Joint-Search Theory:
New Opportunities and New Frictions

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Abstract

Search theory routinely assumes that decisions about the acceptance/rejection of job offers (and, hence, about labor market flows between jobs or across employment states) are made by individuals acting in isolation. In reality, the vast majority of workers are somewhat tied to their partners—in couples or families—and decisions are made jointly. This paper studies, from a theoretical viewpoint, the joint job-search and location problem of a household formed by a couple (e.g., husband and wife) who perfectly pools income. The objective, in the spirit of standard search theory, is to characterize the reservation wage behavior of the couple and compare it to the single-agent search model in order to understand the ramifications of partnerships for individual labor market outcomes and wage dynamics. We focus on two main cases. First, when couples are risk averse and pool income, joint search yields new opportunities—similar to on-the-job search—relative to the single-agent search. Second, when the two spouses in a couple face job offers from multiple locations and a cost of living apart, joint search features new frictions and can lead to significantly worse outcomes than single-agent search.

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1 Introduction

In the year 2000, the labor force participation rate of married women stood at 61%, and in one-third of married couples wives provided more than 40% of household income (US Census, 2000; Raley, Mattingly, and Bianchi, 2006). For these households—which make up a substantial fraction of the population—economic decisions are surely taken jointly by the two spouses. Among such decisions, job search, broadly defined, is arguably one of the most crucial to the economic well-being of a household.

Macroeconomics is rapidly shifting away from the stylized "bachelor model" of the household to models that explicitly recognize the relevance of within-household decisions for aggregate economic outcomes.[1] Surprisingly, instead, since its inception in the early 1970s, search theory has almost entirely focused on the single-agent search problem. The recent survey by Rogerson, Shimer, and Wright (2005), for example, does not contain any discussion on optimal job search strategies of two-person households acting as the decision units. This state of affairs is rather surprising given that Burdett and Mortensen (1977), in their seminal piece entitled "Labor Supply Under Uncertainty," lay out a two-person search model and sketch a characterization of its solution, explicitly encouraging further work on the topic. Their pioneering effort, which remained virtually unfollowed, represents the starting point of our theoretical analysis.

In this paper, we study the job search problem of a couple who faces exactly the same economic environment as in the standard single-agent search problem of McCall (1970) and Mortensen (1970) without on-the-job search, and Burdett (1978) with on-the-job search. A couple is an economic unit composed of two identical individuals linked to each other by the assumption of perfect income pooling. The simple unitary model of a household adopted here is a convenient and logical starting point. It helps us to examine more transparently the role of the labor market frictions and insurance opportunities introduced by joint-search, and it makes the comparison with the canonical single-agent search model especially stark.

From a theoretical perspective, couples would make a joint decision leading to choices different from those of a single agent for several reasons. We start from the two most natural and relevant ones. First, the couple has concave preferences over pooled income. Second, the couple can receive job offers from multiple locations, but faces a utility cost of living apart. In this latter case, deviations from the single-agent search problem occur even for linear preferences. As summarized by the title of our paper, in the first environment joint search introduces new opportunities, whereas in the second it introduces new frictions relative to single-agent search. One appealing feature of our theoretical analysis is that it leads to two-dimensional diagrams in the space of the two spouses' wages (w_1, w_2), where the reservation wage policies can be easily analyzed and interpreted.

In the first environment we study, couples have risk-averse preferences and have access to a

[1] For example, see Aiyagari et al. (2000) on intergenerational mobility and investment in children, Cubeddu and Rios-Rull (2003) on precautionary saving, Blundell et al. (2007) on labor supply, Heathcote et al. (2008) and Lise and Seitz (2008) on economic inequality, and Guner et al. (2009) on taxation.

risk-free asset for saving but are not allowed to borrow. A dual-searcher couple (both members unemployed) will quickly accept a job offer—in fact, more easily than a single unemployed agent. The dual-searcher couple can use income pooling and joint search to its advantage: it initially accepts a lower wage offer (to smooth consumption across states) while, at the same time, not giving up completely the search option (to increase lifetime income), which remains available to the other spouse. Once a worker-searcher couple (one spouse unemployed, the other employed), the pair will be more choosy in accepting the subsequent job offers. We formally show that the gap between the reservation wage of the worker-searcher couple (a function of the employed spouse's wage) and that of the dual-searcher couple (a constant) depends on the degree of absolute risk aversion in preferences and on how absolute risk aversion changes with the level of consumption.

Furthermore, if the searching spouse receives and accepts a job offer, this may trigger a quit by the employed spouse to search for a better job, resulting in a switch between the breadwinner and the searcher within the household. As is well known, this endogenous quit behavior never happens in the corresponding single-agent version of the search model. We call this process—of quit-search-work that allows a couple to climb the wage ladder even in absence of on-the-job search—the "breadwinner cycle." Therefore, one can view joint search as a "costly" version of on-the-job search, even in the formal absence of it. The cost comes from the fact that in order to keep the search option active, the pair must remain a worker-searcher couple, and must not enjoy the full wage earnings of a dual-worker couple as it would be capable of doing in the presence of on-the-job search. Overall, relative to singles, couples spend more time searching for better jobs, which results in longer unemployment durations, but it eventually leads to higher lifetime wages and welfare.

We uncover two "equivalence results" between single-agent search and joint-search outcomes. The first environment requires the presence of on-the-job search with equal search effectiveness on and off the job. The second requires exponential (i.e., constant absolute risk aversion) preferences and loose borrowing limits. In both cases, a risk-averse couple acts like two separate single agents. These equivalence results follow directly from the value added of joint search in terms of climbing the wage ladder and of smoothing consumption, as discussed above. Finally, we also show an intuitive and useful result: the joint-search model is exactly isomorphic to a model where a single agent searches for jobs and she has the possibility of holding multiple jobs.

Our second environment features multiple locations and a flow cost of living apart for each of the spouses in the couple. The couple has to choose reservation functions with respect to "inside offers" (jobs in the current location) and "outside offers" (jobs in other locations). Even with risk-neutral preferences, the search behavior of couples differs from that of single agents in important ways. First, the dual-searcher couple is less choosy than the individual agent because it is effectively facing a worse job offer distribution, since some wage offer configurations are attainable only in different locations—hence, by paying the cost of living apart. Second, there is a region in which the breadwinner cycle is optimal for the couple. For example, a couple who gets a very generous job offer from the outside location could be better off if the currently employed spouse quits and follows the spouse with the job offer to the new location. It should be noted that we also obtain these two results—couples being less picky than singles and the breadwinner cycle—in our previous environment, but for completely different reasons.

The model allows us to formalize what Mincer (1978) called tied-stayers—i.e., workers who turn down a job offer in a different location that they would accept as single—and tied-movers—i.e., workers who accept a job offer in the location of the partner that they would turn down as single. Overall, the disutility of living separately effectively narrows down the job offers that are viable for couples, who end up choosing among a more limited set of job options.

The relevance of a multiple-location joint-search model of the labor market is supported, for example, by Costa and Kahn (2000) who document that highly educated dual-career couples have increasingly relocated to large metropolitan areas in the United States since the 1960s (more so than comparable singles): cities offer a greater and more diverse set of job opportunities, thereby mitigating the frictions associated with joint search. Finally, we also show that this multiple-location model, together with the assumption that women have higher job quit rates than men, can explain why men's unemployment duration is falling in the wage of their spouses whereas the opposite is true for women—a surprising finding that Lentz and Tranaes (2005) labelled the "gender asymmetry puzzle."

The set of propositions proved in the paper formalizes the new opportunities and the new frictions in terms of comparison between the reservation wage functions of the couple and the reservation wage of the single agent. We also provide some illustrative simulations to show that the deviations of joint-search behavior from its single-agent counterpart can be quantitatively substantial. For example, in the one location model with CRRA utility and a coefficient of relative risk aversion equal to four, joint-search generates a job quit rate of 2% per month—a clear sign of the breadwinner cycle in action—and each spouse in a couple earns a lifetime income that is 3% higher than a comparable single agent. In the multiple-location model with risk neutrality, when the disutility cost (of living separately) is equal to 15% of a dual-worker couples' average earnings, more than 50% of all households moving across locations involve a partner who is a tied-mover, and the lifetime income of each spouse in a couple is 6.5% lower than comparable singles.

Only very recently, a handful of papers have started to follow the lead offered by Burdett and Mortensen (1977) into the investigation of household interactions in frictional labor market models. Garcia-Perez and Rendon (2004) numerically simulate a model of family-based job-search decisions to tease out the importance of the added worker effect for consumption smoothing. Dey and Flinn (2008) study quantitatively the effects of health insurance coverage on employment dynamics in a search model where the economic unit is the household. Gemici (2008) estimates a rich structural model of migration and labor market decisions of couples to assess the implications of joint location constraints on labor outcomes and the marital stability of couples. Relative to these contributions, our paper is less ambitious in its quantitative analysis, but it provides a more focused and systematic study of joint-search theory.

From a theoretical perspective, our analysis of the one-location model has useful points of contact with existing results in search theory applied to at least three separate contexts. First, starting from the static analysis of Danforth (1979), a number of papers have studied the role of risk-free wealth in shaping dynamic job-search decisions (e.g., Andolfatto and Gomme, 1996; Greenwood, Gomes, and Rebelo, 2001; Pissarides, 2004; Lentz and Tranaes, 2005; Browning et al., 2007). The income of the spouse differs crucially from risk-free wealth because it is risky (in the presence of exogenous separations) and because it can be optimally controlled by the job-search decision itself. Second, Albrecht, Anderson, and Vroman (2009) study a different type of joint-search decision, that of a committee that votes on an option which gives some value to each member. The authors are interested in drawing a comparison between single-agent search and committee search, in the same spirit as our exercise.[2] Third, as we explain in the main text, there is an analogy with some search models of marriage formation where the flow value of the marriage is a concave function of the sum of the spouses' endowments (e.g., Visschers, 2006).

The rest of the paper is organized as follows. Section 2 describes the single-agent problem which provides the benchmark of comparison throughout the paper. Section 3 develops and fully characterizes the baseline joint-search problem. Section 4 extends this baseline model in a number of directions: nonparticipation, on-the-job search, exogenous separations, and access to borrowing. Section 5 studies an economy with multiple locations and a cost of living apart for the couple. Section 6 provides some illustrative simulations on both models. Section 7 concludes the paper and discusses possible extensions and applications. The Appendix contains the proofs of all our propositions.

2 The Single-Agent Search Problem

We begin by first presenting the sequential job-search problem of a single agent—the well-known McCall-Mortensen model (McCall, 1970; Mortensen, 1970). This model provides a useful benchmark against which we compare the joint-search model that we introduce in the next section. For clarity of exposition, we begin with a very stylized version of this optimal stopping problem and then consider several extensions in Section 4.

Economic environment. Consider an economy populated by single individuals who all participate in the labor force: agents are either employed or unemployed. Time is continuous and there is no aggregate uncertainty. Workers maximize the expected lifetime utility from consumption,

$$E_0 \int_0^{\infty} e^{-rt} u\left(c\left(t\right)\right) dt,$$

where r is the subjective rate of time preference, $c\left(t\right)$ is the instantaneous consumption flow at time t, and $u\left(\cdot\right)$ is the instantaneous utility function, strictly increasing, strictly concave, and smooth.

[2] The similarities, though, stop here more or less. For example, Albrecht, Anderson, and Vroman (2009) also find that committees are less picky than single agents. In our one-location model, this result is due to a consumption-smoothing argument. In their environment, it is due to the negative externality that committee members impose on each other (e.g., voting against when drawing a particularly low value).

An unemployed worker is entitled to an instantaneous benefit, b, and receives wage offers, w, at rate α from an exogenous wage offer distribution, $F(w)$ with support $[0, \infty)$. There is no recall of past wage offers. The worker observes the wage offer, w, and decides whether to accept or reject it. If she rejects the offer, she continues to be unemployed and to receive job offers. If she accepts the offer, she becomes employed at wage w forever, i.e., there are no exogenous separations and no new offers on the job. All individuals are identical in terms of their labor market prospects, i.e., they face the same wage offer distribution and the same arrival rate of offers, α. Finally, we assume that individuals have access to risk-free saving but are not allowed to borrow. As will become clear below, in the present framework individuals face a wage earnings profile that is nondecreasing over the life cycle (without exogenous separation risk), and, therefore, consumption smoothing only requires the ability to borrow but does not benefit from the ability to save. As a result, individuals will optimally set consumption equal to their wage earnings every period even though they are allowed to save.[3]

Value functions. Denote by V and W the value functions of an unemployed and employed agent, respectively. Then, using the continuous time Bellman equations, the problem of a single worker can be written in the following flow value representation:[4]

$$rV = u(b) + \alpha \int \max \{W(w) - V, 0\} \, dF(w), \tag{1}$$

$$rW(w) = u(w). \tag{2}$$

This well-known problem yields a unique reservation wage, w^*, for the unemployed such that for any wage offer above w^*, she accepts the offer and below w^*, she rejects the offer. Furthermore, this reservation wage can be obtained as the solution to the following equation

$$\begin{aligned} u(w^*) &= u(b) + \frac{\alpha}{r} \int_{w^*} [u(w) - u(w^*)] \, dF(w) \\ &= u(b) + \frac{\alpha}{r} \int_{w^*} u'(w) [1 - F(w)] \, dw, \end{aligned} \tag{3}$$

where the second equality comes from integration by parts. This equation shows that the instantaneous utility of accepting a job offer paying the reservation wage (left-hand side, LHS) is equated to the option flow value of continuing to search in the hope of obtaining a better offer in the future (right-hand side, RHS). Since the LHS is increasing in w^* whereas the RHS is a decreasing function of w^*, and they are both continuous, equation (3) uniquely determines the reservation wage, w^*.

3 The Joint-Search Problem

We now study the search problem of a couple facing the same economic environment described above. For the purposes of this paper, a couple is defined as an economic unit composed of two

[3] Nonparticipation, on-the-job search, exogenous job separation, and borrowing are introduced in Section 4.

[4] Below, when the limits of integration are not explicitly specified, they are understood to be those of the support of $F(w)$.

individuals who are ex ante identical in their preferences and in the labor market parameters they face. The two individuals perfectly pool income to purchase a market good which is jointly consumed by the couple. As in the single-search case, households simply consume their (total) income in each period. Couples make their job search decisions in order to maximize their common welfare.

A couple can be in one of three labor market states. First, if both spouses are unemployed and searching, they are referred to as a "dual-searcher couple." Second, if both spouses are employed (an absorbing state), we refer to them as a "dual-worker couple." Finally, if one spouse is employed and the other is unemployed, we refer to them as a "worker-searcher couple." As can perhaps be anticipated, the most interesting state is the last one.

Value functions. Let U denote the value function of a dual-searcher couple, $\Omega(w_1)$ the value function of a worker-searcher couple when the worker's wage is w_1, and $T(w_1, w_2)$ the value function of a dual-worker couple earning wages w_1 and w_2. The flow value in the three states becomes

$$rT(w_1, w_2) = u(w_1 + w_2), \tag{4}$$

$$rU = u(2b) + 2\alpha \int \max\{\Omega(w) - U, 0\}\, dF(w), \tag{5}$$

$$r\Omega(w_1) = u(w_1 + b) + \alpha \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2). \tag{6}$$

The equations determining the first two value functions (4) and (5) are straightforward analogs of their counterparts in the single-agent search problem. When both spouses are employed, their flow value is simply equal to the total instantaneous wage earnings of the household. When they are both unemployed, their flow value is equal to the instantaneous utility of consumption (which equals the total unemployment benefit) *plus* the expected gain in case a wage offer is received. Because both agents sample new offers at rate α, the total offer arrival rate of a dual-searcher couple is 2α.[5]

The value function of a worker-searcher couple is somewhat more involved. As can be seen in equation (6), upon receiving a wage offer (which now arrives at rate α, since only one spouse is unemployed) the couple faces three choices. First, the unemployed spouse can reject the offer, in which case there is no change in the value. Second, the unemployed spouse can accept the job offer and both spouses become employed, which increases the value by $T(w_1, w_2) - \Omega(w_1)$. Third, the unemployed spouse can accept the wage offer $w2$ *and* the employed spouse simultaneously quits his job and starts searching for a better one. In this case, the gain to the couple is $\Omega(w_2) - \Omega(w_1)$.

This latter case is the first important difference between the joint-search problem and the single-agent search problem. In the single-agent search problem, in a stationary environment, once a job offer is accepted, the worker will never choose to quit. In contrast, in the joint-search problem, the reservation wage of each spouse depends on the income of the partner. When this income grows—for example, because of a transition from unemployment to employment—the reservation wage of

[5]Because time is continuous, the probability of both spouses receiving offers simultaneously is negligible and is hence ignored.

the previously employed spouse may also increase, which could lead to exercising the quit option. Below, we return to this "endogenous nonstationarity" implicit in the joint-search problem.

3.1 Characterizing the Couple's Decisions

Before we begin characterizing the solution to the problem, we state the following useful lemma. We refer to Appendix A for all the proofs and derivations.

Lemma 1 $\Omega(w)$ *is a strictly increasing function of* w.

We are now ready to characterize the couple's search behavior. First, for a dual-searcher couple, the reservation wage—which is the same for both spouses by symmetry—is denoted by w^{**} and is determined by the equation

$$\Omega(w^{**}) = U. \tag{7}$$

Because U is a constant and Ω is a strictly increasing function (Lemma 1), w^{**} is a singleton.

