Finance and Development: Limited Commitment vs. Private Information*

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PRELIMINARY AND INCOMPLETE

Abstract

Recent papers argue that financial frictions can explain large cross-country income differences. Financial frictions are usually modeled as arising from a limited commitment problem. We ask whether it matters what the source of frictions is? We develop a framework that is general enough to encompass both frictions arising from limited commitment and from asymmetric information. We argue that asymmetric information frictions have implications that are potentially very different from limited commitment frictions. In particular, limited commitment results in a misallocation of capital across firms with given productivities. In contrast, moral hazard provides a theory for why TFP is endogenously lower at the firm level in developing countries. Finally, and as is well known, limited commitment results in individuals being borrowing constrained whereas under moral hazard they are *savings* constrained, and this has some additional implications for the macro economy. The framework also encompasses mixtures of different friction regimes in different regions of a given economy. This has advantages when mapping models of the macro economy to micro data.

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1 Introduction

There is evidence that even within a given economy, obstacles to trade may vary depending on location. In a companion paper, Karaivanov and Townsend (2011) estimate the financial/information regime in place for households including those running businesses using Townsend Thai data from rural areas (villages) and from urban areas (towns and cities). They do find a difference. A moral hazard constrained financial regime fits best in urban areas and a more limited savings regimes in rural areas.

More generally, there seems to be regional variation. Paulson, Townsend and Karaivanov (2006) find that the decision to become an entrepreneur is based on wealth and talent in both a moral hazard and limited commitment regime, but again the quantitative mapping is distinct. Moral hazard fits best to the data in the Central region but not in the Northeast. Using additional data on repayment of joint liability loans, Ahlin and Townsend (2007) seem to confirm the regional variation (though for not all specifications). Information seems to be a problem in the central area, limited commitment in the Northeast. In more detail, the non-monotone derivative of repayment with respect to loan size in the adverse selection model of Ghatak (1999) is found in the Central region but not the Northeast alone. The negative sign with respect to the joint liability payment of the moral hazard model of Stiglitz (1990), and the model of Ghatak, is found in the Central region. The sign on screening is counter to the Ghatak model in the Northeast. Covariance of outputs raises repayment as in the two information models in the Central region. Ease of monitoring reducing moral hazard and raising repayment in the Central region. Cooperation among borrowers in decision making, which has a positive sign in the moral hazard model of Stiglitz, holds in the Central region. Sanctions for strategic default are especially effective in the Northeast.

Not too surprisingly, the effective financial regime in place depends on the data used. Restricting attention to consumption and income data, the financial regimes are quite good/smooth. This is particularly true for kinship and other village financial networks. Yet investment, cash flow, and firm size data often deliver a simpler, more restrictive financial regime, borrowing and lending, or even savings only, as in a buffer stock model.

As we await the final verdict from the micro data, we begin the next step in this paper and ask what difference the micro financial foundations make for the macro economy. To fix ideas, we consider two regimes of frictions: limited commitment and moral hazard. We study their implications for aggregates like GDP, TFP, capital accumulation, wages and interest rates, but also for micro moments such as the productivity distribution, the size distribution of firms, and the dispersion in the marginal product of capital. We show that all of these look potentially very different depending on the underlying financial regime. We also show that transition dynamics are greatly influenced by the underlying financial regime, with the moral hazard constrained financial contracts slowing down transitions, dramatically.

The bottom line is that the behavior of macro aggregates depends on micro financial underpinnings. This has important implications for the literature studying the role of financial market imperfections in economic development. Most existing literature work with collateral constraints that are either explicitly or implicitly motivated as arising from a limited commitment problem. In contrast, there are much fewer studies that model financial frictions as arising from an asymmetric information problem.[1] In either case, few authors use micro data to discipline their macro models.[2] Even fewer (perhaps none?) use micro data to choose between the myriad of alternative forms of introducing a financial friction into their model. This is a serious shortcoming and the goal of this paper is to make some progress by studying the macroeconomic implications of different micro financial underpinnings suggested by the micro data.[3]

Of course, ideal policy would also depend on micro financial underpinnings. The welfare gains and losses one would compute at a micro level from say subsidies to financial institutions or interest rates will differ from the general equilibrium calculations with market clearing wages and interest rates as endogenous.

2 Households and Intermediaries

We consider an economy populated by a large number of households and intermediaries. Time is discrete. In each period t, a household experiences two shocks: an ability shock, z_t and an additional "production risk shock", ε_t (more on this below). Households have preferences over consumption, c_t and effort, e_t

$$v_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, e_t),$$

Households can access the capital market of the economy only via one of the intermediaries. Intermediaries compete ex-ante for the right to contract with households. Once a household decides to contract with an intermediary he sticks with that intermediary forever. However, the threat of having one's customer poached by another intermediary means that intermediaries make zero expected profits at each point in time.

Consider a household with initial wealth a_0 and income stream $\{y_t\}_{t=0}^{\infty}$ (determined below). When an agent contracts with an intermediary, he gives his entire initial wealth and income stream to the intermediary. The intermediary invests this income at a risk-free interest rate r_t and transfers some consumption, c_t, to the household. A household and an intermediary

[1]Notable exceptions are by Castro, Clementi and Macdonald (2009); Greenwood, Sanchez and Wang (2010*a*,*b*).

[2]Exceptions are Kaboski and Townsend (2011) and Midrigan and Xu (2010)

[3]There are, however, a few papers that compare different micro underpinnings that others have used in macro models. Besides **?**, Meisenzahl (2011) is another example.

therefore form a "risk-sharing group": some of the household's risk is borne by the intermediary according to an optimal contract specified below. The joint budget constraint of such a risk-sharing group is

$$a_{t+1} = y_t - c_t + (1 + r_t)a_t \tag{1}$$

The optimal contract between households and intermediary maximizes households' utility subject to this budget constraint (and incentive constraints specified below). Risk-sharing groups make their decisions taking as given a deterministic sequence of wages and interest rates $\{w_t, r_t\}_{t=0}^{\infty}$, and compete with each other in competitive labor and capital markets.

2.1 Household's Problem

Households can either be entrepreneurs or workers. We denote by $x = 1$ the choice of being an entrepreneur and by $x = 0$ that of being a worker. First, consider entrepreneurs. They get an ability draw z. The evolution of this entrepreneurial talent z is assumed to be exogenous and given by some stationary transition process $\mu(z'|z)$. Denote effort by e, labor hired in the labor market by l, and capital employed by k. Output is given by $z\varepsilon f(k, l)$ where $f(k, l)$ is a span-of-control production function and ε ("production risk") is stochastic with distribution $p(\varepsilon|e)$. An entrepreneur's productivity therefore has two components: his talent, z and production risk, ε, the distribution of which depends on effort. We assume that intermediaries can insure production risk ε but *not* talent z. An entrepreneur hires labor l at a wage w and rents capital k at a rental rate $r + \delta$.[4]

Next, consider workers. A worker sells efficiency units of labor ε in the labor market at wage w. Efficiency units are stochastic and depend on the worker's effort, with distribution $p(\varepsilon|e)$. A worker's ability is fixed over time.

