Aggregate Unemployment and Household Unsecured Debt*

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Abstract

This paper studies the relationship between the availability of unsecured credit to households and unemployment. We extend the Mortensen-Pissarides model to include a goods market with search and financial frictions. Households, who have limited commitment, face endogenous borrowing constraints when financing random consumption opportunities. We show that borrowing limits depend on the sophistication of the financial system, the frequency of liquidity shocks, and the rate of return on (partially) liquid assets that households can accumulate for self insurance. Moreover, firms' expected productivity is endogenous and depends on firms' market power in the goods market and the availability of unsecured credit to consumers. As a result of the complementarity between credit and labor markets, multiple steady states might exist. Across steady states unemployment and debt limits are negatively correlated. We calibrate the model to the U.S. labor and credit markets and illustrate the effects of an expansion in unsecured debt similar to that seen in the U.S. from 1980 to 2008. Under the baseline calibration, the rise in unsecured credit can account for approximately three quarters of the decline in the long-term average unemployment rate.

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"Few doubt the importance of consumer spending to the U.S. economy and its multiplier effect on the global economy, but what is underappreciated is the role of credit-card availability in that spending. Currently, there is roughly $5 trillion in credit-card lines outstanding in the U.S., and a little more than $800 billion is currently drawn upon. While those numbers look small relative to total mortgage debt of over $10.5 trillion, credit-card debt is revolving and accordingly being paid off and drawn down over and over, creating a critical role in commerce in America". Wall Street Journal, March 10, 2009, *Credit cards are the next credit crunch.*

1 Introduction

Average household balances on unsecured loans more than tripled from 1980 to 2007, from roughly 3 to 10 percent of consumption (see Figure 1).[1] In 2007, more than 73 percent of all U.S. households had at least one credit card and roughly 50 percent of all households carried outstanding balances on these accounts.[2] Evidence suggests that unsecured debt has become easier to obtain and limits on credit cards have become increasingly more generous. The expansion of unsecured credit over this time period coincides with a decrease in the share of liquid assets among all assets held by households and a long-term decrease in the unemployment rate.

These trends were abruptly reversed following the 2007-08 financial crisis: the unemployment rate increased from about 4.5 to 10 percent while households use of unsecured credit and liquid assets returned to their 1995 level. These recent changes have led many commentators to speculate about the relationship between the recent credit crunch and the slow recovery and high levels of unemployment following the recession.[3]

The objective of this paper is to provide a tractable dynamic general equilibrium model with trading frictions in which to analyze the relationship between household unsecured debt, liquid assets, and unemployment and the joint behavior of labor and credit markets, both qualitatively and quantitatively. Our

[1]Unsecured debt or non-collateralized debt refers to loans that are not tied to any asset. Unsecured debt primarily consists of revolving accounts, such as credit card loans, see Sullivan (2005).

[2]Source: 2007 Survey of Consumer Finances.

[3]See, e.g., the article in the New-York Times of October 29, 2008, titled "As U.S. economy slows, credit card crunch begins" or the article in the Wall Street Journal of March 10, 2009, titled "Credit cards are the next credit crunch."

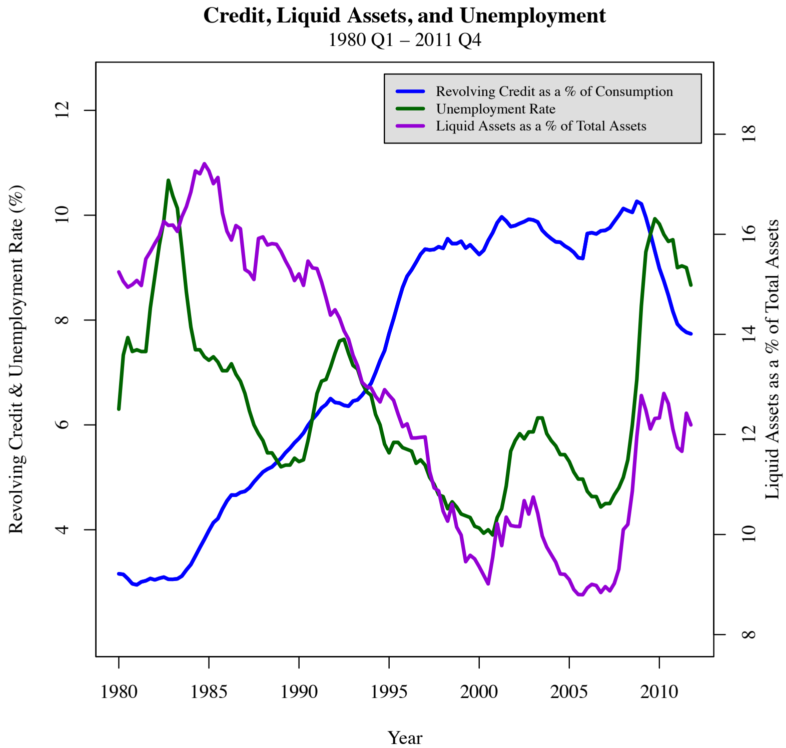

Figure 1: Unemployment, Revolving Credit, and Liquid Assets

starting point is the canonical model of equilibrium unemployment by Mortensen and Pissarides (1994)—MP hereafter. While this model is explicit about the search-matching frictions that prevail in the labor market, trades in the goods market are assumed to be seamless: firms' output can be sold instantly, households have no need for borrowing (and if they do, repayment can be enforced), and there is no role for liquidity. In order to describe household unsecured credit and its relation to labor market outcomes, we incorporate a retail goods market with search frictions and limited commitment—along the lines of Diamond (1987, 1990) and Shi (1996).

The model assumes that frictional labor and goods markets open sequentially, as in Berentsen, Menzio, and Wright (2011). As in MP firms that enter the labor market post vacancies and unemployed workers look for jobs according to a time-consuming process. The output produced by firms can then be sold in a decentralized goods market where retailers and households are matched bilaterally, and households use liquid assets and unsecured debt to finance their purchases. The matching shocks in the decentralized goods market are analogous to liquidity shocks in banking models, except that the frequency of these shocks is endogenous in our analysis. Unsold inventories are traded in a frictionless competitive market where households have linear utility, as in the original MP model. Households value consumption in the decentralized retail market more than they value consumption in the competitive market—e.g., because goods in the retail market can be tailored to the consumer needs—and firms have some market power that allows them to charge a price higher than their marginal cost.

Following Kehoe and Levine (1993), households in the decentralized goods market face endogenous borrowing constraints because they cannot commit to repay their debt—the repayment of the debt must be self-enforcing. In order for unsecured credit arrangements to be incentive feasible some form of punishment must take place if an agent defaults on its obligations. If agents are anonymous and their trading histories are private information, agents cannot be punished from reneging on their debt. Therefore, we will assume that an imperfect record-keeping technology is available that keeps track of defaulting individuals and that makes this information publicly available. If a household defaults, and if default is publicly recorded, then the household is excluded permanently from credit arrangements. The endogenous debt limit that results from this threat increases as households become more patient, as the frequency of trade increases, but it decreases if firms have a higher market power.

We will assume that households are heterogeneous in terms of their access to unsecured credit. Some households' histories of default can be publicly recorded, and therefore these households can be trusted to repay their debt. Other households cannot be monitored or are seen as untrustworthy: those households are not able to borrow. Irrespective of their access to credit, we give the possibility to all households to accumulate liquid assets—assets that can be used in the decentralized goods market as means of payment or collateral to secure one's debt. However, these assets are costly to hold in the sense that their rate of return is less than the rate at which households discount future utility.

We show that the use of liquid assets as means of payment can coexist with the use of unsecured credit.

So our model addresses the challenge in monetary theory of generating coexistence of money—which requires lack of record-keeping—and credit—which requires some record-keeping. For such equilibria to exist the rate of return of liquid assets cannot be too high—so that credit is a better option than money and the punishment from defaulting is sufficiently severe—and it cannot be too low—so that some households have incentives to accumulate liquid assets. Generically, households specialize in their methods of payment. Households with access to unsecured credit do not hold liquid assets and finance their purchases with credit only. Households without access to credit only hold liquid assets as a way to insure themselves against idiosyncratic spending shocks in the retail goods market.

Our model generates the following two-way interaction between the credit and labor markets. First, the aggregate state of the labor market affects the debt limit through the frequency of trade in the retail goods market. Indeed the number of firms participating in the labor market determines the number of retailers in the decentralized goods market. If the labor market is tight—there are a lot of active firms producing output to be sold in the retail market—then households receive frequent opportunities to consume, which makes exclusion from unsecured credit very costly. As a result, an increase in aggregate employment tends to raise households's debt limits. Second, firms' decisions to open vacancies depend on expected sales in the decentralized goods market, which itself depends on the availability of unsecured credit to households. If households can borrow large amounts, then firms can sell a larger fraction of their output in the decentralized goods market where they can charge a positive markup, which raises their expected revenue. Therefore, an increase in debt limits promotes job creation and reduces unemployment.

In the spirit of Kehoe and Levine (2001) we study analytically two limiting economies: one where households have no access to unsecured credit but can accumulate liquid assets to insure themselves against idiosyncratic trading opportunities in the decentralized market; a second one without liquid assets but where agents can borrow up to some endogenous limit. The former case can be interpreted as the pure monetary economy of Berentsen, Menzio, and Wright (2011). In accordance with Rocheteau and Wright (2005, 2013) such an economy can have multiple steady-state equilibria because of the complementarity between households' choice of liquid assets and firms' participation decisions. Moreover, an increase in the rate of return of liquid assets raises output and decreases unemployment at the highest equilibrium.

The second case, which corresponds to a pure credit economy, also has multiple steady-state equilibria. Across equilibria, unemployment and credit limits are negatively correlated. In one equilibrium there is a higher market tightness, i.e., a lower unemployment rate, as well as a higher credit limit. Intuitively, if there are a lot of producers in the market, then the punishment for not repaying one's debt, i.e., the exclusion from the goods market, is large since households would have to forgive a large number of trading opportunities in the future. As default is more costly, the household debt limit increases and, as a result, firms can expect large sales in the decentralized goods market. By a symmetric logic the second equilibrium is one with a high unemployment rate and a low credit limit. Therefore a high unemployment equilibrium with a credit crunch is self-fulfilling.

We calibrate a version of the model to match the U.S. economy pre-Great Recession. We illustrate the equilibrium effects of a reduction in the availability of unsecured credit that match the empirical facts in Figure 1. From 1980 to 2010 unsecured debt increased from 2% to 10% of total consumption spending. Under the baseline calibration, the model predicts that steady state unemployment was 1.7 percentage points higher in 1980. This matches approximately 70% of the movement in trend unemployment between these two time periods. We additionally consider the effects of the credit crunch between 2007 and 2010 in which unsecured debt fell from 10% of consumption to 8%. The model predicts an increase in steady state unemployment of .4 percentage points, from 5.13% to 5.53%. This corresponds to approximately 10% of the total increase in unemployment from 2007 to 2010. The results suggest that the prevalence of unsecured credit can explain long-term movements in the efficiency of the labor market, but are not enough, by itself, to explain short term fluctuations as that seen since 2007.

1.1 Literature

Pairwise credit in a search-theoretic model was first introduced by Diamond (1987a,b, 1990). The environment is similar to Diamond (1982), where agents are matched bilaterally and trade indivisible goods. The punishment for not repaying a loan is permanent autarky. The role of record-keeping technologies to sustain some forms of credit arrangements has been emphasized by Kocherlakota (1998). Kocherlakota and Wallace

(1998) consider the case of an imperfect record-keeping technology where the public record of individual transactions is updated after a probabilistic lag. Nosal and Rocheteau (2011, Chapter 2) and McAndrews, Nosal, and Rocheteau (2011) describe pure credit economies in quasi-linear environments similar to the one in this paper. Sanches and Williamson (2010) were the first to introduce limited commitment to study the coexistence of money and pairwise credit in the Lagos-Wright environment. Gomis-Porqueras and Sanches (2013) study optimal monetary policy in a version of the Sanches-Williamson model focusing on incentive-feasible schemes where all trades are voluntary, including the ones with the government. Gu, Mattesini, Monnet and Wright (2012a,b) study banking and endogenous credit cycles in this type of environment.

Our paper is also closely related to the literature on unemployment and money, e.g., Shi (1998). Our model has a similar structure as in Berentsen, Menzio, and Wright (2011) that extends the quasi-linear environment of Lagos and Wright (2005) to include a frictional labor market. In Rocheteau, Rupert, and Wright (2007) only the goods market is subject to search frictions but unemployment emerges due to indivisible labor. In all these models credit is not incentive feasible. In contrast we introduce an imperfect record keeping technology to make unsecured credit incentive feasible and to allow for the coexistence of liquid assets and credit. Liquid assets are formalized via a storage technology as in Lagos and Rocheteau (2008).

There is a related literature studying unemployment and financial frictions that affect the financing of firms. Wasmer and Weil (2004) add a credit market with search-matching frictions in the Mortensen-Pissarides model. They assume that firms search for investors in order to finance the cost of opening a vacancy. Versions of this model have been calibrated by Petrosky-Nadeau and Wasmer (2012) and Petrosky-Nadeau (2012). Our model differs from that literature in that credit frictions affect households, they take the form of limited commitment instead of search frictions between lenders and borrowers, and a decentralized goods market where unsecured credit is formalized explicitly.

Finally, our paper is related to recent work by Herkenhoff (2013) that focuses on the role of revolving credit as a self-insurance mechanism against unemployment shocks. This mechanism delivers a positive aggregate relationship between unemployment and credit as easy credit conditions lead to a smaller consumption decline upon job loss, higher reservation wages, and therefore higher unemployment. We do not explore this channel in our model, but instead focus on the aggregate relationship between credit and unemployment through its effect on firm productivity.

2 Environment

The set of agents consists of a $[0,1]$ continuum of households and a large continuum of firms. Time is discrete and goes on forever. Each period of time is divided into three stages. In the first stage, households and firms trade indivisible labor services in a labor market (LM) subject to search and matching frictions. In the second stage, they trade consumption goods in a decentralized retail market (DM) with search frictions. In the last stage, debts are settled, wages are paid, and households and firms can trade assets and goods in a competitive market (CM). We take the good traded in the CM as the numéraire good. The sequence of markets in a representative period is summarized in Figure 2.

Figure 2: Timing

The lifetime utility of a household is given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\ell(1-e_t)+\upsilon(y_t)+c_t\right], \tag{1}$$

where $\beta=1/(1+r)\in(0,1)$ is a discount factor, $y_t\in\mathbb{R}_+$ is the consumption of the DM good, $c_t\in\mathbb{R}$ is the consumption of the numéraire good (we interpret $c<0$ as production), and $e_t\in\{0,1\}$ represents the (indivisible) time devoted to work in the first stage, so that ℓ can be interpreted as the utility of leisure or home production.[4] The utility function in the DM, $\upsilon(y_t)$, is twice continuously differentiable, strictly

[4]One can interpret a negative consumption, $c<0$, as borrowing across CMs with perfect enforcement. Alternatively, one can make assumptions to guarantee that $c\geq 0$ always holds. For instance, if households receive exogenous endowments at the beginning of each CM, then one can choose the size of these endowments so that consumption is non-negative.

increasing, and concave. Moreover, $\upsilon(0) = 0$ and $\upsilon'(0) = \infty$. (We need υ to be bounded below to have a well-defined bargaining problem in the DM.)