A worker-searcher couple has two decisions to make. The first decision is whether to accept the job offer to the unemployed spouse (say, spouse 2) or not. The second decision, *conditional* on accepting, is whether the employed spouse (spouse 1) should quit his job or not. Let the current wage of the employed spouse be w_1 and denote the wage offer to the unemployed spouse by w_2.[6]

Accept/reject decision. Let us begin by supposing that it is not optimal to exercise the quit option upon acceptance, $\Omega(w_2) < T(w_1, w_2)$. In this case, a job offer with wage w_2 will be accepted when $T(w_1, w_2) \geq \Omega(w_1)$. Formally, the associated reservation wage function $\phi(w_1)$ solves

$$T(w_1, \phi(w_1)) = \Omega(w_1). \tag{8}$$

Now, in contrast, suppose that it is optimal to exercise the quit option upon acceptance, $\Omega(w_2) \geq T(w_1, w_2)$. Then, the job offer will be accepted when $\Omega(w_2) \geq \Omega(w_1)$. Since Ω is invertible, the associated reservation wage function solves

$$\phi(w_1) = w_1. \tag{9}$$

In this case, $\phi(w_1)$ coincides with the 45^0–line and the optimal rule is very simple: accept the new offer w_2 (and the other spouse will simultaneously quit) whenever it exceeds the current wage w_1. In sum, the worker-searcher reservation wage function $\phi(\cdot)$ is piecewise, being determined by (8) and (9) in different ranges of the domain for w_1. The kink of this piecewise function, which always lies on the 45^0–line in the (w_1, w_2) space, plays a special role in characterizing the behavior of the couple. We denote this point by $(\hat{w}, \hat{w})$, and it satisfies: $T(\hat{w}, \phi(\hat{w})) = \Omega(\hat{w}) = \Omega(\phi(\hat{w}))$. Since $rT(\hat{w}, \hat{w}) = u(2\hat{w})$, $\hat{w}$ solves

$$u(2\hat{w}) = r\Omega(\hat{w}). \tag{10}$$

[6] To better understand the optimal choices of the couple, it is instructive to treat the accept/reject decision of the unemployed spouse and the stay/quit decision of the employed spouse as two separate choices (albeit made simultaneously by the couple).

Stay/quit decision. A quit will never take place if the wage offer w_2 is rejected, as the pair would be worse off. Therefore, the stay/quit decision is nontrivial only when w_2 is accepted. The simplest way to understand this decision is to envision it as the accept/reject decision of the current wage w_1, conditional on retaining the job offer w_2. Formally, the associated indifference condition is $T(\phi(w_2), w_2) = \Omega(w_2)$. The use of the same ϕ function is not accidental: a comparison with equation (8) and the symmetry of T imply that the stay/quit decision is characterized by the same function ϕ defined by (8). In other words, in the (w_1, w_2) space, this decision rule is the mirror image of ϕ with respect to the 45-degree line. In light of this, $\hat{w}$ is the *highest* wage level at which the unemployed spouse of a worker-searcher couple is indifferent between accepting and rejecting an offer and, at the same time, her spouse is indifferent between keeping and quitting his job. To emphasize this feature, we refer to $\hat{w}$ as the "double indifference point."[7]

Taking stock. In light of what we established above, a dual searcher couple accepts any wage offer above w^{**}; a worker-searcher couple accepts any wage offer w_2 above $\phi(w_1)$ and chooses to quit whenever the current wage w_1 is below $\phi(w_2)$. Since a low wage w_1 on the current job makes quitting more attractive, it is immediate that the piece of the ϕ function corresponding to the 45^0 line is relevant in the range $w^{**} \leq w_1 < \hat{w}$, whereas the piece of the ϕ function defined by the indifference condition (8) will be relevant only in the range $w_1 \geq \hat{w}$. Overall, these different reservation rules divide the (w_1, w_2) space into four regions: one in which both spouses work, one where both spouses search, and the remaining two regions in which spouse 1 (2) searches and spouse 2 (1) works.

Characterizing the optimal strategy of the couple means the following: (i) studying the conditions under which $w^{**} < \hat{w}$, a necessary inequality to activate the reservation rule $\phi(w_1) = w_1$; (ii) analyzing the shape of the function ϕ beyond $\hat{w}$; and (iii) ranking $\hat{w}$ relative to the single-agent reservation wage w^*, which is useful for comparing single-agent search to joint-search strategies. Proposition 2 tackles (i). Proposition 3 tackles (ii) and (iii). With this characterization in hand, we offer an intuitive graphic representation of optimal joint-search in the (w_1, w_2) space.

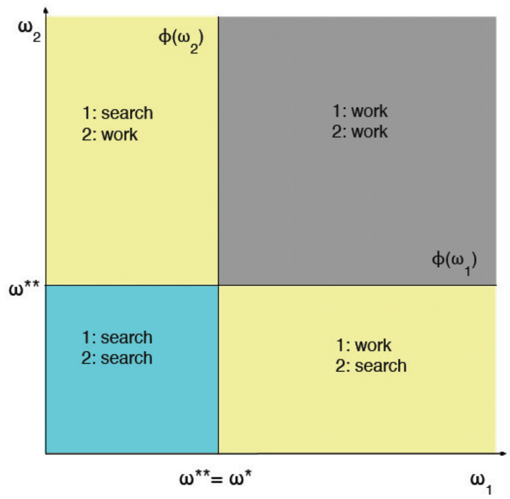
3.2 Risk Neutrality

To provide a benchmark, we begin by presenting the risk-neutral case, then turn to the results with risk-averse agents.

Proposition 1 [Risk Neutrality] *With risk-neutral preferences, i.e., $u'' = 0$, the joint-search problem of the couple reduces to two independent single-agent search problems for the two spouses,*

[7] For any wage above $\hat{w}$, either the employed spouse strictly prefers retaining his job or, if he quits, his unemployed spouse is always strictly better off accepting the job offer than remaining unemployed. So, there can be no longer indifference in both choices.

Figure 1: Reservation Wage Functions with Risk Neutrality



with value functions

$$T(w_1, w_2) = W(w_1) + W(w_2),$$
$$U = 2V,$$
$$\Omega(w_1) = V + W(w_1).$$

The reservation wage function $\phi(\cdot)$ of the worker-searcher couple is constant and is equal to the reservation wage value of a dual-searcher couple (regardless of the wage of the employed spouse), which, in turn, equals the reservation value in the single-agent search problem, i.e., $\phi(w_1) = w^{**} = \hat{w} = w^*$.

Figure 1 shows the relevant reservation wage functions in the (w_1, w_2) space. Throughout the paper, when we discuss worker-searcher couples, we will think of spouse 1 as the employed spouse and display his wage w_1 on the horizontal axis, and think of spouse 2 as the unemployed spouse and display her offer, w_2, on the vertical axis.

As stated in the proposition, the reservation wage function of a worker-searcher couple, $\phi(w_1)$ is simply the horizontal line at w^{**}. Similarly, the reservation wage for the quit decision is the mirror image of $\phi(w_1)$ and is shown by the vertical line at $w_1 = w^{**}$. The intersection of these two lines generates four regions, and the couple displays distinct behaviors in each. No wage below w^{**} is ever accepted by a dual-searcher couple in this model. Therefore, a worker-searcher couple will never be observed with a wage below w^{**}. If the unemployed spouse receives a wage offer $w_2 < w^{**}$, she rejects the offer and continues to search. If she receives an offer higher than w^{**}, she accepts the offer. At this point the employed partner retains his job, and the couple becomes a dual-worker couple.

For things to get interesting in this one-location model, risk aversion must be brought to the fore. In Section 5, we will also see that when the job-search process takes place in multiple locations and there is a cost of living separately for the couple, then even in the risk-neutral case there are important deviations from the single-agent search problem.

3.3 Risk Aversion

We start with a key implication of risk aversion summarized by the following proposition.

Proposition 2 [Breadwinner Cycle] *If u is concave, the reservation wage value of a dual-searcher couple is strictly smaller than the double-indifference point: $w^{**} < \hat{w}$.*

The fact that the reservation wage of a dual-searcher couple is strictly smaller than the double-indifference point activates a region where $\phi(w_1) = w_1$ which, in turn, gives rise to endogenous quits and to dynamics which we label the "breadwinner cycle." To understand how this happens, suppose that $w_1 \in (w^{**}, \hat{w})$ and the unemployed spouse receives a wage offer $w_2 \in (w_1, \hat{w})$. Because $w_2 > w_1 = \phi(w_1)$, the unemployed spouse accepts the offer and becomes employed. However, accepting this wage offer also implies $w_1 < \phi(w_2) = w_2$. Because the threshold for the first spouse to keep his job now exceeds his current wage, he will quit. As a result, spouses simultaneously switch roles and transit from a worker-searcher couple into another worker-searcher couple with a higher wage level. This process repeats itself over and over again—and the identity of the employed spouse (i.e., the breadwinner) alternates—until the employed spouse strictly prefers retaining his job and the pair becomes a dual-worker couple. We return to the breadwinner cycle below.

3.3.1 HARA utility

To obtain sharper predictions on the reservation wage function $\phi(w_1)$ beyond $\hat{w}$, we impose further structure on preferences. Specifically, we now restrict attention to concave preferences in the HARA (hyperbolic absolute risk aversion) class. This class encompasses several well-known utility functions as special cases. Formally, HARA preferences are defined as the family of utility functions that have linear risk tolerance: $-u'(c)/u''(c) = \rho + \tau c$, where ρ and τ are parameters.[8] This class can be further divided into three subclasses depending on the sign of τ. First, when $\tau \equiv 0$, then risk tolerance (and hence absolute risk aversion) is independent of consumption level. This case corresponds to constant absolute risk aversion (CARA) preferences, also known as exponential utility $u(c) = -\rho e^{-c/\rho}$. Second, if $\tau > 0$ then absolute risk tolerance is increasing—and therefore risk aversion is decreasing—with consumption, which is the decreasing absolute risk aversion (DARA) case. A well-known special case of this class is the constant relative risk aversion (CRRA) utility: $u(c) = c^{1-\sigma}/(1-\sigma)$, which obtains when $\rho = 0$ and $\tau = 1/\sigma > 0$. Finally, if $\tau < 0$ risk aversion increases with consumption, and this class is referred to as increasing absolute risk aversion (IARA). A special case of this class is quadratic utility: $u(c) = -(\rho - c)^2$, which obtains when $\tau = -1$.

[8] Risk tolerance is defined as the reciprocal of Pratt's measure of "absolute risk aversion." Thus, if risk tolerance is linear, risk aversion is hyperbolic.

Proposition 3 [HARA Utility] *With HARA preferences, for $w_1 \geq \hat{w}$:*

(*i*) *The reservation wage function satisfies*

$$\phi'(w_1) : \begin{cases} \in (0,1) & \text{if } u \text{ is DARA} \\ = 0 & \text{if } u \text{ is CARA} \\ < 0 & \text{if } u \text{ is IARA.} \end{cases}$$

(*ii*) *The double indifference point satisfies*

$$\hat{w} : \begin{cases} > w^* & \text{if } u \text{ is DARA} \\ = w^* & \text{if } u \text{ is CARA} \\ < w^* & \text{if } u \text{ is IARA.} \end{cases}$$

Before discussing the implications of the proposition, it is interesting to ask why it is the absolute risk aversion that determines the properties of joint-search behavior, as opposed to, for example, relative risk aversion. The reason is that individuals are drawing wage offers from the same probability distribution regardless of the current wage earnings of the couple. As a result, the uncertainty they face –determined by the dispersion in the wage offer distribution– is fixed, making the attitudes of a couple toward a fixed amount of risk—and therefore, absolute risk aversion—the relevant measure.[9]

While Appendix A contains a formal proof of this proposition, it is instructive to sketch the argument behind the proof here. To this end, begin by conjecturing that above a certain wage threshold it is never optimal to exercise the quit option.[10] In this wage range, equation (6) simplifies to

$$r\Omega(w_1) = u(w_1 + b) + \alpha \int_{\phi(w_1)} [T(w_1, w_2) - \Omega(w_1)] \, dF(w_2).$$

Substituting out Ω and T, using equations (4) and (8), shows that the reservation wage function for the unemployed spouse must satisfy

$$u(w_1 + \phi(w_1)) - u(w_1 + b) = \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] \, dF(w_2). \tag{11}$$

Divide both sides by the left-hand side:

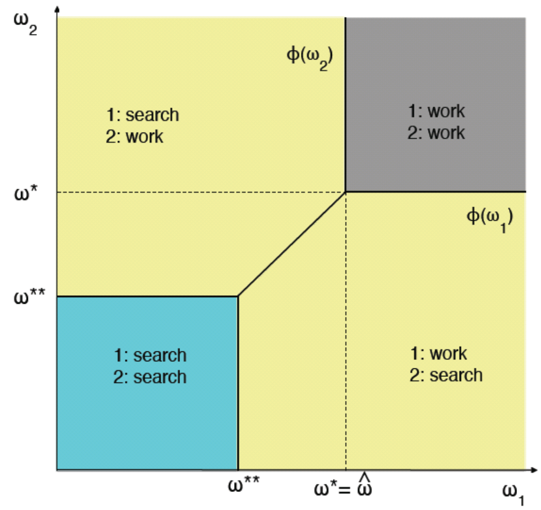
$$1 = \frac{\alpha}{r} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2). \tag{12}$$

Next, applying a well-known result on HARA preferences established by Pratt (1964, Theorem 1), it can easily be shown that the right-hand side of equation (12) is strictly increasing (decreasing) in w_1 under the DARA (IARA) specification, and independent of w_1 under the CARA specification. Also, note that the right-hand side of that equation is strictly decreasing in $\phi(w_1)$. Therefore, for the equality to hold in equation (12), $\phi(w_1)$ should be strictly increasing (decreasing) with DARA (IARA) preferences, and constant with CARA preferences.

[9]If, for example, individuals were to draw wage offers from a distribution that depended on the current wage of a couple, this would make relative risk aversion relevant as well.

[10]In the Appendix, we formally prove that this threshold is $\hat{w}$ for the CARA and IARA cases. For the DARA case, even though the reservation wage function ϕ turns out to be strictly increasing, a finite support for the wage offer distribution and the fact that $\phi' < 1$ will ensure that such a threshold always exists.

Figure 2: Reservation Wage Functions with CARA Preferences



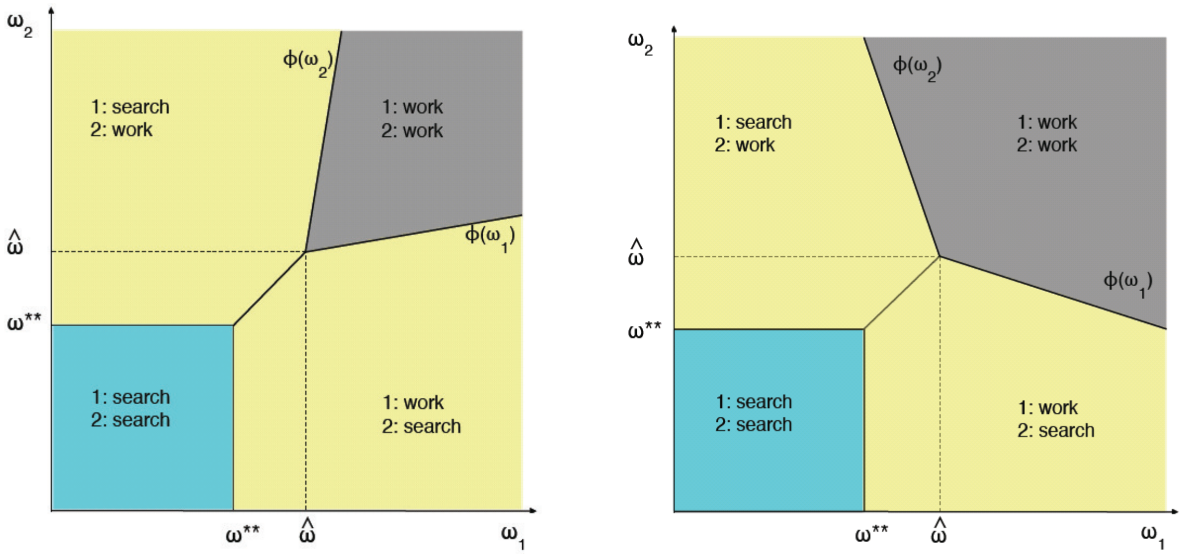
CARA case. Figure 2 provides a visual summary of the contents of this proposition for the CARA case in the wage space.

The reason why the ϕ function is constant and equal to the reservation wage value of the single agent is that, with CARA utility, attitude toward risk does not depend on the consumption (and hence wage) level. As the wage of the employed spouse increases, the couple's absolute risk aversion remains unaffected, implying a constant reservation wage for the unemployed partner.

Combining the results of Propositions 2 and 3, we conclude that the dual searcher couple is less choosy than the single agent $(w^{**} < w^{*})$. With risk aversion, the optimal search strategy involves a trade-off between lifetime income maximization and the desire for consumption smoothing. The former force pushes up the reservation wage, the second pulls it down as risk-averse agents particularly dislike the low income state (unemployment). The dual-searcher couple can use income pooling to its advantage: it initially accepts a lower wage offer (to smooth consumption across states) while, at the same time, not giving up completely the search option (to increase lifetime income) which remains available to the other spouse. In contrast, when the single agent accepts his job he gives up the search option for good, which induces him to be more picky at the start. Notice that joint search plays a role similar to on-the-job search in the absence of it, precisely through the breadwinner cycle.

DARA and IARA cases. Figure 3 illustrates graphically the reservation wage policies in these two cases. Under DARA (IARA) preferences, the reservation function of the worker-searcher couple is increasing (decreasing) with the wage of the employed spouse for wage levels higher than $\hat{w}$. This is because with decreasing (increasing) absolute risk aversion, a couple becomes less (more) concerned about smoothing consumption as household resources increase and, consequently, becomes more (less) picky in its job search.

Figure 3: Reservation Wage Functions for DARA (left) and IARA (right) Preferences



An important feature of DARA preferences —one that complicates the proof of Proposition 3— is that the breadwinner cycle is observed over a wider range of wage values of the employed spouse compared to the CARA and IARA cases. As can be seen in Figure 3 (left panel), ϕ is strictly increasing in w_1. As a result, even when $w_1 > \hat{w}$, a sufficiently high wage offer not only will be accepted by the unemployed spouse but will also trigger the employed spouse to quit.