Putting everything together, the income stream of a household is

$$y = x[z\varepsilon f(k, l) - wl - (r + \delta)k] + (1 - x)w\varepsilon. \tag{2}$$

The joint budget constraint of the risk-sharing group consisting of household and intermediary is given by (1).

The timing is illustrated in Figure 1 and is as follows: first, productivity z is realized. Second, the contract between the intermediary and a household assigns effort, e, occupational choice, x, and – if the chosen occupation is entrepreneurship – capital and labor, k and l. All these choices are conditional on productivity z and assets carried over from the last period, a.

[4] We assume that capital is owned and accumulated by a capital producing sector which then rents it out to entrepreneurs in a capital rental market. See Appendix B for details. That the rental rate equals $r + \delta$ follows from a standard arbitrage argument. This way of stating the problem avoids carrying capital, k, as a state variable in the dynamic program of a risk-sharing group.

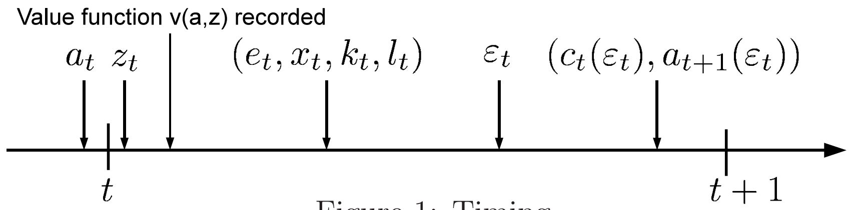

Figure 1: Timing

Third, production risk, ε, is realized which depends on effort through the conditional distribution $p(\varepsilon|e)$. Fourth, the contract assigns the household's consumption and savings, that is functions $c(\varepsilon)$ and $a'(\varepsilon)$. The household's effort choice e can be unobserved depending on the regime we study. All other actions of the household are observed. For instance, there are no hidden savings.

The two state variables are wealth, a, and entrepreneurial ability, z. Recall that z evolves according to some exogenous Markov process $\mu(z'|z)$. It will be convenient below to define the household's expected continuation value by

$$\mathbb{E}_z v(a', z') = \sum_{z'} v(a', z')\mu(z'|z).$$

A contract between a household and an intermediary solves

$$v(a, z) = \max_{e,x,k,l,c(\varepsilon),a'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon), e] + \beta\mathbb{E}_z v[a'(\varepsilon), z']\right\} \quad \text{s.t.}$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{c(\varepsilon) + a'(\varepsilon)\right\} = \sum_{\varepsilon} p(\varepsilon|e)\left\{x[z\varepsilon f(k, l) - wl - (r + \delta)k] + (1 - x)w\varepsilon\right\} + (1 + r)a \tag{3}$$

and also subject to regime-specific constraints specified below. Note that the budget constraint of a risk syndicate in (3) averages over realizations of ε; it does not have to hold separately for every realization of ε. This is because the contract between the household and the intermediary has an insurance aspect which implies that consumption can be different from income less than savings. Such an insurance arrangement can be "decentralized" in various ways. The intermediary could simply make state contingent transfers to the household. Alternatively, intermediaries can be interpreted as banks that offer savings accounts with state-contingent interest payments to households.

In contrast to production risk, talent z is not insurable. *Prior* to the realization of ε, the contract specifies consumption and savings that are *contingent* on ε, $c(\varepsilon)$ and $a'(\varepsilon)$. In contrast, consumption and savings can only depend on talent, z, to the extent that talent has already been observed.[5]

[5] The above dynamic program can be modified to allow for talent to be insured as follows: allow agents to

The contract between intermediaries and households is subject to one of two frictions: private information in the form of moral hazard, or limited commitment. Each friction corresponds to a regime-specific constraint that is added to the dynamic program (3). We specify each in turn.

2.2 Private Information

In this regime, effort e is unobserved. Since the distribution of production risk, $p(\varepsilon|e)$ depends on effort, this gives rise to a standard moral hazard problem: full insurance against production risk would induce the household to exert suboptimal effort. The optimal contract takes this into account in terms of an incentive-compatibility constraint:

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e]+\beta\mathbb{E}_z v[a'(\varepsilon),z']\right\} \geq \sum_{\varepsilon} p(\varepsilon|\hat{e})\left\{u[c(\varepsilon),\hat{e}]+\beta\mathbb{E}_z v[a'(\varepsilon),z']\right\} \;\; \forall e,\hat{e},x \tag{4}$$

This constraint ensures that the value to the household of choosing the effort level assigned by the contract, e, is at least as large as that of any other effort, $\hat{e}$. The optimal contract in the presence of moral hazard solves (3) with the additional constraint (4).

Some readers may be surprised that this optimal dynamic contracting problem features neither promised utility as a state variable nor the usual "promise-keeping" constraint. Appendix C shows that the formulation here is equivalent to a more standard formulation of the contracting problem which uses promised utility as a state variable.

When solving the problem (3) and (4) numerically, we allow for lotteries in the optimal contract to "convexify" the constraint set. See Appendix D for the statement of the problem (3) with lotteries as in Phelan and Townsend (1991).

2.3 Limited Commitment

In this regime, effort e is observed. Therefore, there is no moral hazard problem and the contract consequently provides perfect insurance against production risk, ε. Instead we assume that the friction instead takes the form of a simple collateral constraint:

$$k \leq \lambda a, \quad \lambda \geq 1. \tag{5}$$

This form of constraint has been frequently used in the development literature on financial frictions. It can be motivated as a limited commitment constraint.[6]

trade in assets whose payoff is contingent on the realization of next period's talent z'. On the left-hand side of the budget constraint in (3), instead of $a'(\varepsilon)$, we would write $a'(\varepsilon,z')$ and sum these over future states z' using the probabilities $\mu(z'|z)$.

[6] Consider an entrepreneur with wealth a who rents k units of capital. The entrepreneur can steal a fraction $1/\lambda$ of rented capital. As a punishment, he would lose his wealth. In equilibrium, the financial intermediary will rent capital up to the point where individuals would have an incentive to steal the rented capital, implying a collateral constraint $k/\lambda \leq a$ or $k \leq \lambda a$.

The optimal contract in the presence of limited commitment solves (3) with the additional constraint (5).

2.4 Factor Demands and Supplies

Risk-sharing groups interact in competitive labor and capital markets, taking as given the sequences of wages and interest rates. Denote by by $k(a, z; w, r)$ and $l(a, z; w, r)$ the optimal capital and labor demands of a risk-sharing group with current state (a, z). A worker supplies ε efficiency units of labor to the labor market, so his labor supply is

$$n(a, z; w, r) \equiv [1 - x(a, z)] \sum_{\varepsilon} p(\varepsilon|e(a, z))\varepsilon. \tag{6}$$

Note that we multiply by the indicator for being a worker, $1 - x$, so as to only pick up the efficiency units of labor by people who decide to be workers. Finally, individual capital supply is simply a household's wealth, a.