Each firm is composed of one job. In order to participate in the LM at t, a firm must advertise a vacant position, which costs $k > 0$ units of the numéraire good at $t - 1$.[5] The measure of matches between vacant jobs and unemployed households is given by $m(s_t, o_t)$, where s_t is the measure of job seekers and o_t is the measure of vacant firms (job openings). The measure of job seekers in t is equal to the measure of unemployed households at the end of $t-1$, $s_t = u_{t-1}$. The matching function, m, has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $m(0, o) = m(s, 0) = 0$ and $m(s, o) \leq \min(s, o)$. The job finding probability of an unemployed worker is $p_t = m(s_t, o_t)/s_t = m(1, \theta_t)$ where $\theta_t \equiv o_t/s_t$ is referred to as labor market tightness. We assume that $\lim_{\theta_t \to +\infty} m(1, \theta_t) = 1$, i.e., the job finding probability approaches one when market tightness goes to infinity. The vacancy filling probability for a firm is $f_t = m(u_t, o_t)/o_t = m(1/\theta_t, 1)$. We assume that $\lim_{\theta_t \to 0} m(1/\theta_t, 1) = 1$, i.e., the vacancy filling probability approaches one when market tightness goes to zero.

An existing match is destroyed at a beginning of a period with probability $\delta \in (0, 1)$. A household who is employed in the LM receives a wage in terms of the numéraire good, w, paid in the subsequent CM. A household who is unemployed in the LM receives income in terms of the numéraire good, b, interpreted as unemployment benefits.

Each filled job produces $\bar{z} > 0$ units of goods in the first stage. These $\bar{z}$ units are divided between some (endogenous) amount $y_t \in [0, \bar{z}]$ sold in the DM and the rest, $\bar{z} - y_t$, sold in the CM. This makes y the opportunity cost of selling y in the DM. Let y^* solve $\upsilon'(y^*) = 1$. We assume that $y^* \in (0, \bar{z})$. The assumption $\upsilon'(y) > 1$ for all $y \in [0, y^*)$ captures the notion that households value the opportunities to consume early in the DM more than CM consumption. Therefore, in line with the banking literature, one can interpret an opportunity to consume in the DM as a liquidity shock.

The DM goods market has a similar structure as the LM in that it involves bilateral random matching between retailers (firms) and consumers (households).[6] The matching probabilities for households and firms

[5]We take the view of a recruiting technology that is not labor intensive so that k does not need to be linked to equilibrium wages.

[6]The description of the goods market with search frictions follows Diamond's (1982, 1990) model, except that we assume

are $\alpha(n_t)$ and $\alpha(n_t)/n_t$, respectively, where $n_t = 1 - u_t$ is the measure of participating firms. So $\alpha(n_t)$ is a measure of the frequency of the liquidity shocks in the DM. We assume $\alpha'(n) > 0$, i.e., a tight labor market implies a high frequency of trading opportunities in the goods market. Moreover, $\alpha''(n) < 0$, $\alpha(n) \leq \min\{1, n\}$, $\alpha(0) = 0$, $\alpha'(0) = 1$ and $\alpha(1) \leq 1$.

Households in the DM lack commitment. Therefore, firms are willing to extend credit to households only if the repayment of debt in the subsequent CM is self-enforcing.[7] If a household defaults on its debt, such default is recorded publicly with probability $\rho \in [0, 1]$. The parameter ρ can be interpreted as a measure of the sophistication of the financial system.[8] We assume that only a fraction, ω, of households can be monitored and have access to credit. The remaining households cannot borrow.[9] Moreover, as in Wallace (2005), a monitored household can choose to become unmonitored at any point in time.

Finally, there is a technology allowing households to store goods: each unit of the numéraire good invested in the storage technology at t yields $R < 1 + r$ units of the numéraire good at $t + 1$. (For a similar storage technology, see Lagos and Rocheteau, 2008.) Stored goods (or claims on these goods) are perfectly divisible and portable and can be carried into the DM. If a household has a units of stored goods in the DM, it can transfer up to a fraction $\nu \in [0, 1]$ as means of payment. Therefore, we think of stored goods as (partially) liquid assets, and the parameter ν can be interpreted as the moneyness of the asset. These assets could include currency, demand deposits, shares of mutual funds, and even home equity.[10] The opportunity cost

a constant-return-to-scale matching function. The framework has been adopted in the monetary literature since Kiyotaki and Wright (1989). This description has several advantages for our purpose. First, the notion of credit seems more natural in the context of a market with bilateral relationships. Second, search frictions rationalize the presence of unsold goods (inventories). Third, search frictions generate an endogenous frequency for random consumption opportunities that depends on the state of the labor market. This frequency will play an important role in the determination of credit limits.

[7] Even though we assume that firms lend directly to households, one can think of firms as consolidating the roles of a producer and a financial intermediary such as a bank.

[8] Sanches and Williamson (2010) adopt a similar assumption whereby a fraction of sellers has no monitoring potential. Our assumption treats firms/sellers symmetrically. Williamson (2011) captures the imperfection of the record-keeping mechanism by assuming that sellers have access to a buyer's history only with a given probability. Uninformed sellers might find it optimal to extend credit to buyers, which allows for the possibility of default in equilibrium.

[9] The fact that a group of households does not have access to credit can also be an equilibrium outcome even if those households can be monitored. For instance, if firms believe that some households will not repay their debts, then they won't lend to those households, and it becomes optimal for them not to repay their debts since they are already excluded from future credit transactions.

[10] The partial acceptability of assets due to private information frictions is formalized in Rocheteau (2011), Lester, Postlewaite, and Wright (2012), and Li, Rocheteau, and Weill (2012). Petrosky-Nadeau and Rocheteau (2013) describe a similar economy with a two-sector labor market, a housing market, and home equity loans in the DM retail market to finance consumption. Sanches and Williamson (2010) describe an economy with money and credit. In contrast to our model, engineering a positive return on money requires taxation, and agents can default on their tax liabilities.

of holding liquid assets is $R-(1+r)$, and it can be viewed as a policy parameter.[11]

3 Equilibrium

In the following we focus on steady-state equilibria in which labor market outcomes and loan contracts in the DM are constant over time. We characterize an equilibrium by moving backward from the description of households's choices in the centralized market (CM), to the determination of prices and quantities in the retail goods market (DM), and finally the entry of firms and the determination of wages in the labor market (LM).

3.1 Settlement and competitive markets (CM)

Let $W_e(d,a)$ denote the lifetime expected utility of a monitored household in the CM with debt d from the previous DM, in units of the numéraire good, and a units of liquid assets, where $e \in \{0,1\}$ indicates the labor market status ($e=0$ if the household is unemployed and $e=1$ otherwise). We assume that the debt issued in the DM is repaid in the CM and is not rolled over across periods.[12] Similarly, let U_e be the household's value function in the LM. Then we have

$$W_e(d,a) = \max_{c,a' \geq 0} \{c + (1-e)\ell + \beta U_e(a')\} \tag{2}$$

$$\text{s.t.} \quad c + d + a' = ew + (1-e)b + Ra + \Delta - T. \tag{3}$$

The first two terms between brackets in (2) are the utility of consumption and the utility of leisure, the third term is the continuation value in the next period. Thus, from (2)-(3) the household chooses its net consumption, c, and liquid assets, a', in order to maximize its lifetime utility subject to a budget constraint. The left side of the budget constraint, (3), is composed of the household's consumption, the repayment of its debt, and its purchase of liquid assets, a'. The right side is the household's income associated with its labor status (w if employed and b if unemployed), the gross return of its beginning-of-period liquid assets

[11] Gomis-Porqueras and Sanches (2013) provide a careful analysis of the optimal monetary policy in a related model of money and credit where sellers are heterogeneous in terms of their access to a costly record-keeping technology.

[12] We can extend our theory to allow for debt to be rolled over across periods as follows. A debt contract is defined by a CM payment, Δ, and a probability, $1-\varrho$, that debt is exogenously extinguished. So ϱ can be interpreted as the probability that debt is revolved. Also, if the household gets hit with a liquidity shock, with probability α, the debt contract is terminated. The expected discounted value of this contract is $(1+r)\Delta/[1+r-\varrho(1-\alpha)]$. Setting this value equal to d, we obtain the CM payment, $\Delta = [1-\varrho(1-\alpha)/(1+r)]\, d$.

(Ra), and the profits of the firms (Δ), minus taxes (T). We substitute c from (3) into (2) to obtain

$$W_e(d,a) = Ra - d + ew + (1-e)(\ell + b) + \Delta - T + \max_{a' \geq 0} \left\{ -a' + \beta U_e(a') \right\}. \tag{4}$$

From (4) the value function of the household in the CM is linear in its wealth, $Ra - d$. Moreover, from the linear preferences in CM, the choice of liquid assets for the next period, a', is independent of the assets held at the beginning of the period, a.

Consider next the value function of a household with no access to credit—either the household defaulted on its debt in the past and this default was publicly recorded, or it cannot be monitored. We focus on equilibria where the household is excluded from credit permanently. Firms have no incentive to lend to this household as they anticipate that it would default on its loan; and it is rational for the household to default on its loan as it doesn't expect to get any loans in the future. The value of a household with no access to credit is

$$\tilde{W}_e(a) = Ra + ew + (1-e)(\ell + b) + \Delta - T + \max_{a' \geq 0} \left\{ -a' + \beta \tilde{U}_e(a') \right\}, \tag{5}$$

where $\tilde{U}_e(a')$ is the LM value function of a household with no access to credit. We assume and verify later that the wage paid to a household is independent of its access to credit.

Finally, the expected discounted profits of a firm in the CM with x units of inventories, d units of household's debt, a units of liquid assets, and a promise to pay a wage w, are

$$\Pi(x,d,a,w) = x + d + Ra - w + \beta(1-\delta)J. \tag{6}$$

A firm with x units of inventories can sell x units of numéraire good; the d units of household's debt are worth d units of numéraire good (since there will be no default in equilibrium), and the a units of liquid assets are worth Ra units of numéraire good. So $x + d + Ra$ is the real value of the sales made by the firm within a period across the DM and CM. If the firm remains profitable, with probability $1 - \delta$, then the expected profits of the firm at the beginning of the next period are J.

3.2 Retail goods market (DM)

Consider a match between a firm and a household who holds a units of liquid asset in the DM goods market. A contract is a triple, (y, d, τ), that specifies the output produced by the firm for the household, y, the unsecured debt to be repaid by the household in the next CM, d, and the transfer of liquid assets, τ. The terms of the contract are determined by Kalai's (1977) proportional bargaining solution with $\mu \in [0,1]$ denoting the household's share.[13] This trading mechanism guarantees that the trade is (pairwise) Pareto efficient and it generates an endogenous markup (if $\mu < 1$). The solution is given by:

$$(y, d, \tau) = \arg\max_{y,d,\tau} \left[\upsilon(y) + W_e(d, a - \tau) - W_e(0, a)\right] \tag{7}$$

$$\text{s.t.} \quad \upsilon(y) + W_e(d, a - \tau) - W_e(0, a) = \frac{\mu}{1-\mu}\left[\Pi(\bar{z} - y, d, \tau, w) - \Pi(\bar{z}, 0, 0, w)\right]. \tag{8}$$

According to (7)-(8) the terms of the contract are chosen so as to maximize the household's surplus subject to the constraint that this surplus is equal to $\mu/(1-\mu)$ times the surplus of the firm. The surplus of the household is defined as its utility if a trade takes place, $\upsilon(y) + W_e(d, a - \tau)$, less the utility it obtains if the firm and the household fail to reach an agreement, $W_e(0, a)$. The surplus of the firm is defined in a similar way. From (8) if the firm sells y units of output in the DM its inventories in the following CM are $\bar{z} - y$. The problem (7)-(8) is subject to the debt constraint, $d \leq \bar{d}$, i.e., the household cannot borrow more than a limit, $\bar{d}$, arising from households's lack of commitment, and the feasibility constraint, $\tau \leq \nu a$, i.e., the household cannot transfer more than a fraction of its (partially-)liquid assets.

In Figure 3 we represent graphically the solution to the bargaining problem, where S^F indicates the surplus of the firm and S^H the surplus of the household. Notice that the Pareto frontier of the bargaining set is concave, and it is linear when the match surplus is maximum, i.e., $y = y^*$.[14] Graphically, the solution is at the intersection of the Pareto frontier and the line indicating the relative shares of the household and the firm in the match surplus. To see graphically the role of liquidity in this model, notice that as a or $\bar{d}$ increases the Pareto frontier shifts outward and gets closer to the dashed line. So an increase in liquidity enlarges the set of payoffs that are incentive feasible.

[13] The proportional bargaining solution has several desirable features. First, it guarantees the value functions are concave in the holdings of liquid assets. Second, the proportional solution is monotonic (each player's surplus increases with the total surplus), which means households have no incentive to hide some assets. These results cannot be guaranteed with Nash bargaining (Aruoba, Rocheteau and Waller 2007). Dutta (2012) provides strategic foundations for the proportional bargaining solution.

[14] For the derivation of the Pareto frontier, see Aruoba, Rocheteau, and Waller (2007, Section 3.1).

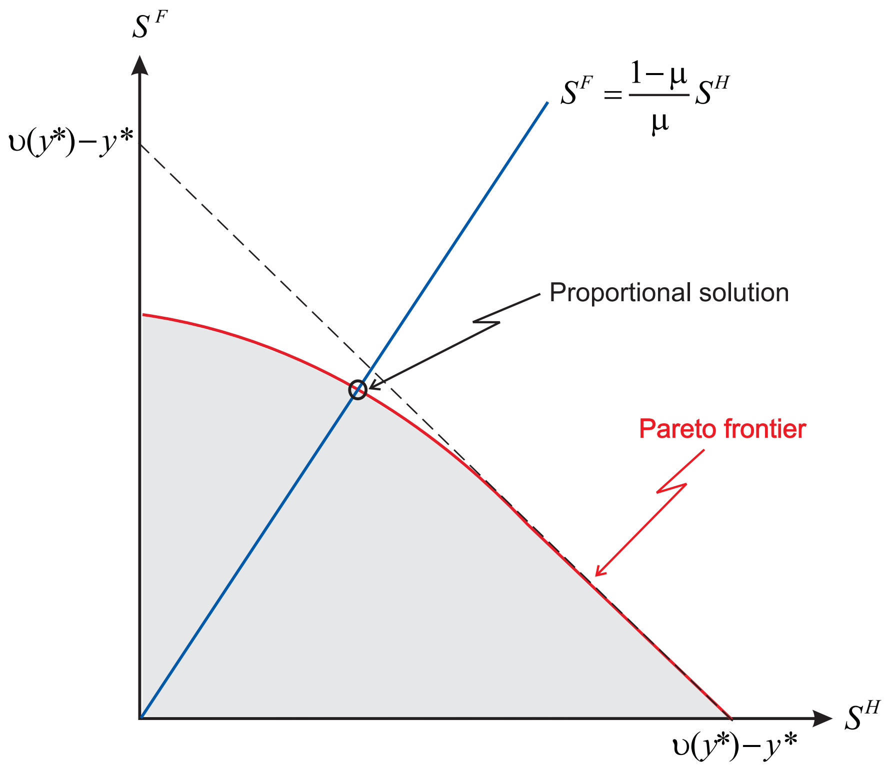

Figure 3: Proportional bargaining in the DM

Using the linearity of W and Π, the solution to the DM pricing problem becomes

$$(y, d, \tau) = \arg\max_{y,d,\tau} [\upsilon(y) - d - R\tau] \tag{9}$$

$$\text{s.t. } d + R\tau = (1 - \mu)\, \upsilon(y) + \mu y \tag{10}$$

$$d \leq \bar{d}, \quad \tau \leq \nu a \tag{11}$$

The bargaining problem can be simplified further by substituting $d + R\tau$ from (10) into (9) to obtain

$$y = \arg\max_{y} \mu \left[\upsilon(y) - y\right] \tag{12}$$

$$\text{s.t. } \quad d + R\tau = (1 - \mu)\upsilon(y) + \mu y \leq \bar{d} + R\nu a. \tag{13}$$

According to (13) the transfer of wealth from the household to the firm is a non-linear function, $(1-\mu)\upsilon(y)+\mu y$, of the output produced by the firm. The price of one unit of DM output is $1+(1-\mu)\left[\upsilon(y)-y\right]/y$, where the second term can be interpreted as the average markup over cost. Given this non-linear pricing rule, output is chosen to maximize the household's surplus, which is a fraction of the total surplus of the match. The solution to the bargaining problem is $y=y^*$ if $(1-\mu)\upsilon(y^*)+\mu y^* \leq \bar{d}+R\nu a$ and $(1-\mu)\upsilon(y)+\mu y=\bar{d}+R\nu a$ otherwise. So provided that the household has enough payment capacity, agents trade the first-best level of output, y^*. If the payment capacity of the household is not large enough, the household borrows up to its limit. If the household does not have access to credit, because it cannot be monitored or has a recorded history of default, then $d=\bar{d}=0$.