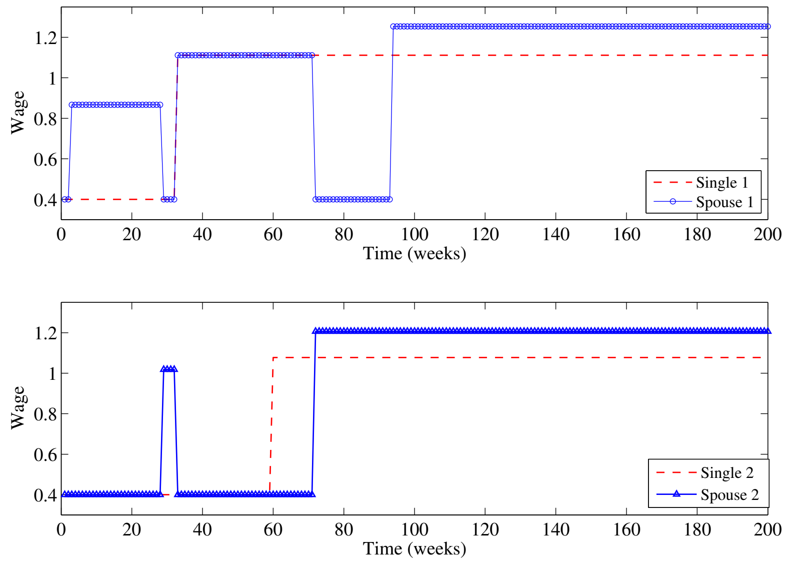
At first blush, it may seem surprising that in the DARA case one cannot rank w^{**} and w^{*} by combining Propositions 2 and 3. After all, the logic used to explain why $w^{**} < w^{*}$ in the CARA case is based on the relative strength of consumption smoothing and income maximization motives. But the argument is more subtle. To see why, consider the one-period gain when deciding whether to accept or reject an offer w. The couple compares $u(b + w)$ to $u(2b)$, whereas the single agent compares $u(w)$ to $u(b)$. The couple makes this comparison at a higher level of consumption and, because of DARA, the couple is less risk averse. This force tends to push w^{**} above w^{*} and does not allow a general ranking.[11] However, when b is very small, $w^{**} < w^{*}$ even in the DARA case, which allows us to state the following result.

Lemma 2 *Suppose u is DARA and $u(0) > -\infty$. Then, if $b = 0$, $w^{**} < w^{*}$.*

To provide a better sense of how the breadwinner cycle works, Figure 4 plots the simulated wage paths of a couple when spouses behave optimally under joint search (lines with markers) and for the same individuals when they act as two unrelated singles (dashed lines).[12] To make the comparison meaningful, the paths are generated using the same simulated sequence of job offers for each individual when he/she is single and when they act as a couple. First, the breadwinner cycle is seen clearly here as spouses alternate between who works and who searches depending on the offers

[11] We verified, by simulation, that with CRRA utility there are parameter configurations where $w^{**} > w^{*}$.
[12] Preferences are assumed to be of the CRRA (hence DARA) class.

Figure 4: Simulated Wage Paths for the Same Individuals as Couple and Singles



received by each spouse. Instead, when faced with the same job offer sequence, the same individuals simply accept a job (agent 1 in period 33 and agent 2 in period 60) and then never quit. Second, in period 29, agent 2 accepts a wage offer of 1.02 when she is part of a couple but rejects the same offer when acting as single, reflecting the fact that dual-searcher couples have a lower reservation wage than single agents. The opposite happens in period 60 when agent 2 accepts a job offer of 1.08 as single but turns it down when married, reflecting the fact that worker-searcher couples are more picky in accepting job offers than single agents. It is also easy to see that in the long run, the wages of both agents are higher under joint search—thanks to the breadwinner cycle, even though it may require a longer search process. Below we provide some illustrative simulations to show that, on average, joint search always yields a higher lifetime discounted value of income.

3.4 Consumption as a private good within the couple

Some goods consumed by the household have features of public goods (e.g., house), others of private goods (e.g., food). In the baseline model we have chosen the former specification. Suppose we take the extreme view that consumption, instead, is a fully private good within the couple, i.e. per-capita intra-period household utility is $u\left(\frac{y_1+y_2}{2}\right)$. One can easily adapt all the proofs and show that all the results stated so far are robust to this extension, the only exception being that, in the CARA case, $\hat{w} < w^*$.[13] Hence, our characterization of optimal joint-search behavior remains fundamentally valid, independently of the degree to which consumption is private within the household.[14]

3.5 An Isomorphism: Search with Multiple Job Holdings

The joint-search framework analyzed so far is isomorphic to a search model with a single agent who can hold multiple jobs at the same time. To see this, suppose that the time endowment of a worker can be divided into two subperiods (e.g., day shift and night shift). The single agent can be (i) unemployed and searching for his first job while enjoying $2b$ units of home production, (ii) working one job at wage w_1 while searching for a second one, or (iii) holding two jobs with wages w_1 and w_2. It is easy to see that the problem faced by this individual is exactly given by the equations (4), (5), and (6) and therefore it has the same solution as the joint-search problem.[15] Consequently, for example, when the agent works in one job and gets a second job offer with a sufficiently high wage offer, he will accept the offer and simultaneously quit the first job to search for a better one. Here, it is not the breadwinners who alternate, but the jobs that the individual works at.

4 Extensions

The baseline joint-search model analyzed so far was intended to provide the simplest deviation from the well-known single-search problem. Despite being highly stylized, this model illustrated some new and potentially important mechanisms that are not operational in the single-agent search problem. In this section, we enrich this basic model in four empirically relevant directions. First, we allow for nonparticipation. Second, we add on-the-job search. Third, we allow for exogenous job separations. Fourth, we allow households to borrow in financial markets.

4.1 Nonparticipation

We now extend the two-state model of the labor market we adopted so far to a three-state model where either spouse can choose nonparticipation. Nonparticipation means that the individual does not search for a job opportunity. Consistent with the rest of the paper, where we interpret b as income, we model the benefit associated to nonparticipation z (with $z > b$) in consumption units (e.g., through home production). We now redefine some of the value functions. First, consider the two configurations where (i) neither spouse participates in the labor force, and (ii) one spouse

[13]When consumption is a private good within the couple, equation (11) becomes

$$u\left(\frac{w_1 + \phi(w_1)}{2}\right) = u\left(\frac{w_1 + b}{2}\right) + \frac{\alpha}{r}\int_{\phi(w_1)}\left[u\left(\frac{w_1 + w_2}{2}\right) - u\left(\frac{w_1 + \phi(w_1)}{2}\right)\right] dF(w_2)$$

and, by exploiting the properties of CARA utility, the result follows immediately.

[14]Incidentally, it is also easy to see that even if consumption is a private good, in the DARA case it is not generally true that $w^{**} < w^*$ as long as $b > 0$.

[15]There is a further implicit assumption here: the arrival rate of job offers is proportional to the nonworking time of the agent (that is, 2α when unemployed and α when working one job).

does not participate and the other is employed at wage w. Because of the absence of randomness, both of these states are absorbing, as is the dual-worker state. Therefore, in the first case we have $rT(z,z) = u(2z)$, and in the second case we have $rT(z,w_2) = u(z+w_2)$. This formulation shows that nonparticipation is equivalent to a job opportunity which pays z (and entails foregoing search) and is always available to the worker.

The value of a dual-searcher couple becomes

$$rU = u(2b) + 2\alpha \int \max\{T(z,w) - U, \Omega(w) - U, 0\} \, dF(w), \tag{13}$$

which shows that upon either spouse finding an acceptable job, the other one has the choice of either continuing to search or dropping out of the labor force. The value when one spouse does not participate and the other is unemployed is

$$r\Omega(z) = u(z+b) + \alpha \int \max\{T(z,w_2) - \Omega(z), \Omega(w_2) - \Omega(z), 0\} \, dF(w_2), \tag{14}$$

which makes clear that when spouse 2 accepts a job offer, spouse 1 can either remain out of the labor force, or start searching. Similarly, the value of a worker-searcher couple where spouse 1 is employed is

$$r\Omega(w_1) = u(w_1+b) + \alpha \int \max\{T(w_1,w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} \, dF(w_2). \tag{15}$$

The choices available to the couple when spouse 2 finds an acceptable job offer are either spouse 1 remains employed at w_1 or spouse 1 quits into unemployment. This state will arise only for $w_1 > z$, since z is always available.[16] As clear from this equation, once the couple reaches this state, nonparticipation will never occur thereafter. This observation is important, since it means that our definitions of w^{**}, $\hat{w}$, and $\phi(w)$ remain unchanged and these functions are independent of z.

Proposition 4 [Joint Search with Nonparticipation] *With either CARA or DARA preferences, the search behavior of a couple can be characterized as follows:*

(i) If $z \leq w^{**}$, the search strategy of the couple is unaffected by nonparticipation, since the latter option is never optimal.

(ii) If $w^{**} < z < \hat{w}$, dual search is never optimal, and whenever a spouse is unemployed, the other is either employed or a nonparticipant. The reservation wage of a nonparticipant-searcher couple is z, and the reservation function of a worker-searcher couple is the same function $\phi(w)$ as in the absence of nonparticipation.

(iii) If $z \geq \hat{w}$, nonparticipation is an absorbing state for both spouses, and search is never optimal.

[16] More precisely, there is a third option in the max operator which is, theoretically, available to spouse 1: quitting into nonparticipation and accepting z forever with a gain $T(z,w_2) - \Omega(w_1)$ for the couple. However, the wage gain associated with spouse 1 keeping his/her current job, $T(w_1,w_2) - \Omega(w_1)$, must be larger, since previously spouse 1 has accepted w_1 when z was available.

Since nonparticipation is like a job offer at wage z that is always available, if $z < w^{**}$ such offer is never accepted by a dual-searcher couple, and nonparticipation is never optimal. When $w^{**} < z < \hat{w}$, then consumption-smoothing motives induce the jobless couple to move one of its members into nonparticipation—say, spouse 1—while spouse 2 is searching with reservation wage $\phi(z) = z$. As soon as a wage offer w_2 larger than z arrives, the unemployed spouse accepts the job and spouse 1 switches into unemployment, since search is equivalent to being employed at $\phi(w_2) \geq \hat{w} > z$. The first inequality follows from the CARA or DARA assumption under which ϕ is a nondecreasing function. If $z \geq \hat{w}$, instead, then both spouses exit the labor force right away and no search occurs. As soon as one chooses not to search, the other spouse reservation wage becomes $\phi(z)$, which is always smaller than z in this region. As a result, nonparticipation is attractive for the other spouse as well.[17]

The joint-search problem is, once again, different from the single-agent search problem. For example, in the CARA case where $\hat{w} = w^{*}$, we can establish that under configuration (ii), a single agent would be always searching and nonemployment would never arise, whereas a jobless couple would choose to move one spouse out of the labor force for consumption-smoothing purposes.

Finally, it is easy to show that the couple will never be in a state where one spouse works and the other is a nonparticipant: this state can only occur in the presence of wealth effects on labor supply, ruled out by our preferences, or in the presence of asymmetries between spouses. However, with IARA preferences, the worker-nonparticipant configuration may be optimal for the couple. Intuitively, since $\phi'(w) < 0$ (recall Figure 3), a wage offer $\tilde{w}$ could arrive—say, to a dual searcher couple—that is, high enough to induce the couple to accept the offer and set the new reservation wage for the unemployed member to $\phi(\tilde{w}) < z$. Thus, the unemployed member immediately exits the labor force.

4.2 On-the-Job Search

Suppose that agents can search both off and on the job. During unemployment, they draw a new wage from $F(w)$ at rate α_u, whereas during employment they sample new job offers from the same distribution F at rate α_e. What we develop below is, essentially, a version of the Burdett (1978) wage ladder model with couples. The flow value functions in this case are

$$rU = u(2b) + 2\alpha_u \int \max\{\Omega(w) - U, 0\}\, dF(w), \tag{16}$$

$$\begin{aligned} r\Omega(w_1) = u(w_1 + b) + \alpha_u \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2) \\ + \alpha_e \int \max\{\Omega(w_1') - \Omega(w_1), 0\}\, dF(w_1'), \end{aligned} \tag{17}$$

[17] In order to save space, we do not represent graphically this version of the model. For the CARA case, for example, it is immediate to see that one can generate the graph with nonparticipation corresponding to configuration (ii) by overlapping a squared area with coordinates $(x, y) = (z, z)$ to Figure 2. This area would substitute the dual-searcher couple with the nonparticipant-searcher couple.

$$rT\left(w_1, w_2\right) = u\left(w_1 + w_2\right) + \alpha_e \int \max\left\{T\left(w_1', w_2\right) - T\left(w_1, w_2\right), 0\right\} dF\left(w_1'\right) \qquad (18)$$
$$+ \alpha_e \int \max\left\{T\left(w_1, w_2'\right) - T\left(w_1, w_2\right), 0\right\} dF\left(w_2'\right).$$

We continue to denote the reservation wage of the dual-searcher couple as w^{**} and the reservation wage of the unemployed spouse in the worker-searcher couple as $\phi\left(w_1\right)$. We now have a new reservation function, that of the employed spouse (in the dual-worker couple and in the worker-searcher couple) which we denote by $\eta\left(w_i\right)$. It is intuitive (and can be proved easily) that under risk neutrality the joint-search problem coincides with the problem of the single agent regardless of offer arrival rates. Below, we prove another equivalence result that holds for any risk-averse utility function but for the special case of symmetric offer arrival rates $\alpha_u = \alpha_e$, i.e., when search is equally effective on and off the job.

Proposition 5 [On-the-job Search with Symmetric Arrival Rates] *If $\alpha_u = \alpha_e$, the joint-search problem yields the same solution as the single-agent search problem, even with concave preferences. Specifically, $w^{**} = w^* = b$, $\phi\left(w_1\right) = w^{**}$ and $\eta\left(w_i\right) = w_i$ for $i = 1, 2$.*

To understand this equivalence result, notice that one way to think about joint search is that it provides a way to climb the wage ladder for the couple even without on-the-job search: when a dual-searcher couple accepts the first job offer, it continues to receive offers, albeit at a reduced arrival rate. Therefore, one can view joint search as a "costly" version of on-the-job search. The cost comes from the fact that, absent on the job search, in order to keep the search option active, the pair must remain a worker-searcher couple and cannot enjoy the full wage earnings of a dual-worker couple as it would be capable of doing with on-the-job search. As a result, when on-the-job search is explicitly introduced and the offer arrival rate is equal across employment states, it completely neutralizes the benefits of joint search and makes the problem equivalent to that of a single agent. The solution is then simply that the unemployed partner should accept any offer above b and the spouse employed at w_1 any wage above its current one.

The preceding proposition provides an alternative benchmark to the baseline model, which had $\alpha_u > \alpha_e \equiv 0$. The empirically relevant case is probably in between these two benchmarks, in which case joint-search behavior continues to be qualitatively different from single search (for example, the breadwinner cycle will be active). We provide some simulations in Section 6.1 below to illustrate these intermediate cases.

4.3 Exogenous Separations

As discussed above, in the absence of exogenous separations, agents optimally choose not to accumulate assets, so a simple no-borrowing constraint ensures that agents live as hand-to-mouth consumers. This is no longer true when exogenous separation risk is introduced, because in this case accumulated assets can be used to smooth consumption when agents lose their jobs. This saving motive, however, significantly complicates the analysis. Thus, to establish some general theoretical results, we rule out savings in this section.

Suppose that jobs are exogenously lost at rate δ and that upon a job loss, workers enter unemployment. Once again, under risk neutrality it is easy to establish that the joint-search problem collapses to the single-agent problem. With risk aversion, however, this is not the case anymore. The modifications to the value functions are immediate, so we omit them. The following proposition states our main result for this framework.

Proposition 6 [CARA/DARA Utility with Exogenous Separations] *With CARA or DARA preferences, no access to financial markets, and exogenous job separation, the search behavior of a couple can be characterized as follows:*

(i) The reservation wage value of a dual-searcher couple satisfies: $w^{**} < \hat{w}$ *(with* $w^{*} < \hat{w}$*), which implies that the breadwinner cycle exists.*

(ii) The reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi(w_1) = w_1$, *and for* $w_1 \geq \hat{w}$, $\phi(w_1)$ *is strictly increasing with* $\phi' < 1$.

Two remarks are in order. First, for DARA preferences, the existence of exogenous separations has *qualitatively* no effect on joint-search behavior, as can be seen by comparing Propositions 3 and 6.[18] Second, and perhaps more interestingly, for CARA preferences $\phi(w_1)$ is no longer independent of the employed spouse's wage but is now increasing with it.[19] In the context of joint-search, the separation risk has two separate meanings. Consider the problem of the worker-searcher couple with current wage w_1 contemplating a new job offer with wage w_2. First, there is the risk associated with the duration of the new job offered to the searching spouse. Second, there is the risk of job loss for the currently employed spouse.[20]

The first effect of exogenous separations is also present in the single-agent search model: if the expected duration of a job is lower (high δ), the unemployed agent reduces her reservation wage for all values of w_1. However, the larger the wage w_1 of the employed spouse, the smaller this effect, since the utility value for the household of the additional wage decreases in w_1. Since $\phi(w_1)$ is weakly increasing in the case $\delta = 0$, with $\delta > 0$ we obtain $\phi'(w_1) > 0$.

[18] The only difference is that here we explicitly rule out saving, whereas previous propositions did not require this assumption, as explained before. Apart from this stronger assumption, the search behavior with DARA utility is the same with and without separations.

[19] Based on the proof of Proposition 6, it is possible to show that under CARA preferences if income during unemployment for each spouse is fully proportional to the wage earned in the last employment spell (an approximation to a UI system with replacement rate used, for example, by Postel-Vinay and Robin, 2002), then the reservation function ϕ is still constant, even with exogenous separations.

[20] In a model with spouse asymmetries in separation rates, this would be even more clear, since we would have a pair (δ_1, δ_2) in the value functions as opposed to just δ.