2.5 Equilibrium

We use the saving policy functions $a'(\varepsilon)$ and the transition probabilities $\mu(z'|z)$ to construct transition probabilities $\Pr(a', z'|a, z)$.[7] Given these transition probabilities and an initial distribution $g_0(a, z)$, we then obtain the sequence $\{g_t(a, z)\}_{t=0}^{\infty}$ from

$$g_{t+1}(a', z') = \Pr(a', z'|a, z)g_t(a, z). \tag{7}$$

Note that we cannot guarantee that the process for wealth and ability (7) has a stationary distribution. While the process is stationary in the z-dimension (recall that the process for z, $\mu(z'|z)$, is exogenous and a simple stationary Markov chain), the process may be non-stationary or degenerate in the a-dimension. That is, there is the possibility that the wealth distribution either fans out forever or collapses to a point mass. In the examples we have computed, this does however not seem to be a problem and (7) always converges.

Once we have found a stationary distribution of states from (7), we check that markets clear. Denote the stationary distribution of ability and wealth by $G(a, z)$. Then market clearing is

$$\int l(a, z; w, r)dG(a, z) = \int n(a, z; w, r)dG(a, z) \tag{8}$$

$$\int k(a, z; w, r)dG(a, z) = \int adG(a, z). \tag{9}$$

The equilibrium factor prices w and r are found in the same way as in Buera-Shin (Appendix A.1).

[7] In the computations we discretize the state space for wealth, a, so this is a simple Markov transition matrix.

3 Parameterization

The next section presents some numerical results. We assume that utility is separable and isoelastic

$$u(c,e) = U(c) - V(e), \quad U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad V(e) = \frac{\chi}{1+\varphi} e^{1+\varphi}$$

and that the production function is Cobb-Douglas

$$\varepsilon z f(k,l) = \varepsilon z k^{\alpha} l^{\gamma}. \tag{10}$$

We assume that $\alpha + \gamma < 1$ so that entrepreneurs have a limited span of control. We use the following parameter values:

$$\alpha = 0.3, \quad \gamma = 0.4, \quad \delta = 0.05, \quad \beta = 1.05^{-1}, \quad \sigma = 1.5, \quad \chi = .5, \quad \varphi = 0.2$$

For our benchmark numerical results, we also set the parameter λ governing the tightness of the collateral constraints (5) to $\lambda = 2$.

4 Limited Commitment vs. Moral Hazard

In this section we compare the moral hazard and limited commitment regimes, and argue that the two have potentially very different implications.

We first present some analytic results that characterize differences in individual savings behavior in the two regimes. These are variants of well-known results in the literature.

Lemma 1 *Let $u(c,e) = U(c) - V(e)$. Solutions to the optimal contracting problem under moral hazard* (3) *and* (4)*, satisfy*

$$U'(c_t) = \beta(1+r)\mathbb{E}_{z,t}\left(\mathbb{E}_{\varepsilon,t}\frac{1}{U'(c_{t+1})}\right)^{-1} \tag{11}$$

where $\mathbb{E}_{z,t}$ and $\mathbb{E}_{\varepsilon,t}$ denote the time t expectation over future values of z and ε.

This is a variant of the inverse Euler equation derived in Rogerson (1985), Ligon (1998) and Golosov, Kocherlakota and Tsyvinski (2003) among others. With a degenerate distribution for productivity, z, our equation collapses to the standard inverse Euler equation. The reason our equation differs from the latter is that we have assumed that productivity, z is not insurable in the sense that asset payoffs are not contingent on the realization of z (see footnote 2). Our equation is therefore a "hybrid" of an Euler equation in an incomplete markets setting and the inverse Euler equation under moral hazard.

If the incentive compatibility constraint (4) is binding, marginal utilities are not equalized across realizations of ε. One well known implication of (11) is that in this case[8]

$$U'(c_t) < \beta(1+r)\mathbb{E}_{z,t}\mathbb{E}_{\varepsilon,t}U'(c_{t+1}). \tag{12}$$

With limited commitment, the Euler equation is instead[9]

$$U'(c_t) = \beta\mathbb{E}_{z,t}\left[U'(c_{t+1})(1+r) + \mu_{t+1}\lambda\right]$$

If the collateral constraint (5) binds, then

$$U'(c_t) > \beta(1+r)\mathbb{E}_{z,t}U'(c_{t+1}) \tag{13}$$

Contrasting (12) for moral hazard and (13) for limited commitment, we can see that in the moral hazard regime individuals are *savings constrained* and in the limited commitment regime, they are instead *borrowing constrained*.[10] The intuition for individuals being savings constrained under moral hazard is that there is an additional marginal cost of saving an extra dollar from period t to period $t+1$: in period $t+1$ an individual works less in response to any given compensation schedule.[11] Therefore the optimal contract discourages savings whenever the incentive compatibility constraint (4) binds. Finally, note that under limited commitment only the savings of entrepreneurs are distorted because only they face the collateral constraint (5). In contrast, under moral hazard the savings decision of both entrepreneurs and workers is distorted because both face the incentive compatibility constraint (4). This will be reflected in the equilibrium interest rate (see Table 5). In particular, the interest rate under moral hazard is higher than the first-best interest rate whereas it is lower than the first-best interest under

[8]This follows because by Jensen's inequality ($1/U'(c_{t+1})$ is a convex function of $U'(c_{t+1})$)

$$\mathbb{E}_{\varepsilon,t}\frac{1}{U'(c_{t+1})} > \frac{1}{\mathbb{E}_{\varepsilon,t}U'(c_{t+1})}.$$

[9]Note that in contrast to (11) no expectation over ε is taken here. This is because there is perfect insurance on ε. Therefore marginal utilities are equalized across ε realizations. More formally, denote by $c(\varepsilon, z, a)$ consumption of an individual who has experienced shocks ε and z and has wealth a. Then $U'(c(\varepsilon, z, a)) = \psi(a, z)$ for all ε, where $\psi(a, z)$ is the Lagrange multiplier on the budget constraint in (3). Since this is true for all ε realizations, of course also $\mathbb{E}_\varepsilon U'(c(\varepsilon, z, a)) = \psi(a, z)$.

[10]In the case where the corresponding constraints do not bind, both (12) and (13) collapse to the standard Euler equation under incomplete markets

$$U'(c_t) = \beta(1+r)\mathbb{E}_{z,t}U'(c_{t+1}).$$

[11]See Rogerson (1985) and Golosov, Kocherlakota and Tsyvinski (2003) for more detailed discussions of this idea.

limited commitment. Individual savings behavior is one prediction in which the two regimes differ dramatically.