The expected discounted utility of a household in the DM is

$$\begin{aligned} V_e(a) &= \alpha(n)\left[\upsilon(y)+W_e\left(d,a-\tau\right)\right]+\left[1-\alpha(n)\right]W_e(0,a) \\ &= \alpha(n)\mu\left[\upsilon\left(y\right)-y\right]+W_e(0,a), \end{aligned} \tag{14}$$

where the terms of trade, (y,d,τ), depend on the household's debt limit and holdings of liquid asset as indicated by the bargaining problem, (12)-(13). According to the first equality in (14), the household is matched with a firm with probability $\alpha(n)$, in which case the household purchases y units of output against d units of debt and τ units of liquid asset. With probability, $1-\alpha(n)$, the household does not have any trading opportunity in the DM, and hence it enters the CM without any debt. The second equality in (14) is obtained by using the linearity of W_e. It says that if the household is matched, with probability $\alpha(n)$, then it enjoys a fraction μ of the match surplus. Similarly, the expected lifetime utility of a household with no access to credit is given by

$$\tilde{V}_e(a)=\alpha(n)\mu\left[\upsilon\left(\tilde{y}\right)-\tilde{y}\right]+\tilde{W}_e(a). \tag{15}$$

According to (15) if the household does not have access to unsecured credit, then it can only spend its liquid assets and consume the quantity, $\tilde{y}$, obtained from the bargaining problem, (12)-(13), with $\bar{d}=0$. Here, we have assumed that a household who defaulted on its debt in the past can choose to be nonmonitored and therefore cannot be excluded from pure monetary trades.

3.3 Labor market (LM)

Households Consider a household who is employed at the beginning of a period. Its lifetime expected utility is

$$U_1(a) = (1-\delta)V_1(a) + \delta V_0(a). \tag{16}$$

With probability, $1-\delta$, the household remains employed ($e = 1$) and offers its labor services to the firm in exchange for a wage in the next CM. With probability, δ, the household loses its job and becomes unemployed ($e = 0$), in which case it will not have a chance to find another job before the next LM in the following period. Substituting V_1 and V_0 by their expressions given by (14),

$$U_1(a) = \alpha(n)\mu\left[\upsilon\left(y\right) - y\right] + (1-\delta)W_1(0,a) + \delta W_0(0,a). \tag{17}$$

The household enjoys an expected surplus in the goods market equal to the first term on the right side of (17). The last two terms are the household's continuation values in the CM depending on its labor status.

The expected lifetime utility of a household who is unemployed at the beginning of the period is

$$U_0(a) = (1-p)V_0(a) + pV_1(a), \tag{18}$$

where p is the job finding probability. Substituting V_1 and V_0 by their expressions given by (14),

$$U_0(a) = \alpha(n)\mu\left[\upsilon\left(y\right) - y\right] + (1-p)W_0(0,a) + pW_1(0,a). \tag{19}$$

By a similar reasoning the value functions for households with no access to credit, $\tilde{U}_e$, solve

$$\tilde{U}_1(a) = \alpha(n)\mu\left[\upsilon\left(\tilde{y}\right) - \tilde{y}\right] + (1-\delta)\tilde{W}_1(a) + \delta\tilde{W}_0(a) \tag{20}$$

$$\tilde{U}_0(a) = \alpha(n)\mu\left[\upsilon\left(\tilde{y}\right) - \tilde{y}\right] + (1-p)\tilde{W}_0(a) + p\tilde{W}_1(a), \tag{21}$$

where $\tilde{y}$ is the DM consumption of a household with no access to credit, $\bar{d} = 0$.

Firms Free entry of firms implies that the cost of opening a vacancy must be equal to the probability of filling the vacancy in the next LM times the discounted value of a filled job, $k = \beta f J$ (assuming there is entry), where $J = \mathbb{E}\left[\Pi(x,d,\tau,w)\right]$ is the expected discounted profits of a filled job. It satisfies

$$J = z - w + \beta(1-\delta)J, \tag{22}$$

where z is the firm's expected revenue in both the DM and CM expressed in numéraire good,

$$z = \bar{z} + \frac{\alpha(n)}{n}(1-\mu)\left\{\omega\left[\upsilon\left(y\right)-y\right]+(1-\omega)\left[\upsilon\left(\tilde{y}\right)-\tilde{y}\right]\right\}. \tag{23}$$

From (22) the value of a filled job is equal to the expected revenue of the firm net of the wage plus the expected discounted profits of the job if it is not destroyed, with probability $1-\delta$. When writing the revenue of the firm in (23) we have conjectured that the level of output traded in a match is identical across households with the same debt limit irrespective of their labor status. If a firm is successful in selling some of its output in the retail market, with probability $\alpha(n)/n$, then it receives a payment (d,τ) but forgoes y in the CM. Therefore, its received payments increase by $d+R\tau-y$, which from (13) is equal to $(1-\mu)\left[\upsilon\left(y\right)-y\right]$. Solving for J we obtain

$$J = \frac{z-w}{1-\beta(1-\delta)}. \tag{24}$$

The value of a job is equal to the discounted sum of the profits where the discount rate is adjusted by the probability of job destruction.

Wage The wage is determined by bargaining between the household and the firm. As is standard in the literature, we adopt Nash/Kalai bargaining as our solution. The wage is set to divide the match surplus according to the following rule, $V_1(a)-V_0(a)=\lambda J/(1-\lambda)$, where $\lambda\in[0,1]$ is the household's bargaining power in the labor market. The firm's surplus, J, is given by (24). We conjecture that employed and unemployed workers face the same debt limit and hold the same quantity of assets. As a result, from (14) the surplus of a household from being employed, $V_1(a)-V_0(a)=W_1(0,0)-W_0(0,0)$, is independent of the household's asset holdings or borrowing capacity. Therefore, we will assume that the household holds its optimal level of liquid assets and we will omit this argument in the value functions. From (4), (14), and (16) the value of an employed household solves

$$V_1 = w + \varpi + \beta\left[(1-\delta)V_1+\delta V_0\right], \tag{25}$$

where

$$\varpi = \alpha(n)\mu\left[\upsilon\left(y\right)-y\right]+(R-1)a+\Delta-T. \tag{26}$$

From the first two terms on the right side of (25) the flow utility from being employed is the sum of the wage paid by the firm, the expected surplus in the DM goods market, the net return from liquid assets, and firms' profits net of taxes. The third term on the right side of (25) describes the transitions in the next LM. With probability, $1-\delta$, the household remains employed in the following period and enjoys the discounted utility βV_1; with complement probability, δ, the households loses its job and enjoys the discounted utility βV_0. From (25) the value from being employed is

$$V_1 = \frac{w + \varpi + \beta\delta V_0}{1-\beta(1-\delta)}.$$

Substract V_0 on both sides to obtain the surplus of an employed worker,

$$V_1 - V_0 = \frac{w + \varpi - (1-\beta)V_0}{1-\beta(1-\delta)}. \tag{27}$$

The term, $(1-\beta)V_0 - \varpi$, is the household's reservation wage, i.e., it is the wage that makes a household indifferent between being employed and being unemployed. Therefore, according to (27), the surplus of a household is equal to the discounted sum of the difference between the wage paid by the firm and the household's reservation wage. From (24) and (27) the wage determined by the Nash/Kalai solution solves

$$w + \varpi - (1-\beta)V_0 = \frac{\lambda}{1-\lambda}\left(z - w\right).$$

Solving for w this gives

$$w = \lambda z + (1-\lambda)\left[(1-\beta)V_0 - \varpi\right]. \tag{28}$$

The wage is a weighted average of the firm's expected revenue, z, and the worker's reservation wage, $(1-\beta)V_0 - \varpi$. Using the same reasoning as above, the expected discounted utility of an unemployed household is

$$V_0 = \ell + b + \varpi + \beta V_0 + \beta p(V_1 - V_0). \tag{29}$$

From Nash/Kalai bargaining, $V_1 - V_0 = \lambda J/(1-\lambda)$ and from free entry $J = k/\beta f$. Therefore, from (29), the value of an unemployed household can be reexpressed as

$$(1-\beta)V_0 = \ell + b + \varpi + \frac{\lambda}{1-\lambda}\theta k. \tag{30}$$

Substitute $(1-\beta)V_0$ from (30) into (28) to obtain

$$w = \lambda z + (1-\lambda)(\ell + b) + \lambda\theta k. \tag{31}$$

The expression for the wage, (31), is identical to the one in Pissarides (2000). The wage is a weighted average of firm's revenue, z, and household's flow utility from being unemployed, $\ell + b$, augmented by a term proportional to firms' average recruiting expenses per vacancy, $\upsilon k/u$. By the same reasoning as above the same wage is paid to households with no access to credit.

Market tightness

The ratio of vacant jobs per unemployed worker is determined by the free-entry condition according to which $k = \beta f J$ where J is given by (24). Substituting w by its expression from (31) into (24) and using that $\beta = 1/(1+r)$,

$$\frac{(r+\delta)\,k}{f} = (1-\lambda)(z - \ell - b) - \lambda\theta k. \tag{32}$$

If $(r+\delta)k > (1-\lambda)(z-\ell-b)$, (32) determines a unique $\theta > 0$ for a given z. The financial frictions affect firms' entry decision through z, their expected revenue. If credit is more limited, then households have a lower payment capacity, sales in the DM goods market, y, fall, which reduces z (provided that $\mu < 1$). As z is reduced, fewer firms find it profitable to enter the market.

3.4 Liquidity

A household's optimal holdings of liquid assets is obtained by substituting $U_e(a)$ given by (17)-(19) into (4), i.e., a solves

$$\max_{a \geq 0} \left\{ -(1-\beta R)\, a + \beta\alpha(n)\mu \left[\upsilon(y) - y \right] \right\}, \tag{33}$$

where y is the solution to the DM bargaining problem, (12)-(13), i.e., $(1-\mu)\upsilon(y) + \mu y = \bar{d} + R\nu a$ if the solution is less than y^* and $y = y^*$ otherwise. According to (33) households choose their holdings of liquid assets in order to maximize their expected surplus in the DM net of the cost of holding assets, which is approximately equal to the difference between the gross rate of time preference, β^{-1}, and the gross rate of return of liquid assets, R. The problem in (33) is independent of the labor status of the household, which establishes that both employed and unemployed households will hold the same quantity of liquid assets conditional on facing the same debt limit.

From the bargaining solution $dy/da = \nu R/\left[(1-\mu)\upsilon'(y)+\mu\right]$ if $\bar{d}+R\nu a < (1-\mu)\upsilon(y^*)+\mu y^*$. Therefore, the first-order condition associated with (33) is

$$-\left(1+r-R\right)+\alpha(n)\mu\nu R\left[\frac{\upsilon'\left(y\right)-1}{(1-\mu)\upsilon'(y)+\mu}\right]\leq 0, \tag{34}$$

with equality if $a > 0$. The first term on the left side of (34) is the opportunity cost of holding liquid assets. The second term on the left side of (34) is the liquidity premium of the asset, i.e., the expected marginal benefit from holding liquid assets in the DM. This expected marginal benefit is computed as follows. With probability, $\alpha(n)$, the household has an opportunity to spend its marginal unit of asset in the DM; this marginal unit buys dy/da units of DM output, which is valued at the marginal surplus of the household in a DM meeting, $\mu\left[\upsilon'\left(y\right)-1\right]$.

A household with no access to credit solves a similar portfolio problem as in (33) where y is replaced with $\tilde{y}$ and $(1-\mu)\upsilon(\tilde{y})+\mu\tilde{y} = R\nu\tilde{a}$. After rearranging terms, it becomes:

$$\max_{\tilde{y}\geq 0}\left\{\left[\alpha(n)\nu\mu-\left(\frac{1+r}{R}-1\right)(1-\mu)\right]\upsilon\left(\tilde{y}\right)-\left[\frac{1+r}{R}-1+\alpha(n)\nu\right]\mu\tilde{y}\right\}. \tag{35}$$

Using that $\upsilon'(0)=\infty$, a necessary and sufficient condition for $\tilde{y} > 0$ is that the first term between squared brackets in (35) is positive, i.e., $\alpha(n)\nu\mu > \left[\left(1+r\right)/R-1\right](1-\mu)$. It is optimal for households with no access to credit to hold liquid assets if the holding cost of those assets, $1+r-R$, is not too large. Moreover, the higher the frequency of trades, $\alpha(n)$, the higher the household's bargaining power, μ, and the more likely it is that households will accumulate liquid assets.

3.5 Borrowing constraint

Consider a household with debt level, d, and labor status, e, in the CM. The incentive compatibility constraint for the repayment of the household's debt is

$$-d+W_e(0,a)\geq\rho\tilde{W}_e(a)+(1-\rho)W_e(0,a), \tag{36}$$

where $\tilde{W}_e$ is the value of a household who is excluded permanently from credit transactions. The left side of (36) is the expected lifetime utility of the household if it does not default: the household pays back its debt and enters the CM with a units of liquid asset and future access to credit. The right side is the expected lifetime utility of the household if it defaults. With probability, ρ, the identity of the defaulting household is publicly known, and as a result the household is banned from future credit but it can keep trading with liquid assets (because he can choose to be non-monitored).[15] Its continuation value is $\tilde{W}_e(a)$. If the default is not recorded, with probability $1-\rho$, then the household's public trading history shows no event of default, which allows the household to keep its line of credit. In this case its continuation value is $W_e(0,a)$.