The second effect is related to the event that the currently employed spouse might lose his job. If the couple turns down the offer at hand and the job loss indeed occurs, its earnings will fall from $w_1 + b$ to $2b$ for a net change of $b - w_1 < 0$. Clearly, this income loss (and, therefore, the fall in consumption) is larger, the higher is the current wage of the employed spouse. If instead the couple accepts the job offer and spouse 1 loses his job, earnings will change from $w_1 + b$ to $b + w_2$, for a net change of $w_2 - w_1$. On the one hand, setting the reservation wage to $\phi(w_1) = w_1$ would completely insure the downside risk of spouse 1 losing his job (because then $w_2 - w_1 \geq 0$). At the same time, letting the reservation wage rise this quickly with w_1 reduces the probability of an acceptable offer and increases the probability that the searcher will still be unemployed when spouse 1 loses his job. As a result, the optimal policy balances these two considerations to provide the best self-insurance to the couple and, consequently, have $\phi(w_1)$ rise with w_1, but less than one for one: $\phi' < 1$.[21]

4.4 Borrowing in Financial Markets

With few exceptions, search models with risk-averse agents and a borrowing-saving decision do not allow analytical solutions.[22] One such exception is when preferences display CARA and agents have access to a risk-free asset. This environment has been recently used in previous work to obtain analytical results in the context of the single-agent search problem (e.g., Acemoglu and Shimer (1999), and Shimer and Werning (2008)). Following this tradition, we start from the CARA framework studied in Section 3.3.1, extended to allow for borrowing. Before analyzing the joint-search problem, it is useful to recall here the solution to the single-agent problem.

Single-agent search problem. Let a denote the asset position of the individual. Assets evolve according to the law of motion,

$$\frac{da}{dt} = ra + y - c, \tag{19}$$

where r is the risk-free interest rate, y is income (equal to w during employment and b during unemployment), and c is consumption. The value functions for the employed and unemployed single agent are, respectively:

$$rW(w,a) = \max_c \{u(c) + W_a(w,a)(ra + w - c)\}, \tag{20}$$

$$rV(a) = \max_c \{u(c) + V_a(a)(ra + b - c)\} + \alpha \int \max\{W(w,a) - V(a), 0\} \, dF(w), \tag{21}$$

where the subscript denotes the partial derivative. These equations reflect the non-stationarity due to the change in assets over time. For example, the second term in the RHS of (20) is $(dW/dt) = (dW/da) \cdot (da/dt)$. And similarly for the second term in the RHS of (21).

[21] This mechanism is closely related to Lise (2007), in which individuals climb the wage ladder but fall to the same unemployment benefit level upon layoff. As a result, in his model, the savings rate increases with the current wage level, whereas this increased precautionary savings demand manifests itself as delayed offer acceptance in our model.

[22] Some examples in which the decision maker is an individual are Costain (1999), Lentz and Tranaes (2005), Rendon (2006), Browning, Crossley, and Smith (2007), Lise (2007), Rudanko (2008), Krusell et al. (2009), and Lentz (2009).

We begin by conjecturing that $rW(w,a) = u(ra+w)$. If this is the case, then the first-order condition (FOC) determining optimal consumption for the agent gives $u'(c) = u(ra+w)$, which confirms the conjecture and establishes that the employed individual consumes his current wage plus the interest income on the risk-free asset. Let us now guess that $rV(a) = u(ra+w^*)$. Once again, it is easy to verify this guess through the FOC of the unemployed agent. Substituting this solution back into equation (21) and using the CARA assumption yields

$$w^* = b + \frac{\alpha}{r} \int_{w^*} \left[u(w-w^*) + \rho \right] dF(w), \tag{22}$$

which shows that w^* is the reservation wage and is independent of wealth. Therefore, the unemployed worker consumes the reservation wage plus the interest income on his wealth. This result highlights an important point: the asset position of an unemployed worker deteriorates and, in presence of a debt constraint, she may hit it. As in the rest of the papers cited above which use this setup, we abstract from this possibility. The implicit assumption is that borrowing constraints are "loose," and by this we mean they do not bind along the solution for the unemployed agent.

Joint-search problem. When the couple searches jointly for jobs, the asset position of the couple still evolves based on (19), but now $y = 2b$ for the dual-searcher couple, $b+w_1$ for the worker-searcher couple, and w_1+w_2 for the employed couple. The value functions become

$$rT(w_1,w_2,a) = \max_c \left\{ u(c) + T_a(w_1,w_2,a)(ra+w_1+w_2-c) \right\}, \tag{23}$$

$$rU(a) = \max_c \left\{ u(c) + U_a(a)(ra+2b-c) \right\} + \alpha \int \max\left\{ \Omega(w,a) - U(a), 0 \right\} dF(w), \tag{24}$$

$$\begin{aligned} r\Omega(w_1,a) = {} & \max_c \left\{ u(c) + \Omega_a(w_1,a)(ra+w_1+b-c) \right\} \\ & + \alpha \int \max\left\{ T(w_1,w_2,a) - \Omega(w_1,a), \Omega(w_2,a) - \Omega(w_1,a), 0 \right\} dF(w_2). \end{aligned} \tag{25}$$

Solving this problem requires characterizing the optimal consumption policy for the dual-searcher couple $c_u(a)$, for the worker-searcher couple $c_{eu}(w_1,a)$, and for the dual-worker couple $c_e(w_1,w_2,a)$, as well as the reservation wage functions, now potentially a function of wealth too, which must satisfy, as usual: $\Omega(w^{**}(a),a) = U(a)$, $T(w_1,\phi(w_1,a),a) = \Omega(w_1,a)$, and $\Omega(\phi(w_1),a) = \Omega(w_1,a)$. The following proposition characterizes the solution to this problem.

Proposition 7 [CARA Utility with Borrowing-Saving] *With CARA preferences, access to risk-free borrowing and lending, and "loose" debt constraints, the search behavior of a couple can be characterized as follows:*

(i) The optimal consumption policies are: $c_u(a) = ra + 2w^{**}$, $c_{eu}(w_1,a) = ra + w^{**} + w_1$, *and* $c_e(w_1,w_2,a) = ra + w_1 + w_2$.

(ii) The reservation function ϕ *of the worker-searcher couple is independent of* (w_1,a) *and equals* w^{**}, *so there is no breadwinner cycle.*

(iii) The reservation wage w^{**} of the dual-searcher couple equals w^*, the reservation wage of the single agent.

The main message of this proposition could perhaps be anticipated by the fact that borrowing effectively substitutes for the consumption smoothing provided within the household, making the latter redundant. Each spouse can implement search strategies that are independent from the other spouse's actions and, as a result, each acts as in the single-agent model. Of course, to the extent that borrowing constraints bind or preferences deviate from CARA, the equivalence result no longer applies.

5 Joint Search with Multiple Locations

The importance of the geographical dimension of job search is undeniable. For the single-agent search problem, accepting a job in a different market could require a relocation cost high enough to induce the agent to turn down the offer. In the joint-search problem, this spatial dimension introduces an additional and interesting search friction with important ramifications as we show in this section. Basically, a couple is likely to suffer from the disutility of living apart if spouses work in different locations. This cost can easily rival or exceed the physical cost of relocation, since it is a flow cost as opposed to the latter, which is arguably better thought of as a one-time cost.

To analyze the joint-search problem with multiple locations, we extend the framework proposed in Section 2 by introducing a fixed flow cost of living separately for a couple. The introduction of location choice leads to important changes in the search behavior of couples compared to a single agent, *even* with risk neutrality. To make this comparison sharper, we focus precisely on the risk-neutral case. Furthermore, many of these changes are not favorable to couples, which serves to show that joint search can itself create new frictions as opposed to the new opportunities analyzed in the first part of the paper.[23]

To keep the analysis tractable, we first consider agents that search for jobs in two symmetric locations and provide a theoretical characterization of the solution. In the next subsection, we examine the more general case with $L(> 2)$ locations that is more suitable for a meaningful calibration, and provide some results based on numerical simulations.

5.1 Two Locations

Environment. A couple is an economic unit composed of a pair of risk-neutral spouses $(1, 2)$. The economy has two locations. Couples incur a flow resource cost, denoted by κ, if the two spouses live apart. Denote by i the "inside" location, i.e., the location where the couple resides, and by o the

[23] This friction raises the issue of whether the couple should split. While the interaction between labor market frictions and changes in marital status is a fascinating question, it is beyond the scope of this paper. Here we assume that the couple has committed to stay together or, equivalently, that there is enough idiosyncratic non-monetary value in the marriage to justify continuing the relationship.

"outside" location. Unemployed individuals receive job offers at rate α_i from the current location and at rate α_o from the outside location, e.g., job search in the inside location is more effective with $\alpha_i > \alpha_o$. The two locations have the same wage offer distribution F. We assume away moving costs: the aim of the analysis is the comparison with the single-agent problem, and such costs would also be borne by the single agent.

A couple can be in one of four labor market states. First, if both spouses are unemployed and searching, they are referred to as a "dual-searcher couple." Second, if both spouses are employed in the same location (in which case they will stay in their jobs forever) we refer to them as a "dual-worker couple," but if they are employed in different locations we refer to them as a "separate dual-worker couple" (another absorbing state). Finally, if one spouse is employed and the other one is unemployed, we refer to them as a "worker-searcher couple." Because of symmetry in locations, couples with searchers have no advantage from living separately, so they will choose to live in the same location. Let $U, T(w_1, w_2), S(w_1, w_2)$, and $\Omega(w_1)$ be the value of these four states, respectively. Then, we have

$$rT(w_1, w_2) = w_1 + w_2 \tag{26}$$

$$rS(w_1, w_2) = w_1 + w_2 - \kappa \tag{27}$$

$$rU = 2b + 2(\alpha_i + \alpha_o) \int \max\{\Omega(w) - U, 0\} dF(w) \tag{28}$$

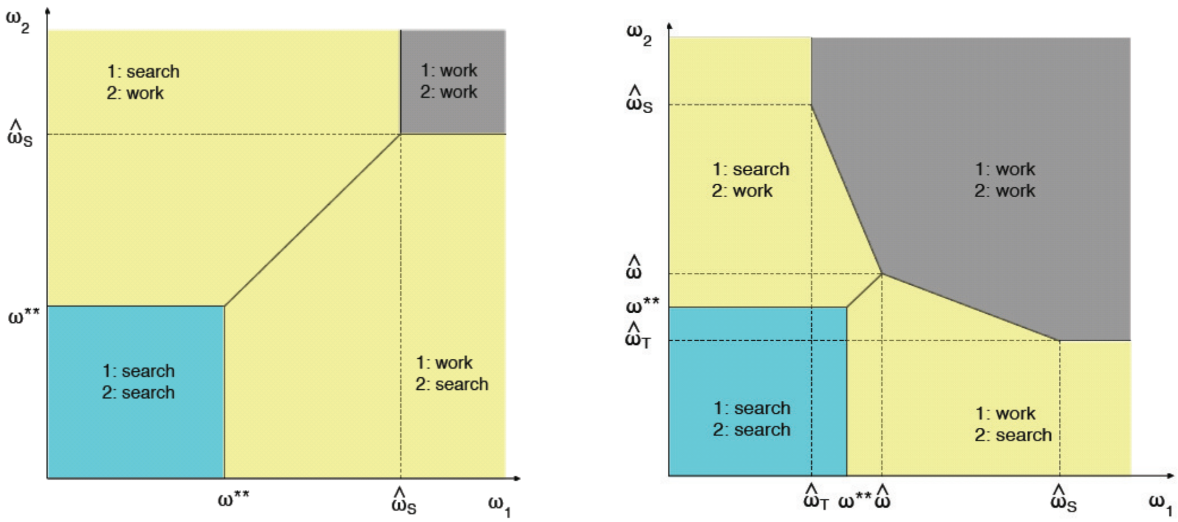
$$\begin{aligned} r\Omega(w_1) = w_1 + b + \alpha_i \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \\ + \alpha_o \int \max\{S(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2). \end{aligned} \tag{29}$$

The first three value functions are easily understood and do not require explanation. The value function for a worker-searcher couple now has to account separately for inside and outside offers. If an inside offer arrives, the choice is the same as in the one-location case, since no cost of living separately is incurred. If, however, an outside offer is received, the unemployed spouse may accept the job, in which case the couple has two options: either it chooses to live separately incurring cost κ, or the employed spouse quits and follows the newly employed spouse to the new location to avoid the cost.

The decision of the dual-searcher couple is entirely characterized by the reservation wage w^{**}. For the worker-searcher couple, let $\phi_i(w_1)$ and $\phi_o(w_1)$ be the reservation functions corresponding to inside and outside offers. Once again, these functions are piecewise with one piece corresponding to the 45-degree line. By inspecting equation (29), it is immediate that, as in the one-location case, the same functions $\phi_i(w_2)$ and $\phi_o(w_2)$ characterize the quitting decision.

Single-agent search. Before proceeding further, it is straightforward to see that the single-search problem with two locations is the same as the one-location case, with the appropriate modification to the reservation wage to account for separate arrival rates from two locations. In the risk-neutral

Figure 5: Reservation Wage Functions for Outside (Left) and Inside (Right) Offers



case, we have

$$w^* = b + \frac{\alpha_i + \alpha_o}{r} \int_{w^*} [1 - F(w)]\, dw. \tag{30}$$

Recall that in the one-location case, risk neutrality resulted in an equivalence between the single-search and joint-search problems. As the next proposition shows, this result no longer holds in the two-location case, whenever there is a positive cost κ of living apart.

Proposition 8 [Two-Location with Risk Neutrality] *With risk neutrality, two locations, and $\kappa > 0$, the search behavior of a couple can be characterized as follows. There is a wage value*

$$\hat{w}_S = b + \kappa + \frac{\alpha_i}{r} \int_{\hat{w}_S - \kappa} [1 - F(w)]\, dw + \frac{\alpha_o}{r} \int_{\hat{w}_S} [1 - F(w)]\, dw$$

and a corresponding value $\hat{w}_T = \hat{w}_S - \kappa$ such that:

(i) $w^{**} \in (\hat{w}_T, \hat{w})$, whereas $w^* \in (\hat{w}, \hat{w}_S)$. Therefore, $w^{**} < w^*$, which implies that the breadwinner cycle exists.

(ii) For outside offers, the reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}_S$, $\phi_o(w_1) = w_1$, and for $w_1 \geq \hat{w}_S$, $\phi_o(w_1) = \hat{w}_S$.

(iii) For inside offers, the reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi_i(w_1) = w_1$, for $w_1 \in (\hat{w}, \hat{w}_S)$, $\phi_i(w_1)$ is strictly decreasing, and for $w_1 \geq \hat{w}_S$, $\phi_i(w_1) = \hat{w}_T$.

The first useful result is that the dual-searcher couple is less choosy than the individual agent because it is effectively facing a worse job offer distribution: some wage offer configurations are attainable only in different locations, hence by paying the cost of living apart. Figure 5 graphically show the reservation wage functions for outside offers and inside offers, respectively. As seen in these figures, the reservation wage functions for both inside and outside offers are quite different from the corresponding ones of the model with one location (Figure 1). In particular, the reservation wage functions for both inside and outside offers now depend on the wage of the employed spouse at least when $w_1 \in (w^{**}, \hat{w}_S)$. This has several implications.

Consider first outside offers for a worker-searcher couple where one spouse is employed at $w_1 < \hat{w}_S$ (left panel). The couple will reject wage offers below w_1, but when faced with a wage offer above w_1, the employed worker will quit his job and follow his spouse to the outside location. The new wage offer is too high to be foregone, but the cost κ is too large to justify living apart while being employed at such wages. In this region, the breadwinner cycle is active "across locations." In contrast, when $w_1 > \hat{w}_S$ if the couple receives a wage offer $w_2 > \hat{w}_S$, it will bear the cost of living separately in order to maintain both high wages.

Comparing the right panel for inside offers to the left panel (outside offers), it is immediate that the range of wages for which inside offers are accepted by a worker-searcher couple is larger, since no cost κ has to be paid. Interestingly, the reservation function $\phi_i(w_1)$ now has three distinct pieces. For w_1 large enough, it is constant, as in the single-agent case. In the intermediate range $(\hat{w}, \hat{w}_S)$ the function is decreasing. This phenomenon is linked to the reservation function for outside offers ϕ_o, which is increasing in this range: as the wage w_1 from employment in the inside location rises, the expected gains from search accruing through outside offers are lower (it takes a higher outside wage offer w_2 to induce the employed spouse to quit) and the reservation wage for inside offers falls. For w_1 small enough, the reservation function $\phi_i(w_1)$ is increasing and equal to the wage of the employed spouse. In this region, the breadwinner cycle is again active. However, if the wage offer is high enough, the couple accepts it and retains its current wage becoming a dual-worker couple.

In this multiple location model, we obtained two results that were also present in our previous environment with one location and risk-aversion: (i) the couple being less picky than the individual, and (ii) the breadwinner cycle. As explained, the analogy stops here, since the economic intuition is completely different in the two models.

Tied-movers and tied-stayers. In a seminal paper, Mincer (1978) has studied empirically the job-related migration decisions of couples in the United States (during the 1960s and 1970s). Following the terminology introduced by Mincer, we refer to a spouse who rejects an outside offer that she would accept when single as a "tied-stayer." Similarly, we refer to a spouse who follows her spouse to the new destination even though her individual calculus dictates otherwise as a "tied-mover." Using data from the 1962 Bureau of Labor Statistics (BLS) survey of unemployed persons, Mincer estimated that "22 percent or two-thirds of the wives of moving families would be tied-movers, while 23 percent out of 70 percent of wives in families of stayers declared themselves to be tied-stayers" (page 758).[24]

[24] More precisely, Mincer (1978) defines an individual to be a tied-stayer (a tied-mover) if the individual cites his/her spouse's job as the main reason for turning down (accepting) a job from a different location: Mincer wrote (page 758):

Figure 6: Tied-Stayers and Tied-Movers in the Joint-Search Model

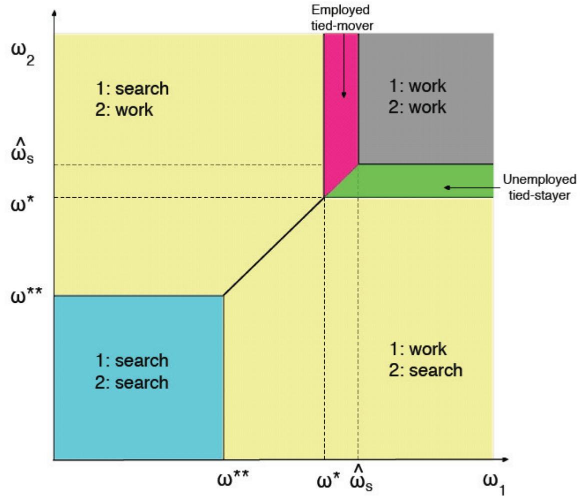


Figure 6 re-draws the reservation wage functions for outside offers and indicates the regions that give rise to tied-stayers and tied-movers in our model. First, if the wage of the employed spouse, w_1, is higher than w^*, then the unemployed spouse rejects outside offers and stays in the current location for all wage offers less than $\phi_i(w_1)$. In contrast, a single agent would accept all offers w_2 above w^*, which is less than $\phi_i(w_1)$ by Proposition 8. Therefore, an unemployed spouse who rejects an outside wage offer $w_2 \in (w^*, \phi_i(w_1))$ is formally a tied-stayer (as shown in Figure 6).