Next, we present some numerical results that illustrate further differences between the moral hazard and limited commitment regimes. Figure 2 plots the distributions of the marginal product of capital in the two regimes. In the limited commitment regime (left panel), the

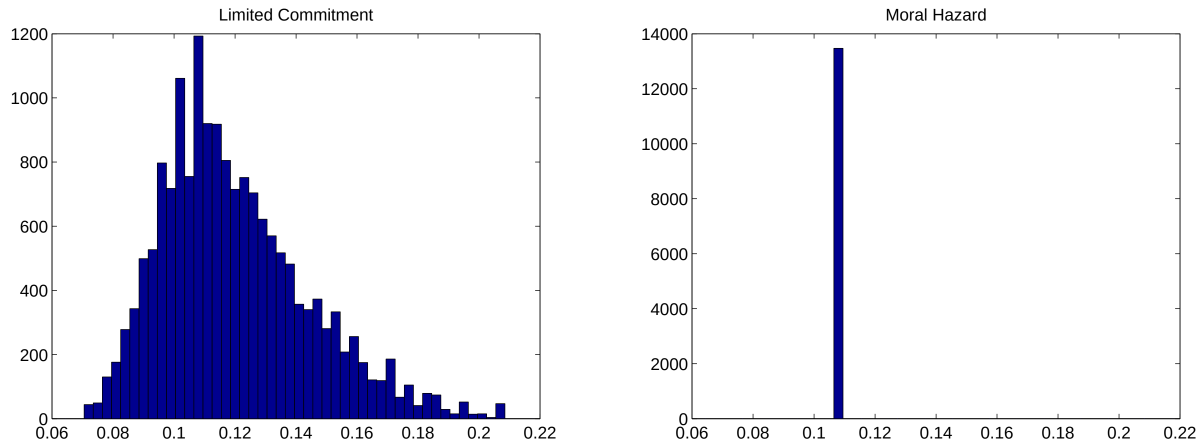

Figure 2: Distribution of Marginal Products of Capital.

presence of collateral constraints (5) implies that marginal products of capital are not equalized across individual firms, that is capital is misallocated. In contrast, in the moral hazard regime marginal products of capital *are* equalized across firms so that the distribution of marginal products is degenerate (right panel). This is because firms don't face any constraints that limit the amount of capital they can rent and so all of them rent capital until their expected marginal product equals the user cost of capital[12]

$$z\bar{\varepsilon}(e)f_k(k,l) = r + \delta, \quad \bar{\varepsilon}(e) \equiv \sum_{\varepsilon} \varepsilon p(\varepsilon|e) \tag{14}$$

If not in a misallocation of capital, how then will the presence of moral hazard manifest itself in our economy? Figure 3 has the answer: in the moral hazard economy, TFP is endogenously lower at the firm level. Recall that firm-level TFP is the product of "ability" and "production risk" and production risk depends on effort with probability distribution $p(\varepsilon|e)$. Ex-ante firm-level TFP is then given by $z\bar{\varepsilon}(e)$ where $\bar{\varepsilon}(e) \equiv \sum_{\varepsilon} \varepsilon p(\varepsilon|e)$ is expected production risk given an

[12]Similarly, entrepreneurs hire labor to equate the expected marginal product of labor to the wage, $z\bar{\varepsilon}(e)f_l(k,l) = w$. Hence, even though entrepreneurs bear some of the production risk, ε, under the optimal contract they behave as if they are risk neutral. This is because risk neutral intermediaries find it optimal to first maximize expected profits and to then assign ε-dependent consumption to entrepreneurs to make sure they expend the optimal amount of effort given incentive constraints. Since intermediaries pool risk over a large number of households, the expectation in (14) should be thought of as an integral over the population and not an expectation for the individual.

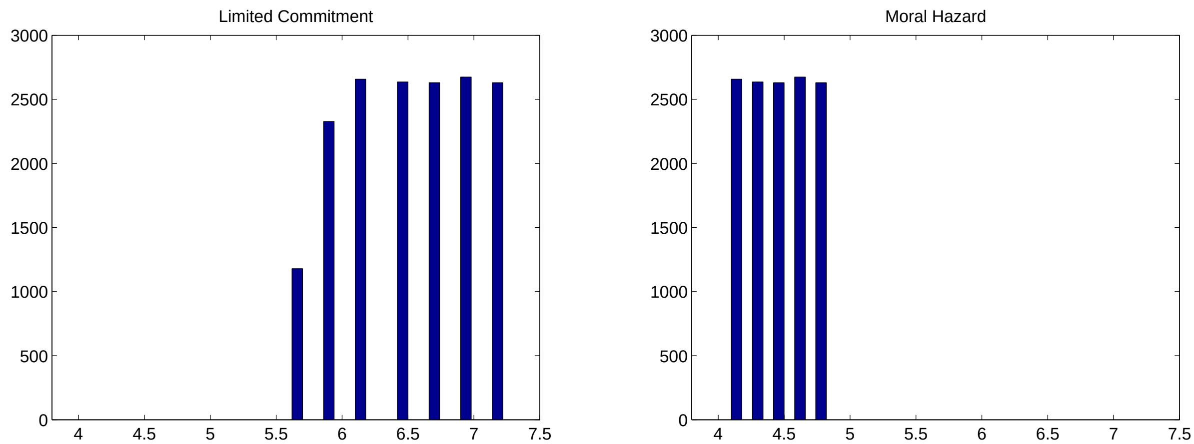

Figure 3: Distribution of Firm-level TFP.

effort choice, e. In the limited commitment regime (left panel), everyone exerts high effort so the distribution of TFP is simply given by the (exogenous) ability distribution (the stationary distribution of the Markov process $\mu(z'|z)$). In contrast, in the moral hazard economy (right panel), some individuals exert low effort which then results in lower expected production risk component, $\bar{\varepsilon}(e)$. As a result firm-level TFP is lower and more dispersed in the moral hazard economy.

Finally, figure 4 plots the distribution of firm size as measured by a firm's number of employees. In the moral hazard regime, firms are smaller on average than in the limited commitment regime. This is an immediate implication of the fact that firm-level TFP is lower in the moral

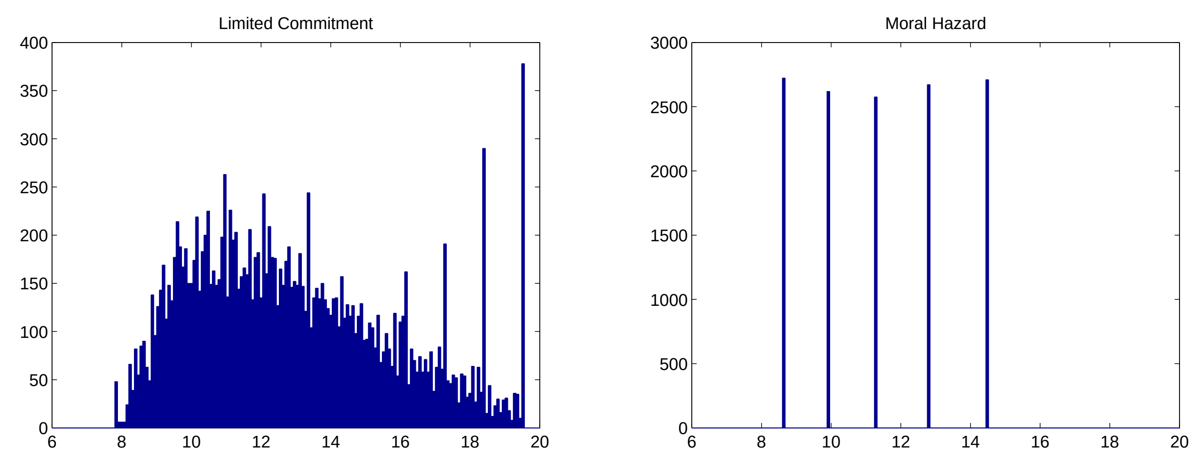

Figure 4: Firm-Size (Employee) Distribution.

hazard economy.