Using the linearity of W_e and $\tilde{W}_e$, from (4) and (5), the household credit constraint, (36), can be reexpressed as

$$d \leq \bar{d} \equiv \rho \left[W_e(0,0) - \tilde{W}_e(0) \right]. \tag{37}$$

For repayment to be incentive compatible the household's debt cannot be greater than the expected cost from defaulting, which is equal to the probability of losing access to credit, ρ, times the difference between the lifetime utility of a household with access to credit, W_e, and the lifetime utility of a household with no access to credit, $\tilde{W}_e$. From (37) $\bar{d}$ is independent from the quantity of assets held by the household when entering the CM. Using (4) and (5), the debt limit can be rewritten as

$$\bar{d} = \rho \left\{ \max_{a \geq 0} \left[-a + \beta U_e(a) \right] - \max_{\tilde{a} \geq 0} \left[-\tilde{a} + \beta \tilde{U}_e(\tilde{a}) \right] \right\}. \tag{38}$$

From (38) the possibility offered to households to self-insure against idiosyncratic shocks in the DM by holding liquid assets will affect debt limits.

In order to characterize the debt limit it is useful to introduce the following two thresholds for the gross rate of return of liquid assets:

$$\bar{R} \equiv \frac{\rho(1+r)}{r\nu + \rho} \tag{39}$$

$$\underline{R} \equiv \frac{(1-\mu)(1+r)}{\alpha(n)\nu\mu + 1 - \mu}. \tag{40}$$

[15]Our assumption is consistent with the one in Kehoe and Levine (1993) according to which an agent who defaults on a contract cannot be excluded from spot markets trading. It is also consistent with Wallace (2005) who assumes that monitored people who defect can join the ranks of the nonmonitored people and suffer no further punishment Similarly, in Aiyagari and Williamson (2000) an agent who defaults can trade with money in the future. In contrast, Sanches and Williamson (2010) assume that if a buyer defaults, then sellers will refuse to take his money.

We show in the next Lemma that $\bar{R}$ is an upper bound for the gross rate of return on liquid assets above which the repayment of credit is not incentive compatible. From (35) the threshold, $\underline{R}$, is the gross rate of return below which households do not want to accumulate liquid assets.

Proposition 1 ***(Endogenous debt limit)*** *For given n, the debt limit, $\bar{d}$, is a solution to*

$$r\bar{d} = \Gamma(\bar{d}), \tag{41}$$

where

$$\begin{aligned}\Gamma(\bar{d}) \equiv \rho \max_{a \geq 0} \left\{ -\left(1+r-R\right) a + \alpha(n)\mu \left[\upsilon\left(y\right) - y\right] \right\} \\ - \rho \max_{\tilde{a} \geq 0} \left\{ -\left(1+r-R\right) \tilde{a} + \alpha(n)\mu \left[\upsilon\left(\tilde{y}\right) - \tilde{y}\right] \right\}.\end{aligned} \tag{42}$$

There exists a $\bar{d} > 0$ solution to (41) if and only if $r < \rho\alpha(n)\mu/(1-\mu)$ (i.e., $\underline{R} < \bar{R}$) and $R \leq \bar{R}$. Moreover, if $R < \bar{R}$, then this solution is unique; if $R = \bar{R}$, then any $\bar{d} \in [0, R\nu\tilde{a}]$ is a solution.

As conjectured earlier, the debt limit is independent of the household's employment status.[16] The determination of the debt limit, $\bar{d}$, is represented in Figure 4. The line, $r\bar{d}$, is the return to the household from having access to a line of credit of size $\bar{d}$. The curve, $\Gamma(\bar{d})$, represents the flow cost from defaulting on one's debt if the debt limit for future DM trades is equal to $\bar{d}$. This cost is equal to the probability of being caught, ρ, times the loss from not being eligible for a loan in the future. The first term on the right side of (42) is the expected DM surplus of a household with access to credit net of the cost of holding liquid assets. The second term gives a similar expression for households with no access to credit. The punishment from defaulting increases with the size of the credit line, i.e., Γ is upward sloping. Moreover, $\Gamma(0) = 0$. If a household anticipates that it will not have access to credit in the future, then there is no cost from defaulting. As a consequence, there always exists an equilibrium with no unsecured credit.[17]

For a further characterization of Γ we distinguish two cases. First, if the size of the credit line, $\bar{d}$, is less than the payment capacity of a household with no access to credit, $R\nu\tilde{a}$, then Γ is linear. Indeed, as shown in the bottom panels of Figure 4, households with a credit limit of $\bar{d} < R\nu\tilde{a}$ choose asset holdings so as equalize their payment capacity with the one of households with no credit line, $R\nu a + \bar{d} = R\nu\tilde{a}$. This result

[16] This does not rule out the existence of other equilibria where $\bar{d}$ would be a function of e.

[17] For a similar result, see Sanches and Williamson (2010).

comes from the observation that both types of households face the same trade-off at the margin, captured by (34), in terms of the cost and benefit from holding liquid assets. As a result, the cost from defaulting is equal to the probability, ρ, times the quantity of liquid assets that the household has to accumulate to replace the credit line, $\tilde{a} - a = \bar{d}/R\nu$, where the cost of holding one unit of liquid asset is equal to $1 + r - R$. Hence, the slope of Γ is $\rho(1 + r - R)\bar{d}/R\nu$.

Second, households with a credit limit, $\bar{d} > R\nu\tilde{a}$, find it optimal to hold no liquid assets (from (34)), and Γ is strictly concave (provided that $\bar{d}$ is not too large). To see this, notice from (13) that $\partial y/\partial \bar{d} = 1/\left[(1 - \mu)\upsilon'(y) + \mu\right]$, and hence

$$\frac{\partial \left[\upsilon(y) - y\right]}{\partial \bar{d}} = \frac{\upsilon'(y) - 1}{(1 - \mu)\upsilon'(y) + \mu} = \frac{1}{1 - \mu + 1/\left[\upsilon'(y) - 1\right]}, \tag{43}$$

which is decreasing in y when $y < y^*$. So the surplus from a DM trade, $\upsilon(y) - y$, is strictly concave in $\bar{d}$.

For unsecured debt to emerge the slope of Γ at $\bar{d} = 0$ must be greater than r. The expression for $\Gamma'(0)$ depends on whether households with no access to credit find it optimal to hold liquid assets. If $R \leq \underline{R}$, then households with no access to credit choose not to accumulate liquid assets, $\tilde{a} = 0$. In that case, from (42) and (43) $\Gamma'(0) = \rho\alpha(n)\mu/(1 - \mu)$, and hence a necessary condition for credit to be sustainable is $r < \rho\alpha(n)\mu/(1 - \mu)$, as indicated in Proposition 1. Households must be sufficiently patient and care enough about the future punishment in case of default for the repayment of debt to be self-enforcing. The threshold for the rate of time preference below which unsecured credit is incentive compatible increases with the probability of being punished in case of default, ρ, with the frequency of liquidity shocks, α, and with the household's market power in the DM, μ.

If $R > \underline{R}$, households with no access to credit accumulate liquid assets, $\tilde{a} > 0$. This possibility of self-insurance lowers the cost of defaulting, and hence the condition for credit to be incentive incompatible is more stringent. In that case $\Gamma'(0) = \rho(1 + r - R)/R\nu$ so that $r < \Gamma'(0)$ can be reexpressed as $R < \bar{R}$. So unsecured credit can be sustained in equilibrium if the rate of return of liquid assets is not too close to the rate of time preference, $R/(1 + r) < \rho/(r\nu + \rho)$. Graphically, the curve representing Γ intersects the curve representing $r\bar{d}$ from above. See the left panel of Figure 4. In contrast, if R is greater than $\bar{R}$, then the cost of defaulting is too small to sustain unsecured credit and Γ is located underneath the line $r\bar{d}$. See right panel of Figure 4. Finally, there is a knife-edge case where $r\bar{d}$ and $\Gamma(\bar{d})$ coincide so that there are a continuum of debt limits, $\bar{d} \in [0, R\nu\tilde{a}]$. For any $\bar{d} \in [0, R\nu\tilde{a}]$, the flow value of credit is $r\bar{d}$ and the flow cost from defaulting is $r\bar{d}$, which makes the debt limit indeterminate.

The following Corollary provides a condition on the rate of return of liquid assets for the coexistence of credit and money.

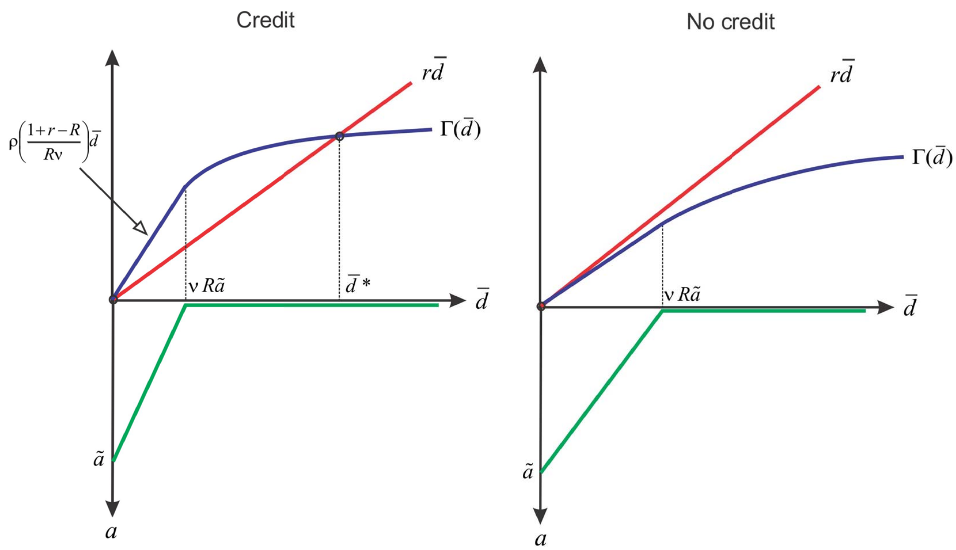

Figure 4: Debt Limit

Corollary 1 ***(Coexistence of money and credit)*** *A necessary condition for the coexistence of money and credit is $R \in (\underline{R}, \bar{R}]$. If $R > \bar{R}$, then unsecured credit is not incentive compatible. If $R < \underline{R}$, then agents do not hold liquid assets.*

As shown in Proposition 1 if $R > \bar{R}$, then unsecured credit is not incentive compatible and, as a result, all households choose the same holdings of liquid assets, $a = \tilde{a}$. The economy in this case corresponds to a pure monetary economy. Households' payment capacity, $R\nu a = R\nu\tilde{a}$, increases with R. See the upward-sloping dashed lines in Figure 5 for all $R > \bar{R}$. When R is exactly equal to $1+r$, there is no cost of holding liquid assets (which is equivalent to a Friedman-rule outcome) and households accumulate enough liquidity to finance y^*, i.e., $R\nu a \geq (1-\mu)\upsilon(y^*) + \mu y^*$. In Figure 5 the dashed lines become vertical at $R = 1+r$.

If $R < \bar{R}$, then unsecured credit is incentive compatible. As R increases, the punishment from being excluded from credit is lower since defaulters can accumulate liquid assets at a lower cost. Graphically, the curve representing Γ in Figure 4 shifts downward and the debt limit decreases. In Figure 5, the plain line representing $\bar{d}$ is downward sloping. For all $R \in (\underline{R}, \bar{R})$ there is coexistence of money and credit: some households pay with liquid assets only while other households pay with credit only.[18] Moreover, households with access to credit have a larger payment capacity than households who trade with liquid assets only (graphically, the plain curve is located above the dashed one). Finally, from (35) if $R < \underline{R}$, then the cost of holding liquid assets is so high that even households with no access to credit choose not to hold any liquidity. In Figure 5 the dashed lines are horizontal at 0. The economy is a pure credit economy.

The following Corollary shows how the aggregate state of the labor market affects credit limits.

Corollary 2 ***(Credit limits and the labor market)*** *Assume $r < \rho\alpha(n)\mu/(1-\mu)$ and $R < \bar{R}$. The solution, $\bar{d} > 0$, to (41) is increasing with n.*

The state of the labor market affects debt limits through the frequency of consumption opportunities in the DM goods market, α. If employment is high, then there are a large number of firms participating in the retail market and, as a result, households have frequent trading opportunities. Such frequent trading opportunities imply that the cost from defaulting on one's debt is high, and therefore the debt limit is high. In Figure 4, as n increases, the curve representing Γ moves upward for all $\bar{d} > R\nu\tilde{a}$.

3.6 Steady states with unsecured credit

We consider steady-state equilibria with unsecured debt, $\bar{d} > 0$. As we have shown above, in any such equilibrium households who have access to credit will not accumulate liquid assets, $a = 0$ (except in the

[18] In order to have households hold money and use credit one could assume that there is a probability that a firm in the retail market is not able to identify a consumer and therefore cannot extend credit for him. For a related assumption, see Sanches and Williamson (2010).

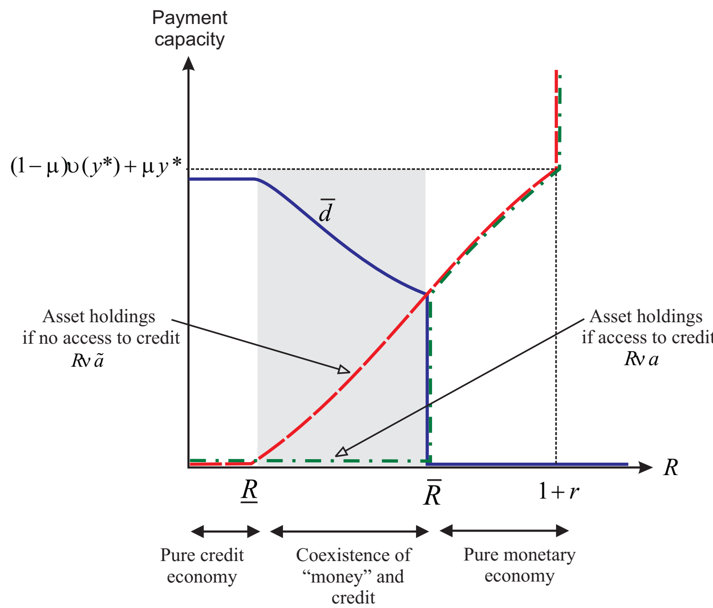

Figure 5: Debt Limit vs. Rate of Return

knife-edge case where $R = \bar{R}$), because their payment capacity is greater than the one of households with no access to credit, $y > \tilde{y}$, where

$$(1-\mu)\upsilon(y) + \mu y = \min\left[(1-\mu)\upsilon(y^*) + \mu y^*, \bar{d}\right]. \tag{44}$$

From (34) the choice of liquid assets by households with no access to credit solves

$$\begin{aligned} \alpha(n)\mu\nu\left[\frac{\upsilon'(\tilde{y})-1}{(1-\mu)\upsilon'(\tilde{y})+\mu}\right] &= \frac{1+r-R}{R} \text{ if } \frac{1+r-R}{R} < \frac{\alpha(n)\nu\mu}{1-\mu} \\ \tilde{y} &= 0 \text{ otherwise,} \end{aligned} \tag{45}$$

where

$$\tilde{a} = \frac{(1-\mu)\upsilon(\tilde{y}) + \mu\tilde{y}}{\nu R}. \tag{46}$$

The steady-state level of unemployment in the LM, u, is such that the flow in unemployment is equal to the flow out of unemployment, i.e., $pu = \delta(1-u)$, which gives

$$u = \frac{\delta}{m(1,\theta)+\delta}. \tag{47}$$

From (23) and (32), labor market tightness (assuming it is positive) solves

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\theta},1\right)}+\beta\lambda\theta k=(1-\lambda)\left\{\frac{\alpha(1-u)}{1-u}(1-\mu)\left\{\omega\left[\upsilon\left(y\right)-y\right]+(1-\omega)\left[\upsilon\left(\tilde{y}\right)-\tilde{y}\right]\right\}+\bar{z}-\ell-b\right\}. \qquad (48)$$

Definition 1 *A steady-state equilibrium with unsecured credit is a list* $(u,\theta,y,\tilde{y},\tilde{a},\bar{d})$ *that solves (41)-(48) with* $\bar{d}>0$.

