (1-\beta) \left[ \frac{(R-\mathcal{B})(\theta^2\bar{Y} - \theta) - \theta(R-\mathcal{B}) + \mathcal{B} - 1}{R-\mathcal{B}} \right] Z \tag{104}$$

Then, the remaining analysis of the efficiency of rehypothecation and the externality of the individualized decision is analogous to the previous non-fungible collateral case.