There is a region in which the *employed* spouse is a tied-mover. Suppose the wage of the employed spouse, w_1, is between w^* and $\hat{w}_S$, and the unemployed spouse receives an outside wage offer higher than w_1, then the unemployed spouse accepts the offer, the employed spouse quits the job, and both move to the other location. The employed spouse would not move to the other location if she were single, since she would not be searching any longer, so the employed spouse is a tied-mover (see Figure 6).

Both sets of choices involve potentially large concessions by each spouse compared to the situation where he/she were single, but they are optimal from a joint decision perspective. This feature opens the possibility of welfare costs of being in a couple versus being single with respect to job search, an aspect of the model which we analyze quantitatively, through simulation, in the next section.

Finally, we note that the isomorphism to the single-agent search model with multiple job holding extends to this set up as well. It is enough to think of κ as a commuting cost the agent would incur when holding two jobs in different locations.

"The unemployed were asked whether they would accept a job in another area comparable with the one they lost. A positive answer was given by 30 percent of the married men, 21 percent of the single women, and only 8 percent of the married women. Most people who said no cited family, home, and relatives as reasons for the reluctance to move. However, one quarter of the women singled out their husbands' job in the present area as the major deterrent factor."

6 Some Illustrative Simulations

In this section, our goal is to gain some sense about the quantitative differences in labor market outcomes between the single-agent search and the joint-search economy. We begin with the one-location model and then we turn to the multiple location model.

6.1 Single Location Model

Our benchmark is the case of CRRA utility (more common than CARA or IARA in macroeconomics) and exogenous separations. Later we add on-the-job search. The economy is characterized by the following set of parameters: $\{b, r, \rho, \delta, F, \alpha_u, \alpha_e\}$. When on-the-job search is not allowed, we simply set $\alpha_e = 0$ and $\alpha \equiv \alpha_u$.

We first simulate labor market histories for a large number of individuals acting as singles, then compute their optimal choices and some key statistics: reservation wage w^*, mean wage, unemployment rate, and unemployment duration. Second, we pair individuals together and treat them as couples solving the joint-search problem in exactly the same economy (i.e., same set of parameters). We use the same sequence of wage offers and separation shocks for each agent in both economies. The interest of the exercise lies in comparing the key labor market statistics across economies. For example, it is not obvious whether the joint-search model would have a higher or lower unemployment rate: for the dual-searcher couples, $w^{**} < w^*$, but for the worker-searcher couple $\phi(w)$ is above w^* at least for large enough wages of the employed spouse.

Calibration. We calibrate the model (with singles) to replicate some salient features of the US economy. The time period in the model is set to one week of calendar time. The coefficient of relative risk aversion ρ will vary from zero (risk neutrality) up to eight in simulations. The weekly net interest rate, r, is set equal to 0.001, corresponding to an annual interest rate of 5.3%. Wage offers are drawn from a log-normal distribution with standard deviation $\sigma = 0.1$ and mean $\mu = -\sigma^2/2$ so that the average wage is always normalized to one. We set $\delta = 0.0054$, which corresponds to a monthly employment-unemployment (exogenous) separation rate of 0.02. For each risk aversion value, the offer arrival rate, α_u, is recalibrated to generate an unemployment rate of roughly 0.055.[25] For the model with on-the-job search, we set the offer arrival rate on the job, α_e, to match a monthly employment-employment transition rate of 0.02. Finally, the value of leisure b is set to 0.40, i.e., 40% of the mean of the wage offer distribution.

Table 1 reports the results of our simulation. The first two columns confirm the statement in Proposition 1 that under risk neutrality the joint-search problem reduces to the single-search problem. Let us now consider the case with $\rho = 2$. The reservation wage of the dual-searcher couple is almost 25% lower than in the single-search economy. And this is reflected in the much

[25] As risk aversion goes up, w^{**} falls and unemployment duration decreases. So, to continue matching an unemployment rate of 5.5%, we need to decrease the value of α_u. For example, for $\rho = 0$, $\alpha_u = 0.4$ and for $\rho = 8$, $\alpha_u = 0.12$.

Table 1: Single versus Joint Search: CRRA Preferences

	$\rho = 0$		$\rho = 2$		$\rho = 4$		$\rho = 8$	
	Single	Joint	Single	Joint	Single	Joint	Single	Joint
Res. wage (w^* or w^{**})	1.02	1.02	0.98	0.75	0.81	0.58	0.60	0.48
Res. wage ($\phi(1)$)	–	n/a	–	1.03	–	0.94	–	0.84
Double ind. ($\hat{w}$)	–	1.02	–	1.02	–	0.94	–	0.82
Mean wage	1.06	1.06	1.07	1.10	1.01	1.05	1.001	1.01
Mm ratio	1.04	1.04	1.09	1.47	1.23	1.81	1.67	2.10
Unemp. rate	5.5%	5.5%	5.4%	7.6%	5.4%	7.7%	5.3%	5.6%
Unemp. duration	9.9	9.9	9.7	12.6	9.8	13.3	9.6	10
Dual-searcher	–	6	–	4.7	–	7.7	–	7.1
Worker-searcher	–	9.8	–	14.2	–	13.6	–	9.6
Quits/Separations	–	0%	–	11.1%	–	5.5%	–	0.7%
EQVAR- cons.	–	0%	–	4.5%	–	14%	–	26%
EQVAR- income	–	0%	–	1.1%	–	2.8%	–	0.7%

shorter unemployment durations of dual-searcher couples. At the same time, though, the reservation wage of worker-searcher couples is always higher than w^*. In the second row of the table, we report the reservation wage of the worker-searcher couple at the mean wage offer. Indeed, for these couples, unemployment duration is higher. Overall, this second effect dominates and the joint-search economy displays a longer average unemployment duration—12.6 weeks instead of 9.7—and a considerably higher unemployment rate, 7.6% instead of 5.4%.

Comparing the mean wage tells a similar story. The job-search choosiness of worker-searcher couples dominates the insurance motive of dual-searcher couples, and the average wage is higher in the joint-search model. The ability of the couple to climb higher up the wage ladder is reflected in the endogenous quit rate (leading to the breadwinner cycle), which is sizeable, 11.1% of all separations are quits. Indeed, the region in which the breadwinner cycle is active is rather big, as measured by the gap between w^{**} and $\hat{w}$, which is equal to 2.7 times the standard deviation of the wage offer.

The next four columns in Table 1 display how these statistics change as we increase the coefficient of relative risk aversion. As is clear from the first row, in the case when $\rho = 0$ the difference between w^* and w^{**} is zero. As ρ goes up, both reservation wages fall. Clearly, higher risk aversion implies a stronger demand for consumption smoothing, which makes agents accept job offers more quickly. However, the gap between w^* and w^{**} first grows but then shrinks. Indeed, as $\rho \to \infty$, it must be true that $w^* = w^{**} = b$, so the two economies converge again. As for $\phi(1)$, it falls as risk aversion increases, which means that for higher values of ρ, the worker-searcher couple accepts job offers more quickly, thus reducing unemployment. Indeed, at $\rho = 8$ the unemployment rate and the mean wage are almost the same in the two economies.

We also report a measure of frictional wage dispersion, the mean-min ratio (Mm), defined as the

Table 2: Single versus Joint Search: CRRA Preferences and On-the-Job Search

	$\rho = 0$		$\rho = 2$		$\rho = 2$		$\rho = 4$	
	$\alpha_u = 0.2$		$\alpha_u = 0.1$		$\alpha_u = 0.11$		$\alpha_u = 0.11$	
	$\alpha_e = 0.03$		$\alpha_e = 0.1$		$\alpha_e = 0.02$		$\alpha_e = 0.02$	
	Single	Joint	Single	Joint	Single	Joint	Single	Joint
Res. wage (w^* or w^{**})	0.98	0.98	0.40	0.40	0.78	0.67	0.62	0.54
Res. wage ($\phi(1)$)	–	0.98	–	0.40	–	0.85	–	0.74
Double ind. ($\hat{w}$)	–	0.98	–	0.40	–	0.87	–	0.80
Mean wage	1.13	1.13	1.16	1.16	1.08	1.09	1.08	1.09
Mm ratio	1.15	1.15	2.90	2.90	1.38	1.63	1.74	2.02
Unemp. rate	5.4%	5.4%	5.4%	5.4%	5.3%	5.8%	5.3%	5.4%
Unemp. duration	9.8	9.8	10.5	10.5	9.7	10.6	9.6	9.8
Dual-searcher	–	7.0	–	7.7	–	7.1	–	7.0
Worker-searcher	–	9.4	–	9.9	–	10.2	–	9.3
EU Quits/Separations	–	0%	–	0%	–	0.9%	–	0.2%
EQVAR-cons.	–	0%	–	4.6%	–	4.1%	–	15%
EQVAR-income	–	0%	–	0%	–	0.2%	–	0.1%

ratio between the mean wage and the lowest wage, i.e., the reservation wage. Hornstein, Krusell, and Violante (2009) demonstrate that the sequential search model with homogeneous workers, when plausibly calibrated, generates very little frictional wage dispersion. The fifth row of Table 1 confirms this result. It also confirms the finding in Hornstein et al. that the Mm ratio increases with risk aversion. What is novel here is that the joint-search model generates more frictional dispersion: the reservation wage for the dual-searcher couple is lower, but the couple can climb the wage distribution faster which translates into a higher average wage.

Next, we discuss two separate measures of the welfare effects of joint search in the simulated economy. Recall that the jointly searching couple has two advantages: first, it can smooth consumption better, and second, it can get higher earnings. The first measure of welfare gain is the standard consumption-equivalent variation and embeds both advantages. The second is the change in lifetime income from being married, which isolates the second aspect—the novel one.[26] The consumption-based measure of welfare gain is very large, not surprisingly. What is remarkable is that also the gains in terms of lifetime income can be very large—for example, around 2.8% for the case $\rho = 4$. As risk aversion goes up, the welfare gains from family insurance keep increasing, but as explained above, the ones stemming from better search opportunities fade away.

Table 2 presents the results when on-the-job search is introduced into this environment. The first four columns simply confirm the theoretical results established in previous sections. For example,

[26]To make the welfare comparison between singles and couples meaningful, we assume that each spouse consumes half of the household's income (as opposed to "all income" assumed in the theoretical analysis). Recall that, with DARA preferences, this alternative assumption does not affect any of our theoretical results.

when agents are risk neutral, on-the-job search has no additional effect, and both the single-agent and joint search problems yield the same solution regardless of parameter values. Similarly, as shown in Proposition 5 when on-the-job search is as effective as search during unemployment ($\alpha_e = \alpha_u$), then, again, single-agent and joint search coincide.

Overall, comparing these results to those in Table 1 shows that the effects of joint search on labor market outcomes are qualitatively the same as before, but they become much smaller quantitatively. This is perhaps not surprising in light of the discussion in Section 4, where we argued that joint search is a partial substitute for on-the-job search (or a costly version of it). Therefore, once on-the-job search is available, having a search partner is not so useful any longer to obtain higher earnings. But it obviously remains an effective means to smooth consumption, as evident from the last two lines of the table.

6.2 Multiple Location Model

The two-location case serves as a convenient benchmark to illustrate all the key mechanisms. For the simulation exercise, we extend the framework described above to multiple locations and allow exogenous separations. Specifically, consider an economy with L geographically separate symmetric labor markets. Firms in each location generate offers at flow rate ψ. A fraction θ of total offers are distributed equally to the $L-1$ outside locations and the remaining $(1-\theta)$ is made to the local market.[27] The value functions corresponding to this economy are provided in Appendix B and are straightforward extensions of the value functions in (26)–(29).

The number of locations, L, is set to nine representing the number of US census divisions and θ is set to $1 - 1/L$, implying that firms make offers to all locations with equal probability. The remaining parameter values are exactly the same as in the one-location model with risk neutrality (see Section 6.1).

Table 3 presents the simulation results. A comparison of the first two columns confirms that the single-agent and joint search problems are equivalent when there is no disutility from living apart ($\kappa = 0$). The third and fourth columns show the results when $\kappa = 0.1$ and $\kappa = 0.3$, respectively—representing a flow cost equal to 10% and 30% of the mean offered wage. First, the reservation wages are in line with our theoretical results in Proposition 8: $\hat{w}_T < w^{**} < w^* < \hat{w}_S$. Second, the presence of the cost κ makes outside offers less appealing, inducing the couple to reject some offers that a single person would accept. As a result, the unemployment rate is higher in the joint-search economy. For example, when $\kappa = 0.3$ the unemployment rate is 13.7% compared to 5.5% in the single-agent model. However, the average duration of unemployment is not necessarily longer under joint search: when $\kappa = 0.1$ the average duration falls to 9.8 weeks from 9.9 weeks in the single agent case, but rises to 13 weeks when κ is further raised to 0.3. The next two rows decompose

[27] The assumption that there is a very large number of individuals in each location, combined with the fact that the environment is stationary (i.e., no location-specific shocks), implies that we can take the number of workers in each location as constant, despite the fact that workers are free to move across locations and across employment states depending on the offers they receive.

Table 3: Single versus Joint Search: Nine Locations and Risk-Neutral Preferences

	$\kappa = 0$		$\kappa = 0.1$	$\kappa = 0.3$
	Single	Joint	Joint	Joint
Res. wage (w^* or w^{**})	1.02	1.02	0.97	0.94
$\hat{w}_T$	–	1.02	0.95	0.88
Double indiff. point ($\hat{w}$)	–	1.02	0.99	0.97
$\hat{w}_S$	–	1.02	1.04	1.13
Res. wage ($\phi_i(1)$)	–	n/a	0.98	0.95
Mean wage	1.058	1.058	1.06	1.045
Mm ratio	1.04	1.04	1.09	1.11
Unemployment rate	5.5%	5.5%	6.9%	13.7%
Unemployment duration	9.9	9.9	9.8	13.0
Dual-searcher	–	6.5	3.3	3.0
Worker-searcher	–	9.3	12.9	28.0
Movers (% of population)	0.5%	0.5%	0.7%	1.3%
Stayers (% of population)	1.1%	1.1%	1.5%	3.4%
Tied-movers/Movers	–	0%	29%	56%
Tied-stayers/Stayers	–	0%	11%	23%
Quits/Separations	–	0%	23%	50%
EQVAR-income	–	0%	−0.8%	−6.5%

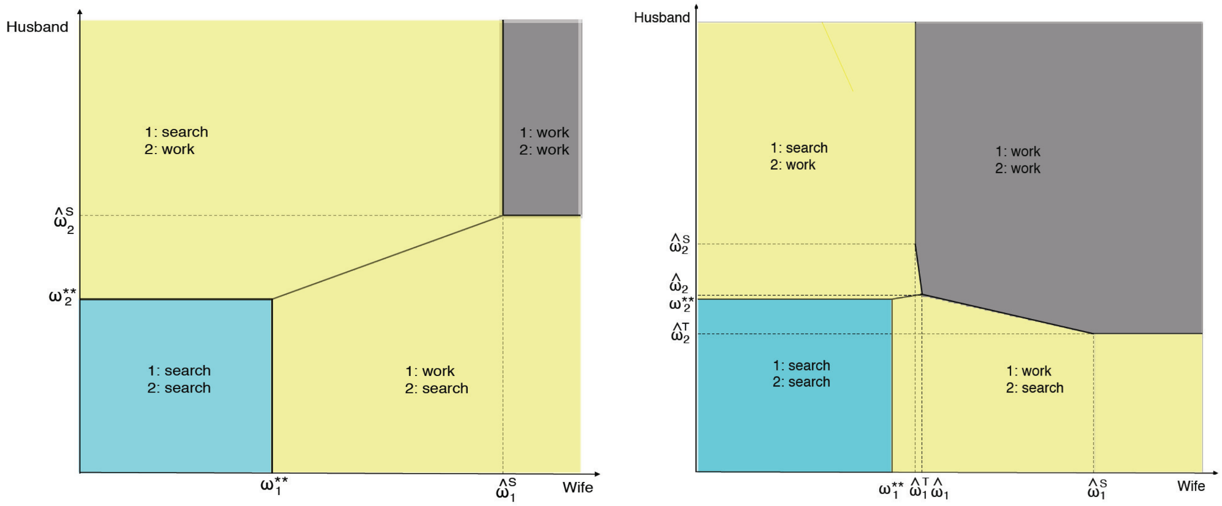
average unemployment duration into the component experienced by dual-searcher couples and by worker-searcher couples. The duration of the former group is shorter than that of single agents (since $w^{**} < w^*$) and gets even shorter as κ increases (falls from 6.5 weeks to 3 weeks in column 4). However, because worker-searcher couples face a smaller number of feasible job offers from outside locations, they have much longer unemployment spells: 12.9 weeks when $\kappa = 0.1$ and 28 weeks when $\kappa = 0.3$, compared to 9.3 weeks when $\kappa = 0$. Overall, there are more people who are unemployed at any point in time, and some of these unemployed workers—those in worker-searcher families—stay unemployed for much longer than they would have had they been single, while trying to resolve their joint-location problem.