Assuming $\bar{z}>\ell+b+\frac{(r+\delta)k}{1-\lambda}$, let $\theta_0>0$ be the unique solution to

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\theta_0},1\right)}+\beta\lambda\theta_0 k=(1-\lambda)\left(\bar{z}-\ell-b\right).$$

Denote $n_0=m(1,\theta_0)/\left[m(1,\theta_0)+\delta\right]$ the associated level of employment. The following proposition provides sufficient conditions for the existence of an equilibrium with credit.

Proposition 2 ***(Existence of an equilibrium with unsecured credit)*** *If* $\bar{z}>\ell+b+(r+\delta)k/(1-\lambda)$, $R<\bar{R}$, *and* $r<\rho\alpha(n_0)\mu/(1-\mu)$, *then there exists a steady-state equilibrium with unsecured credit,* $\bar{d}>0$.

A sufficient condition for the labor market to be active is that labor productivity is sufficiently high relative to workers' utility of leisure and unemployment benefits. This is a standard existence condition for an active equilibrium in the Mortensen-Pissarides model. For unsecured credit to emerge, it must also be the case that the rate of return on liquid assets is not too high since otherwise the threat of exclusion from credit transactions would not be strong enough to sustain households' incentives to repay unsecured credit.

Notice that Proposition 2 only gives sufficient conditions for existence. An equilibrium might exist even if $\bar{z}<\ell+b+(r+\delta)k/(1-\lambda)$ because effective productivity, z, is endogenous and depends on the availability of credit. Moreover, in contrast to the Mortensen-Pissarides model, the equilibrium might not be unique because of the complementarity between the credit limit in the goods market and aggregate employment. We will illustrate this last point in the following section.

4 Two limiting economies

Before we turn to a quantitative exploration of the model we will characterize the set of equilibria and some comparative statics for two limiting economies: (i) A pure monetary economy where households do not have access to unsecured debt ($\omega = 0$ or $\rho = 0$); (ii) A pure credit economy where all households have access to unsecured credit ($\omega = 1$) but there is no liquid asset ($R = 0$ or $\nu = 0$).

4.1 Pure monetary economies

Suppose that there is no record keeping technology that can keep track of households' defaults ($\omega = 0$ or $\rho = 0$). Because of lack of commitment, credit is not incentive feasible as households would always renege on their debts. Therefore, households must hold liquid assets in order to trade in the DM. We assume that households can transfer all their liquid assets in a match, $\nu = 1$.[19]

A steady-state monetary equilibrium can be reduced to a pair, $(\tilde{y}, \theta)$, that solves (45) and (48) with $\omega = 0$ and $n = 1 - u$, i.e.,

$$\frac{1+r}{R} = 1 + \alpha(n)\mu\left[\frac{\upsilon'(\tilde{y}) - 1}{(1-\mu)\upsilon'(\tilde{y}) + \mu}\right] \tag{49}$$

$$\frac{(r+\delta)k}{m\left(\frac{1}{\theta}, 1\right)} + \beta\lambda\theta k = (1-\lambda)\left\{\frac{\alpha(n)}{n}(1-\mu)\left[\upsilon(\tilde{y}) - \tilde{y}\right] + \bar{z} - \ell - b\right\}, \tag{50}$$

where $n(\theta) = m(1,\theta)/[m(1,\theta)+\delta]$. We study each equilibrium condition in turn.

Let us start with the equilibrium condition for the choice of liquid assets, (49). If $\alpha\mu/(1-\mu) \leq (1+r-R)/R$, where α is evaluated at $n = 1/(1+\delta)$, then for all $\theta \geq 0$ there is no $\tilde{y} > 0$ solution to (49). So a necessary condition for a monetary equilibrium to exist is that households have enough bargaining power in the DM,

$$\mu > \frac{1+r-R}{1+r-R\left[1-\alpha\left(\frac{1}{1+\delta}\right)\right]}. \tag{51}$$

Assume (51) holds and define $\underline{\theta} > 0$ as the solution to

$$\frac{\mu\alpha\left[n(\underline{\theta})\right]}{1-\mu} = \frac{1+r-R}{R}. \tag{52}$$

According to (49) and (52), for all $\theta \leq \underline{\theta}$, $\tilde{y} = 0$ and, for all $\theta > \underline{\theta}$, $\tilde{y}$ is an increasing function of θ. Indeed, as $n(\theta)$ increases, the frequency of trading opportunities in the DM increases, which raises the non-pecuniary return of liquid assets and gives households higher incentives to accumulate them. As market tightness tends

[19]This limiting economy is analogous to the pure currency economy studied in Berentsen, Menzio, and Wright (2011) except that the medium of exchange takes the form of capital in our model, as in Lagos and Rocheteau (2008), instead of fiat money.

to infinity, $n(\theta)$ approaches $1/(1+\delta)$, and y approaches some upper bound, $\bar{y}$. Finally, as R tends to $1+r$, i.e., the cost of holding liquid assets approaches 0, $\underline{\theta}$ tends to 0 and $\tilde{y}$ tends to y^* for all θ. Indeed, if liquidity is costless, all households want to hold $Ra \geq \mu y^* + (1-\mu)\upsilon(y^*)$ irrespective of the frequency at which trading opportunities in the DM occur. In the left panel of Figure 7 we represent the equilibrium condition, (49), by LD (demand for liquidity).

Let us turn to the vacancy supply condition, (50). First, we assume that

$$(1-\lambda)\left\{(1-\mu)\left[\upsilon\left(y^*\right)-y^*\right]+\bar{z}-\ell-b\right\} > (r+\delta)k. \tag{53}$$

Condition (53) guarantees that if households have enough liquid assets to buy the first-best level of output in the DM, $\tilde{y} = y^*$, then the labor market is active, $\theta > 0$. If, in addition, $(1-\lambda)\left(\bar{z}-\ell-b\right) < (r+\delta)k$, then the labor market is active only if there is a minimum amount of trade in the DM. Define $\underline{y} \in (0, y^*)$ as the solution to

$$(1-\lambda)\left\{(1-\mu)\left[\upsilon\left(\underline{y}\right)-\underline{y}\right]+\bar{z}-\ell-b\right\} = (r+\delta)k. \tag{54}$$

For all $y \leq \underline{y}$, $\theta = 0$; for all $y > \underline{y}$, $\theta > 0$ is an increasing function of y for all $y < y^*$, and it reaches a maximum at $y = y^*$. Under these conditions there is an inactive equilibrium with $\tilde{y} = \theta = 0$, and if an active equilibrium exists, then there is an even number of active equilibria (except for degenerate cases where the equilibrium is unique). In the left panel of Figure 7 we represent the equilibrium condition, (50), by VS (vacancy supply).

If $(1-\lambda)\left(\bar{z}-\ell-b\right) > (r+\delta)k$, then the firm's productivity is high enough to cover its entry cost even if the DM is inactive. Denote $\underline{\underline{\theta}} > 0$ the solution to (50) with $\tilde{y} = 0$, i.e.,

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\underline{\underline{\theta}}}, 1\right)} + \beta\lambda k \underline{\underline{\theta}} = (1-\lambda)\left(\bar{z}-\ell-b\right). \tag{55}$$

This is the market tightness that would prevail if the DM shuts down. Any solution to (49)-(50) is such that $\theta \geq \underline{\underline{\theta}}$. Let $\hat{R} < 1+r$ be the solution to $\underline{\underline{\theta}} = \underline{\theta}(\hat{R})$. If $R > \hat{R}$, then $\underline{\underline{\theta}} > \underline{\theta}(R)$ and any equilibrium is such that $\tilde{y} > 0$. We recapitulate our characterization of the pure monetary economy in the following proposition.

Proposition 3 ***(Pure monetary equilibrium)*** *Consider an economy with no unsecured credit ($\omega = 0$ and/or $\rho = 0$) and suppose (51) and (53) hold.*

1. *If $\bar{z} - \ell - b < (r+\delta)k/(1-\lambda)$, then there is an inactive equilibrium, $(\theta, \tilde{y}) = (0, 0)$. If R is sufficiently close to $1+r$, there are also an even number of equilibria with both an active DM, $\tilde{y} > 0$, and an active LM, $\theta > 0$.*

2. *If $\bar{z} - \ell - b > (r+\delta)k/(1-\lambda)$, then all equilibria have an active LM, $\theta > 0$. Moreover, if $R > \hat{R}$, then all equilibria have an active DM.*

The left panel of Figure 7 provides a graphical representation of part 1 of Proposition 3. There is an inactive equilibrium and two active equilibria. The logic for the multiplicity of equilibria is based on strategic complementarities between firms' entry decision and households' choice of liquid assets.[20] If households accumulate a lot of liquid assets, then firms' expected profits in the DM are high and a lot of firms participate. But if aggregate employment is high, the frequency of consumption opportunities is also high, and households have incentives to accumulate a large quantity of liquid assets. By the same logic, there is an equilibrium with a small number of firms and with few liquid assets held by households. So the aggregate liquidity in the economy and unemployment are negatively correlated across equilibria.

From part 2 of Proposition 3 we learn that the labor market is active in any equilibrium provided that firms' productivity is sufficiently high. Moreover, the retail goods market will also be active if the rate of return of liquid assets is sufficiently close to the rate of time preference.

Finally, we can study the effects of monetary policy on aggregate liquidity and the labor market, where monetary policy is described by the choice of R. From (49) as the rate of return on liquid assets increases, households increase their asset holdings for a given θ. Graphically, in Figure 7, the curve LD moves to the right. At the highest equilibrium both market tightness and DM output increase. Consequently, a higher interest rate leads to lower unemployment and higher output.

4.2 Pure credit economies

We now assume that all households have access to credit, $\omega = 1$, and there is no liquid asset, $R = 0$ (goods cannot be stored) or $\nu = 0$ (capital goods are not portable or cannot be transfered in the DM, let

[20] For a similar result in the context of an economy with fiat money and free entry of sellers, see Rocheteau and Wright (2005).

say, because they can be counterfeited at no cost). From Proposition 1 the credit limit is a solution to $r\bar{d} = \rho\alpha(n)\mu\left[\upsilon(y) - y\right]$ where $(1-\mu)\upsilon(y) + \mu y = \min\left[(1-\mu)\upsilon(y^*) + \mu y^*, \bar{d}\right]$. If the credit limit binds, then y solves

$$y = \left\{1 - \frac{r}{\left[r + \rho\alpha(n)\right]\mu}\right\}\upsilon(y). \tag{56}$$

Provided that $r < \rho\alpha\mu/(1-\mu)$, then there is a unique $y > 0$ solution to (56). If the solution to (56) is greater than y^*, then $\bar{d} > (1-\mu)\upsilon(y^*) + \mu y^*$ and $y = y^*$. Market tightness is determined by

$$\frac{(r+\delta)\,k}{m\left(\frac{1}{\theta}, 1\right)} + \beta\lambda\theta k = (1-\lambda)\left\{\frac{\alpha(n)}{n}(1-\mu)\left[\upsilon\left(y\right) - y\right] + \bar{z} - \ell - b\right\}. \tag{57}$$

We can reduce an equilibrium to a pair, $(\theta, y) \in \mathbb{R}_+ \times [0, y^*]$, that solves the conditions above.

We first describe an equilibrium where households' borrowing constraint in the DM is not binding, which implies $y = y^*$. It requires the right side of (56) to be greater than the left side when both are evaluated at $y = y^*$, i.e.,

$$\left[\frac{\rho\alpha(n^*) - r(1-\mu)/\mu}{r + \rho\alpha(n^*)}\right]\frac{\upsilon(y^*)}{y^*} \geq 1, \tag{58}$$

where $n^* = n(\theta^*)$ is the employment level given by (57) when $y = y^*$. Assuming (53) holds, $\theta^* > 0$ and $n^* > 0$. From (58) credit is abundant if the match surplus in the DM is sufficiently large, if households have sufficient market power in the goods market (i.e., μ is high), and if the financial system is sufficiently sophisticated (i.e., ρ is high). A necessary condition for the debt constraint not to bind is $\rho\alpha\left(n^*\right) > r(1-\mu)/\mu$. In Figure 6, we describe a case with multiple equilibria where the high equilibrium is such that the debt limit is not binding. Notice that the curve representing the credit limit as a function of market tightness, labelled CL, becomes vertical when θ is above a threshold. Indeed, for high values of θ, $\bar{d} > (1-\mu)\upsilon(y^*) + \mu y^*$ and households have enough borrowing capacity to finance the purchase of y^*. In this case the CL curve intersects the vacancy-supply curve, VS, at its maximum.

We now study equilibria where borrowing constraints do bind. We analyze each equilibrium condition in turn starting with the condition for the DM output level, (56). Define $\underline{\theta} < \infty$ as the solution to $\rho\alpha(n)\mu = r(1-\mu)$, i.e.,

$$\frac{m(1, \underline{\theta})}{m(1, \underline{\theta}) + \delta} = \alpha^{-1}\left[\frac{r}{\rho}\left(\frac{1-\mu}{\mu}\right)\right], \tag{59}$$

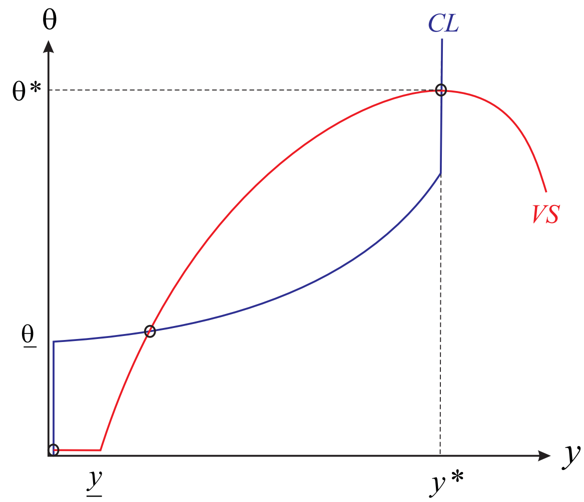

Figure 6: Multiple equilibria in which the debt limit at the high steady state is not binding.

if $r < \rho\alpha\left[1/(1+\delta)\right]\mu/(1-\mu)$, and $\underline{\theta} = +\infty$ otherwise. For all $\theta \leq \underline{\theta}$, $y = 0$. Below a threshold for market tightness, unsecured credit cannot be sustained. Above $\underline{\theta}$, y increases with θ because $\alpha(n)$ is an increasing function of θ. As θ tends to infinity, y approaches $\bar{y}$, where $\bar{y}$ solves (56) with $n = 1/(1+\delta)$. In Figure 7 we represent the equilibrium condition, (56), by a curve labelled CL (credit limit).