These results have important implications for measurement. For instance, consider an econometrician examining data generated by the moral hazard economy who measures gaps

	FB	LC	MH	Mix -LC	Mix - MH
GDP	1	0.8230	0.8046	0.8136	
TFP	1	0.9418	0.9187	0.9216	
Capital Stock	1	0.7496	0.7617	0.7825	
Wage	1	0.9282	0.9134	0.9240	
Interest Rate (%)	4.08	3.52	4.66	3.75	
% Entrepreneurs	26.98	37.36	35.33	6.67	52.58

Table 1: Comparison of Regimes

in marginal products of capital across individual firms. This econometrician would observe no capital misallocation and may therefore (erroneously) conclude that there is no friction in the capital market.

Finally, consider the savings behavior in the two economies, in particular the speed of individual transitions. One convenient way of summarizing this speed of transition is to compare the eigenvalues of the transition matrix $\Pr(a', z'|a, z)$ defined in (7) for the two economies. The eigenvalue governing the speed of convergence in the limited commitment economy is 0.9465 with a corresponding half life of $-\log(2)/\log(0.9465) \approx 12.6$ years whereas in the moral hazard economy this eigenvalue is 0.994 which implies a half-life of 115.2 years.[13] Individual transitions are therefore much slower in the moral hazard economy.

5 Mixtures of Moral Hazard and Limited Commitment

The previous section compared two economies: one in which all agents were subject to the moral hazard friction and another in which all agents were subject to the limited commitment friction. However, there is no reason why a given economy should be subject to only one imperfection. For example, Paulson, Townsend and Karaivanov (2006) find that for Thailand, moral hazard fits better in and around Bangkok and limited commitment better in the Northeast (see also Karaivanov and Townsend, 2011). Therefore, we ask in this section: what happens if both frictions are present in the same economy? We report results for an economy in which fifty percent of the population is subject to moral hazard and the other fifty percent is subject to limited commitment. We argue that such *mixture* regimes are different from simple convex combinations of the pure moral hazard and pure limited commitment economies. Table 1 reports some aggregate statistics of the economy with both regimes.

There is a perhaps surprising interaction of occupational choice with the two frictions: individuals who are subject to moral hazard are more likely to become entrepreneurs. Intuitively,

[13] The speed of convergence is determined by the largest eigenvalue that is less than one (see e.g. Stokey, Lucas and Prescott, 1989).

this is because in the limited commitment regime, the friction only affects entrepreneurs but not workers.

Finally, note that the interest rate under moral hazard is higher than the first-best interest rate whereas it is lower than the first-best interest under limited commitment. This is an implication of the different savings behavior at the micro level as described by the savings distortions in equations (12) and (13). Under moral hazard, individuals are *savings* constrained; in general equilibrium, the interest rate therefore adjusts upwards so as to equilibrate savings supply and demand. Under limited commitment, individuals are *borrowing* constrained so the opposite is true.

6 Transition Dynamics

Now the environment is non-stationary, and w_t and r_t, as well as the value function carry time indices:

$$V_t(a,z) = \max_{e,x,k,l,c(\varepsilon),a'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e) \left\{ u[c(\varepsilon),e] + \beta \mathbb{E}_z V_{t+1}[a'(\varepsilon),z'] \right\} \quad \text{s.t.}$$

$$\sum_{\varepsilon} p(\varepsilon|e) \left\{ c(\varepsilon) + a'(\varepsilon) \right\} \leq \sum_{\varepsilon} p(\varepsilon|e) \left\{ x[z\varepsilon f(k,l) - w_t l - (r_t+\delta)k] + (1-x)w_t\varepsilon] \right\} + (1+r_t)a$$

and subject to regime specific constraints as in sections 2.2 and 2.3. For computation, we follow the strategy in Buera-Shin, Appendix A.2. That is, the value function is computed by simple backward induction. We first compute the value function of the stationary equilibrium above, and let

$$V_T(a,z) = v(a,z)$$

We then compute $V_{T-1}(a,z)$ taking as given $V_T(a,z)$. Proceeding by backward induction, we can compute the entire sequence $V_t(a,z)$ for $t = T-1, T-2, ..., 1$. No value or policy function iteration is needed.

The equilibrium is defined in the same way as in the preceding section, with the exception that prices and the joint distribution of wealth and ability now carry t subscripts.

6.1 An Experiment

We conduct the following experiment: the economy starts in a steady state in which all individuals are subject to the moral hazard regime. At time $t = 10$, however, the nature of the friction changes: the entire population is suddenly subject to limited commitment rather than moral hazard. Given that Karaivanov and Townsend (2011) and others estimate different frictions for different geographical regions, this can be interpreted as a large migration of the entire population from an area where moral hazard is prevalent to another where the friction is limited commitment. This is an admittedly rather stark experiment but it serves to illustrate some of the features of transitions in our setup. We later also conduct experiments where only a fraction of the population switches regime.

Figure 5 presents time paths for the aggregate capital stock, TFP, GDP and the number of entrepreneurs in the economy.