We next turn to the impact of joint search on the migration decision of couples. In our context, we define to be "movers" dual-searcher and worker-searcher couples who move to another location because one of the spouses accepts an outside job offer.[28] Similarly, we define a couple to be a "stayer"if either member of the couple turns down an outside job offer.

Using this definition, the fraction of movers in the population is 0.52% per week when $\kappa = 0$; it rises to 0.74% when $\kappa = 0.1$ and to 1.26% when $\kappa = 0.3$. Part of the rise in the moving rate is mechanically related to the rise in the unemployment rate with κ: because there is no on-the-job

[28] However, consider a dual-worker couple in which spouses live in separate locations. If one of the spouses receives a separation shock and becomes unemployed, she will move to her spouse's location. In this case, the household is not considered to be a mover, since the move did not occur in order to accept a job.

Figure 7: Reservation Wage Functions for Outside (Left) and Inside (Right) Offers When a Wife Has a Higher Separation Rate than Her Husband



search, individuals only get job offers when they are unemployed, which in turn increases the number of individuals who accept offers and move. Notice also that while the fraction of movers appears high in all three cases, this is not surprising given that we are completely abstracting from the physical costs of moving. Perhaps more striking is the fact that almost 56% of all movers are tied-movers when $\kappa = 0.3$, using the definition in Mincer (1978) described above. The fraction of tied-stayers is also sizeable: 21% in the high-friction case. The fraction of employment-to-unemployment transitions that are due to voluntary quits is as high as 50% when $\kappa = 0.3$.

Finally, a comparison of lifetime wage incomes shows that the friction introduced by the spatial dimension in joint search can be substantial: it reduces the lifetime income of a couple by about 0.8% (per person) compared to a single agent when $\kappa = 0.1$ and by 6.5% when $\kappa = 0.3$. Overall, these results show that with multiple locations, joint-search behavior can deviate substantially from the standard single-agent search.

6.2.1 A Solution to the Lentz-Tranaes "Gender Asymmetry Puzzle"?

Lentz and Tranaes (2005) and Lentz (2009) have estimated empirically, from Danish data, how the unemployment duration of spouses in married couples depends on the earned income of their partners. One would expect a positive relationship. The data, instead, reveal a surprising "gender asymmetry": while the unemployment duration of the wife (and therefore, the couple's reservation wage) is *increasing* in the husband's wage, the unemployment duration of the husband is *decreasing* in the wife's wage.

In this section, we show that the joint-search framework with multiple locations is able to qualitatively replicate this pattern of the data to the extent that married women have a higher exogenous job separation rate than married men. The multiple location model has the potential of

Table 4: Estimates (S.D.) from Simulated Multiple Location Model

	Wife's Unemp. duration	Husband's Unemp. duration
Intercept	1.530	2.230
term	(0.069)	(0.045)
Coefficient on	0.186	−0.097
spouse's wage	(0.067)	(0.046)

generating reservation wages (and unemployment duration) declining in the spouse's income—recall Figure 5, right panel. Gender-specific differences in separation rates could arise due to unexpected shocks to household's home production needs (such as childrearing, etc.) that may require the wife to quit her job (more so than the husband), or to women being overrepresented in more volatile occupations or sectors (e.g., retail sales).

Figure 7 plots the reservation wage functions of a couple (for outside and inside offers) under this assumption of different separation rates. The left panel shows that, when the unemployed wife in a worker-searcher couple receives an offer from the outside location, except for a very small range of husband's wages, she either turns it down or she accepts it and the couple lives apart, as a dual-worker couple, until one of the two employment spells terminates. Instead, when the unemployed husband receives an outside offer, there is a large range of wife's wages where the wife chooses to quit her job and the couple moves to the new location as a worker-searcher couple. This asymmetry in behavior is induced by the larger separation rate for the wife. It is rarely the optimal choice for the husband to quit a high wage job to follow his wife on a precarious (even though temporarily high paying) job in a different location.

This asymmetry in the response to outside offers between husband and wife translates into different reservation functions for inside offers. A larger wage of the employed wife reduces the husband's value of search in outside locations since a higher wage offer is now needed to induce the wife to quit her job. This, in turn, makes the husband less picky towards inside offers. This force, which induces a negative relationship between husband's unemployment duration and his spouse's income, is not at work for the wife since her value of search in outside locations (i.e., the reservation wage with respect to outside offers) is roughly independent of the husband's wage.

To investigate this idea further, we simulate a multiple location economy inhabited by couples. The parameter values are the same as above, aside from the weekly separation rate for males and females (δ_1, δ_2) and the fraction of offers coming from the outside location (θ). We set $\delta_1 = 0.002$, $\delta_2 = 0.007$, and $\theta = 0.4$. The cost of living apart, κ, is set to 0.3. Lentz and Tranaes (2005) estimate the dependence of unemployment duration (d_i) of the jobless spouse (individual i) on a vector of control variables and on the wage (w_{-i}) of the employed spouse (individual $-i$) in worker-searcher couples. Abstracting from the exogenous control variables which play no role in our set up, they specify the statistical model $\log(d_i) = C_i + \beta_i w_{-i} + \varepsilon_i$, where C_i is a constant and ε_i is an orthogonal error term.[29] We simulated the labor market histories of couples with a total of 12,865 unemployment spells, the same number of observations in Lentz and Tranaes (2005). We ran the above regression separately on husband and wives who experience unemployment spells, excluding right-censored spells.

The results of the regression, reported in Table 4, show that the multiple location model with plausible asymmetries in separation rates can offer a resolution to the "gender asymmetry puzzle": husbands' unemployment duration depends negatively on wives' wages, whereas wives' unemployment duration depends positively on husband's wages. Both effects are statistically significant.

7 Conclusions

This paper characterizes theoretically the joint job-search behavior of couples in a variety of economic environments. So far, search theory has almost exclusively focused on the single-agent problem, ignoring the ramifications of joint search for labor market dynamics. Interestingly, the "equivalence" results demonstrated in the paper suggest that, in specific contexts, the predictions of joint search theory align well with those of single-agent search. For example, when borrowing limits are generous, when couples have large wealth, or when search on the job is almost as effective as during unemployment, then the optimal search strategies of couples can be very close to those of singles.

On the one hand, these results may justify, in those contexts, abstracting from within-household interactions in the study of labor markets. On the other hand, they provide a guide for future empirical work by identifying circumstances where the predictions of joint-search (e.g., the breadwinner cycle) should be sharper. In light of our results, we conjecture that deviations from single-agent search behavior in the data are more likely to be detectable among young and poor households, who are closer to hand-to-mouth consumers. Furthermore, empirically, one would expect the network of labor market contacts and opportunities (and hence the effectiveness of on-the-job search) to increase with skill level and with occupational experience. As a result, deviations from single-agent search should be more evident among inexperienced and uneducated couples.

As it is often the case in theoretical analyses, we had to strike a balance between generality and tractability to make sharp statements about optimal joint-search behavior. Structural empirical

[29] In our model, we can express the exit rate of person i when the spouse is employed at wage w_{-i} as

$$\lambda_i = (1-\theta)\,\psi\left[1 - F\left(\phi_T^i\left(w_{-i}\right)\right)\right] + \theta\psi\left[1 - F\left(\phi_S^i\left(w_{-i}\right)\right)\right].$$

Furthermore, if we assume F is a Weibull distribution, then the relation above becomes

$$\lambda_i = (1-\theta)\,\psi\exp\left(-\frac{\phi_T^i\left(w_{-i}\right)}{\eta}\right)^k + \theta\psi\exp\left(-\frac{\phi_S^i\left(w_{-i}\right)}{\eta}\right)^k,$$

where η and k are parameters of the distribution. This exit rate formula can be approximated as $\lambda_i \simeq A_i\exp\left(-\beta_i w_{-i}\right)$. Since expected duration is $\bar{d}_i = 1/\lambda_i$, we obtain the expression in the main text, with $C_i = -\log\left(A_i\right)$, which is the functional form used by Lentz and Tranaes in their estimation. Hence, up to this approximation, their specification is consistent with our joint-search model.

analysis of the data may require richer models. However, knowing the properties of the reservation wage functions in special cases (like ours) provides guidance towards the numerical solution and the interpretation of simulation-based results in these more complex joint-search environments. From a theoretical viewpoint, there are additional forces that could influence joint-search decisions in the labor market beyond those studied in this paper. Some examples include complementarity/substitutability of leisure between spouses (Burdett and Mortensen, 1977), or consumption-sharing rules within the family that deviate from full income pooling, as in the collective model (Chiappori, 1992), or the option given to the couple to split and break up the marriage (Aiyagari, Greenwood, and Guner, 2000), or fundamental asymmetries between men and women (something we started exploring with reference to the Lentz-Tranaes gender asymmetry puzzle). A search-based analysis of labor and marriage market dynamics with general preferences, asymmetric spouses, multiple locations, and more sophisticated models for within-household consumption distribution is an ambitious research project. The recent paper by Gemici (2008) represents a significant step in this direction. One key challenge in this research program is the access to micro data with household-level, high-frequency information on the detailed labor market histories of both members of the couple and on their geographical movements. A more feasible task is the structural estimation of a search model to understand patterns of multiple job holding, an environment that we showed to be isomorphic to joint search, under some assumptions. The survey data needed for such task are more readily available.

Combining such rich model with the right data would allow one to shed light on the quantitative importance of joint search for a host of empirical issues, not least the design of unemployment compensation and other policies where the key trade-off is between offering consumption insurance and providing work incentives. There is a growing literature on optimal unemployment insurance in search models (e.g., Acemoglu and Shimer, 1999; Lentz, 2009; Shimer and Werning 2008). Generous and long-lasting benefits are often advocated on the basis that workers are poorly insured against layoff risk and that short-lived benefits would induce the unemployed to accept jobs where they are mismatched, hence lowering productive efficiency in the economy. Joint search is both an additional channel of household consumption smoothing and a vehicle to select better jobs, and therefore explicitly recognizing that the job search process is often joint might limit the scope for copious or long-lived unemployment benefits. Finally, policies such as the U.S. Earned Income Tax Credit, or the British Working Families' Tax Credit, are typically evaluated within single-agent search models (e.g., Shephard, 2010). Future research should recognize explicitly that their impact on labor market outcomes depends crucially on how they affect the *joint* job search incentives of spouses within households targeted by the policy (e.g., Shephard, 2010).

A Proofs

Proof. [Lemma 1] Rewrite equation (6) using equation (4):

$$r\Omega(w) = u(w+b) + \alpha g(w), \tag{31}$$

where

$$g(w) \equiv \int \max\left\{\frac{u(w+w_2)}{r} - \Omega(w), \Omega(w_2) - \Omega(w), 0\right\} dF(w_2).$$

We construct the proof by contradiction. Let us assume $\Omega(w)$ is nonincreasing in w, then obviously $g(w)$ is nondecreasing in w. Since u is strictly increasing in w, the right-hand side of the equation (31) becomes strictly increasing in w, which results a contradiction. Hence, $\Omega(w)$ is strictly increasing. ■

Proof. [Proposition 1] From the definition of the worker-searcher reservation wage function, when the quit option is not exercised, ϕ has to satisfy equation (8). We conjecture that under risk neutrality this option is never exercised. This allows us to disregard the second term inside the max operator in (6). Substituting (6) and (4) into (8), and using the fact that workers are risk neutral, the equation characterizing $\phi(w_1)$ becomes

$$\phi(w_1) = b + \frac{\alpha}{r}\int_{\phi(w_1)} [w_2 - \phi(w_1)]\, dF(w_2).$$

It is clear that $\phi(w_1)$ does not depend on w_1, and the above equation is exactly equation (3) of the single-agent search problem. So, $\phi(w_1) = w^* = \hat{w}$. As a result, $\phi(w_2) = w^*$ as well, confirming the guess that the employed spouse never quits, since quits occur only if the current wage w_1 is below $\phi(w_2)$.

Now we establish that $w^{**} = w^*$. Equations (5) and (7), and simple integration by parts, yield[30]

$$r\Omega(w^{**}) = rU = 2b + \frac{2\alpha}{r}\int_{w^{**}} r\Omega'(w)[1 - F(w)]\, dw. \tag{32}$$

At $w_1 = w^*$, we can rewrite equation (6) in the following way:

$$r\Omega(w^*) = w^* + b + \frac{\alpha}{r}\int_{w^*} r\Omega'(w)[1 - F(w)]\, dw. \tag{33}$$

Subtracting (32) from (33) multiplied by 2 and using the fact that $r\Omega(w^*) = 2w^*$ yields

$$r[\Omega(w^*) - \Omega(w^{**})] = \frac{2\alpha}{r}\int_{w^*}^{w^{**}} r\Omega'(w)[1 - F(w)]\, dw.$$

Since Ω is strictly increasing, $w^* \leq w^{**}$ implies $\Omega(w^*) \leq \Omega(w^{**})$, but then the above equation in turn implies that $w^{**} = w^*$. ■

[30] It is straightforward to see that Ω is almost everywhere differentiable as long as u is strictly increasing and everywhere differentiable, as assumed. We will use the differentiability of Ω throughout the proofs.

Proof. [Proposition 2] Since $r\Omega(\hat{w}) = u(2\hat{w})$ and $r\Omega(w^{**}) = rU$, using (5) we obtain

$$r \ \left[\Omega(\hat{w}) - \Omega(w^{**})\right] = u(2\hat{w}) - u(2b) - 2\alpha \int_{w^{**}} \left[\Omega(w) - \ \Omega(w^{**})\right] dF(w). \tag{34}$$

At $w_1 = \hat{w}$, we can write equation (6) as

$$r\Omega(\hat{w}) = u(\hat{w} + b) + \alpha \int_{\hat{w}} \max\left\{T(\hat{w}, w) - \Omega(\hat{w}), \Omega(w) - \Omega(\hat{w})\right\} dF(w).$$

Multiplying the above equation by 2 and using equation (10), we arrive at

$$u(2\hat{w}) = 2u(\hat{w} + b) - u(2\hat{w}) + 2\alpha \int_{\hat{w}} \max\left\{T(\hat{w}, w) - \Omega(\hat{w}), \Omega(w) - \Omega(\hat{w})\right\} dF(w).$$

Substituting this expression for $u(2\hat{w})$ into the RHS of equation (34) delivers

$$\begin{aligned} r \ \left[\Omega(\hat{w}) - \Omega(w^{**})\right] &= 2u(\hat{w} + b) - u(2\hat{w}) - u(2b) \\ &+ 2\alpha \int_{\hat{w}} \max\left\{T(\hat{w}, w) - \Omega(\hat{w}), \Omega(w) - \Omega(\hat{w})\right\} dF(w) \\ &- 2\alpha \int_{w^{**}} \left[\Omega(w) - \Omega(w^{**})\right] dF(w) \end{aligned}$$

Now, by concavity of u, using Jensen's inequality we have $2u(\hat{w} + b) - u(2\hat{w}) - u(2b) > 0$. Suppose, ad absurdum, $w^{**} \geq \hat{w}$. Then, the RHS of the above equation is strictly positive, but the LHS is nonpositive, which is a contradiction. Therefore, $w^{**} < \hat{w}$. ■

Proof. [Proposition 3] To prove part (i), we start with the conjecture that for $w_1 \geq \hat{w}$, the employed spouse does not find optimal quitting his job for any wage offer w_2.[31] Using this conjecture in equation (6), substituting (6) into (8), and using $rT(w_1, w_2) = u(w_1 + w_2)$ we arrive at:

$$u(w_1 + \phi(w_1)) = u(w_1 + b) + \frac{\alpha}{r} \int_{\phi(w_1)} \left[u(w_1 + w_2) - u(w_1 + \phi(w_1))\right] dF(w_2).$$

Rearranging, we get

$$1 = \frac{\alpha}{r} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)}\right] dF(w_2) \tag{35}$$

We now study the shape of $\phi(w_1)$. Pratt (1964, Theorem 1) shows that if u is in the HARA family, for any $k > 0$ and m, n, p, q such that $p < q \leq m < n$, we have

$$f'(k) \begin{cases} > 0 & \text{if } u \text{ is DARA} \\ = 0 & \text{if } u \text{ is CARA} \\ < 0 & \text{if } u \text{ is IARA} \end{cases},$$

[31] The proof of this proposition focuses on the range $w_1 \geq \hat{w}$. We take as given that, below $\hat{w}$, the function ϕ is the 45^0 line, an intuitive fact. A complete, but also more cumbersome, proof is available upon request.

where

$$f(k) = \frac{u(n+k) - u(m+k)}{u(q+k) - u(p+k)}.$$

Let's set aside the DARA case for now. Setting $p = w_1 + b$, $q = m = w_1 + \phi(w_1)$, and $n = w_1 + w_2$, it is straightforward to see that the expression inside the integral in equation (35) is independent of w_1 in the CARA case and strictly decreasing in w_1 in the IARA case, for any $w_2 > \phi(w_1)$. Moreover, given that u is strictly increasing and the integral in (35) is positive, the RHS of the equation (35) is strictly decreasing in $\phi(w_1)$. Therefore, for the equality in equation (35) to hold, $\phi(w_1)$ must be independent of w_1 in the CARA case and strictly decreasing in w_1 in the IARA case.

We now verify our conjecture that the employed spouse does not quit when the unemployed accepts the offer. This conjecture is easy to check for CARA and IARA preferences. In these two cases, ϕ is nonincreasing for $w \geq \hat{w}$, implying $\phi(w_2) \leq \hat{w}$. But since a quit occurs only for $w_1 < \phi(w_2)$, and we are in the range $w_1 > \hat{w}$, quits never indeed occur.