Let us turn to the vacancy-supply condition, (57), represented by the curve labelled VS in Figure 7. It gives a positive relationship between θ and y for all $y \leq y^*$. Intuitively, if households have a higher payment capacity, they buy more output in the DM, and firms have more incentives to participate. Assume $(1-\lambda)(\bar{z} - \ell - b) < (r+\delta)k$. For all $y \leq \underline{y}$, where $\underline{y}$ is defined in (54), $\theta = 0$. Under this condition, there is an inactive equilibrium with $\theta = y = 0$ and an even number of equilibria (possibly zero) across which $\theta > 0$ and $y > 0$ are positively correlated. If $(1-\lambda)(\bar{z} - \ell - b) > (r+\delta)k$, then $\theta > 0$ for all $y \geq 0$. In this case any equilibrium has an active labor market, even if the DM is inactive. Define $\underline{\underline{\theta}} > 0$ as the solution to (55), i.e., the market tightness when the DM is inactive, and

$$\hat{\rho}(\mu) = \frac{r(1-\mu)}{\mu\alpha\left[\frac{m(1,\underline{\underline{\theta}})}{m(1,\underline{\underline{\theta}})+\delta}\right]} \tag{60}$$

$$\hat{\mu} = \frac{r}{r+\alpha\left[\frac{m(1,\underline{\underline{\theta}})}{m(1,\underline{\underline{\theta}})+\delta}\right]}. \tag{61}$$

From (60) the quantity, $\hat{\rho}(\mu)$, is defined as the level of monitoring such that $\underline{\theta} = \underline{\underline{\theta}}$. For all $\rho > \hat{\rho}(\mu)$, credit can be sustained if $\theta = \underline{\underline{\theta}}$. From (61) the threshold, $\hat{\mu}$, is defined such that $\hat{\rho}(\hat{\mu}) = 1$. Therefore, for all $\mu > \hat{\mu}$ and $\rho > \hat{\rho}(\mu)$, $\underline{\theta} < \underline{\underline{\theta}}$ and any equilibrium has both an active labor market and an active retails goods market since $\theta > \underline{\underline{\theta}} > \underline{\theta}$.

Proposition 4 ***(Pure credit equilibrium)*** *Suppose (53) holds.*

1. *If (58) holds, then there exists an equilibrium with $y = y^*$ and $\theta = \theta^*$.*
2. *If $\bar{z}-\ell-b < (r+\delta)k/(1-\lambda)$, then there is an inactive equilibrium with $(\theta, y) = (0, 0)$. If r is sufficiently close to 0, there are also an even number of active equilibria with both an active DM, $y > 0$, and an active LM, $\theta > 0$.*
3. *If $\bar{z} - \ell - b > (r+\delta)k/(1-\lambda)$, then any equilibrium has an active LM, $\theta > 0$. Moreover, if $\mu > \hat{\mu}$ and $\rho > \hat{\rho}(\mu)$, then all equilibria have an active DM, $y > 0$.*

In Figure 7 we represent the determination of an equilibrium under the condition in Part 2 of Proposition 4. There is an inactive equilibrium where households do not have access to credit and firms do not participate in the labor market, and two active equilibria. Credit limits and unemployment rates are negatively correlated across equilibria. Intuitively, if there are a lot of firms in the retail market, then the punishment for not repaying one's debt, i.e., the exclusion from the DM market, is large since households would have to forgive a large number of trading opportunities. As a result, households can borrow a large amount and firms can expect large profits in the DM. In Figure 6 the high equilibrium is such that liquidity is abundant in the sense that borrowing constraints are not binding. By a symmetric logic there is also an equilibrium with a high unemployment rate and a low credit limit. So our model provides a theory of high unemployment due to a self-fulfilling credit crunch.

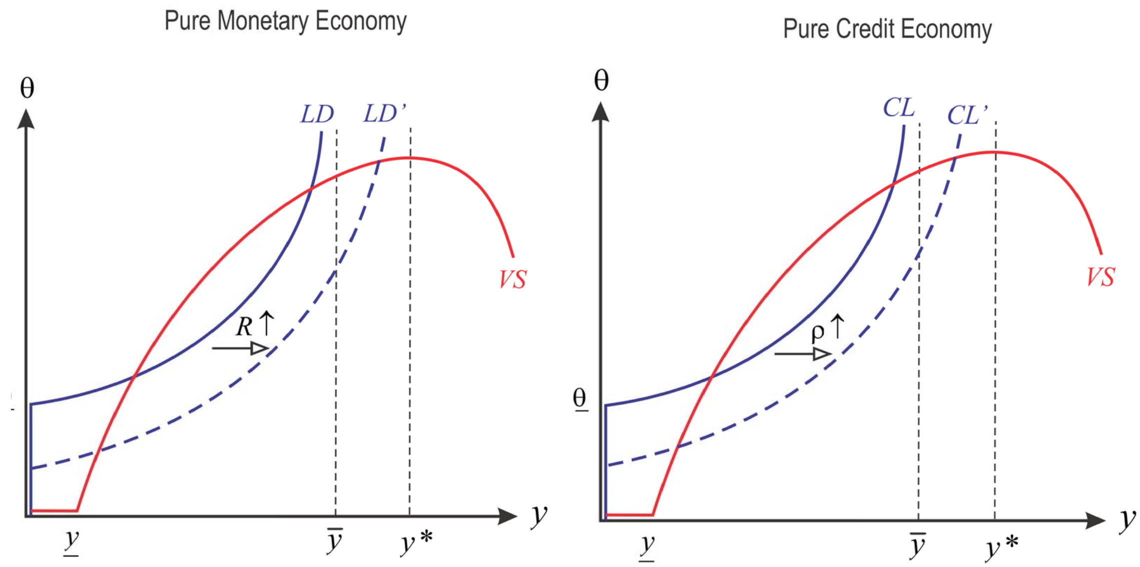

Figure 7: Limiting economies and multiple equilibria

As in the case of a pure monetary economy, the labor market is active in any equilibrium provided that productivity is sufficiently high. Moreover, the retail goods market is active in any equilibrium if the record-keeping technology is sufficiently effective and households are sufficiently patient. An improvement in the sophistication of the financial system (i.e. an increase in ρ) affects the credit limit condition (56) indicated as a rightward shift in the CL' curve in Figure 7. As ρ increases, the expected loss from default is higher which tightens household's incentive compatibility constraint for all levels of unemployment and labor market tightness. This leads to higher levels of DM trade, increasing labor market tightness, and a decrease in steady state unemployment.

5 Quantitative Analysis

We now turn to a quantitative evaluation of the theory. To start, the model is calibrated to match features of the U.S. economy between 2000 and 2008, characterized by high levels of unsecured credit and low unemployment. We then exogenously reduce the availability of credit in accordance with the observed data between 2008 and 2011 to find the extent to which it affects the unemployment rate. We also analyze a similar experiment and show to what extent the expansion of credit over the period 1980-2008 can explain the long-term decrease in unemployment.

5.1 Calibration

The model period is chosen to be a month. Accordingly, we set $r = .003$, implying $\beta = 0.997$. All empirical targets represent monthly averages over the time period 2000-2008.

Labor market. The calibration of the labor market follows Shimer (2005) and Berentsen, Menzio, Wright (2011) in the context of a monetary model. The matching function is Cobb-Douglas, $m(s, o) = As^{\eta}o^{1-\eta}$. We set the elasticity with respect to job seekers to $\eta = 0.5$, in the middle of the suggested range from empirical surveys (see, e.g., Petrolongo and Pissarides, 2001). The households' bargaining power corresponds to an egalitarian solution, $\lambda = 0.5$. It is also the value consistent with constrained-efficiency (Hosios, 1990) in an economy with non-binding borrowing constraints.

The matching efficiency parameter, A, the job destruction rate, δ, and the vacancy posting cost, k, are jointly determined by targeting the job finding probability, the unemployment rate, and labor market tightness. The unemployment rate comes from the Current Population Survey (CPS) and averaged 5.1% between 2000-2008, $u = 0.051$. We estimate the job finding probability according to $p_t = 1 - (U^L_{t+1}/U_t)$, where U^L is the number of workers unemployed over 5 weeks and U is the total number of unemployed workers from the CPS.[21] We find that $p = 0.36$. Labor market tightness is given by the Job Openings and Labor Turnover Survey (JOLTS).[22] From 2000-2007, there were 0.523 job openings for every unemployed worker, $\theta = 0.51$. Using the moments above, labor market matching efficiency is given by $A = p\theta^{\eta-1} = 0.50$. From (47) the job destruction rate is $\delta = pu/(1-u) = 0.019$. Finally, k is given by the vacancy supply condition, (57), which yields vacancy costs as a percentage of monthly wages to be 9.2%.

The remaining two parameters associated with the labor market are the value of leisure, ℓ, and unemployment income, b. We interpret b to represent unemployment benefits in the US and so we set the replacement

[21] See Shimer (2012) for the structural foundation of this equation.

[22] It is standard to normalize either the measure of vacancies or labor market tightness to one (see Shimer (2005) or Berensten, Menzio, and Wright (2011)). Since our time period allows the use of JOLTS, we instead use some measure of vacancies to job seekers. This choice does not affect the quantitative results.

ratio to one half, i.e., $b = 0.5w$. The value of leisure, ℓ, is an important parameter to pin down the response of equilibrium unemployment to productivity. Shimer (2005) sets $\ell = 0$ and finds that the unemployment rate is not responsive enough to changes in productivity. Alternatively, Hagedorn and Manovskii (2008) show that if this parameter is calibrated to match hiring costs, then $b + \ell$ should be set to 95% of wages, which results in the response of unemployment to productivity that is more in line with data. Since the effects of credit on unemployment in our model are channeled through changes in productivity, z, we follow Hagedorn and Manovksii (2008) and set $b + \ell/w = .95$, or $\ell = 0.45w$.[23]

Credit and Goods Market The DM matching function takes the form $\alpha(n) = \epsilon\sqrt{n}$, which corresponds to a Cobb-Douglas specification where households and firms have an equal contribution to the matching process. The household's bargaining weight, μ, is set to match the retail markup of 40% as discussed in Faig and Jerez (2005).

We focus on revolving unsecured credit, which primarily consists of credit and charge cards, as debt limits for such loans are observable in the data. To target the eligibility of households to unsecured credit, we choose ω to represent the fraction of US households that had a non-zero debt limit. The Survey of Consumer Finances (SCF) reports that the average percentage of households having at least one unsecured credit account with a non-zero limit. Over 2000-2007, this averaged 74.7%. Therefore, $\omega = 0.747$.

The efficiency parameter of the DM matching function, ϵ, is chosen to match the average credit utilization rate, defined as the fraction of the unsecured debt limit outstanding. In the model, average debt outstanding is given by $\omega\alpha(n)\bar{d}$ while the average debt limit is $\omega\bar{d}$, which implies a credit utilization rate is equal to $\alpha(n)$. Mian and Suffi (2010) report a median credit card utilization rate of 23.4% in the fourth quarter of 2006. Hence, we target $\alpha = 0.23$. This implies that households receive a liquidity shock on average every 4.3 months.

The monitoring probability, ρ, is chosen to target the average amount of monthly consumption financed through unsecured credit. In the model, aggregate consumption is given by $C = \alpha(n)(1 - \mu)\{\omega[\upsilon(y) - y] +$

[23] An important difference between our methodology and that of Hagedorn and Manovskii (2008) is that we let unemployment benefits b vary with wages in the experiments we consider. Doing so dampens the response of unemployment to productivity changes and is more in line with the movements in the two variables over the long run.

$(1-\omega)[\upsilon(\tilde{y})-\tilde{y}]\} + n\bar{z} - \theta uk$ and consumption out of unsecured credit is $\omega\alpha(n)\bar{d}$. We target $\omega\alpha(n)\bar{d}/C$ as the ratio of new monthly charges on credit and charge cards given by the SCF and total monthly household consumption expenditures as reported in the Consumer Expenditure Survey (CEX). Over 2000-2008, the average household had \$845 of new charges on credit cards and total consumption expenditures of \$3,681 during a month. Therefore, we set $\omega\alpha(n)\bar{d}/C = 0.23$, which results in $\rho = 0.30$.

We take M2 as our measure of liquid assets and the rate of return on liquid assets, R, is chosen to target the real user cost of M2 monetary services as published by the St. Louis Federal Reserve. The real user cost is defined as the discounted interest rate spread between a benchmark rate and the own rate of return on assets included in M2.[24] In the model the real user cost is given by $(1 + r - R)/(1 + r)$. Over 2000-2008, the real user cost averaged 0.055%. This implies $R = 1.0025$.

Utility over DM consumption is given by, $\upsilon(y) = \upsilon_0 y^{1-\gamma}/(1-\gamma)$. The level and elasticity parameters in the utility function, (υ_0, γ), are set such that the relationship between the demand for liquid assets, $R\tilde{a}$, and the opportunity cost of holding them, $(1 + r - R)/(1 + r)$, matches the data. In the model, aggregate holdings of liquid assets normalized by consumption are given by $(1-\omega)[(1-\mu)\upsilon(\tilde{y}) + \mu\tilde{y}]/\nu C$, where aggregate consumption C is defined above. The demand for liquid assets depends on the opportunity cost $(1 + r - R)/(1 + r)$ through $\tilde{y}$ given by (46). We set υ_0 to match average M2/C and γ to match the empirical elasticity of M2/C to the real user cost, which we estimate to be -0.17. Together these imply $(\upsilon_0, \gamma) = (1.42, 0.03)$. Figure 8 shows the estimated demand for liquid assets.[25]

The targets discussed above are sufficient to pin down all but one parameter, ν, which determines the acceptability of liquid assets as a means of payment in the DM. We first solve for a range of ν that is consistent with the coexistence of liquid assets and credit in equilibrium. We find ν must range between .001 and .1. In the baseline calibration, we set $\nu = .05$ and show a sensitivity analysis around this target.[26] Table 1 reports the calibrated parameters.

[24] The benchmark rate is given as the upper envelope of a set of rates including the Baa corporate bond yield and other short-term money market rates plus a small liquidity premium. See Anderson and Jones (2011) for a review of the Monetary Services Index (MSI).

[25] The estimated money demand curve fits the empirical relationship well until 1991, when there was an apparent shift down in the demand for liquid assets.