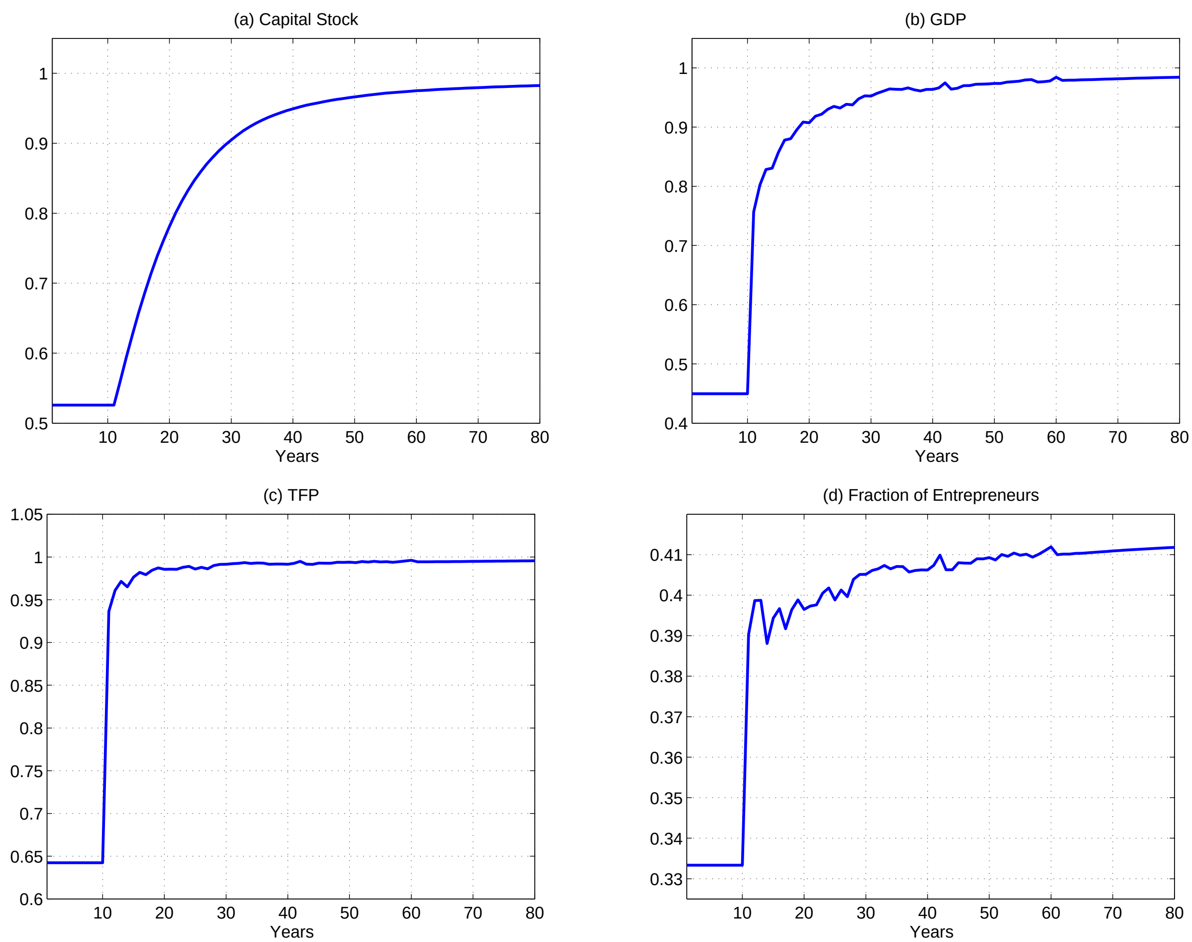

Figure 5: Transition Dynamics: Quantities

Figure 6 presents the time paths for the wage and interest rate. All transition dynamics are due to the endogenous evolution of the wealth distribution. Figure 7 contrasts the initial wealth distribution – the stationary distribution of the moral hazard regime – and the terminal wealth distribution – the stationary distribution of the limited commitment regime. It can be seen that the two are quite different.

7 Conclusion

More research is needed that makes use of micro data and takes seriously the micro financial underpinnings of macro models. One likely reason for the relative scarcity of such studies is

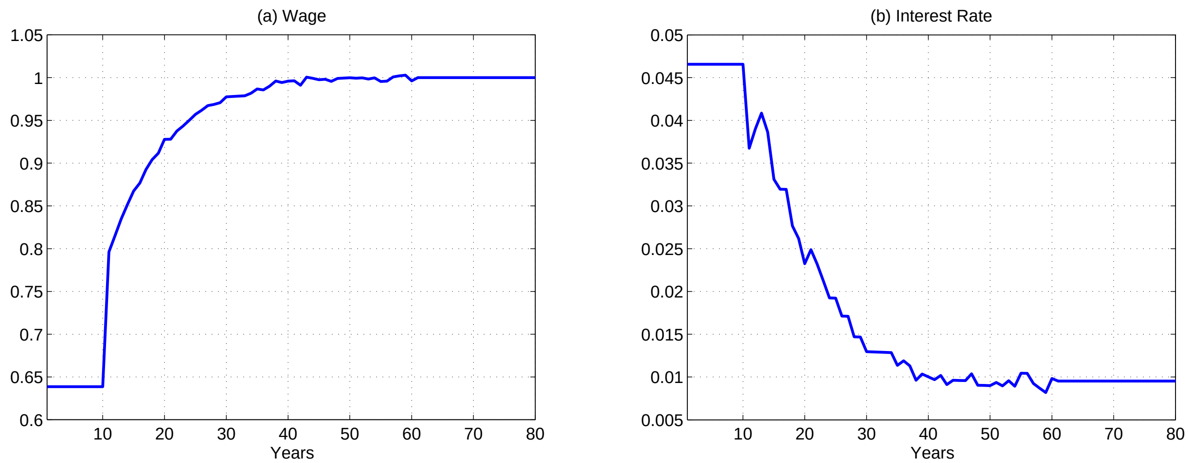

Figure 6: Transition Dynamics: Prices

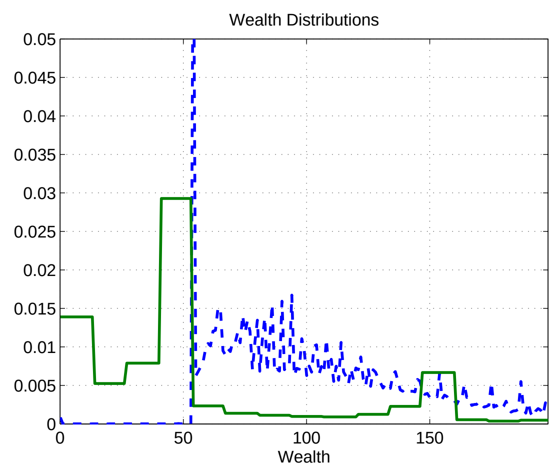

Figure 7: Initial (solid line) and terminal (dashed line) wealth distribution

the lack of reliable balance sheet data for firms, including smaller household-firms and , and household wage earners in developing countries. The Townsend Thai data used by Karaivanov and Townsend (2011) are an exception. The collection of more such data is a very worthwhile project and there are plans to collect such data countrywide. However, an obvious recommendation for Thailand, and for other countries, is to collect the balance sheet data equivalent to manufacturing censuses that many developing are now collecting quite effectively.

Not only does the financial sector matter for real variables, including growth and inequality, but also the details of financial contracts matter for the macro economy. This joins what have been largely two distinct literatures – macro development and micro development – into a coherent whole. The macro development literature needs to take into account the contracts we see on the ground and the micro development literature needs to take into account general equilibrium effects of interventions.

Appendix

A Proof of Lemma 1

The Lagrangean for (3) and (4) is

$$\mathcal{L} = \sum_{\varepsilon} p(\varepsilon|e)\left\{U(c(\varepsilon)) - V(e) + \beta \mathbb{E}_z v[a'(\varepsilon), z']\right\}$$
$$+ \psi\left[(1+r)a + \sum_{\varepsilon} p(\varepsilon|e)\left\{x[z\varepsilon f(k,l) - wl - (r+\delta)k] + (1-x)w\varepsilon\right\} - \sum_{\varepsilon} p(\varepsilon|e)\left\{c(\varepsilon) + a'(\varepsilon)\right\}\right]$$
$$+ \sum_{e,\hat{e},x} \mu(e,\hat{e},x)\left[\sum_{\varepsilon} p(\varepsilon|e)\left\{U(c(\varepsilon)) - V(e) + \beta \mathbb{E}_z v[a'(\varepsilon), z']\right\} - \sum_{\varepsilon} p(\varepsilon|\hat{e})\left\{U(c(\varepsilon)) - V(\hat{e}) + \beta \mathbb{E}_z v[a'(\varepsilon), z']\right\}\right]$$