We now turn to the DARA case. Let's conjecture that ϕ is strictly increasing in the range $w_1 > \hat{w}$, so the employed spouse may find it optimal to quit the job if the unemployed partner receives a sufficiently high wage offer. Then, for $w_1 \geq \hat{w}$, the equation characterizing $\phi(w_1)$ becomes

$$\begin{aligned} 1 = & \frac{\alpha}{r} \int_{\phi(w_1)}^{\phi^{-1}(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \\ & + \frac{\alpha}{r} \int_{\phi^{-1}(w_1)} \left[\frac{r\Omega(w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2), \end{aligned} \tag{36}$$

where, in writing the equation above, we are exploiting the inverse function ϕ^{-1} which is defined since ϕ is strictly increasing. Moreover, we have recognized explicitly that for $w_2 > \phi^{-1}(w_1)$ a quit may occur, and the couple stays a worker-searcher with roles reversed. This implies that in this range, $\Omega(w_2) > T(w_1, w_2) = u(w_1 + w_2)$, as the second row of the above equation shows.

For any w_1, we can find an $\varepsilon > 0$, sufficiently small, such that $\int_{\phi^{-1}(w_1)} r\Omega(w_2)\, dF(w_2) \geq \int_{\phi^{-1}(w_1)} u(w_1 + w_2 + \varepsilon)\, dF(w_2)$. Then, for such an $\varepsilon > 0$, using the DARA property of Pratt's theorem, we get

$$\begin{aligned} 1 \;=\; & \frac{\alpha}{r} \int_{\phi(w_1)}^{\phi^{-1}(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \\ & + \frac{\alpha}{r} \int_{\phi^{-1}(w_1)} \left[\frac{r\Omega(w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \\ < \; & \frac{\alpha}{r} \int_{\phi(w_1)}^{\phi^{-1}(w_1)} \left[\frac{u(w_1 + w_2 + \varepsilon) - u(w_1 + \varepsilon + \phi(w_1))}{u(w_1 + \phi(w_1) + \varepsilon) - u(w_1 + b + \varepsilon)} \right] dF(w_2) \\ & + \frac{\alpha}{r} \int_{\phi^{-1}(w_1)} \left[\frac{r\Omega(w_2) - u(w_1 + \phi(w_1) + \varepsilon)}{u(w_1 + \phi(w_1) + \varepsilon) - u(w_1 + b + \varepsilon)} \right] dF(w_2). \end{aligned}$$

Moreover, since

$$\begin{aligned}
1 = {} & \frac{\alpha}{r}\int_{\phi(w_1+\varepsilon)}^{\phi^{-1}(w_1+\varepsilon)}\left[\frac{u\left(w_1+w_2+\varepsilon\right)-u\left(w_1+\phi\left(w_1+\varepsilon\right)+\varepsilon\right)}{u\left(w_1+\varepsilon+\phi\left(w_1+\varepsilon\right)\right)-u\left(w_1+b+\varepsilon\right)}\right]dF\left(w_2\right) \\
& +\frac{\alpha}{r}\int_{\phi^{-1}(w_1+\varepsilon)}\left[\frac{r\Omega\left(w_2\right)-u\left(w_1+\phi\left(w_1+\varepsilon\right)+\varepsilon\right)}{u\left(w_1+\phi\left(w_1+\varepsilon\right)+\varepsilon\right)-u\left(w_1+b+\varepsilon\right)}\right]dF\left(w_2\right),
\end{aligned}$$

then $\phi(w_1) < \phi(w_1+\varepsilon)$ for $\varepsilon > 0$ sufficiently small, implying that $\phi\left(w_1\right)$ is strictly increasing in w_1.

We now prove that, in the DARA case, the slope of ϕ is strictly less than one. Let us assume $\phi' > 1$. This means that for $w_1 > \hat{w}$, $\phi\left(w_1\right) > \phi^{-1}\left(w_1\right)$. For any $w_1 > \hat{w}$, if the wage offer $w_2 > \phi\left(w_1\right)$, the unemployed accepts the offer, or $T\left(w_1, w_2\right) > \Omega\left(w_1\right)$. But since $w_2 > \phi\left(w_1\right) > \phi^{-1}\left(w_1\right)$, the employed quits the job at the same time, or $\Omega\left(w_2\right) > T\left(w_1, w_2\right) > \Omega\left(w_1\right)$. With the same logic, one can see that if $w_2 \in \left(w_1, \phi\left(w_1\right)\right)$, we get $\Omega\left(w_2\right) > \Omega\left(w_1\right) > T\left(w_1, w_2\right)$. If $w_2 \in \left(\phi^{-1}\left(w_1\right), w_1\right)$, we have $\Omega\left(w_1\right) > \Omega\left(w_2\right) > T\left(w_1, w_2\right)$ and if $w_2 < \phi^{-1}\left(w_1\right)\left(w_1\right)$, we have $\Omega\left(w_1\right) > T\left(w_1, w_2\right) > \Omega\left(w_2\right)$. Hence, if $w_2 > w_1$, then the unemployed accepts the job and the employed quits the job, forcing the reservation wage to be w_1. Hence $\phi\left(w_1\right) = w_1$, resulting in $\phi' = 1$, a contradiction.

We now prove part (ii), i.e., the relation between $\hat{w}$ and w^*. Consider the CARA case first. If u belongs to the CARA family, then $u\left(c_1+c_2\right) = -u\left(c_1\right)u\left(c_2\right)/\rho$. Using this property, we can write equation (35) as:

$$\begin{aligned}
1 &= \frac{\alpha}{r}\int_{\phi(w_1)}\left[\frac{u\left(w_2\right)-u\left(\phi\left(w_1\right)\right)}{u\left(\phi\left(w_1\right)\right)-u\left(b\right)}\right]dF\left(w_2\right), \\
u\left(\phi\left(w_1\right)\right) &= u\left(b\right)+\frac{\alpha}{r}\int_{\phi(w_1)}\left[u\left(w_2\right)-u\left(\phi\left(w_1\right)\right)\right]dF\left(w_2\right),
\end{aligned}$$

which is exactly equation (3) characterizing the reservation wage of the single agent. Since $\phi\left(w_1\right)$ is constant, $\hat{w} = w^*$.

We now turn to the DARA case. Equation (6) evaluated at $\hat{w}$ can be written as

$$r\Omega\left(\hat{w}\right) = u\left(\hat{w}+b\right)+\alpha\int_{\hat{w}}\left[\Omega\left(w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w\right),$$

because at wage $\hat{w}$ the employed spouse quits his job whenever the unemployed spouse accepts her job offer, by virtue of the fact that ϕ is strictly increasing, as shown above. Since $r\Omega\left(\hat{w}\right) = u\left(2\hat{w}\right)$, we can rewrite the above equation as

$$\begin{aligned}
u\left(2\hat{w}\right)-u\left(\hat{w}+b\right) &= \frac{\alpha}{r}\int_{\hat{w}}\left[r\Omega\left(w\right)-u\left(2\hat{w}\right)\right]dF\left(w\right) \\
&> \frac{\alpha}{r}\int_{\hat{w}}\left[rT\left(\hat{w}, w\right)-u\left(2\hat{w}\right)\right]dF\left(w\right) \\
&= \frac{\alpha}{r}\int_{\hat{w}}\left[u\left(\hat{w}+w\right)-u\left(\hat{w}+\hat{w}\right)\right]dF\left(w\right).
\end{aligned}$$

Rearrange the above equation to get

$$
\begin{aligned}
1 &> \frac{\alpha}{r}\int_{\hat{w}}\left[\frac{u(\hat{w}+w)-u(\hat{w}+\hat{w})}{u(\hat{w}+\hat{w})-u(\hat{w}+b)}\right]dF(w) \\
&> \frac{\alpha}{r}\int_{\hat{w}}\left[\frac{u(w)-u(\hat{w})}{u(\hat{w})-u(b)}\right]dF(w),
\end{aligned}
$$

where the second inequality uses the property of DARA utility. We know from equation (3) that

$$
1=\frac{\alpha}{r}\int_{w^*}\left[\frac{u(w)-u(w^*)}{u(w^*)-u(b)}\right]dF(w),
$$

and since its RHS is a strictly decreasing function of w^*, it is easy to see that $w^* < \hat{w}$.

Finally, we turn to the IARA case. In this case, we can write equation (6) evaluated at $\hat{w}$ as

$$
\begin{aligned}
r\Omega(\hat{w})-u(\hat{w}+b) &= \frac{\alpha}{r}\int_{\hat{w}}\left[rT(\hat{w},w)-u(2\hat{w})\right]dF(w). \\
&= \frac{\alpha}{r}\int_{\hat{w}}\left[u(\hat{w}+w)-u(\hat{w}+\hat{w})\right]dF(w).
\end{aligned}
$$

because at wage $\hat{w}$ the employed spouse does not quit his job whenever the unemployed spouse accepts her job offer, by virtue of the fact that ϕ is strictly decreasing, as shown above. Rearranging the equation above and comparing it to the single agent reservation wage equation yields

$$
\begin{aligned}
\frac{\alpha}{r}\int_{w^*}\left[\frac{u(w)-u(w^*)}{u(w^*)-u(b)}\right]dF(w)=1 &= \frac{\alpha}{r}\int_{\hat{w}}\left[\frac{u(\hat{w}+w)-u(\hat{w}+\hat{w})}{u(\hat{w}+\hat{w})-u(\hat{w}+b)}\right]dF(w) \\
&< \frac{\alpha}{r}\int_{\hat{w}}\left[\frac{u(w)-u(\hat{w})}{u(\hat{w})-u(b)}\right]dF(w)
\end{aligned}
$$

where the inequality in the second line descends from the IARA property. Therefore, $\hat{w} < w^*$. This concludes the proof.

■

Proof. [Lemma 2] The reservation wage for the dual-searcher couples, w^{**}, is characterized by equation (7):

$$
\Omega(w^{**})=U.
$$

Using equations (5) and (6) and the properties of the reservation wage function for the worker-searcher couple in the DARA case summarized in Proposition 3, we can rewrite the above equation as

$$
u(w^{**}+b)-u(2b)=\alpha\int_{w^{**}}\Omega'(w)\left[1-F(w)\right]dw. \tag{37}
$$

Similarly, in the single-search problem, the reservation wage is characterized by the following equation:

$$
u(w^*)-u(b)=\alpha\int_{w^*}\frac{u'(w)}{r}\left[1-F(w)\right]dw. \tag{38}
$$

Once we set $b=0$, showing $\Omega'(w) < \frac{u'(w)}{r}$ will be sufficient to prove $w^{**} < w^*$.

In the DARA case, Ω is a piecewise function characterized by the following equations

$$r\Omega(w) = u(w+b) + \begin{cases} \alpha \int_w \Omega'(w')[1-F(w')]\,dw' & \text{if } w < \hat{w} \\ \alpha \int_{\phi(w)}^{\phi^{-1}(w)} [T(w,w') - \Omega(w)]\,dF(w') + \alpha \int_{\phi^{-1}(w)} [\Omega(w') - \Omega(w)]\,dF(w') & \text{if } w \geq \hat{w} \end{cases}$$

For $w < \hat{w}$, it is immediate to show that $\Omega'(w) < \frac{u'(w)}{r}$ once we set $b = 0$. For $w \geq \hat{w}$, taking the derivative of both sides yields

$$r\Omega'(w) = u'(w+b) + \alpha \int_{\phi(w)}^{\phi^{-1}(w)} \left[T_1\left(w,w'\right) - \Omega'(w)\right] dF\left(w'\right) - \alpha \int_{\phi^{-1}(w)} \Omega'(w)\,dF\left(w'\right)$$

where we used the fact that $T(w,\phi(w)) = \Omega(w)$, and $T\left(w,\phi^{-1}(w)\right) = \Omega\left(\phi^{-1}(w)\right)$ to cancel out some terms. Collecting terms in $\Omega'(w)$, we arrive at:

$$\Omega'(w) = \frac{u'(w+b) + \frac{\alpha}{r}\int_{\phi(w)}^{\phi^{-1}(w)} u'(w+w')\,dF(w')}{r + \alpha[1-F(\phi(w))]}.$$

Evaluating the above expression at $b = 0$, and using $u'(w+w') < u'(w)$ by the strict concavity of u, it is immediate to we prove that also in this wage range $\Omega'(w) < \frac{u'(w)}{r}$ which concludes the proof. ■

Proof. [Proposition 4] There are three cases to consider.

(i) Consider a dual-searcher couple. Recall that by definition of w^{**}, $U = \Omega(w^{**}) > T(w^{**},w^{**}) > T(z+w^{**}) > T(z+w)$ for all $w < w^{**}$. Hence, no wage offer below w^{**} is accepted by the searching couple, since dual search always dominates. For wage offers above w^{**}, $T(z,w) < T(w^{**},w) < \Omega(w)$ since under CARA or DARA ϕ is a nondecreasing function. Therefore, a dual-searcher couple that samples an offer above w^{**} becomes a worker-searcher couple. Simple inspection of equation (15) shows that the worker-searcher couple will never transit through nonparticipation. It remains to be proved that being a dual nonparticipant couple is also dominated. This is straightforward, since $U = \Omega(w^{**}) > T(w^{**},w^{**}) > T(z,z)$. Dual search dominates dual nonparticipation. Hence, nonparticipation never occurs.

(ii) Since $U = \Omega(w^{**}) < \Omega(z)$, search-nonparticipation is always preferred to dual search. Since we are in the range $z < \hat{w}$, where quitting is optimal, we know that $\phi(z) = z$. As soon as the searcher receives a job offer higher w than z, she becomes employed and the couple becomes a worker-searcher couple. From that point onward, the dynamics are as in the baseline model.

(iii) Under this configuration, $U = \Omega(w^{**}) < \Omega(\hat{w}) < \Omega(z)$, which proves that search- nonparticipation is always preferred to dual search. However, we can write $\Omega(z) = T(z,\phi(z)) \leq T(z,z)$, since above $\hat{w}$ we have $\phi(z) \leq z$. Thus, both members enter the nonparticipation pool, which is an absorbing state. ■

Proof. [Proposition 5] Let us conjecture that $\phi(w_1) = w^{**}$ for any value of w_1, i.e., $T(w^{**},w_2) = \Omega(w_2)$. This implies that the quit option is never exercised, since any observed w_1 will be greater than or equal to w^{**}. So, one can disregard the second argument in the max operator in (17). Evaluating (17) at w^{**} yields

$$r\Omega\left(w^{**}\right) = u\left(w^{**} + b\right) + 2\alpha_u \int \max\left\{\Omega\left(w\right) - \Omega\left(w^{**}\right), 0\right\} dF\left(w\right),$$

where we have used the fact that $\alpha_e = \alpha_u$ and the conjecture. Since $\Omega\left(w^{**}\right) = U$, comparing the above equation to (16) yields that $w^{**} = b$. We now verify our conjecture. From (18) evaluated at $w_2 = w^{**}$:

$$\begin{aligned} rT\left(w_1, w^{**}\right) &= u\left(w_1 + b\right) + \alpha_e \int \max\left\{T\left(w_1', w^{**}\right) - T\left(w_1, w^{**}\right), 0\right\} dF\left(w_1'\right) \\ &+ \alpha_u \int \max\left\{T\left(w_1, w_2'\right) - T\left(w_1, w^{**}\right), 0\right\} dF\left(w_2'\right) \\ &= u\left(w_1 + b\right) + \alpha_e \int \max\left\{\Omega\left(w_1'\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_1'\right) \\ &+ \alpha_u \int \max\left\{T\left(w_1, w_2'\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_2'\right) \\ &= r\Omega\left(w_1\right), \end{aligned}$$

which confirms our conjecture, since $T\left(w^{**}, w_2\right) = \Omega\left(w_2\right)$ implies that $\phi\left(w_2\right) = w^{**}$. Finally, from equation (18), it is immediate that $\eta\left(w_i\right) = w_i$, which completes the proof. ■

Proof. [Proposition 6] We begin with part (ii). The value functions (4) and (6) modified to allow for exogenous separations are

$$rT\left(w_1, w_2\right) = u\left(w_1 + w_2\right) + \delta\left[\Omega\left(w_1\right) - T\left(w_1, w_2\right)\right] + \delta\left[\Omega\left(w_2\right) - T\left(w_1, w_2\right)\right] \tag{39}$$

$$\begin{aligned} r\Omega\left(w_1\right) &= u\left(w_1 + b\right) - \delta\left[\Omega\left(w_1\right) - U\right] \\ &+ \alpha \int \max\left\{T\left(w_1, w_2\right) - \Omega\left(w_1\right), \Omega\left(w_2\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_2\right). \end{aligned} \tag{40}$$

From the definition of reservation function ϕ for the worker-searcher couple, $T\left(w_1, \phi\left(w_1\right)\right) = \Omega\left(w_1\right)$, we have:

$$rT\left(w_1, \phi\left(w_1\right)\right) = u\left(w_1 + \phi\left(w_1\right)\right) + \delta\left[\Omega\left(\phi\left(w_1\right)\right) - \Omega\left(w_1\right)\right] = r\Omega\left(w_1\right).$$

Let us assume that there is a wage value w_1 beyond which the employed worker never quits. Then, in this range $\phi\left(w_1\right)$ is a nonincreasing function. Using this property, the second option in the max operator in equation (40) becomes irrelevant, and we are left with the following equation:

$$\begin{aligned} u\left(w_1 + \phi\left(w_1\right)\right) &= u\left(w_1 + b\right) + \alpha \int_{\phi(w_1)} \left[T\left(w_1, w_2\right) - T\left(w_1, \phi\left(w_1\right)\right)\right] dF\left(w_2\right) - \delta\left[\Omega\left(\phi\left(w_1\right)\right) - U\right] \\ &= u\left(w_1 + b\right) + h\left(\phi\left(w_1\right)\right) \\ &+ \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} \left[u\left(w_1 + w_2\right) - u\left(w_1 + \phi\left(w_1\right)\right)\right] dF\left(w_2\right), \end{aligned} \tag{41}$$

where

$$h(x) = \frac{\alpha\delta}{r+2\delta}\int_x [\Omega(w_2) - \Omega(x)]\, dF(w_2) - \delta[\Omega(x) - U]$$

with h decreasing in x. Rearrange equation (41) as

$$1 = \frac{\alpha}{r+2\delta}\int_{\phi(w_1)} \left[\frac{u(w_1+w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)}\right] dF(w_2) + \frac{h(\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)}. \quad (42)$$

Since $\phi(w_1)$ is a decreasing function of w_1, then, for any $\tilde{w}_1 > w_1$, we have

$$0 \le \frac{u(w_1+w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)} \le \frac{u(\tilde{w}_1+w_2) - u(\tilde{w}_1+\phi(w_1))}{u(\tilde{w}_1+\phi(w_1)) - u(\tilde{w}_1+b)} \le \frac{u(\tilde{w}_1+w_2) - u(\tilde{w}_1+\phi(\tilde{w}_1))}{u(\tilde{w}_1+\phi(\tilde{w}_1)) - u(\tilde{w}_1+b)},$$

where the first weak inequality stems from the fact that u is CARA or DARA, and the second from the fact that ϕ is weakly decreasing. Overall, the above condition implies the first term in equation (42) is an increasing function of w_1.