[26] At first, the suggested range for ν may seem to be too small; however, it is important to consider that this parameter captures various costs that are not directly taken into account in the model. For instance, Saches and Williamson (2010)

Description	Value	Source/Target
Labor Market		
Unemployment benefits, b	0.53	$b = .5w$
Value of leisure, ℓ	0.48	$(b+\ell)/w = .95$, Hagedorn and Manovskii (2008)
Job destruction rate, δ	0.019	Unemployment rate, CPS
Vacancy cost, k	0.10	Job finding probability Shimer (2012), CPS
Elasticity of LM matching function, η	0.50	Petrolongo & Pissarides (2001)
LM bargaining, λ	0.50	Hosios condition
LM matching efficiency, A	0.50	Vacancy rate, JOLTS
Credit & Goods Market		
DM production, $\bar{z}$	1	Normalization
Access to unsecured credit, ω	0.74	Fraction that hold at least 1 credit card, SCF
DM bargaining, μ	0.13	Retail Markup 40% , Faig & Jerez (2005)
Elasticity of DM matching function, ψ	0.50	Equal contribution to goods market matching
DM matching efficiency, ϵ	0.24	Credit card utilization rate, Mian and Sufi (2010)
Detection Rate, ρ	0.30	Debt to consumption, SCF & CEX
Return on Liquid Assets, R	1.0025	Real user cost of M2, St. Louis Federal Reserve
Utility level parameter, υ_0	1.42	M2 to consumption, FRB H.6 & NIPA
Risk aversion, γ	0.03	Elasticity of M2 to real user cost

Table 1: Parameter Values

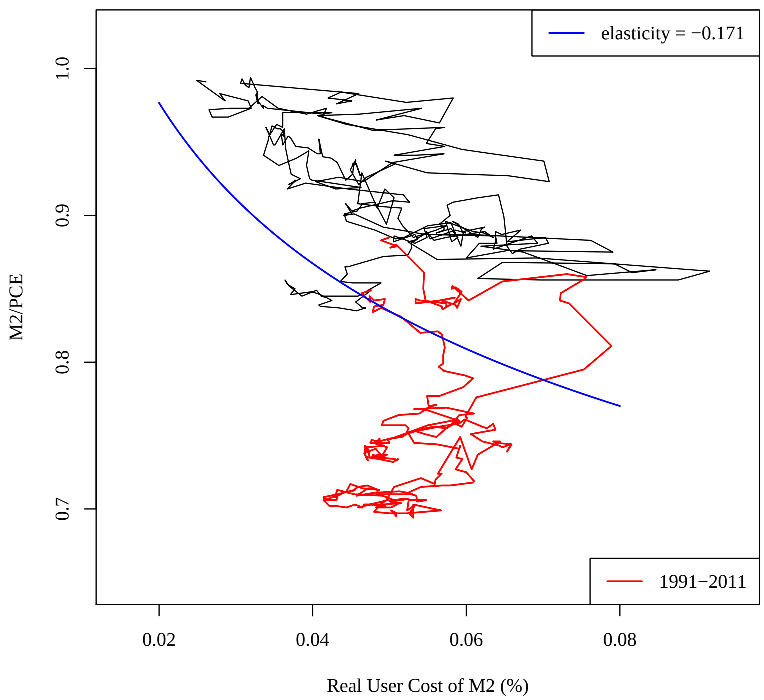

Figure 8: Demand for Liquid Assets

5.2 Quantitative Results

The US experienced a large expansion of the availability and use of household unsecured credit from 1980 to 2008. Credit as a percentage of consumption more than tripled. This coincided with a general decline in the unemployment rate over this time period. For instance, the unemployment rate averaged 7.2% in 1980 compared to 5.1% over 2000-2008 as used in the benchmark calibration. The Great Recession reversed these trends and led to a fall in households' access to unsecured credit in conjunction with an increase in the unemployment rate.[27]

In this section, we use the calibrated model to analyze the effects of an exogenous change in unsecured

emphasize the possibility of theft in anonymous transactions.

[27]See Figure 1.

credit. We consider two experiments: (i) a change in credit conditions to mimic those in 1980 and (ii) a change in credit conditions to mimic those in 2011, after the Great Recession. We will vary two parameters ω and ρ. In the model, ω controls the measure of households that have access to the unsecured credit market while ρ affects the endogenous debt limit for those households who maintain access to credit.

Compared to 2008, unsecured credit as a percentage of consumption was 70.6% lower in 1980. We engineer this reduction in credit by first decreasing ω to 65%, the fraction of households in the 1983 SCF that reported having at least one credit card. We then decrease ρ by 5.1% to generate the total fall in the ratio of unsecured credit to consumption of 70.6%. The results are presented in Table 2.

	Baseline	Decrease ω		Decrease ρ		Decrease both		% change
		Level	% change	Level	% change	Level	% change	2008-1980
Credit & Goods Market								
Credit to Con., $\alpha(n)\omega\bar{d}/C$	0.23	0.20	-13.3	0.08	-66.0	0.07	-70.6	-70.6
M2 to Cons., $(1-\omega)R\tilde{a}/C$	0.74	1.01	36.3	0.68	-8.10	0.93	25.5	18.3
Agg. productivity, z	1.07	1.06	-0.81	1.03	-4.28	1.02	-4.45	
Liquidity shocks, $\alpha(n)$	0.23	0.23	-0.11	0.23	-0.83	0.23	-0.89	
Labor Market								
Unemployment rate (%)	5.13	5.34	4.18	6.69	30.6	6.82	33.0	46.0
Job Finding Rate, p	0.36	0.34	-4.23	0.27	-24.7	0.26	-26.2	16.1

Table 2: Unemployment and Credit, 1980-2008

The difference in the availability of unsecured credit between 1980 and 2008 is largely attributable to changes in borrowing limits. As indicated in the first row of Table 2, the fall in the fraction of households with any access to credit, ω, generates only 13.3% of the total 70.6% difference in the credit to consumption ratio. The remainder is generated by decreases in borrowing limits.

Holdings of liquid assets increase as a result of the restriction of credit. The model slightly over accounts for the total difference in M2 holdings as a percentage of consumption between 1980 and 2008, as indicated in the last two columns of row 2, but in general captures the trade-off between the utilization of credit and money in the aggregate. Measured productivity decreases as the absence of credit leads to lost consumption opportunities. The model predicts that productivity decreases by 4.45%.

The difference in credit conditions has significant implications for unemployment in the long run. The model predicts that credit differences can account for 71% of the total difference in the average unemployment rate between 1980 and the benchmark 2000-2008.

Table 3 reports the results of a similar experiment in which we generate an exogenous contraction of credit to mimic the US before and after the Great Recession. Unsecured credit as a percentage of consumption fell by 22.9% between 2008 and 2011. As before, we generate this decline by first decreasing ω to 67.9% to mimic the fraction of households in the 2010 SCF that reporting having at least one credit card. We then decrease ρ by 0.8% to generate the 22.9% fall in credit to consumption.

	Baseline	Decrease ω		Decrease ρ		Decrease both		% change
		Level	% change	Level	% change	Level	% change	2008-2011
Credit & Goods Market								
Credit to Con., $\alpha(n)\omega\bar{d}/C$	0.23	0.21	-9.58	0.20	-15.1	0.18	-22.9	-22.9
M2 to Cons., $(1-\omega)R\tilde{a}/C$	0.74	0.94	26.1	0.73	-1.28	0.92	24.4	11.0
Agg. productivity, z	1.07	1.07	-0.58	1.06	-0.95	1.06	-1.44	
Liquidity shocks, $\alpha(n)$	0.23	0.23	-0.08	0.23	-0.13	0.23	-0.21	
Labor Market								
Unemployment rate (%)	5.13	5.28	2.95	5.38	4.95	5.53	7.80	74.1
Job Finding Rate, p	0.36	0.35	-3.31	0.34	-4.97	0.33	-7.63	-46.6

Table 3: Unemployment and Credit, 2008-2011

The model predicts the fall in unsecured credit between 2008 and 2011 led to a decline in aggregate productivity of 1.4%. This generates an increase in equilibrium unemployment from 5.13% to 5.53%, or 7.8% of the total 74% increase in unemployment over this time period. In the steady-state, the reduction in household's ability to finance purchases with unsecured credit explains about 10% of the total increase in unemployment over this time period.

The effects are robust to alternative calibration strategies presented in tables 4 and 5 in the Appendix. Varying ν within the range $(0.035, 0.065)$ affects the substitution between liquid assets and credit, but does not alter the magnitude of the effect on productivity and unemployment. Considering a smaller target for the retail markup of 30% also delivers similar results, though the effect on unemployment and productivity are slightly dampened.

Figures 9 and 10 present the comparative static effects of changes in the two credit parameters ω and ρ. The vertical, dotted-lines represent the baseline parameters. As ω decreases to zero, there is an aggregate reallocation of the means of payment towards liquid assets. From Proposition 2, $y > \tilde{y}$ and so consumption falls as a result. In the extreme, as credit markets freeze, productivity falls by 5.9% and unemployment increases from 5.1% to 7.8%. The effects of changes in the detection rate, ρ are similar though highly nonlinear. Similarly, as ρ falls far enough and credit is no longer incentive feasible, productivity falls by 6.6% and unemployment increases to 8.5%.

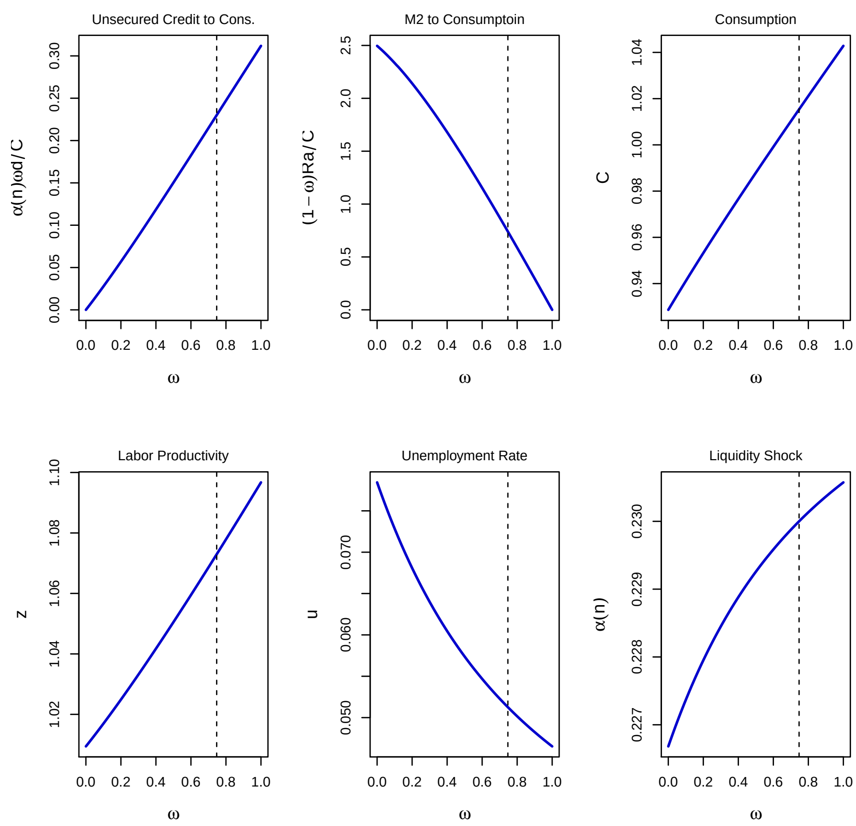

Figure 9: Steady-state effect of a changes in ω

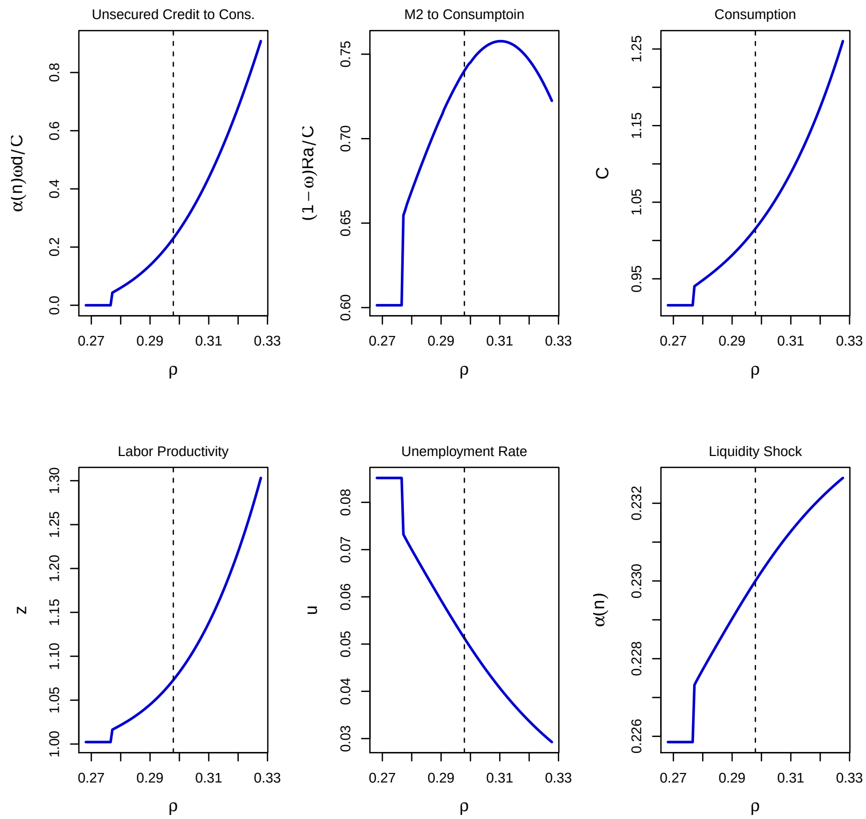

Figure 10: Steady-state effect of a changes in detection rate, ρ

The Response of Unemployment to Productivity Shocks: The Role of Unsecured Credit In this section, we analyze the role of the unsecured credit market in amplifying exogenous shocks in the model. Specifically, we return to a question posed by Petrosky-Nadeau (2009), and others, which asks to what extent do credit frictions amplify the response of unemployment to productivity shocks. We analyze this question in the context of the steady-state. Consider a decrease in the exogenous component of productivity, $\bar{z}$. In partial equilibrium, holding unsecured credit fixed, there is an increase in equilibrium unemployment. The value of a filled job decreases, firms post less vacancies causing unemployment to rise. This is just the standard mechanics of Mortensen-Pissarides. In general equilibrium, however, an increase in unemployment also lowers households access to unsecured credit. This causes a further drop in measured productivity, z

and a larger increase in unemployment. Figures 11 and 12 highlight this channel.

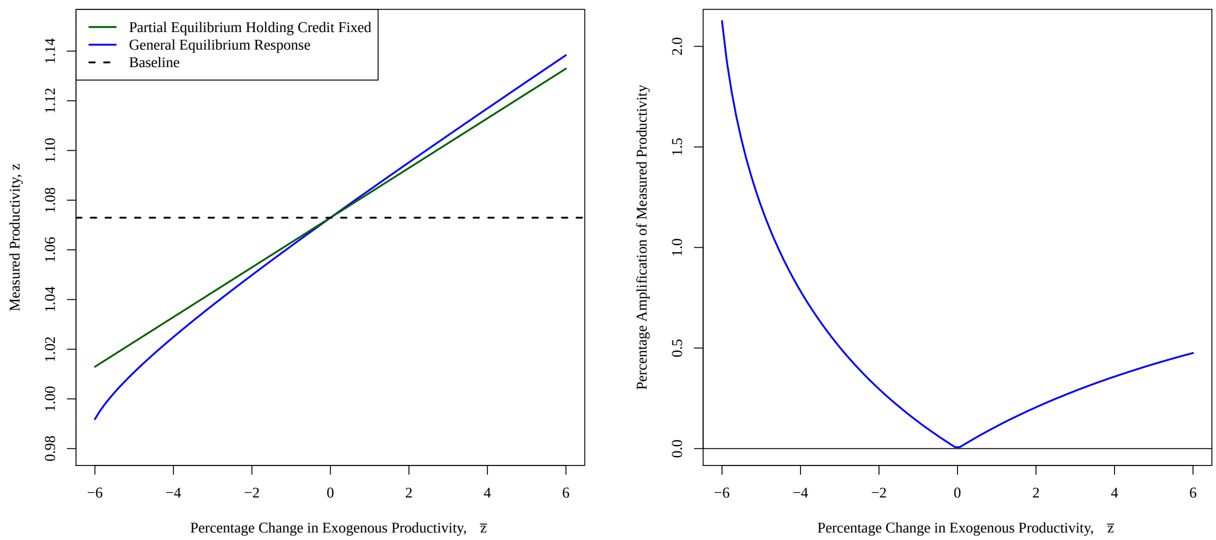

Figure 11: The credit amplification channel on measured productivity

The first graph of each figure shows the response in equilibrium steady-state measured productivity and unemployment, respectively, for an exogenous change in $\bar{z}$. The green line represents the partial equilibrium effect holding credit constant while the blue line represents the full general equilibrium movement. The difference between the two lines is caused by the endogenous response of credit. The graph on the right of each figure highlights the magnitude of this amplification. The y-axis represents the absolute distance between the green and blue lines in percentage terms.