The first-order conditions with respect to $c(\varepsilon)$ and $a'(\varepsilon)$ are

$$\psi p(\varepsilon|e) = p(\varepsilon|e)U'(c(\varepsilon)) + \sum_{e,\hat{e},x} \mu(e,\hat{e},x)[p(\varepsilon|e) - p(\varepsilon|\hat{e})]U'(c(\varepsilon)) \tag{15}$$

$$\psi p(\varepsilon|e) = p(\varepsilon|e)\beta \mathbb{E}_z v_a(a'(\varepsilon), z') + \sum_{e,\hat{e},x} \mu(e,\hat{e},x)[p(\varepsilon|e) - p(\varepsilon|\hat{e})]\beta \mathbb{E}_z v_a(a'(\varepsilon), z') \tag{16}$$

Rearranging

$$\frac{p(\varepsilon|e)}{U'(c(\varepsilon))} = \frac{1}{\psi}\left[p(\varepsilon|e) + \sum_{e,\hat{e},x} \mu(e,\hat{e},x)[p(\varepsilon|e) - p(\varepsilon|\hat{e})]\right] \tag{17}$$

$$\frac{p(\varepsilon|e)}{\beta \mathbb{E}_z v_a(a'(\varepsilon), z')} = \frac{1}{\psi}\left[p(\varepsilon|e) + \sum_{e,\hat{e},x} \mu(e,\hat{e},x)[p(\varepsilon|e) - p(\varepsilon|\hat{e})]\right] \tag{18}$$

Summing (17) over ε,

$$\sum_{\varepsilon} \frac{p(\varepsilon|e)}{U'(c(\varepsilon))} = \frac{1}{\psi}$$

The envelope condition is

$$v_a(a, z) = \psi(1 + r) = (1 + r) \left(\sum_{\varepsilon} \frac{p(\varepsilon|e)}{U'(c(\varepsilon))} \right)^{-1} \tag{19}$$

From (17) and (18)

$$U'(c(\varepsilon)) = \beta \mathbb{E}_z v_a(a'(\varepsilon), z') \tag{20}$$

Combining (19) and (20) yields (11).□

B Capital Accumulation

The purpose of this section is to spell out in detail how capital accumulation works in our economy. We assume that there is a representative capital producing firm that issues bonds, B_t, and dividends, D_t, invests, I_t, to accumulate capital, K_t which it rents out to households at a rental rate R_t. The budget constraint of capital producer is then

$$B_{t+1} + I_t + D_t = R_t K_t + (1 + r_t) B_t, \quad K_{t+1} = I_t + (1 - \delta) K_t$$

The entire debt of the capital producer is held by the intermediary and hence the debt market clearing condition is

$$B_t + \int a dG_t(a, z) = 0 \tag{21}$$

The capital producer maximizes

$$V_0 = \sum_{t=0}^{\infty} \frac{D_t}{(1 + r)^t}.$$

subject to

$$K_{t+1} + B_{t+1} + D_t = (R_t + 1 - \delta) K_t + (1 + r) B_t \tag{22}$$

It is easy to show that this maximization implies the no arbitrage condition $R_t = r_t + \delta$.[14] Therefore the budget constraint (22) is

$$D_t = (1 + r)(K_t + B_t) - K_{t+1} - B_{t+1}$$

[14]Defining cash-on-hand, $M_t = (R_t + 1 - \delta) K_t + (1 + r) B_t$, the associated dynamic program is

$$V(M) = \max_{K', B'} \; M - K' - B' + (1 + r)^{-1} V[(R' + 1 - \delta) K' + (1 + r') B']$$

The first order conditions imply $R' = r' + \delta$.

and so the present value of profits is

$$V_t = \sum_{s=0}^{\infty} \frac{D_{t+s}}{(1+r)^s} = (1+r)(K_t + B_t) \quad \text{all } t.$$

Zero profits implies $K_t + B_t = 0$ for all t. Using bond market clearing (21), this implies

$$K_t = \int a dG_t(a,z), \quad \forall t.$$

C Wealth vs Promised Utility as a State Variable

The optimal contract in (31) to (32) uses as state variables wealth, a and ability, z. We here show how to derive this contract from a more standard formulation of the dynamic contracting problem where the state variables are promised utility and ability.

C.1 More Standard Formulation with Promised Utility

Consider the following problem: maximize the household's utility

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau, e_\tau)$$

subject to providing profits of at least π_t to the intermediary

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{y_\tau - c_\tau}{(1+r)^{\tau-t}} \geq \pi_t \tag{23}$$

and regime-specific constraints. Note that this is the dual to the perhaps even more standard formulation of maximizing intermediary profits subject to delivering a given level of promised utility to the household. In our formulation instead, we maximize household utility subject to delivering "promised utility" π_t *to the intermediary.*

The associated dynamic problem is:

$$V(\pi, z) = \max_{e,x,c(\varepsilon),\pi'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e) \left\{ u[c(\varepsilon), e] + \beta \mathbb{E} V[\pi'(\varepsilon), z'] \right\} \quad \text{s.t.} \tag{24}$$

$$\sum_{\varepsilon} p(\varepsilon|e) \left\{ u[c(\varepsilon), e] + \beta \mathbb{E} V[\pi'(\varepsilon), z'] \right\} \geq \sum_{\varepsilon} p(\varepsilon|\hat{e}) \left\{ u[c(\varepsilon), \hat{e}] + \beta \mathbb{E} V[\pi'(\varepsilon), z'] \right\} \ \forall e, \hat{e} \tag{25}$$

The "promise-keeping" constraint now says that the optimal contract has to deliver *expected* profits π *to the intermediary*:

$$\sum_{\varepsilon} p(\varepsilon|e) \left\{ x[z\varepsilon f(k,l) - wl - (r+\delta)k] + (1-x)w\varepsilon - c(\varepsilon) + (1+r)^{-1}\pi'(\varepsilon) \right\} = \pi. \tag{26}$$

C.2 Formulation in Main Text, (3)

The budget constraint of a risk-sharing syndicate (1) can be written in present-value form as

$$0 = \pi_t + a_t(1+r), \quad \text{for all } t \quad \text{where} \quad \pi_t \equiv \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{y_\tau - c_\tau}{(1+r)^{\tau-t}} \tag{27}$$

are the intermediary's expected future profits. In the recursive formulation of the dual:

$$\pi = -a(1+r), \quad \pi'(\varepsilon) = -a'(\varepsilon)(1+r) \tag{28}$$

By using this simple relationship, the promise-keeping constraint (26) becomes

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{a'(\varepsilon) - x[z\varepsilon f(k,l) - wl - (r+\delta)k] - (1-x)w\varepsilon\right\} = (1+r)a$$

This is exactly the budget constraint (32) used above. Similarly, use (28) in $V(\pi, z)$ to define

$$v(a,z) = V[-(1+r)a, z].$$

We then arrive at the formulation of the problem in equations (31) to (32). Using wealth as a state variable instead of promised utility is therefore a simple change of variables.

D Numerical Solution: Optimal Contract with Lotteries

When solving the optimal contract under moral hazard (3) and (4) numerically, we allow for lotteries as in Phelan and Townsend (1991). This section formulates the associated dynamic program.