Since h is decreasing in x, and $\phi(\tilde{w}_1) \le \phi(w_1)$ for $\tilde{w}_1 > w_1$, we have

$$\frac{h(\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)} < \frac{h(\phi(\tilde{w}_1))}{u(\tilde{w}_1+\phi(\tilde{w}_1)) - u(\tilde{w}_1+b)},$$

because the right hand side has a weakly greater numerator and a strictly smaller denominator than the left-hand side. And we reach the following contradiction:

$$\begin{aligned} 1 &= \frac{\alpha}{r+2\delta}\int_{\phi(w_1)} \left[\frac{u(w_1+w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)}\right] dF(w_2) + \frac{h(\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)} \\ &< \frac{\alpha}{r+2\delta}\int_{\phi(\tilde{w}_1)} \left[\frac{u(\tilde{w}_1+w_2) - u(\tilde{w}_1+\phi(\tilde{w}_1))}{u(\tilde{w}_1+\phi(\tilde{w}_1)) - u(\tilde{w}_1+b)}\right] dF(w_2) + \frac{h(\phi(\tilde{w}_1))}{u(\tilde{w}_1+\phi(\tilde{w}_1)) - u(\tilde{w}_1+b)} \\ &= 1, \end{aligned}$$

where the last equality follows from the fact that the RHS in the second line is like the RHS in the first line evaluated at $\tilde{w}_1$ instead of w_1. We conclude that $\phi(w_1)$ is strictly increasing in w_1. Once we have established this result, the same arguments used in the proof of Proposition 3 apply here to prove that $\phi' < 1$. ■

Proof. [Proposition 7] We conjecture that $rT(w_1, w_2, a) = u(ra + w_1 + w_2)$. Then the RHS of equation (23) becomes

$$\max_c \left\{u(c) + u'(ra + w_1 + w_2)(ra + w_1 + w_2 - c)\right\}.$$

The FOC implies $u'(c) = u'(ra + w_1 + w_2)$, so $c_e(a, w_1, w_2) = ra + w_1 + w_2$. If we plug this optimal consumption function back into equation (23), we arrive at $rT(w_1, w_2, a) = u(ra + w_1 + w_2)$, which confirms the conjecture.

Similarly, let us guess that $r\Omega(w_1, a) = u(ra + w_1 + \phi(w_1))$. Again, plugging this guess into the RHS of equation (25), the FOC implies $c_{eu}(w_1, a) = ra + w_1 + \phi(w_1, a)$. Substituting this function back into (25) gives

$$
\begin{aligned}
r\Omega\left(w_1,a\right) = & \, u(ra + w_1 + \phi\left(w_1,a\right)) + u'\left(ra + w_1 + \phi\left(w_1,a\right)\right)\left(b - \phi\left(w_1,a\right)\right) \\
& + \frac{\alpha}{r}\int \max\left\{u\left(ra + w_1 + w_2\right) - u\left(ra + w_1 + \phi\left(w_1,a\right)\right),\right. \\
& \left. u\left(ra + w_2 + \phi\left(w_1,a\right)\right) - u\left(ra + w_1 + \phi\left(w_1,a\right)\right), 0\right\} dF\left(w_2\right).
\end{aligned}
$$

Using the CARA property of u, we can simplify the RHS and rewrite the above equation as

$$
\begin{aligned}
r\Omega\left(w_1,a\right) &= u(ra + w_1 + \phi\left(w_1,a\right))\left[1 - \left(b - \phi\left(w_1,a\right)\right)/\rho\right. \\
& \left. -\frac{\alpha}{r}\int \max\left\{u\left(w_2 - \phi\left(w_1,a\right)\right)/\rho + 1, u\left(w_2 - w_1\right)/\rho + 1, 0\right\} dF\left(w_2\right)\right].
\end{aligned}
$$

Now, using the definition of ϕ and the expression for $rT\left(w_1, \phi\left(w_1,a\right), a\right)$ in the above equation, we have

$$
\phi\left(w_1,a\right) = b + \frac{\alpha}{r}\int \left[u\left(\max\left\{w_2 - \phi\left(w_1,a\right), w_2 - w_1, 0\right\}\right) + \rho\right] dF\left(w_2\right).
$$

As in the CARA case without saving, conjecture that there is a value w_1 such that beyond that value the quitting option is never exercised. Then, in this range we can ignore from the second argument in the max operator and rewrite

$$
\phi\left(w_1,a\right) = b + \frac{\alpha}{r}\int_{\phi\left(w_1,a\right)} \left[u\left(w_2 - \phi\left(w_1,a\right)\right) + \rho\right] dF\left(w_2\right), \tag{43}
$$

which implies that ϕ is a constant function, independent of $\left(w_1,a\right)$. Moreover, comparing (43) to the equivalent equation for the single-agent problem (22) yields that $\phi\left(w_1,a\right) = w^*$.

Finally, let us turn to U and conjecture that $rU\left(a\right) = u\left(ra + 2w^{**}\right)$. Substituting this guess into equation (24) and taking the FOC leads to the optimal policy function $c_u\left(a\right) = ra + 2w^{**}$, which confirms the guess. Then, using the CARA assumption, equation (24) becomes

$$
\begin{aligned}
rU\left(a\right) &= u\left(ra + 2w^{**}\right) - u\left(ra + 2w^{**}\right)\left(2b - 2w^{**}\right)/\rho - \frac{2\alpha}{r}u\left(ra + 2w^{**}\right)\int_{w^{**}}\left[\rho u\left(w - w^{**}\right) + 1\right] dF\left(w\right) \\
&= u\left(ra + 2w^{**}\right)\left[1 - \left(2b - 2w^{**}\right)/\rho - \frac{2\alpha}{r}\int_{w^{**}}\left[\rho u\left(w - w^{**}\right) + 1\right] dF\left(w\right)\right]
\end{aligned}
$$

and using $rU\left(a\right) = u\left(ra + 2w^{**}\right)$ we arrive at

$$
w^{**} = b + \frac{\alpha}{r}\int_{w^{**}}\left[u\left(w - w^{**}\right) + \rho\right] dF\left(w\right),
$$

which, once again, compared to (22) implies that $w^{**} = w^*$. This concludes the proof. ■

Proof. [Proposition 8] We first prove parts (ii) and (iii), which establish the behavior of the reservation wage functions. The reservation function for an outside offer satisfies $S\left(w_1, \phi_o\left(w_1\right)\right) =$

$\Omega(w_1)$. As before, we begin by conjecturing that the quit option is never exercised beyond a certain wage threshold. In this range, from the definition of $\phi_o(w_1)$:

$$\begin{aligned}\phi_o(w_1) &= b+\kappa+\alpha_i\int_{\phi_i(w_1)}[T(w_1,w_2)-\Omega(w_1)]\,dF(w_2)+\alpha_o\int_{\phi_o(w_1)}[S(w_1,w_2)-\Omega(w_1)]\,dF(w_2)\\ &= b+\kappa+\alpha_i\int_{\phi_i(w_1)}T_2(w_1,w_2)(1-F(w_2))\,dw_2+\alpha_o\int_{\phi_o(w_1)}S_2(w_1,w_2)(1-F(w_2))\,dw_2\\ &= b+\kappa+\frac{\alpha_i}{r}\int_{\phi_i(w_1)}[1-F(w_2)]\,dw_2+\frac{\alpha_o}{r}\int_{\phi_o(w_1)}[1-F(w_2)]\,dw_2, \end{aligned} \tag{44}$$

where the second line is obtained through integration by parts and the third line uses the risk neutrality assumption, which assures $T_2(w_1,w_2)=S_2(w_1,w_2)=\frac{1}{r}$.

We now turn to inside offers. The reservation function for an inside offer satisfies $T(w_1,\phi_i(w_1))=\Omega(w_1)$. We keep analyzing the region of w_1 above $\hat{w}_S$ where we know the employed worker does not quit upon receiving outside offers. From the definition of $\phi_i(w_1)$:

$$\begin{aligned}\phi_i(w_1) &= b+\alpha_i\int_{\phi_i(w_1)}[T(w_1,w_2)-\Omega(w_1)]\,dF(w_2)+\alpha_o\int_{\phi_o(w_1)}[S(w_1,w_2)-\Omega(w_1)]\,dF(w_2)\\ &= b+\frac{\alpha_i}{r}\int_{\phi_i(w_1)}[1-F(w_2)]\,dw_2+\frac{\alpha_o}{r}\int_{\phi_o(w_1)}[1-F(w_2)]\,dw_2, \end{aligned} \tag{45}$$

where the second line is derived exactly as for the outside offer case.

Combining equations (44) and (45), we can verify that $\phi_o(w_1)$ and $\phi_i(w_1)$ are independent of w_1, and $\phi_i(w_1)=\phi_o(w_1)-\kappa$ for $w_1\geq\hat{w}_S$. This confirms the conjecture and yields $\hat{w}_T=\hat{w}_S-\kappa$.

Let us extend our analysis of inside offers to the region in which w_1 is lower than $\hat{w}_S$. Here, the reservation function ϕ_i satisfies

$$\phi_i(w_1)=b+\frac{\alpha_i}{r}\int_{\phi_i(w_1)}[1-F(w)]\,dw+\frac{\alpha_o}{r}\int_{w_1}^{\hat{w}_S}\Omega'(w_2)[1-F(w_2)]\,dw_2,$$

since the employed worker will quit upon receiving outside offers. Clearly, $\phi_i(w_1)$ is decreasing in w_1 over this region. We conclude that for $w_1\geq\hat{w}_S$, we have $\phi_i(w_1)=\hat{w}_T$ and in the range $[\hat{w},\hat{w}_S)$ the function ϕ_i is decreasing, with $\hat{w}$ denoting the double indifference point, i.e., the intersection with the 45-degree line. As usual, below $\hat{w}$, $\phi_i(w_1)=w_1$. This completes the proof of parts (ii) and (iii).

We next prove part (i) of the proposition: $w^{**}\in(\hat{w}_T,\hat{w})$ and $w^*\in(\hat{w},\hat{w}_S)$, so $w^{**}<w^*$. It is also useful to recall that $\hat{w}_T<\hat{w}<\hat{w}_S$.

Step 1: We first show $w^{**}\in(\hat{w}_T,\hat{w})$. Equation (29) evaluated at the point $w_1=\hat{w}_T$ becomes

$$r\Omega(\hat{w}_T)=\hat{w}_T+b+(\alpha_i+\alpha_o)\int_{\hat{w}_T}\Omega'(w)[1-F(w)]\,dw. \tag{46}$$

The reservation wage of the dual-searcher couple w^{**} is characterized by the equation

$$r\Omega(w^{**})=2b+2(\alpha_i+\alpha_o)\int_{w^{**}}\Omega'(w)(1-F(w))\,dw. \tag{47}$$

Now subtract equation (46) multiplied by 2 from equation (47) and get

$$r\left[\Omega\left(w^{**}\right)-\Omega\left(\hat{w}_T\right)\right]=r\Omega\left(\hat{w}_T\right)-2\hat{w}_T+2\left(\alpha_i+\alpha_o\right)\int_{w^{**}}^{\hat{w}_T}\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw.$$

Suppose $w^{**}\leq\hat{w}_T$, then the LHS of the above equation is negative or zero. The second term of the RHS is positive. The term $r\Omega\left(\hat{w}_T\right)-2\hat{w}_T$ is also positive because for $w_1=\hat{w}_T$, the employed worker would prefer to quit his job rather than remain employed (more precisely, he strictly prefers it for an outside offer, but he is indifferent for an inside offer). Therefore the RHS is positive, which is a contradiction. So $w^{**}>\hat{w}_T$.

Step 2: Similarly, consider equation (29) evaluated at $w_1=\hat{w}$. Note that at $w_1=\hat{w}$, for inside offers the employed spouse never exercises the quit option, whereas for outside offers, she does. So, equation (29) evaluated at $w_1=\hat{w}$ becomes

$$r\Omega\left(\hat{w}\right)=\hat{w}+b+\frac{\alpha_i}{r}\int_{\hat{w}}\left[1-F\left(w\right)\right]dw+\frac{\alpha_o}{r}\int_{\hat{w}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw.$$

Also note that since $\hat{w}$ is the double indifference point for inside offers, $r\Omega\left(\hat{w}\right)=2\hat{w}$. Again, subtract this last equation multiplied by 2 from equation (47) to get

$$r\left[\Omega\left(w^{**}\right)-\Omega\left(\hat{w}\right)\right]=\frac{2\alpha_i}{r}\left[\int_{w^{**}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw-\int_{\hat{w}}\left[1-F\left(w\right)\right]dw\right]+2\frac{\alpha_o}{r}\int_{w^{**}}^{\hat{w}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw.$$

Now, suppose $w^{**}\geq\hat{w}$. Then the LHS becomes nonnegative. The last term in the RHS is negative. From the definition of $\phi_i\left(w_1\right)$, $r\Omega\left(w_1\right)=rT\left(w_1,\phi_i\left(w_1\right)\right)=w_1+\phi_i\left(w_1\right)$. Thus, $\phi_i'\left(w_1\right)=r\Omega'\left(w_1\right)-1$. But since we have proved that $\phi_i'\left(w_1\right)\leq 0$ above $\hat{w}$, we have that $r\Omega'\left(w_1\right)\leq 1$. Therefore, the first term in the RHS must also be negative, which delivers a contradiction and leads to $w^{**}<\hat{w}$. Steps 1 and 2 establish that $w^{**}\in\left(\hat{w}_T,\hat{w}\right)$.

Step 3: We next prove $w^{*}\in\left(\hat{w},\hat{w}_S\right)$. Combining equation (29) evaluated at $\hat{w}$ with the fact that $r\Omega\left(\hat{w}\right)=2\hat{w}$, we have

$$\hat{w}=b+\frac{\alpha_i}{r}\int_{\hat{w}}\left[1-F\left(w\right)\right]dw+\frac{\alpha_o}{r}\int_{\hat{w}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw.$$

Subtracting this equation from equation (30), we get

$$w^{*}-\hat{w}=\frac{\alpha_i}{r}\int_{w^{*}}^{\hat{w}}\left[1-F\left(w\right)\right]dw+\frac{\alpha_o}{r}\left[\int_{w^{*}}\left[1-F\left(w\right)\right]dw-\int_{\hat{w}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw\right].$$

Suppose $w^{*}\leq\hat{w}$, then the LHS becomes non-positive, but the RHS is strictly positive since $r\Omega'\left(w\right)\leq 1$, a contradiction. Thus, $w^{*}>\hat{w}$.

Step 4: Finally we show that $w^{*}<\hat{w}_S$. Rewrite the equation for $\hat{w}_S$ as

$$\hat{w}_S=b+\kappa+\frac{\alpha_1}{r}\int_{\hat{w}_S-\kappa}\left(1-F\left(w\right)\right)dw+\frac{\alpha_2}{r}\int_{\hat{w}_S}\left(1-F\left(w\right)\right)dw.$$

Subtracting equation (30) from the equation defining $\hat{w}_S$, we get

$$\hat{w}_S-w^{*}=\kappa+\frac{\alpha_i}{r}\int_{\hat{w}_S-\kappa}^{w^{*}}\left[1-F\left(w\right)\right]dw+\frac{\alpha_o}{r}\int_{\hat{w}_S}^{w^{*}}\left[1-F\left(w\right)\right]dw.$$

Suppose $w^* \geq \hat{w}_S$, then the LHS is non-positive. However, since $\kappa > 0$, the RHS is strictly positive. Thus, $w^* < \hat{w}_S$. Therefore, $w^* \in (\hat{w}, \hat{w}_S)$, and the proof is complete. ■

B Multiple Locations Case: Value Functions

Below, we report value functions for the economy with multiple locations and exogenous separations that we simulate in Section 6.2. The value of a couple of employed spouses who currently live together is

$$rT(w_1, w_2) = w_1 + w_2 - \delta [T(w_1, w_2) - \Omega(w_1)] - \delta [T(w_1, w_2) - \Omega(w_2)],$$

and the value of a couple whose members are employed but currently live in different locations is

$$rS(w_1, w_2) = w_1 + w_2 - \kappa - \delta [S(w_1, w_2) - \Omega(w_1)] - \delta [S(w_1, w_2) - \Omega(w_2)].$$

We now turn to the worker-searcher couple. First, the unemployed spouse receives offers at rate $(1-\theta)\psi$ from the current location, in which case the couple faces the same options as in the one-location problem. Second, the same spouse receives outside offers at rate $\theta\psi$, in which case (i) the unemployed spouse can choose to accept the offer, the employed spouse would keep his job, and the couple could live separately, (ii) the household can accept the offer, and the currently employed spouse would quit his job, or (iii) the offer could be rejected. The value for a worker-searcher couple is therefore

$$\begin{aligned} r\Omega(w_1) = w_1 + b + (1-\theta)\psi \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) \\ + \theta\psi \int \max\{S(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\} dF(w_2) - \delta [\Omega(w_1) - U], \end{aligned}$$

and the value for an unemployed couple is

$$rU = 2b + 2\psi \int \max\{\Omega(w) - U, 0\} dF(w).$$

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