First, the existence of unsecured credit can have a significant amplification effect on measured productivity and unemployment. For instance, consider a negative 4% shock to exogenous productivity, $\bar{z}$. Holding unsecured credit constant, measured productivity, z, falls by 3.7% and unemployment increases from 5.1% to 6.4%. As debt limits adjust, productivity decreases an additional 0.8% and unemployment increases further to 6.9%. Therefore the additional amplification due to unsecured credit is 7.8%. Secondly, the effects are highly asymmetric. Negative productivity shocks lead to a significantly larger amplification caused

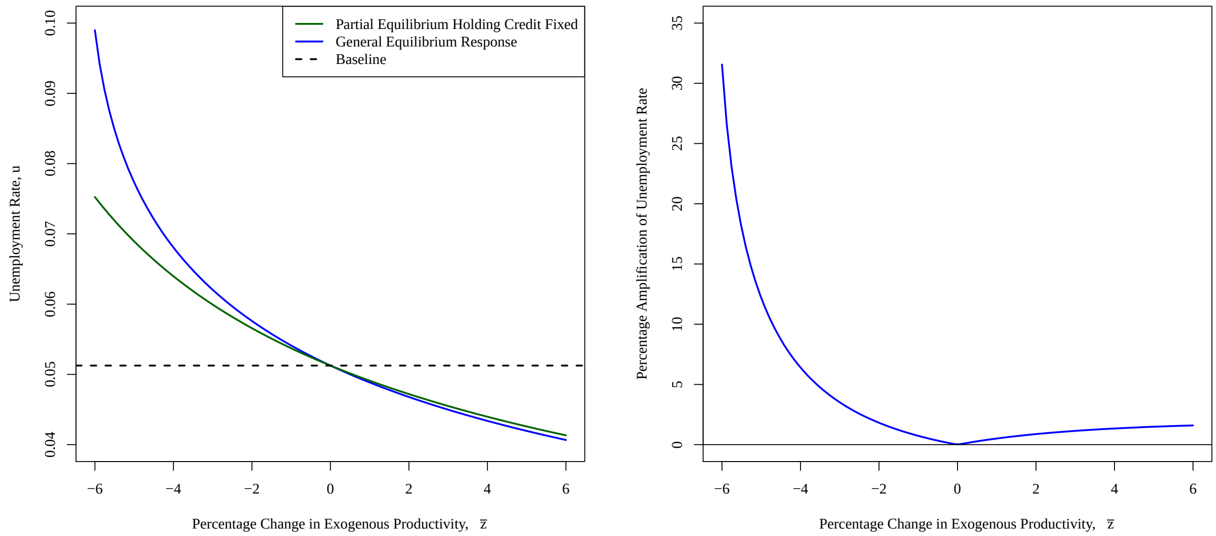

Figure 12: The role of credit in the amplification of unemployment to productivity shocks

by unsecured credit. For instance, when exogenous productivity decreases by 6 percent, unsecured credit amplifies the effect on unemployment by around 30%. A positive 6% shock to exogenous productivity is only amplified by 1.6%.

6 Conclusion

This paper studies the relationship between aggregate unemployment and household unsecured credit. We develop a theory that endogenizes the role of unsecured credit in firm productivity. We show the possibility of multiple steady-state equilibria in which there exists a negative relationship between unemployment and unsecured debt. The key mechanism delivering the result is the complementarity between the endogenous borrowing limit and firms' entry decisions. Additionally, the model allows for the coexistence of credit and liquid assets.

The model is also amendable to quantitative analysis. As an example, we illustrate the labor market effects of the contraction in credit as seen in the US between 2007 and 2010. Although the baseline calibration explains a small fraction of the concurrent increase in the aggregate unemployment rate, we highlight the potential of large effects on unemployment of a larger credit crunch. An important channel that leads to the negative response in unemployment, is the size of the labor surplus for unemployed workers.

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Appendix

Proof of Proposition 1 First we derive Equation (41) that determines the debt limit. Let $\Lambda_e = [-a + \beta U_e(a)] - \left[-\tilde{a} + \beta \tilde{U}_e(\tilde{a})\right]$, where a and $\tilde{a}$ are the optimal choices of liquid assets of households with and without access to credit, respectively. From (17) and (19) it can be checked that

$$
\begin{aligned}
U_1(a) &= \alpha(n)\mu\left[\upsilon(y) - y\right] + (R-1)a + \Delta - T + (1-\delta)\left[w + \beta U_1(a)\right] + \delta\left[\ell + b + \beta U_0(a)\right] \\
\tilde{U}_1(\tilde{a}) &= \alpha(n)\mu\left[\upsilon(\tilde{y}) - \tilde{y}\right] + (R-1)\tilde{a} + \Delta - T + (1-\delta)\left[w + \beta \tilde{U}_1(\tilde{a})\right] + \delta\left[\ell + b + \beta \tilde{U}_0(\tilde{a})\right] \\
U_0(a) &= \alpha(n)\mu\left[\upsilon(y) - y\right] + (R-1)a + \Delta - T + p\left[w + \beta U_1(a)\right] + (1-p)\left[\ell + b + \beta U_0(a)\right] \\
\tilde{U}_0(\tilde{a}) &= \alpha(n)\mu\left[\upsilon(\tilde{y}) - \tilde{y}\right] + (R-1)\tilde{a} + \Delta - T + p\left[w + \beta \tilde{U}_1(\tilde{a})\right] + (1-p)\left[\ell + b + \beta \tilde{U}_0(\tilde{a})\right].
\end{aligned}
$$

After some calculation,

$$
\begin{aligned}
\Lambda_1 &= -(1-\beta R)(a-\tilde{a}) + \beta\alpha(n)\mu\left\{\left[\upsilon(y) - y\right] - \left[\upsilon(\tilde{y}) - \tilde{y}\right]\right\} + (1-\delta)\beta\Lambda_1 + \delta\beta\Lambda_0 \\
\Lambda_0 &= -(1-\beta R)(a-\tilde{a}) + \beta\alpha(n)\mu\left\{\left[\upsilon(y) - y\right] - \left[\upsilon(\tilde{y}) - \tilde{y}\right]\right\} + p\beta\Lambda_1 + (1-p)\beta\Lambda_0.
\end{aligned}
$$

It follows that $\Lambda_1 = \Lambda_0 = \Lambda$ where

$$
\Lambda = \frac{-(1+r-R)(a-\tilde{a}) + \alpha(n)\mu\left\{\left[\upsilon(y) - y\right] - \left[\upsilon(\tilde{y}) - \tilde{y}\right]\right\}}{r}. \tag{62}
$$

Thus, the cost of losing access to credit is the same for employed and unemployed households. From (38) and (62), $\bar{d}$ is a fixed point to (41). Next, we prove the claims in Proposition 1 by distinguishing the case where households with no access to credit hold liquid assets (case 1) from the case where they don't (case 2).

Case 1: $\tilde{a} > 0$. From (34) this requires $(1+r-R)/R < \alpha(n)\nu\mu/(1-\mu)$, i.e., $R > \underline{R}$.

The left side of (41), $r\bar{d}$, is linear. Let us turn to the right side of (41), $\Gamma(\bar{d})$. If $\bar{d} \leq R\nu\tilde{a}$, then the debt limit is less than the payment capacity of households with no access to credit. It follows from (34) that households with access to credit will choose the same payment capacity as the one of households with no access to credit, i.e., $\bar{d} + R\nu\hat{a} = R\nu\tilde{a}$ and $y = \tilde{y}$, since they face the same marginal condition for the choice of liquid assets. Consequently, the right side of (41) is linear, $\Gamma(\bar{d}) \equiv \rho(1+r-R)\,\bar{d}/R\nu$. Since $\Gamma(0) = 0$, $\bar{d} = 0$ is a solution to (41).

If $\bar{d} > R\nu\tilde{a}$, then the debt limit is greater than the payment capacity of households with no access to credit. Consequently, from (34), households with access to credit choose not to accumulate liquid assets, $a = 0$, and the derivative of Γ is

$$\Gamma'(\bar{d}) \equiv \rho\alpha(n)\mu \left[\frac{\upsilon'(y) - 1}{(1-\mu)\upsilon'(y) + \mu} \right] \geq 0.$$

In that case $\Gamma(\bar{d})$ is a concave function of $\bar{d}$. For all $\bar{d} \geq (1-\mu)\upsilon(y^*) + \mu y^*$, $y = y^*$ and $\Gamma'(\bar{d}) = 0$. A necessary and sufficient condition for a unique $\bar{d} > 0$ solution to (41) to exist is that the slope of the left side of (41) is less than the slope of the right side of (41) evaluated at $\bar{d} = 0$, i.e., $r < \Gamma'(0)$. This condition can be rewritten as $r < \rho\left(\frac{1+r-R}{R\nu}\right)$ or $R < \bar{R}$.

If $R = \bar{R}$, then $r = \Gamma'(0)$ and $\Gamma(\bar{d}) = r\bar{d}$ for all $\bar{d} \leq R\nu\tilde{a}$. Moreover, for all $\bar{d} > R\nu\tilde{a}$, $\Gamma(\bar{d})$ is located underneath $r\bar{d}$ so that there is no solution to $\Gamma(\bar{d}) = r\bar{d}$. This proves the claim that if $R = \bar{R}$, then any $\bar{d} \in [0, R\nu\tilde{a}]$ is a solution to (41).

Case 2: $\tilde{a} = 0$. This happens if $\frac{1+r-R}{R} \geq \frac{\alpha(n)\nu\mu}{1-\mu}$, i.e., $R \leq \underline{R}$. In this case (41) can be reexpressed as

$$r\bar{d} = \rho\alpha(n)\mu\left[\upsilon(y) - y\right].$$

It follows that $\bar{d} > 0$ iff $r < \frac{\rho\alpha(n)\mu}{1-\mu}$. Finally, notice that $\underline{R} < \bar{R}$ iff $r < \frac{\rho\alpha(n)\mu}{1-\mu}$.

Putting together cases 1 and 2, $\bar{d} > 0$ is unique iff $\underline{R} < \bar{R}$ and $R < \bar{R}$, as claimed in Proposition 1.

Proof of Proposition 2 We establish first that n, y and $\tilde{y}$ are monotone, continuous functions of θ. From (47), $n = 1 - u = m(1,\theta)/[m(1,\theta) + \delta]$ is a continuous and increasing function of θ. Therefore, from (45), $\tilde{y}$ is a continuous and non-decreasing function of θ. By virtue of the assumption, $r < \rho\alpha(n_0)\mu/(1-\mu)$, for all $\theta \geq \theta_0$ the condition, $r < \rho\alpha(n)\mu/(1-\mu)$, holds. Therefore, from Proposition 1, for all $\theta \geq \theta_0$ and $R < \bar{R} \equiv \rho(1+r)/(r\nu + \rho)$, there exists a unique $\bar{d} > 0$ solution to (41). Moreover, $\bar{d}$ is continuous and increasing (Corollary 2) with θ. From (44) for all $\theta \geq \theta_0$, y is continuous and increasing with θ. Let us turn to the mapping defining equilibrium market tightness:

$$\begin{aligned}\Psi(\theta) \quad \equiv \quad & \frac{(r+\delta)\,k}{m\left(\frac{1}{\theta}, 1\right)} + \beta\lambda\theta k \\ & -(1-\lambda)\left\{ \frac{\alpha\left[n(\theta)\right]}{n(\theta)}(1-\mu)\left\{\omega\left[\upsilon\left(y(\theta)\right) - y(\theta)\right] + (1-\omega)\left[\upsilon\left(\tilde{y}(\theta)\right) - \tilde{y}(\theta)\right]\right\} + \bar{z} - \ell - b \right\}.\end{aligned}$$

From (48) an equilibrium value for θ solves $\Psi(\theta) = 0$. From the assumption that $(1-\lambda)(\bar{z} - \ell - b) > (r + \delta)k$, and using the definition of θ_0, it can be checked that

$$\Psi(\theta_0) = -(1-\lambda)\left\{\frac{\alpha\left[n(\theta_0)\right]}{n(\theta_0)}(1-\mu)\left\{\omega\left[\upsilon\left(y(\theta_0)\right) - y(\theta_0)\right] + (1-\omega)\left[\upsilon\left(\tilde{y}(\theta_0)\right) - \tilde{y}(\theta_0)\right]\right\}\right\} < 0.$$

Since $\Psi(\infty) = +\infty$, there exists a $\theta > \theta_0$ solution to $\Psi(\theta) = 0$. Moreover, since $r < \rho\alpha(n_0)\mu/(1-\mu) \leq \rho\alpha(n)\mu/(1-\mu)$, the corresponding debt limit is positive, $\bar{d} > 0$.

	$\nu = .035$	$\nu = .065$	Markup = 20%	
	Decrease both	Decrease both	Decrease both	% change
	% change	% change	% change	2008-1980
Credit & Goods Market				
Credit to Con., $\alpha(n)\omega\bar{d}/C$	-70.6	-70.6	-70.6	-70.6
M2 to Cons., $(1-\omega)R\tilde{a}/C$	7.01	33.3	31.0	18.3
Agg. productivity, z	-4.63	-4.34	-3.58	
Liquidity shocks, $\alpha(n)$	-0.94	-0.85	-0.64	
Labor Market				
Unemployment rate (%)	34.6	31.3	23.7	40.0
Job Finding Rate, p	-27.1	-25.1	-20.2	16.1

Table 4: Unemployment and Credit, 1980-2008, Sensitivity Analysist

	$\nu = .035$	$\nu = .065$	Markup = 20%	
	Decrease both	Decrease both	Decrease both	% change
	% change	% change	% change	2008-2011
Credit & Goods Market				
Credit to Con., $\alpha(n)\omega\bar{d}/C$	-22.9	-22.9	-22.9	-22.9
M2 to Cons., $(1-\omega)R\tilde{a}/C$	19.9	26.2	25.4	11.0
Agg. productivity, z	-1.51	-1.36	-1.14	
Liquidity shocks, $\alpha(n)$	-0.21	-0.20	-0.16	
Labor Market				
Unemployment rate (%)	8.21	7.33	6.02	74.1
Job Finding Rate, p	-8.00	-7.19	-5.99	-46.6

Table 5: Unemployment and Credit, 2008-2011, Sensitivity Analysis