D.1 Simplification

Capital and labor only enter the problem in (3) through entrepreneurial profits. We can make use of this fact to reduce the dimensionality of the problem. Entrepreneurs solve the following profit maximization problem.

$$\max_{k,l} \bar{\varepsilon}(e) z f(k,l) - (r+\delta)k - wl, \quad \bar{\varepsilon}(e) \equiv \sum_{\varepsilon} p(\varepsilon|e)\varepsilon.$$

With the functional form assumption in (10), the first-order conditions are

$$\alpha z \bar{\varepsilon}(e) k^{\alpha-1} l^{\gamma} = r + \delta, \quad \gamma z \bar{\varepsilon}(e) k^{\alpha} l^{\gamma-1} = w$$

These can be solved for the optimal factor demands given effort, e, talent, z and factor prices w and r.

$$k^*(e,z;w,r) = (\bar{\varepsilon}(e)z)^{\frac{1}{1-\alpha-\gamma}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}}$$

$$l^*(e,z;w,r) = (\bar{\varepsilon}(e)z)^{\frac{1}{1-\alpha-\gamma}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{1-\alpha}{1-\alpha-\gamma}}$$

Realized (as opposed to expected) profits are

$$\Pi(\varepsilon,z,e;w,r) = z\varepsilon k(e,z;w,r)^{\alpha} l(e,z;w,r)^{\gamma} - wl(e,z;w,r) - (r+\delta)k(e,z;w,r)$$

Substituting back in from the factor demands, realized profits are

$$\Pi(\varepsilon,z,e;w,r) = \left(\frac{\varepsilon}{\bar{\varepsilon}(e)} - \alpha - \gamma\right)(z\bar{\varepsilon}(e))^{\frac{1}{1-\alpha-\gamma}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \tag{29}$$

The budget constraint in (3) can then be written as

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{c(\varepsilon) + a'(\varepsilon)\right\} = \sum_{\varepsilon} p(\varepsilon|e)\left\{x\Pi(\varepsilon,z,e;w,r) + (1-x)w\varepsilon]\right\} + (1+r)a. \tag{30}$$

D.2 Linear Programming Representation

Denote by

$$d = (c,\varepsilon,e,x)$$

the household's consumption, output, effort, and occupational choice respectively. This is the vector of control variables. A contract between the intermediary and a household specifies a probability distribution over the vector

$$(d,a') = (c,\varepsilon,e,x,a')$$

given (a,z). Denote this probability distribution by $\pi(d,a'|a,z)$. The associated dynamic program then is a linear programming problem where the choice variables are the probabilities $\pi(d,a'|a,z)$:

$$v(a,z) = \max_{\pi(d,a'|a,z)} \sum_{D,A} \pi(d,a'|a,z)\left\{u(c,e) + \beta\mathbb{E}v(a',z')\right\} \quad \text{s.t.} \tag{31}$$

$$\sum_{D,A} \pi(d,a'|a,z)\left\{a'+c\right\} = \sum_{D,A} \pi(d,a'|a,z)\left\{x\Pi(\varepsilon,e,z;w,r) + (1-x)w\varepsilon\right\} + (1+r)a. \tag{32}$$

$$\sum_{(D\backslash E),A} \pi(d,a'|a,z)\left\{u(c,e) + \beta\mathbb{E}v(a',z')\right\} \geq \sum_{(D\backslash E),A} \pi(d,a'|a,z)\frac{p(\varepsilon|\hat{e})}{p(\varepsilon|e)}\left\{u(c,\hat{e}) + \beta\mathbb{E}v(a',z')\right\} \ \forall e,\hat{e},x \tag{33}$$

$$\sum_{T,C,A} \pi(d,a'|a,z) = p(\varepsilon|e) \sum_{T,C,\varepsilon,A} \pi(d,a'|a,z), \quad \forall \varepsilon,e,x \tag{34}$$

(32) is the analogue of (30). The set of constraints (34) are the Bayes consistency constraints.[15]

[15] (34) is derived from the timing of the problem as follows. A lottery with probabilities $\Pr(e,x)$ first determines an occupational choice, x, and an effort, e, for each household. Then a second lottery with probabilities

Variable	grid size	grid range
Wealth, a	30	[0.1, 90]
Productivity, z	20	$[0.08, 0.16]$
Consumption, c	30	$[c_{\min}(w,r), c_{\max}(w,r)]$
Efficiency, q	2	[1, 18]
Effort, e	2	[0, 1]

Table 2: Variable Grids

D.3 Bounds on Consumption Grid

To solve the optimal contracting problem, we follow Prescott and Townsend (1984) and Phelan and Townsend (1991) and constrain all variables to lie on discrete grids. It is necessary to adjust those grids when prices change. In particular, the boundaries of the consumption grid are chosen as

$$c_{\min}(w,r) = ra_{\min} + \max\{\Pi(\varepsilon_{\min}, z_{\min}, e_{\min}; w, r), w\varepsilon_{\min}\},$$

$$c_{\max}(w,r) = ra_{\max} + \max\{\Pi(\varepsilon_{\max}, z_{\max}, e_{\max}; w, r), w\varepsilon_{\max}\},$$

for any given (w,r), and where the profit function Π is defined in (29). These are the minimum and maximum levels of consumption that can be sustained given that $a'(\varepsilon) = a$ in (3). Table 2 lists our choices of grids.

E Algorithm for Computing Transition Dynamics

We construct the market clearing wage and interest rate sequences using the method described in Appendix A.2 of Buera and Shin (2010). Consider period t. The goal is to "find w_t that clears the labor market." Proceed as follows: Fix $w_t^{i,j}, r_t^i$ and $V_{t+1}(\cdot)$ (and therefore $\{w_t, r_t\}_{s=t+1}^T$). Compute $V_t(a,z)$ and the corresponding optimal policy functions $k(a,z;w_t^{i,j}, r_t^i)$, $l(a,z,w_t^{i,j}, r_t^i)$, and $n(a,z;w_t^{i,j}, r_t^i)$. This is relatively fast because it's essentially a static moral hazard problem

$\Pr(c,\varepsilon,a'|e,x)$ determines the remaining variables. Of course, nature plays a role in this second lottery since the conditional probabilities $p(\varepsilon|e)$ are technologically determined. It is therefore required that

$$\sum_{T,C,A} \Pr(c,\varepsilon,a'|e,x) = \Pr(\varepsilon|e,x) = p(\varepsilon|e). \tag{35}$$

We have that

$$\Pr(c,\varepsilon,a'|e,x) = \frac{\pi(c,\varepsilon,e,x,a')}{\sum_{T,C,\varepsilon,A} \pi(c,\varepsilon,e,x,a')} \tag{36}$$

Combining (35) and (36), we have

$$\frac{\sum_{T,C,A} \pi(c,\varepsilon,e,x,a')}{\sum_{T,C,A,\varepsilon} \pi(c,\varepsilon,e,x,a')} = p(\varepsilon|e),$$

which is (34) above.

(i.e. ns linear programs). Then check labor market clearing and update $w_t^{i,j+1}$ using the bisection algorithm. With the new $w_t^{i,j+1}$ recompute the value function still taking as given $V_t(\cdot)$, and so on...

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